LHCSpin and Polarimetry

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A study of forward proton proton spin dependence and its implications for the high energy behaviour of amplitudes relating to polarimetry is given

OUTLINE

- Polarised p, d, and ^{3}He beams provide polarised up and down quarks

  Musgrave *et al.*, PoS PSTP **2017** (2018) 020

- Polarimety for a fixed polarised target can use L–R recoil asymmetries
  - examine helicity amplitudes for p p↑ and p ^{3}He↑ elastic reactions
  - the analysing power in the CNI region of $q^2 = -t$ can reach 4 %

- Review asymmetries in the electromagnetic hadronic interference region

  Kopeliovich and Lapidus, Yad Fiz 19 (1974) 340

- Express A_{N} and A_{NN} in terms of values of the hadronic ratios r_5 and r_2

- Analysis of exchanges at 100 and 255 GeV leads to asymmetry prediction

Elastic Amplitudes and CNI Analysing Powers

Neglecting the Coulomb phase δ_C, the real part ratio ρ, hadronic spin-flip and form factor effects, the spin nonflip f and spin-flip g amplitudes are

$$f = \frac{\alpha}{t} + i\,\frac{\sigma_{\rm tot}}{8\,\pi} \qquad \text{and} \qquad g = \frac{\mu - 1}{2}\,\frac{q}{m}\,\frac{\alpha}{t}$$

indicating that the analysing power reaches an extremum at momentum transfer $t_e = -8\sqrt{3}\,\pi\,\alpha/\sigma_{\rm tot}$ arising from the following $q^2 = -t$ dependence

$$A_{\rm N} = \frac{2\,{\rm Im}\,(\,f^*\,g\,)}{|\,f\,|^2 + |\,g\,|^2} \;=\; \frac{\mu - 1}{m}\,(-3\,t_e)^{1/2}\,\frac{(t/t_e)^{3/2}}{3\,(t/t_e)^2 + 1}$$

where $|\,g\,|^2$ has been ignored relative to the larger $|\,f\,|^2$ in the CNI region. The extreme value of the pp$\rightarrow$pp analysing power in the interference region

$$A_{\rm N}^{\rm e} \;=\; (\mu - 1)\,(-3\,t_e)^{1/2}/4\,m \;\approx\; 4.5\,\%$$

Kinematics for p p↑ Elastic Collisions

- Polarimetry at HJET BNL used p↑ p↑ $\rightarrow$ p p at 100 GeV and 255 GeV
- Best results occurred for recoil kinetic energies: $2.0 < T_{\mathrm{R}} < 5.3$ MeV
- Systematic errors below 2 MeV and inelastic effects above 5.5 intruded
- Peak analysing power appeared at $T = 1.4$ MeV, $t = -0.0015$ GeV/c^2
- At 255 GeV, $\sigma_{\mathrm{tot}} = 39.2$ mb; and at 7 TeV, $\sigma_{\mathrm{tot}} \approx 47$ mb, a factor 1.2
- Recoil energies for 7 TeV may be best for values: $1.7 < T_{\mathrm{R}} < 4.6$ MeV
- Protons to recoil over angles: $30 < \theta < 50$ mrad, at: 6% $< v/c <$ 10%

Amplitudes and Asymmetries for p ^{3}He Elastic Collisions

$$
\begin{aligned}
\phi_1 &= \langle + + \,|M|\, + + \rangle, & \phi_4 &= \langle + - \,|M|\, - + \rangle \propto -t \\
\phi_2 &= \langle + + \,|M|\, - - \rangle, & \phi_5 &= \langle + + \,|M|\, + - \rangle \propto \sqrt{-t} \\
\phi_3 &= \langle + - \,|M|\, + - \rangle, & \phi_6 &= \langle + + \,|M|\, - + \rangle \propto \sqrt{-t}
\end{aligned}
$$

