

# SM tests with $e$ , $\mu$ , $\tau$ magnetic moments

Massimo Passera  
INFN Padova

Workshop on electromagnetic dipole moments  
of unstable particles  
Milano 3-4 Oct 2019

- **e**: Testing new physics with the electron  $g-2$
- **$\mu$** : The muon  $g-2$ : recent theory progress
- **$\tau$** : The tau  $g-2$ : opportunities or fantasies?

$$g = 2$$

- Uhlenbeck and Goudsmit in 1925 proposed for electrons

$$\begin{aligned}\vec{\mu} &= g \frac{e}{2m} \vec{s} \\ g &= \underline{2} \quad (\text{not } 1!)\end{aligned}$$

- Dirac 1928:

$$(i\partial_\mu - eA_\mu) \gamma^\mu \psi = m\psi$$

- A Pauli term in Dirac's eq would give a deviation...

$$a \frac{e}{2m} \sigma_{\mu\nu} F^{\mu\nu} \psi \quad \rightarrow \quad g = 2(1 + a)$$

...but there was no need for it!  $g=2$  stood for ~20 yrs.

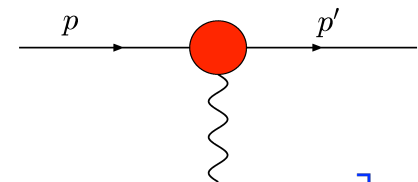
- **Kusch and Foley 1948:**

$$\left(\frac{g_e}{2}\right)^{\text{exp}} \equiv 1 + a_e^{\text{exp}} = 1.00119 \pm 0.00005$$

- **Schwinger 1948 (triumph of QED!):**

$$\left(\frac{g_e}{2}\right)^{\text{th}} \equiv 1 + a_e^{\text{th}} = 1.00116 \dots$$

- **We keep studying the lepton- $\gamma$  vertex:**



$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p') \left[ \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2) + \dots \right] u(p)$$

$$F_1(0) = 1 \quad F_2(0) = a_l$$

A pure “quantum correction” effect!

# Testing new physics with the electron $g-2$

# The QED prediction of the electron g-2



$$a_e^{\text{QED}} = + (1/2)(\alpha/\pi) - 0.328\,478\,444\,002\,55(33)(\alpha/\pi)^2$$

Schwinger 1948 Sommerfeld; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\,478\,965\,579\,193\,78\dots$$

$O(10^{-18})$  in  $a_e$

$$A_2^{(4)}(m_e/m_\mu) = 5.197\,386\,68(26) \times 10^{-7}$$

$$A_2^{(4)}(m_e/m_\tau) = 1.837\,98(33) \times 10^{-9}$$

$$+ 1.181\,234\,016\,816(11)(\alpha/\pi)^3$$

$O(10^{-19})$  in  $a_e$

Kinoshita; Barbieri; Laporta, Remiddi, ..., Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\,241\,456\,587\dots$$

$$A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,62(27) \times 10^{-6}$$

$$A_2^{(6)}(m_e/m_\tau) = -6.5830(11) \times 10^{-8}$$

$$A_3^{(6)}(m_e/m_\mu, m_e/m_\tau) = 1.909\,82(34) \times 10^{-13}$$

$$- 1.9113213917(12)(\alpha/\pi)^4$$

$O(10^{-20})$  in  $a_e$

Kinoshita & Lindquist '81, ..., Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2015 & 2017; Kurz, Liu, Marquard & Steinhauser 2014. Laporta, arXiv:1704.06996 (mass independent term)

$$+ 6.73(16)(\alpha/\pi)^5$$

**Complete Result! (12672 mass indep. diagrams!)**

Aoyama, Hayakawa, Kinoshita, Nio, 2012, 2019. Volkov 1909.08015:  $A_1^{(10)}$ [no left loops] at variance.

$1.1 \cdot 10^{-14}$  in  $a_e$  NB:  $(\alpha/\pi)^6 \sim O(10^{-16})$

The SM prediction is:

$$a_e^{\text{SM}}(\alpha) = a_e^{\text{QED}}(\alpha) + a_e^{\text{EW}} + a_e^{\text{HAD}}$$

The EW (1&2 loop) term is: Czarnecki, Krause, Marciano '96, Jegerlehner 2017

$$a_e^{\text{EW}} = 0.3053(23) \times 10^{-13}$$

The Hadronic contribution, at LO+NLO+NNLO, is:

Nomura & Teubner '12, Jegerlehner 2017; Krause'97; Kurz, Liu, Marquard & Steinhauser 2014

$$a_e^{\text{HAD}} = 16.93(12) \times 10^{-13}$$

$$a_e^{\text{HLO}} = +18.490(108) \times 10^{-13}$$

$$a_e^{\text{HNLO}} = [-2.213(12)_{\text{vac}} + 0.37(5)_{\text{lbl}}] \times 10^{-13}$$

$$a_e^{\text{HNNLO}} = +0.28(1) \times 10^{-13}$$

Which value of  $\alpha$  should we use to compute  $a_e^{\text{SM}}$ ?

- The 2008 measurement of the electron g-2 is:

$$a_e^{\text{EXP}} = 11596521807.3 (2.8) \times 10^{-13} \quad \text{Hanneke et al, PRL100 (2008) 120801}$$

vs. old (factor of 15 improvement,  $1.8\sigma$  difference):

$$a_e^{\text{EXP}} = 11596521883 (42) \times 10^{-13} \quad \text{Van Dyck et al, PRL59 (1987) 26}$$

- Equate  $a_e^{\text{SM}}(\alpha) = a_e^{\text{EXP}}$  → “g<sub>e-2</sub>” determination of alpha:

$$\alpha^{-1} = 137.035\,999\,150 (33) \quad [0.24 \text{ ppb}]$$

- Compare it with the present best determination of alpha:

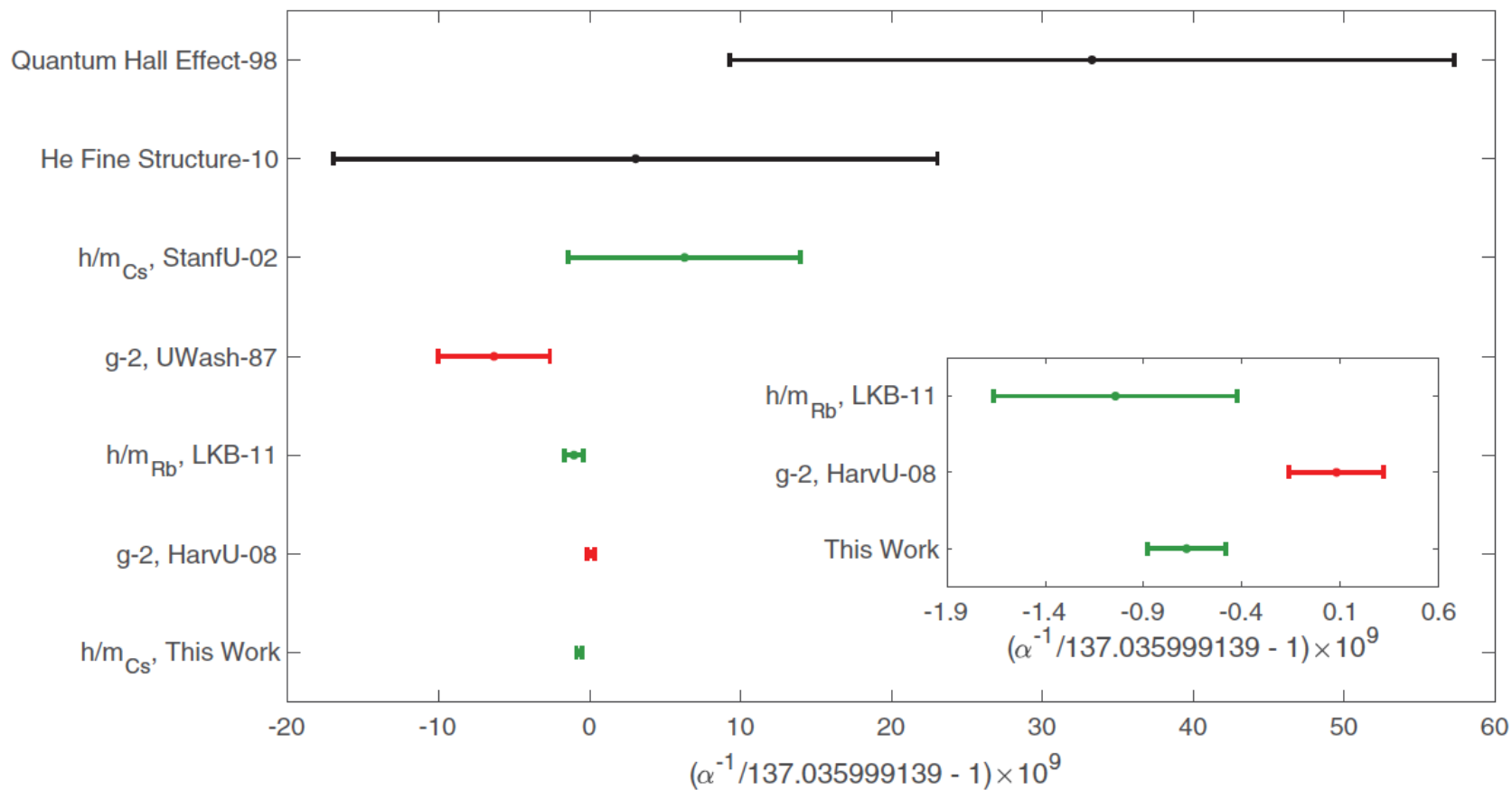
$$\alpha^{-1} = 137.035\,999\,046 (27) \quad [0.20 \text{ ppb}] \quad \text{Science 360 (2018) 191 (Cs)}$$

(was  $\alpha^{-1} = 137.035\,998\,995 (85) [0.62 \text{ ppb}]$  PRL106 (2011) & CODATA 2016 )

**2.4 sigma discrepancy**



# Determinations of alpha



Richard H. Parker, Chenghui Yu, Weicheng Zhong, Brian Estey, Holger Müller  
 Science 360 (2018) 191

- Using  $\alpha = 1/137.036\,999\,046\,(27)$  [Cs 2018], the SM prediction for the electron g-2 is:

$$a_e^{\text{SM}} = 115\,965\,218\,16.1\,(0.1)\,(0.1)\,(2.3) \times 10^{-13}$$

$\delta C_5^{\text{qed}}$

$\delta a_e^{\text{had}}$

from  $\delta\alpha$

- The (EXP - SM) difference is:

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -8.8\,(3.6) \times 10^{-13}$$

i.e. 2.4 sigma difference. Note the negative sign!  
(the 5-loop contrib. to  $a_e^{\text{QED}}$  is  $4.6 \times 10^{-13}$ )

- The present sensitivity is  $\delta\Delta a_e = 3.6 \times 10^{-13}$ , ie ( $10^{-13}$  units):

$$\underbrace{(0.1)_{\text{QED5}}, \quad (0.1)_{\text{HAD}}, \quad (2.3)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\text{EXP}}}}_{(0.2)_{\text{TH}}}$$

- The  $(g-2)_e$  exp. error may soon drop below  $10^{-13}$  and work is in progress to further reduce the error induced by  $\delta\alpha \rightarrow$   
**sensitivity below  $10^{-13}$  may be reached with ongoing exp work**
- In a broad class of BSM theories, contributions to  $a_l$  scale as

$$\frac{\Delta a_{l_i}}{\Delta a_{l_j}} = \left( \frac{m_{l_i}}{m_{l_j}} \right)^2 \quad \text{This Naive Scaling leads to:}$$

$$\Delta a_e = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13}; \quad \Delta a_\tau = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.8 \times 10^{-6}$$

- The sensitivity in  $\Delta a_e$  may soon drop below  $10^{-13}$ ! This will bring  $a_e$  to play a pivotal role in probing new physics in the leptonic sector.
- NP scenarios exist which **violate Naive Scaling**. They can lead to larger effects in  $\Delta a_e$  and contributions to EDMs, LFV or lepton universality breaking observables.

