

# MAGNETIC MOMENT OF CHARMED BARYON

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*Workshop on electromagnetic dipole moments of unstable particles, 3-4 October  
2019, at LAL*

# MAGNETIC MOMENT OF ELEMENTARY PARTICLES

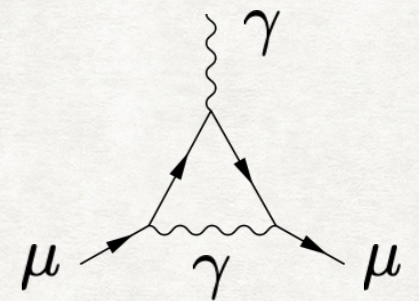
## LEPTON, PROTON AND QUARK

- The spin **1/2 particle** such as leptons (electron, muon...) have a magnetic moment of the form

$$\mu = \frac{g |e| Q}{2 2m}$$

$$g_{\text{electron}} = 2.00231930436182 \pm (2.6 \times 10^{-13})$$

where  $Q$  and  $m$  are the charge and mass of the particle.



- The **g factor is 2 at the classic level** while it is slightly modified by the quantum effect. This correction is called **anomalous magnetic moment** and defined as  $a = (g-2)/2$ .
- There is a longstanding question of **muon anomalous magnetic moment**: the experiment is **3.6 sigma away** from experiment (hint of new physics?)

$$a_{\mu}^{\text{exp.}} = 116592091(54)(33) \times 10^{-11}$$
$$a_{\mu}^{\text{the.}} = 116591803(1)(42)(26) \times 10^{-11}$$

**3.6 $\sigma$  effect!**

# MAGNETIC MOMENT OF ELEMENTARY PARTICLES

## LEPTON, PROTON AND QUARK

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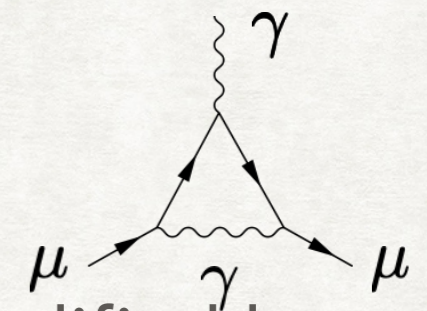
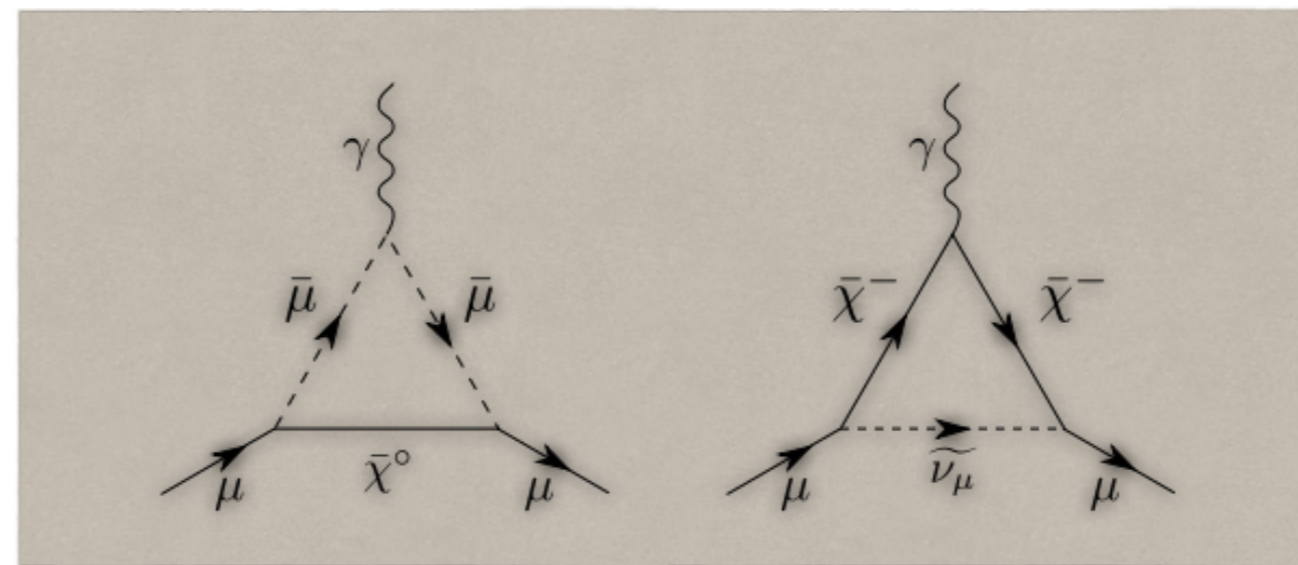
$$\mu = \frac{g |e| Q}{2} \frac{1}{2m}$$

$$g_{\text{electron}} = 2.00231930436182 \pm (2.6 \times 10^{-13})$$

where  $Q$  and  $m$  are

- The **g factor** is the quantum effect of the **magnetic moment** and depends on the particle's spin and mass.

