Channeling physics overview

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Overview

Assumptions of the presently dominating
dechennling model

Essential simulation issues

Single and multiple scattering interrelation

To the "exact" formulae

Real origin and application scope of the
dechanneling length notion

The book

V.M. Biryukov, Y.A. Chesnokov, V.I. Kotov,
Crystal Channeling and Its Application at High-Energy Accelerators (*Springer, Berlin,* 2010)

proclaims a (too) simple theory,

but also honestly reveals its problems.

Presently dominating model boils down to:

instant nuclear dechanneling

(*nuclear corridor*)

universal electron dechanneling length,

adopted from the conventional energy loss formula

“Channeling means dechanneling”

In channeling theory

the only considerable difficulties

arise in the specific treating of

incoherent scattering processes

responsible for dechanneling

Conventional formula for nuclear multiple scattering

$$\langle \Theta^2 \rangle = 2\pi n l \left(\frac{2\alpha Z z}{pv} \right)^2 ln \left(\frac{\theta_{max}}{\theta_{min}} \right)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2e^2 Z z}{pv} \right)^2 \frac{1}{(\theta^2 + \theta_{min}^2)^2}$$

$$\theta_{min}^{Quant} = \frac{\hbar}{p R_{at}}$$

$$\theta_{max}^{Quant} = \frac{\hbar}{p R_{nucl}}$$

both θ_{min} and θ_{max}

should be redefined in crystals!

Electron scattering and ionization loss formula *H. Bethe*

$$T \equiv E_{e^-} = \frac{p_\perp^2}{2m} \qquad p_\perp = \theta / E_p \qquad \frac{d\sigma}{dT} = \frac{C}{T^2} \qquad \langle \theta^2 \rangle \propto \int T \frac{d\sigma}{dT} dT \propto \ln\left(\frac{T_{max}}{T_{min}}\right)$$

$$\left\langle \frac{(\delta\theta_s)^2}{\delta z} \right\rangle = \frac{m_e}{2p^2} \left(\frac{\delta E}{\delta z}\right)_{am} \frac{n_e(x)}{n_{am}}$$

$$\left(\frac{\delta E}{\delta z}\right)_{am} = 4\pi N_A r_e^2 m_e c^2 \frac{Z}{A} \rho \left(\frac{Z_i}{\beta}\right)^2 \left(\ln \frac{2m_e c^2 \beta^2 \gamma^2}{I} - \beta^2 - \frac{\delta}{2}\right)$$

T_{max}

$$\ln I = \frac{2m}{Z} \sum_n \omega_{n0} |x_{n0}|^2 \ln \omega_{n0}$$

Both upper and lower limits cause problems!

Multiple scattering approach is invalid at any cutoff (from Biryukov)

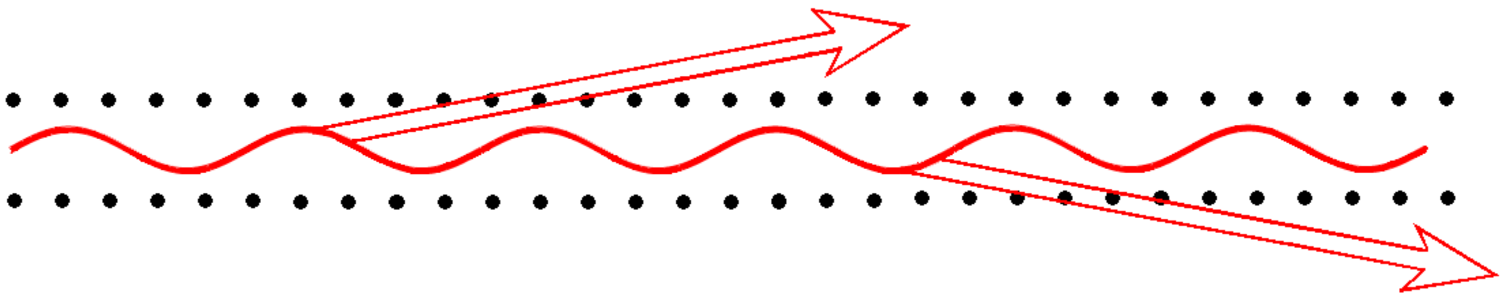
$$\frac{d^2N}{dT\,dz} = \frac{D\rho_e(x)}{2\beta^2}\frac{1}{T^2} \, .$$

$$T_{\max} = \frac{2m_e c^2\beta^2\gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2} \approx 2m_e c^2\beta^2\gamma^2 \qquad (1.52)$$

$$\theta^2_{\mathrm{rms}} = \frac{2m_e}{p^2}\left\langle \sum_i T_i(1 - T_i/T_{\max}) \right\rangle \, . \qquad (1.56)$$

24 1. Channeling Phenomenon

Although θ^2_{rms} depends on $T_{\max}$ via $\ln(T_{\max}/I)$, it will be shown below that removal of the energy transfers of catastrophic collisions from (1.56) reduces the diffusion coefficient by a factor of 2–3.



Multiple scattering approach is invalid at any cutoff (from Biryukov)

Although $\theta^2_{\rm rms}$ depends on $T_{\rm max}$ via $\ln(T_{\rm max}/I)$, it will be shown below that removal of the energy transfers of catastrophic collisions from (1.56) reduces the diffusion coefficient by a factor of 2–3.

Nevertheless, some authors [28, 34, 35] use the maximal possible value for $T_{\rm max}$, given by (1.52). Others [43–44] try a 'cutoff', defining $T_{\rm max}$ as the energy transfer T_c causing the angular kick of the order of θ_c (or even much smaller [45]), because at higher T the diffusion approach is certainly invalid.

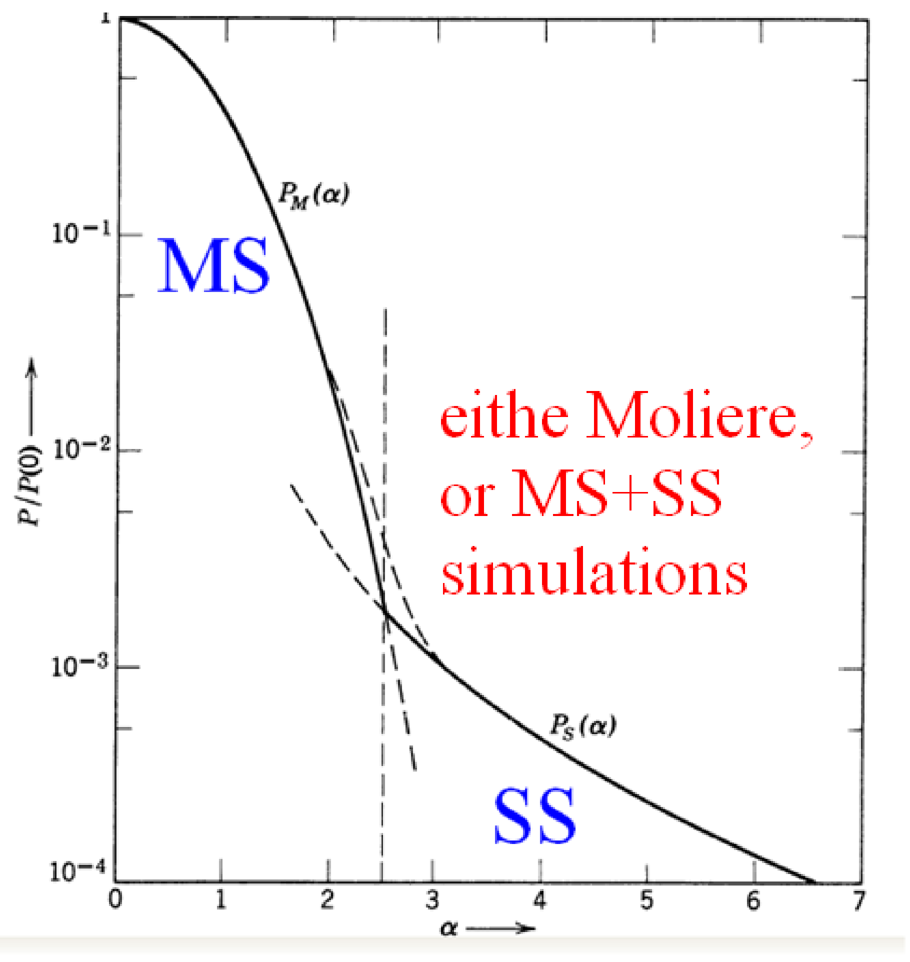
The weak points of both choices may be clarified with a numerical exam-

28. Ohtsuki Y.-H. and Nitta H. in *Relativistic Channeling* R.A. Carrigan, Jr. and J. Ellison, eds. (Plenum, NY, 1987) p.59

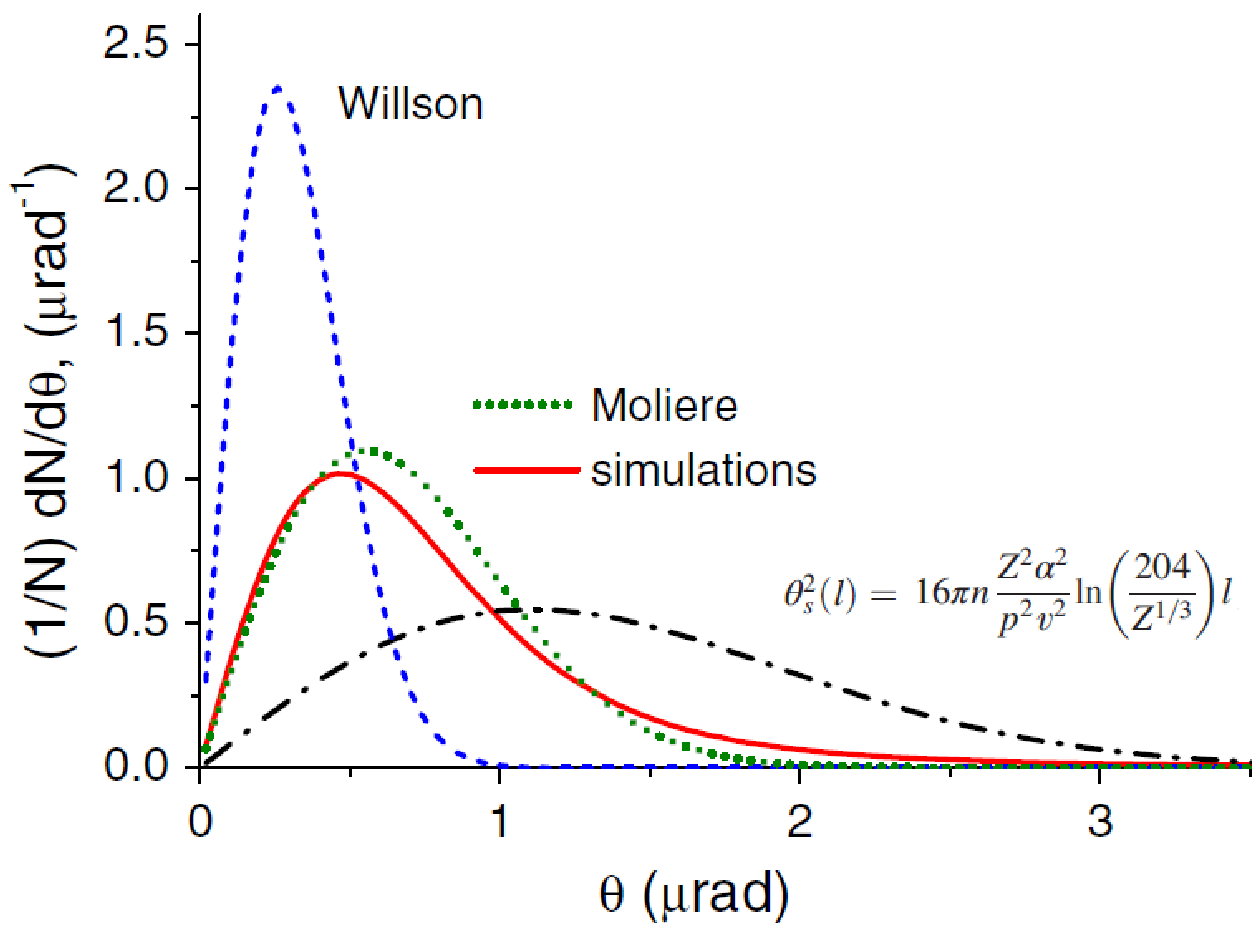
34. Taratin A.M., Vorobiev S.A. *Nucl. Instr. and Meth. B* **47**, 247 (1990)
35. Kudryashov N.A., Petrovsky C.V., Strikhanov M.N. *Yad. Fiz.* **48**, 666 (1988); *Zh. Tekh. Fiz.* **59**, 68 (1989)

43. Taratin A.M., Vorobiev S.A. *Zh. Tekh. Fiz.* **55**, 1598 (1985)
44. Taratin A.M. et al. *Phys. Stat. Sol.(b)* **100**, 273 (1980)
45. Taratin A.M. and Vorobiev S.A., *Phys. Stat. Sol.(b)* **107**, 521 (1981)

