Optimisation of parameters for single-event based TOF FBP image reconstruction

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Motivation: time resolution

Typical scintillation crystals (LSO:Ce, LYSO:Ce, LaBr$_3$:Ce) exhibit coincidence resolving time **(CRT)** of ~ 100 ps. The lowest actual value is **214 ps** (Biograph Vision scanner, Siemens).

In **Jagiellonian PET (J-PET)**, plastic scintillators are superior time-wise: **70 ps** – for 1-meter strips [Moskal P et al. PMB 2016]. Despite lower detection efficiency, smaller statistics/times of scan are required due to time-of-flight **(TOF)** available.
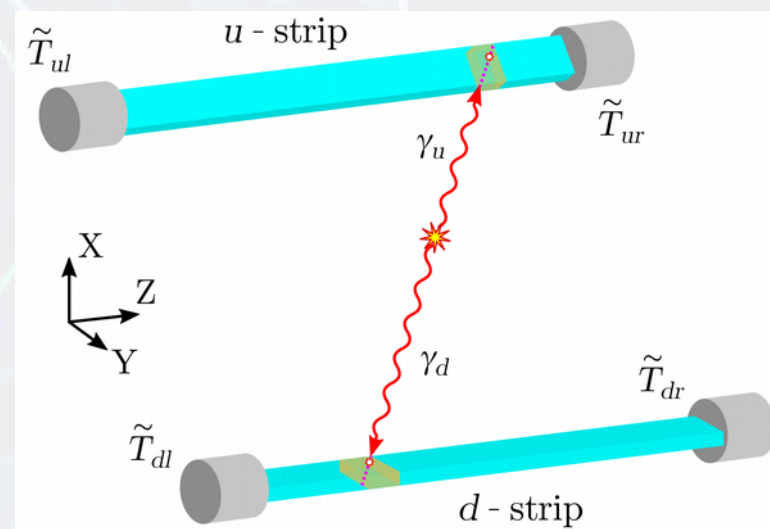
The main smearing factor is the readout – photomultipliers (PMs) attached at each end: silicon PM (SiPM) or tube PM (PMT).

TOF and CRT below 100 ps – analytical reconstruction methods may outperform iterative ones [V Westerwoudt et al. IEEE Trans. Nucl. Sci. 2014].

Motivation: real time imaging

Lower statistics compared to non-TOF methods and small CRT open up prospects for image reconstruction on the fly during real time scans.

A platform based on Field Programm-able Gate Array (**FPGA**) System-on-Chip (SoC) **has already been implemented** for J-PET [G Korcyl et al. IEEE Trans. Med. Im. 2018]. It performs event building, filtering, coincidence search and so-called Region-Of-Response **(ROR)** reconstruction. Filtered back projection **(FBP)** has not been implemented, but recent reports suggest a number of solutions.

35-second scan

In order to boost the performance, _only small fraction_ of field-of-view **(FOV)** is processed for each event.
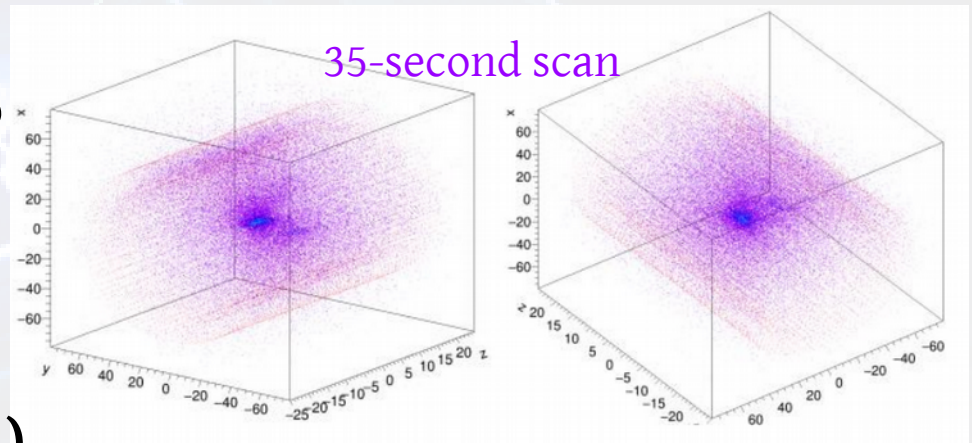
Single-event TOF FBP in image space