- There is a long-standing discrepancy in the muon's **magnetic moment**: the experiment is **3.6 sigma away** from experiment (hint of new physics?)



modified by **new magnetic**

**magnetic**

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$$a_{\mu}^{\text{the.}} = 116591803(1)(42)(26) \times 10^{-11}$$

**3.6σ effect!**

# MAGNETIC MOMENT OF ELEMENTARY PARTICLES

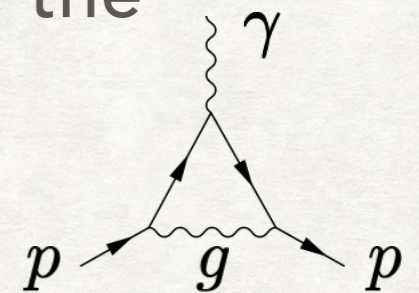
## LEPTON, PROTON AND QUARK

- The proton magnetic moment is also measured very precisely. But **how do we interpret** this result?

$$g_{\text{proton}} = 5.585694702(17)$$

- If we consider the proton to be a fundamental particle, the magnetic moment can be written as

$$\mu = \frac{g_P}{2} \frac{|e|}{2m_P}$$



- $g \gg 2$  can be understood by a large strong interaction effect?
- The proton is **not a fundamental particle**. So this may not be the solution...

$\mu_M = |e|/2M_P$  is called the nuclear magneton

# PROTON MAGNETIC MOMENT IN QUARK MODEL

- In quark model, magnetic moment of proton is a sum of the magnetic moment of the constituent quark (up-up-down) with fully symmetric spin configuration.

$$\mathbf{M} = \sum_q \mathbf{M}_q \quad \mathbf{M}_q = \mu \frac{e_q}{e} \sigma_q$$

$$\mu = |e|/2M_q$$

where  $q$  is the constituent quark and  $\sigma$  is the spin operator

$$\Psi_{\text{spin+flavor}}^{\text{proton}} = [2u \uparrow u \uparrow d \downarrow - u \downarrow u \uparrow d \uparrow - u \uparrow u \downarrow d \uparrow + 2u \uparrow d \downarrow u \uparrow - u \downarrow d \uparrow u \uparrow - u \uparrow d \uparrow u \downarrow + 2d \downarrow u \uparrow u \uparrow - d \uparrow u \downarrow u \uparrow - d \uparrow u \uparrow u \downarrow] / \sqrt{18},$$

- Then the magnetic moment of proton is computed as

$$\begin{aligned} \mu_p &= \langle \phi_P | \mathbf{M}_u + \mathbf{M}_u + \mathbf{M}_d | \phi_P \rangle \\ &= \frac{1}{18} (4 \times 3(2e_u - e_d) + 6 \times e_d) \mu \\ &= \mu = \frac{|e|}{2m_q} \end{aligned}$$

Similar to the previous result but now, the denominator is not proton mass but quark mass!

Using the constituent quark mass  $m_q = 1/3 m_P$ , we find  $g_q = 1.86$

# PROTON MAGNETIC MOMENT IN QUARK MODEL

- In addition, since it is known that the large portion of the proton spin is actually carried by the gluons, gluon contributions to the magnetic moment has to be considered.
- On the other hand, the quark model can predict the relation of proton and neutron/Lambda magnetic moment without quark mass dependence

$$\mu_N = -\frac{2}{3}\mu_p, \quad \mu_\Lambda = -\frac{1}{3}\mu_p$$

$g_{\text{proton}} = 5.585694702(17)$
$g_{\text{neutron}} = -3.82608545(90)$
$g_{\text{lambda}} = -1.226(8)$

which is **satisfied well by the experiment.**

- In any case, the proton magnetic moment is an input for various theoretical computations, it is important to measure it very precisely.

# CHARMED BARYON MAGNETIC MOMENT

- Proposal to measure directly MDM of charm baryons discussed during the workshop
- It will be the first direct measurement of the magnetic moment of the charmed baryon.
- Different from light baryon, spin is known to be carried mostly by the heavy quark (charm quark) —> direct connection to the charm quark (anomalous) magnetic moment.
- LHCb are producing many new results on charmed baryon (e.g. discovery of doubly charged charmed baryon!) and charmed baryon spectroscopy is becoming very interesting.