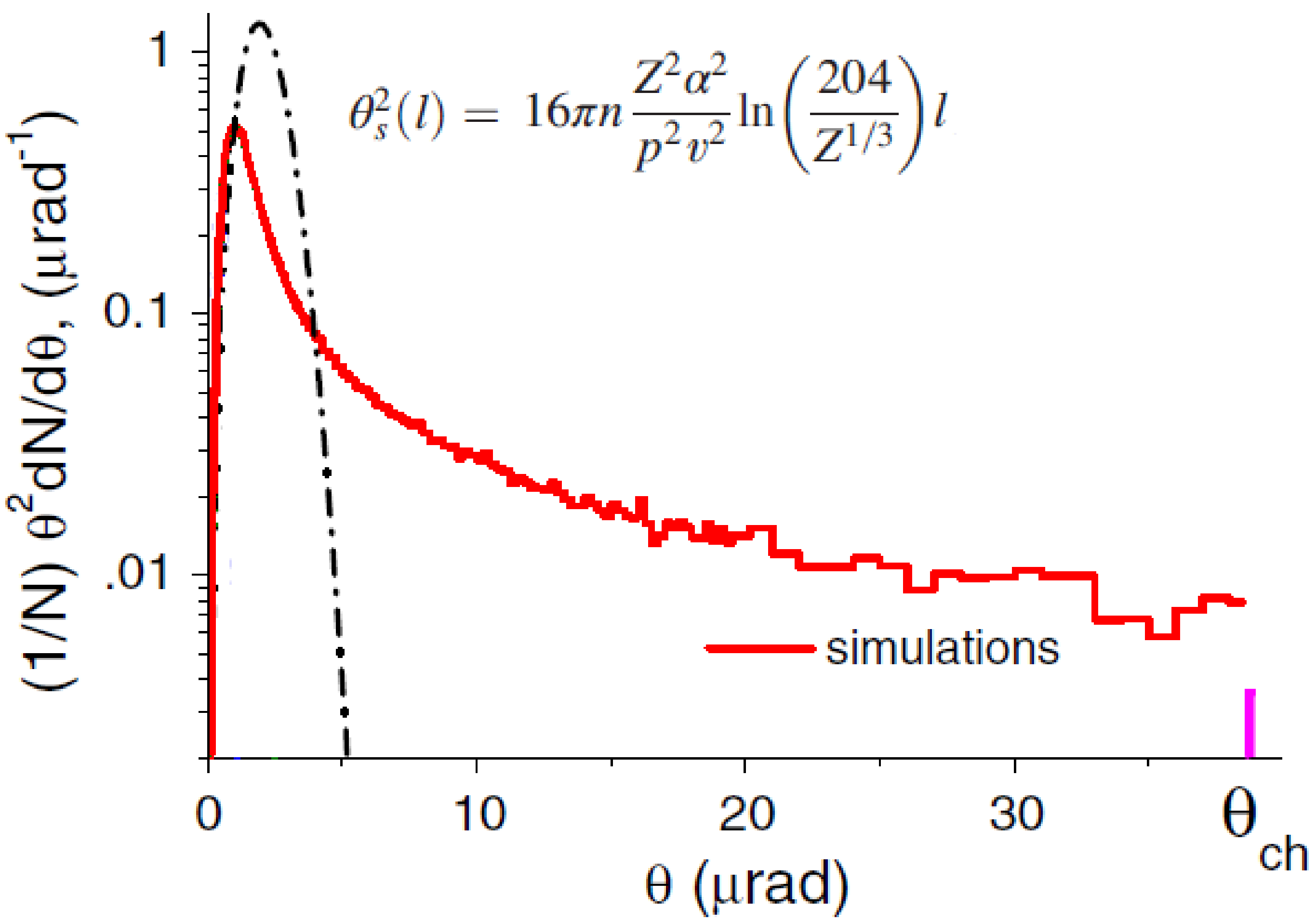
Multiple **S**cattering, **S**ingle **S**catting and **intermediate angle region** have to be adequately described



from J D Jackson

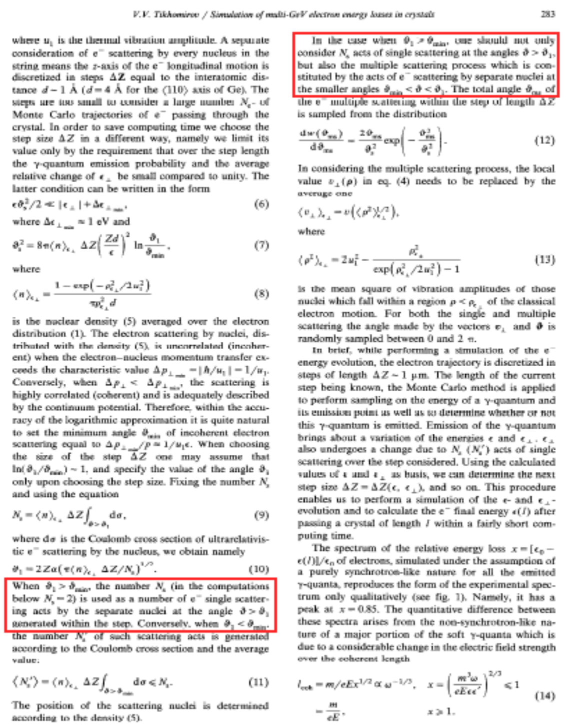


Although θ^2_{rms} depends on $T_{\max}$ via $\ln(T_{\max}/I)$, it will be shown below that removal of the energy transfers of catastrophic collisions from (1.56) reduces the diffusion coefficient by a factor of 2–3.



Gaussian with the notorious RMS angle
is completely unedquate at small lengths

Our solution – to simulate **jointly** large angle SS with small angle MS



where u_1 is the thermal vibration amplitude. A separate consideration of e^- scattering by every nucleus in the string means the z-axis of the e^- longitudinal motion is discretized in steps ΔZ equal to the interatomic distance $d \sim 1$ Å ($d = 4$ Å for the ⟨110⟩ axis of Ge). The steps are too small to consider a large number N_{e^-} of Monte Carlo trajectories of e^- passing through the crystal. In order to save computing time we choose the step size ΔZ in a different way, namely we limit its value only by the requirement that over the step length the γ-quantum emission probability and the average relative change of $\epsilon_\perp$ be small compared to unity. The latter condition can be written in the form

$$\epsilon \vartheta_s^2/2 \ll |\epsilon_\perp| + \Delta\epsilon_{\perp\min}, \quad (6)$$

where $\Delta\epsilon_{\perp\min} \approx 1$ eV and

$$\vartheta_s^2 = 8\pi \langle n \rangle_{\epsilon_\perp} \Delta Z \left(\frac{Zd}{\epsilon}\right)^2 \ln\frac{\vartheta_1}{\vartheta_{\min}}, \quad (7)$$

where

$$\langle n \rangle_{\epsilon_\perp} = \frac{1 - \exp(-\rho_{\epsilon_\perp}^2/2u_1^2)}{\pi\rho_{\epsilon_\perp}^2 d} \quad (8)$$

is the nuclear density (5) averaged over the electron distribution (1). The electron scattering by nuclei, distributed with the density (5), is uncorrelated (incoherent) when the electron–nucleus momentum transfer exceeds the characteristic value $\Delta p_{\perp\min} = |\hbar/u_1| = 1/u_1$. Conversely, when $\Delta p_\perp < \Delta p_{\perp\min}$, the scattering is highly correlated (coherent) and is adequately described by the continuum potential. Therefore, within the accuracy of the logarithmic approximation it is quite natural to set the minimum angle $\vartheta_{\min}$ of incoherent electron scattering equal to $\Delta p_{\perp\min}/p \approx 1/u_1\epsilon$. When choosing the size of the step ΔZ one may assume that $\ln(\vartheta_1/\vartheta_{\min}) \sim 1$, and specify the value of the angle ϑ_1 only upon choosing the step size. Fixing the number N_s and using the equation

$$N_s = \langle n \rangle_{\epsilon_\perp} \Delta Z \int_{\vartheta > \vartheta_1} d\sigma, \quad (9)$$

where $d\sigma$ is the Coulomb cross section of ultrarelativistic e^- scattering by the nucleus, we obtain namely

$$\vartheta_1 = 2Z\alpha(\pi \langle n \rangle_{\epsilon_\perp} \Delta Z/N_s)^{1/2}. \quad (10)$$

When $\vartheta_1 > \vartheta_{\min}$, the number N_s (in the computations below $N_s = 2$) is used as a number of e^- single scattering acts by the separate nuclei at the angle $\vartheta > \vartheta_1$ generated within the step. Conversely, when $\vartheta_1 < \vartheta_{\min}$, the number N_s' of such scattering acts is generated according to the Coulomb cross section and the average value:

$$\langle N_s' \rangle = \langle n \rangle_{\epsilon_\perp} \Delta Z \int_{\vartheta > \vartheta_{\min}} d\sigma \leqslant N_s. \quad (11)$$

The position of the scattering nuclei is determined according to the density (5).

In the case when $\vartheta_1 > \vartheta_{\min}$, one should not only consider N_s acts of single scattering at the angles $\vartheta > \vartheta_1$, but also the multiple scattering process which is constituted by the acts of e^- scattering by separate nuclei at the smaller angles $\vartheta_{\min} < \vartheta < \vartheta_1$. The total angle ϑ_{ms} of the e^- multiple scattering within the step of length ΔZ is sampled from the distribution

$$\frac{dw(\vartheta_{ms})}{d\vartheta_{ms}} = \frac{2\vartheta_{ms}}{\vartheta_s^2} \exp\left(-\frac{\vartheta_{ms}^2}{\vartheta_s^2}\right). \quad (12)$$

In considering the multiple scattering process, the local value $v_\perp(\rho)$ in eq. (4) needs to be replaced by the average one

$$\langle v_\perp \rangle_{\epsilon_\perp} = v(\langle \rho^2 \rangle_{\epsilon_\perp}^{1/2}),$$

where

$$\langle \rho^2 \rangle_{\epsilon_\perp} = 2u_1^2 - \frac{\rho_{\epsilon_\perp}^2}{\exp(\rho_{\epsilon_\perp}^2/2u_1^2) - 1} \quad (13)$$

is the mean square of vibration amplitudes of those nuclei which fall within a region $\rho < \rho_{\epsilon_\perp}$ of the classical electron motion. For both the single and multiple scattering the angle made by the vectors $v_\perp$ and $\boldsymbol{\vartheta}$ is randomly sampled between 0 and 2π.

In brief, while performing a simulation of the e^- energy evolution, the electron trajectory is discretized in steps of length $\Delta Z \sim 1$ μm. The length of the current step being known, the Monte Carlo method is applied to perform sampling on the energy of a γ-quantum and its emission point as well as to determine whether or not this γ-quantum is emitted. Emission of the γ-quantum brings about a variation of the energies ϵ and $\epsilon_\perp$. $\epsilon_\perp$ also undergoes a change due to N_s (N_s') acts of single scattering over the step considered. Using the calculated values of ϵ and $\epsilon_\perp$ as basis, we can determine the next step size $\Delta Z = \Delta Z(\epsilon, \epsilon_\perp)$, and so on. This procedure enables us to perform a simulation of the ϵ- and $\epsilon_\perp$-evolution and to calculate the e^- final energy $\epsilon(l)$ after passing a crystal of length l within a fairly short computing time.

The spectrum of the relative energy loss $x = [\epsilon_0 - \epsilon(l)]/\epsilon_0$ of electrons, simulated under the assumption of a purely synchrotron-like nature for all the emitted γ-quanta, reproduces the form of the experimental spectrum only qualitatively (see fig. 1). Namely, it has a peak at $x = 0.85$. The quantitative difference between these spectra arises from the non-synchrotron-like nature of a major portion of the soft γ-quanta which is due to a considerable change in the electric field strength over the coherent length

$$l_{coh} = m/eE\chi^{1/2} \propto \omega^{-1/3}, \quad \chi = \left(\frac{m^3\omega}{eE\epsilon\epsilon'}\right)^{2/3} \leqslant 1$$
$$= \frac{m}{eE}, \quad \chi \geqslant 1. \quad (14)$$

In the case when $\vartheta_1 > \vartheta_{\min}$, one should not only consider N_s acts of single scattering at the angles $\vartheta > \vartheta_1$, but also the multiple scattering process which is constituted by the acts of e^- scattering by separate nuclei at the smaller angles $\vartheta_{\min} < \vartheta < \vartheta_1$.

When $\vartheta_1 > \vartheta_{\min}$, the number N_s (in the computations below $N_s = 2$) is used as a number of e^- single scattering acts by the separate nuclei at the angle $\vartheta > \vartheta_1$ generated within the step.

V. V. Tikhomirov. *Simulation of Multi-GeV electron energy losses in crystals*. *Nucl. Instrum. and Meth*. **1989**. V. B36. P. 282-285.

Enjoying the strength of logarithmic approximation in combining the **overlapping** MS and SS processes

Threshold transverse energy for electron scattering

$$\Delta \varepsilon_{\perp}^{thr} = \frac{p_{\perp}^{2}}{2\varepsilon} = \frac{p_{\perp}^{2}}{2m}\frac{m}{\varepsilon} = \frac{m}{\varepsilon}T < \frac{m}{\varepsilon}T_{\min} \sim 10\frac{m}{\varepsilon}I \sim 10^{-3}eV(\varepsilon = 1\,TeV)$$

Threshold transverse energy for nuclear scattering

$$\Delta\varepsilon_{\perp}^{thr} \le \frac{(\theta_{th}p)^{2}}{2\varepsilon} = \frac{(3\hbar/u_{1})^{2}}{2\varepsilon} \sim 10^{-3}eV(\varepsilon = 1\,TeV)$$

The range of possible momentum transfers can be divided into two parts. The two parts overlap with $\Delta\varepsilon_{\perp}^{thr} \ll \varepsilon_{\perp} \ll V_{max}$; this allows an exact joining of the results for each part separately.

Nuclear dechanneling

is most important

at "SELDOM energies" !

Combining MS and SS processes

$$\ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right) = \ln\left(\frac{\theta_{th}}{\theta_{\min}}\right) + \ln\left(\frac{\theta_{\max}}{\theta_{th}}\right)$$

MS, $\langle\theta^2\rangle$

SS, dσ

How to calculate $\langle\theta^2\rangle$?

$\langle\theta^2\rangle$ can be **negative** !