TOF FBP (filtering by a function $W(v_s)$ in Fourier space): 5 variables: $s, \phi, \zeta, \theta, t$.

p – projections
$h(t)$ – TOF kernel

$$p_i^F(s, \phi, \zeta, \theta, t) = \mathcal{F}^{-1}\{W(\nu_s)\mathcal{F}[p_i(s, \phi, \zeta, \theta)]\} \cdot h(t - t_i).$$

$$p_i(s, \phi, \zeta, \theta) = \begin{cases} 1, & s = s_i \cap \phi = \phi_i \cap \zeta = \zeta_i \cap \theta = \theta_i, \\ 0, & \text{otherwise.} \end{cases}$$

The dimensionality is in fact reduced to 2 dimensions (s, t) for one event or bin:
3D TOF FBP (arbitrary voxel $\boldsymbol{v}$):
[Conti M et al. PMB 2005]

$$f(\boldsymbol{v}) = \sum_{i=1}^{N_{\text{LOR}}} f_i(\boldsymbol{v}) = \sum_{i=1}^{N_{\text{LOR}}} \mathcal{B}\{p_i^F(s, \phi_i, \zeta_i, \theta_i, t)\}.$$

Forward and inverse Fourier transform, a filter in frequency domain... *too cumbersome!*

The alternative: *treat all lines-of-response (LORs) independently (**no bins**).* One LOR reflects *one point* on a sinogram. Back-projection $B\{p^F_i\}$ is substituted by two kernels in image space, applied to i-th event: $h(t)$-related Gaussian along a LOR and Ram-Lak (ramp) filter $w(s) = F^{-1}W(v_s)$ – in perpendicular direction.

$$q(n) = \begin{cases} \dfrac{1}{4} & n = 0 \\ \dfrac{-1}{n^2\pi^2} & n \text{ is odd} \\ 0 & n \text{ is even} \end{cases}$$