Hadronic ϕ_1, ϕ_2, ϕ_3 are nonzero at $t = 0$, and $\phi_6 = -\phi_5$ for p p $\to$ p p

$$
\begin{aligned}
(k\sqrt{s}\,/\,2\pi)\,\sigma_{\rm tot} &= \mathrm{Im}\,[\phi_1(s,0) + \phi_3(s,0)] \\
\frac{2k^2 s}{\pi}\frac{d\sigma}{dt} &= |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 2|\phi_5|^2 + 2|\phi_6|^2 \\
A_{\rm N}\,\frac{2k^2 s}{\pi}\frac{d\sigma}{dt} &= \mathrm{Im}\,[\,(\phi_1 + \phi_2 + \phi_3 - \phi_4)^*\phi_5\,] \\
A_{\rm NN}\,\frac{2k^2 s}{\pi}\frac{d\sigma}{dt} &= \mathrm{Re}\,[\,\phi_1^*\,\phi_2 - \phi_3^*\,\phi_4 - 2\phi_5^*\,\phi_6\,]
\end{aligned}
$$

Scattering of Identical and Non-identical Fermions

- For the elastic reactions $\mathrm{p\,p} \to \mathrm{p\,p}$ and ${}^3\mathrm{He}\,{}^3\mathrm{He} \to {}^3\mathrm{He}\,{}^3\mathrm{He}$, $\phi_6 = -\phi_5$
- For non-identical $\mathrm{p}\,{}^3\mathrm{He} \to \mathrm{p}\,{}^3\mathrm{He}$ or $\mathrm{p}\,{}^{13}\mathrm{C} \to \mathrm{p}\,{}^{13}\mathrm{C}$, in general, $\phi_6 \neq -\phi_5$

  NHB, Gotsman, Leader, Phys Rev D18 (1978) 694
- The p-C polarimeter uses a thin carbon ribbon with a 2 MHZ event rate
- Here $\mathrm{p}\,{}^{12}\mathrm{C} \to \mathrm{p}\,{}^{12}\mathrm{C}$ and ${}^3\mathrm{He}\,{}^{12}\mathrm{C} \to {}^3\mathrm{He}\,{}^{12}\mathrm{C}$ have just two amplitudes

The proton carbon polarimeter requires calibration from a H-jet polarimeter operating at a 90 Hz rate to achieve a $\delta P \approx 2\%$ statistical accuracy for an 8-hour RHIC store. A. Poblaguev *et al.*, PoS PSTP **2017** (2018) 022

Interference Region Elastic Amplitudes

Amplitudes including Coulomb contributions and the Bethe-Soloviev phase $\delta_{\mathrm{C}} = -\,Z^2\alpha\left(\,\ell\mathrm{n}\,|Bt/2\,+\,4t/\Lambda^2|\,-\,\gamma\,\right)$, for t close to $t_{\mathrm{c}} = -8\,\pi Z^2\alpha/\sigma_{\mathrm{tot}}$

$$\begin{aligned}
\phi_1 \;\approx\; \phi_3 \;&=\; \frac{k\sqrt{s}}{4\pi}\,\sigma_{\mathrm{tot}}\left(i\,+\,\rho\,-\,\frac{t_{\mathrm{c}}}{t}\,e^{i\,\delta_{\mathrm{C}}}\right)\\
\phi_2 \;&=\; \frac{k\sqrt{s}}{4\pi}\,\sigma_{\mathrm{tot}}\left(2\,r_2-\frac{\kappa^2\,t_{\mathrm{c}}}{4m_{\mathrm{p}}^2}\,e^{i\,\delta_{\mathrm{C}}}\right)\\
\frac{m_{\mathrm{p}}}{\sqrt{-t}}\,\phi_5 \;&=\; \frac{k\sqrt{s}}{4\pi}\,\sigma_{\mathrm{tot}}\left(r_5\,-\,\frac{\kappa\,t_{\mathrm{c}}}{2\,t}\,e^{i\,\delta_{\mathrm{C}}}\right)
\end{aligned}$$

The hadronic amplitude $\phi_4 \,\propto\, t$ is ignored, but $\phi_4^{\mathrm{em}} = -\phi_2^{\mathrm{em}}$ is kept below.
The CM momentum is given by: $k^2 \;=\; s-2\,m^2-2\,\widetilde{m}^2+\left(m^2-\widetilde{m}^2\right)^2/s$