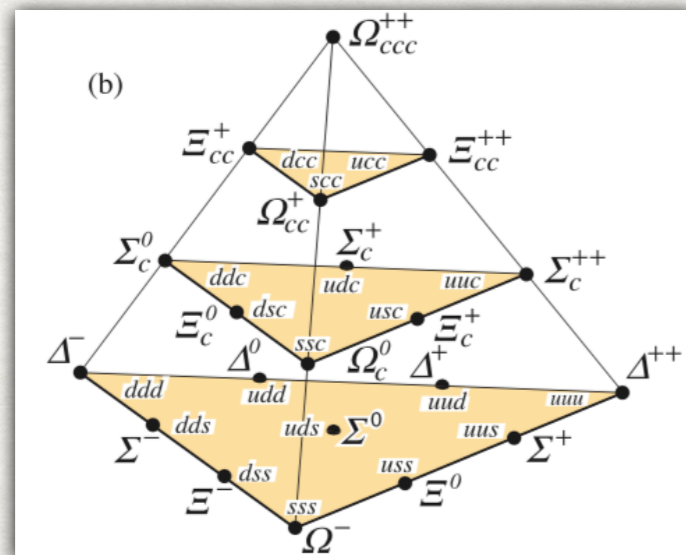
# BARYON SPECTROSCOPY

## CHARMED BARYONS

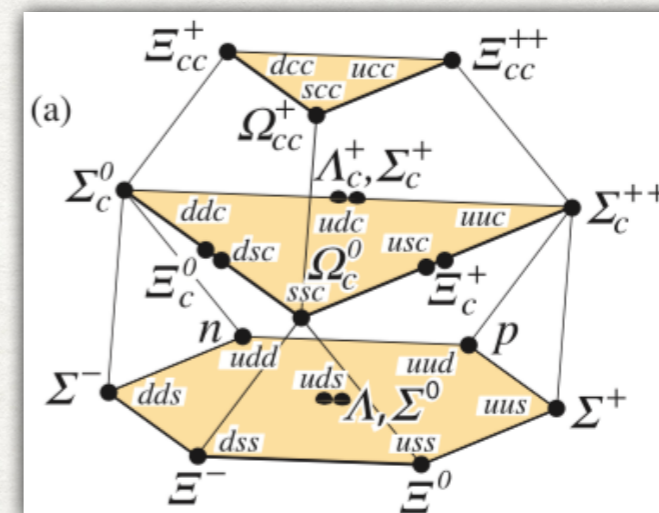
- Let us now include charm quark. SU(4) symmetry for 3 quark states

$$4 \otimes 4 \otimes 4 = 20_s \oplus 20_s \oplus 20_a \oplus 4_a$$

- Considering charmed baryons, this results in 6 spin 3/2 baryons and 9 spin 1/2 baryons.
- Different from the light baryons, not only  $\Lambda_c$  ( $udc$ ) and  $\Sigma_c^+$  ( $udc$ ), but also  $\Xi_c^+$  ( $usc$ ) and  $\Xi_c^0$  ( $dsc$ ) are no longer degenerated.
- New names are NOT given to these particles but to distinguish them, they are sometimes called ( $\Xi_{c1}, \Xi_{c2}$ ) or ( $\Xi_c, \Xi_c'$ ).



