

A photograph of a grand hallway with a frescoed ceiling and a red banner. The fresco depicts several figures in classical attire. The banner is red with white Chinese characters. The hallway has ornate architectural details and a decorative railing.

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Quantum Mechanics

欢迎你们来到乌迪内大学

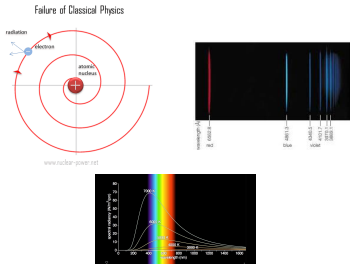
Summer School on Particle Physics



Introduction

In late XIX century, classical mechanics proved to be unable to explain some well known phenomena:

- 1 Black body radiation
- 2 Stability of the atom
- 3 Spectral series of Hydrogen



“On the theory of the Energy distribution law of the Normal Spectrum”

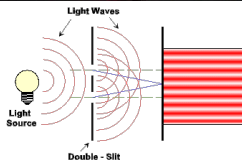
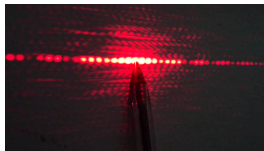
1900, Max Planck

This was the start of a big revolution in Physics, i.e. of **Quantum Mechanics**.



Black Body Spectrum

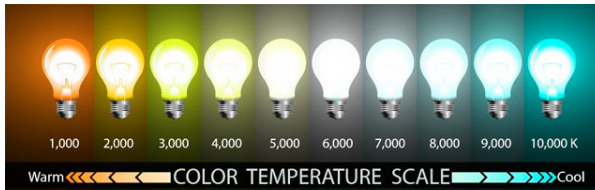
Light behaves as a wave ...



... **but** ...



Black Body Spectrum



- < 1900 K: Candle light
- 2700–3300 K: Warm white
- 2200–3400 K: Incandescent light bulbs
- > 3900 K: Fluorescent lamps
- 4000–5000 K: Neutral white
- 5100–5400 K: Midday sun in summer
- 6500 K: Standard light C (xenon test lamp)
- 5000–6800 K: Daylight white
- > 9000 K: Midday blue sky in December

$$B \longleftrightarrow ? \longleftrightarrow T$$

$$B = \begin{pmatrix} \text{Output power/unit wavelength} \\ \approx \text{Intensity spectrum} \end{pmatrix}$$
$$T = \text{Temperature}$$

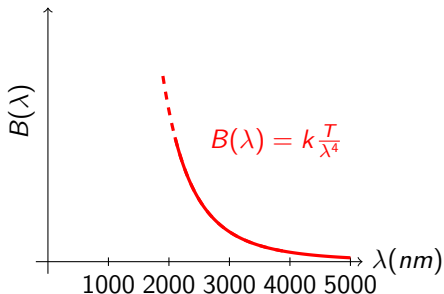


Black Body Spectrum

Assuming light is a wave, classical mechanics predicts:

$$B(\lambda) = \frac{2ck_B T}{\lambda^4} \propto \frac{T}{\lambda^4}$$

“Ultra-violet catastrophe”





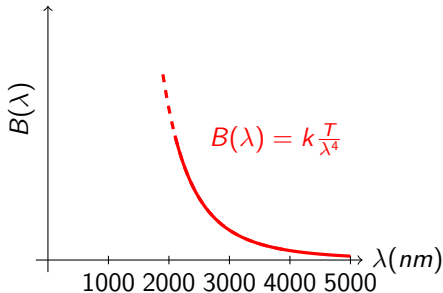
Planck's proposal

Light is emitted in quanta of energy

$$E = h \nu$$

- ν (Greek letter: "nu") is the light's frequency ($\nu = \frac{c}{\lambda}$)
- h is the Planck's constant

$$h = 6.62606957 \times 10^{-34} \text{ Js}$$





Planck's proposal

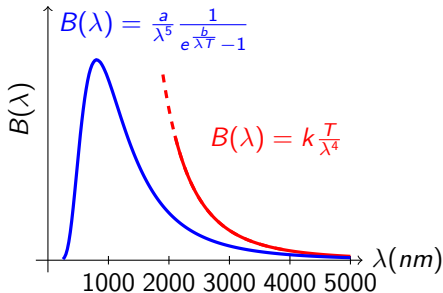
Light is emitted in quanta of energy

$$E = h \nu$$

Predicted spectrum becomes:

$$B(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

- Same behaviour at high λ
- **Dumping factor**
 - heals $\lambda \rightarrow 0$





Planck's proposal

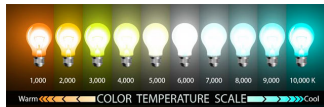
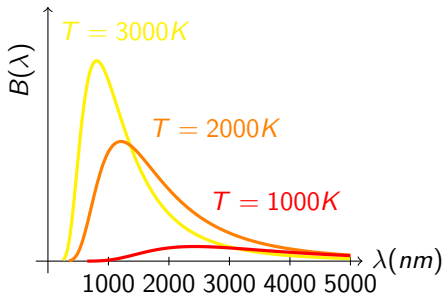
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- Same behaviour at high λ
- **Dumping factor**
 - heals $\lambda \rightarrow 0$
 - agrees with observations



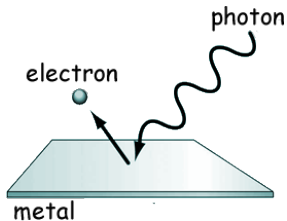


Plank & Einstein

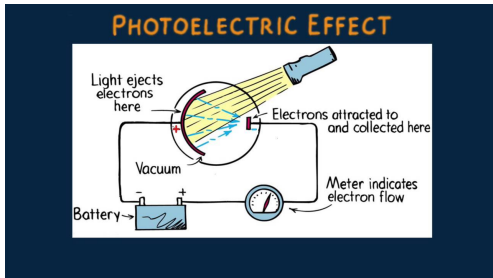
- Plank's proposed $E = h\nu$ as a *mathematical* assumption
- Einstein even further: light *is composed* by quanta, later called "photons"



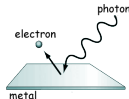
- Photoelectric effect, Nobel Prize 1921



Photoelectric Effect



- Shining ultraviolet light on the metal plate
 - gives flow of negative charge (*Hertz, 1887*)

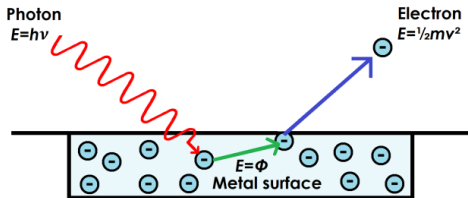


- Brightness \propto current, but ...
- Flow can be stopped with a specific voltage V_0
 - **independent** of the **brightness**
 - depends only on the frequency (*Lenard, 1902*)



Photoelectric Effect

- Light is actually **made up** out of particles “photons” (*Einstein, 1905*)
 - of energy $E = h\nu$
- Kinetic energy of the emitted electrons is the energy left over after the electron has been lifted over the work function barrier





Wave or particle?

So we have seen that light behaves

- as a **wave** (interference, diffraction ...)
- and **also** is made of **particles** (photoelectric effect, black body radiation, ...)

“Wave-particle duality”

...not yet the end of the story!





Atom structure

early 20th century

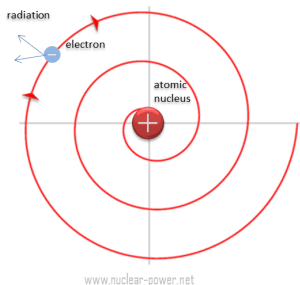
Rutherford, 1911, **atom** seen as

- diffuse cloud of e^-
- dense positively charged nucleus

How can be an atom stable?

- Electrons on circular (accelerated!) orbit
 - Radiates photons (Larmor formula)
- loose energy. Catastrophe!