One LOR

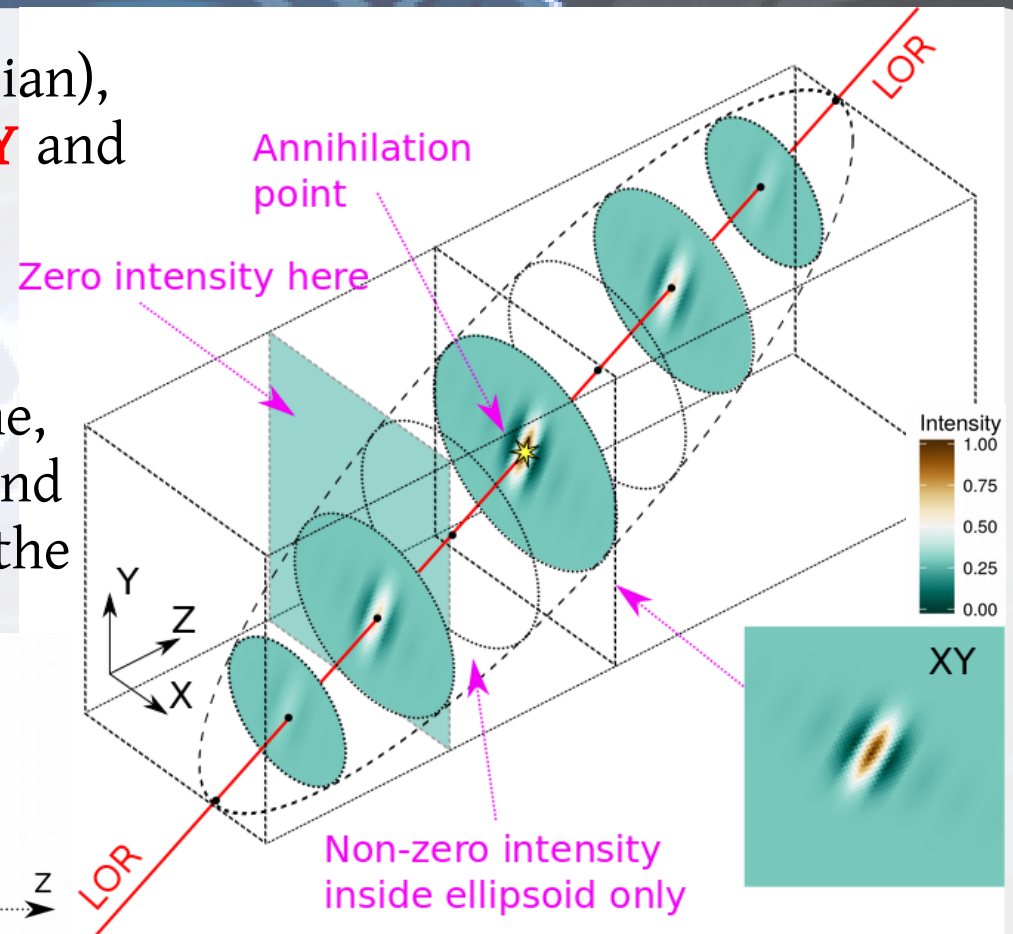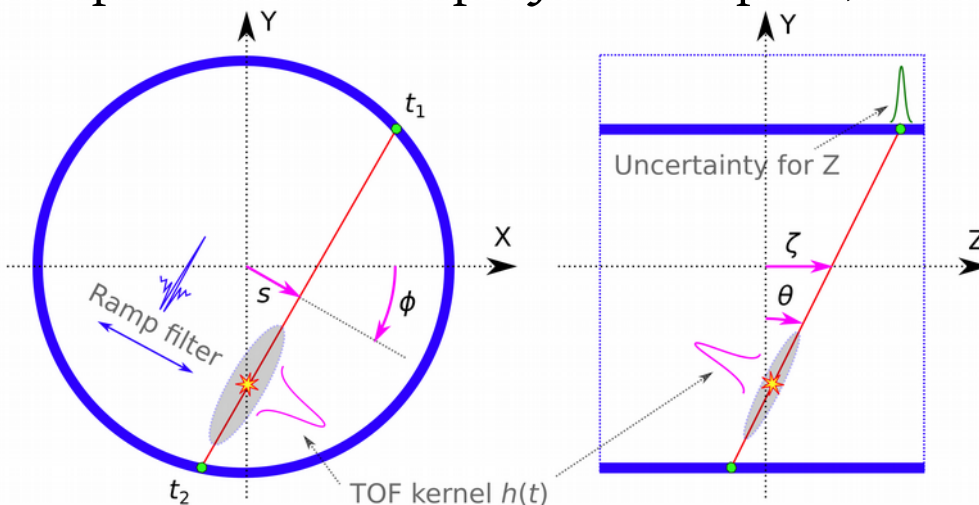Intensity

Pixel ID for displacement
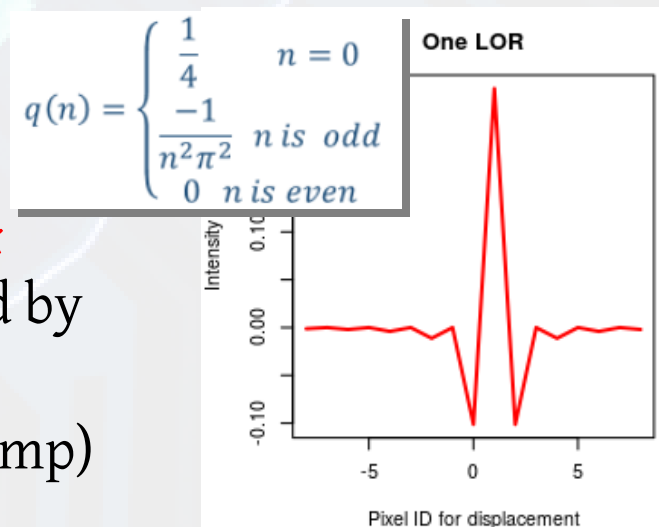
3D asymmetrical kernel in image space

Model: **TOF kernel along LOR** (Gaussian), **Ram-Lak filter normal to LOR in XY** and a **small Gaussian along Z** (3rd kernel, depends on Δz between slices).

Update intensity within a small volume, limited by at least $\pm 3.3\sigma$ for Gaussian and $\pm 9.0\Delta s$ for Ram-Lak (Δs – sampling for the displacement s in projection space).

Annihilation point

Zero intensity here

Intensity
1.00
0.75
0.50
0.25
0.00

XY

Non-zero intensity inside ellipsoid only

The ROR volume of the ellipsoid is much smaller than the whole FOV.

