Quark Gluon plasma: The perfect liquid with heavy quark impurities

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Synopsis

• Why we think the Quark-Gluon Plasma is a liquid
  RHIC white papers,nucl-th/0405013

• Why heavy quarks seem to confirm it

• Why heavy quarks can be use to link QGP hydrodynamics to QCD
  Based on GT, J. Noronha, 1004.0237, in press, PLB
What RHIC was built for:

- Deconfined quark–gluon plasma
- Color superconductivity models

It did not find a conclusive proof for it!

But it became famous...

WHY?!
What kind of "medium" is created in nuclear collisions?

A "dust"
Particles ignore each other, their path is independent of initial shape

A "fluid"
Particles continuously interact. Expansion determined by density gradient (shape)

Quantitatively distinguished by mean free path, viscosity
Hydrodynamics predicts flow eccentricity as a function of number of particles ($\sim$ area of overlap region). Parametrized by 2nd Fourier component, $v_2$

$$E \frac{dN}{d^3p} = \sum_n E \frac{dN}{dp_z p_T dp_T} (1 + 2v_n \cos(n\phi))$$
Data described by \textit{ideal hydrodynamics} (mean free path between particle collisions is \textit{zero}!)

Calculations using ideal hydrodynamics


P. Romatschke, PRL99:172301, 2007
What should viscosity be?

Microscopic picture: "collective" effects implemented from perturbative dynamics via Boltzmann equation (neglecting quantum correction):

\[
\left( \frac{1}{m} p^\mu \frac{\partial}{\partial x^\mu} + F^\mu \frac{\partial}{\partial p^\mu} \right) f(x, p) = C^{2\text{body}}[f] + C^{3\text{body}}[f] + \ldots
\]

\[C^{2\text{body}} = \int d^3[X, X', P, P'] \sigma(P, P' \leftrightarrow p, p') [f(X, P) f(X', P') - f(x, p) f(X', P')]\]

**Ideal hydro:** \(C = 0\) (Gain=Loss) \(f = \Upsilon e^{-\frac{p^\mu u^\mu}{T}}\) always, \((T, u_\mu\text{ change})\)

**Non-ideal:** Expand \(C[f]\) around \(f - f_{eq}, \equiv \text{Knudsen n.K} = l_{mf} \partial_\mu u_\nu\)
So the small parameter for hydro is the Knudsen Number $K = l_{mfp} \partial_\mu u_\nu$

Ideal hydro $O(K^0)$, Navier-Stokes $O(K^1)$, Israel-Stewart $O(K^2)$. Note $K$ "really" a "tensor". (Grad expansion):

$$f = f_{eq} \left[ \frac{u^\mu p_\mu}{T} \right] \left[ 1 + \underbrace{\epsilon}_{O(K^1)[\zeta]+\text{higher}} + \underbrace{\epsilon_\mu}_{O(K^1)[\eta]+\text{higher}} p^\mu + \underbrace{\epsilon_{\mu\nu}}_{O(K^2)+\text{higher}} p^\mu p^\nu + ... \right]$$

Plug into Boltzmann equation use $H$-theorem and obtain $\epsilon_\mu$ in terms of $\eta \partial u$ etc.. For first order, we can show that

$$\eta = \frac{4}{5} \langle p \rangle n = \frac{1}{5} \langle p \rangle s l_{mfp}, \quad \zeta = \left( c_s^2 - \frac{1}{3} \right)^2 \eta$$

Last relation relies on 1 reaction, broken if elastic and inelastic collisions equivalent to Kubo formulae in perturbative case!
So \( \eta \sim e l_{mfp} \sim s T l_{mfp} \)

Note: This means that \( \eta/s \) is a "pure" number in natural units (no scale)! It reflects the "readiness of thermalization" of the system, the speed at which the degrees of freedom \( s \) rethermalize when disturbed (by a flow gradient). (NB: Superfluid has low \( \eta \) but also low \( s \).)

