Wave-function Collapse

as a

Resolution

of a

Tension between

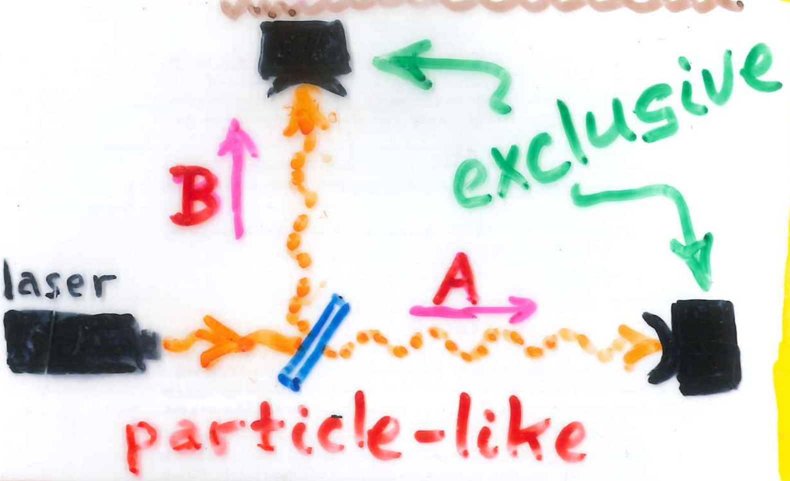
General Relativity

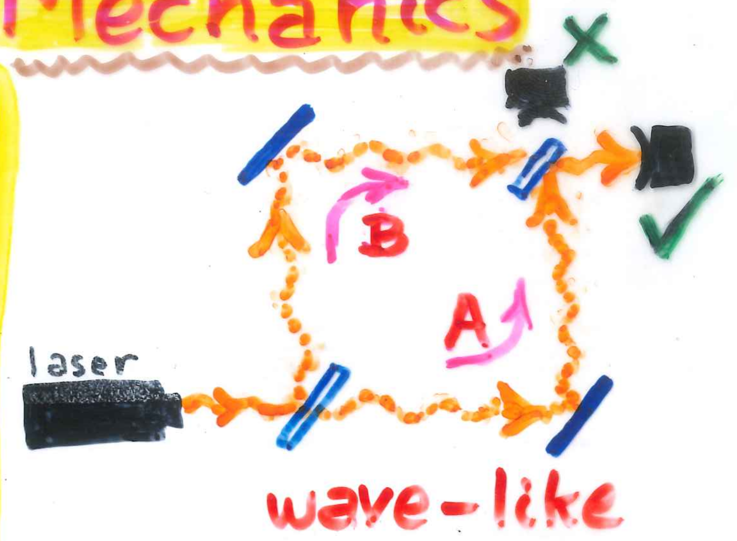
and

Quantum Mechanics

Roger Penrose, Oxford

Quantum Mechanics

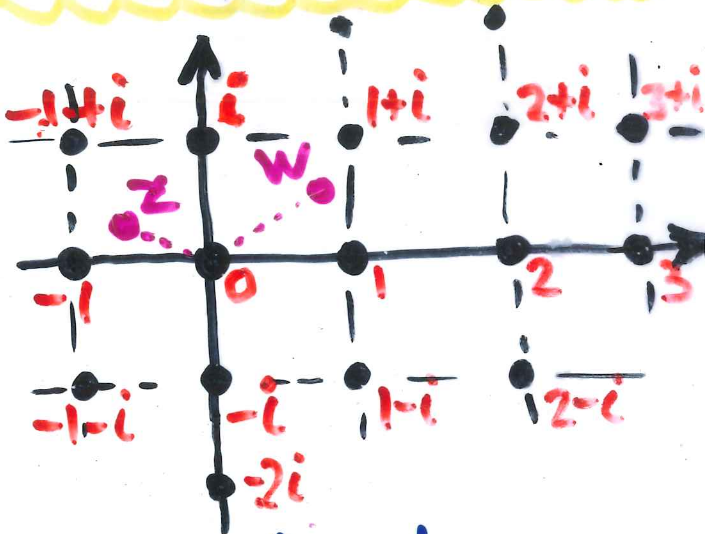




State = w × (alternative A) + z × (alt. B)

Quantum superposition

w and z are complex numbers (involve $i = \sqrt{-1}$)



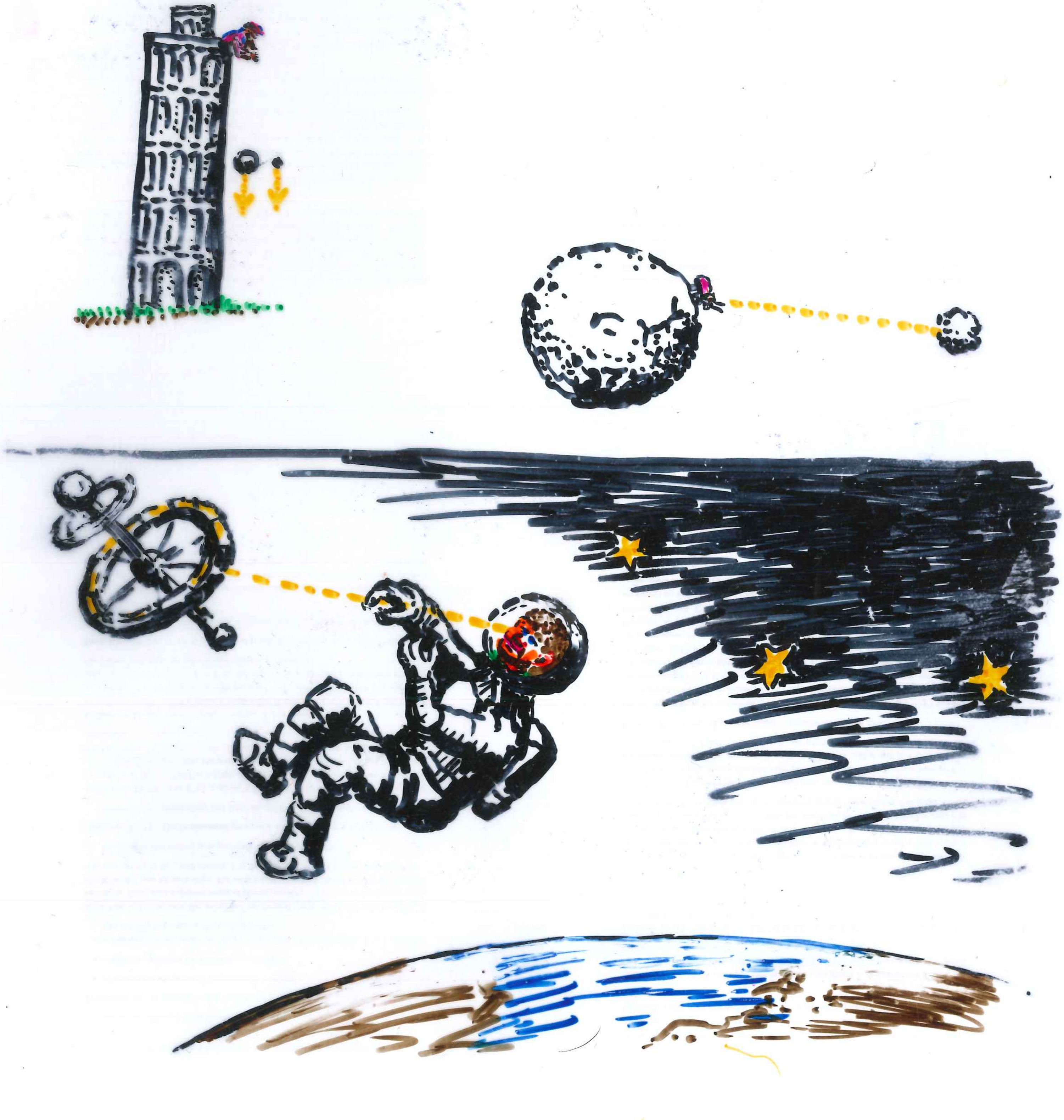
U Schrödinger's equation

Linear: w, z remain constant; A and B evolve irrespective of supn.

R Measurement process: state jumps to individual element of superposition with probability given by $|w|^2 : |z|^2$

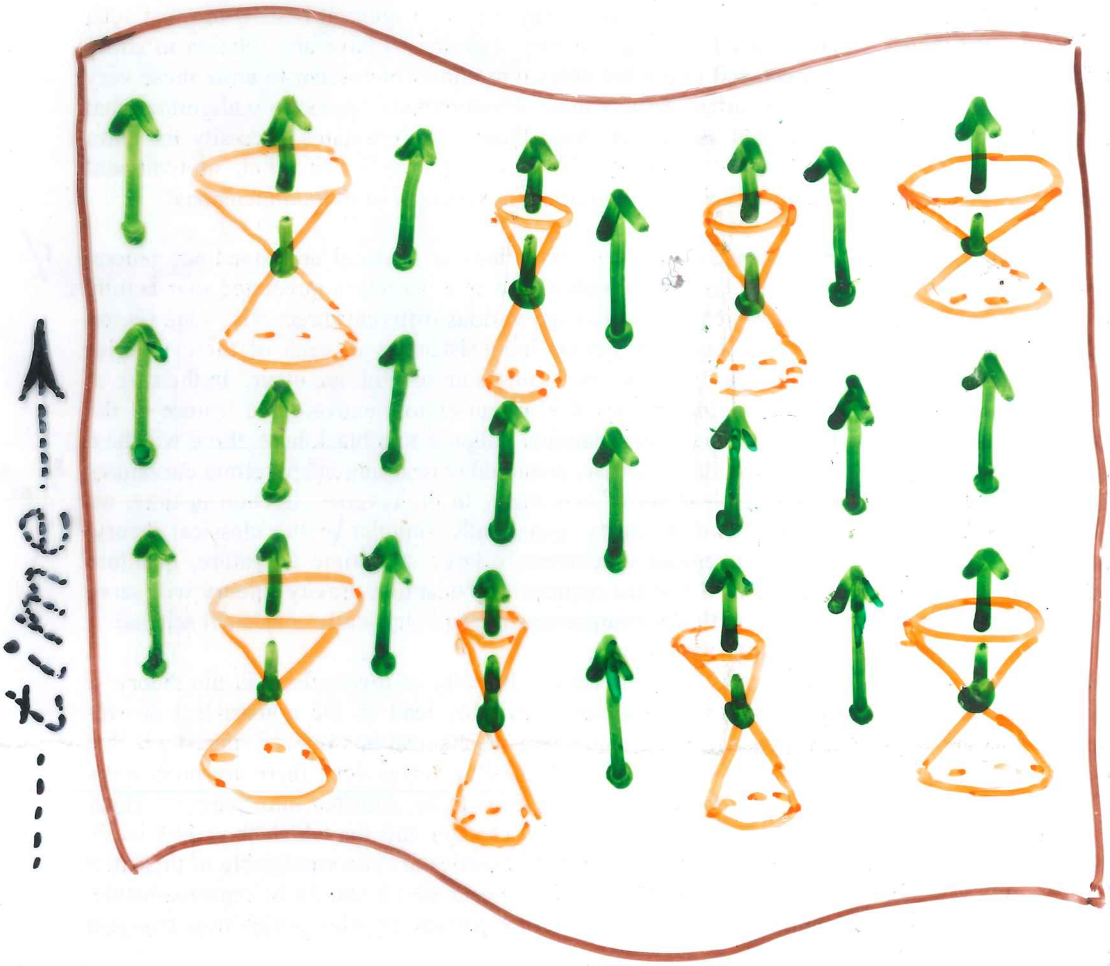
Measurement paradox: treating measuring device as a quantum system — why does not whole syst. evolve continuously by **U**?