Kernel optimisation problem

We may define Gaussians from CRT and number of slices (σ_{TOF} and σ_z), but is this an optimal solution?

From similarities with multivariate kernel density estimation (**KDE**), *applied directly to annihilation positions, estimated from TOF*.

For a d-dimensional dataset $\mathbf{X}_1, ..., \mathbf{X}_n$ of the size n:

$$\hat{f}_{n\boldsymbol{H}}(\boldsymbol{x}) = n^{-1} \sum_{i=1}^{n} |\boldsymbol{H}|^{-1/2} K\left[\boldsymbol{H}^{-1/2}(\boldsymbol{x} - \mathbf{X}_i)\right]$$

(2D) wikipedia

$\boldsymbol{x} = (x_1, ..., x_d)^T$, $K(\cdot)$ – *spherically symmetric* kernel, $\boldsymbol{H}$ – *bandwidth matrix*

Bandwidth selection – min. of non closed-form *mean integrated squared error* (MISE):

$$\underset{\mathbf{H}}{\arg\min} \, \mathrm{MISE}(\mathbf{H}) = \mathrm{E}\left[\int (\hat{f}_{n\boldsymbol{H}}(\boldsymbol{x}) - f(\boldsymbol{x}))^2 \, d\boldsymbol{x}\right]$$

TOF FBP   TOF KDE (samse)

Intensity
1.00
0.75
0.50
0.25
0.00

Sum of asymptotic mean squared error (**SAMSE**) – elements of **H** $\ll \sigma_{TOF} < \sigma_z$!

[Duong T et al. J. Npar. Stat 2003]

(example for 1-mm source, ideal scanner [Kowalski P et al. PMB 2018])
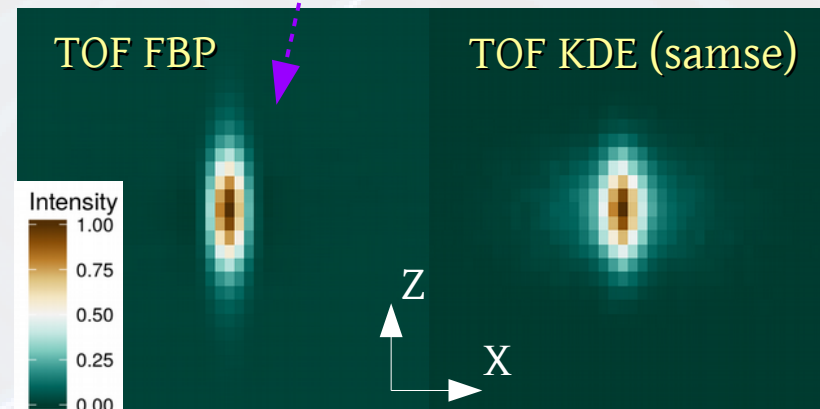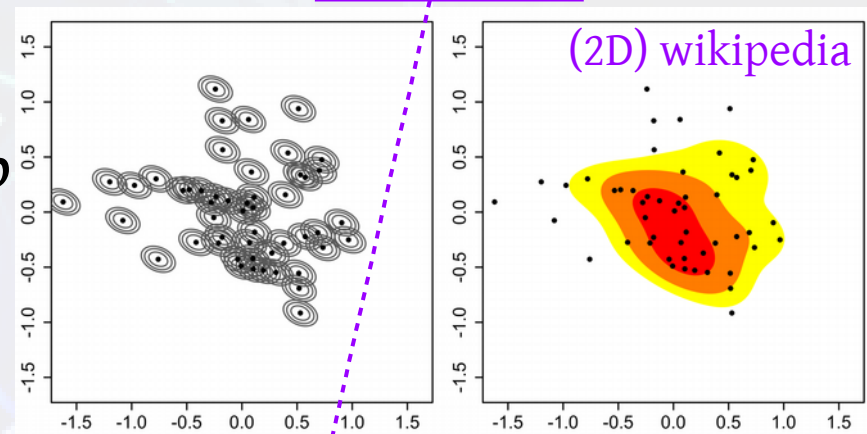
Mixing KDE with Ram-Lak filter?