It might be counter-intuitive that a low \( l_{mfp} \) (ie, a lot of reinteractions) mean low \( \eta \). But viscosity is a "diffusion" of momentum due to the finiteness of \( l_{mfp} \). When \( l_{mfp} \) small, MANY collisions prevent diffusion
**η** and perturbation theory

Perturbation theory means, generally, **weak** coupling constant. I.e., a large mean free path and a large viscosity

\[
\frac{\eta}{s} \sim e l_{mfp} \sim \frac{T}{\sigma_{crosssection}} \sim \frac{1}{\alpha^2 \ln \alpha}_{\text{perturbation theory}} \sim \text{any sensible } \alpha
\]

\[\eta/s < 1\] would require a \(\alpha\) too large for calculation to work!

Attempts to lower this by many-body effects (**3 ↔ 2 collisions, Plasma instabilities**). But low experimental viscosity (see later!) encourages us to look beyond perturbation theory

In this regime **cannot** use Boltzmann equation (No scattering approximation for each collision, no matrix elements but...
How low can the viscosity be? Lets forget we cant use the Boltzmann equation at strong coupling! A rough estimate: (Danielewicz and Gyulassy, 1987)

\[ l_{mfp} \geq \langle \lambda_{de Broglie} \rangle \sim 1/\langle p \rangle \]

If one plugs this into the Boltzmann equations and calculates viscosity the usual way, a lower limit is obtained

\[ \eta/s \geq 1/12 \]

but this procedure is less than rigorous: Remember, we cant use Boltzmann!
A way to make this (a bit!) more rigorous: Hydrodynamics (and viscosity) from AdS/CFT

The AdS-CFT correspondence: Every $\left\langle \hat{O}_{CFT} \right\rangle$ a 4D $N_{susy} = 4$ Gauge theory with $N_c$ colors and T’hoof coupling $\lambda$, can be calculated by translating to a 10D string theory, with 5 Anti-DeSitter ($\Lambda < 0$) dimensions, 5 dimensions compactified on a sphere, and a string coupling constant of $g_s = \lambda / (4\pi N_c)$

- **dictionary** between $\hat{O}_{CFT}$ and $\hat{O}_{ADS}$ can be worked out

- **Links strongly coupled** CFT to weakly coupled perturbative string theory. 
  Infinitely strongly coupled CFT $\Leftrightarrow$ classical supergravity.
Entropy density Can be extracted from the entropy of the Black hole:
\[ s = \frac{3}{4}s_{SB} \]

\[ \eta \] Can be gotten with the Kubo formula, via the linearized theory of perturbations of a Black hole in AdS-space \( \eta \sim \lim_{w \to 0} e^{iwx} \langle h_{\mu\nu}(0) h_{\mu\nu}(x) \rangle \). Plugging in the numbers we get the famous “limit”
\[ \frac{\eta}{s} = \frac{1}{4\pi} \]

(Compare with Kinetic theory limit of \( 1/15\pi \)).

NB: It seems the bound is violated for more complicated dual theories.
not clear if \( \eta/s \) can go to 0.
Hydrodynamics gives us an edge to probe QCD thermal physics. But...

P. Huovinen et al
ARNPS.56:163,2006
nucl–th/0605008

Different EoSs
(And we know EOS from lattice)

No one knows from first principles \( \eta/s \) in QCD (rather than \( \mathcal{N} = 4 \) SYM), and \( v_2 \) insensitive to Equation of state (which we do know). Need additional “lever” on EoS to link calculable QCD and data! Such as heavy quarks
Hydrodynamics and heavy quarks

A "heavy" particle in a medium can be thought of as undergoing Brownian motion (hit infinitely many times by light medium particles, each collision exchanging a small amount of momentum). Langevin equation approach gives, for diffusion coefficient $D$

$$D \sim \mathcal{O}(1) \frac{\eta}{e + p}$$

In other words,

$$\tau_{\text{light}} \sim \frac{\eta}{p + e}, \quad \tau_{\text{heavy}} \sim \frac{M}{T} \tau_{\text{light}}$$

Thus **Heavy quark thermalization ⇔ Stricter limits on $\eta$**

**Problem:** Experimentally, not “heavy quarks” but non-photonic electrons
Heavy quarks (nonphotonic electrons)  
PHENIX,PRL 98 172301 (2007) 

Light quarks  
STAR,NPA 757, 102 (2005) 

$v_2$ of heavy and light quarks same within error bar!!!!
• Heavy quarks thermalized with the rest of the system (as light quarks)

• When Diffusion coefficient calculated, its compatible with a $\eta/s \leq 0.1$

Heavy quark measurements uphold ”perfect fluid” idea. HUGE uncertainty from inability to distinguish $c/b$, ignorance of resonance admixture, but...
Leads to interesting consequences (GT+Noronha, 1004.0237, in press, PLB)

what you need to know...