Differential cross section close to $t = t_{\rm c}$ with hadronic slope parameter B

$$\frac{16\pi}{\sigma_{\rm tot}^2}\frac{d\sigma}{dt}\,e^{-Bt} \;=\; \left(\frac{t_{\rm c}}{t}\right)^2 - \,2\,(\,\rho + \delta_{\rm C} + \epsilon\,)\,\frac{t_{\rm c}}{t} \,+\, 1 \,+\, \rho^2$$

Spin dependent observables in interference region of momentum transfer

$$\frac{m_{\rm p}\,A_{\rm N}}{\sqrt{-t}}\,\frac{8\,\pi}{\sigma_{\rm tot}^2}\,\frac{d\sigma}{dt}\,e^{-Bt} \;=\; \left[\frac{\kappa}{2}\,(1 + {\rm Im}\,r_2 - \delta_{\rm C}\,\rho) - {\rm Im}\,r_5 + \delta_{\rm C}\,{\rm Re}\,r_5\right]\,\frac{t_{\rm c}}{t}$$

$$- \,(1 + {\rm Im}\,r_2)\,{\rm Re}\,r_5 \,+\, (\rho + {\rm Re}\,r_2)\,{\rm Im}\,r_5$$

$$A_{\rm NN}\,\frac{8\,\pi}{\sigma_{\rm tot}^2}\,\frac{d\sigma}{dt}\,e^{-Bt} \;=\; -\left[{\rm Re}\,r_2 \,+\, \delta_{\rm C}\,{\rm Im}\,r_2\right]\frac{t_{\rm c}}{t} \,+\, \left(\kappa\,t_{\rm c}\,/m_{\rm p}^2\right){\rm Re}\,r_5$$

$$+ \,{\rm Im}\,r_2 \,+\, \rho\,(\,{\rm Re}\,r_2 \,-\, \kappa^2 t_{\rm c}/4m_{\rm p}^2\,)$$

For protons, $\kappa = \mu_p - 1$; for He-3 (h), $\kappa = \mu_h/Z - m_{\rm p}/m_h$, with $Z = 2$.

A polarimeter requires a process with nonvanishing high energy polarization

- Spin one photon exchange suggests the Primakoff or a Coulomb effect
- Helium-3 scattering reaches about $-3\,\%$ asymmetry in the CNI region

A spin half hadron of mass m, charge Ze, magnetic moment μ scattering elastically off a charge $Z'e$ has an asymmetry that involves an interference

$$2\ \mathrm{Im}\left[\frac{Z\,Z'}{137\,t}\ +\ (\rho+i)\,\frac{\sigma_{\mathrm{tot}}}{8\,\pi}\right]^{*}\frac{\kappa\,\sqrt{-t}}{2\,m_{\mathrm{p}}}\left[\frac{Z\,Z'}{137\,t}+(\mathrm{Re}\,r_5+i\ \mathrm{Im}\,r_5)\,\frac{\sigma_{\mathrm{tot}}}{8\,\pi}\right]$$

of helicity nonflip and flip amplitudes with electromagnetic and hadronic elements and σ_{tot} relates to the hadronic particles of charges Ze and $Z'e$.

Including the spin averaged denominator, the asymmetry is proportional to

$$A_{\rm N} \ \propto \ \frac{\sqrt{x}}{x^2+3}\,, \qquad x \ = \ \frac{t_{\rm e}}{t}\,, \qquad t_{\rm e} \ = \ -\frac{8\,\pi\,\sqrt{3}\ |\,Z\,Z'\,|}{137\,\sigma_{\rm tot}(s)} \ = \ \sqrt{3}\,t_{\rm c}$$

the extremum value of which occurs at $x = 1$, that is, at transfer $t = t_{\rm e}$.

