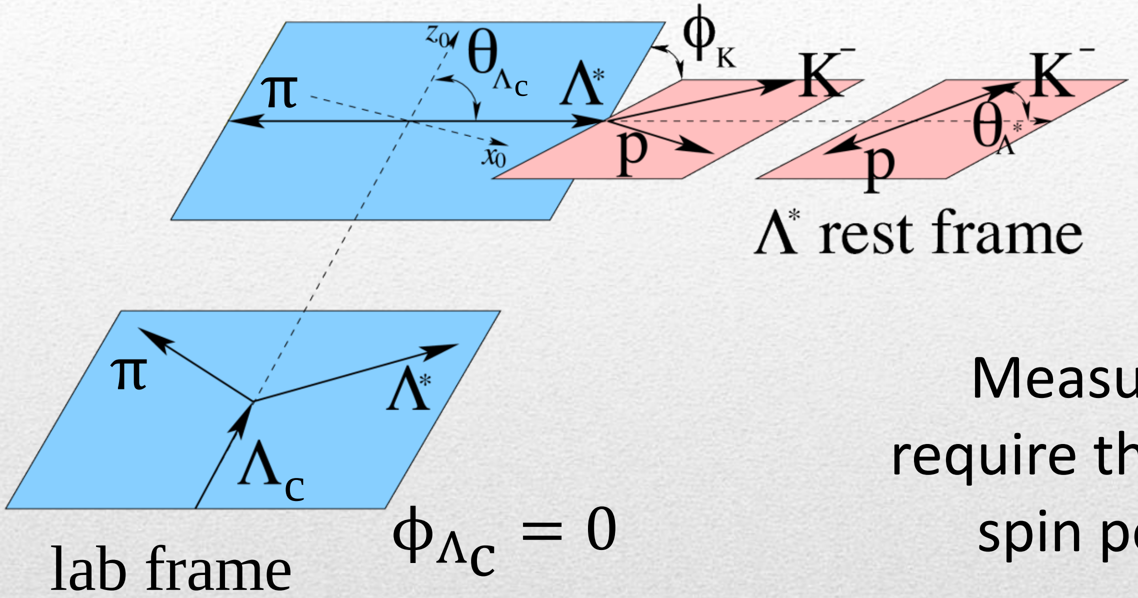
Amplitude analysis for EDM/MDM of heavy baryons

Alessandro Pilloni

Milano, October 3rd, 2019

Heavy baryon polarization



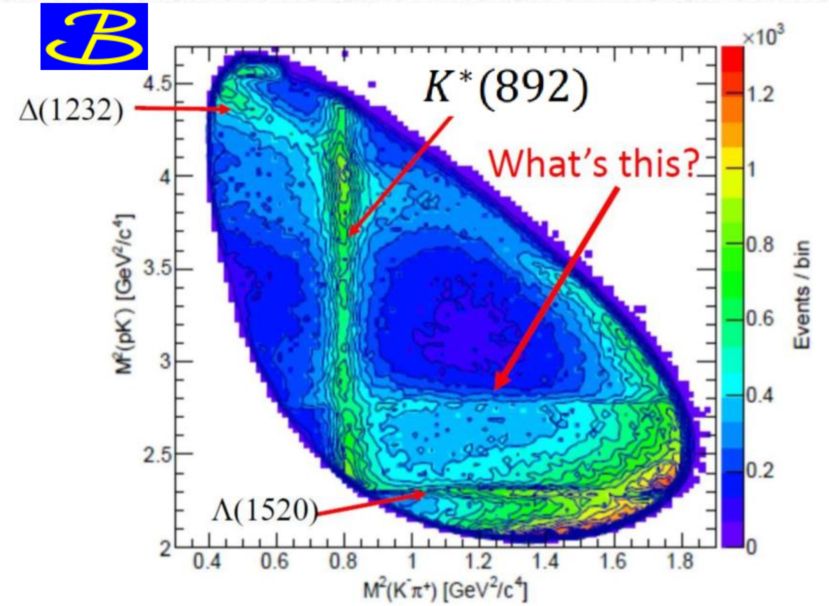
Measurements of EDM/MDM require the reconstruction of the spin polarization of the heavy baryon

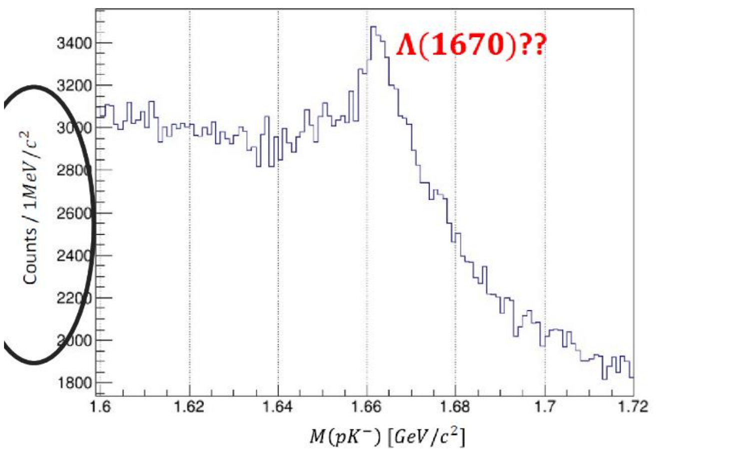
I discuss the $\Lambda_c^+ \to p\, K^- \pi^+$ 3-body decay

2 Dalitz variables $(m_{Kp}, \theta_{\Lambda^*})$: encode resonance dynamics

3 Euler angles $(\phi_{\Lambda_c}, \theta_{\Lambda_c}, \phi_K)$: encode information about polarization

$\Lambda_c^+ \to p\, K^- \pi^+$





Belle, PRL117, 011801

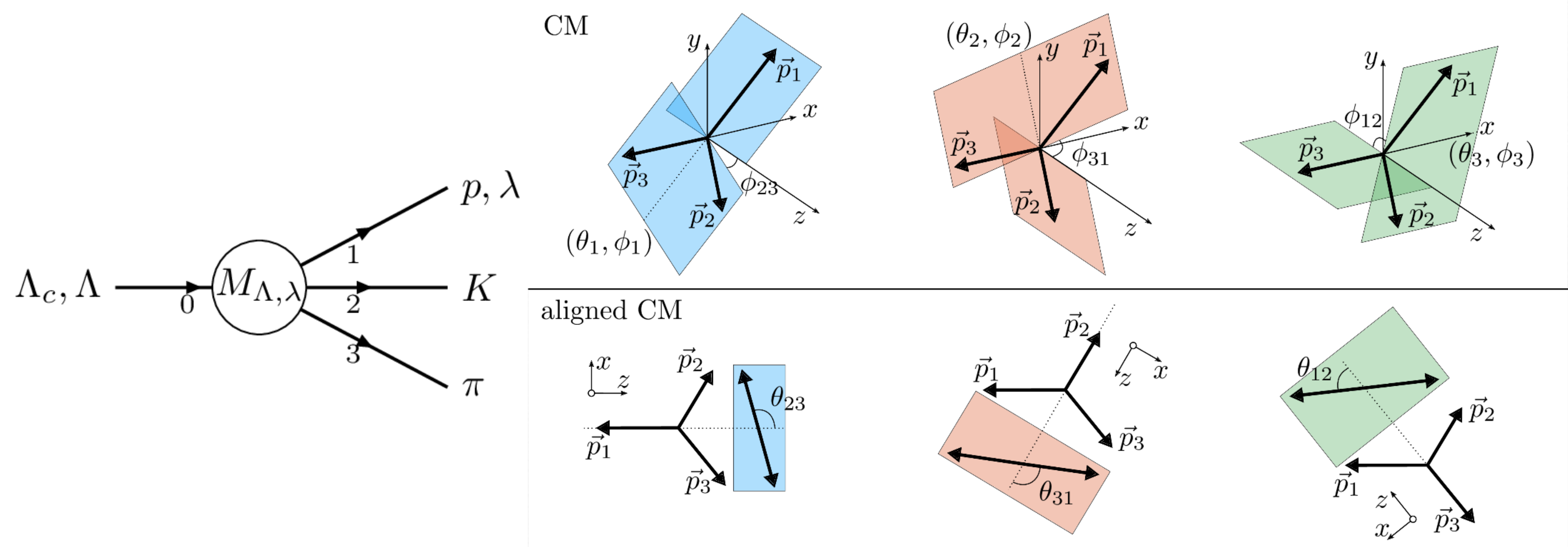
The dynamics in the Dalitz is extremely rich
Resonances appear in all subchannels

$$\Lambda_c^+ \to \begin{Bmatrix} K^{*0}\, p \\ \Delta^{++} K^- \\ \Lambda^{*0} \pi^+ \end{Bmatrix} \to p K^- \pi^+$$