Widsom from cooking: The thermal and transport properties of water change when you salt it! (Boiling point goes up, heat capacity goes down!)
Charm and beauty abundance, what you should remember

• Its mass is $\gg T$ for most (all?) of the evolution, even at the LHC

• It should be far $\sim 10^2$ above chemical equilibrium

• Its still very dilute ($f_c = N_c/N_{ch} \ll 1$)
  (NB: Throughout this talk, by ”charm” I mean ”charm+beauty”, as all the calculations apply to both. However this is a correction, as $N_b \ll N_c$)

• It fluctuates event by event in a Poissonian manner

• It should to a good approximation be conserved, as annihilation probability $\sim f_c^2$ (need quark and antiquark close)
R. Vogt, 0709.253

pQCD estimate:

\[ N_{\text{charm}} \sim 10^{1-2} \text{ in Pb-Pb} \]

HUGE uncertainties, NOT including rescattering, saturation

"higher end" favoured by data

Using \( \frac{dN}{dy} \sim N_{\text{part}} \ln \sqrt{s} \) and \( N_c \sim N_{\text{coll}} \frac{\Delta y}{4\pi\Lambda_{QCD}^2} \sigma_{X \rightarrow c\bar{c}} \) we get

\[ \frac{N_c}{N_{\text{ch}}} \sim 10^{-(3-2)} \text{ on average!} \]
Of course $N_c/N_{ch}$ fluctuates

- Charm fluctuates with a binomial distribution $\leftrightarrow$ Poissonian
- $dN/dy$ fluctuations above poissonian (KNO? Glauber?)
- Charm produced by collisions, $dN/dy$ by wounded nucleons

\[
\sigma_{N_c/N_{ch}} = \frac{\langle (\Delta N_c)^2 \rangle}{\langle N_c \rangle^2} + \frac{\langle (\Delta N_{ch})^2 \rangle}{\langle N_{ch} \rangle^2} - 2 \frac{\langle N_{ch} \Delta N_c \rangle}{\langle N_{ch} \rangle \langle N_c \rangle} \frac{\Delta N_{part} \Delta N_{coll}}{\langle N_{ch} \rangle \langle N_c \rangle} \sim \text{Poisson} - \text{ve binomial}
\]

complicated, but both observable e-by-e!
So, with enough luminosity, can bin events according to $N_c/N_{ch}$.
In an ideal homogeneous fluid $\tilde{\rho}$ locally conserved to a good approximation.
And we can do this *quantitatively* to a good approximation from lattice data!

If we add an “infinitely heavy quark” to the system, the renormalized free energy (excluding the quark mass) is given by the Polyakov loop expectation value

$$F_{\text{total}} = F_{\text{plasma}} - TN_c \ln \ell + O\left(\frac{T \ln \ell}{m_c}\right)$$

The last correction is big at the LHC, but less then unity, and small until $T \approx T_c$. Also no difference between quark and anti-quark non-perturbatively. Hence, in the dilute limit free energy density depends on $\tilde{\rho}$

$$f = f_{\text{plasma}} - T\tilde{\rho}s \ln \ell$$

Note $\ln \ell \to -\infty$ at confinement
And now we are set!

\[ P = - f, \quad \epsilon = - T^2 \frac{d F}{dT^2}, \quad c_s^2 = \frac{dP}{d\epsilon} \]

And the Polyakov loop expectation value is known from the lattice!

Bazavov et al 0903.4379
A consistent decrease of the speed of sound, directly proportional to the “flavoriness” of the medium. Effect is greatest close to $T_c$ (due to rapid variation in $\ell$) but always present. Physically intuitively clear: Admixture of heavy particles slows systems response to pressure.
What happens when $T < T_c$?