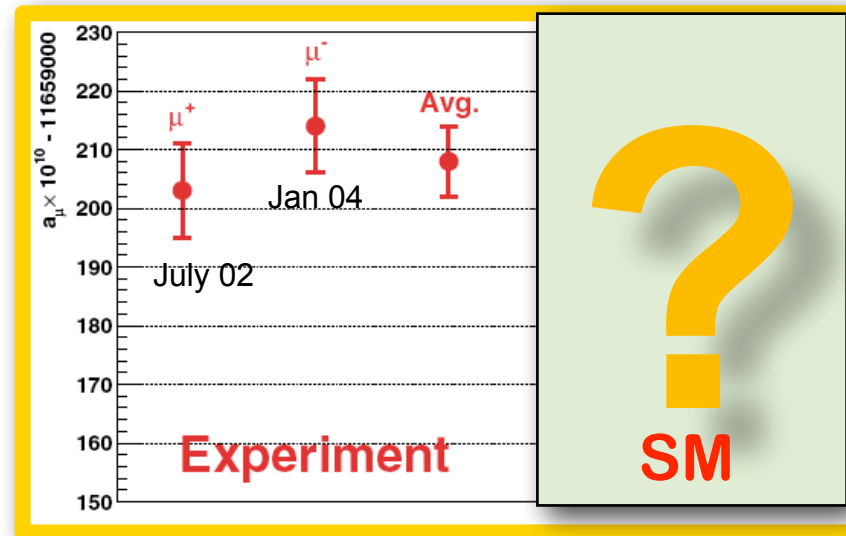
Giudice, Paradisi & MP, JHEP 2012

Crivellin, Hoferichter, Schmidt-Wellenburg, PRD 2018

- One real scalar with a mass of  $\sim 250-1000$  MeV could explain the deviations in  $a_\mu$  and  $a_e$ , through one- and two-loop processes, respectively.

Davoudiasl & Marciano, PRD 2018

# The muon $g-2$ : recent theory progress



- BNL 821:  $a_\mu^{\text{EXP}} = (116592089 \pm 54_{\text{stat}} \pm 33_{\text{sys}}) \times 10^{-11}$  [0.5ppm].
- New muon g-2 experiments at:
  - Fermilab E989: aims at  $\pm 16 \times 10^{-11}$ , ie 0.14ppm.  
First two data taking completed. Analysis in progress.  
First result expected very soon with  $\sim$  BNL E821 precision.
  - J-PARC proposal: phase-1 start with 0.46ppm (TDR 2017).
- Are theorists ready for this (amazing) precision? Not yet!

See Venanzoni's talk

# The muon g-2: the QED contribution



$$a_{\mu}^{\text{QED}} = (1/2)(\alpha/\pi) \quad \text{Schwinger 1948}$$

$$+ 0.765857426 (16) (\alpha/\pi)^2$$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$$+ 24.05050988 (28) (\alpha/\pi)^3$$

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek; MP '04;  
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$$+ 130.8780 (60) (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '04, '05;  
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;  
Steinhauser et al. 2013, 2015 & 2016 (all electron &  $\tau$  loops, analytic);  
Laporta, PLB 2017 (mass independent term). **COMPLETED!**

$$+ 750.86 (88) (\alpha/\pi)^5 \quad \text{COMPLETED!}$$

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta, ...  
Aoyama, Hayakawa, Kinoshita, Nio 2012, 2015, 2017 & 2019.

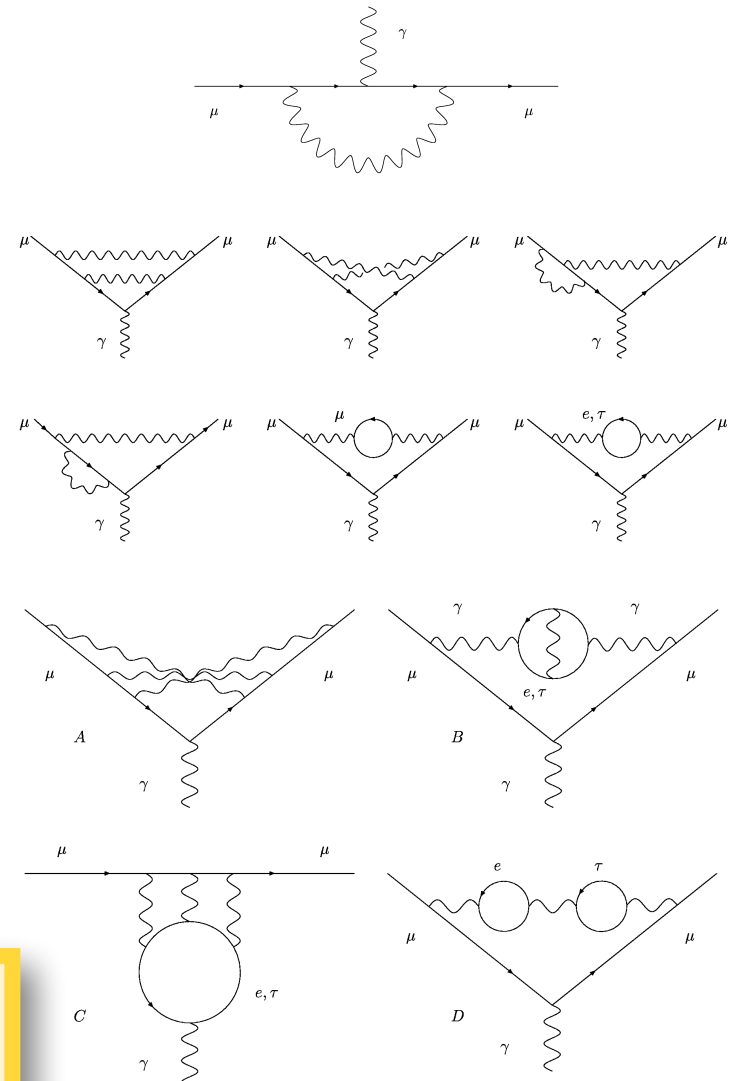
Volkov 1909.08015:  $A_1^{(10)}$ [no lept loops] at variance, but negligible  $\Delta$ .