Each chain prefers a variable choice
A proper amplitude analysis is mandatory

Reference frames

M. Mikhasenko, AP et al. (JPAC), to appear



The 5 variables of each chain can be expressed in terms of the variables of another chain

Isobar model in the helicity formalism

The natural choice for the amplitude looks

$$
\begin{aligned}
M_{\Lambda,\lambda}(5\text{ variables}) \stackrel{?}{=} & \sum_s^{K^*\to K\pi} \sum_\tau H^{0\to(23),1}_{\tau,\lambda} D^{1/2*}_{\Lambda,\tau-\lambda}(\phi_1,\theta_1,0)\, X_s(\sigma_1)\, H^{(23)\to 2,3}_{0,0} D^{s*}_{\tau,0}(\phi_{23},\theta_{23},0) \\
& + \sum_s^{\Delta\to\pi p} \sum_{\tau,\lambda} H^{0\to(31),2}_{\tau,0} D^{1/2*}_{\Lambda,\tau}(\phi_2,\theta_2,0)\, X_s(\sigma_2)\, H^{(31)\to 3,1}_{0,\lambda} D^{s*}_{\tau,-\lambda}(\phi_{31},\theta_{31},0) \\
& + \sum_s^{\Lambda\to pK} \sum_{\tau,\lambda} H^{0\to(12),3}_{\tau,0} D^{1/2*}_{\Lambda,\tau}(\phi_3,\theta_3,0)\, X_s(\sigma_3)\, H^{(12)\to 1,2}_{\lambda,0} D^{s*}_{\tau,\lambda}(\phi_{12},\theta_{12},0)
\end{aligned}
$$

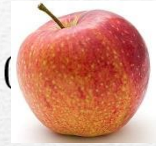
with the cross section given by the trace with the spin-density matrix

$$
d\sigma \sim \sum_{\Lambda,\Lambda',\lambda} \rho_{\Lambda,\Lambda'} M_{\Lambda,\lambda} M^*_{\Lambda',\lambda}
$$

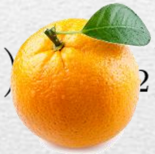
Isobar model in the helicity formalism

The natural choice for the amplitude looks

$$M_{\Lambda,\lambda}(5 \text{ variables}) \stackrel{?}{=} \sum_{s}^{K^*\to K\pi} \sum_{\tau} H^{0\to(23),1}_{\tau,\lambda} D^{1/2*}_{\Lambda,\tau-\lambda}(\phi_1,\theta_1, (\quad {}_1)\, H^{(23)\to 2,3}_{0,0} D^{s*}_{\tau,0}(\phi_{23},\theta_{23},0)$$



$$+ \sum_{s}^{\Delta\to\pi p} \sum_{\tau,\lambda} H^{0\to(31),2}_{\tau,0} D^{1/2*}_{\Lambda,\tau}(\phi_2,\theta_2,0) \quad {}_2)\, H^{(31)\to 3,1}_{0,\lambda} D^{s*}_{\tau,-\lambda}(\phi_{31},\theta_{31},0)$$



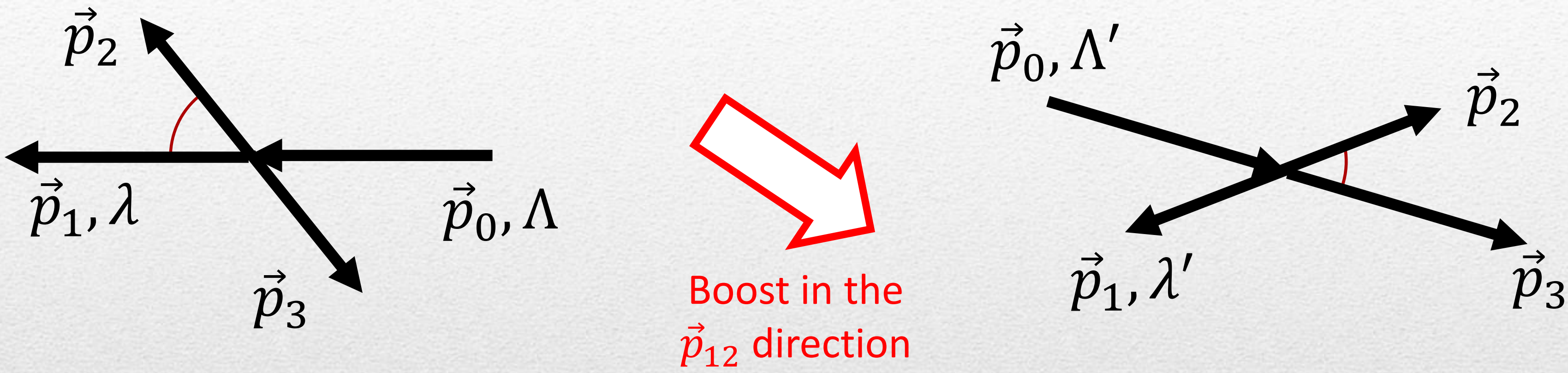
$$+ \sum_{s}^{\Lambda\to pK} \sum_{\tau,\lambda} H^{0\to(12),3}_{\tau,0} D^{1/2*}_{\Lambda,\tau}(\phi_3,\theta_3, \quad {}_3)\, H^{(12)\to 1,2}_{\lambda,0} D^{s*}_{\tau,\lambda}(\phi_{12},\theta_{12},0)$$

with the cro atrix

No! The helicities have not the same meaning in the different chains!

Wigner rotations

Let's analyze the process in the decay plane



(23) rest frame

(12) rest frame

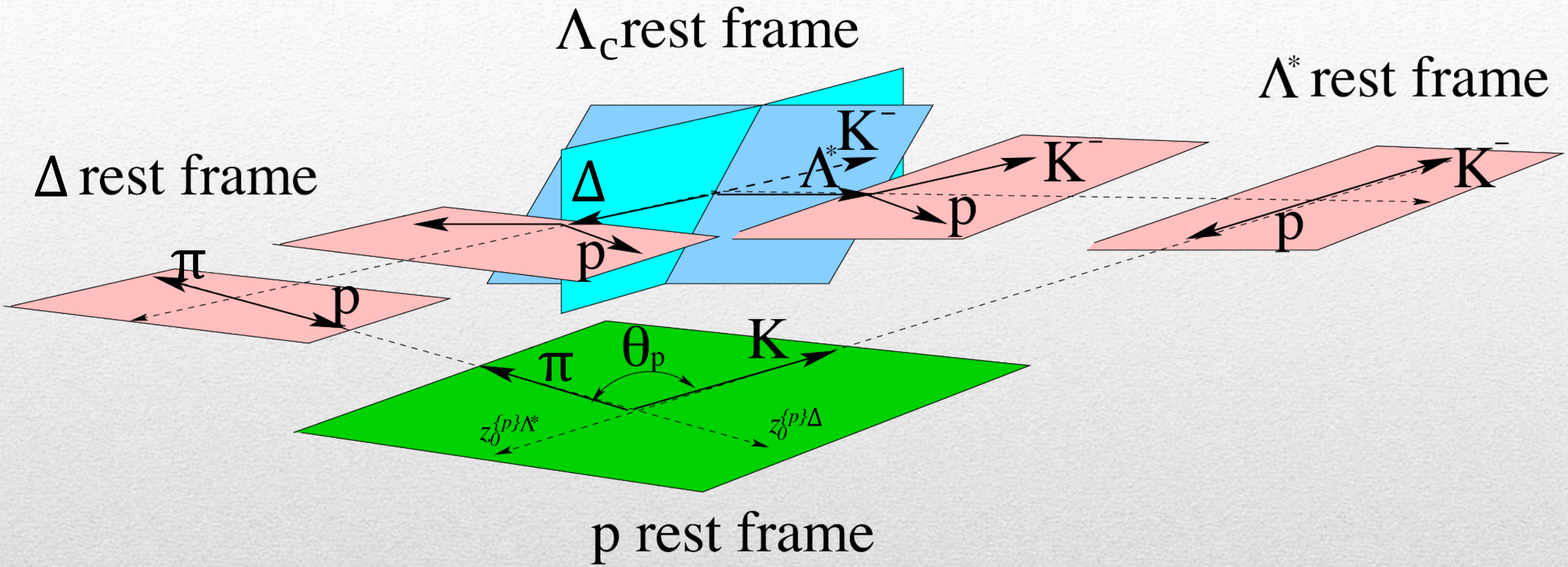
Helicities are not Lorentz invariant

A boost disaligned with the momentum mixes (rotates) the helicities

$\downarrow$

Additional d-functions: Wigner rotations

Wigner rotations



The new rotation is about the y axis, the angle is defined in the rest frame of the p or Λ_c