Galilei – Einstein
Principle of Equivalence



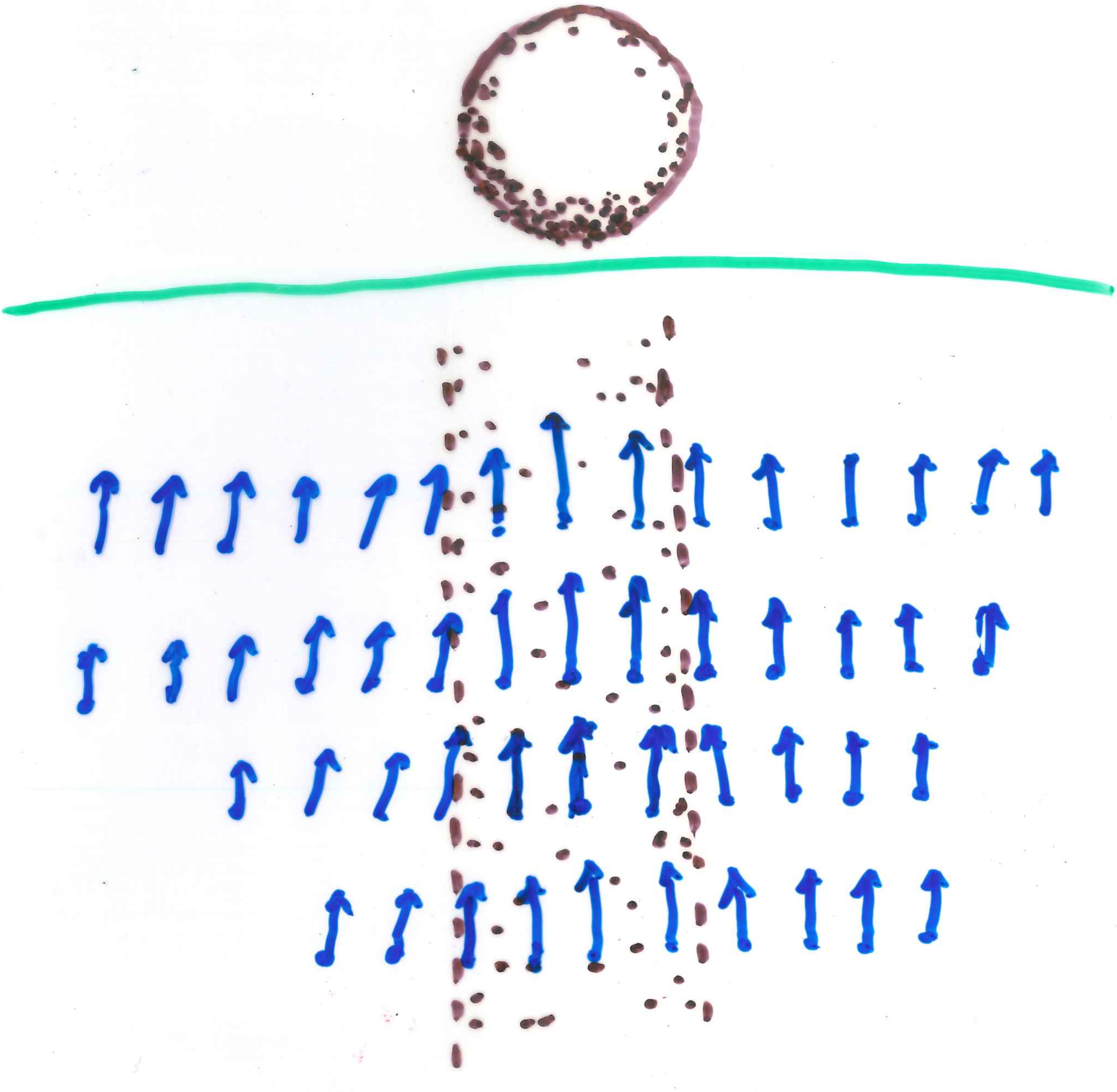
Killing Vector

Generator of continuous symmetry

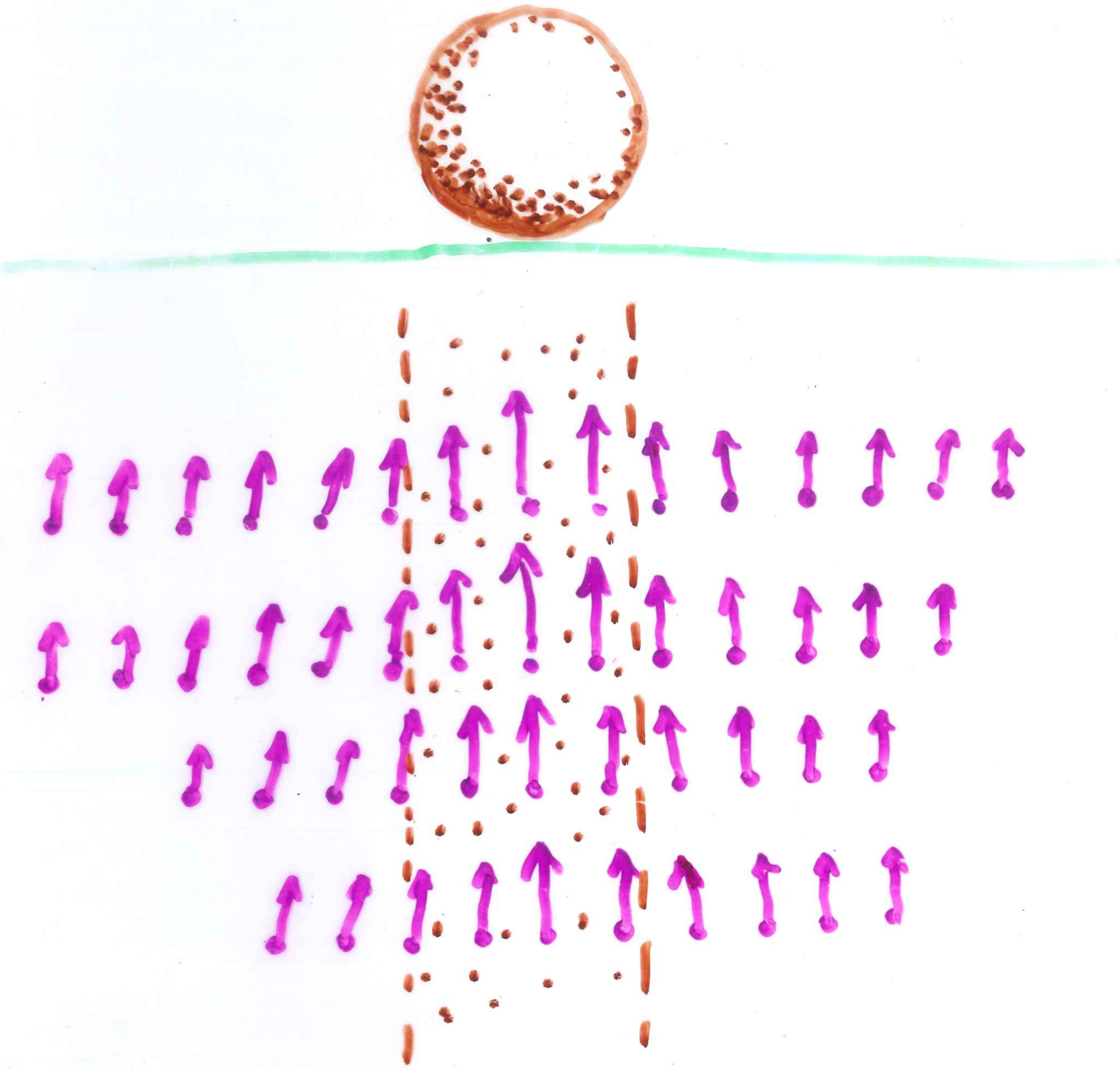


Vector field which preserves metric structure

The "$\partial/\partial t$" that occurs in Schrödinger's equation must be replaced, in a static curved space-time by the appropriate Killing vector T. (T is timelike:



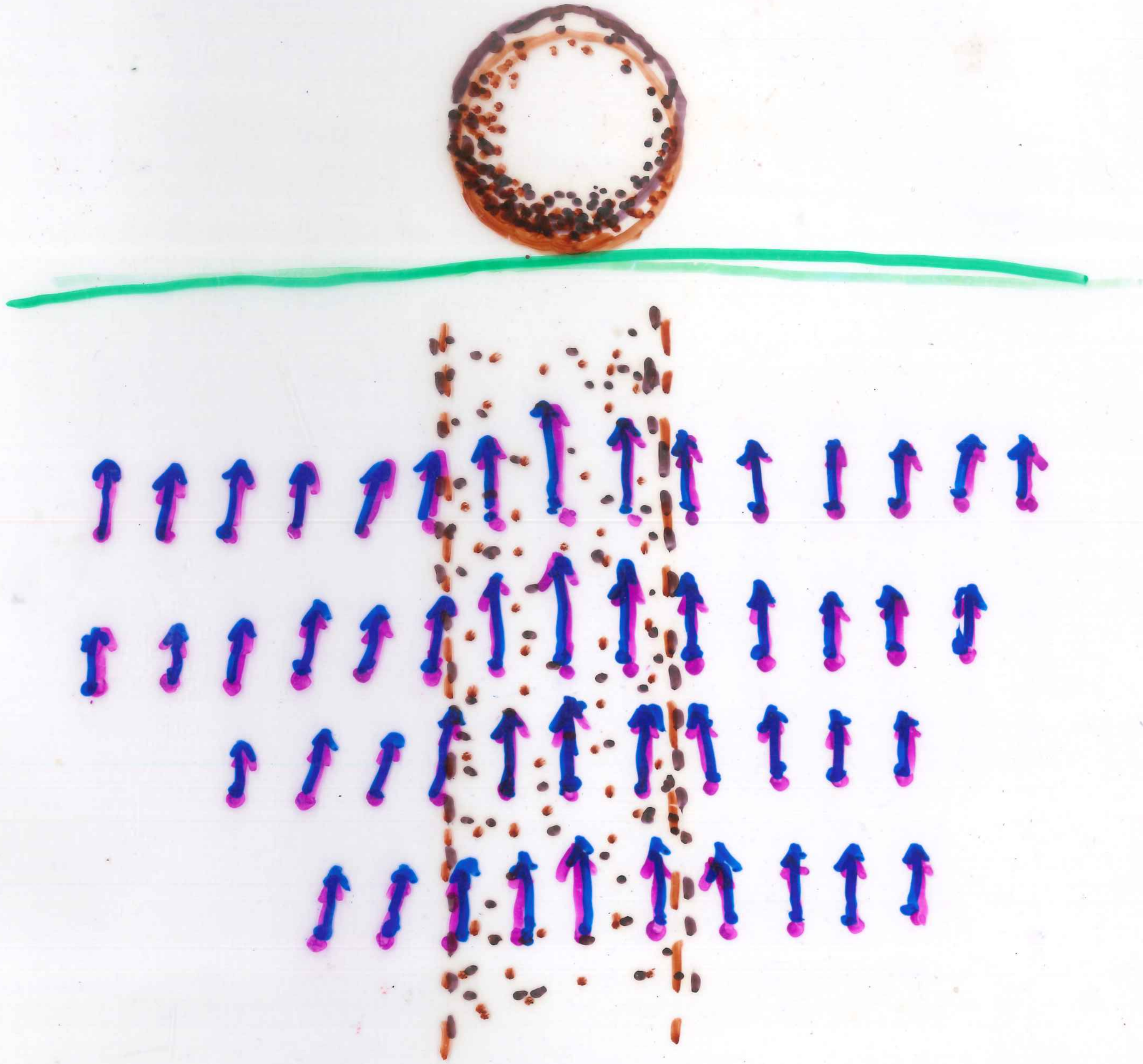
$$\frac{\partial}{\partial t}$$



$$\frac{\partial}{\partial t'}$$

$t' = t$

But: $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \frac{\partial x}{\partial t}\frac{\partial}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial}{\partial y} + \frac{\partial z}{\partial t}\frac{\partial}{\partial z}$