Adding up, I get:

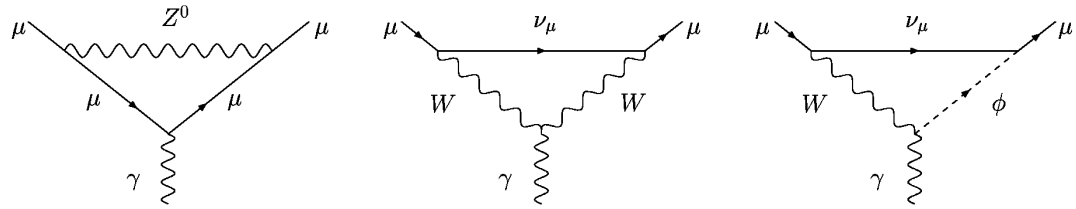
$$a_{\mu}^{\text{QED}} = 116584718.933 (20)(23) \times 10^{-11}$$

from coeffs, mainly from 4-loop unc ← → from  $\alpha$  (Cs)

with  $\alpha = 1/137.035999046(27) [0.2\text{ppb}]$  2018



## ● One-loop term:



$$a_{\mu}^{\text{EW}}(1\text{-loop}) = \frac{5G_{\mu}m_{\mu}^2}{24\sqrt{2}\pi^2} \left[ 1 + \frac{1}{5} (1 - 4\sin^2\theta_W)^2 + O\left(\frac{m_{\mu}^2}{M_{Z,W,H}^2}\right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

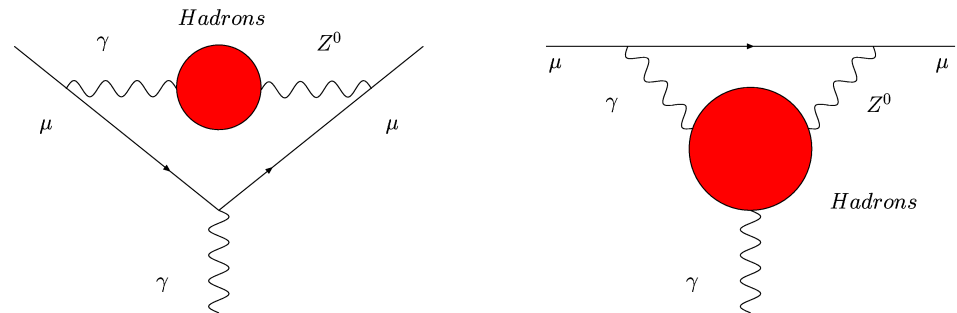
## ● One-loop plus higher-order terms:

$$a_{\mu}^{\text{EW}} = 153.6 (1.0) \times 10^{-11}$$

with  $M_{\text{Higgs}} = 125.6 (1.5) \text{ GeV}$

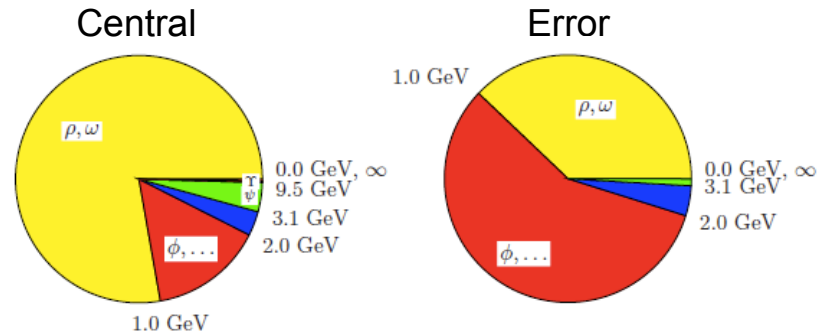
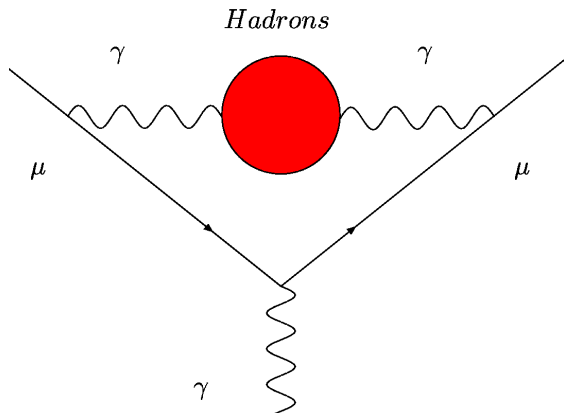
Hadronic loop uncertainties and 3-loop nonleading logs.

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrossi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019:  $152.9(1.0)e-11$ .