Nuclear dechanneling

is most important

at TeV energies

How to calculate $\left\langle \theta^2 \right\rangle$

Incoherent scattering reduction in crystals

Eur. Phys. J. C manuscript No.
(will be inserted by the editor)

Broad angular anisotropy of multiple scattering in a Si crystal

A. Mazzolari [1], A. Sytov [a,1,2], L. Bandiera [1], G. Germogli [1,2], M. Romagnoni [1,2], E. Bagli [1], V. Guidi [1,2], V. V. Tikhomirov [3], D. De Salvador [4,5], S. Carturan [4,5], C. Durigello [4,5], G. Maggioni [4,5], M. Campostrini [5], A. Berra [6,7], V. Mascagna [6,7], M. Prest [6,7], E. Vallazza [7], W. Lauth [8], P. Klag [8], M. Tamisari [1,9]

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Reduction of incoherent scattering and characterizing r.m.s. angle:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{coh}}{d\Omega} + \frac{d\sigma_{inc}}{d\Omega}$$

$$\frac{d\sigma_{coh}}{d\Omega} = exp(-q^2u_1^2)\sum_{j=1}^{N}|e^{i\mathbf{q}\mathbf{r}_{j0}}|^2\frac{d\sigma_{at}}{d\Omega}$$

$$\frac{d\sigma_{inc}}{d\Omega} = \frac{d\sigma_{am}}{d\Omega} - \frac{d\sigma_1}{d\Omega}$$

$$\frac{d\sigma_1}{d\Omega} = exp(-q^2u_1^2)\frac{d\sigma_{am}}{d\Omega}$$

$$d\sigma \xrightarrow{d\sigma_{coh}=0} d\sigma_{am} - d\sigma_1$$

Angular measure of incoherent scattering reduction

$$\langle \vartheta_{x'}^2 \rangle_1 = n_{at} l \int \vartheta_{x'}^2 \frac{d\sigma_1}{d\Omega} d\Omega = n_{at} l \int \frac{q_{x'}^2}{p^2} \frac{d\sigma_1}{d\Omega} d\Omega$$

$$= \frac{\theta_1^2}{2} \left[\left(1 + \frac{u_1^2}{R^2}\right) e^{\frac{u_1^2}{R^2}} E_1 \left(\frac{u_1^2}{R^2}\right) - 1 \right]$$

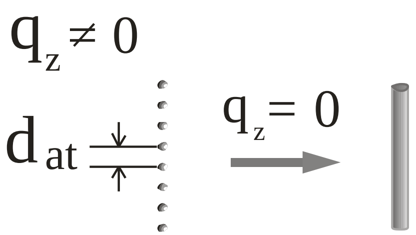
Incoherent scattering reduction

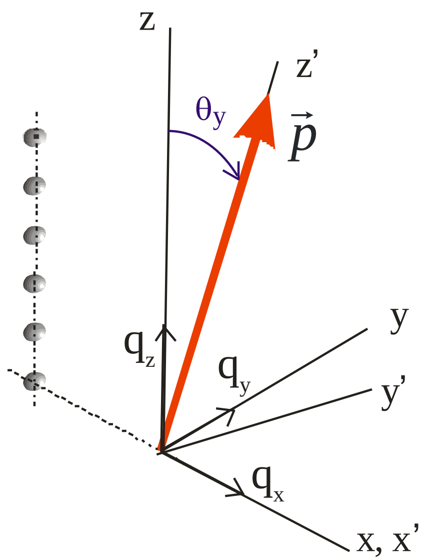
is ubiquitous

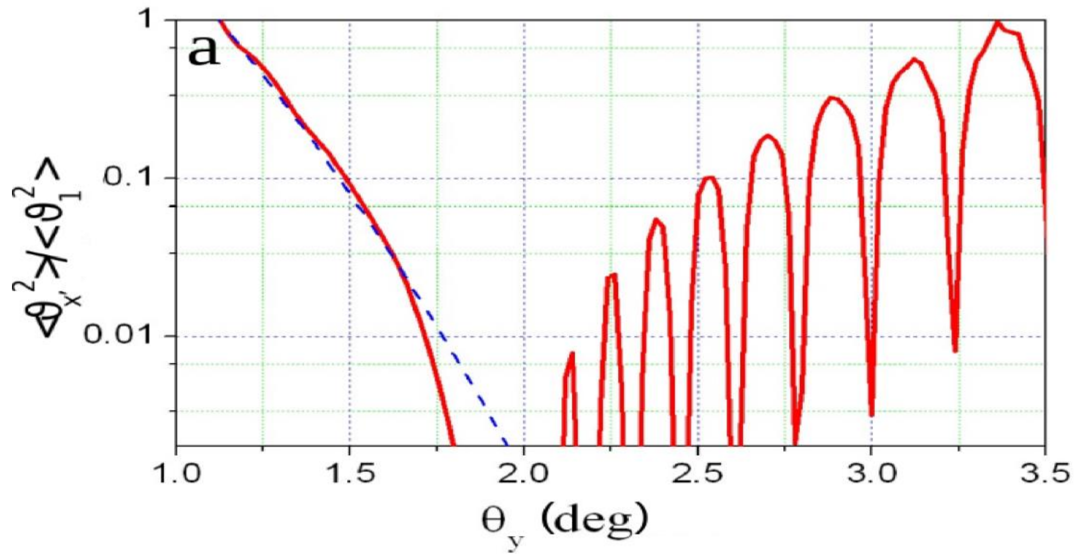
but difficult to extract

$100 \text{ MeV} - 7 \cdot 10^{12} \text{ eV}$

Coherent scattering by separate **atoms** is suppres-sed at small angle incidence w.r.t. crystal **axes**







$$\frac{10\,\theta_{ch}^{pl}\, d_{ax}}{\pi u_1} \approx 18\ mrad < \theta_y < \frac{\pi u_1}{d_{at}} \approx 43\ mrad$$

Cross section of the coherent scattering by **strings**

$$\left\{ \begin{array}{c} \langle \vartheta_{x'}^2 \rangle_{coh} \\ \langle \vartheta_{y'}^2 \rangle_{coh} \end{array} \right\} = \int \left\{ \begin{array}{c} \vartheta_{x'}^2 \\ \vartheta_{y'}^2 \end{array} \right\} \frac{d\sigma_{coh}}{d\Omega} d\Omega = \frac{1}{p^2} \int \left\{ \begin{array}{c} q_{x'}^2 \\ q_{y'}^2 \end{array} \right\} \frac{d\sigma_{coh}}{d\Omega} d\Omega$$

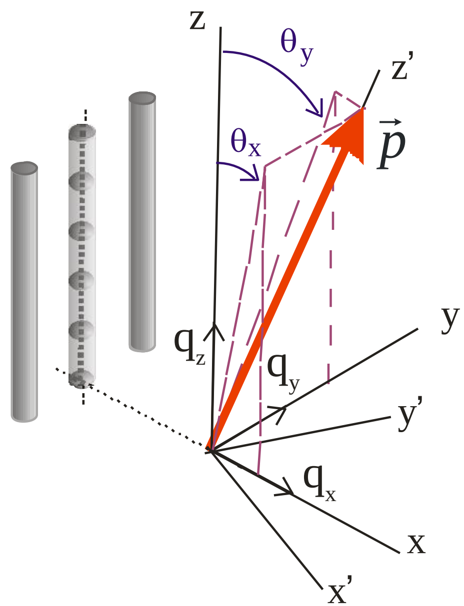
$$= \left(\frac{2\alpha Z}{\theta_0 p v} \right)^2 \left(\frac{2\pi}{d_{at}} \right)^3 \frac{n_{at}}{8} l \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \sum_{n_3=-\infty}^{\infty} |S|^2 \left\{ \begin{array}{c} (\theta_{01} q_{2n_2} - \theta_{02} q_{1n_1})^2 \\ q_{3n_3}^2 \end{array} \right\}$$

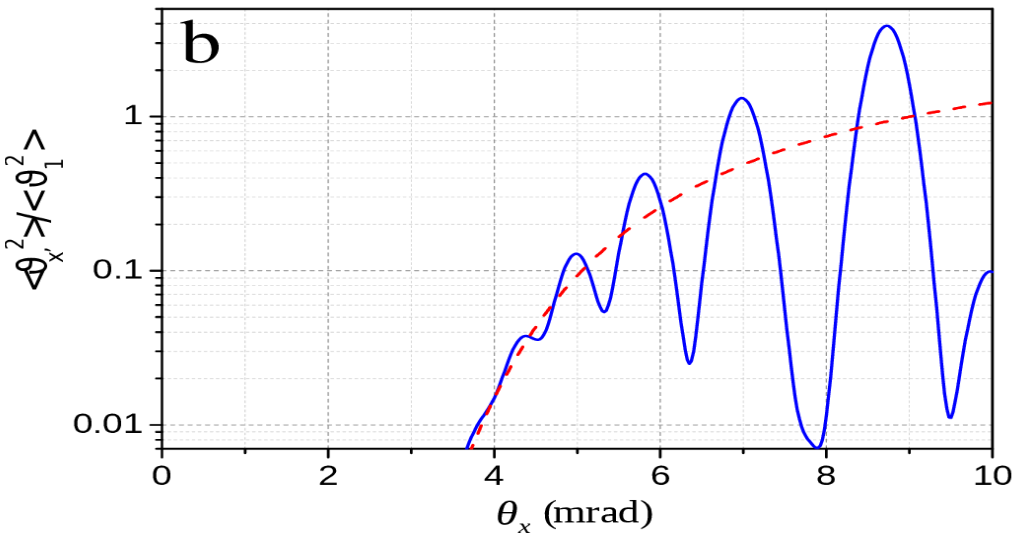
$$\times \frac{q_{n_1 n_2}}{q_{n_1 n_2 n_3}} \frac{\underline{\exp\left(-q_{n_1 n_2 n_3}^2 u_1^2\right)}}{\left(q_{n_1 n_2 n_3}^2 + R^{-2}\right)^2} \frac{1}{\sqrt{2\pi}\delta q_{n_1 n_2 n_3}} \exp\left[-\underline{(q_{3n_3} + q_{1n_1}\theta_{01} + q_{2n_2}\theta_{02})^2 / 2\delta^2 q_{n_1 n_2}^2}\right]$$

$$q_{n_1 n_2}^2 = q_{1n_1}^2 + q_{2n_2}^2 \,, \quad q_{n_1 n_2 n_3}^2 = q_{n_1 n_2}^2 + q_{3n_3}^2 .$$

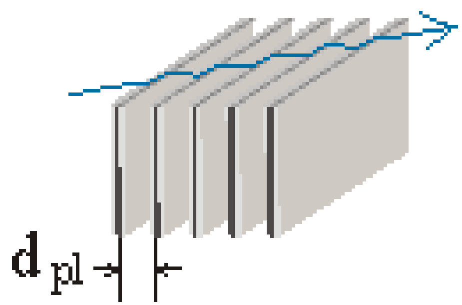
$$\frac{10\theta_{ch}^{pl} d_{ax}}{\pi u_1} \approx 18 \ mrad < \theta_y < \frac{\pi u_1}{d_{at}} \approx 43 \ mrad$$

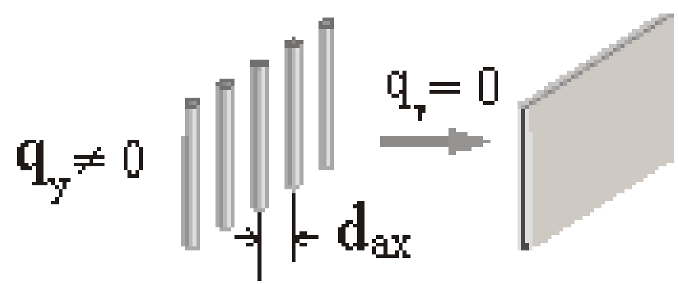
Coherent scattering by separate **strings** is suppressed at small angle incidence w.r.t. crystal **planes**





$$10\,\theta_{ch}^{pl} \approx 2\ mrad < \theta_x < \frac{\pi u_1}{d_{ax}}\theta_y \approx 4.3\ mrad$$





Cross section of the coherent scattering by **planes**

$$\langle \vartheta_{x'}^2 \rangle_{coh} = \int \vartheta_{x'}^2 \frac{d\sigma_{coh}}{d\Omega}\, d\Omega = \frac{1}{p^2} \int q_{x'}^2 \frac{d\sigma_{coh}}{d\Omega}\, d\Omega$$

$$= \left(\frac{2\alpha Z}{\theta_0 p v}\right)^2 \frac{2\pi}{d_{at}} \frac{2\pi}{d_{ax}} \frac{2\pi}{d_{pl}} n_{at} l \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} (\theta_{0x} q_{ym} - \theta_{0y} q_{xk})^2$$

