Spin dependent of structure functions and target mass corrections

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With recent advances in the precision of inclusive lepton-nuclear scattering experiments, it has become apparent that comparable improvements are needed in the accuracy of the theoretical analysis tools. In particular, when extracting parton distribution functions in the large-x region, it is crucial to correct the data for effects associated with the non-zero mass of the target. We present here an updated, more accurate, version of our calculations based on the Jacobi polynomial expansion method on performing a global fit to the existing data by imposing target mass correction effects. The effect of these corrections are studied and polarized structure functions with and without them are compared.

1. Introduction

As the precision of the recent lepton-hadron scattering data has improved, it is vital for the theoretical analysis to keep pace. For example, the calculation of the Wilson coefficients has progressed to encompass next-to-leading order (NLO) quantum chromodynamics (QCD) and beyond. It is important, therefore, to consider all sources of corrections which may contribute at a comparable magnitude, such as electroweak radiative corrections, quark mass effects, and target mass corrections. In this review, we will focus on the problem of target mass corrections (TMCs)[1].

One of the features of polarized DIS is that a lot of the present data are in the preasymptotic region ($Q^2 \sim 1-5\text{GeV}^2, 4\text{GeV}^2 < W^2 < 10\text{GeV}^2$). While in the unpolarized case we can cut the low Q^2 and W^2 data in order to minimize the less known target mass correction and higher twist effects, it is impossible to perform such a procedure for the present data on the spin-dependent structure functions without losing too much information. TMCs effects are arising from purely kinematic effects associated with finite values of the quantity $4M^2x^2/Q^2$. These are also powers in $1/Q^2$ corrections, which can be calculated without using models[2]. In this note we present numerical results which illustrate the main features of the TMCs to the spin structure function g_1 valid in the preasymptotic DIS region. We consider that their knowledge is useful and important in the QCD analyses of the present and future data on polarized DIS at moderate energies.

In this work, we employ the results of our previous QCD fit [3] to analyze the target mass corrections to the spin structure functions in the DIS region.

2. Polarized structure functions

To extrapolate the spin structure function g_1, which fitted to high Q^2 proton, deuteron and neutron data, down to a low value of Q^2 by means of the Altarelli-Parisi evolution equation, we have to confront the QCD prediction with the experimental data at low Q^2, and we impose the significant QCD corrections by taking into account the effects of the target mass. In the low-Q^2 region ($Q^2 \sim$ few GeV^2), we cannot simply neglect the order of Q^2/M^2 with M being the nucleon mass. To get the target mass corrections to the nucleon spin structure function g_1, one may follow the methd proposed by Georgi and Politzer in the case of unpolarized structure function[4].

The recent calculations show that [5]-[8]

$$g_1^{TMCs}(x,Q^2) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dn x^{-n}\sum_{j=0}^{\infty}(\frac{M^2}{Q^2})^j\frac{n(n+j)!}{j!(n-1)!(n+2j)^2} \times M_1^{n+2j}(Q^2;M=0), \quad (1)$$

where $M_1^n(Q^2;M=0)$ is the nth Cornwall-Norton (CN) moment of g_1

$$M_1^n(Q^2;M=0)=\int_0^1 dx x^{n-1}g_1(x,Q^2;M=0), \quad (2)$$

calculated in the perturbative QCD where all the mass terms $O(M^n/Q^n)$ are neglected, namely, the nucleon mass M vanishes. In CN moments, different spin operators contribute to the twist-two moment.

