

Parameters:

$$\text{Al Junction parameters} \quad J_c := 100 \frac{\text{A}}{\text{cm}^2} \quad \text{Area} := 10 \mu\text{m}^2 \quad C_s := 0.085 \frac{\text{F}}{\text{m}^2}$$

$$\Delta := 0.001 \cdot e.c \cdot V \quad R_{sg} \text{over} R_{nn} := 40$$

$$I_0 := J_c \cdot \text{Area} = (1 \cdot 10^4) \text{ nA} \quad R_{nn} := \frac{\pi \cdot \Delta}{2 \cdot e.c \cdot I_0} = 0.157 \text{ k}\Omega \quad \gamma_{dc} := 0.97$$

$$C_J := C_s \cdot \text{Area} = 0.85 \text{ pF}$$

$$R_J := R_{nn} \cdot R_{sg} \text{over} R_{nn} = 6.283 \text{ k}\Omega$$

$$\omega_p := \sqrt{\frac{2 \cdot e.c}{\hbar} \cdot \frac{I_0}{C_J}}$$

$$\text{Resonator parameters} \quad Z_R := 100 \text{ }\Omega \quad f_R := 14 \text{ GHz} \quad Q := 1000$$

$$\text{coupling capacitor} \quad C_c := 0.0002 \text{ pF} \quad E_{ph} := h \cdot f_R = (9.276 \cdot 10^{-24}) \text{ J}$$

$$\text{JJ bias current amplitude} \quad I_{dc} := \gamma_{dc} \cdot I_0 = (9.7 \cdot 10^3) \text{ nA}$$

$$\text{photon current amplitude}$$

$$\text{Temperature} \quad T := 0.04 \text{ K} \quad I_{ac} := \sqrt{E_{ph} \cdot \frac{\omega_p}{Z_R}} = 132.435 \text{ nA} \quad I_{ac} := 50 \text{ nA}$$

Constants

$$\Phi_0 := 2.06785 \cdot 10^{-15} \text{ Wb} \quad K_B := k = (1.381 \cdot 10^{-23}) \frac{\text{J}}{\text{K}}$$

Derived parameters

Resonator

$$\omega_R := f_R \cdot 2 \cdot \pi \quad C_R := \frac{1}{\omega_R \cdot Z_R} = 0.114 \text{ pF} \quad L_R := \frac{Z_R}{\omega_R} = 0.001 \mu\text{H}$$

$$BW := \frac{f_R}{Q} = 14 \text{ MHz} \quad R_R := Q \cdot Z_R = 100 \text{ k}\Omega$$

MW photon

initial normalized inductor flux for energy = 1 photon

$$\varphi_{in0} := 2 \cdot \pi \cdot \frac{\sqrt{2 \cdot E_{ph} \cdot L_R}}{\Phi_0} = 0.441$$

Josephson junction

$$U(x) := 1 - \cos(x) - \gamma_{dc} \cdot x \quad \phi_{max} := \pi - \arcsin(\gamma_{dc}) = 1.816 \quad \phi_{min} := \arcsin(\gamma_{dc}) = 1.325$$

$$x := 0.8, 0.81..2.2$$

$$\omega := \sqrt{\frac{2 \cdot e.c}{\hbar} \cdot \frac{I_0}{C_J}} \quad f := \frac{\omega_p}{\omega} = 30.091 \text{ GHz} \quad f_{max} := f \cdot \sqrt[4]{(1 - \gamma_{dc})^2} = 14.837 \text{ GHz}$$