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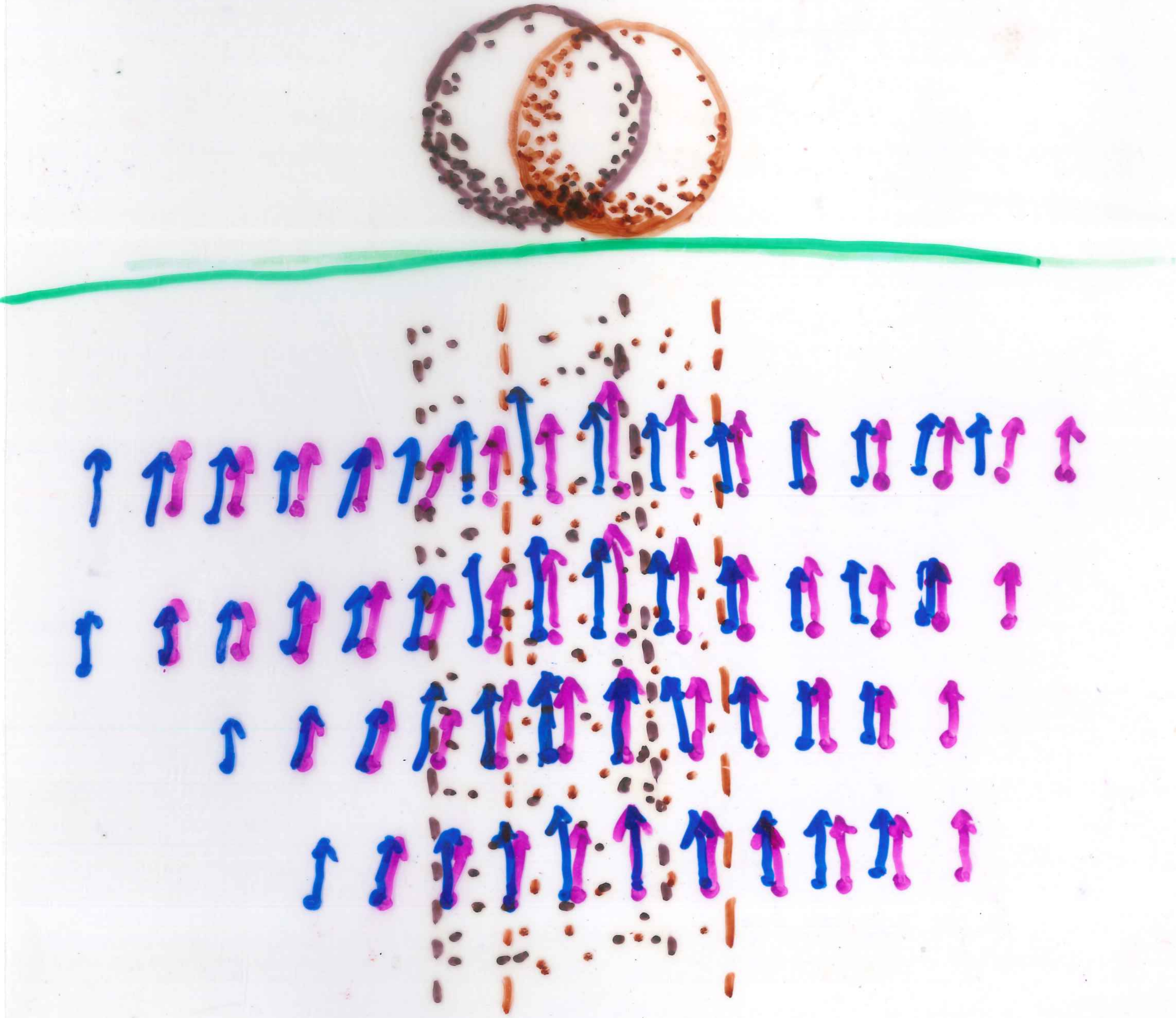

$$\frac{\partial}{\partial t} \quad \frac{\partial}{\partial t'}$$

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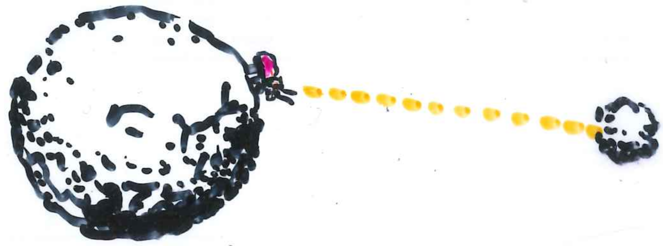
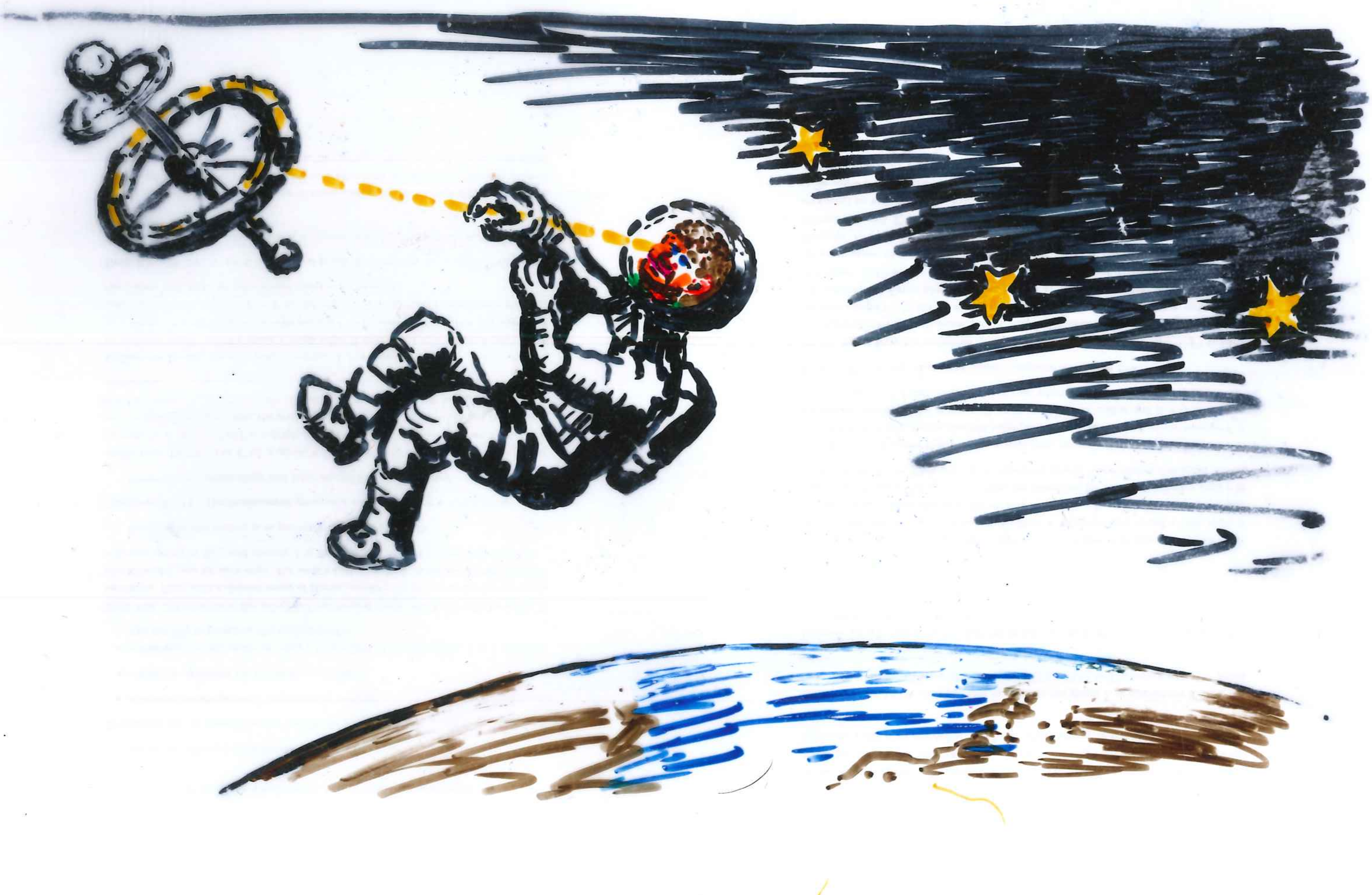
Galilei – Einstein Principle of Equivalence

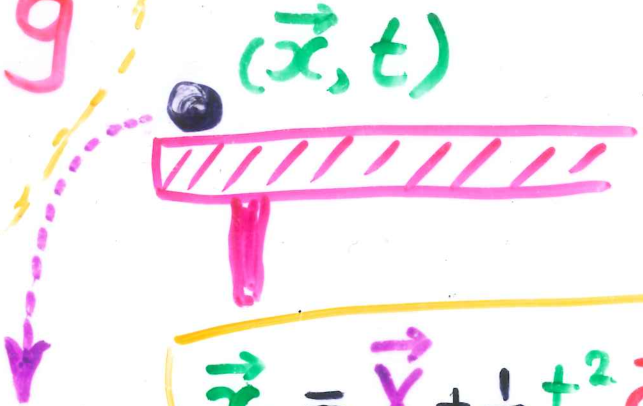




Principle of Equivalence and QM

($c = \infty$) Const. grav. field $\vec{g}$

$(\vec{x}, t)$-system, fixed on table



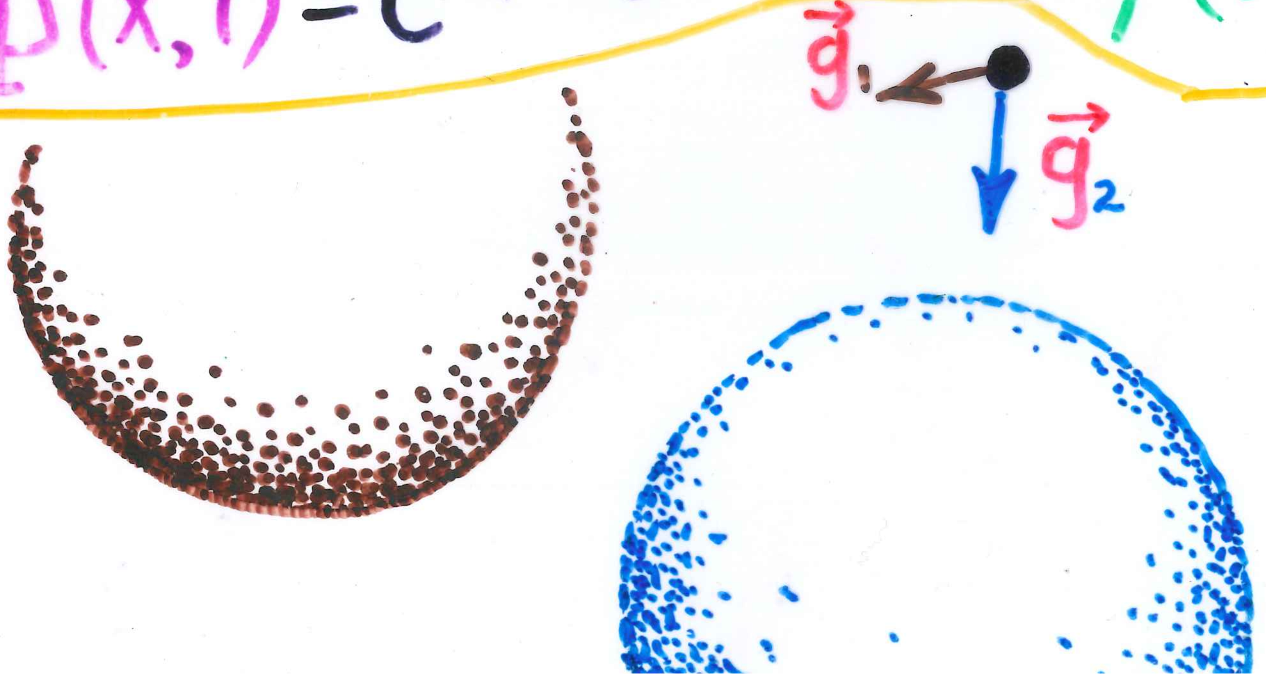
$(\vec{X}, t)$-syst. free fall, so grav. field $= 0$

$(\vec{X}, T)$

$$\vec{x} = \vec{X} + \tfrac{1}{2} t^2 \vec{g}$$
$$t = T$$

$$\Psi(\vec{X}, T) = e^{\frac{iM}{\hbar}\left(\frac{t^3 g^2}{6} - t\,\vec{x}\cdot\vec{g}\right)} \psi(\vec{x}, t)$$