The explicit twist-2 expression of g_1 with the TMCs is

$$g_1^{TMCs}(x,Q^2)=\frac{xg_1(\xi,Q^2;M=0)}{\xi(1+4M^2x^2/Q^2)^{3/2}} + \frac{4M^2x^2}{Q^2}\frac{x+\xi}{\xi(1+4M^2x^2/Q^2)^2} \times \int_\xi^1 \frac{d\xi'}{\xi'}g_1(\xi',Q^2;M=0) - \frac{4M^2x^2}{Q^2}\frac{(2-4M^2x^2/Q^2)}{2(1+4M^2x^2/Q^2)^{5/2}} \times \int_\xi^1\frac{d\xi'}{\xi'}\int_{\xi'}^1\frac{d\xi''}{\xi''}g_1(\xi'',Q^2;M=0), \quad (3)$$

where for massless quarks, the parton light-cone fraction is given by the Nachtmann variable [9]

$$\xi=\frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}, \quad (4)$$

with the Bjorken $x=Q^2/(Q^2+W^2-M^2)$ (W being the center-of-mass energy). Note that the Nachtmann variable ξ has a more general meaning than the Bjorken variable x. It represents the fraction of the total momentum carried by the parton with a finite Q^2 and with a non-vanishing target mass. Therefore, this variable partly contains the target mass corrections. If the nuclear mass vanishes or the momentum transfer is large enough $M^2/Q^2 \ll 1$, the Nachtmann variable turns to be the Bjorken one.

Similarly, the spin structure function g_2 with twist-2 contribution and with the TMCs is

$$g_2^{TMCs}(x,Q^2)=-\frac{xg_1(\xi,Q^2;M=0)}{\xi(1+4M^2x^2/Q^2)^{3/2}} + \frac{x(1-4M^2x\xi/Q^2)}{\xi(1+4M^2x^2/Q^2)^2} \times\int_\xi^1\frac{d\xi'}{\xi'}g_1(\xi',Q^2;M=0) + \frac{3}{2}\frac{4M^2x^2/Q^2}{(1+4M^2x^2/Q^2)^{5/2}} \times\int_\xi^1\frac{d\xi'}{\xi'}\int_{\xi'}^1\frac{d\xi''}{\xi''}g_1(\xi'',Q^2;M=0). \quad (5)$$

The above g_2^{TMCs} satisfies the well-known Wandzura-Wilczek (WW) relation [10]

$$g_2^{TMCs}(x,Q^2) = -g_1^{TMCs}(x,Q^2) + \int_x^1\frac{g_1^{TMCs}(y,Q^2)}{y}dy. \quad (6)$$

This relation is not affected by the target mass corrections.

3. Nachtmann moments

The Nachtmann moments of the nucleon spin structure functions, shown in the literature [11, 12], are defined to factor out the target mass dependence of the structure functions in a way such that its CN moments would equal the moments of the corresponding parton distributions. In other words the difference between the CN and Nachtmann moments comes from the trace terms appearing in the matrix elements of the operators of definite spin, which are disregarded in the CN moments, but kept in the Nachtmann moments [13].

Piccione and Ridolfi [5] have compared CN and Nachtmann moments. They argued that Nachtmann moments are not directly applicable in a full analysis of the polarized DIS data because the target mass corrected reduced matrix elements of the relevant operators in operator production expansion (OPE), like a_n and d_n, were expressed in terms of the polarized structure functions, if taking the TMCs into account; these expressions reduce to the moments of the structure functions

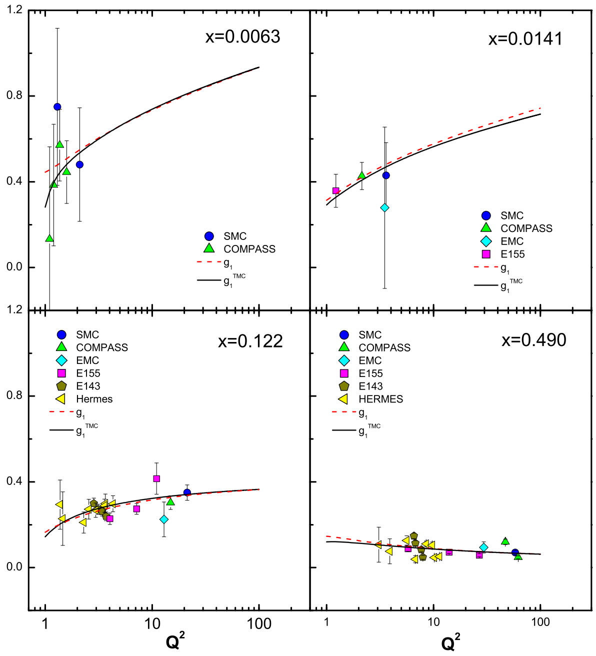

Figure 1. The spin-dependent structure functions $g_1(x,Q^2)$ and $g_1^{TMC}(x,Q^2)$ as a function of Q^2 in different values of x. The experimental data are compared to the fit result with the statistical error band.