Simulation of a DC biased JJ coupled to a RLC resonator with AC input current

2

$$w_{pbias} := 2 \cdot \pi \cdot f_{pbias}$$

$$Z_J := \frac{1}{w_{pbias} \cdot C_J} = 12.62 \text{ } \Omega$$

$$L_J := \frac{Z_J}{w_{pbias}} = (1.354 \cdot 10^{-4}) \text{ } \mu\text{H}$$

$$E_J := I_0 \cdot \frac{\Phi_0}{2 \cdot \pi} = (3.291 \cdot 10^{-21}) \text{ } \textbf{J}$$

$$E_C := \frac{e \cdot c^2}{2 \cdot C_J} = (1.51 \cdot 10^{-26}) \text{ } \textbf{J}$$

$$I_{ac2} := \sqrt{2 \cdot \frac{E_{ph}}{L_J}} = 370.2 \text{ } \textbf{nA}$$

$$E_T := K_B \cdot T = (5.523 \cdot 10^{-25}) \text{ } \textbf{J}$$

$$E_{pbias} := h \cdot f_{pbias} = (9.831 \cdot 10^{-24}) \text{ } \textbf{J}$$

$$E_b := E_J \cdot (U(\phi_{max}) - U(\phi_{min})) = (3.229 \cdot 10^{-23}) \text{ } \textbf{J}$$

$$\frac{E_b}{E_T} = 58.477 \quad \frac{E_b}{E_{pbias}} = 3.285 \quad \frac{E_J}{E_{pbias}} = 334.768 \quad \frac{E_J}{E_T} = 5.959 \cdot 10^3 \quad \frac{E_J}{E_C} = 2.18 \cdot 10^5$$

$$ER_T := f_{pbias} \cdot e^{\frac{-E_b}{E_T}} = (5.958 \cdot 10^{-16}) \text{ } \textbf{Hz}$$

$$T_{cross} := f_{pbias} \cdot \frac{h}{2 \cdot \pi \cdot K_B} = 0.113 \text{ } \textbf{K}$$

$$ER_Q := f_{pbias} \cdot e^{-\pi \cdot \sqrt{\frac{E_{pbias}}{E_C} \cdot (1 - \gamma_{dc}^2)}} = 51.055 \text{ } \textbf{Hz}$$

$$ER_{Q2} := f_{pbias} \cdot e^{-\pi \cdot \sqrt{\frac{\left(E_b - \frac{E_{pbias}}{2}\right)}{E_C} \cdot (1 - \gamma_{dc}^2)}} = (1.116 \cdot 10^{-4}) \text{ } \textbf{Hz}$$

Energy levels in JJ

$$N_{levels} := \frac{E_b}{E_{pbias}} = 3.285$$

$$E_{lev}(n) := h \cdot f_{pbias} \cdot \left(0.5 + n + \frac{5 \cdot \left(n^2 + n + \frac{11}{30}\right)}{72 \cdot N_{levels}} \right) \quad n := 0, 1..N_{levels}$$

$$f_{10} := \frac{(E_{lev}(1) - E_{lev}(0))}{h} = 15.464 \text{ } \textbf{GHz}$$

$$E_{lev}(n) = \begin{bmatrix} 4.992 \cdot 10^{-24} \\ 1.524 \cdot 10^{-23} \\ 2.59 \cdot 10^{-23} \\ 3.698 \cdot 10^{-23} \end{bmatrix} \text{ } \textbf{J}$$

$$f_{21} := \frac{(E_{lev}(2) - E_{lev}(1))}{h} = 16.091 \text{ } \textbf{GHz}$$

$$\frac{E_{lev}(n)}{E_J} = \begin{bmatrix} 0.002 \\ 0.005 \\ 0.008 \\ 0.011 \end{bmatrix}$$

$$f_{32} := \frac{(E_{lev}(3) - E_{lev}(2))}{h} = 16.719 \text{ } \textbf{GHz}$$

$$E_0 := \frac{E_{lev}(0)}{E_J} + U(\phi_{min}) = -0.527 \quad E_1 := \frac{E_{lev}(1)}{E_J} + U(\phi_{min}) = -0.524 \quad E_2 := \frac{E_{lev}(2)}{E_J} + U(\phi_{min}) = -0.521$$

AC current simulating a photon

$$t0 := 500 \quad dur := 20$$

$$\psi(t) := 2.66 \cdot \sin\left(\frac{\omega_R}{\omega_p} \cdot (t - t0)\right) \cdot \frac{1}{\sqrt[2]{dur^1 \cdot 2 \cdot \pi}} \cdot e^{\frac{-(t - t0)^2}{2 \cdot dur^2}}$$

$$Int := \int_{t0 + 3 \cdot dur}^{t0 + 3 \cdot dur} \psi(t)^2 dt = 0.998 \quad Iac = 50 \text{ } \textbf{nA}$$