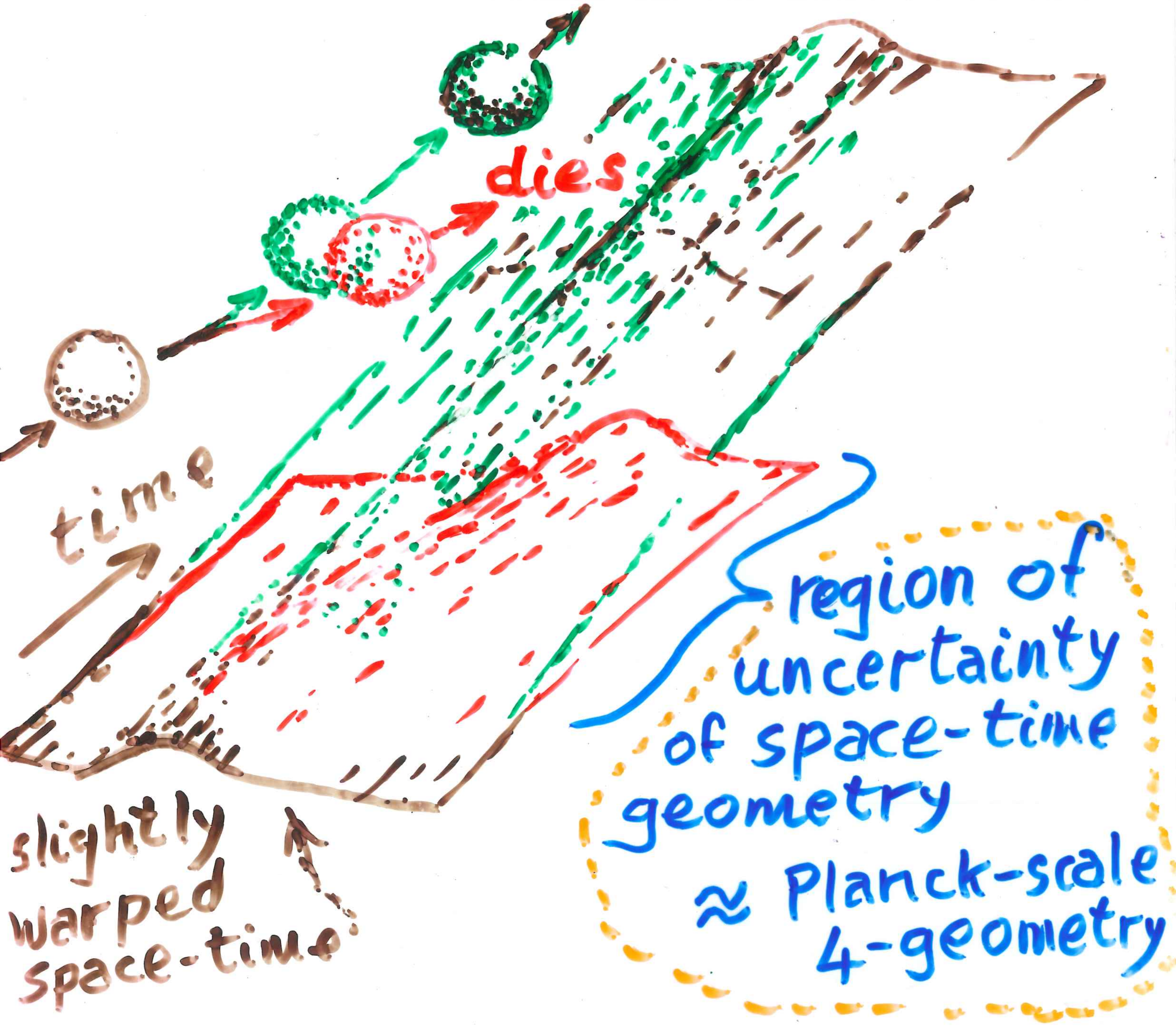
Colella
Overhauser
Werner



$c \to \infty$ limit of Unruh Vacuum

$$\int (\vec{g}_1 - \vec{g}_2)^2 d^3x = E_G$$

= grav. self-energy of difference mass distr



Planck distance $\approx 10^{-35}$ m

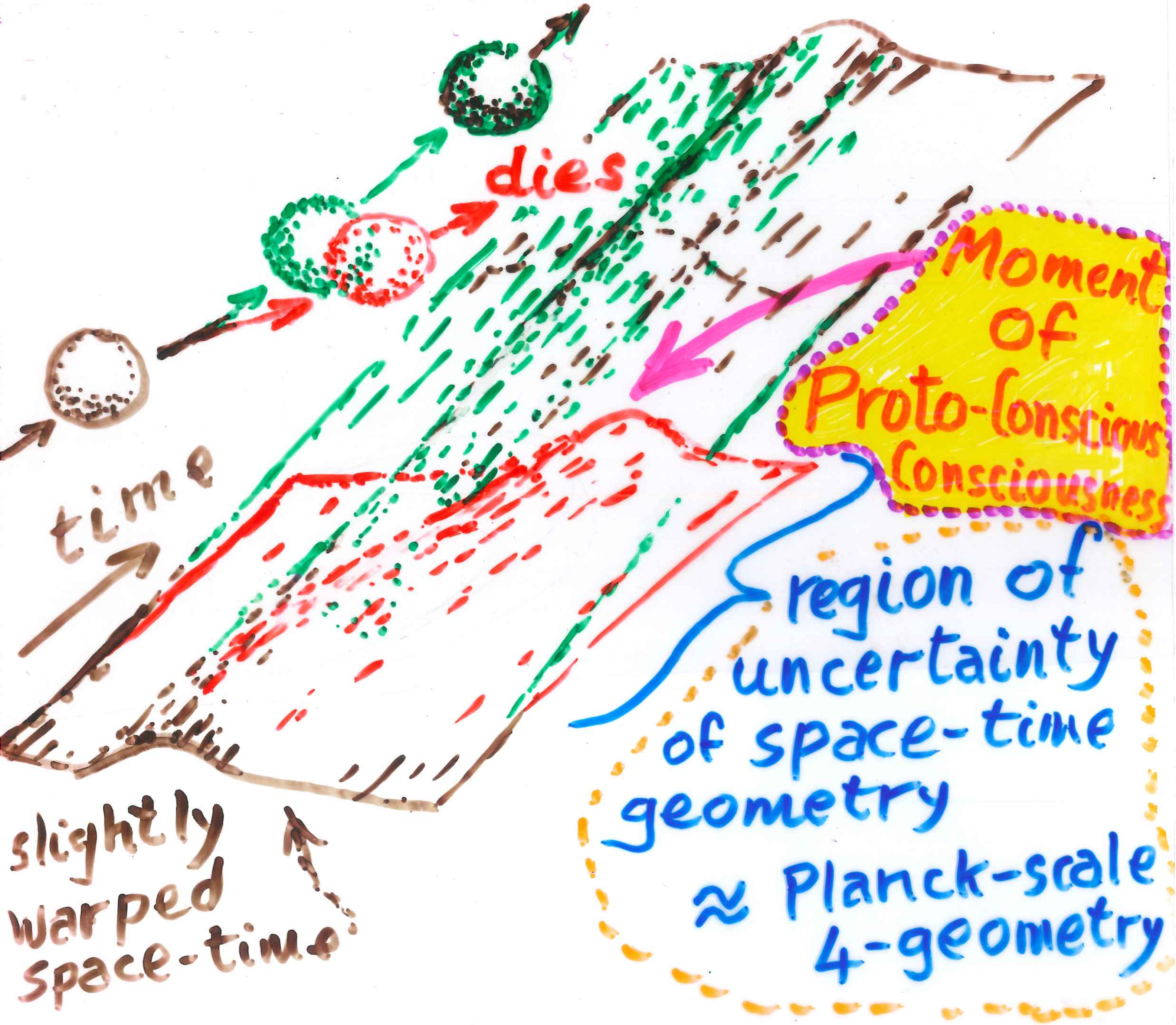
Planck time $\approx 10^{-43}$ s

Planck const. small

$$\tau \approx \frac{\hbar}{E_G}$$

small, since grav. const. is small

decay time



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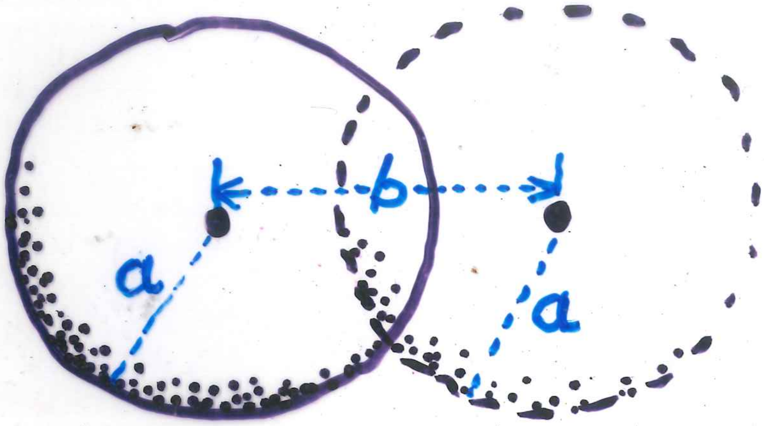
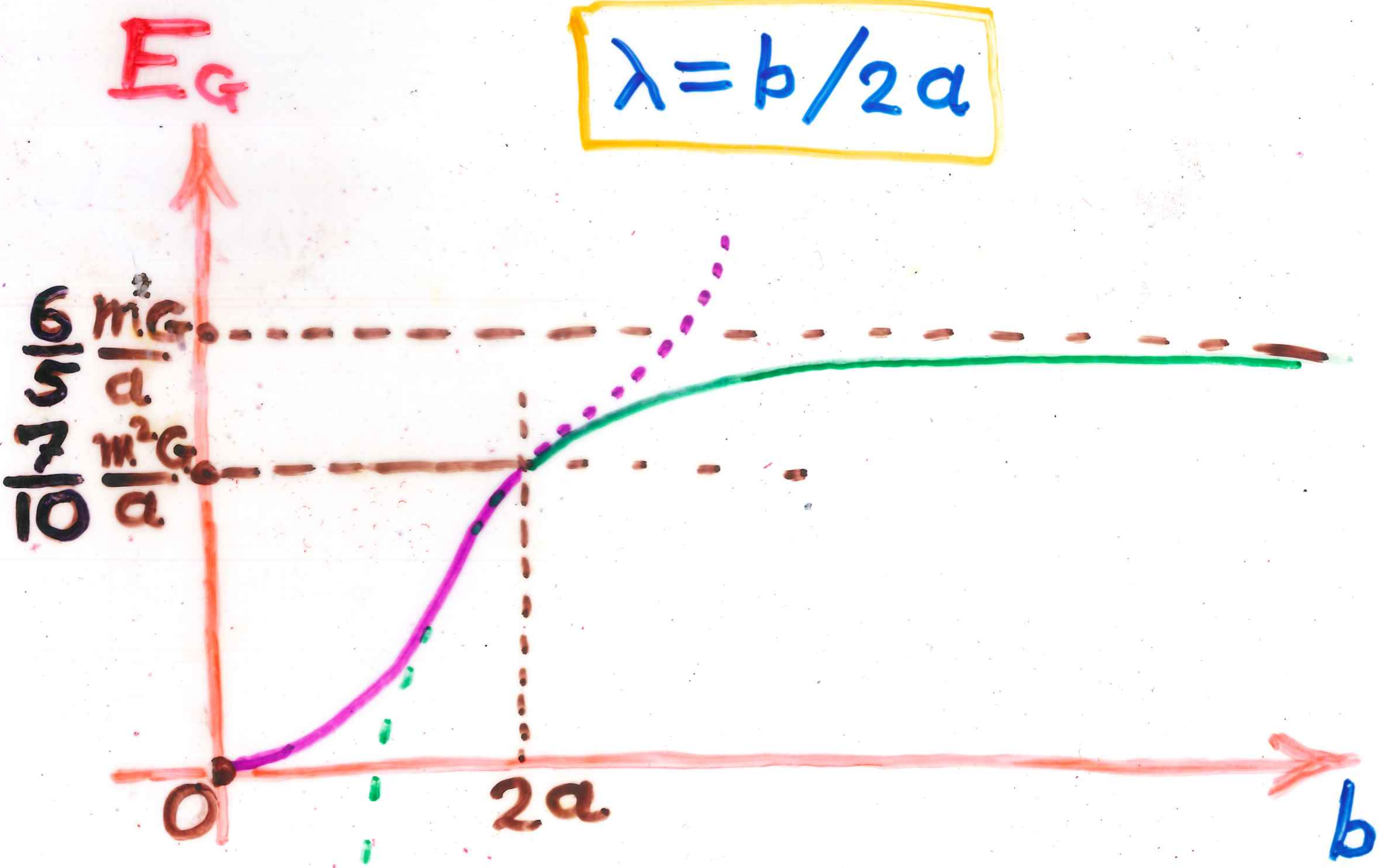
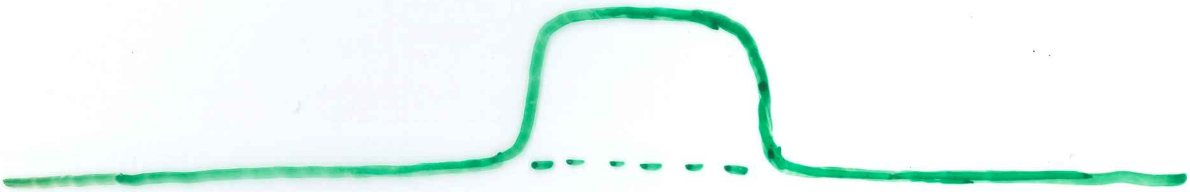
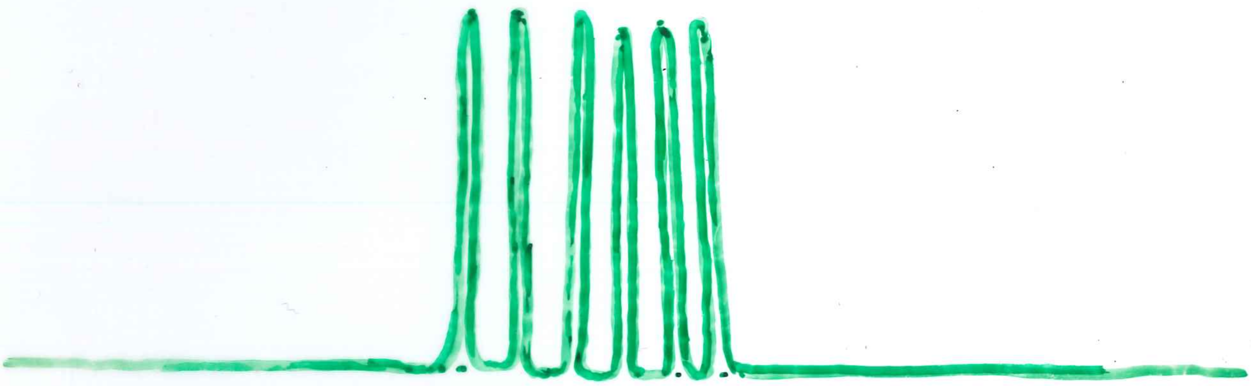


