UNIVERSITÀ DEGLI STUDI DI UDINE
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PARTICLE PHYSICS 粒子物

Relativity 1

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- Introduction to Einstein's special relativity
- Consequences: relativity of simultaneity, time and space dilatation
- Lorentz transforms

Why Special Relativity

Why do we need special relativity to describe the behavior of elementary particles in accelerators?

- Because elementary particles typically have large kinetic energies and speeds, often approaching $c = 2.998 \times 10^8$ m/s, the speed of light. In such regime, classical mechanics no longer applies.

- Because reactions leading to creation or destruction of particles may occur. The *rest energy*, proportional to the mass of the particles, must be taken into account in the global balance of energies. Such concept is unknown to classical mechanics.

Einstein's special relativity is the theory that properly describes the kinematics and energetics of elementary particles in accelerators.

Einstein's Relativity Principle

Special relativity is based upon Einstein's *Relativity Principle*:

- *The laws of physics are valid in all inertial reference frames,* and
- *The speed of light in vacuum is the same in all inertial reference frames, independently upon the speed of the source*

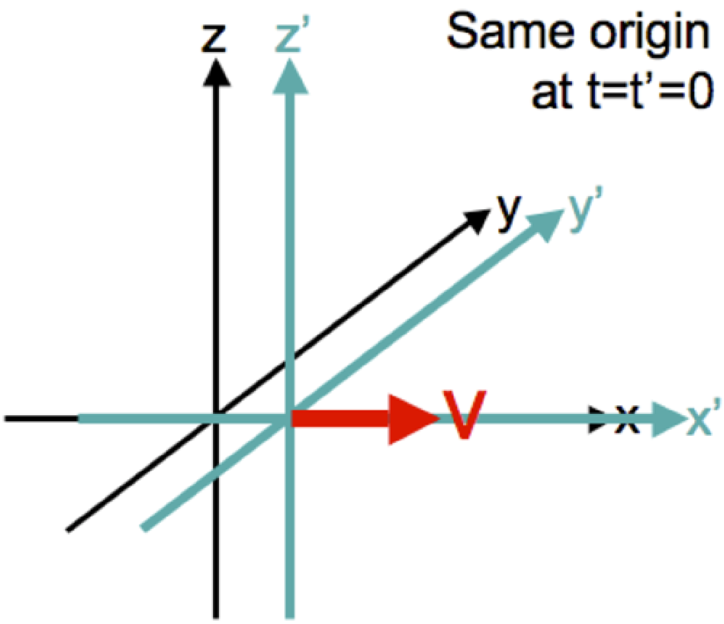
Such principle is not compatible with Galilean transforms (valid only as limit case for speeds $v << c$), and with Newton's idea of absolute time.

If we accept such principle, there is no reason to assume that the light propagates into a medium (the "ether") in order to reconcile Maxwell's equations with Galilean transforms.

We need to introduce *Lorentz transforms* and the concept of *space-time*.

Reminder: Galilean Transforms

Transformation rules between an inertial system $\mathcal{S}$ (described by x, y, z, t) and another one, $\mathcal{S}'$ (described by x', y', z', t') traveling with velocity V along x with respect to $\mathcal{S}$:



$$\begin{cases} x' &= x - Vt \\ y' &= y \\ z' &= z \\ t' &= t \end{cases}$$

(origins are assumed to coincide at $t = 0$). The *velocity addition rules* follows: a particle with velocity $\mathbf{v}$ in $\mathcal{S}$ has velocity $\mathbf{v}' = \mathbf{v} - \mathbf{V}$ in $\mathcal{S}'$

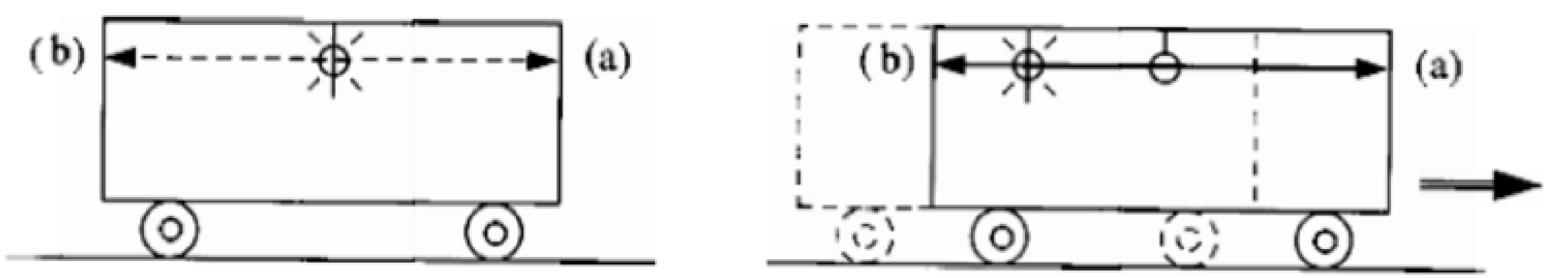
The inverse transform from $\mathcal{S}'$ to $\mathcal{S}$ is obtained reversing the sign of V.