J=3/2  
totally-symmetric  
decuplet

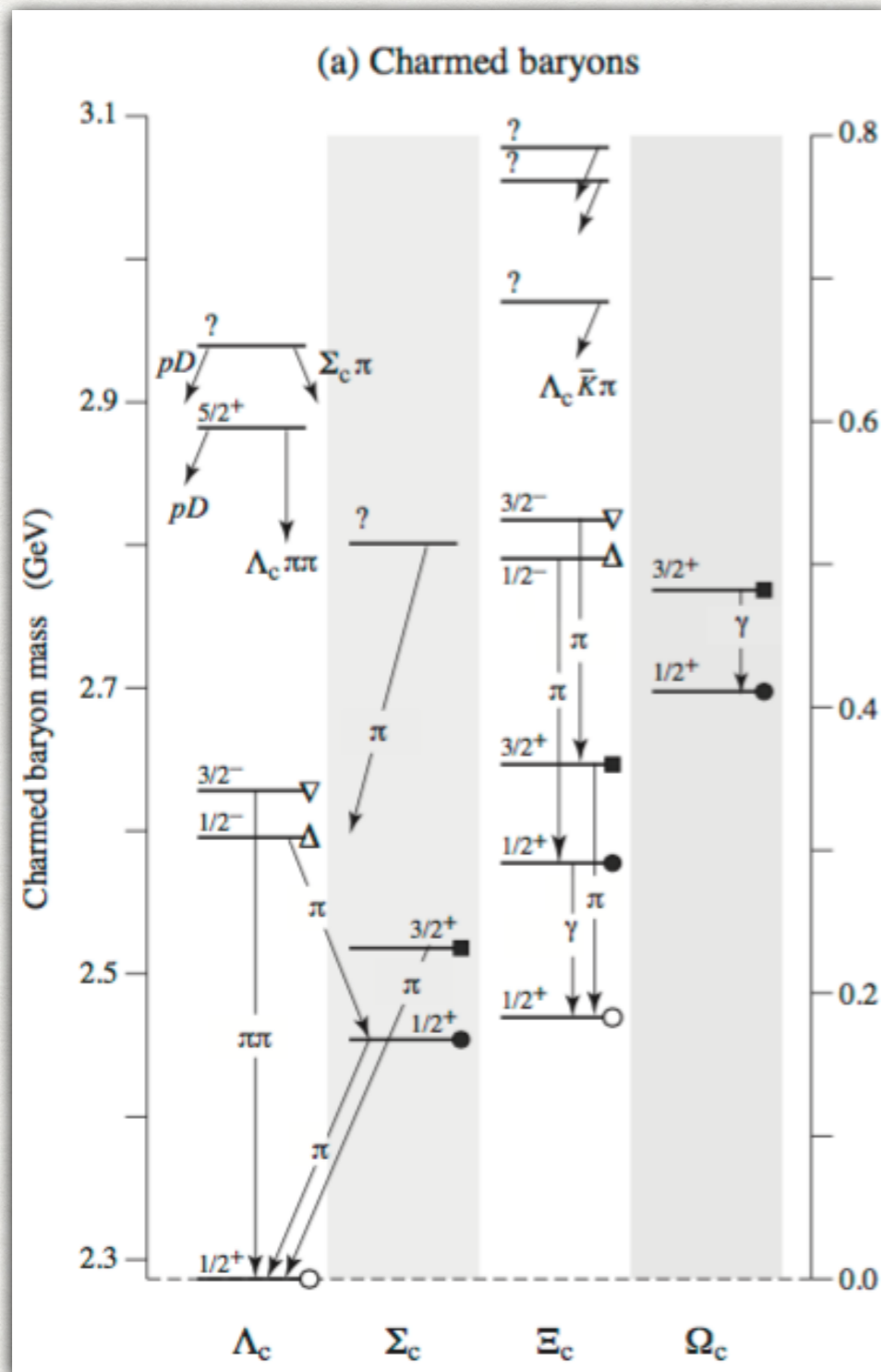


J=1/2  
mixed-symmetry  
octet



# QUESTION OF TWO $\Xi_c$ STATES

$\Xi_{c1}^+$ ,  $\Xi_{c2}^{+??}$



At heavy quark limit :

- : the anti-symmetric 1/2 ( $\Lambda_c, \Xi_{c1}^+, \Xi_{c1}^0$ )
- : the symmetric 1/2 ( $\Sigma_c^0, \Xi_{c2}^+, \Xi_{c2}^0$ )
- : the symmetric 3/2 ( $\Sigma_c^{0*}, \Xi_c^{+*}, \Xi_c^{0*}$ )

But this has never been confirmed...  
The observed state can be a mixture of  $\Xi_{c1}$  and  $\Xi_{c2}$ . How can we distinguish?

**We show below that magnetic moment, which measures directly the quark spin-configuration, is the most powerful tool to distinguish different charmed baryon states!**

# $\Lambda_c$ MAGNETIC MOMENT

- We compute the  $\Lambda_c$  ( $udc$ , spin anti-symmetric state) magnetic moment in the quark model. The result turns out that  $\Lambda_c$  magnetic moment is equal to the charm quark magnetic moment.

$$\begin{aligned}\mu_{\Lambda_c} &= \langle \phi_{\Lambda_c} | \mathbf{M}_u + \mathbf{M}_d + \mathbf{M}_c | \phi_{\Lambda_c} \rangle \\ &= \mu_c\end{aligned}$$

- Using the definition

$$\mu_{\Lambda_c} \left( = \frac{g_{\Lambda_c}}{2} \frac{|e|Q_c}{2m_P} \right) = \mu_c \left( = \frac{g_c}{2} \frac{|e|Q_c}{2m_c} \right)$$

the measurement of  $g_{\Lambda_c}/2$  can be translated to the charm quark magnetic moment  $g_c/2$

$$\frac{g_{\Lambda_c}}{2} = \frac{m_{\Lambda_c}}{m_c} \frac{g_c}{2}$$

- If the measured  $g_{\Lambda_c}/2$  is far from  $m_{\Lambda_c}/m_c \sim 1.3-1.9$  ( $m_c = 1.2-1.8$  GeV) then, that is an indication of large anomalous magnetic moment of charm quark.
- Nevertheless, the quantitative statement is model-dependent (interactions of heavy quark and photons inside the hadron, ...).

$$\frac{\mu(\Lambda_c^+)}{\mu_N} = 0.37-0.42,$$

Brown mock argument...

# OTHER CHARMED BARYON MAGNETIC MOMENT

- Spin anti-symmetric state

$$\mu_{\Xi_c^{0,+}} = \mu_c$$

~0.39 N.M

which is the same as  $\Lambda_c$  magnetic moment

*Corrections to the relation  
Savage et al PLB326 ('94)  
Banuls et al PRD61 ('00)*

- Spin symmetric state

$$\mu_{\Sigma_c^+} = -\frac{1}{3}\mu_c + \frac{2}{3}\mu_u + \frac{2}{3}\mu_d, \quad \mu_{\Sigma_c^0} = -\frac{1}{3}\mu_c + \frac{4}{3}\mu_d$$

~0.54 N.M

@SU(3) limit  
 $\mu_{\Sigma_c} = \mu_{\Xi_c'}$

~  
1.46 N.M.