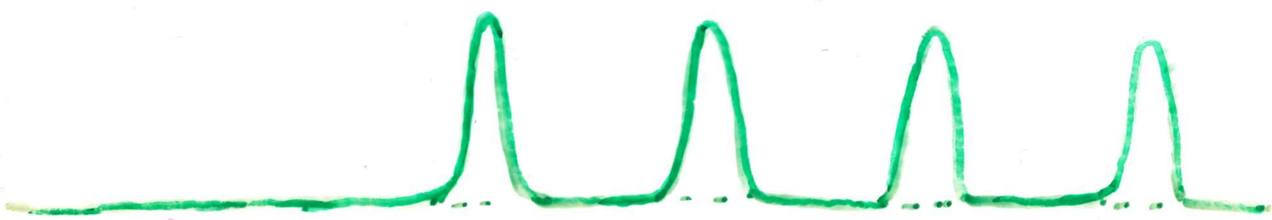
$$E_G = \begin{cases} \dfrac{m^2 G}{a}\left(2\lambda^2 - \dfrac{3}{2}\lambda^3 + \dfrac{1}{5}\lambda^5\right) & \boxed{0 \leq \lambda \leq 1} \\[2ex] \dfrac{m^2 G}{a}\left(\dfrac{6}{5} - \dfrac{1}{2\lambda}\right) & \boxed{1 \leq \lambda} \end{cases}$$

$$\boxed{\lambda = b/2a}$$





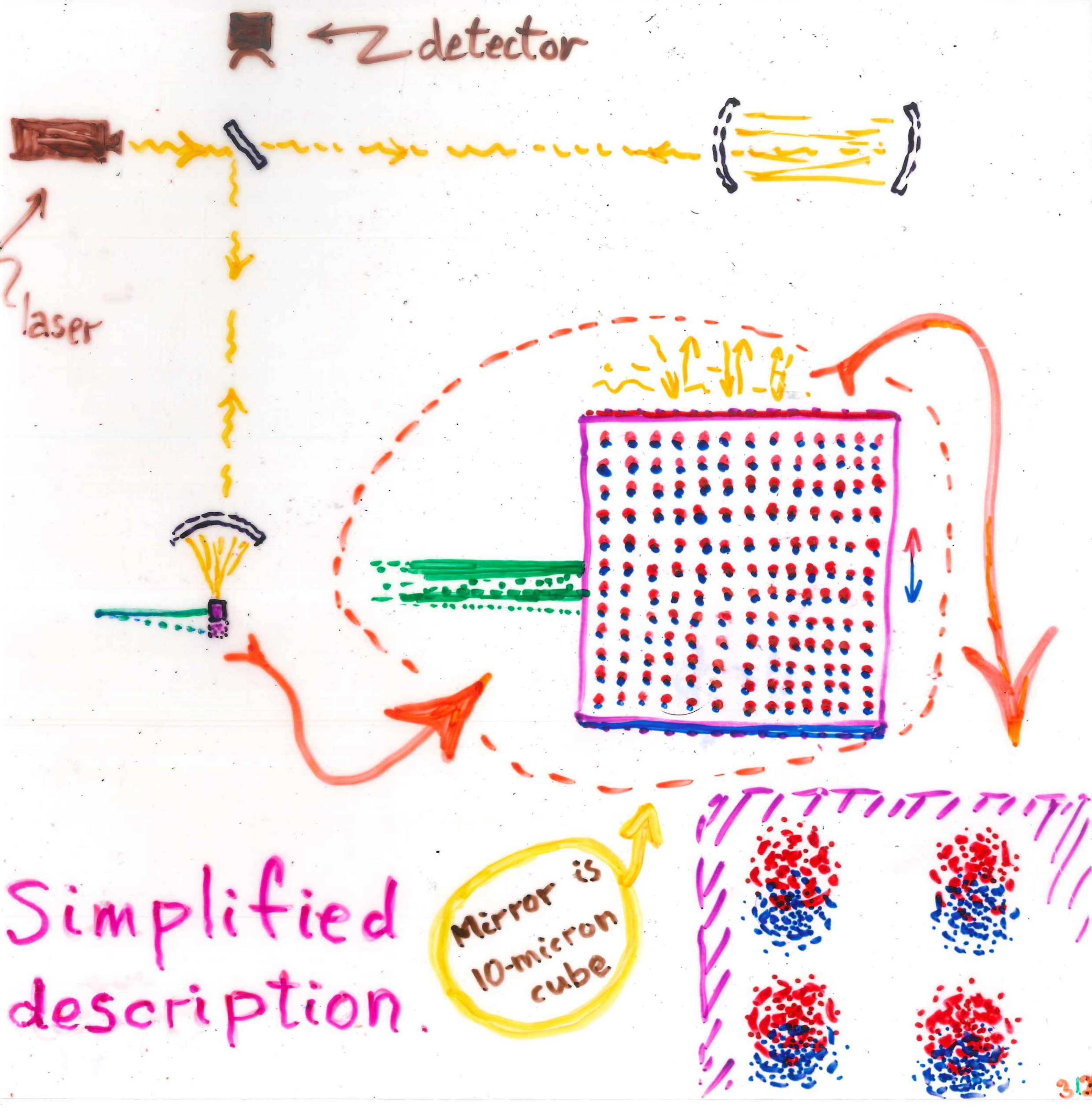




A Proposed Experiment

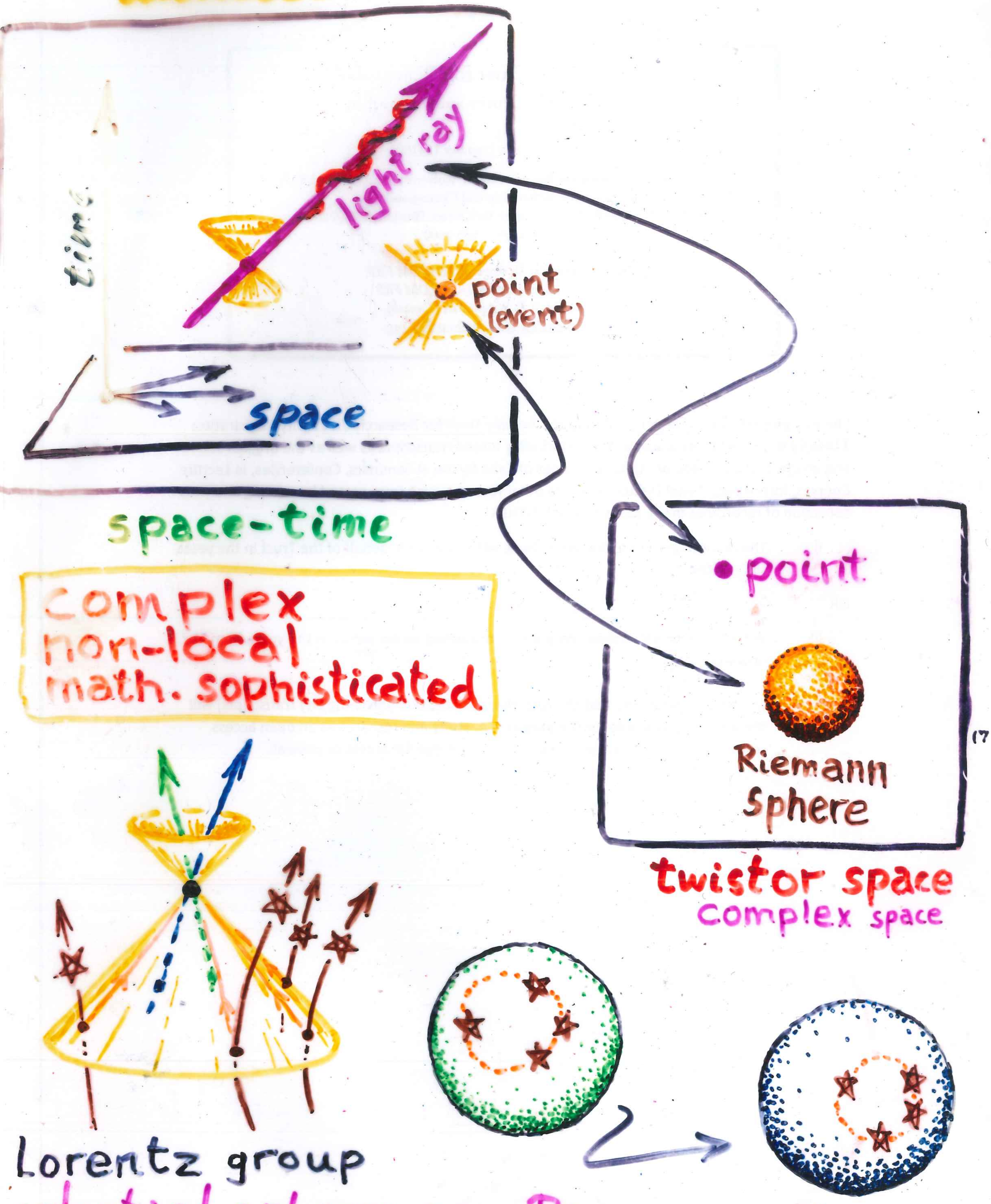
aimed at eventually testing Gravitational OR

Dirk Bouwmeester, Will Marshall, Christoph Simon, ..., & RP
(& early input from Johannes Dapprich, Anton Zeilinger, ...)



Simplified description.