The optimum value of 3% to 4 % varies slowly with energy s as $1/\sqrt{\sigma_{\rm tot}(s)}$

It is either a maximum or minimum depending on the sign of constant κ

$$A_{\rm N}^{\rm opt} \ = \ \frac{\kappa}{4m_{\rm p}}\sqrt{-3\,t_{\rm e}}\,, \qquad \kappa \ = \ \frac{\mu}{Z} - \frac{m_{\rm p}}{m}$$

The value of κ is 1.793 (anomalous μ) for protons and -1.398 for helions.

Hadronic helicity flip amplitudes and two photon exchange are ignored here.

Quantities related to proton carbon collisions may be compared to those of the more familiar proton proton case with the same incident fermion, viz,

$$\frac{t_{\rm e}^{\rm pC}}{t_{\rm e}^{\rm pp}} \;=\; \frac{6\,\sigma_{\rm tot}^{\rm pp}}{\sigma_{\rm tot}^{\rm pC}} \;\approx\; 0.74\,, \qquad\qquad \frac{A_{\rm N}^{\rm pC}}{A_{\rm N}^{\rm pp}} \;=\; \left(\frac{t_{\rm e}^{\rm pC}}{t_{\rm e}^{\rm pp}}\right)^{1/2} \;\approx\; 0.86$$

With distinct incident fermions, by contrast, helion carbon and proton carbon scattering have extremum momentum transfer and asymmetry ratios

$$\frac{t_{\rm e}^{\rm hC}}{t_{\rm e}^{\rm pC}} \;=\; \frac{2\,\sigma_{\rm tot}^{\rm pC}}{\sigma_{\rm tot}^{\rm hC}} \;\approx\; 1.0\,, \qquad\qquad \frac{A_{\rm N}^{\rm hC}}{A_{\rm N}^{\rm pC}} \;=\; \frac{\kappa_{\rm h}}{\kappa_{\rm p}}\left(\frac{t_{\rm e}^{\rm hC}}{t_{\rm e}^{\rm pC}}\right)^{1/2} \;\approx\; -0.78$$

The same would be approximately true if the target carbon particle C here were replaced throughout by another ion such as a proton p or a helion ^{3}He