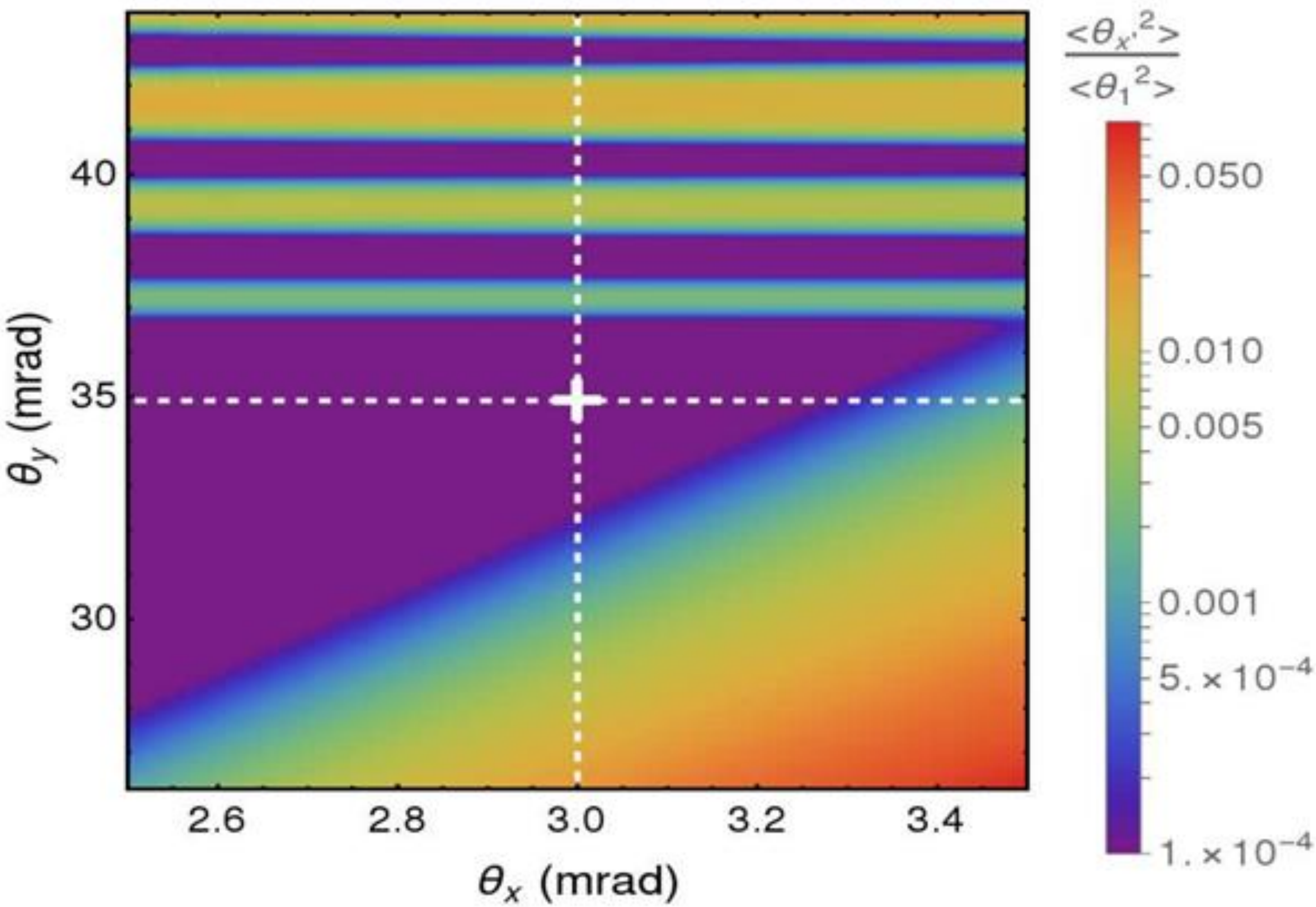
$$\times \frac{\exp\left(-q_{mk}^2 u_1^2\right)}{(q_{mk}^2 + R^{-2})^2} \frac{1}{\sqrt{2\pi}\delta q_{mk}} \exp\left[-(q_{xk}\theta_{0x} + q_{ym}\theta_{0y})^2 / 2\delta^2 q_{mk}^2\right],$$

$$q_{mk}^2 = q_{xk}^2 + q_{ym}^2,\ d_{pl} = d_{ax} = d' = d_{at}/2\sqrt{2},$$

$$q_{xk} = 2\pi k / d_{pl} \text{ and } q_{ym} = 2\pi m / d_{ax}, \quad k, m = 0, \pm 1, \pm 2, \ldots$$

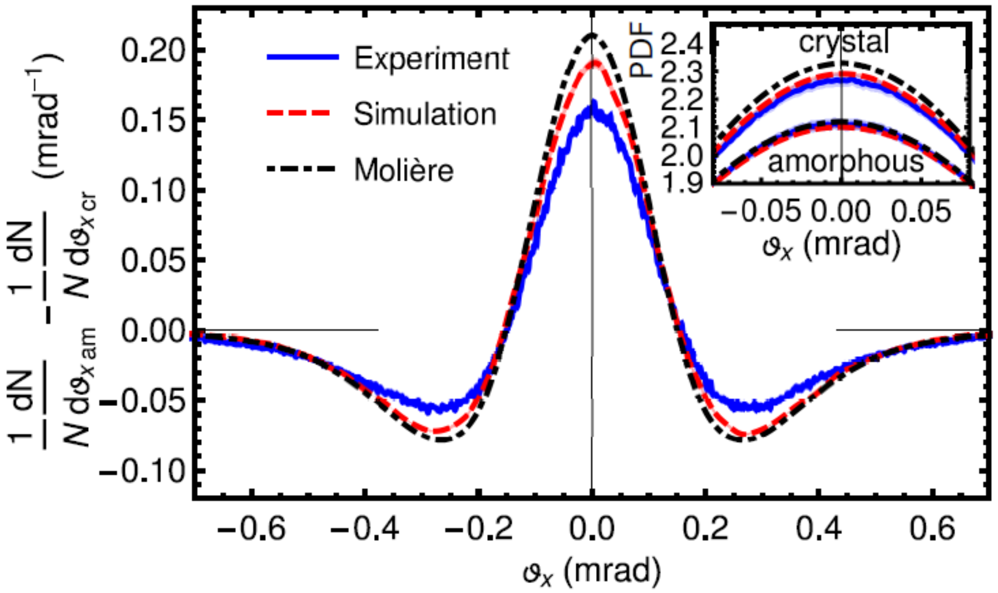
$$10\theta_{ch}^{pl} \approx 2\ mrad < \theta_x < \frac{\pi u_1}{d_{ax}} \theta_y \approx 4.3\ mrad$$

2D plot of incoherent scattering deduction and optimal incidence direction choice



Direct observation of the incoherent scattering reduction

$$d\sigma = d\sigma_{at}\left[1-\exp\left(\frac{-q^2\overline{u^2}}{\hbar^2}\right)\right]$$



$$\frac{\sigma_{inc}}{\sigma_{am}} = xe^{x}E_1(x) \sim 0.2 \div 0.4$$

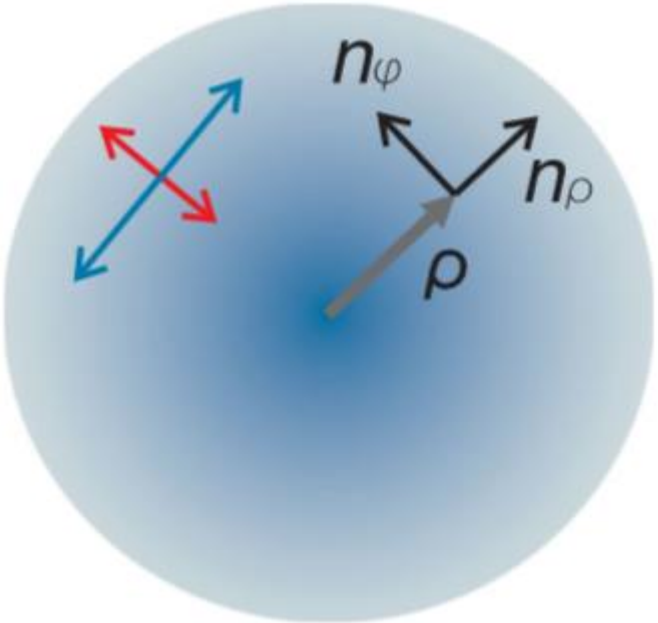
$$x = \left(\frac{u_1}{R}\right)^2$$

The difference of the angular distributions of deflected beam by a silicon crystal, aligned at θ_x = 3 mrad, θ_y = 34.9 mrad (2◦) and amorphous membranes for experiment (solid).

$$d\sigma = d\sigma_{at}\left[1-\exp\left(\frac{-q^2\overline{u^2}}{\hbar^2}\right)\right]$$ is applicable only in plane wave (straight-line) approximation, relevant to the high-above-barrier motion

For both near-above-barrier and channeling motion new approach is needed

Local theory of incoherent scattering in crystals



V. V. Tikhomirov. *Quantum features of high energy particle incoherent scattering in crystals.* Phys. Rev. Acc. and Beams. 22, 054501 (2019).

High energy scattering is quantum in nature

$$\frac{d\sigma}{d\Omega} = \left(\frac{2e^2 Zz}{pv}\right)^2 \frac{1}{(\theta^2 + \theta_{min}^2)^2}$$

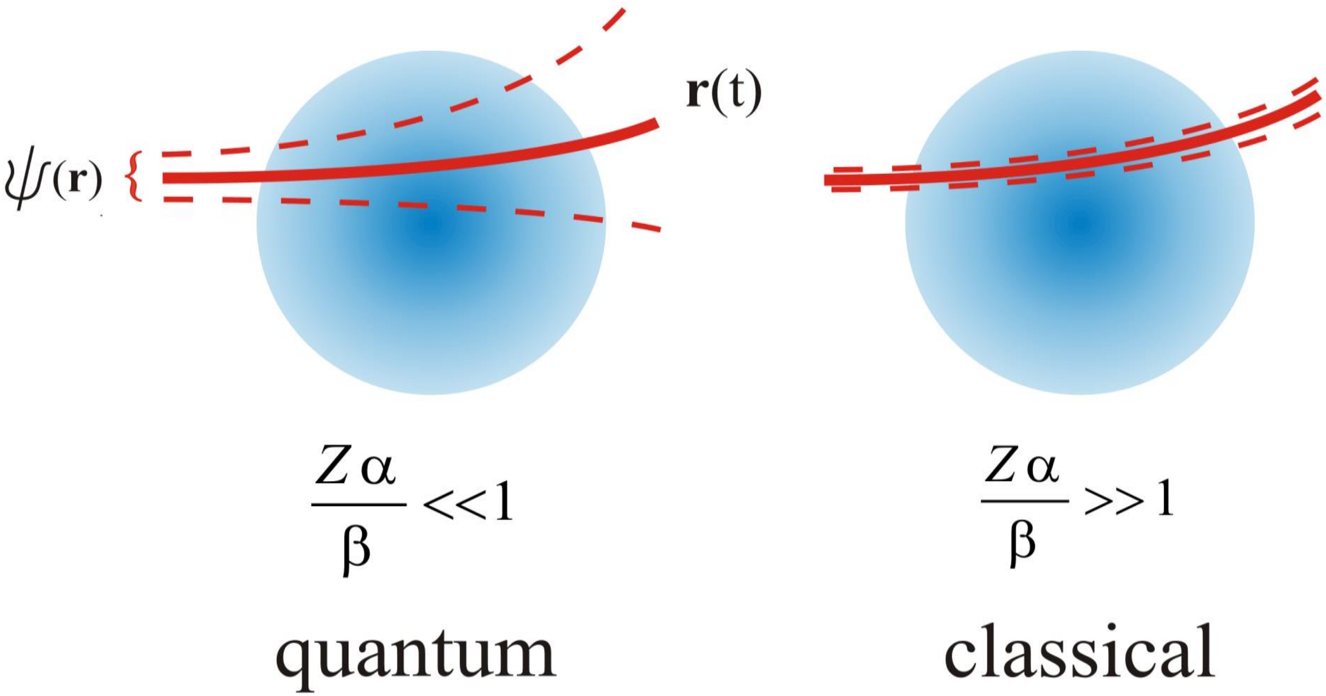
$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \pi \left(\frac{2e^2 Zz}{pv}\right)^2 \frac{1}{\theta_{min}^2}$$

$$\theta_{min}^{Class} = \frac{e^2 Zz}{pvR_{at}}$$

$$\theta_{min}^{Quant} = \frac{\hbar}{pR_{at}}$$

$$\frac{\sigma^{Class}}{\sigma^{Quant}} = \left(\frac{\theta_{min}^{Quant}}{\theta_{min}^{Class}}\right)^2 = \left(\frac{\beta}{\alpha Zz}\right)^2 \sim 100 \text{ !!!} \quad (for\ p \to Si, \beta \to 1)$$

Classical *vs* quantum scattering



V. V. Tikhomirov. *Quantum features of high energy particle incoherent scattering in crystals.* Phys. Rev. Acc. and Beams. 22, 054501 (2019).