Isobar model in the decay plane

The natural choice for the amplitude looks

$$O^{\nu}_{\lambda}(\{\sigma\}) = \sum_s^{K^*\to K\pi} \sum_\tau H^{0\to(23),1}_{\tau,\lambda} \delta_{\nu,\tau-\lambda}\, X_s(\sigma_1)\, H^{(23)\to 2,3}_{0,0} d^s_{\tau,0}(\theta_{23})$$
$$+ \sum_s^{\Delta\to\pi p} \sum_{\tau,\lambda'} H^{0\to(31),2}_{\tau,0} d^{1/2}_{\nu,\tau}(\hat\theta_{2(1)})\, X_s(\sigma_2)\, H^{(31)\to 3,1}_{0,\lambda'} d^s_{\tau,-\lambda'}(\theta_{31})\, d^{1/2}_{\lambda',\lambda}(\zeta^1_{2(1)})$$
$$+ \sum_s^{\Lambda\to pK} \sum_{\tau,\lambda'} H^{0\to(12),3}_{\tau,0} d^{1/2}_{\nu,\tau}(\hat\theta_{3(1)})\, X_s(\sigma_3)\, H^{(12)\to 1,2}_{\lambda',0} d^s_{\tau,\lambda'}(\theta_{12})\, d^{1/2}_{\lambda',\lambda}(\zeta^1_{3(1)})$$

$\zeta^1_{2(1)}$ is the angle between π and Λ_c in the p rest frame

$\zeta^1_{3(1)}$ is the angle between K and Λ_c in the p rest frame

$\hat\theta_{2(1)}$ is the angle between K and p in the Λ_c rest frame

$\hat\theta_{3(1)}$ is the angle between π and p in the Λ_c rest frame

Isobar model in the decay plane

The natural choice for the amplitude looks

$$O^{\nu}_{\lambda}(\{\sigma\}) = \sum_{s}^{K^*\to K\pi} \sum_{\tau} H^{0\to(23),1}_{\tau,\lambda}\delta_{\nu,\tau-\lambda}\, X_s(\sigma_1)\, H^{(23)\to 2,3}_{0,0} d^s_{\tau,0}(\theta_{23})$$
$$+ \sum_{s}^{\Delta\to\pi p} \sum_{\tau,\lambda'} H^{0\to(31),2}_{\tau,0} d^{1/2}_{\nu,\tau}(\hat\theta_{2(1)})\, X_s(\sigma_2)\, H^{(31)\to 3,1}_{0,\lambda'} d^s_{\tau,-\lambda'}(\theta_{31})\, d^{1/2}_{\lambda',\lambda}(\zeta^1_{2(1)})$$
$$+ \sum_{s}^{\Lambda\to pK} \sum_{\tau,\lambda'} H^{0\to(12),3}_{\tau,0} d^{1/2}_{\nu,\tau}(\hat\theta_{3(1)})\, X_s(\sigma_3)\, H^{(12)\to 1,2}_{\lambda',0} d^s_{\tau,\lambda'}(\theta_{12})\, d^{1/2}_{\lambda',\lambda}(\zeta^1_{3(1)})$$

All the angles are polar: no azimuthal phases
all these functions can be expressed in term of the
Mandelstam variables $\sigma_1 = m^2_{K\pi}$, $\sigma_2 = m^2_{p\pi}$, $\sigma_3 = m^2_{pK}$
Now all the spins are aligned correctly

Isobar model out of the decay plane

Now that everything is aligned with the K^* chain,
we can use the Euler angles of chain 1 for the full thing

$$M_{\Lambda,\lambda}(5 \text{ variables}) = \sum_{\nu} D^{1/2*}_{\Lambda,\nu}(\phi_1, \theta_1, \phi_{23}) \times O^{\nu}_{\lambda}(2 \text{ variables})$$

In this form it is straightforward to check that, for unpolarized Λ_c, the cross section does not depend on the 3 Euler angles

Recipes to build an amplitude

M. Mikhasenko, AP, J. Nys *et al.* (JPAC), EPJC78, 3, 229
AP, J. Nys, M. Mikhasenko *et al.* (JPAC), EPJC78, 9, 727

The literature abounds with discussions on the optimal approach to construct the amplitudes for the hadronic reactions

- Helicity formalism

  Jacob, Wick, Annals Phys. 7, 404 (1959)

- Covariant tensor formalisms

  Chung, PRD48, 1225 (1993)
  Chung, Friedrich, PRD78, 074027 (2008)
  Filippini, Fontana, Rotondi, PRD51, 2247 (1995)
  Anisovich, Sarantsev, EPJA30, 427 (2006)

The common lore is that the former one is nonrelativistic,
especially when expressed in terms of LS couplings
and the latter takes into account the proper relativistic corrections

How helicity formalism works

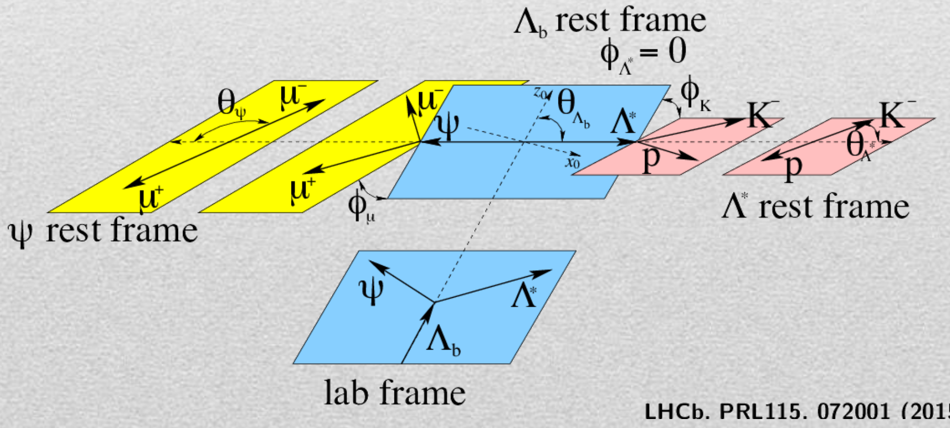
- Helicity formalism enforces the constraints about rotational invariance
- It allows us to fix the angular dependence of the amplitude
- What about energy dependence?

Example: $B \to \psi K^* \to \pi K$

$$\mathcal{M}^{K^*}_{\Delta\lambda_\mu} \equiv \sum_n \sum_{\lambda_{K^*}} \sum_{\lambda_\psi} \mathcal{H}^{B\to K^*_n \psi}_{\lambda_{K^*},\,\lambda_\psi} \delta_{\lambda_{K^*},\lambda\psi}$$

$$\mathcal{H}^{K^*_n \to K\pi} D^{J_{K^*_n}}_{\lambda_{K^*},0}(\phi_K,\theta_{K^*},0)^*$$

$$R_{K^*{}_n}(m_{K\pi}) D^{1}_{\lambda_\psi,\,\Delta\lambda_\mu}(\phi_\mu,\theta_\psi,0)^*,$$



LHCb. PRL115. 072001 (2015)

Each set of angles is defined in a different reference frame

How tensor formalism works

The method is based on the construction of explicitly covariant expressions.