Consequences of Relativity Principle

Einstein's Relativity Principle has rather surprising consequences, that can be demonstrated on the basis of simple *gedankenexperimente* ("though experiments"):

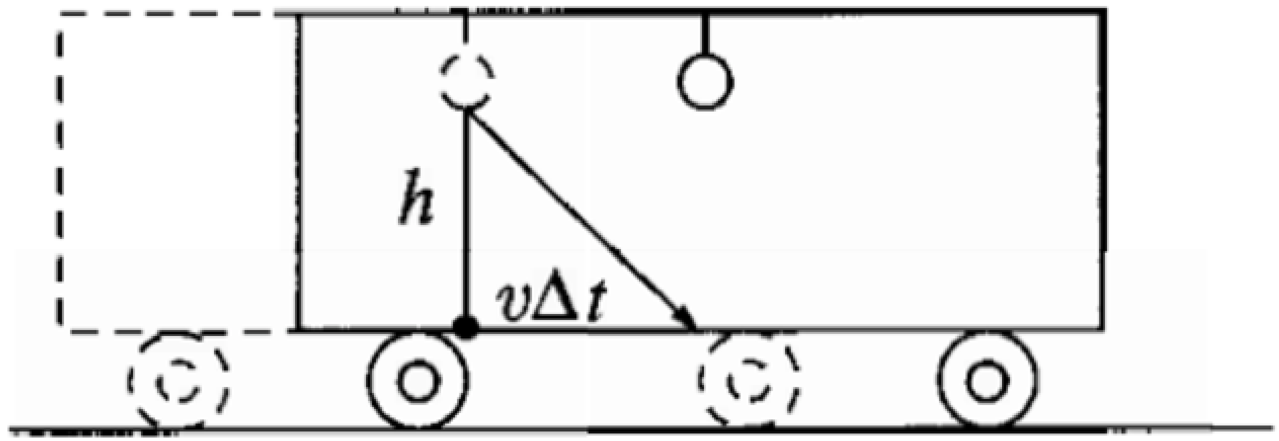
- **Relativity of simultaneity:** *Two events that are simultaneous (happen at the same time) in an inertial system may not be simultaneous in another inertial system*

In the reference frame of the moving observer, the ray of light hits simultaneously the two walls; in the laboratory (observer at rest) reference frame, this does not happen.



Consequences of Relativity Principle (2)

- **Time Dilatation:** *A moving clock runs slower*



Let us consider a ray of light that hits the floor. This happen after $\Delta t' = h/c$ in the reference frame of the moving observer, in $\Delta t = \sqrt{h^2 + (V\Delta t)^2}/c$ in the reference frame for the fixed observer. Thus:

$$\Delta t' = \sqrt{1 - V^2/c^2}\Delta t \equiv \gamma^{-1}\Delta t < \Delta t$$

where we have introduced the factor $\gamma = 1/\sqrt{1 - V^2/c^2}$.