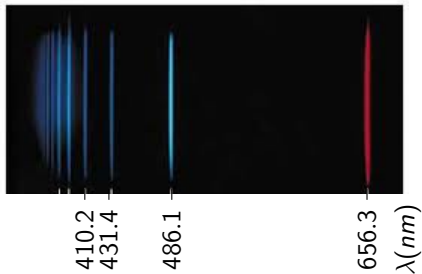
Failure of Classical Physics





Atom structure

Bohr



Hot hydrogen emits light in a set of spectral lines ("*Balmer series*")

⇒ set of lines in the visible spectrum

Solution (Bohr, 1913)

⇒ Electron's **angular momentum L quantized** in units of $\hbar = \frac{h}{2\pi}$



Bohr's radius

in one slide!

So if we start from the classical relation for electron's motion:

$$F_{em} = m_e a, \quad \left(a = \frac{v^2}{r} \right)$$

(circular orbit), and remembering Coulomb's law

$$F_{em} = k_C \frac{Ze^2}{r^2}$$

we get

$$F_{em} = k_C \frac{Ze^2}{r^2} = m_e \frac{v^2}{r} = m_e a$$

And finally using Bohr's assumption $L = m_e v r = n \hbar$:

Quantized Bohr's radii

$$r_n = \frac{n^2 \hbar^2}{Z k_C e^2 m_e}$$



Put together:

- 1 Photons are quanta of light, with $E = pc = h\nu = \frac{hc}{\lambda}$
⇒ light has also a particle behaviour
- 2 Electrons have quantized angular momenta $L = n\hbar$

But then also **particles** can behave as waves: “**matter-waves**” with wavelength

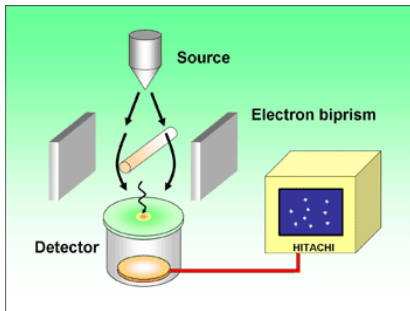
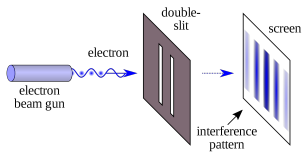
De Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Davisson and Germer **measured** $\lambda_e = \frac{h}{m_e v}$ using diffraction.



Electrons as waves



Further reading and bibliography [here](#)



Wave-like nature of light/particles described by

$$\psi(t, \mathbf{x}) \neq \psi^*(t, \mathbf{x})$$

Probability:

$$P \propto |\psi(t, \mathbf{x})|^2, \quad \int dx dt |\psi(t, \mathbf{x})|^2 = 1$$



Uncertainty principle

Heisenberg

Only one of the “position” or “momentum” can be measured accurately at a single moment within the instrumental limit.

It is impossible to measure both the position and momentum simultaneously with unlimited accuracy.

..or

$\Delta x \rightarrow$ uncertainty in position

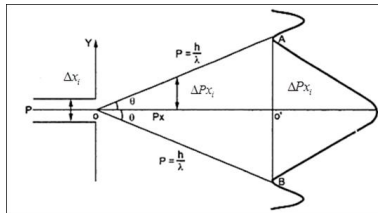
$\Delta p_x \rightarrow$ **uncertainty in momentum**

then

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \quad \left(\hbar = \frac{h}{2\pi} \right)$$

The product $\Delta x \Delta p_x$ of an object is greater than or equal to 2

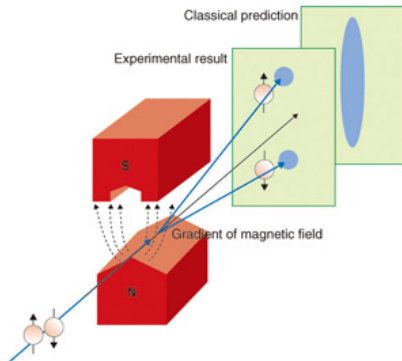
(equivalent to state that interference is intrinsic)





Stern & Gerlach, 1922:

- Electrons are deviated by a magnetic field, in different directions
- Two “types” of electrons \Leftrightarrow
spin $s = \pm \frac{1}{2}$



- Fermions: half-integer spin (electrons, ...)
- Bosons: integer spin (photons, ...)

Scalars have zero spin (Higgs)