Quantum *vs* classical cross sections

Relativistic analog to

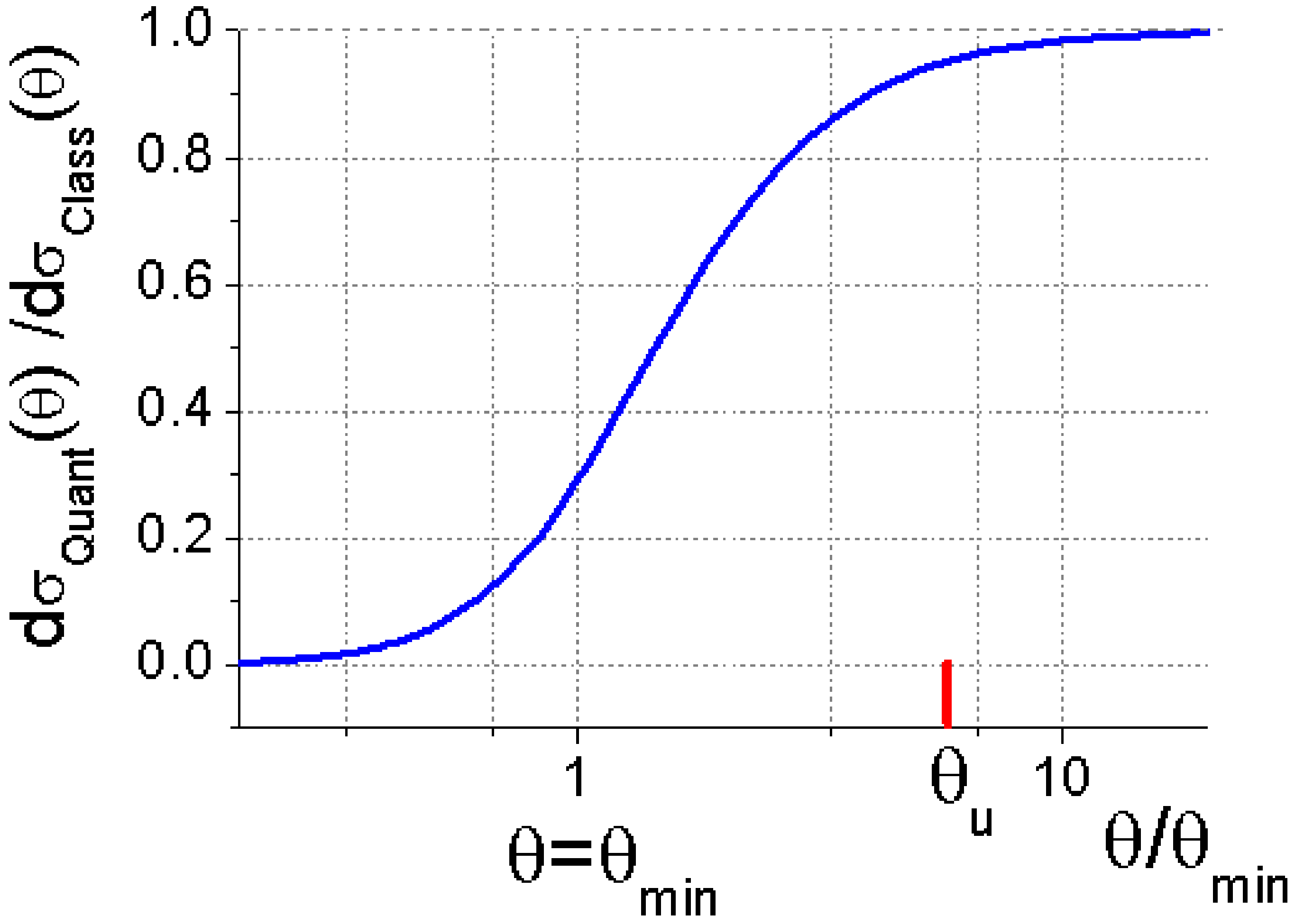
C. LEHMANN and G. LEIBFRIED. Higher order momentum approximations in classical collision theory. Zeitschrift fiir Physik 172, 465--487 (1963).

$$\theta(\rho) \approx \frac{2Z\alpha}{\varepsilon a \rho} \int_{\rho}^{\infty} \frac{\exp(-r/a)}{\sqrt{r^2-\rho^2}}\, r\,dr = \frac{2Z\alpha}{\varepsilon a} K_1(\rho/a)$$

$$\frac{d\sigma_{Class}}{d\Omega} \approx \frac{\rho(\theta)}{\theta}\left|\frac{d\theta(\rho)}{d\rho}\right|^{-1} = \frac{4Z^2\alpha^2}{\varepsilon^2\theta^4}\left[\frac{b}{K_1'(b)K_1^3(b)}\right], \qquad b = \rho/a$$

$$\frac{d\sigma_{Quant}}{d\Omega} \approx \frac{4Z^2\alpha^2}{\varepsilon^2\theta^4}\frac{1}{\left(1+\hbar^2/\theta^2 p^2 a^2\right)^2} = \frac{4Z^2\alpha^2}{\varepsilon^2\theta^4}\left(1+\hbar^2/\theta^2 p^2 a^2\right)^{-2}$$

Indeed, classical approach **overestimates** scattering



Typical scattering angles $\theta_{\min}$ and θ_u

Momentum transfer $q_\perp = \hbar Z^{1/3} / a_0$

and scattering angle $\theta_{\min} = \hbar Z^{1/3} / a_0 p$

are determined by the **atom screening** radius $a_0 / Z^{1/3}$

Momentum transfer $q_\perp = \hbar / u_1$

and scattering angle $\theta_u = \hbar / u_1 p \approx (2 \div 3)\, \theta_{\min}$

are determined by atomic vibration amplitude u_1

which determines **additional effective screening**

(scattering suppression by correlations)

Wigner function approach to the local incoherent scattering treatment

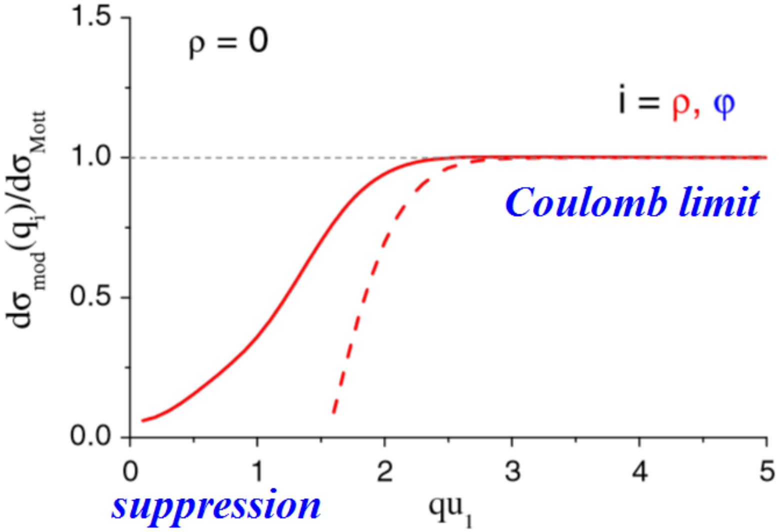
$$W(\boldsymbol{\rho},\boldsymbol{q}) = \frac{1}{\pi^2}\int \psi^*(\boldsymbol{\rho}+\boldsymbol{\chi})\psi(\boldsymbol{\rho}-\boldsymbol{\chi})\exp(2i\boldsymbol{q}\boldsymbol{\chi})d^2\chi$$

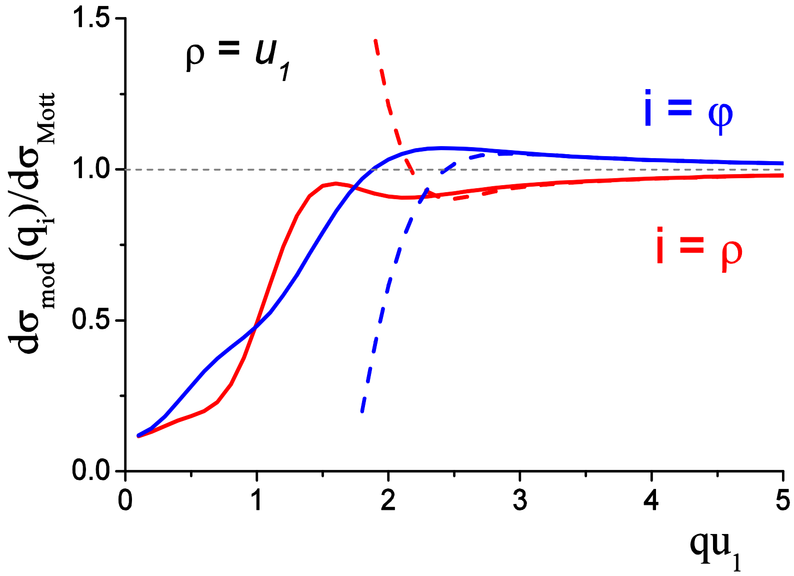
$$\frac{d\Sigma(\boldsymbol{\rho})}{d\boldsymbol{q}} = \frac{W(\boldsymbol{\rho},\boldsymbol{q})}{d\boldsymbol{q}} = \frac{4Z^2\alpha^2}{\pi^2 v^2}\frac{1}{d}\left\{\int \cos(2\boldsymbol{\kappa}\boldsymbol{\rho})\frac{\exp(-2\kappa^2u_1^2)-\exp[-(q^2+\kappa^2)u_1^2]}{[(\boldsymbol{q}+\boldsymbol{\kappa})^2+\kappa_{sc}^2][(\boldsymbol{q}-\boldsymbol{\kappa})^2+\kappa_{sc}^2]}d^2\kappa\right.$$

$$\left. + \underbrace{\cos(2\boldsymbol{q}\boldsymbol{\rho})}_{interference}\left[\frac{\pi\ln(\sqrt{q^2/\kappa_{sc}^2+1}+q/\kappa_{sc})}{q\sqrt{q^2+\kappa_{sc}^2}}\underbrace{\exp[-2q^2u_1^2]}_{nonlicality} - \int\frac{\exp[-(q^2+\kappa^2)u_1^2]d^2\kappa}{[(\boldsymbol{q}+\boldsymbol{\kappa})^2+\kappa_{sc}^2][(\boldsymbol{q}-\boldsymbol{\kappa})^2+\kappa_{sc}^2]}\right]\right\}$$

V. V. Tikhomirov. *Quantum features of high energy particle incoherent scattering in crystals.* Phys. Rev. Acc. and Beams. 22, 054501 (2019).

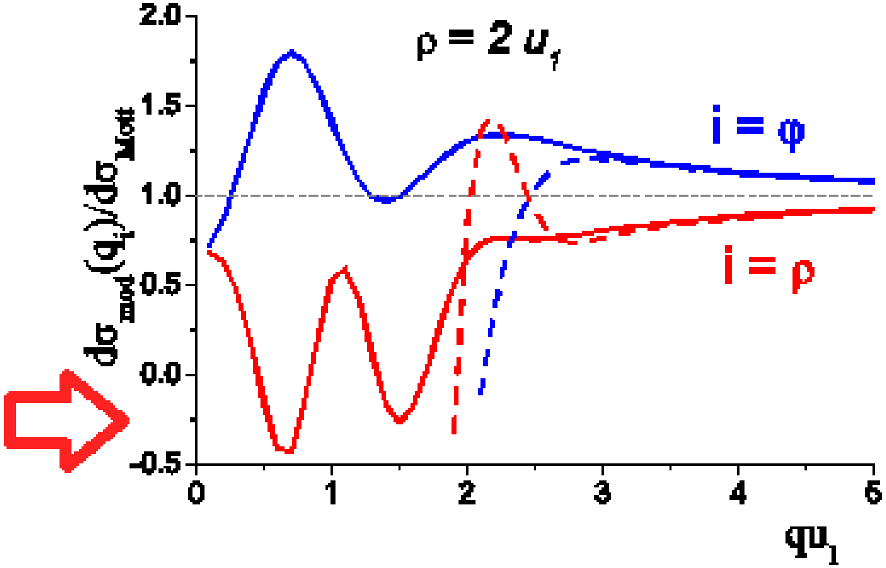
Local treatment of incoherent scattering in crystals



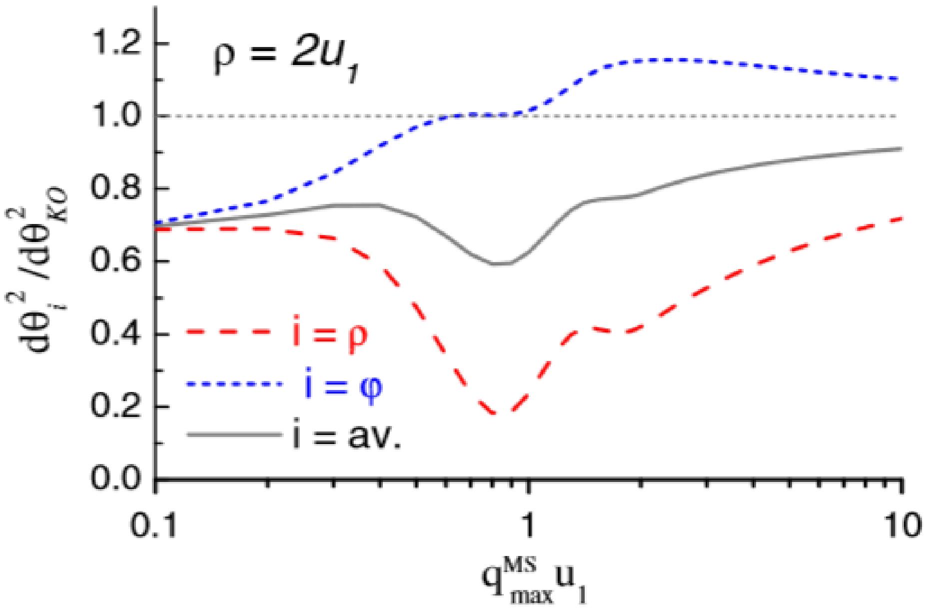


Modified cross section dependence on the momentum transfer for radial (i . ρ) and azimuthal (i . φ) scattering directions,

V. V. Tikhomirov. *Quantum features of high energy particle incoherent scattering in crystals.* Phys. Rev. Acc. and Beams. 22, 054501 (2019).