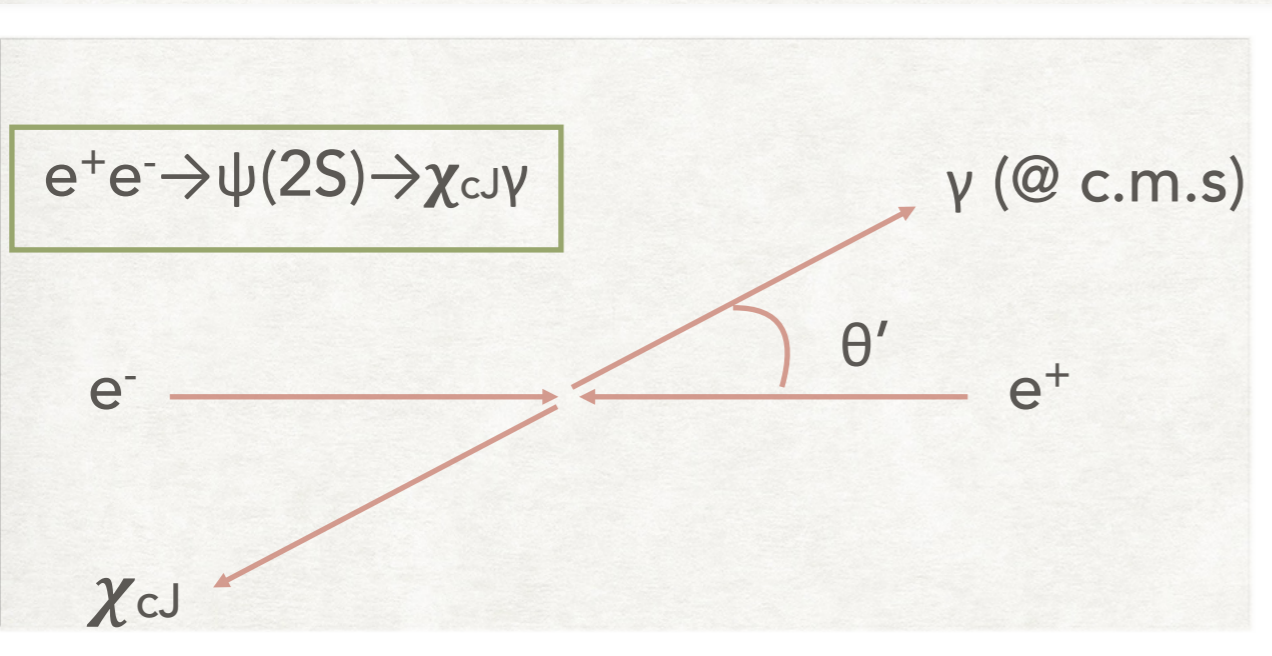
$$\mu_{\Xi_c'^+} = -\frac{1}{3}\mu_c + \frac{2}{3}\mu_u + \frac{2}{3}\mu_s, \quad \mu_{\Xi_c'^0} = -\frac{1}{3}\mu_c + \frac{2}{3}\mu_d + \frac{2}{3}\mu_s$$

**If heavy quark limit is correct and  $\Xi_c$  ( $\Xi_c'$ ) state is purely anti-symmetric (symmetric) state, we would observe**