- To describe the decay $a \to bc$, we first consider the polarization tensor of each particle, $\varepsilon^i_{\mu_1\dots\mu_{j_i}}(p_i)$
- We combine the polarizations of b and c into a "total spin" tensor $S_{\mu_1\dots\mu_S}(\varepsilon_b, \varepsilon_c)$
- Using the decay momentum, we build a tensor $L_{\mu_1\dots\mu_L}(p_{bc})$ to represent the orbital angular momentum of the bc system, orthogonal to the total momentum of p_a
- We contract S and L with the polarization of a

Tensor $\times R_X(m)$ which contain resonances and form factors

What do we know?

- Energy dependence is not constrained by symmetry
- Still, there are some known properties one can enforce

$$R_X(m) = B'_{L^X_{\Lambda_b^0}}(p, p_0, d) \left(\frac{p}{M_{\Lambda_b^0}}\right)^{L^X_{\Lambda_b^0}}$$
$$\mathrm{BW}(m|M_{0X}, \Gamma_{0X})\, B'_{L_X}(q, q_0, d) \left(\frac{q}{M_{0X}}\right)^{L_X}$$

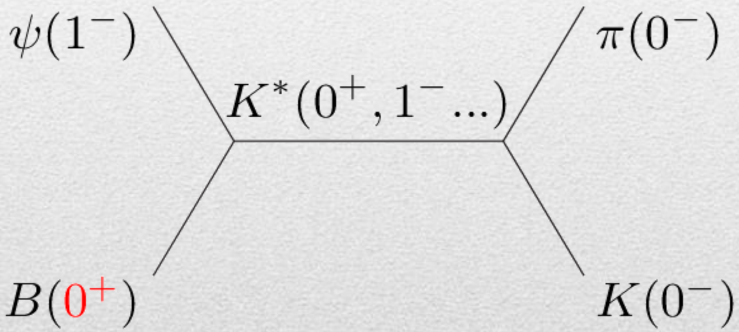
- Kinematical singularities: e.g. barrier factors (known)
- Left hand singularities (need model, e.g. Blatt-Weisskopf)
- Right hand singularities = resonant content (Breit Wigner, K-matrix...)

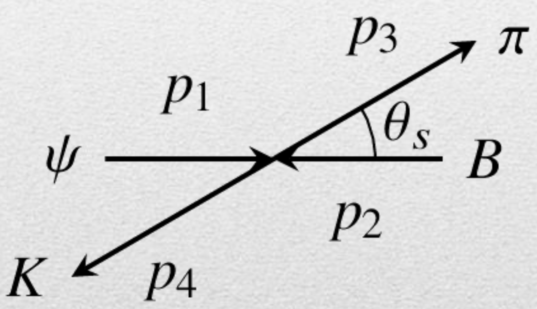
Kinematics

- Kinematical singularities appear because of the spin of the external particle involved
- We can write the most general covariant parametrization of the amplitude as
  tensor of external polarizations $\otimes$ scalar amplitudes
- Scalar amplitudes must be kinematical singularities free
- They can be matched to the helicity amplitudes
- We can get the minimal energy dependent factor
- Any other additional energy factor would be model-dependent

$B \to \psi\, \pi K$

To consider the effect of spin, let's consider $B \to \psi\pi K$
We focus on the parity violating amplitude for the K^* isobars, scattering kinematics





$$p = \text{incoming 3-momentum in the COM} = \frac{\lambda_{12}^{1/2}}{2\sqrt{s}}$$

$$= \frac{\sqrt{[s-(m_1+m_2)^2]\,[s-(m_1-m_2)^2]}}{2\sqrt{s}}$$

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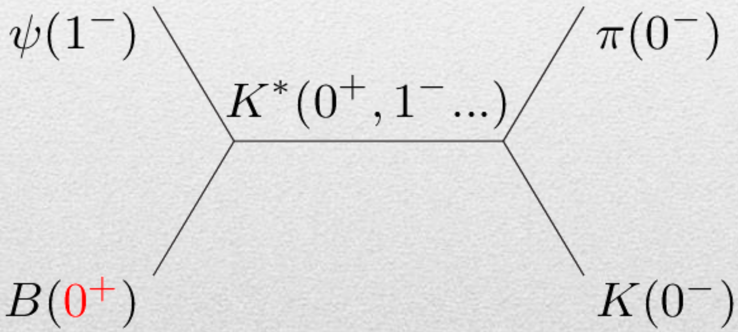

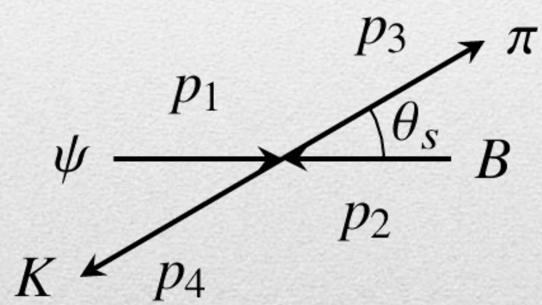

$$q = \text{outgoing 3-momentum in the COM} = \frac{\lambda_{34}^{1/2}}{2\sqrt{s}}$$

$$= \frac{\sqrt{[s-(m_3+m_4)^2]\,[s-(m_3-m_4)^2]}}{2\sqrt{s}}$$

$B \to \psi\,\pi K$

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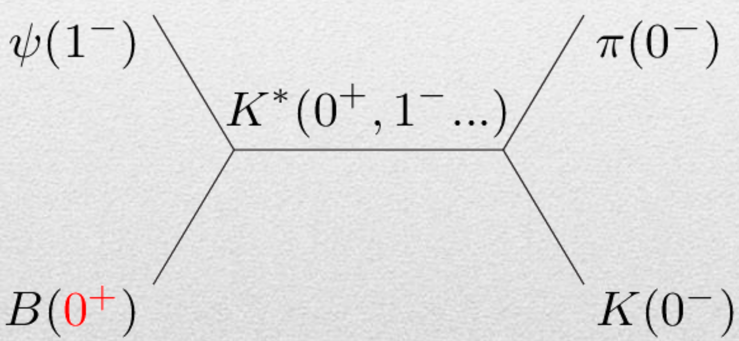
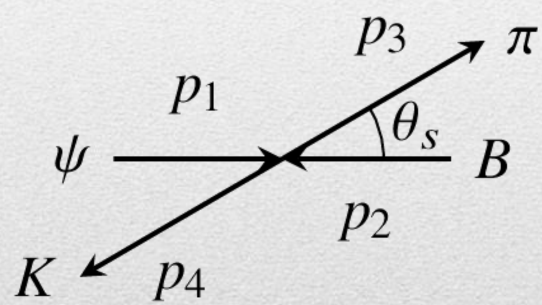
We focus on the parity violating amplitude for the K^* isobars, scattering kinematics





$$z_s = \text{cosine of the scatt. angle in the COM}$$
$$= \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\lambda_{12}^{1/2}\lambda_{34}^{1/2}} = \frac{\text{polynomial}}{\lambda_{12}^{1/2}\lambda_{34}^{1/2}}$$

Helicity amplitudes

$$A_\lambda = \frac{1}{4\pi}\sum_{j=|\lambda|}^{\infty}(2j+1)A^j_\lambda(s)\, d^j_{\lambda 0}(z_s)$$

$$d^j_{\lambda 0}(z_s) = \hat{d}^j_{\lambda 0}(z_s)\xi_{\lambda 0}(z_s), \qquad \xi_{\lambda 0}(z_s) = \left(\sqrt{1-z_s^2}\right)^\lambda$$

$\hat{d}^j_{\lambda 0}(z_s)$ is a polynomial of order $j-|\lambda|$ in z_s,

The kinematical singularities of $A^j_\lambda(s)$ can be isolated by writing

$$A^j_0 = \frac{m_1}{p\sqrt{s}}\,(pq)^j\,\hat{A}^j_0 \qquad \text{for } j \geq 1,$$

$$A^j_\pm = q\,(pq)^{j-1}\,\hat{A}^j_\pm \qquad \text{for } j \geq 1,$$

$$A^0_0 = \frac{p\sqrt{s}}{m_1}\,\hat{A}^0_0 \qquad \text{for } j = 0,$$

Identify covariants

Two helicity couplings $\to$ two independent covariant structures
Important: we are not imposing any intermediate isobar

$$A_\lambda(s,t) = \varepsilon_\mu(\lambda, p_1)\left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3+p_4)^\mu\right] C(s,t) + \varepsilon_\mu(\lambda,p_1)(p_3+p_4)^\mu B(s,t)$$

$$C(s,t) = \frac{1}{4\pi\sqrt{2}}\sum_{j>0}(2j+1)(pq)^{j-1}\hat{A}^j_+(s)\,\hat{d}^j_{10}(z_s)$$

$$B(s,t) = \frac{1}{4\pi}\hat{A}^0_0 + \frac{1}{4\pi}\frac{4m_1^2}{\lambda_{12}}\sum_{j>0}(2j+1)(pq)^j\left[\hat{A}^j_0(s)\hat{d}^j_{00}(z_s) + \frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2}\hat{A}^j_+(s)\,z_s\hat{d}^j_{10}(z_s)\right]$$