SAMSE could not be employed for asymmetrical TOF FBP kernel with the additional non-Gaussian filter (Ram-Lak or other). A distinct model might be required for the minimisation of $\mathrm{MISE}(\mathbf{H}) = \mathrm{E}\left[\int (\hat{f}_{n\mathbf{H}}(\boldsymbol{x}) - f(\boldsymbol{x}))^2 d\mathbf{x}\right]$

But what if we use the elements of $\mathbf{H}$ for the estimation of σ_{TOF} along each LOR?

Ram-Lak applied here:
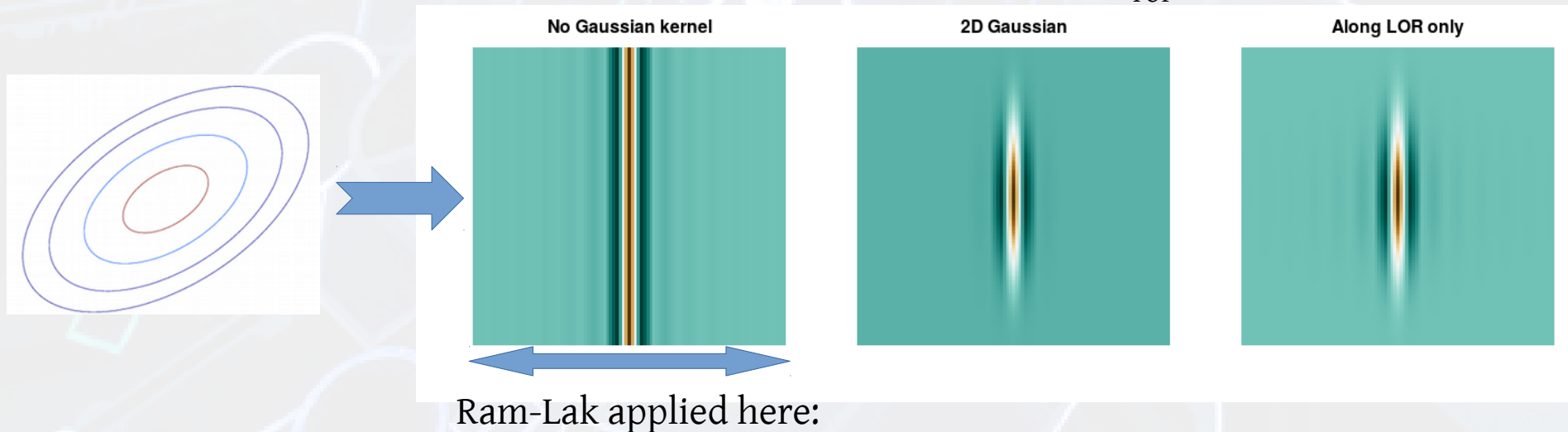
KDE kernel could be applied over the whole FOV or restricted along Ram-Lak direction.

Other possibilities: tiny TOF kernel (small σ_{TOF}) and/or σ_Z, different filter (Hann, Hamming), apodisation (cutoff max frequency for $W(v_s)$ function) etc.

Image quality for IEC NEMA phantom

IEC NEMA phantom, simulated in GATE (at the centre of the scanner, *true coincidences* only (data size 10 mln.) *Ideal geometry*: 384 strips, R=43.73 cm, **SiPM** (CRT=235 ps)

Image quality parameters: Contrast recovery coefficient (CRC), Background variation (BV), Signal-to-noise ratio (SNR) [NEMA-NU-2, 2012]

Attenuation correction: each LOR is treated as a projector, attenuation path is estimated via Siddon algorithm (intersecting length of a ray with each voxel) [R Li et al., Journ. Comp. Sci. 2010]:

Update intensity: $I = I_0 \exp(-\mu x)$, $\mu^{PET}(H_2O) = 0.096$ cm^{-1}

Attenuation map: all phantom volume filled with radioactive liquid, without cold spheres/capillaries.