$$\mu_{\Lambda_c} = \mu_{\Xi_c^0} \gg \mu_{\Xi_c'^0}$$

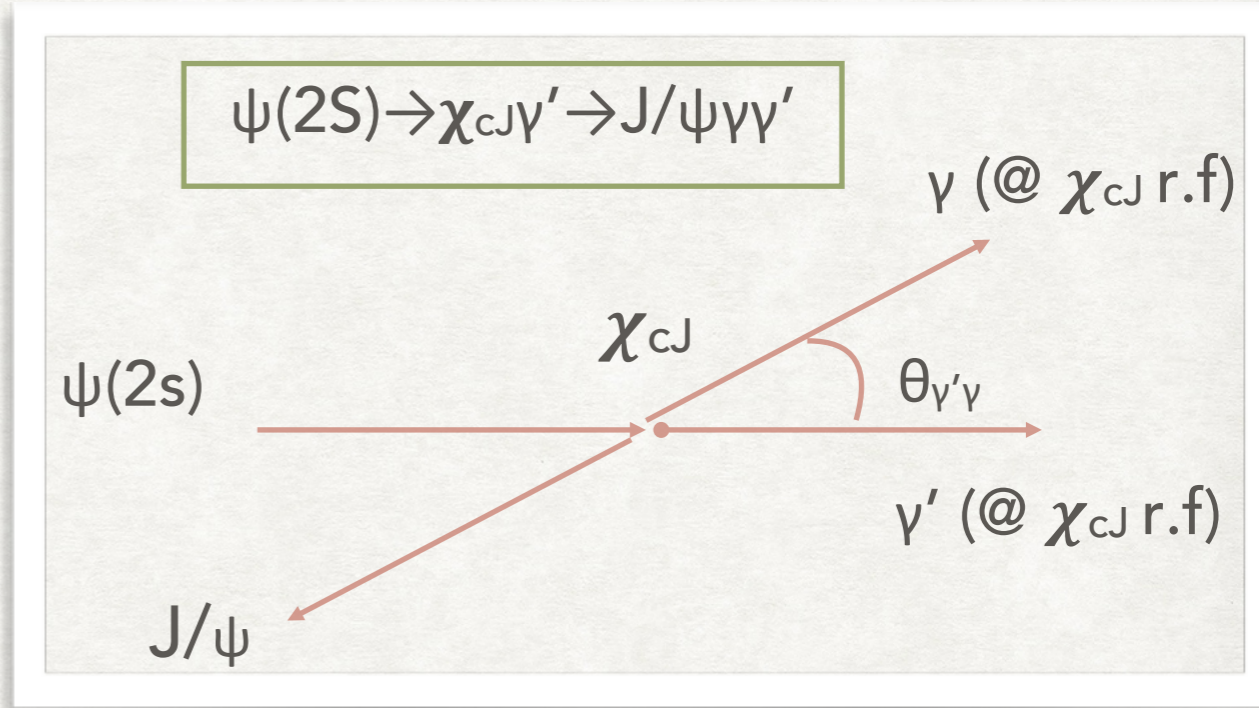
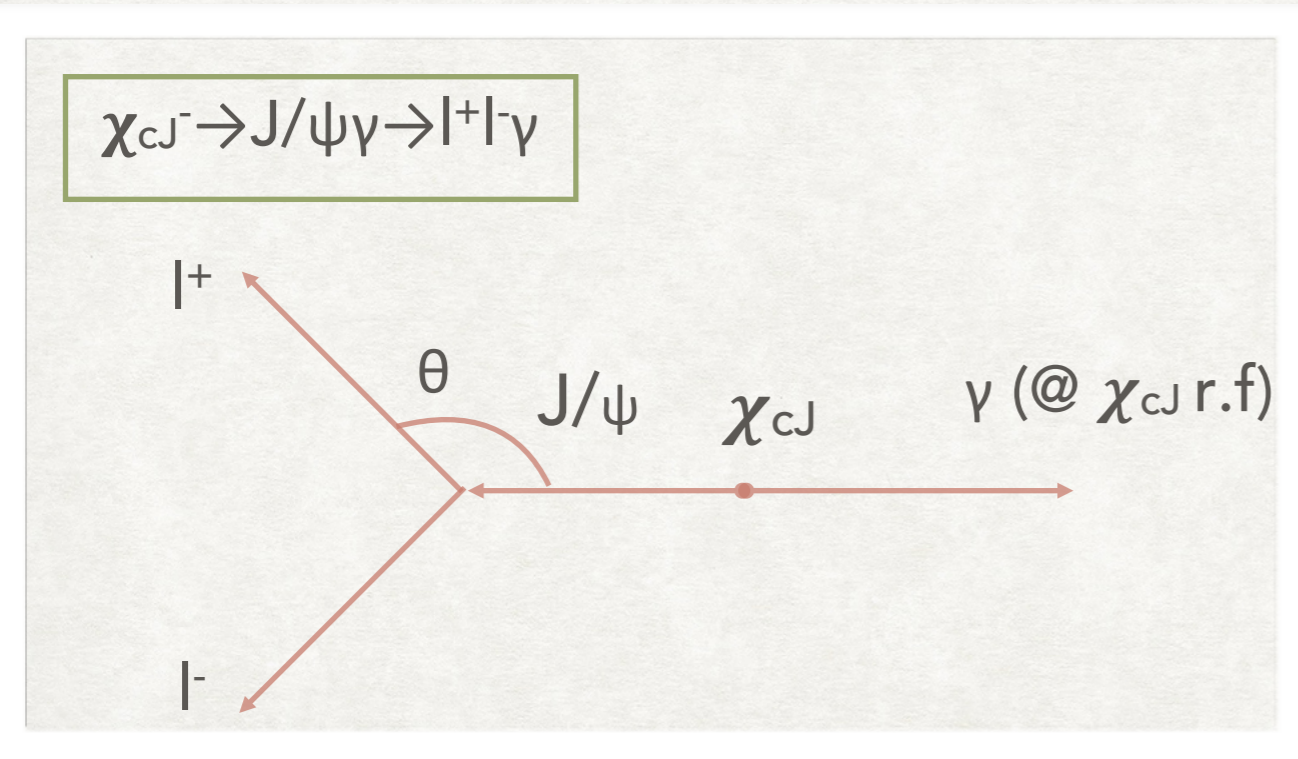
# PREDICTING $\Lambda_c$ MAGNETIC MOMENT WITH BESIII RESULT

Karl et al PR D13 '76



$$\begin{matrix} C & (\frac{\vec{r}}{2}, \frac{\vec{v}}{2}, \vec{\sigma}_1) \\ \bar{C} & (-\frac{\vec{r}}{2}, -\frac{\vec{v}}{2}, \vec{\sigma}_2) \end{matrix}$$

The charm quark magnetic moment can be determined by the charmonium radiative decays



5 angles to disentangle different contributions  
 $(\theta, \varphi, \theta', \theta_{\gamma'\gamma}, \varphi_{\gamma'\gamma})$

# PREDICTING $\Lambda_C$ MAGNETIC MOMENT WITH BESIII RESULT

$a_i^J, b_i^J =$  helicity amplitudes

arXiv: 1701.01197 (also see CLEO 0910.0046)

TABLE I. Fit results for  $a_{2,3}^J$  and  $b_{2,3}^J$  for the process of  $\psi(3686) \rightarrow \gamma_1 \chi_{c1,2} \rightarrow \gamma_1 \gamma_2 J/\psi$ ; the first uncertainty is statistical, and the second is systematic. The  $\rho_{a_{2,3}^J b_{2,3}^J}^J$  are the correlation coefficients between  $a_{2,3}^J$  and  $b_{2,3}^J$ .