Everything looks fine but the λ_{12} in the denominator
The brackets must vanish at $\lambda_{12}=0 \Rightarrow s = s_\pm = (m_1 \pm m_2)^2$,
$\hat{A}^j_+$ and $\hat{A}^j_0$ cannot be independent

General expression and comparison

$$\hat{A}^j_+ = \langle j-1,0;1,1|j,1\rangle g_j(s) + f_j(s)$$

$$\hat{A}^j_0 = \langle j-1,0;1,0|j,0\rangle \frac{s+m_1^2-m_2^2}{2m_1^2} g'_j(s) + f'_j(s)$$

$g_j(s_\pm) = g'_j(s_\pm)$, and $f_j(s), f'_j(s) \sim O(s-s_\pm)$
All these four functions are free of kinematic singularity.

Comparison with tensor formalisms ($j=1$)

$$g_1 = g'_1 = \frac{4\pi}{3} g_S, \quad f_1 = \frac{2\pi\lambda_{12}}{3s} g_D, \quad f'_1 = -\frac{4\pi\lambda_{12}}{3s}\frac{s+m_1^2-m_2^2}{m_1^2} g_D.$$

If the g_S, g_D are the usual Breit-Wigner, the g', f' are fine

There is no unique recipe to build the right amplitude, but one can ensure the right singularities to be respected

$\Lambda_b \to \psi\, p\, K$

$$A^{j-}_{+,++}(s) = p^{j-1/2} q^{j-1/2} \Big[\langle \tfrac{1}{2}, \tfrac{1}{2}; 1, -1 | \tfrac{1}{2}, -\tfrac{1}{2} \rangle \langle \tfrac{1}{2}, -\tfrac{1}{2}; j-\tfrac{1}{2}, 0 | j, -\tfrac{1}{2} \rangle\, g_{j-}(s) + \langle \tfrac{1}{2}, \tfrac{1}{2}; 1, -1 | \tfrac{3}{2}, -\tfrac{1}{2} \rangle \langle \tfrac{3}{2}, -\tfrac{1}{2}; j-\tfrac{1}{2}, 0 | j, -\tfrac{1}{2} \rangle\, h_{j-}(s) + p^2\, f_{j+}(s) \Big]$$

$$A^{j-}_{+,+0}(s) = p^{j-1/2} q^{j-1/2} \Big[\langle \tfrac{1}{2}, \tfrac{1}{2}; 1, 0 | \tfrac{1}{2}, \tfrac{1}{2} \rangle \langle \tfrac{1}{2}, \tfrac{1}{2}; j-\tfrac{1}{2}, 0 | j, \tfrac{1}{2} \rangle \frac{E_\psi}{m_\psi}\, g'_{j-}(s) + \langle \tfrac{1}{2}, \tfrac{1}{2}; 1, 0 | \tfrac{3}{2}, \tfrac{1}{2} \rangle \langle \tfrac{3}{2}, \tfrac{1}{2}; j-\tfrac{1}{2}, 0 | j, \tfrac{1}{2} \rangle \frac{E_\psi}{m_\psi}\, h'_{j-}(s) + p^2\, f'_{j-}(s) \Big]$$

$$A^{j-}_{+,+-}(s) = p^{j-1/2} q^{j-1/2} \Big[\langle \tfrac{3}{2}, \tfrac{3}{2}; j-\tfrac{1}{2}, 0 | j, \tfrac{3}{2} \rangle \left(\frac{1}{C}\, g''_{j-}(s) + h''_{j-}(s) \right) + p^2\, f''_{j-}(s) \Big]$$

For baryons the idea is similar,
but the expressions are messier

$$A^{j+}_{+-,+}(u) = p_u^{j+1/2} q_u^{j-3/2} \Big[\langle \tfrac{3}{2}, \tfrac{3}{2}; j-\tfrac{3}{2}, 0 | j, \tfrac{3}{2} \rangle g''_{j+}(u) + q_u^2\, f''_{j+}(u) \Big]$$

$$A^{j-}_{++,+}(u) = p_u^{j-1/2} q_u^{j-1/2} \Big[\frac{E_p + m_p}{2 m_p} \Big(\langle \tfrac{1}{2}, \tfrac{1}{2}; 1, -1 | \tfrac{1}{2}, -\tfrac{1}{2} \rangle \langle \tfrac{1}{2}, -\tfrac{1}{2}; j-\tfrac{1}{2}, 0 | j, -\tfrac{1}{2} \rangle\, g_{j-}(u) + \langle \tfrac{1}{2}, \tfrac{1}{2}; 1, -1 | \tfrac{3}{2}, -\tfrac{1}{2} \rangle \langle \tfrac{3}{2}, -\tfrac{1}{2}; j-\tfrac{1}{2}, 0 | j, -\tfrac{1}{2} \rangle\, h_{j-}(u) \Big) + q_u^2\, f_{j+}(u) \Big]$$

Conclusions

- Measuring electromagnetic moments of baryons requires a thorough understanding of the amplitudes involved
- The formalism gets more complicated when multiple particles with spin are involved, in particular with baryons
- The Euler angles can be meaningfully included only if the spins of the different chains are properly aligned
- The minimal energy depence can be deduced from first principles

Thank you

BACKUP

Crossing symmetry in tensor formalisms

- The process $B \to \bar{D}\pi\pi$ is composed of scalar particles only

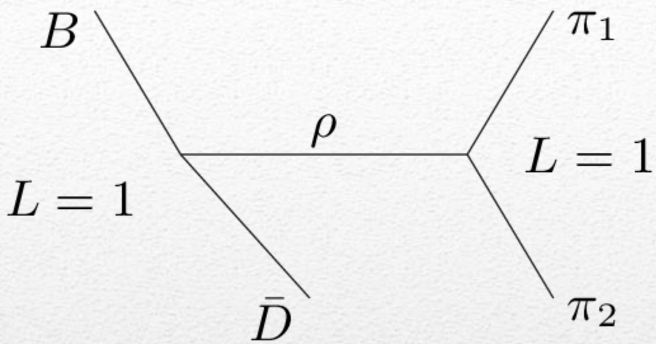
LHCb, PRD92, 032002 (2015)

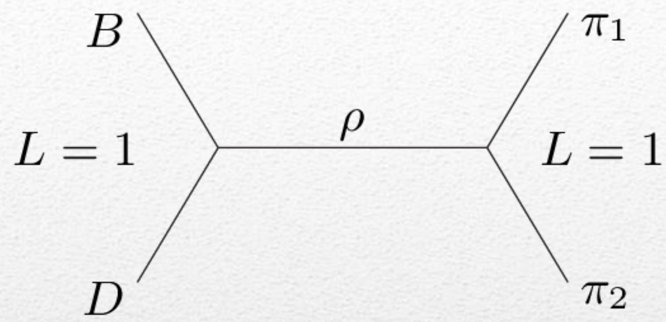
- One defines the helicity angle in the isobar rest frame, then the amplitude is Lorentz Invariant
- Let's consider the ρ intermediate state, $B \to \bar{D}\rho(\to \pi\pi)$

$$A = \frac{m_B^2 + s - m_D^2}{2m_B^2} \cos\theta \times qp = \frac{E_\rho^{(B)}}{m_B} \cos\theta \times qp$$

- The factors p and q are the $L = 1$ expected barrier factors. The additional factor is analytical in s, not a kinematical singularity. Why is it there?