Reference reconstructions:
– Non-TOF **FBP 3DRP, OSMAPOSL** (OSEM-One Step Late) from STIR framework [K Thielemans et al., PMB 2012]
– **TOF KDE** – no filters, symmetric 3D kernel

Attenuation map (XY)
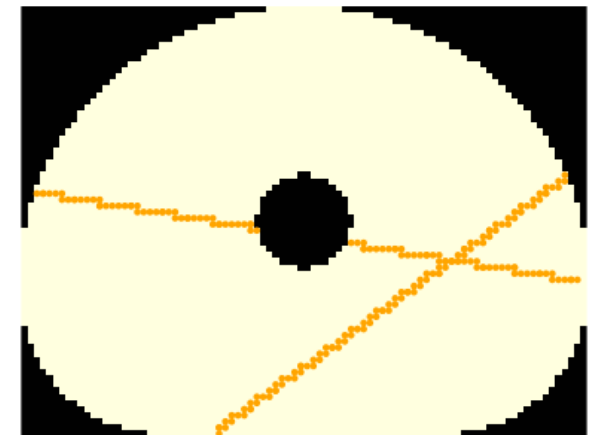
Results: KDE + Ram-Lak

Estimate 3D kernel from KDE (with $\mathbf{H}_{samse}$) and reconstruct for 3 cases:

1) *Substitute* by Ram-Lak in XY plane for each LOR,

2) *Multiply* with Ram-Lak (ramp * Gaussian)

3) *Pure* KDE (**alternate attenuation used!**)

Cuts are made across the centres of spheres in XY and XZ plane
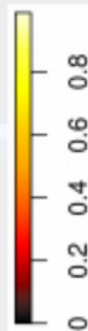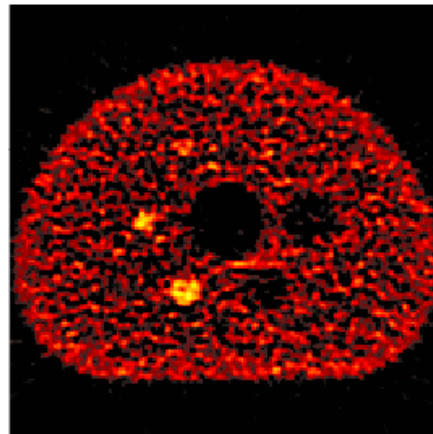
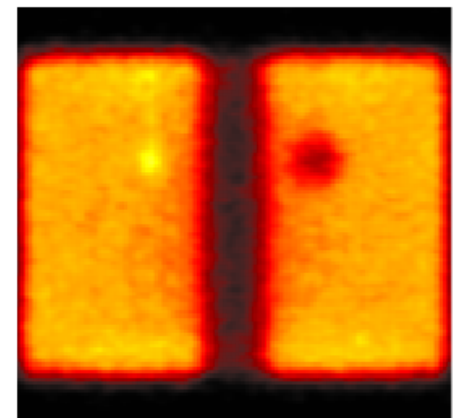Along LOR only (XY) Full 'samse' kernel (XY) KDE 'samse', no RL filter (XY)
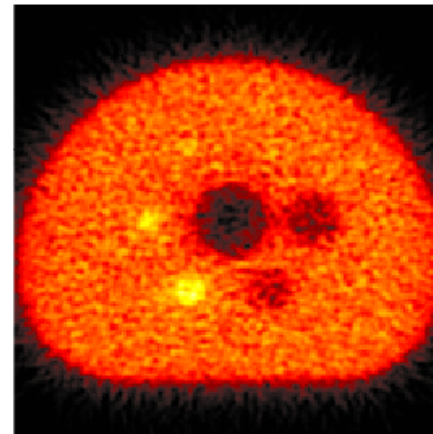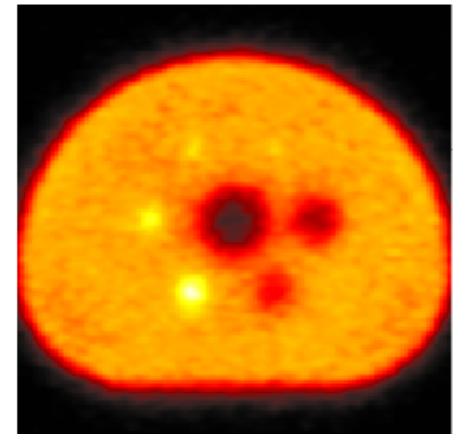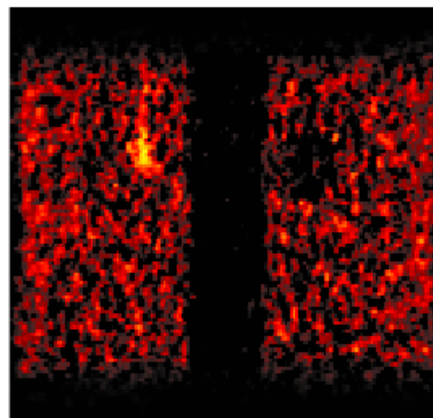
Along LOR only (XZ) Full 'samse' kernel (XZ) KDE 'samse', no RL filter (XZ)
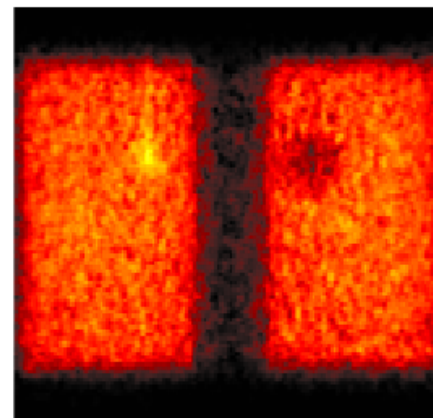
Results: CRT-related TOF kernel