$\chi_{c1}$	$a_2^1 = -0.0740 \pm 0.0033 \pm 0.0034, b_2^1 = 0.0229 \pm 0.0039 \pm 0.0027$ $\rho_{a_2 b_2}^1 = 0.133$
$\chi_{c2}$	$a_2^2 = -0.120 \pm 0.013 \pm 0.004, b_2^2 = 0.017 \pm 0.008 \pm 0.002$ $a_3^2 = -0.013 \pm 0.009 \pm 0.004, b_3^2 = -0.014 \pm 0.007 \pm 0.004$ $\rho_{a_2 b_2}^2 = -0.605, \rho_{a_2 a_3}^2 = 0.733, \rho_{a_2 b_3}^2 = -0.095$ $\rho_{a_3 b_2}^2 = -0.422, \rho_{b_2 b_3}^2 = 0.384, \rho_{a_3 b_3}^2 = -0.024$

	theory
$b_2^1/b_2^2 = 1.35 \pm 0.72,$	$\leftarrow 1.000 \pm 0.015$
$a_2^1/a_2^2 = 0.617 \pm 0.083.$	$\leftarrow 0.676 \pm 0.071$

Extracting anomalous magnetic moment

$$1 + \kappa = - \frac{4m_c}{E_{\gamma_2}[\chi_{c1} \rightarrow \gamma_2 J/\psi]} a_2^1$$

$$= 1.140 \pm 0.051 \pm 0.053 \pm 0.229,$$

$$= \frac{g_c}{2}$$

error from charm mass  
 $mc = 1.5 \pm 0.3 \text{ GeV}$

# PREDICTING $\Lambda_c$ MAGNETIC MOMENT

- Using the BES III data, we can predict magnetic moment of  $\Lambda_c$

$$g_c/(2m_c) = 0.76 \pm 0.05 \text{ GeV}^{-1} \rightarrow \mu_{\Lambda_c} = (0.48 \pm 0.03) \mu_N$$

*without charm mass ambiguity.*

This value should be compared with theoretical predictions (where charm quark mass is often obtained from other observable)

$$\frac{\mu(\Lambda_c^+)}{\mu_N} = 0.37-0.42,$$

which implies a slightly higher charm anomalous magnetic moment ( $g_c > 2$ ?).

**Higher precision measurements on BOTH at a few % precision desired!**

magnetic moment of  $\Lambda_c$

&

charm radiative decays

# CONCLUSIONS

- Charm quark magnetic moment has **never been measured directly**.
- A new idea to measure the magnetic moment of charmed baryon  $\mu_{\Lambda_c}$ , using the bent-crystal is proposed (a high precision expected).  $\mu_{\Lambda_c}$  can be translated to the magnetic moment of charm quark,  $\mu_c$ .
- We showed that the measurement of various charmed baryon, such as  $\Xi_c$  can provide **important information on the charmed baryon spectroscopy**.
- **Charmonium radiative decay** can indirectly provide the charm quark magnetic moment. Using BESIII result, we made an estimate on  $\mu_{\Lambda_c}$ . We found that the obtained value of  $\mu_{\Lambda_c}$  is **slightly higher** than the theory predictions.
- Thus, **we conclude that a few % level precision for both  $\Lambda_c$  polarisation and the radiative charmonium decays are desired**.
- We are currently working on LHCb measurement of polarisation and weak parameter of  $\Lambda_c$ , which is a crucial factor for  $\mu_{\Lambda_c}$  determination.

# BARYON SPECTROSCOPY

## LIGHT BARYONS

- Let us start with the light baryons. SU(3) symmetry for 3 quark states

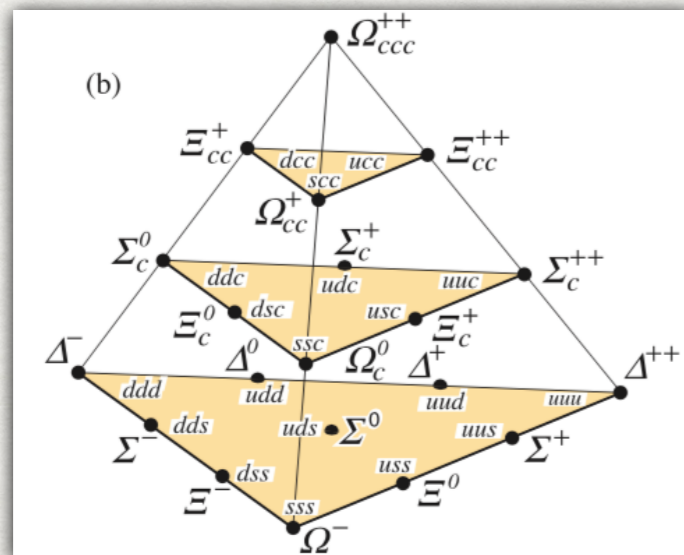
$$3 \otimes 3 \otimes 3 = 10_s \oplus 8_s \oplus 8_a \oplus 1_a$$

where s and a corresponds to symmetric and anti-symmetric of flavour, e.g.