in the massless limit, but do not have a simple parton model interpretation in the case of $M \neq 0$. Thus, they claimed that CN moments have the advantage that the moments of the polarized structure functions are expressed as the functions of the reduced operator matrix elements and the effects of the TMCs on the nucleon spin structure functions, which are of pure kinematical origin, can be explicitly seen.

4. Target mass corrections

We perform target mass corrections on our previous analysis of existing polarized deep inelastic scattering world data in the framework of QCD at next-to-leading order [3]. Our method to determine PPDFs is based on Jacobi polynomial expansion method

$$xg_1^{N_{max}}(x,Q^2) = x^{\beta}(1-x)^{\alpha} \times \sum_{n=0}^{N_{max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)\mathbf{M}[xg_1, j+2], \quad (7)$$

where $\mathbf{M}[xg_1, j+2]$ is the moment of parton structure function in Mellin-N space. Here, we consider $N_{max} = 9$, $\alpha = 3.0$ and $\beta = 0.5$ to get the most accurate xg_1. Detailed information about Jacobi application in polarized and unpolarized structure function can be obtained respectively in Ref. [14]-[16].

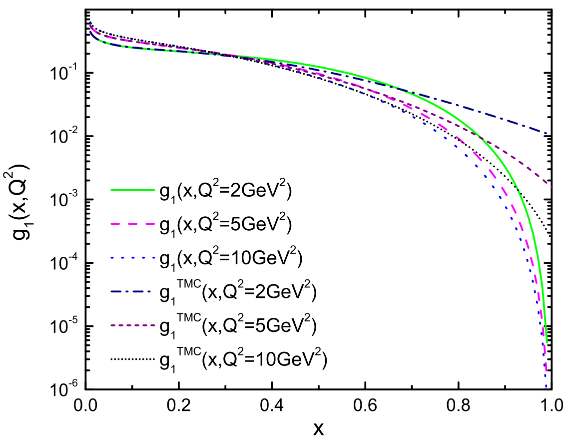

Figure 2. Comparison $g_1(x,Q^2)$ with $g_1^{TMCs}(x,Q^2)$. The solid ($Q^2 = 2$ GeV^2), dashed ($Q^2 = 5$ GeV^2) and dotted ($Q^2 = 10$ GeV^2) curves are results without the target mass corrections; whereas dash-dotted($Q^2 = 2$ GeV^2), short-dashed ($Q^2 = 5$ GeV^2) and short-dotted ($Q^2 = 10$ GeV^2) curves represent the results with TMCs, respectively.

The fit quality of our g_1 model and effects of considering target mass corrections as a function of Q^2 in different values of x are shown in Fig. 1. To see the effects of the TMCs in Figs. 2, 3, we explicitly show the comparisons of $g_1(x,Q^2)$ with $g_1^{TMCs}(x,Q^2)$, and of $-g_2(x,Q^2)$