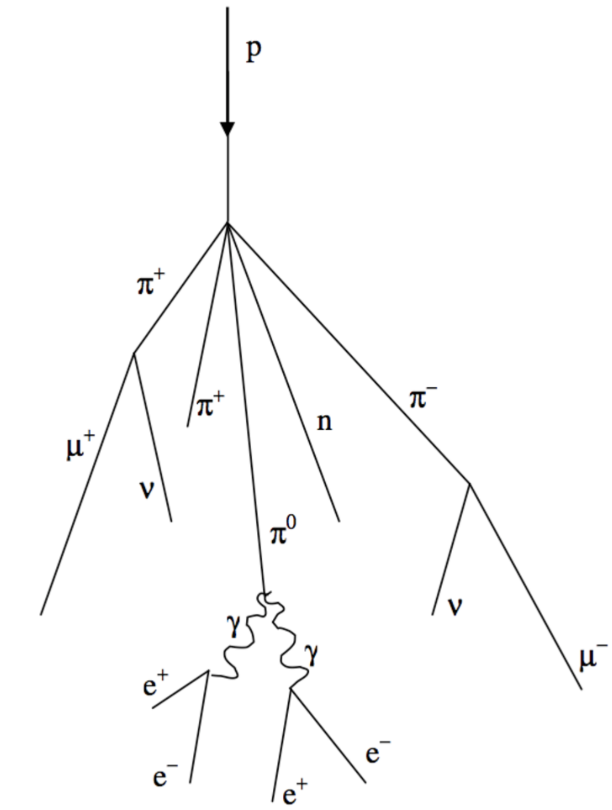
Time Dilatation in action: cosmic rays

Muons (μ^+, μ^-): components of cosmic rays, generated by heavier particles (mesons) in the high atmosphere (~ 15 Km)

Muons' energy is such that $v \sim c$. The half life of a muon is $\tau_0 = 2.2\mu s$. During such time, a muon with speed c travels a distance:

$$s = c\tau_0 = 2.2 \times 10^{-6}\text{s} \cdot 3 \times 10^8\text{m/s} = 660\text{m}.$$

The flux of muons is however easily measurable at ground level. How is that possible?



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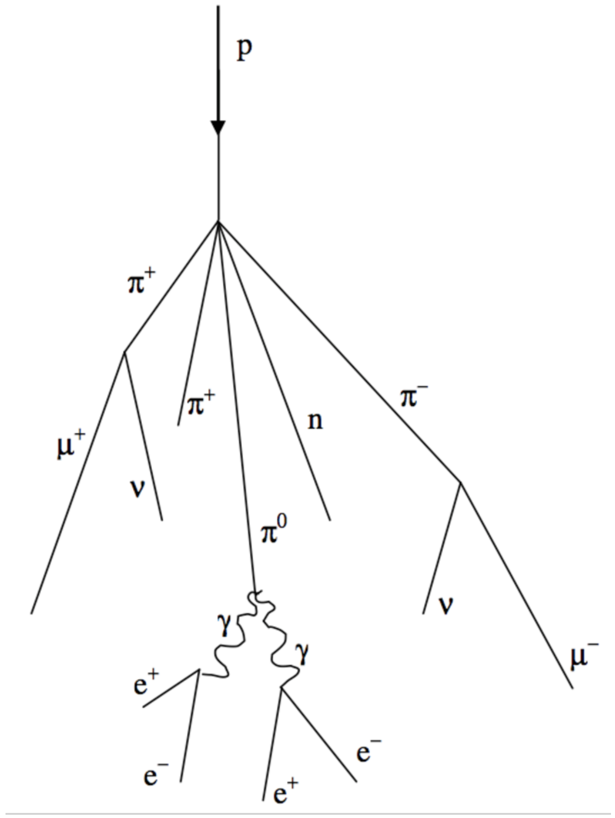
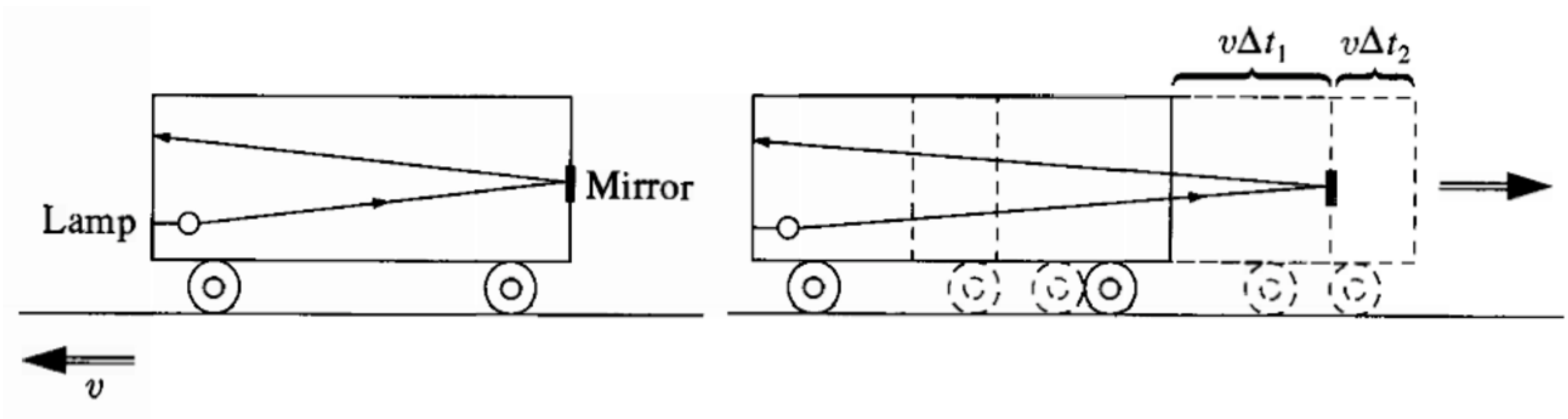