However, effective cross section can become **negative**



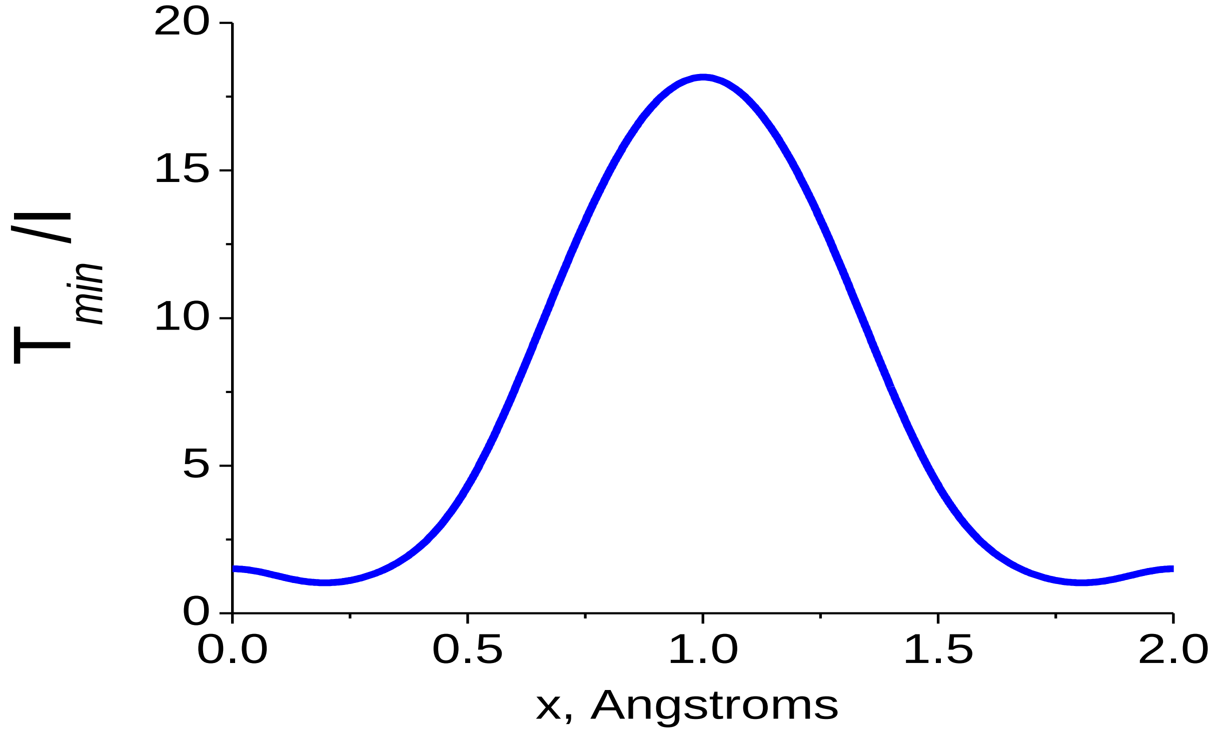
Mean square angle

$$\frac{d\theta_i^2(\boldsymbol{\rho})}{dz} = \int_{q<q_{\max}^{\mathrm{MS}}} \frac{d\Sigma(\boldsymbol{\rho})}{d\boldsymbol{q}} \frac{q_i^2}{p^2} d^2q$$

should be used instead

"Logarithm problem" solution for electron scattering

$$\frac{dE(p)}{dz} = \frac{4\pi Z_1^2 e^4}{mv^2} N \left\{ Z_2 \log\left(\frac{T_{min}}{I}\right) + \sum_{K \neq 0} e^{iK\cdot p} \langle 0 | e^{-iK\cdot r} | 0 \rangle \log\left(\frac{2mv}{\hbar K}\right) \right\} > 0$$



Starting from H. Esbansen and J. A. Golovchenko. *Energy loss of fast channeled particles*. Nuclear Physics. A298 (1978)382.

Overlook

"Problems" of the presently dominating model

Essential simulation issues

Single and multiple scattering interrelation

To the "exact" formulae

Dechanneling length notion justification and application scope

There is no ground to expect that a channeling fraction, evaluated by any averaging method, will exponentially depend on the particle penetration depth, justifying an introduction of a dechanneling length independent of the latter. In fact, only the theory

V.V. Beloshitsky, M.A. Kumakhov, V.A. Muralev, Radiat. Eff. 20, 95–109 (1973),

describes the collective statistical particle behavior and finds the lowest eigen number inverse to dechaneling length

V. V. Tikhomirov. *Quantitative theory of channeling particle diffusion in transverse energy in the presence of nuclear scattering and direct evaluation of dechanneling length.* Eur. Phys. J. C (2017) 77:483

New features of the diffusion equation is the catastrophic scattering

$$\frac{\partial F}{\partial z} = -\frac{\partial}{\partial \varepsilon_\perp}\left(\frac{\Delta\varepsilon_\perp}{\Delta z}F\right) + \frac{1}{2}\frac{\partial^2}{\partial \varepsilon_\perp^2}\left(\frac{(\Delta\varepsilon_\perp)^2}{\Delta z}F\right) - wF,$$

$$w = w\left(\varepsilon_\perp, x\right) = \int' \mathrm{d}\Sigma$$

$$\varepsilon'_\perp = \varepsilon[v_x(x)+\theta_x]^2/2+V(x) = \varepsilon_\perp+\varepsilon v_x(x)\theta_x+\varepsilon\theta_x^2/2$$

V. V. Tikhomirov. *Quantitative theory of channeling particle diffusion in transverse energy in the presence of nuclear scattering and direct evaluation of dechanneling length.* EPJ. C (2017) 77:483

Please, do not ask for simulating any experiment with this approach!

It was mostly aimed to clarify the dechanneling length role!

Correct Monte Carlo is superiorly better!

General solution of the diffusion equation

$$\frac{\partial F}{\partial z} = -\frac{\partial}{\partial \varepsilon_\perp}\left(\frac{\Delta\varepsilon_\perp}{\Delta z}F\right) + \frac{1}{2}\frac{\partial^2}{\partial \varepsilon_\perp^2}\left(\frac{(\Delta\varepsilon_\perp)^2}{\Delta z}F\right) - wF,$$

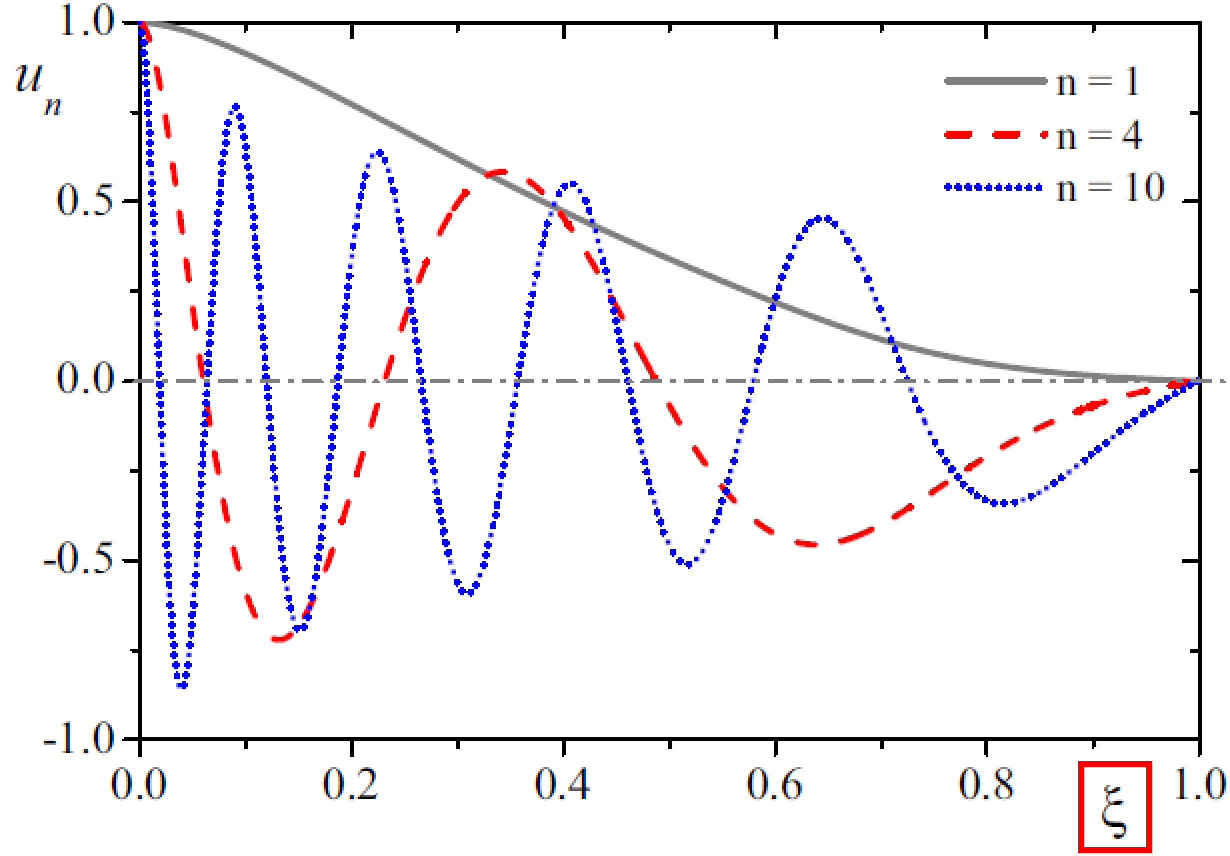
$$w = w(\varepsilon_\perp, x) = \int' \mathrm{d}\Sigma$$

$$\varepsilon_\perp' = \varepsilon[v_x(x)+\theta_x]^2/2+V(x) = \varepsilon_\perp+\varepsilon v_x(x)\theta_x+\varepsilon\theta_x^2/2$$

$$P_{\mathrm{ch}}(z) = \sum_{n=1}^{\infty} c_n \bar{u}_n \exp(-\lambda_n z)$$

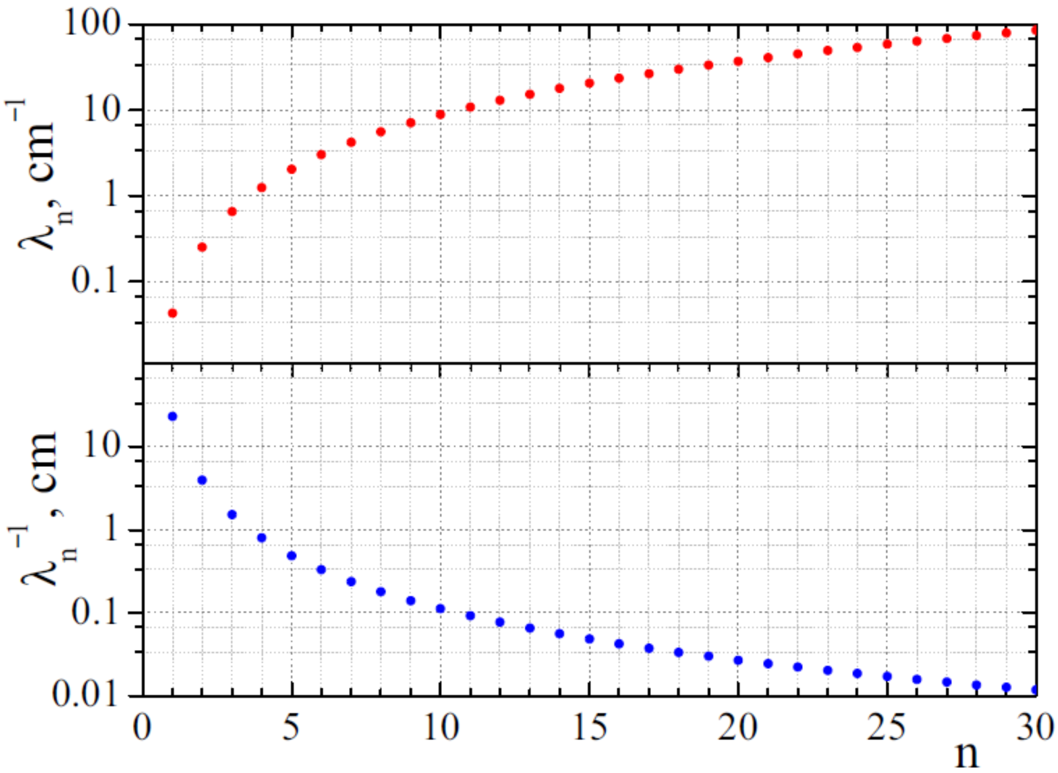
V. V. Tikhomirov. *Quantitative theory of channeling particle diffusion in transverse energy in the presence of nuclear scattering and direct evaluation of dechanneling length.* Eur. Phys. J. C (2017) 77:483

Eigen energy distributions are very peculiar!



$$\xi'(\varepsilon_{\perp}) = \int_{0}^{\varepsilon_{\perp}} T(\varepsilon_{\perp})\, d\varepsilon_{\perp} \sim \overline{T}\varepsilon_{\perp}$$

Eigen values and corresponding lengths



400 GeV Si (110)

$$P_{\rm ch}(z) = \sum_{n=1}^{\infty} c_n \bar{u}_n \exp\left(-\lambda_n z\right)$$

To the critical transverse coordinate

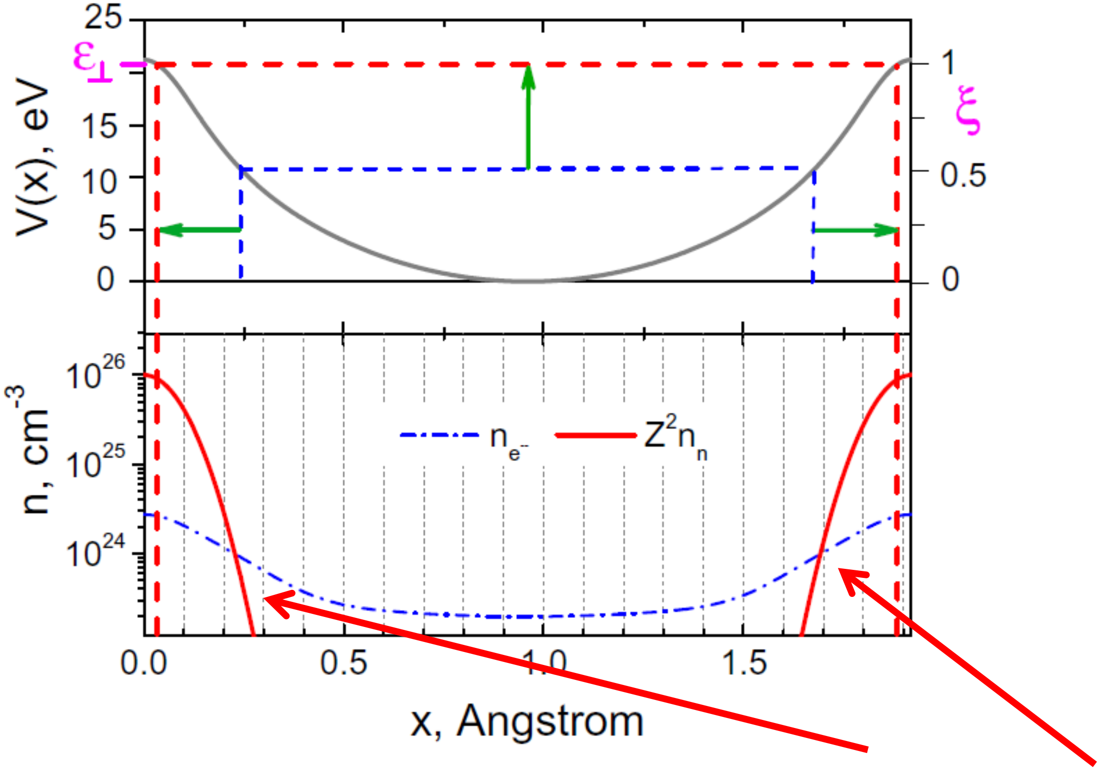
The scattering from the nuclei rapidly removes the particle from the channeling mode; a particle approaching the atomic plane as close as $\sim a_{\mathrm{TF}}$ may be considered as lost from the channeling mode. For the channeled particle the critical transverse coordinate

$$x_c \approx \frac{d_p}{2} - a_{\mathrm{TF}} \qquad (1.27)$$

p. 20: Limiting $x_c \leq d_p/2 - a_{\mathrm{TF}}$, one may consider only scattering from electrons.