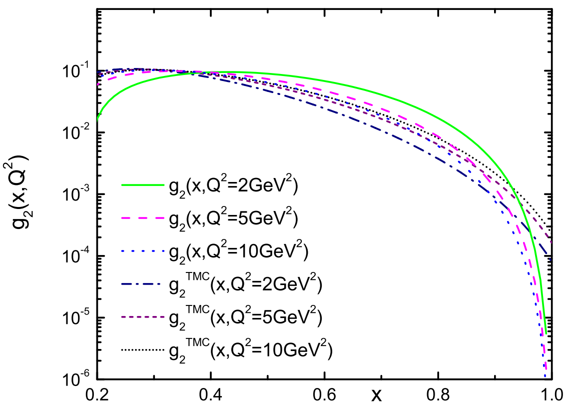

Figure 3. Comparison $-g_2(x, Q^2)$ with $-g_2^{TMCs}(x, Q^2)$. The solid ($Q^2 = 2$ GeV^2), dashed ($Q^2 = 5$ GeV^2) and dotted ($Q^2 = 10$ GeV^2) curves are results without the target mass corrections; whereas dash-dotted($Q^2 = 2$ GeV^2), short-dashed ($Q^2 = 5$ GeV^2) and short-dotted ($Q^2 = 10$ GeV^2) curves represent the results with TMCs, respectively.

with $-g_2^{TMCs}(x, Q^2)$. We see that the TMCs play a significant role. They enlarge the values of the spin structure functions, particularly in the large x region. The figures reasonably show that the smaller the momentum transfer Q^2 is, the larger the effects of the TMCs are. Moreover, we see that in the limit $x \longrightarrow 1$, $g_{1,2}^{TMCs}$ do not vanish, although $g_{1,2}(x, Q^2; M = 0) \longrightarrow 1$ [8]. This problem was discussed in Ref.[17] for the unpolarized case; the conclusion reached there is that in the large x region dynamical higher twist corrections become important and cannot be neglected any more.

5. Conclusion

We have presented in this review a survey of the key issues pertaining to target mass corrections (TMCs) in inclusive lepton-nuclear scattering structure functions and their impact on the analysis of experimental results. As illustrated by our structure function results with and without TMC terms, our calculations based on the Jacobi polynomials method are in good agreement with experimental data. Our results show that TMCs play a remarkable role on the nucleon spin structure functions, specially in the large x and low Q^2 region. A reliable extraction of PDFs therefore demands an accurate description of the TMCs.

REFERENCES

1. I. Schienbein *et al.*, J. Phys. G **35**, 053101 (2008) [arXiv:0709.1775 [hep-ph]].
2. A. V. Sidorov and D. B. Stamenov, deep inelastic scattering," Mod. Phys. Lett. A **21**, 1991 (2006) [arXiv:hep-ph/0604092].
3. F. Arbabifar, A. Khorramian, S. Atashbar Tehrani, A. Najafgholi, will be published in Int. J. Mod. Phys. A, (2010).
4. H. Georgi and H. D. Politzer, Phys. Rev. D **14**, 1829 (1976).
5. A. Piccione and G. Ridolfi, Nucl. Phys. B **513**, 301 (1998) [arXiv:hep-ph/9707478].
6. J. Blumlein and A. Tkabladze, Nucl. Phys. B **553**, 427 (1999) [arXiv:hep-ph/9812478].
7. Y. B. Dong, Nucl. Phys. A **744**, 293 (2004).
8. Y. B. Dong, Phys. Lett. B **641**, 272 (2006).
9. O. Nachtmann, Nucl. Phys. B **63**, 237 (1973).
10. S. Wandzura and F. Wilczek, Phys. Lett. B **72**, 195 (1977).
11. S. Wandzura, Nucl. Phys. B **122**, 412 (1977).
12. S. Matsuda and T. Uematsu, Nucl. Phys. B **168**, 181 (1980).
13. F. M. Steffens and W. Melnitchouk, Phys. Rev. C **73**, 055202 (2006) [arXiv:nucl-th/0603014].
14. S. Atashbar Tehrani and A. N. Khorramian, JHEP **0707**, 048 (2007) [arXiv:0705.2647 [hep-ph]].
15. A. N. Khorramian and S. A. Tehrani, Phys. Rev. D **78**, 074019 (2008) [arXiv:0805.3063 [hep-ph]].
16. A. N. Khorramian, H. Khanpour and S. A. Tehrani, Phys. Rev. D **81**, 014013 (2010) [arXiv:0909.2665 [hep-ph]].
17. A. De Rujula, H. Georgi and H. D. Politzer, Phys. Rev. D **15**, 2495 (1977).