Crossing symmetry in tensor formalisms





The tensor amplitude is given by $p_D^{(B)} \cdot p_\pi^{(\rho)}$, where $p_D^{(B)}$ is the breakup momentum in the B frame, and $p_\pi^{(\rho)}$ the decay momentum in the isobar frame

$$A = \frac{m_{B^0}^2 + s - m_{h_3}^2}{2m_{B^0}^2} pq \cos\theta$$

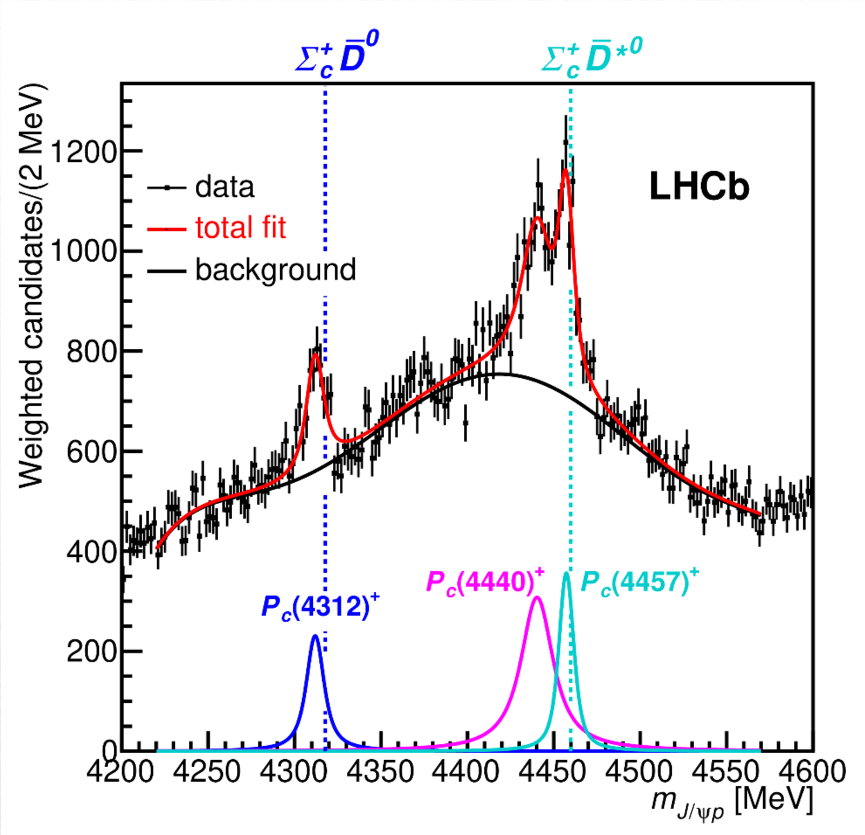
However, one can consider the scattering process just in the isobar rest frame.

$$A = pq\cos\theta$$

By crossing symmetry the amplitudes must be the same.

The usual implementation fails crossing symmetry

New pentaquarks discovered



LHCb, PRL 122, 222001

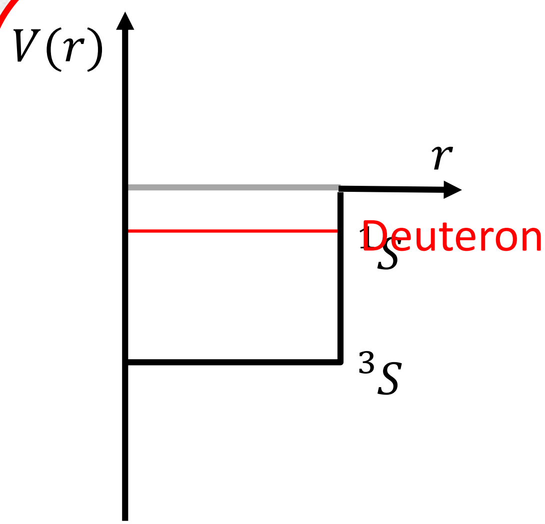
A higher statistics 1D analysis is able to resolve the fine structure of the $P_c(4450)$ peak.

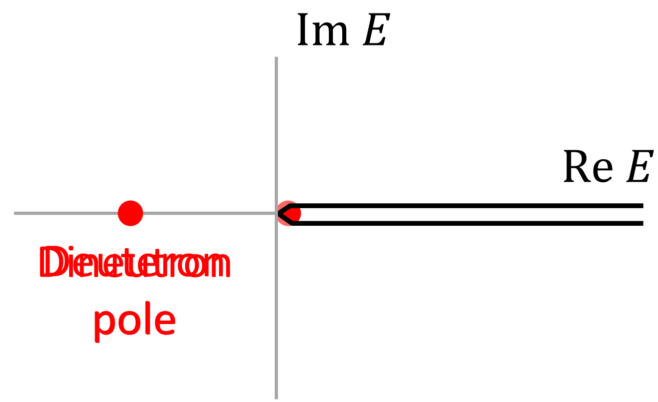
Moreover, a new isolated $P_c(4312)$ at the $\Sigma_c^+ \bar{D}^0$ threshold appears

The old assignment of quantum numbers might be jeopardized by this new finding

Bound and virtual states

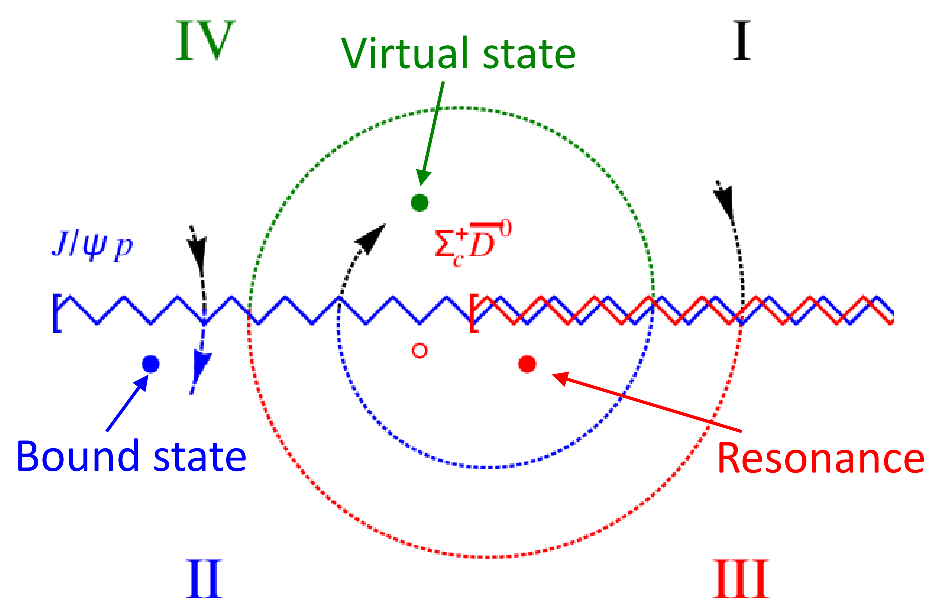
Example from pn scattering
Decreasing the potential strength,
the pole goes to the 2nd sheet (dineutron),
Bound states on the real axis
the pole reaches the threshold
towards 1st sheet (deuteron)





Bound and virtual states

The situation is more complicated when more channels are open



Minimal(istic) model

(LHCb data)

Fernandez-Ramirez, AP *et al.* (JPAC), PRL 123, 092001

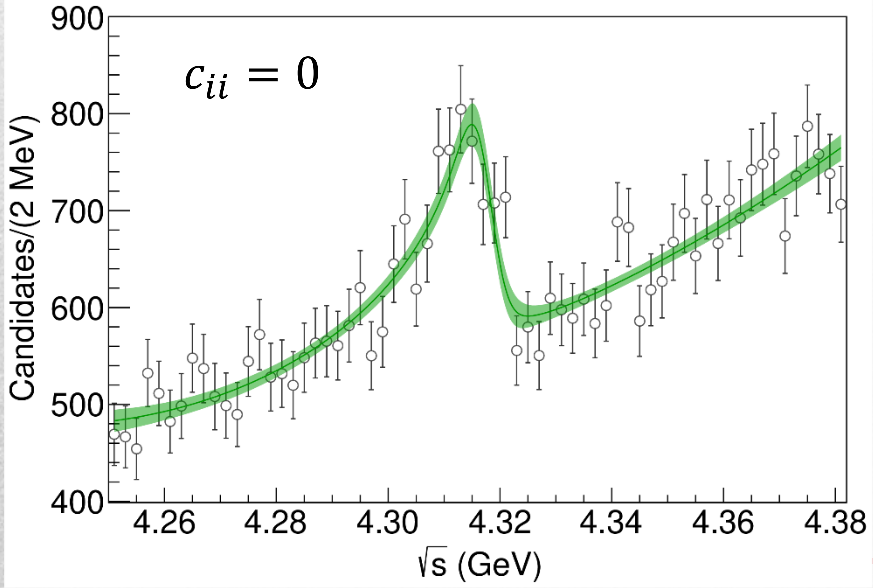
$$\frac{dN}{d\sqrt{s}} = \rho(s)\left[|F(s)|^2 + b_0 + b_1 s\right]$$