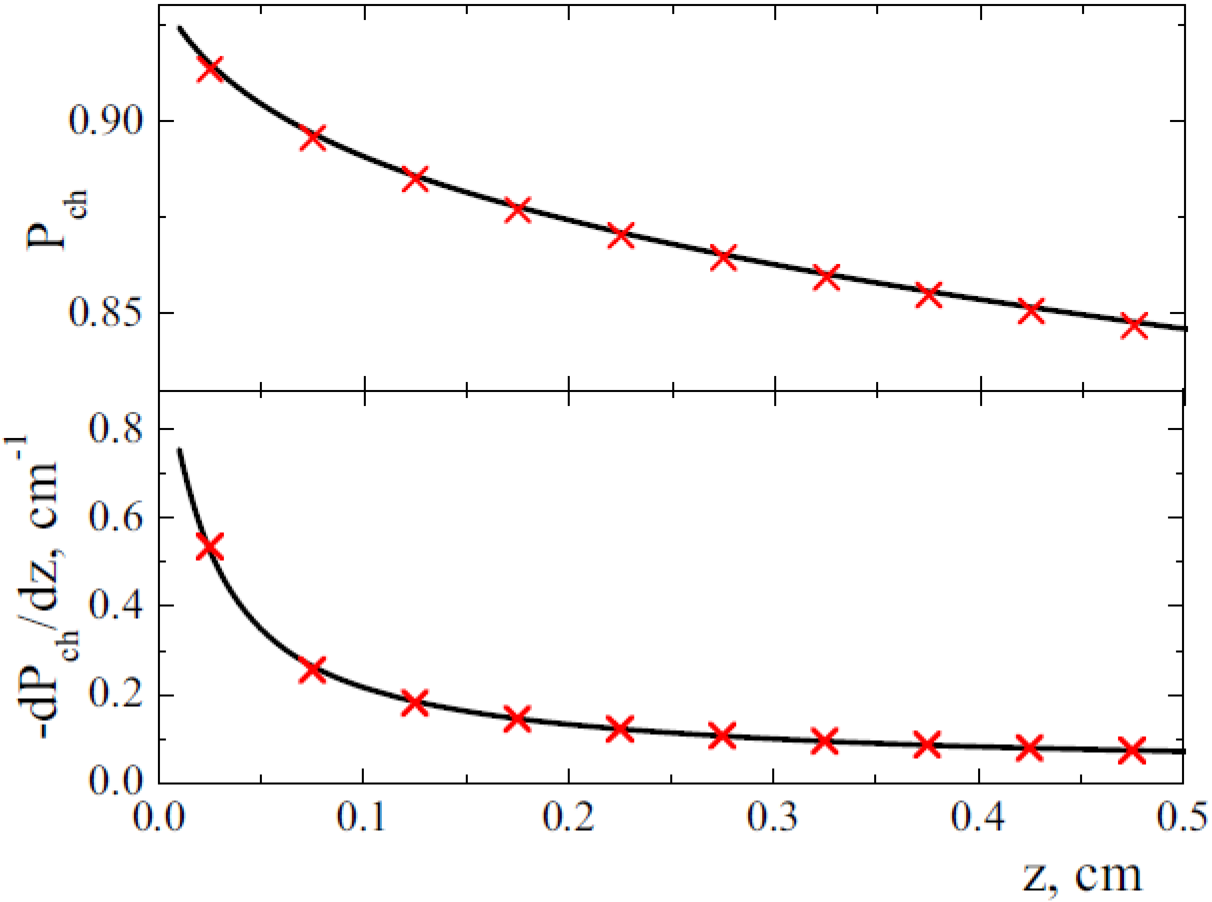
V.M. Biryukov, Y.A. Chesnokov, V.I. Kotov, *Crystal Channeling and Its Application at High-Energy Accelerators* (Springer, Berlin, 2010)

In fact, the ideas of instant *nuclear dechanneling and "nuclear corridor"* came from the **MeV**-energy ion channeling and are absolutely inadequate in the **Tev-**energy region.



Electron and nuclear scattering is **comparable** in the most actual transverse coordinate regions.

Nucler dechanneling is essentially nonexponential



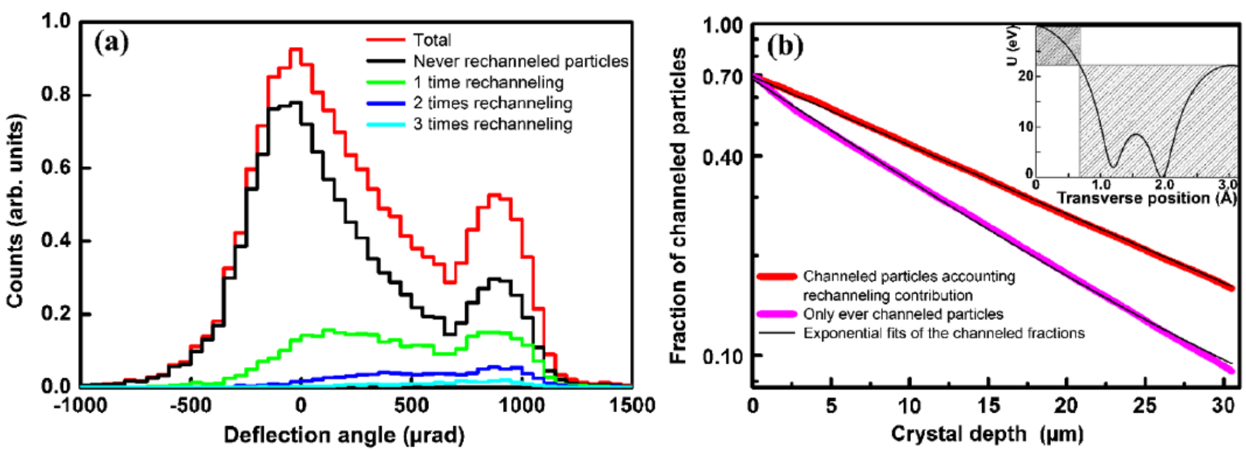
$$P_{\rm ch}(z) = \sum_{n=1}^{\infty} c_n \bar{u}_n \exp\left(-\lambda_n z\right) \xrightarrow{z \ll \lambda_1} 0.954 - 0.137\sqrt[3]{z}$$

400 GeV Si (110)

Negatively charged particle dechanneling is quite peculiar

Table 2 Dechanneling lengths for protons and electrons of different energies

e^-/p	ε, GeV	l_{dech}, cm	λ_2/λ_1	Δl_{dech}, %	N_{ch0}/N_0
p	400	23.1	6.0	0.61	0.895
p	6500	303.6	5.7	0.34	0.895
p	10^5	3936.0	5.6	0.18	0.895
e^-	1	6.0×10^{-4}	7.8	130.0	0.33
e^-	10	50.0×10^{-4}	6.9	78.0	0.39
e^-	100	0.044	6.4	46.0	0.44
e^-	1000	0.38	6.1	28.5	0.49



PRL 2014 855 MeV Si (111)

Effective dechanneling length

$$-P'_{\mathrm{ch}}(z) = \sum_{n=1}^{\infty} c_n \bar{u}_n \lambda_n \exp(-\lambda_n z)$$

$$P''_{\mathrm{ch}}(z) = \sum_{n=1}^{\infty} c_n \bar{u}_n \lambda_n^2 \exp(-\lambda_n z)$$

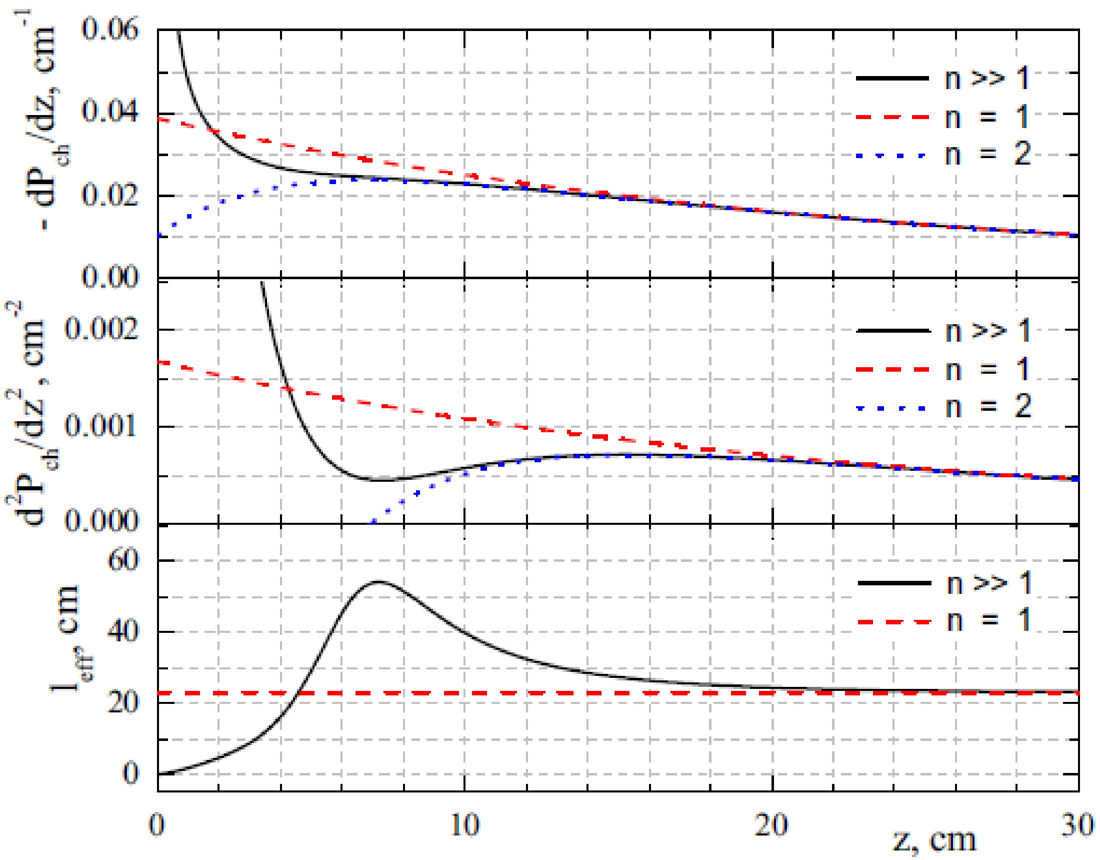
$$l_{\mathrm{dech}}^{\mathrm{eff}}(z) = -\frac{P'_{ch}(z)}{P''_{ch}(z)}$$

$$P''_{ch}(z) = n_1 e^{-\lambda_1 z} + n_2 e^{-\lambda_2 z} + n_3 e^{-\lambda_3 z}$$

$$\simeq 0.89\left(\mathrm{e}^{-z/188} - \underline{4.3}\,\mathrm{e}^{-z/35} + \underline{25}\,\mathrm{e}^{-z/14}\right)$$

7 TeV Si (110)

Effective dechanneling length **inconstancy**



400 GeV Si (110)

V. V. Tikhomirov. *Quantitative theory of channeling particle diffusion in transverse energy in the presence of nuclear scattering and direct evaluation of dechanneling length.* EPJ. C (2017) 77:483

Nuclear Inst. and Methods in Physics Research B 438 (2019) 38–41

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Dechanneling of high energy particles in a long bent crystal

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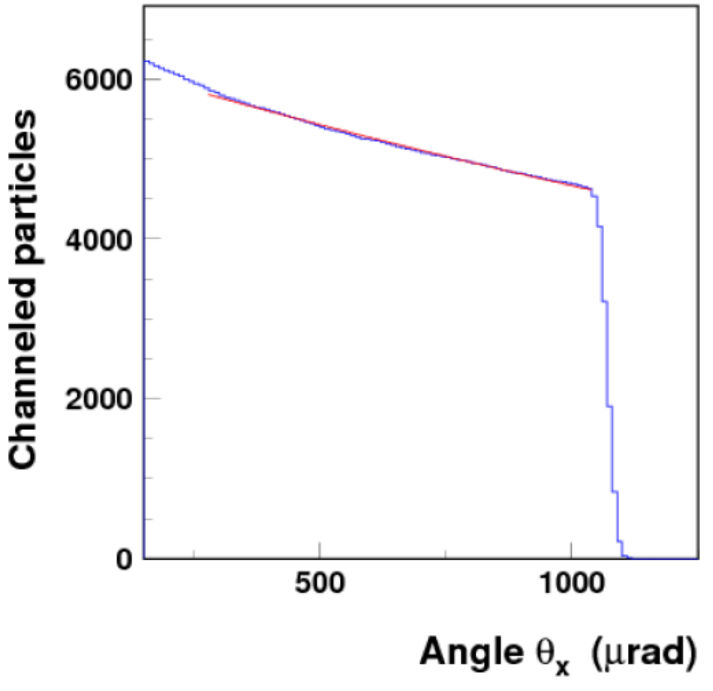
A B S T R A C T

Experimental results on deflection of a 180 GeV/c π^+-meson beam by a 23 mm long bent silicon crystal are analyzed to study the dechanneling process of particles due to multiple scattering.