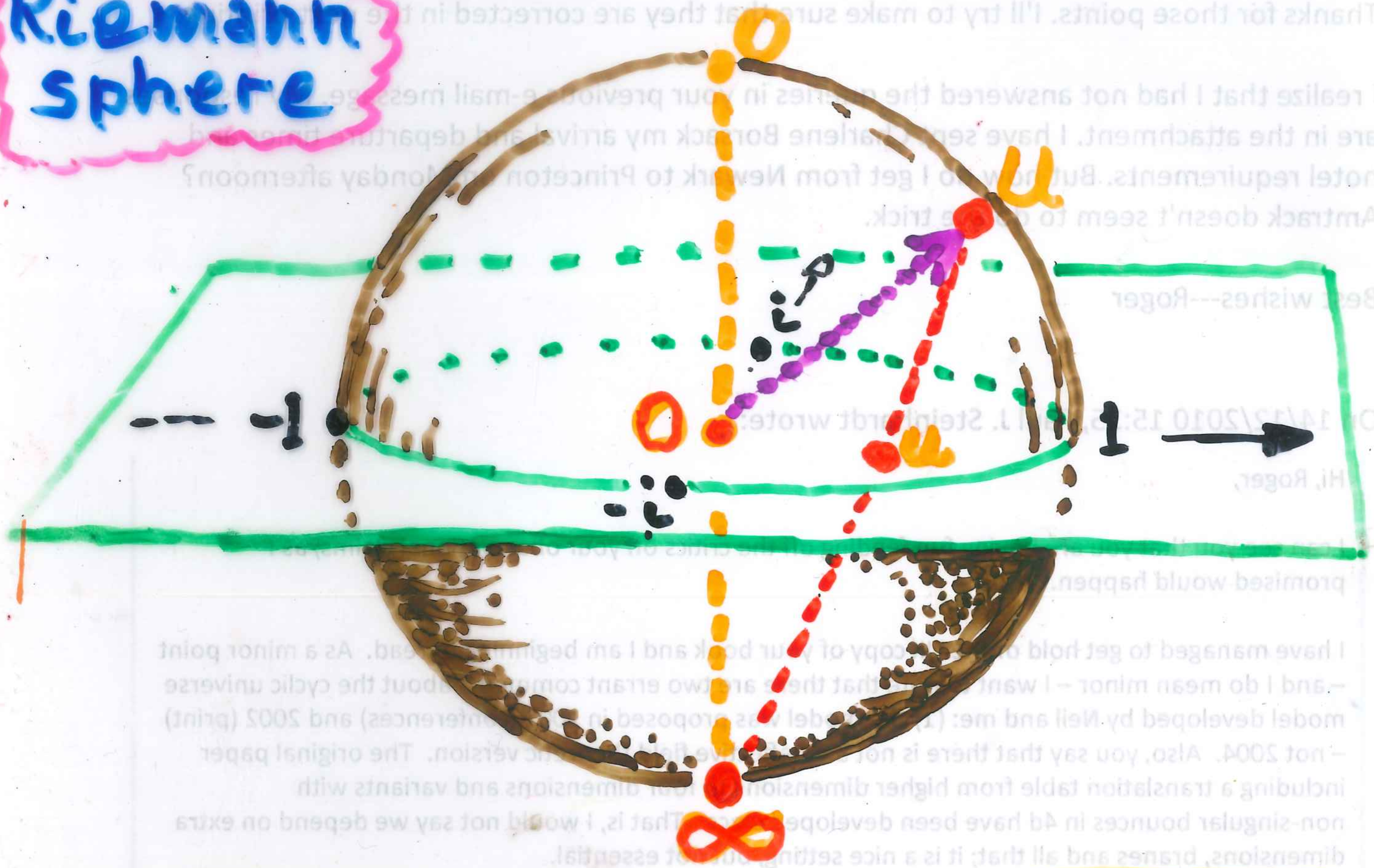
Twistor Theory



celestial spheres are Riemann spheres

Stereographic projection

Riemann sphere



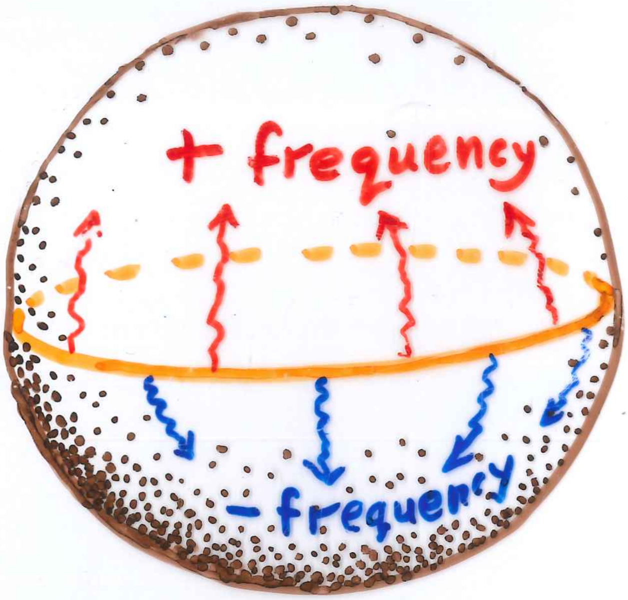
Spin $\frac{1}{2}$ particle (e.g. electron proton neutron quark)

$$u = \frac{z}{w}$$

Riemann sphere

Positive/Negative Frequency Splitting

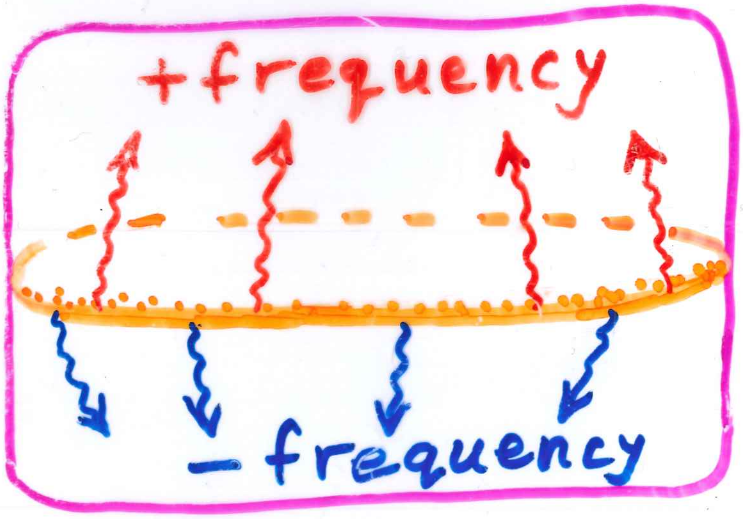
Riemann sphere $\mathbb{CP}^1$



Splitting of H^0s (ordinary functions) by holomorphic extension

real axis

Projective twistor space $\mathbb{PN} = \mathbb{CP}^3$



splitting of H^1s, by holomorphic extension

$\mathbb{PN}$ (null twistors)

This realized (finally!) one of the very early motivations behind twistor theory

$$\begin{pmatrix} Z^0 \\ Z^1 \end{pmatrix} = \frac{i}{\sqrt{2}} \underbrace{\begin{bmatrix} r^0 + r^3 & r^1 + i r^2 \\ r^1 - i r^2 & r^0 - r^3 \end{bmatrix}}_{r^{AA'}} \begin{pmatrix} Z^2 \\ Z^3 \end{pmatrix}$$

$$\omega^A = i \, r^{AA'} \, \pi_{A'}$$

$$Z^\alpha = (\omega^A, \pi_{A'})$$

$$\boxed{\text{incidence: } \omega = i r \cdot \pi}$$

shift of origin

$$\overset{Q}{\omega}{}^A = \overset{O}{\omega}{}^A - i q^{AA'} \overset{O}{\pi}_{A'}$$

$$\overset{Q}{\pi}_{A'} = \overset{O}{\pi}_{A'}$$

complex conjugate twistor

$$\boxed{\bar{Z}_\alpha = (\bar{\pi}_A, \bar{\omega}^{A'})}$$

Null twistor

$$Z^\alpha \bar{Z}_\alpha = 0 \quad \longleftarrow \text{equation of } \mathbb{P}N$$

Momentum-angular mom. for massless ptcle.

p_a = 4-momentum $\quad \left. \begin{array}{l} p_a p^a = 0 \\ p_0 > 0 \end{array} \right\}$

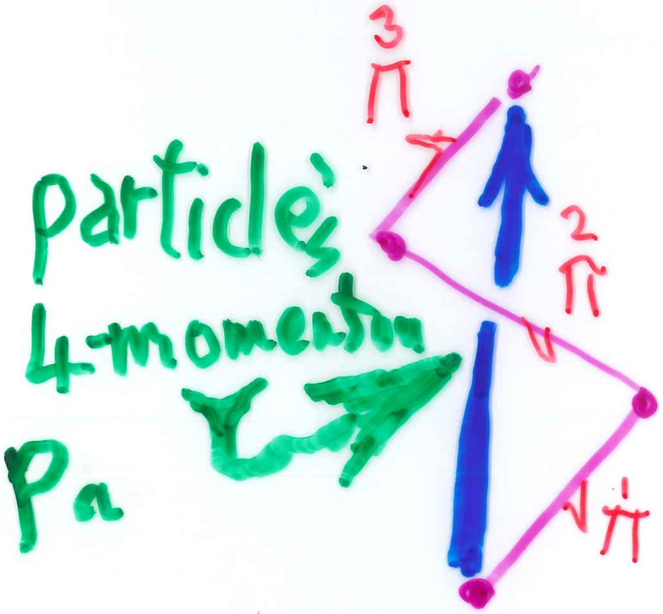
M^{ab} = 6-angular momentum

Pauli-Lubanski spin vector $\quad S_a = \frac{1}{2} e_{abcd} \overbrace{P^b M^{cd}}^{*(p \wedge M)} = \underbrace{s}_{\text{helicity}} p_a$

$$p_a = \pi_{A'} \bar{\pi}_A, \quad M^{ab} = i \omega^{(A} \bar{\pi}^{B)} \varepsilon^{A'B'} - i \varepsilon^{AB} \bar{\omega}^{(A'} \pi^{B')}, \quad 2s = Z^\alpha \bar{Z}_\alpha$$

Note: (...) denotes symmetrization

Massive particles



Use several twistors

Add their [momentum-angular mom] expressions, to get one for massive particle

Freedom in doing this gives twistor internal symmetry group

Suggests:
2 twistors: leptons
3 twistors: baryons

Massless Particles of Arbitrary Spin

Wave equations:

$s=0$:

$$\Box \phi = 0$$

$s>0$:

$$\phi_{\underbrace{A'B'\ldots L'}_{2s}} = \phi_{(A'B'\ldots L')}$$

$$\nabla^{AA'}\phi_{A'B'\ldots L'} = 0$$

$s<0$:

$$\phi_{\underbrace{AB\ldots L}_{2s}} = \phi_{(AB\ldots L)}$$

$$\nabla^{AA'}\phi_{AB\ldots L} = 0$$

	2×helicity	Twistor hom.	Dual Twistor hom.
graviton	2	-6	2
photon	1	-4	0
massles anti-neutrino	1/2	-3	-1
Scalar	0	-2	-2
massles neutr.	-1/2	-1	-3
photon	-1	0	-4
graviton	-2	2	-6

Incidence: $\omega^A = i r^{AA'} \pi_{A'}$

Twistor: $Z^\alpha = (\omega^A, \pi_{A'})$

PN: $Z^\alpha \bar{Z}_\alpha = 0$, where $\bar{Z}_\alpha = (\bar{\pi}_A, \bar{\omega}^{A'})$

Linearized gravity (can be complex)

$$\nabla^{AA'} \psi_{ABCD} = 0, \quad \nabla^{AA'} \tilde{\psi}_{A'B'C'D'} = 0.$$

Left-handed anti-self-dual

Right-handed self-dual

$$\psi_{ABCD}(r) = \oint_{\omega = ir\pi} \frac{\partial}{\partial \omega^A} \frac{\partial}{\partial \omega^B} \frac{\partial}{\partial \omega^C} \frac{\partial}{\partial \omega^D} f(Z)\, \pi\, d\pi$$

hom. deg +2

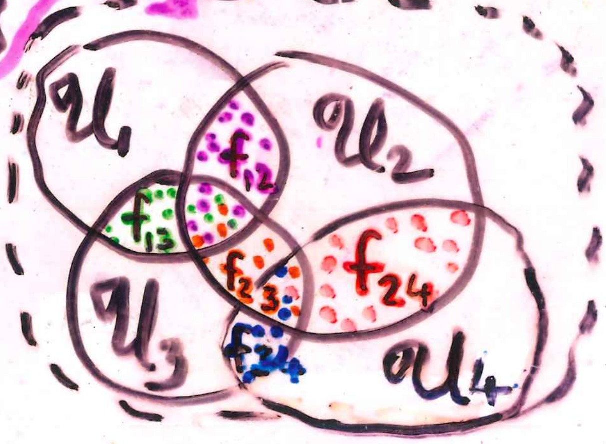
$$\tilde{\psi}_{A'B'C'D'}(r) = \oint_{\omega = ir\pi} \pi_{A'} \pi_{B'} \pi_{C'} \pi_{D'} \tilde{f}(Z)\, \pi\, d\pi$$