**Polyakov loop method can no longer be used** as $\ell \to 0$, so

$$-T \ln \ell \to \infty \gg m_{\text{quark}}$$

In confined phase, strings can break, so Polyakov loop not anymore a measure of the free energy

**The Hadron resonance gas model**, if it also applies to heavy quarks, gives us a way to estimate the effect of salting by a mixture of dilute heavy mesons in a sea of pions. Using the textbook formulae for mixtures of ideal gases,

$$c_s^2 \sim c_{s\pi}^2 + \tilde{\rho} c_{sD}^2 \sim \frac{1}{3} + \tilde{\rho} \frac{5T}{3m}$$

Parametrically, this contains *two* small parameters, $\tilde{\rho}, T/m$, and hence its smaller than the Polyakov estimate which only has $\tilde{\rho}$. 
So, ”salting” response to the equation of state only arises in the deconfined regime, since dominating effect Polyakov-loop driven. Same argument shows salting response small $\mathcal{O}(\tilde{\rho}/T \rho M)$ @ weak coupling! need sQGP.
This qualitative behaviour is unique to asymptotically free confining theories.

An example: $\mathcal{N} = 4$ SYM (Not confining nor asymptotically free). There, $\ln \ell \sim \sqrt{\lambda}$ so

$$F = \frac{3}{4} \left( F_{SB} - \tilde{\rho} T \sqrt{\lambda} s_{sb} \right)$$

Do the calculation, $c_s$ independent of $\tilde{\rho}$! (Everything cancels out. Makes sense: Quark infinitely heavy, and conformal invariance exact!).
Relating $c_s$ to physical observables... By Taylor-expanding around $c_s$ ...

$$\langle p_T \rangle \sim T_f + m_\pi f(N_{part}, \sqrt{s}) \langle c_s \rangle_\tau^\alpha , \quad (\alpha \simeq 2)$$

$c_s$ comes as limiting speed of the rarefaction wave in a shock

**Exact** in a shallow shock (sound wave)

**Approximate** in a deep shock, as rarefaction wave, carrying bulk of $p_T$, moves with $c_s$ in rest frame
So we expect to have a correlation between $\hat{\rho}$ and $\langle p_T \rangle$ whose slope is rigorously calculable from lattice and hydro.
This is not conservation of energy (and is not contaminated by it)

\[ \frac{dN}{dy} \] at mid-rapidity is controlled by soft \( (x \ll 1) \) partons

**charm** at mid-rapidity is controlled by harder \( (x \sqrt{s} > \Lambda_{QCD}) \) partons

So the energy to create charm at mid-rapidity is created at significant \( (> 1) \) forward or backward rapidity. If soft physics local in rapidity (Bjorken picture) conservation of energy will not lower \( \langle p_T \rangle \) at mid-rapidity!
Effect anti-correlated with $N_{part}$ fluctuations

A hotter system than average has more flow and more charm. Correlation not perfect, as $dN/dy \sim N_{part}, N_{cc} \sim N_{coll} \neq N_{part}$
Fluctuations larger in smaller systems, $\sigma \tilde{\rho} \sim \left(\frac{dN}{dy}\right)^{-1}$

Star HBT results indicate p–p flows ~ Au–Au

And so does blast wave fit if energy exactly conserved

At RHIC, $p - p$ events with $dN/dy \sim 10$, $2c\bar{c}$ pairs have $\tilde{\rho} \sim 10\%$
People speculate weather p-p also flows at RHIC. This might be way to tell!

M.Lisa and Z.Chajecki
QM2005
QM2009
Data from STAR collab.
Conclusions

• RHIC seems to have produced a "perfect" fluid

• Viscosity much below naive perturbative expectations

• Likely window into non-perturbative physics

• Heavy quarks...
  - Validate the perfect liquid evidence via Langevin-analysis
  - Response of equation of state to "chemical" impurities Reliable link between data and non-perturbative QCD!

RHIC made the perfect soup. The LHC could make a soup which is not only perfect, but also flavorful!