Dechanneling length measurement in a not long enough crystal



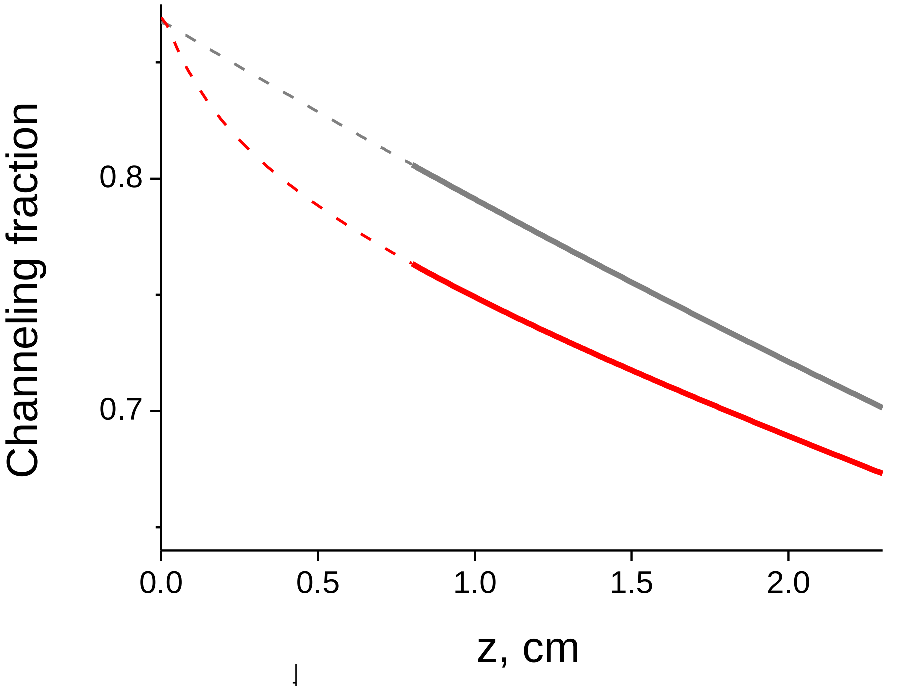
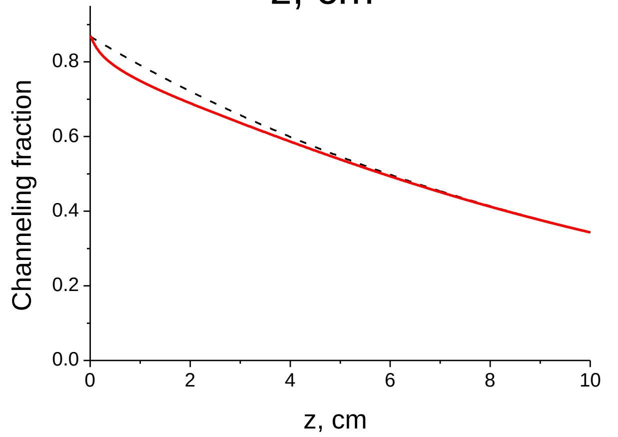
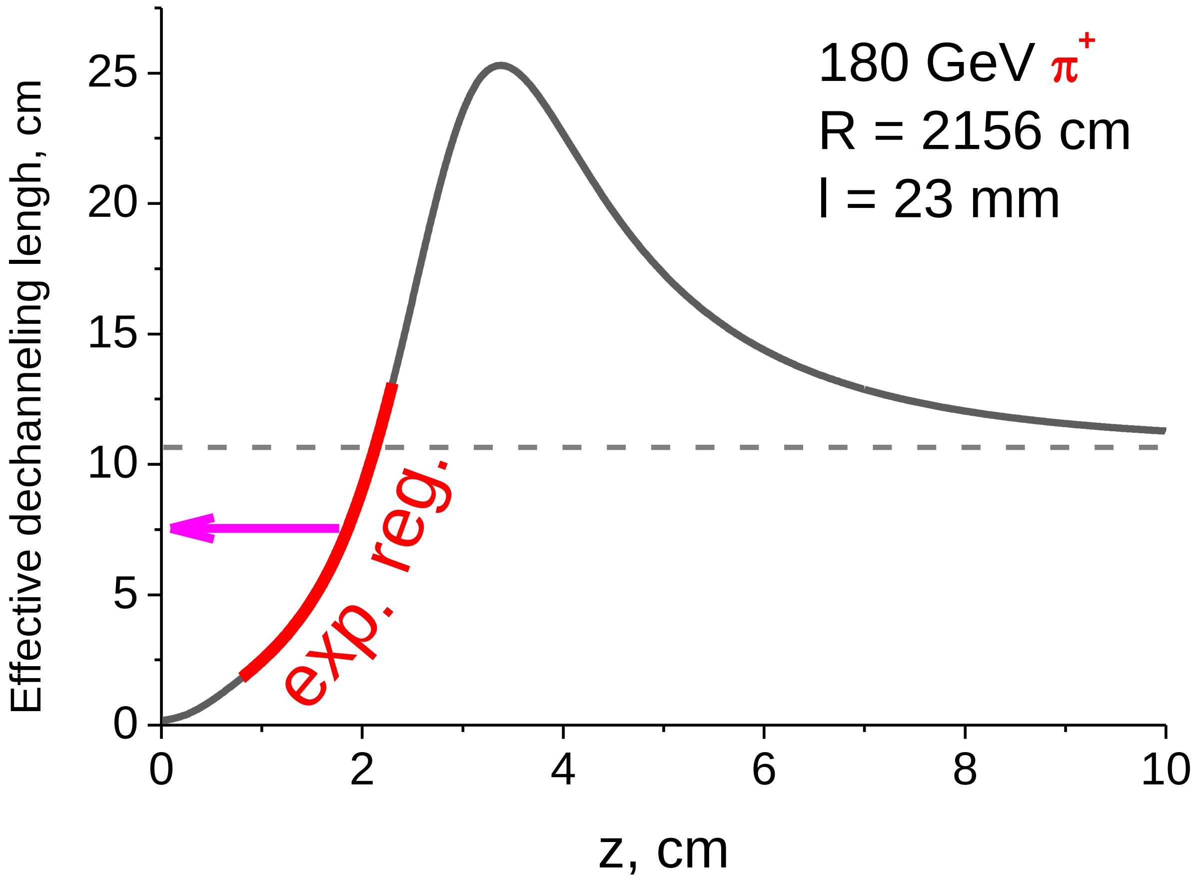


Fig. 4. The calculated dependence ...
an exponential fit ... gives a
dechanneling length (75.06 ± 0.75) mm.



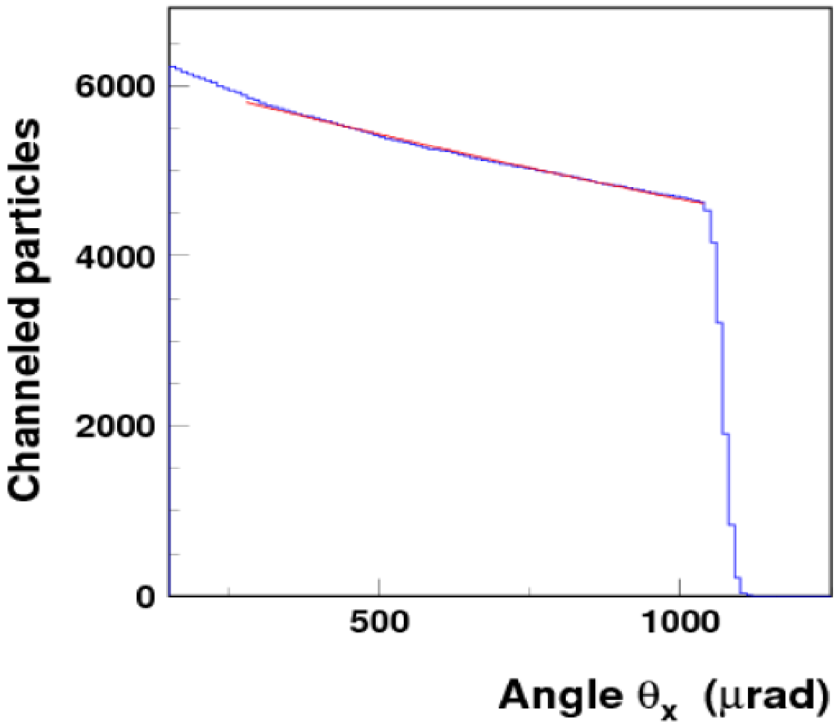
Dechanneling length measurement in a not long enough crystal 11 cm vs 7.5 cm



W. Scandale,.. A.M. Taratin,.. Dechanneling of high energy particles in a long bent crystal. NIM B 438 (2019)38.

Instant nuclear dechanneling came from Mev-energy ions

At TeV energies it takes many centimeters and can not be considered as an instant event!



The 8 mm region is excluded from the consideration due to the evidently ***non-exponential*** *behavior*

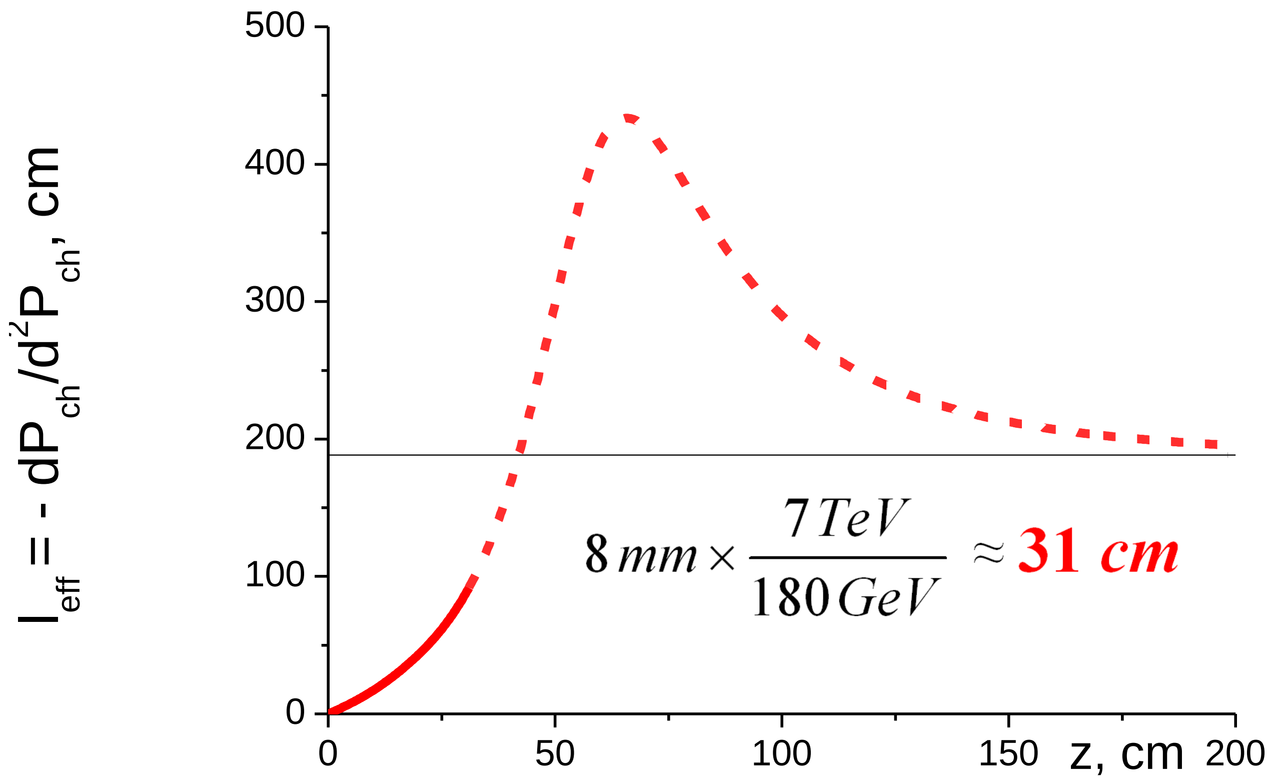
8 mm at 180 GeV

correspond to

31 cm at 7 TeV

“Transition region” length at 7 TeV

according to W. Scandale et al NIM B 438 (2019)38



No practical sense in asymptotic L_{dech} use at all**!**

At practical lengths
bent crystals will be
used solely in the
nuclear dechanneling
regime at TeV energies

A theoretical background
has been developed
to make the channelind
(dechanneling) simulations
science, instead of art.

Who will check?

Who will continue?

Who will implement?

Thank you for attention!