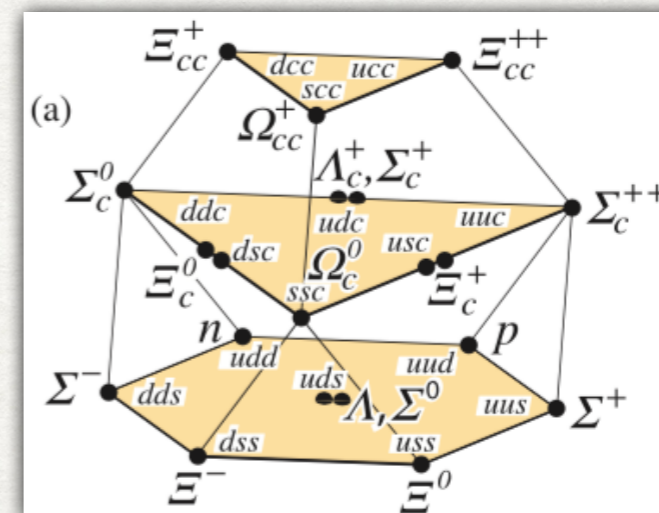
*Symmetric*:  $uuu, ddd, \frac{1}{\sqrt{2}} [ud + du] u, \frac{1}{\sqrt{2}} [us + su] u \dots$

*Anti - Symmetric*:  $\frac{1}{\sqrt{2}} [ud - du] u, \frac{1}{\sqrt{2}} [us - su] u \dots$

- This results in 10 spin 3/2 baryons and 8 spin 1/2 baryons



J=3/2  
totally-symmetric  
decuplet



J=1/2  
mixed-symmetry  
octet



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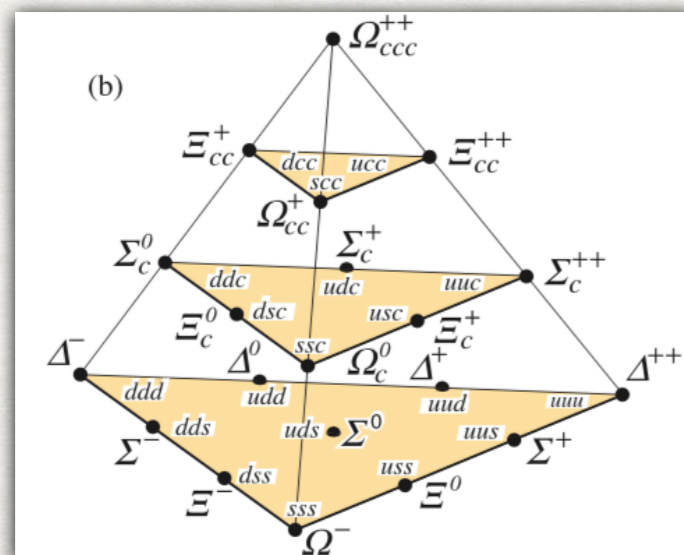
### NOTE ON OCTET STATES:

8s and 8a states are mixed.

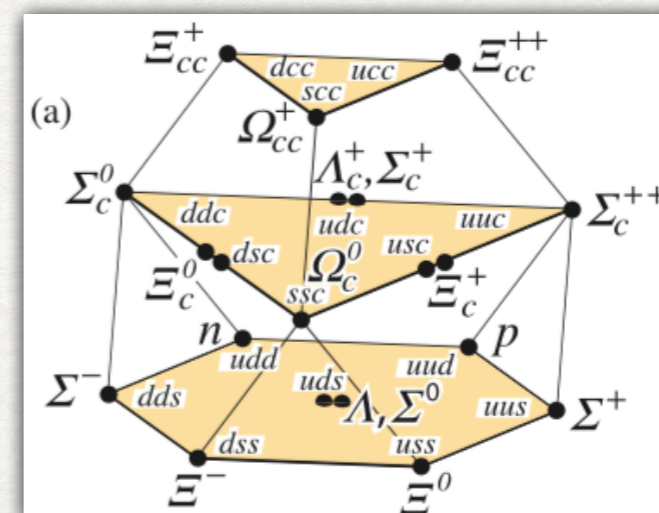
The  $qqq'$  states (like proton= $uud$ ), multiplying with the spin symmetry, they turn out to have exactly the same wave function.

For  $\Lambda$  and  $\Sigma^0$  ( $uds$  states), the wave function is different:  $\Lambda$  is anti-symmetric and  $\Sigma^0$  is symmetric.

- This



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totally-symmetric  
decuplet



J=1/2  
mixed-symmetry  
octet