# The muon g-2: the Hadronic LO contribution (HLO)



F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)}$$

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} K(s) R(s)$$

$$a_\mu^{\text{HLO}} = 6894.6 (32.5) \times 10^{-11}$$

F. Jegerlehner, arXiv:1711.06089

$$= 6939 (40) \times 10^{-11}$$

Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921

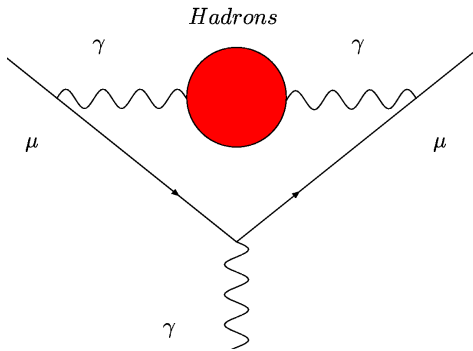
$$= 6932.6 (24.6) \times 10^{-11}$$

Keshavarzi, Nomura, Teubner, arXiv:1802.02995

**Radiative Corrections are crucial.** S. Actis et al, Eur. Phys. J. C66 (2010) 585

**Lots of progress in lattice calculations.** Muon g-2 Theory Initiative

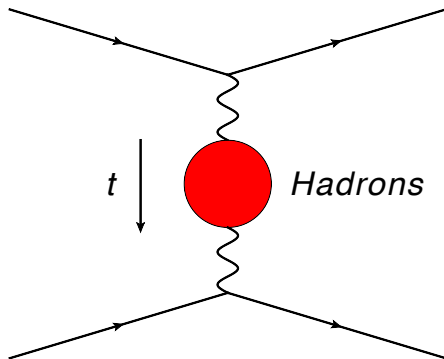
- At present, the leading hadronic contribution  $a_\mu^{\text{HLO}}$  is computed via the **timelike** formula:



$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds K(s) \sigma_{\text{had}}^0(s)$$

$$K(s) = \int_0^1 dx \frac{x^2 (1-x)}{x^2 + (1-x)(s/m_\mu^2)}$$

- Alternatively, exchanging the  $x$  and  $s$  integrations in  $a_\mu^{\text{HLO}}$



$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{\text{had}}(t)$  is the hadronic contribution to the running of  $\alpha$  in the **spacelike** region:  $a_\mu^{\text{HLO}}$  can be extracted from scattering data!

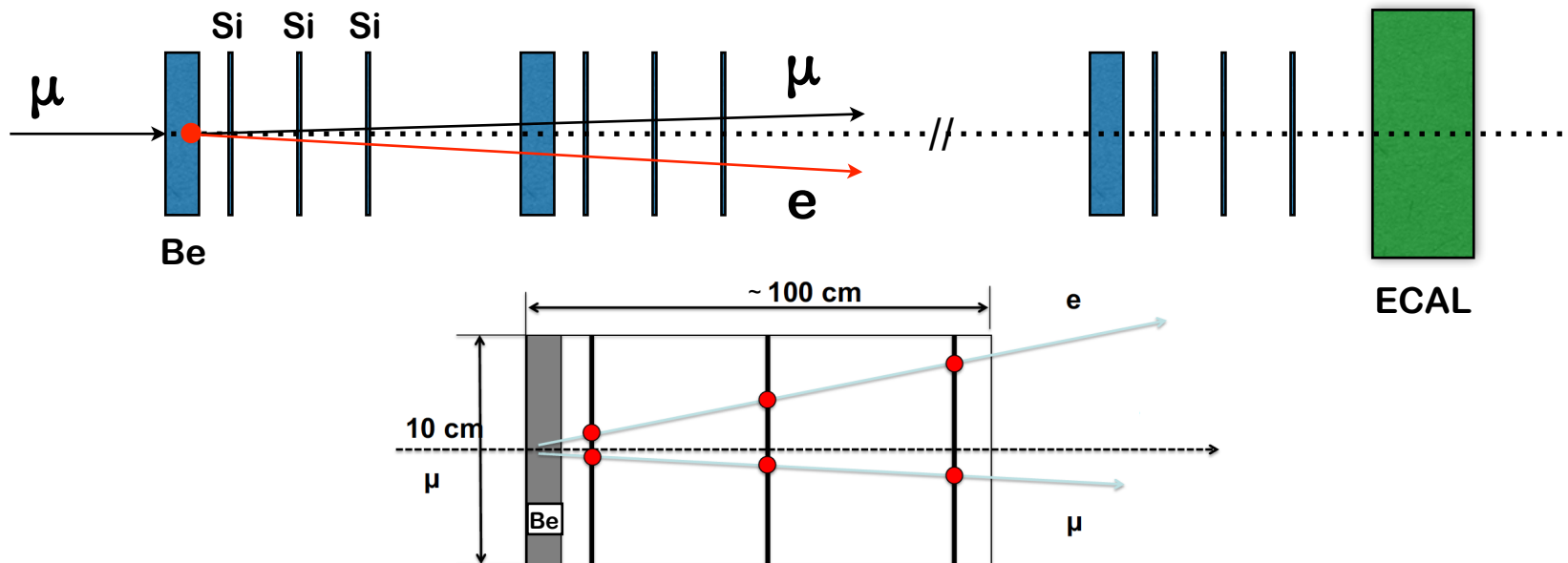
# Muon-electron scattering: The MUonE Project

Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna,  
Nicosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni

EPJC 2017 - arXiv:1609.08987



- $\Delta\alpha_{\text{had}}(t)$  can be measured via the **elastic scattering  $\mu e \rightarrow \mu e$** .
- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.

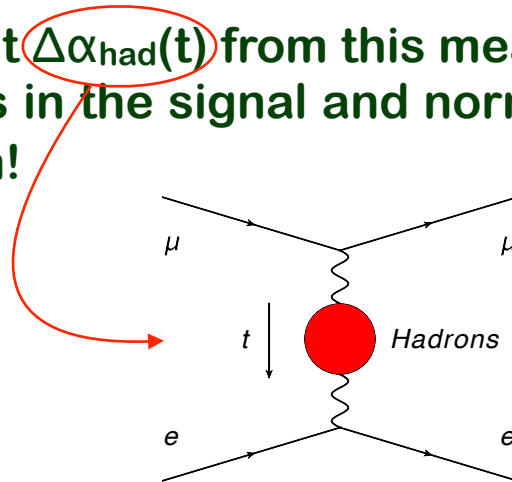


- State-of-the-art Si detectors:  $\sim 20\mu\text{m}$  hit resolution/1m  $\rightarrow \sim 0.02\text{mrad}$  expected angular resolution. ECAL and  $\mu$  filter at the end for PID.