The half life of muons refers to the *reference frame of the muon* (that is: moving with it). In a *laboratory reference frame* (fixed on the Earth):

$$\tau = \frac{\tau_0}{\sqrt{1 - v^2/c^2}} = \gamma\tau_0 >> \tau_0!$$

Consequences of Relativity Principle (3)

- **Length contraction:** *A moving object becomes shorter* (only in the direction of velocity)

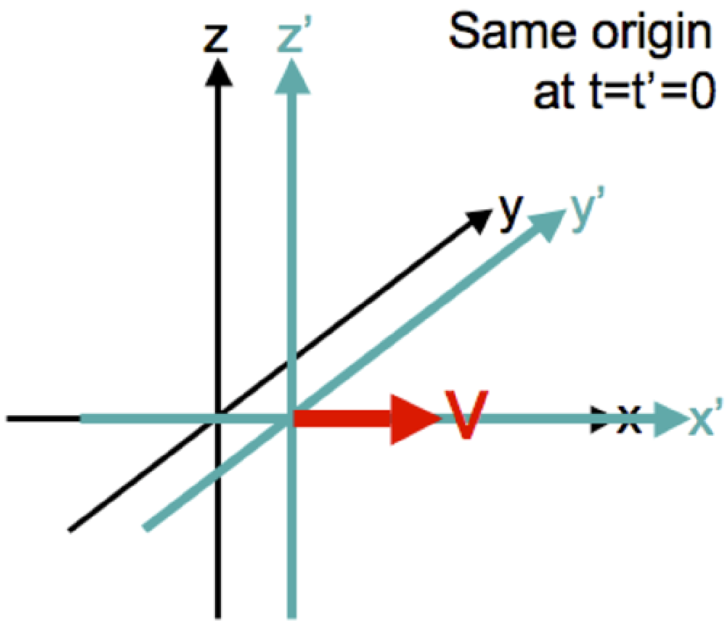


A light ray is reflected by the wall in a time $\Delta t' = 2\Delta x'/c$ for the moving observer, $\Delta t = \Delta t_1 + \Delta t_2$, where $\Delta t_1 = (\Delta x + v\Delta t_1)/c$, $\Delta t_2 = (\Delta x - v\Delta t_2)/c$, for the fixed observer. One finds $\Delta t = 2\gamma^2\Delta x/c$. Finally, since $\Delta t = \gamma\Delta t'$:

$$\Delta x' = \gamma\Delta x = \frac{1}{\sqrt{1 - V^2/c^2}}\Delta x$$

Lorentz Transforms

Transformation rules between an inertial system $\mathcal{S}$ (described by x, y, z, t) and a $\mathcal{S}'$ one (described by x', y', z', t') traveling with speed V along x with respect to S:



$$\begin{cases} x' &= \gamma(x - Vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - \frac{V}{c^2}x) \end{cases}$$

(origins are assumed to coincide at $t = 0$)

In the limit $\gamma = 1$, $V/c << 1$, we recover Galilean transforms. The inverse transform from $\mathcal{S}'$ to $\mathcal{S}$ is obtained by reversing the sign of V.

Velocity addition rules

Let us assume that a particle travels in $\mathcal{S}$ for a distance dx in a time dt: $v_x = dx/dt$. In $\mathcal{S}'$ it travels for a a distance $dx' = \gamma(dx - V dt)$ in a time $dt' = \gamma\left(dt - (V/c^2)dx\right)$. It follows that:

$$v'_x = \frac{dx'}{dt'} = \frac{\gamma(dx - V dt)}{\gamma\left(dt - (V/c^2)dx\right)} = \frac{v_x - V}{1 - v_x V/c^2}$$

The is *Einstein's velocity addition rule*. In the limit $V/c << 1$, it reduces to the usual (Galilean) rule: $v'_x = v_x - V$. If $v_x = c$, also $v'_x = c$. In no case can one exceed the speed of light c.

Einstein's second Relativity Principle can actually be reformulated as follows:

- *For all inertial reference frames, there is a finite limit speed c for physical objects*