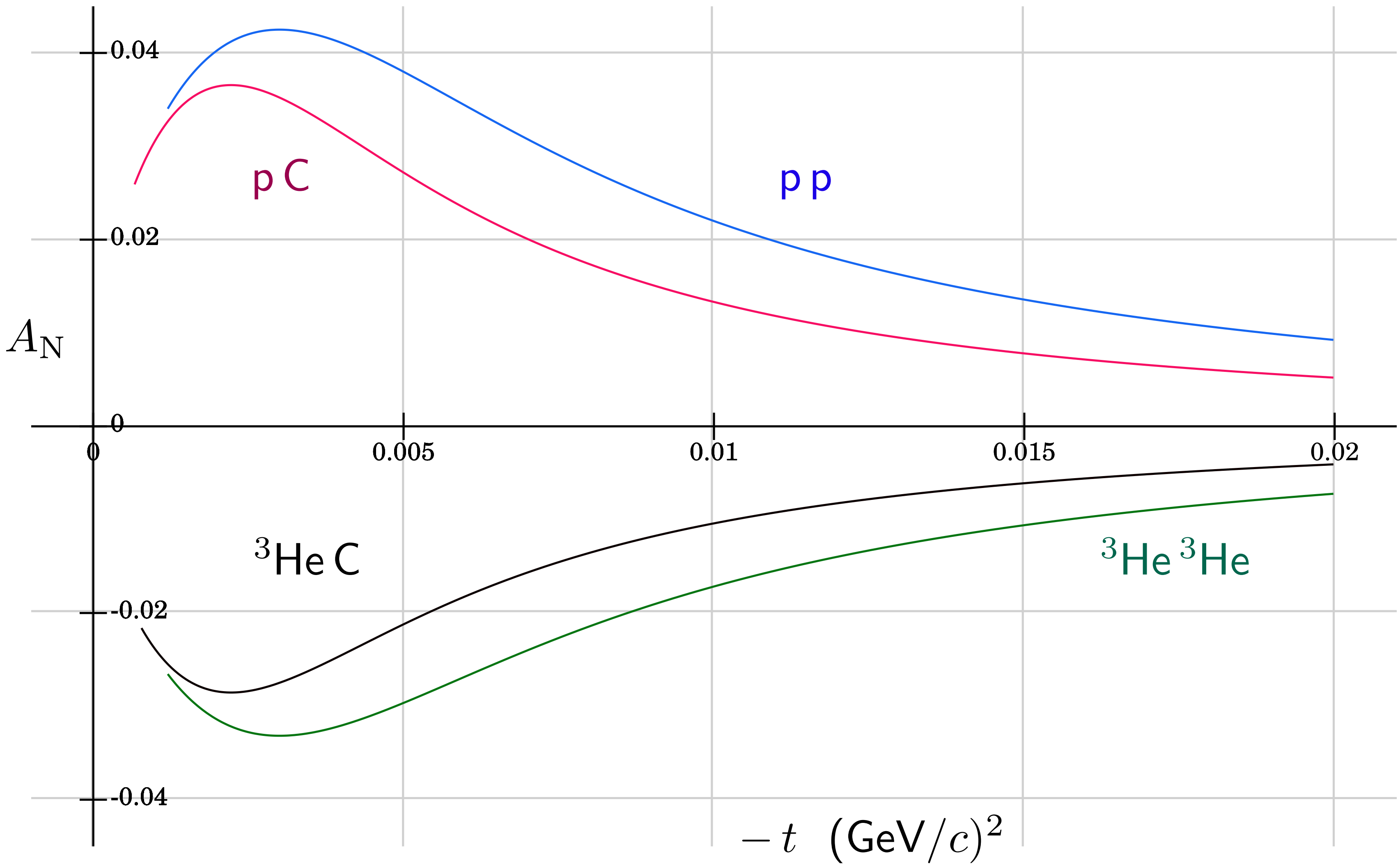

Figure 1: Analyzing power A_{N} versus invariant momentum transfer (-t) in $(\mathrm{GeV}/c)^2$ for (1) p p and p h scattering, (2) p C scattering, (3) h C scattering, (4) h h and h p scattering

The extremum value of t has first order corrections in the Coulomb phase δ_{C}, the hadronic non-flip real part ratio ρ, and the helicity-flip ratio r_5

$$t_{\mathrm{e}} \;:\quad 1 \,-\, (\rho + \delta_{\mathrm{C}})/\sqrt{3} \,-\, (\mathrm{Re}\, r_5 \,-\, \rho\, \mathrm{Im}\, r_5)\, 4/\sqrt{3}$$

Another factor with small items δ, ρ, r_5, multiplies the extremum of A_{N}

$$A_{\mathrm{N}} \;:\quad 1 \,+\, (\rho + \delta_{\mathrm{C}})\sqrt{3}\,/\,2 \,-\, (\sqrt{3}\,\mathrm{Re}\, r_5 \,+\, \mathrm{Im}\, r_5)$$

The extremum value of A_{NN} occurs at $t = t_{\mathrm{c}} = t_{\mathrm{e}}/\sqrt{3}$, approximately. Polarized proton nucleus scattering has been studied over a range of momentum transfers Kopeliovich and Trueman, Phys Rev D **64** (2001) 034004

Hadronic spin flip and Coulomb phase effects have been treated in detail in

NB, Kopeliovich, Leader, Soffer, Trueman, Phys Rev D59 (1999) 114010

The pp2pp experiment at STAR (RHIC BNL) has shown that the elastic pp

- hadronic single helicity-flip amplitude is small at $\sqrt{s} = 200$ GeV

L. Adamczyk *et al.* [STAR Collaboration], Phys Lett B 719, 62 (2013)

The acceleration of Helium-3 nuclei to high energy has been discussed.

W. W. MacKay, AIP Conference Proceedings 980, 191 (2008)

Helium-3 ions have been accelerated in the AGS at BNL (Haixin Huang).
The Helium-3 carbon cross section at the AGS appears to be twice that for proton carbon scattering.