Simulation of a DC biased JJ coupled to a RLC resonator with AC input current

3

$$E := I_0^2 \cdot \frac{Z_R}{\omega_p} \cdot \int_{t_0 - 3 \cdot dur}^{t_0 + 3 \cdot dur} \gamma_{ac}(t)^2 dt = (1.32 \cdot 10^{-24}) J$$

$$\gamma_{ac}(t) := \frac{I_{ac}}{I_0} \cdot \psi(t)$$

$$E_{ph} = (9.276 \cdot 10^{-24}) J$$

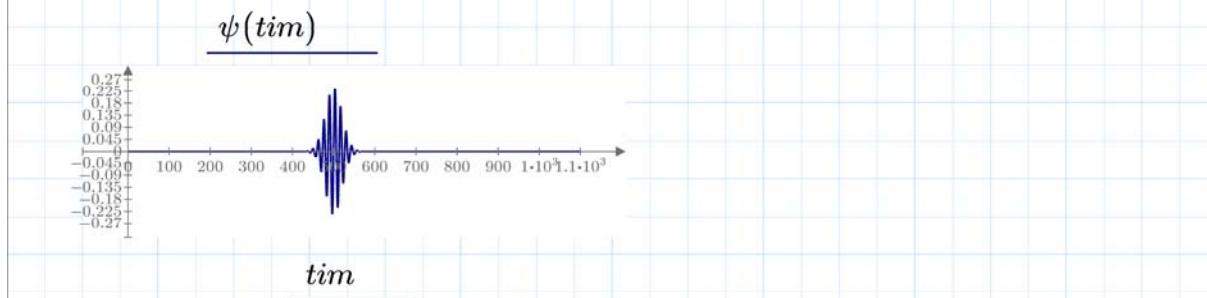
ODE Parameters

$$tfin := 1100 \quad nsteps := 10000 \quad tim := 0, \frac{tfin}{nsteps} \dots tfin$$

$$\tau := \frac{1}{\omega_p} = 5.289 \text{ ps} \quad duration := tfin \cdot \tau = 5.818 \text{ ns}$$

$$\alpha := \frac{1}{\omega_p \cdot R_J \cdot C_J} = 9.903 \cdot 10^{-4} \quad k := \frac{C_c}{C_J} = 2.353 \cdot 10^{-4} \quad \beta := \frac{C_J}{C_R} = 7.477 \quad \delta := \frac{\omega_R}{\omega_p} = 0.465$$

$$\varepsilon := \frac{(1 + \beta \cdot k)}{1 + \beta \cdot k + k} = 1 \quad \nu := \frac{k}{1 + \beta \cdot k + k} = 2.348 \cdot 10^{-4} \quad \mu := \frac{(1 + k)}{1 + \beta \cdot k + k} = 0.998$$



Vincigliatori ipotizzati	$v'(t) = -\alpha \cdot \varepsilon \cdot v(t) - \varepsilon \cdot \sin(\phi(t)) - \delta \cdot \frac{\nu}{Q} \cdot v_r(t) - \delta^2 \cdot \nu \cdot \varphi(t) + \beta \cdot \nu \cdot \gamma_{ac}(t) + \varepsilon \cdot \gamma_{dc}$
Soltore	$v_r'(t) = -\delta \cdot \frac{\mu}{Q} \cdot v_r(t) - \delta^2 \cdot \mu \cdot \varphi(t) - \alpha \cdot \beta \cdot \nu \cdot v(t) - \beta \cdot \nu \cdot \sin(\phi(t)) + (\beta \cdot k \cdot \nu + 1) \cdot \frac{\beta}{1 + \beta \cdot k} \cdot \gamma_{ac}(t) + \beta \cdot \nu \cdot \gamma_{dc}$
	$\phi'(t) = v(t) \quad v(0) = 0 \quad \phi(0) = \phi_{min} \quad v_r(0) = 0 \quad \varphi(0) = 0$
	$\begin{bmatrix} Vj \\ Vr \\ Fj \\ Fr \end{bmatrix} := \text{odesolve} \left(\begin{bmatrix} v(t) \\ v_r(t) \\ \phi(t) \\ \varphi(t) \end{bmatrix}, tfin, nsteps \right)$