- **Statistics:** With CERN's 150 GeV muon beam M2 ( $1.3 \times 10^7 \mu/s$ ), incident on 40 15mm Be targets (total thickness 60cm), 2 years of data taking ( $2 \times 10^7$  s/yr)  $\rightarrow$  integrated luminosity  $\mathcal{L}_{\text{int}} \sim 1.5 \times 10^7 \text{ nb}^{-1}$ .
- With this  $\mathcal{L}_{\text{int}}$  we estimate that measuring the shape of  $d\sigma/dt$  we can reach a statistical sensitivity of  $\sim 0.3\%$  on  $a_\mu^{\text{HLO}}$ , ie  $\sim 20 \times 10^{-11}$ .
- **Systematics:** Systematic effects must be known at  $\lesssim 10\text{ppm!}$

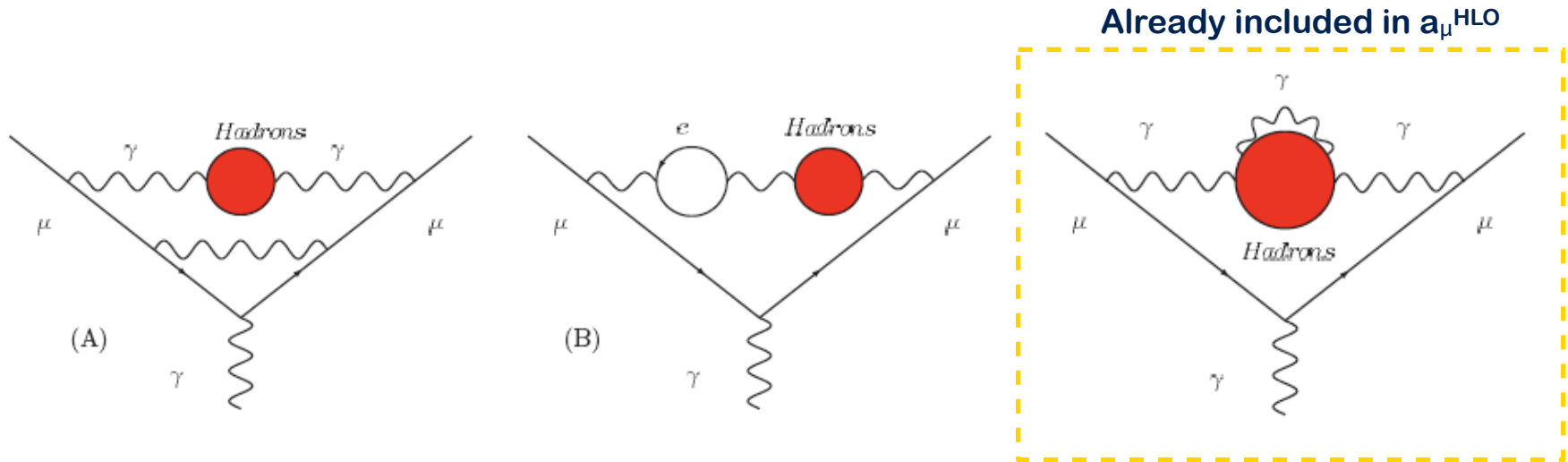


- **Theory:** To extract  $\Delta\alpha_{\text{had}}(t)$  from this measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at  $\lesssim 10\text{ppm!}$



- **MUonE**: a proposal for a new experiment at CERN to measure the leading hadronic contribution to the muon  $g-2$  via  $\mu e$  scattering.
- **Very challenging experiment! Test beams @ CERN in 2017 & 2018**
- **Positive report from CERN's "Physics Beyond Colliders" WG.**
- **June 2019: Letter of Intent submitted to CERN's SPSC for Pilot Run in 2021. Under review.**
- **Lots of ongoing experimental & theoretical progress...**

- HNLO: Vacuum Polarization**

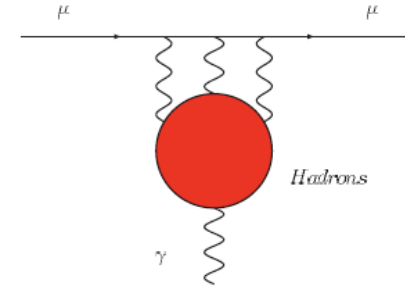


**$O(\alpha^3)$  contributions of diagrams containing hadronic vacuum polarization insertions:**

$$a_\mu^{\text{HNLO}}(\text{vp}) = - 98.2 (4) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011, Jegerlehner 2017, Keshavarzi, Nomura, and Teubner 2018

## ● HNLO: Light-by-light contribution



This term had a troubled life! Nowadays:

$a_{\mu}^{\text{HNLO}}( b ) = +80 (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_{\mu}^{\text{HNLO}}( b ) = +136 (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_{\mu}^{\text{HNLO}}( b ) = +105 (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$a_{\mu}^{\text{HNLO}}( b ) = +100 (29) \times 10^{-11}$	Jegerlehner, arXiv:1705.00263

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

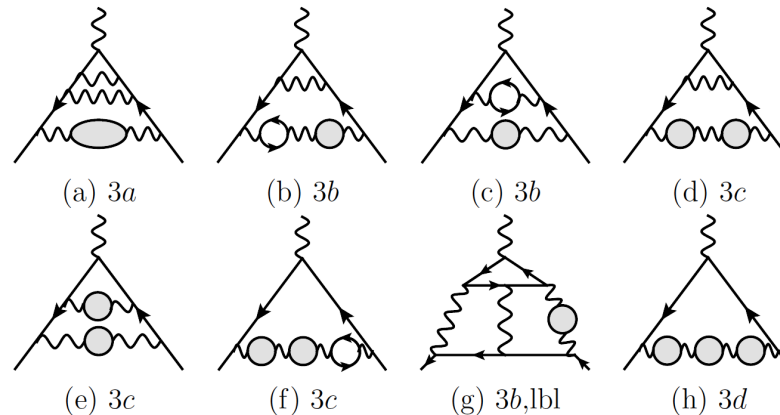
**Improvements expected in the  $\pi^0$  transition form factor** A. Nyffeler 1602.03398

**The HLbL contribution can be expressed in terms of observables in a dispersive approach.** Colangelo et al, 2014-15-17; Vanderhaeghen et al, 2014.