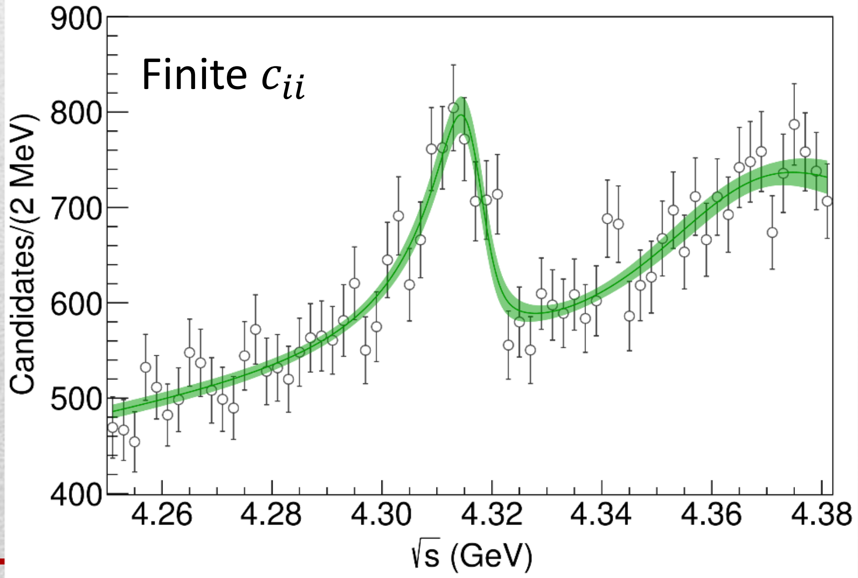
Effective range expansion

$$F(s) = (N_1 + N_2 s)\, T_{11}(s)$$

We can set $c_{ii} = 0$ to reduce to the scattering length approximation

$$T(s) = \begin{pmatrix} m_{11} - c_{11}s - i\rho_1(s) & m_{12} \\ m_{12} & m_{22} - c_{22}s - i\rho_2(s) \end{pmatrix}^{-1}$$





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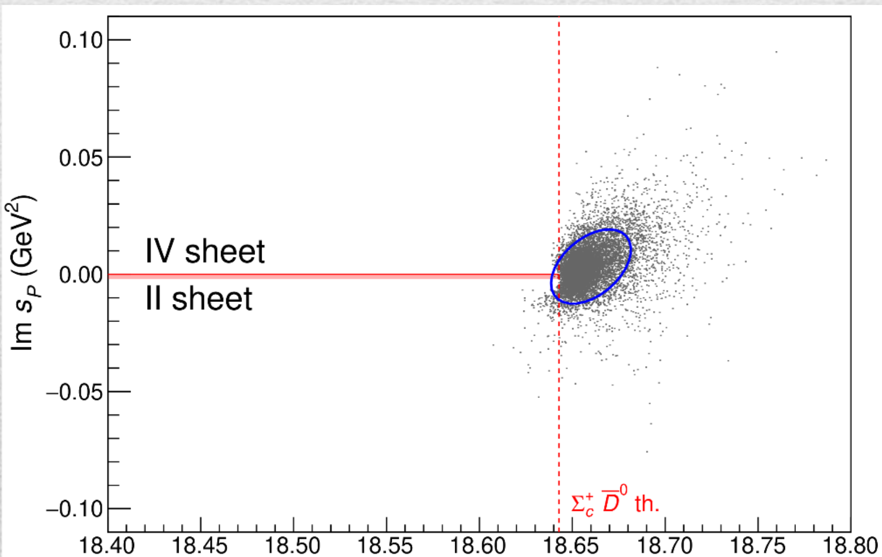
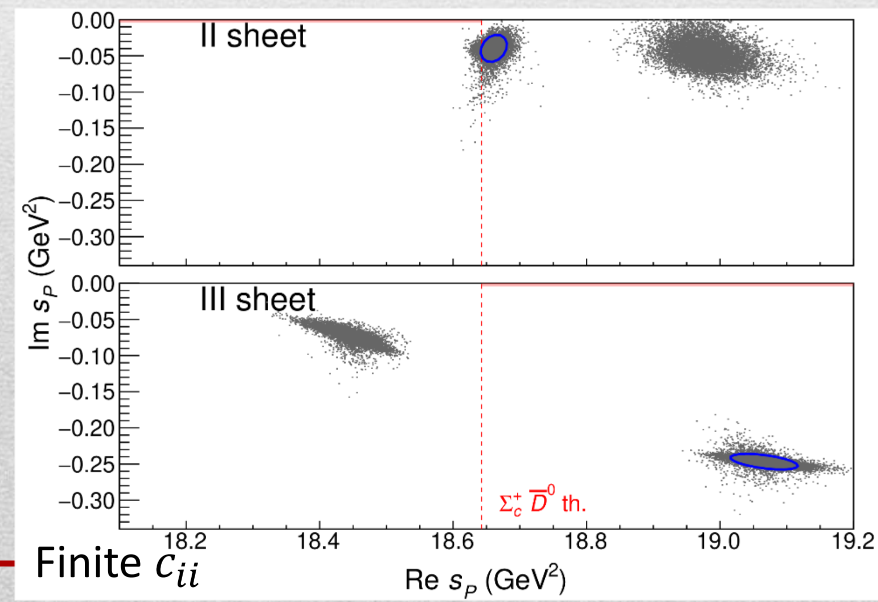


$c_{ii} = 0$



Finite c_{ii}

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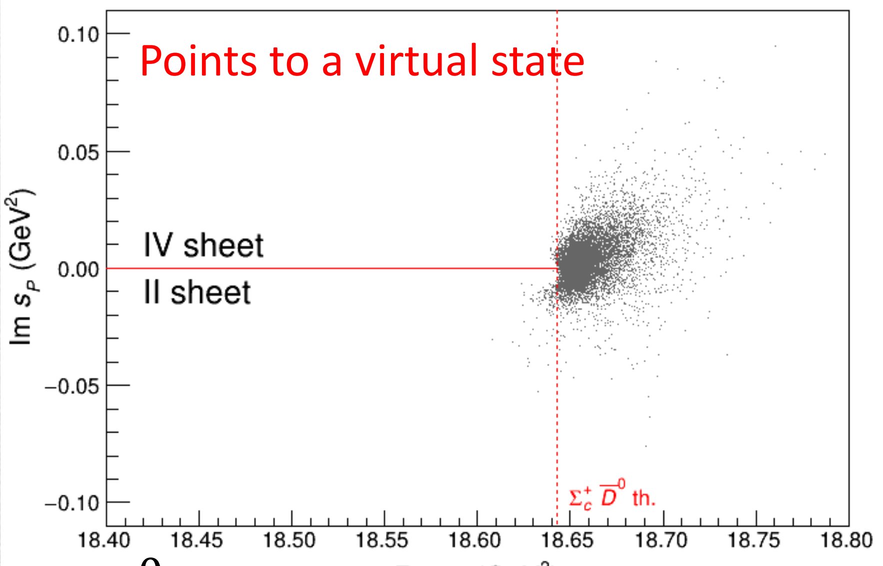