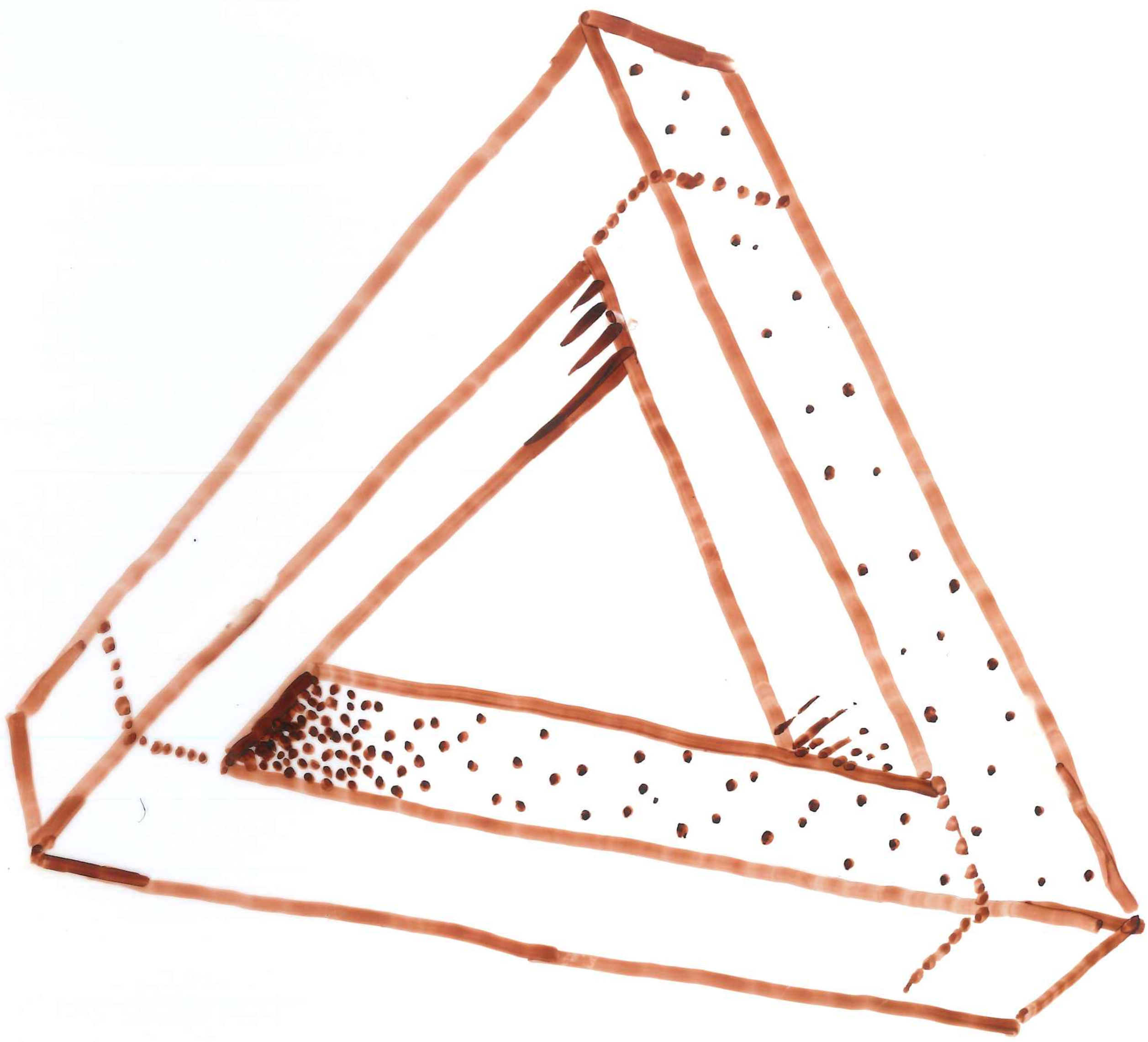
hom. deg. −6

covering $\{\mathcal{U}_i\}$

Really, f and $\tilde{f}$ are representatives of

COHOMOLOGY





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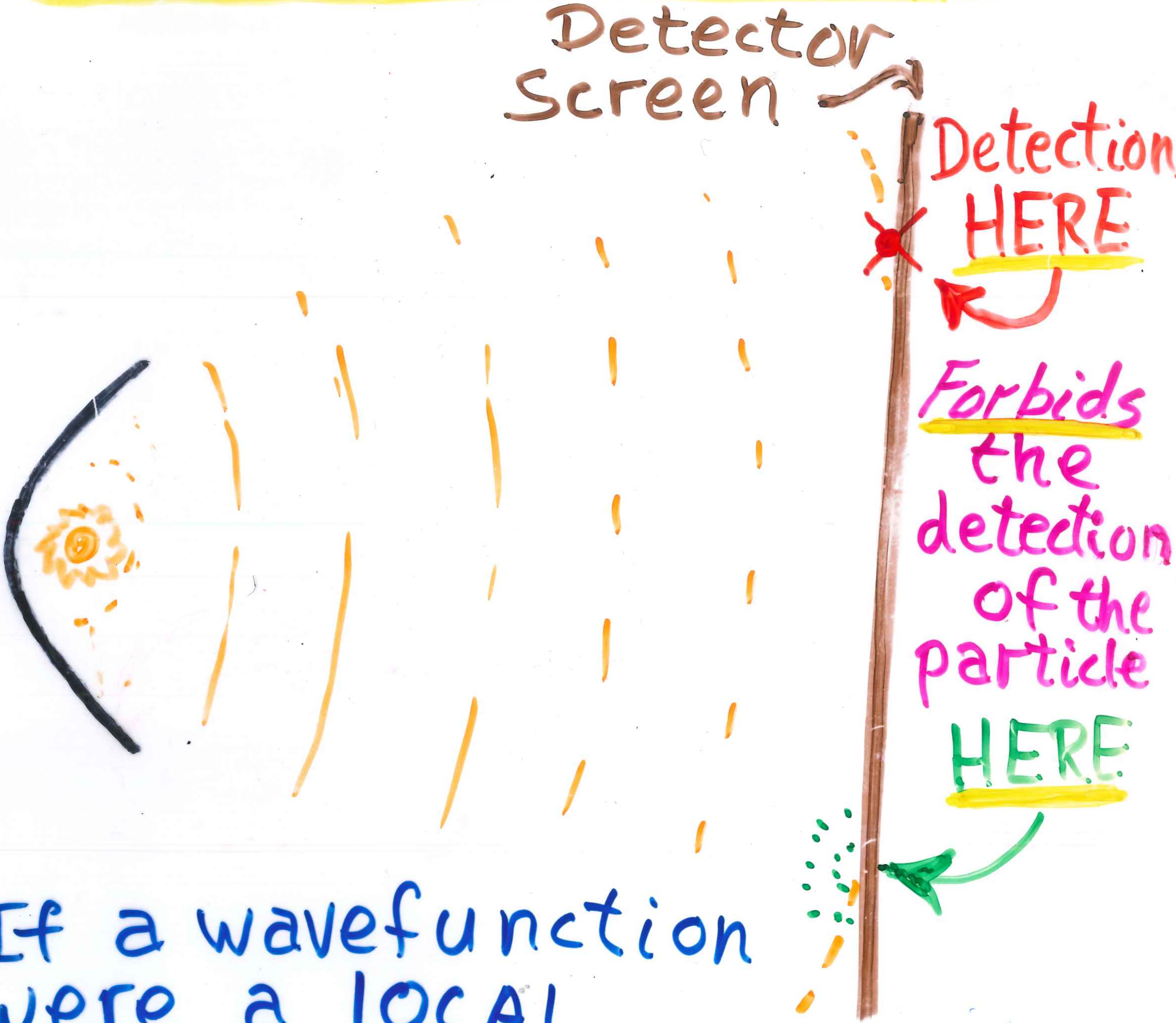
Non-Locality in the Wavefunction of a Single Particle

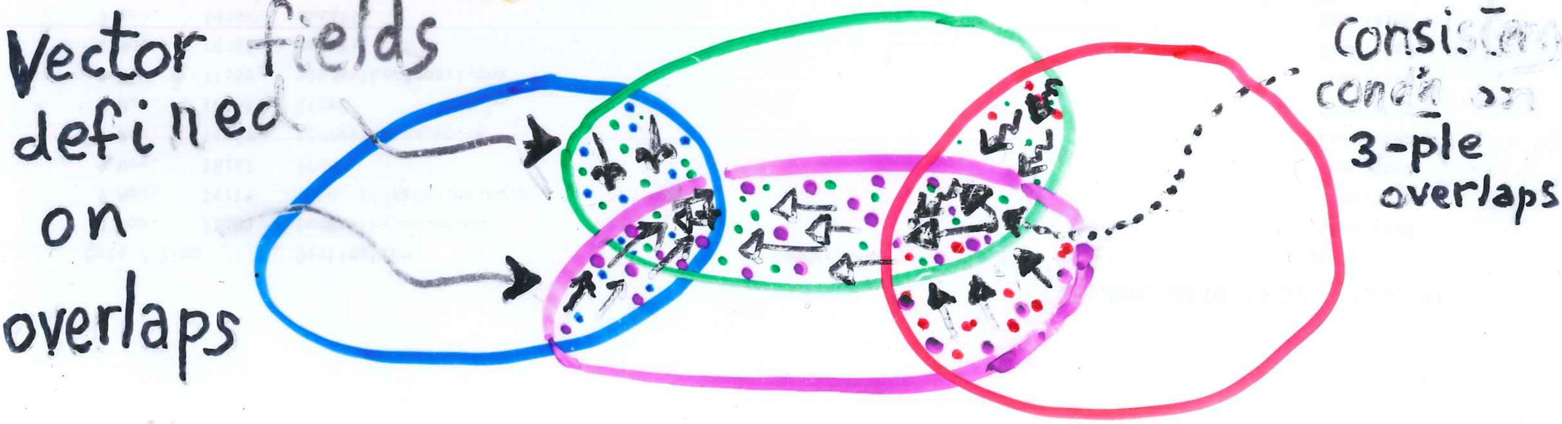


If a wavefunction were a LOCAL disturbance, then detections at both places (or nowhere) could occur — would seem to imply superluminal communication

"Non-linearization" of 1st (sheaf) Cohomology

• Construction of a bundle (Ward constr'n) (1977)

• Construction of a curved manifold (R.P. 1976)

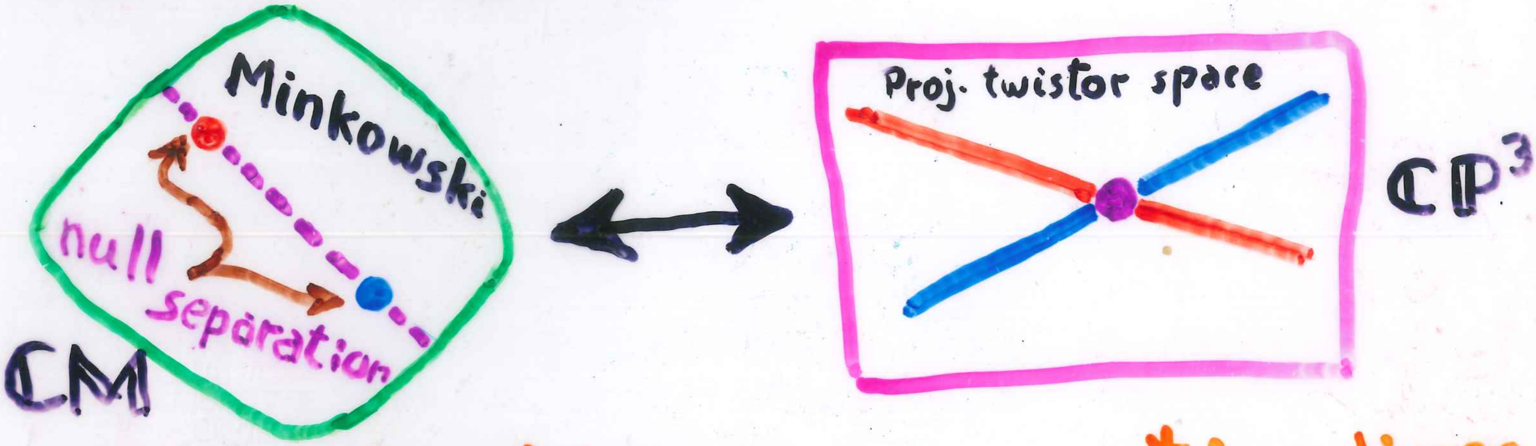


General Relativity

Numerous special applications
(e.g. Woodhouse-Mason: stationary axi-symm.)

As part of general programme:
"non-linear graviton construction" [R.P. '76]

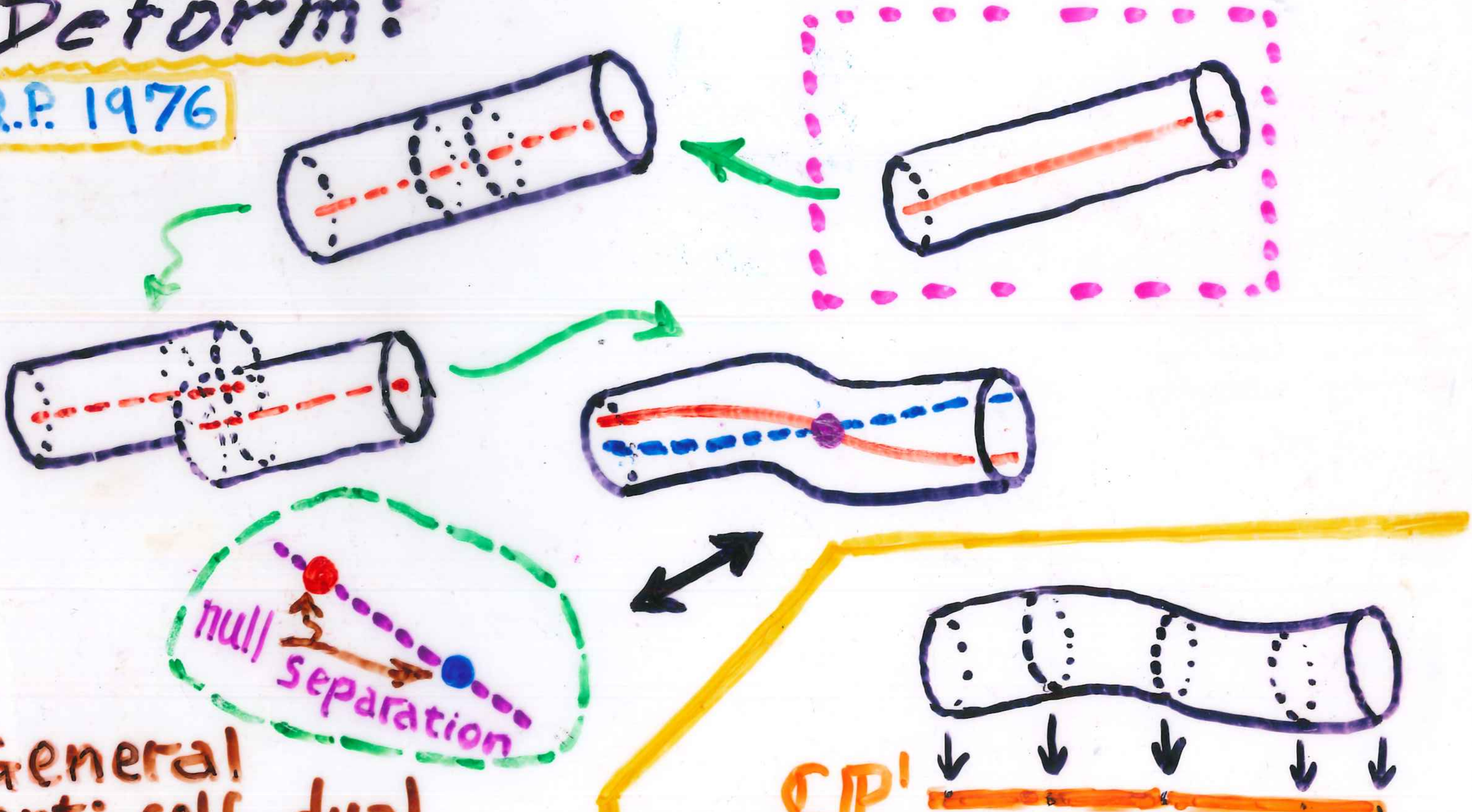
N.B. for flat space:



null separation ⟷ meeting lines

Deform:

R.P. 1976



General
anti-self-dual
complex
"space-time"

General anti-self-dual
Ricci-flat complex "space-time"

Infinity twistors

Provide the space-time metric for Minkowski space $\mathbb{M}$ or (Anti) de Sitter $\mathbb{D}$

$$I_{\alpha\beta} = \begin{pmatrix} \frac{\Lambda}{6}\varepsilon_{AB} & 0 \\ 0 & \varepsilon^{A'B'} \end{pmatrix}, \quad I^{\alpha\beta} = \begin{pmatrix} \varepsilon^{AB} & 0 \\ 0 & \frac{\Lambda}{6}\varepsilon_{A'B'} \end{pmatrix}$$

Λ = cosmological constant > 0

$I_{\alpha\beta} = \overline{I^{\alpha\beta}}$, $I_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta} I^{\gamma\delta}$ Skew

$I^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta} I_{\gamma\delta}$, $I_{\alpha\beta} I^{\beta\gamma} = -\frac{\Lambda}{6}\delta_\alpha^\gamma$

inverse if $\Lambda \neq 0$

Complex symplectic structure

$$\Theta = I_{\alpha\beta}\, dZ^\alpha \wedge dZ^\beta + I^{\alpha\beta}\, dW_\alpha \wedge dW_\beta$$

Sympl. potential: $I_{\alpha\beta} Z^\alpha dZ^\beta + I^{\alpha\beta} W_\alpha dW_\beta$

degenerate if $\Lambda = 0$

Googly Problem

(Note: a "googly" is a cricket ball bowled with an action that would appear to give it a left-handed spin, whereas the ball actually spins right handed.)

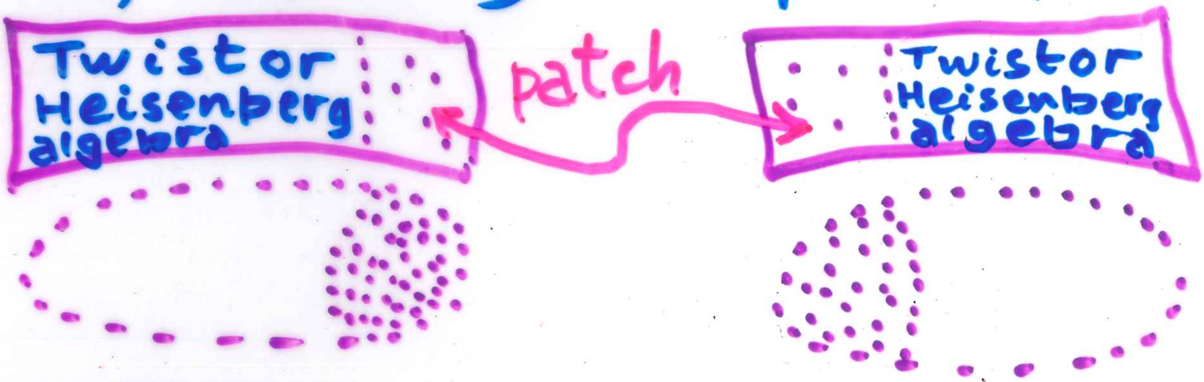
Geometrical coding of −6 (& −4) homogeneity

The "wavefunction" viewpoint, with regard to twistor functions demands that we adopt the chiral procedure of either a twistor ($Z^{\cdot}$) or a dual twistor ($W_{\cdot}$) description, rather than, say, an ambitwistor-type (($Z^{\cdot}, W_{\cdot}$) with $Z^{\alpha}W_{\alpha}=0$) procedure — which is more analogous to a classical-physics description.

Moreover, we cannot get away with using a different twistor space for each helicity, since we need to describe, say, plane-polarized photons, which are superpositions of the two, In twistor theory, holomorphicity plays a crucial role, and this is indeed retained through this twistor quantization procedure

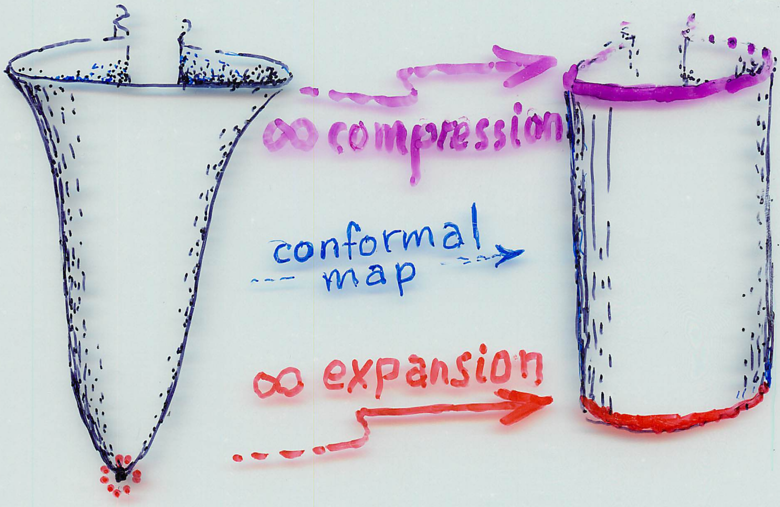
$$\boxed{\bar{Z}_\alpha = -\partial/\partial Z^\alpha \,, \quad \partial/\partial \bar{Z}_\alpha = Z^\alpha} \quad \hbar = 1$$

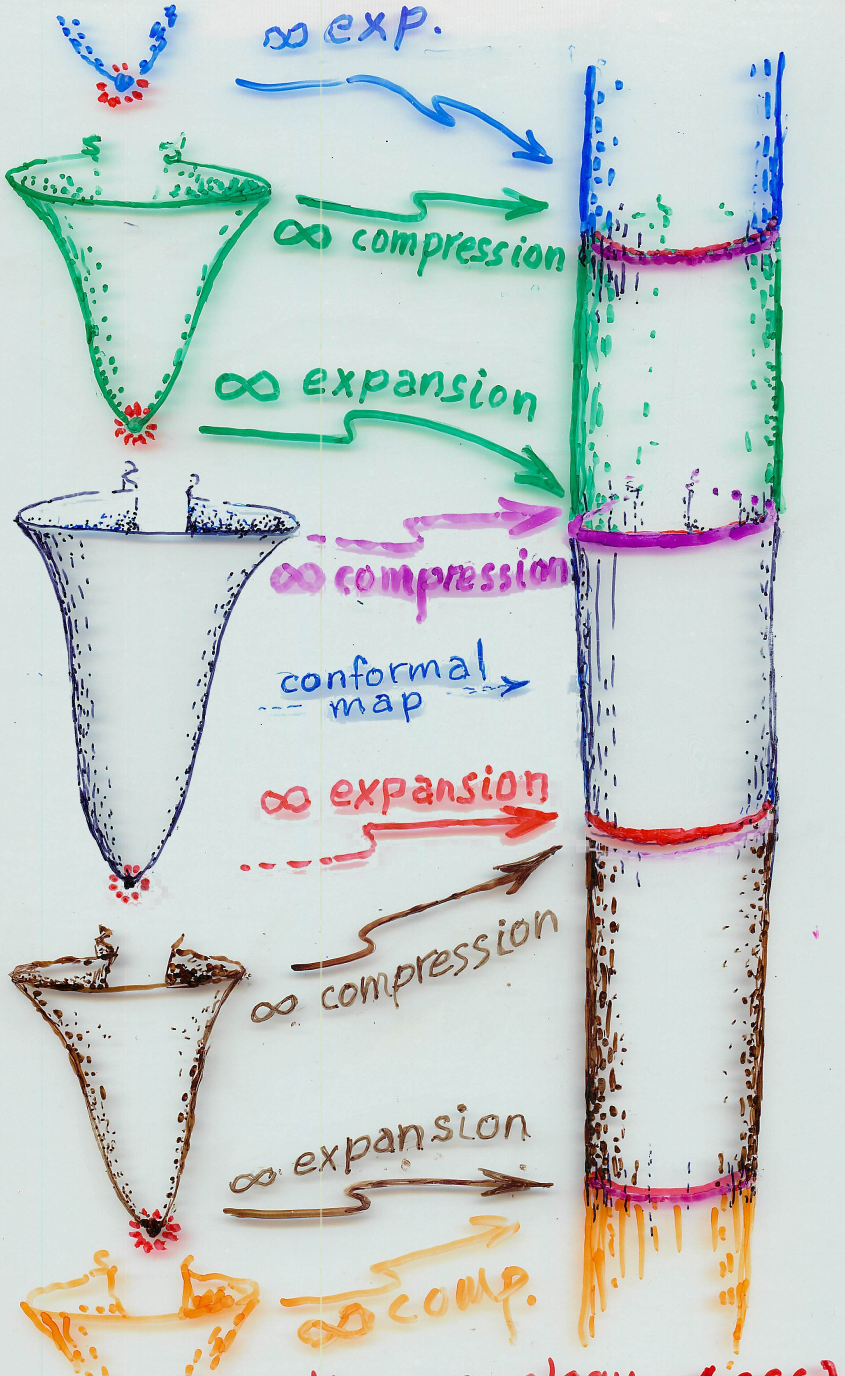
Now, we try to patch:



The patching could proceed somewhat like this:

This is the general idea, but the patching could involve sublties

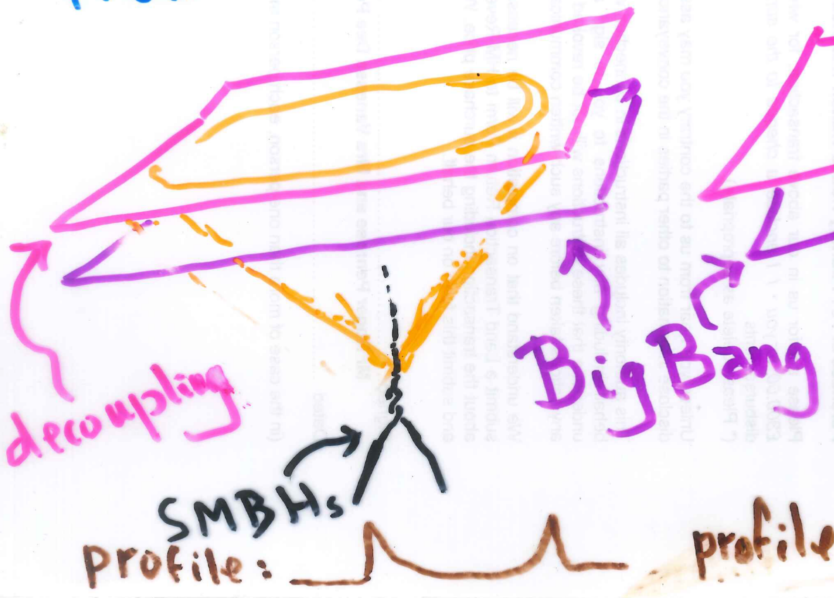

∞ compression
conformal map
∞ expansion



conformal cyclic cosmology (ccc)

Two types of signal from CCC

① Grav. radn. from SMBH collision



② Hawking radn. from SMBH