**Lots of progress on the lattice.** See Muon g-2 Theory Initiative



## HNNLO: Vacuum Polarization



$O(\alpha^4)$  contributions of diagrams containing hadronic vacuum polarization insertions:

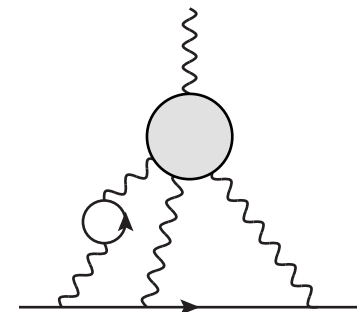
$$a_{\mu}^{\text{HNNLO}}(\text{vp}) = 12.4 (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

## HNNLO: Light-by-light

$$a_{\mu}^{\text{HNNLO}}(|b|) = 3 (2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



## Comparisons of the SM predictions with the measured g-2 value:

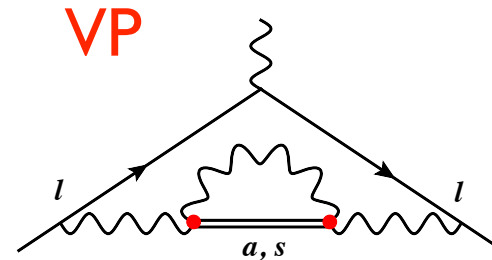
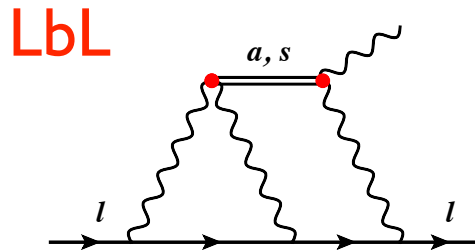
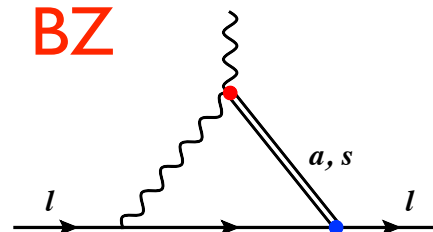
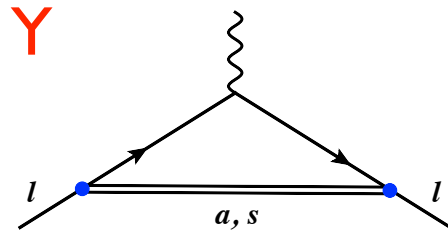
$$a_{\mu}^{\text{EXP}} = 116592091 (63) \times 10^{-11}$$

E821 – Final Report: PRD73 (2006) 072 with latest value of  $\lambda_{\mu} = \mu_{\mu}/\mu_p$  from CODATA'10

$a_{\mu}^{\text{SM}} \times 10^{11}$	$\Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}}$	$\sigma$
116 591 784 (44)	$307 (77) \times 10^{-11}$	4.0 [1]
116 591 829 (49)	$262 (80) \times 10^{-11}$	3.3 [2]
116 591 822 (38)	$269 (74) \times 10^{-11}$	3.6 [3]

with the hadronic light-by-light  $a_{\mu}^{\text{HNLO}}(|b|) = 100 (29) \times 10^{-11}$  of F. Jegerlehner arXiv:1705.00263, and the hadronic leading-order of:

- [1] F. Jegerlehner, arXiv:1711.06089.
- [2] Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921.
- [3] Keshavarzi, Nomura, Teubner, arXiv:1802.02995.



- Both scalar and pseudoscalar ALPs can solve  $\Delta a_\mu$  for masses  $\sim [100\text{MeV}-1\text{GeV}]$  and couplings allowed by current experimental constraints.
- They can be tested at present low-energy  $e^+e^-$  experiments, via dedicated  $e^+e^- \rightarrow e^+e^- + \text{ALP}$  &  $e^+e^- \rightarrow \gamma + \text{ALP}$  searches.

Marciano, Masiero, Paradisi, MP, arXiv:1607.01022

# The tau g-2: opportunities or fantasies?

The Standard Model prediction of the tau g-2 is:

$$\begin{aligned}
 a_{\tau}^{\text{SM}} &= 117324 \quad (2) && \times 10^{-8} && \text{QED} \\
 &+ 47.4 \quad (0.5) && \times 10^{-8} && \text{EW} \\
 &+ 337.5 \quad (3.7) && \times 10^{-8} && \text{HLO} \\
 &+ 7.6 \quad (0.2) && \times 10^{-8} && \text{HHO (vac)} \\
 &+ 5 \quad (3) && \times 10^{-8} && \text{HHO (lbl)}
 \end{aligned}$$

$$a_{\tau}^{\text{SM}} = 117721 (5) \times 10^{-8}$$

Eidelman & MP  
2007

$(m_{\tau}/m_{\mu})^2 \sim 280$ : great opportunity to look for New Physics, and a “clean” NP test too...

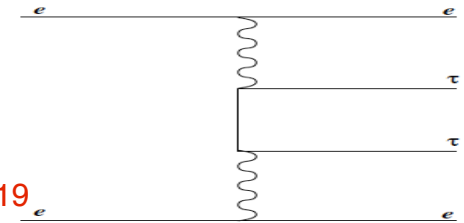
	Muon	Tau
$a_{\text{EW}} / a_{\text{H}}$	1/45	1/7
$a_{\text{EW}} / \delta a_{\text{H}}$	3	10

... if only we could measure it!!