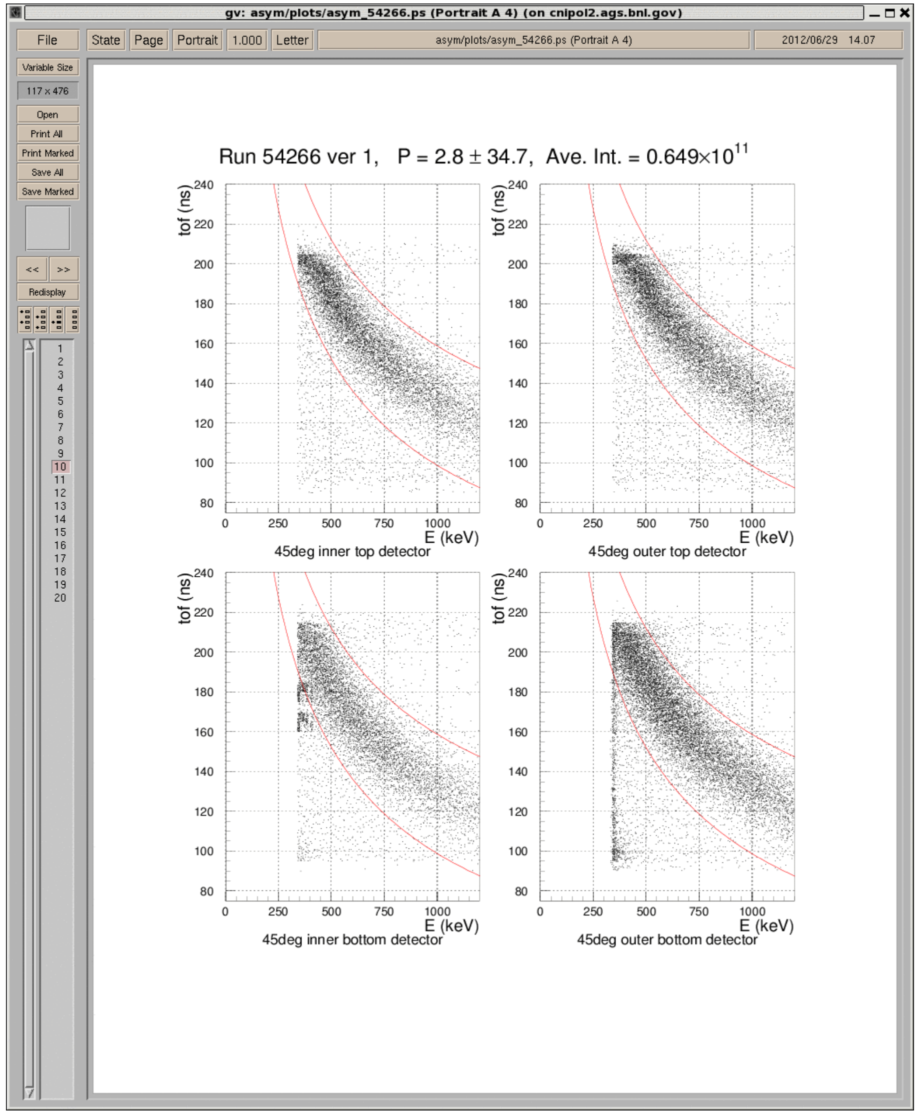

Figure 2: Time of flight of carbon recoils (on y-axis) versus the recoil kinetic energy of Helium-3 (on x-axis) as measured at the AGS. The 3He-C events are double those of p-C.

CONCLUSIONS

Probing the spin structure of hadrons increases an understanding of QCD

There is great potential for studies using polarized up and down quarks

Proton polarimetry is mature now and polarized ^{3}He may be forthcoming

The ^{3}He–C analyzing power is $\approx -78\%$ of A_N for p-C in the CNI region

A polarized ^{3}He jet and beam would enable an absolute ^{3}He polarimeter

Single and double helicity flip pp amplitudes are known at many energies

Extrapolation of amplitudes to other high energies is becoming possible