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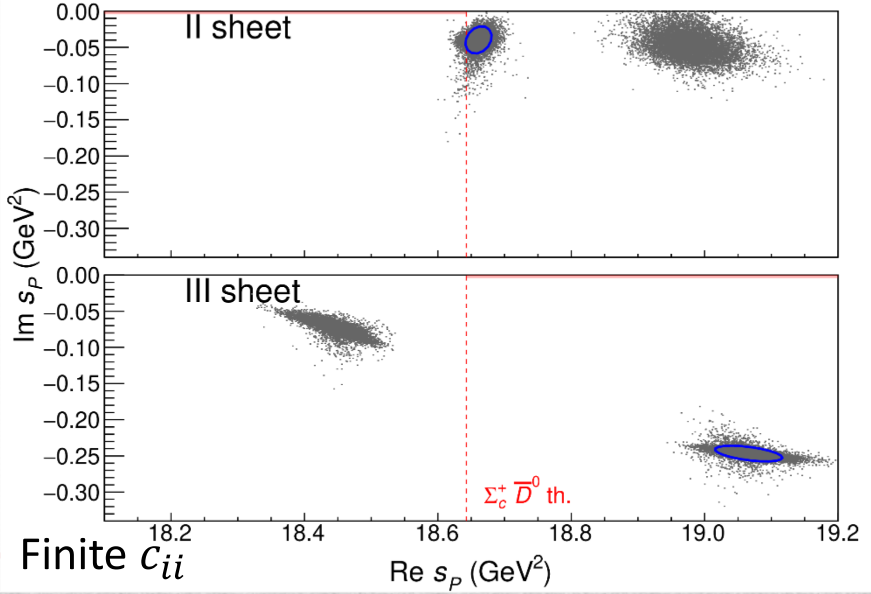

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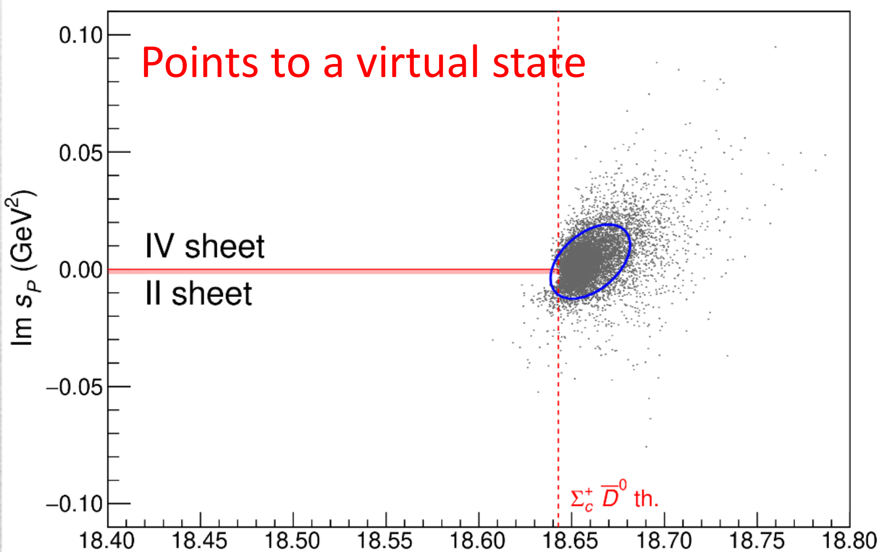

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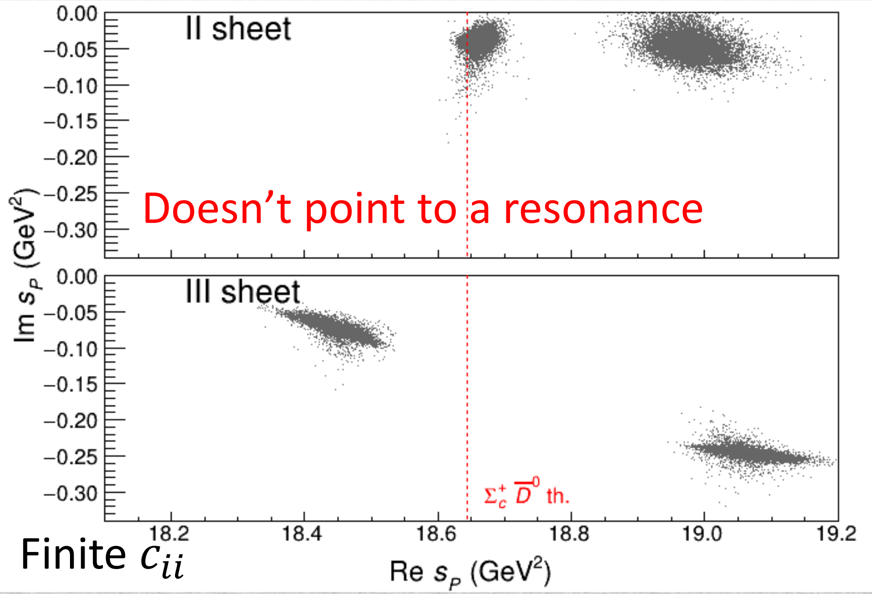

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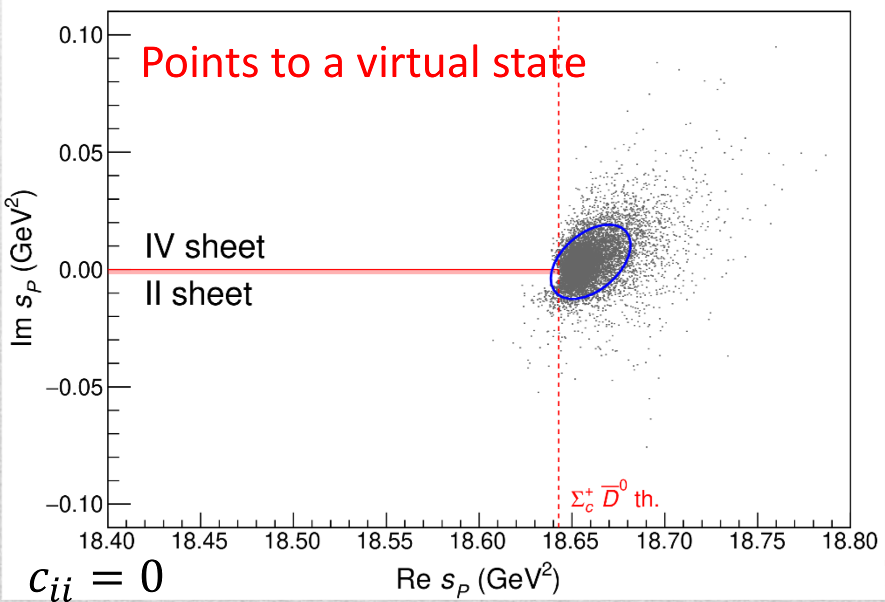

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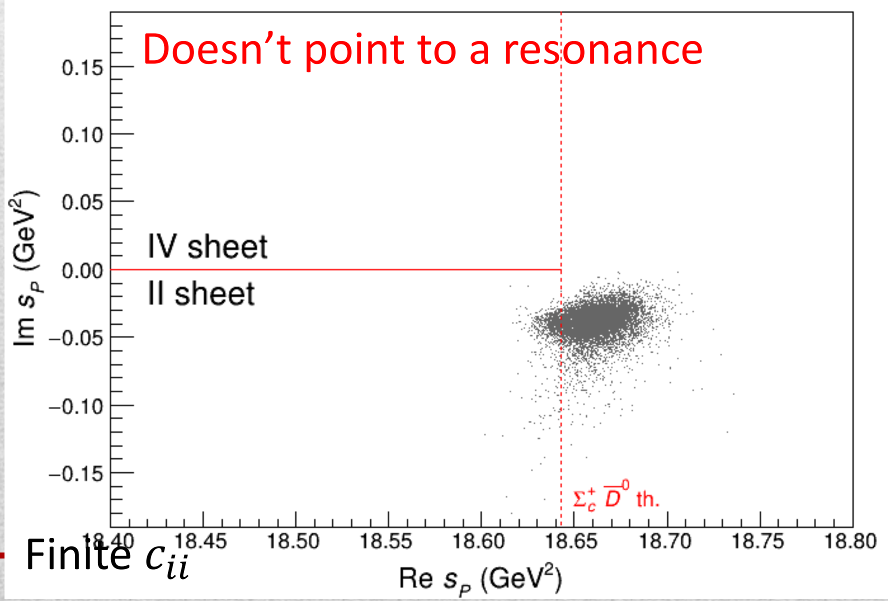

Finite c_{ii}

Conclusions & prospects

The study of exotic heavy quark sector is a challenging task
Experiments are very prolific! Constant feedback on predictions

- Study of spectra and decay patterns will improve our understanding, new data expected by, BESIII, LHCb, Belle II
- Diquark models are useful to describe the XYZ sector, although the role of thresholds must be somehow included
- A more refined study of amplitudes provides a complimentary tool to give insights on the nature of some of the states

Thank you