- The very short mean life of the tau ( $2.9 \times 10^{-13}$  s) makes it very difficult to determine  $a_\tau$  measuring its spin precession in a magnetic field.
- DELPHI's result, from  $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$  total cross-section measurements at LEP 2 (the PDG value):

$$a_\tau = -0.018 (17)$$

PDG 2019



- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

$$-0.007 < a_\tau^{NP} < 0.005 \quad (95\% \text{ CL})$$

González-Sprinberg et al 2000

- Bernabéu et al, proposed the measurement of  $F_2(q^2=M_\gamma^2)$  from  $e^+e^- \rightarrow \tau^+\tau^-$  production at B factories.

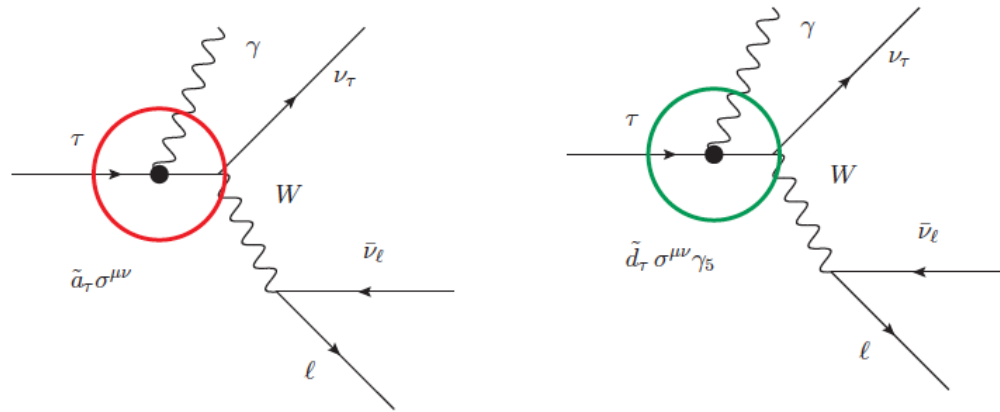
NPB 790 (2008) 160

- Direct probe of tau dipole moments with bent crystals:

See Fomin's & Ruiz Vidal's talks

- $a_\tau$  via the radiative leptonic decays  $\tau \rightarrow e\bar{\nu}\nu\gamma$ ,  $\tau \rightarrow \mu\bar{\nu}\nu\gamma$  comparing the theoretical prediction for the differential decay rates with precise data from high-luminosity B factories:

$$d\Gamma = d\Gamma_0 + \left(\frac{m_\tau}{M_W}\right)^2 d\Gamma_W + \frac{\alpha}{\pi} d\Gamma_{\text{NLO}} + \tilde{a}_\tau d\Gamma_a + \tilde{d}_\tau d\Gamma_d$$



- Detailed feasibility study performed in Belle-II conditions: we expect a (modest) improvement of the present PDG bound.

Eidelman, Epifanov, Fael, Mercolli, MP, arXiv:1601.07987 (JHEP 2016)

B.R. of radiative  $\tau$  leptonic decays ( $\omega_0 = 10$  MeV)

	$\tau \rightarrow e\bar{\nu}\nu\gamma$	$\tau \rightarrow \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\text{LO}}$	$1.834 \times 10^{-2}$	$3.663 \times 10^{-3}$
$\mathcal{B}_{\text{NLO}}^{\text{Inc}}$	$-1.06 (1)_n(10)_N \times 10^{-3}$	$-5.8 (1)_n(2)_N \times 10^{-5}$
$\mathcal{B}_{\text{NLO}}^{\text{Exc}}$	$-1.89 (1)_n(19)_N \times 10^{-3}$	$-9.1 (1)_n(3)_N \times 10^{-5}$
$\mathcal{B}^{\text{Inc}}$	$1.728 (10)_{\text{th}}(3)_{\tau} \times 10^{-2}$	$3.605 (2)_{\text{th}}(6)_{\tau} \times 10^{-3}$
$\mathcal{B}^{\text{Exc}}$	$1.645 (19)_{\text{th}}(3)_{\tau} \times 10^{-2}$	$3.572 (3)_{\text{th}}(6)_{\tau} \times 10^{-3}$
$\mathcal{B}_{\text{EXP}}^{\dagger}$	$1.847 (15)_{\text{st}}(52)_{\text{sy}} \times 10^{-2}$	$3.69 (3)_{\text{st}}(10)_{\text{sy}} \times 10^{-3}$

$\mathcal{O}(10\%)$  RC  $\leftarrow$

( $n$ ): numerical errors

( $N$ ): uncomputed NNLO corr.

$$\sim (\alpha/\pi) \ln r \ln(\omega_0/M) \times \mathcal{B}_{\text{NLO}}^{\text{Exc/Inc}}$$

$\dagger$  BABAR - PRD 91 (2015) 051103

(th): combined ( $n$ )  $\oplus$  ( $N$ )

( $\tau$ ): experimental error of  $\tau$

lifetime:  $\tau_{\tau} = 2.903(5) \times 10^{-13}$  s




[Agreement with MEG's  $\mu \rightarrow e\nu\nu\gamma$  2016]

	$\tau \rightarrow e\bar{\nu}\nu\gamma$	$\tau \rightarrow \mu\bar{\nu}\nu\gamma$
$\Delta^{\text{Exc}}$	$2.02 (57) \times 10^{-3} \rightarrow 3.5\sigma$	$1.2 (1.0) \times 10^{-4} \rightarrow 1.1\sigma$

Fael, Mercolli, MP, 1506.03416 (JHEP 2015)



# Conclusions

-  **Electron g-2:**  $\Delta a_e$  @ “–”  $2.4 \sigma$ . NP sensitivity limited only by exp uncertainties in  $\alpha$  &  $a_e$ . The  $\Delta a_e$  sensitivity will soon drop below  $10^{-13} \rightarrow a_e$  will play a pivotal role in probing NP in lepton sector.
-  **Muon g-2:**  $\Delta a_\mu \sim 3.5 - 4 \sigma$ . New upcoming measurement. QED & EW ready. Lots of progress in the hadronic sector, but not yet ready. MUonE: recent proposal to measure the leading hadronic contribution to the muon g-2 via  $\mu e$  scattering at CERN.
-  **Tau g-2:** unknown.