Average Energy in the RLC resonator after pulse

$$Ef := \frac{\Phi_0^2}{2 \cdot 4 \cdot \pi^2 \cdot L_R \cdot 3 \cdot dur} \cdot \int_{t_0 + 3 \cdot dur}^{t_0 + 6 \cdot dur} Fr(t)^2 dt = (5.176 \cdot 10^{-24}) J$$

$$Ev := \frac{C_R \cdot \Phi_0^2 \cdot \omega_p^2}{2 \cdot 4 \cdot \pi^2 \cdot 3 \cdot dur} \cdot \int_{t_0 + 3 \cdot dur}^{t_0 + 6 \cdot dur} Vr(t)^2 dt = (5.219 \cdot 10^{-24}) J$$

tati | $t := 2 \cdot t0$

Simulation of a DC biased JJ coupled to a RLC resonator with AC input current

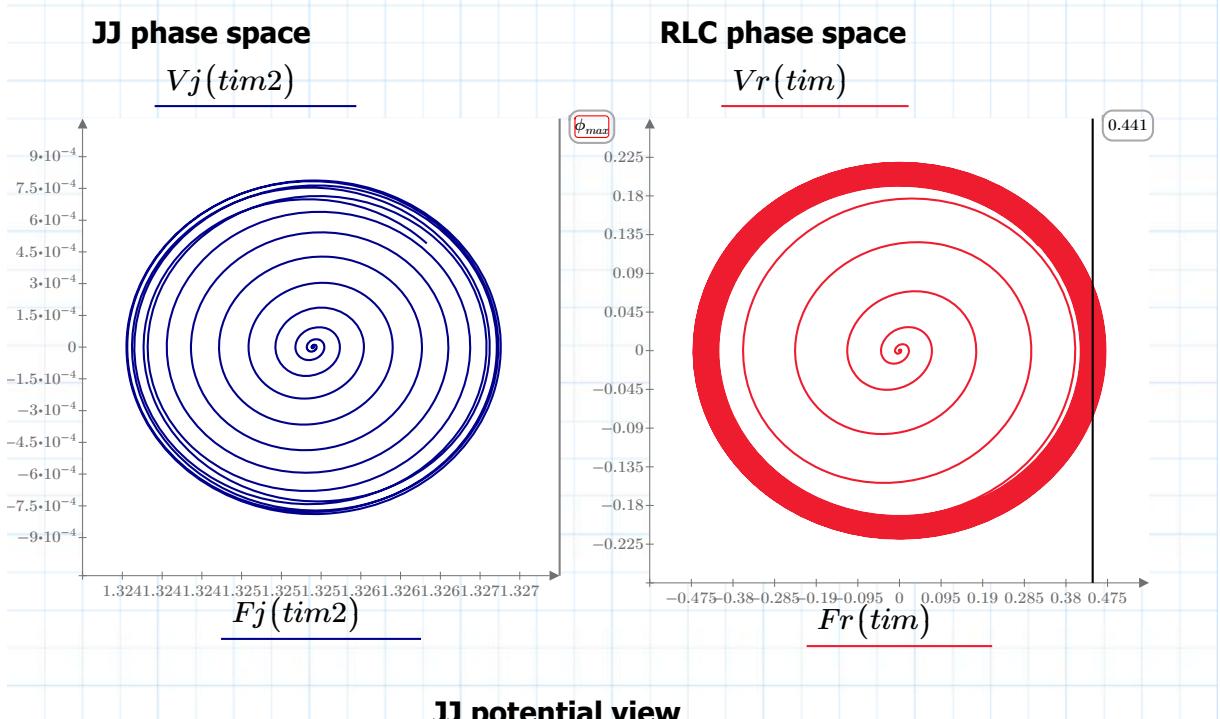
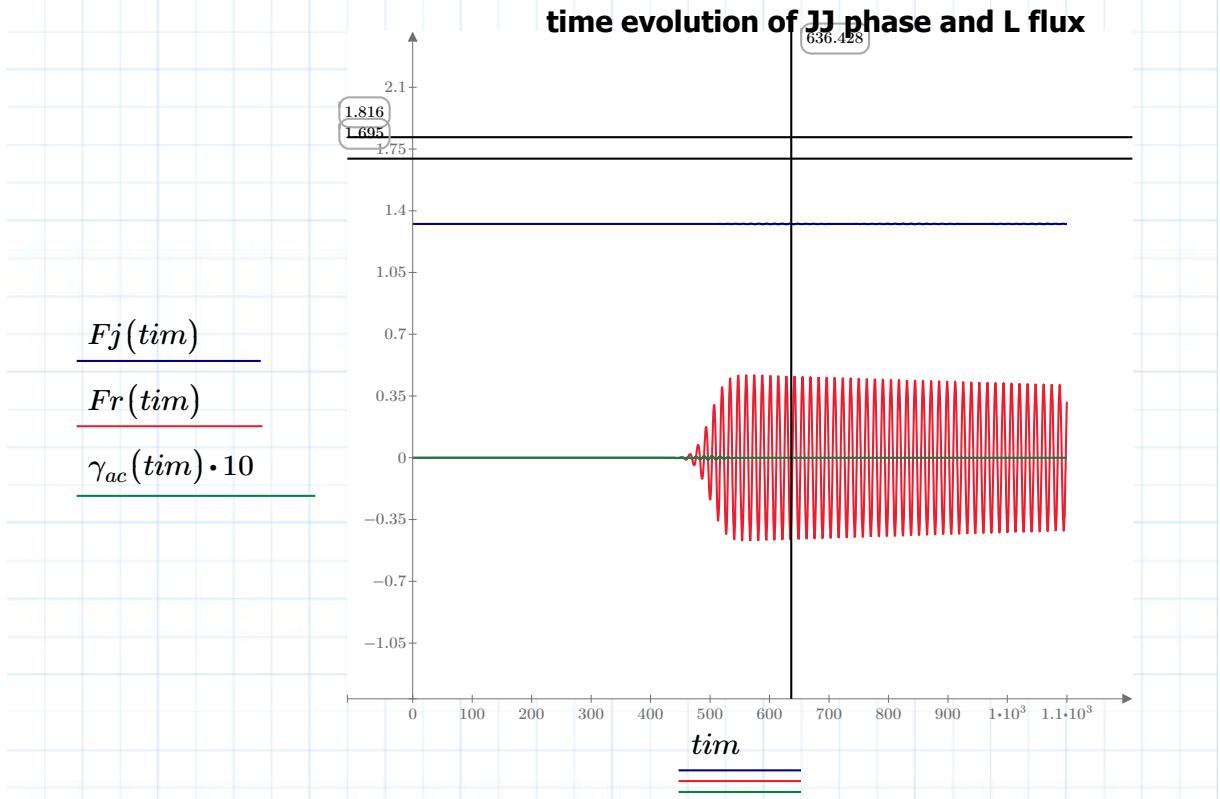
4

$$Fj(t) = \phi_{max}$$

$$Evj := \frac{\varPhi_0 \cdot I_0}{2 \cdot \pi \cdot 6 \cdot dur} \cdot \int_{t0 - 3 \cdot dur}^{t0} \gamma_{ac}(t) \cdot Vr(t) dt = (8.985 \cdot 10^{-26}) \text{ J}$$

$$tsw := \text{find}(t) = ?$$

$$tim2 := 0, \frac{t0}{nsteps}..t0 + 7 \cdot dur + 6$$



Simulation of a DC biased JJ coupled to a RLC resonator with AC input current