σ_{TOF} and σ_Z are defined from CRT and slice width along Z, respectively.

It is worth comparing with the case of σ_{TOF} set to a smaller value: one can see *visibly better axial resolution.*

(!) The example to the right is obtained using Hamming filter!

Sensitivity correction helps with the consistency on the edges of phantom, mainly across Z
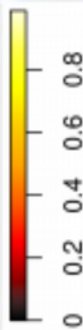
From CRT (Hamming), XZ — σ_{TOF} ~ 1.5 cm (CRT)
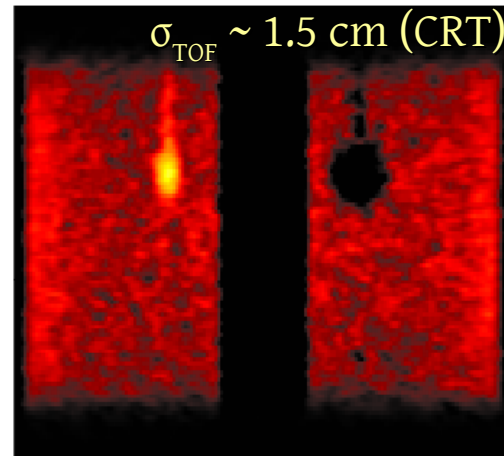
Small sigma_tof (Hamming), XZ — σ_{TOF} ~ 0.5 cm
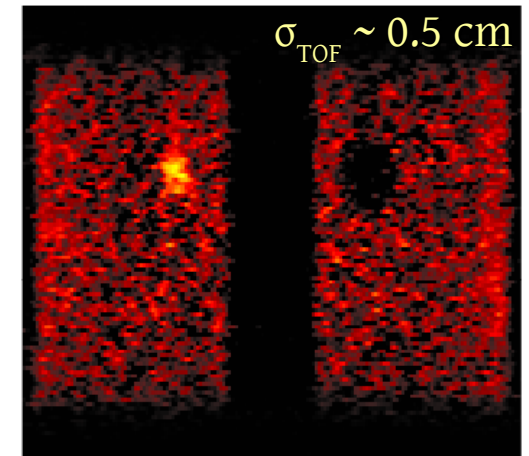
Adjusted by Sens. matrix
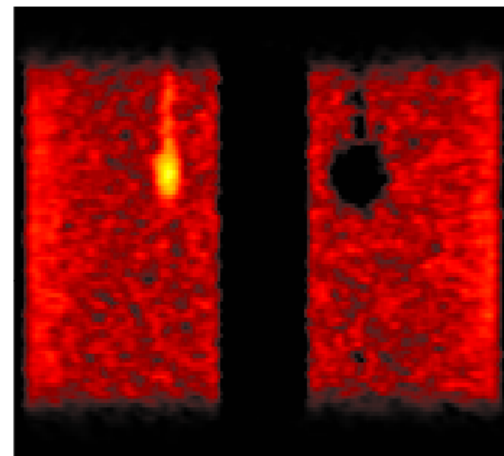
Sens. matrix (XZ)

Results: filter selection

+ apodisation (cutoff frequency for $W(v_s)$ function)
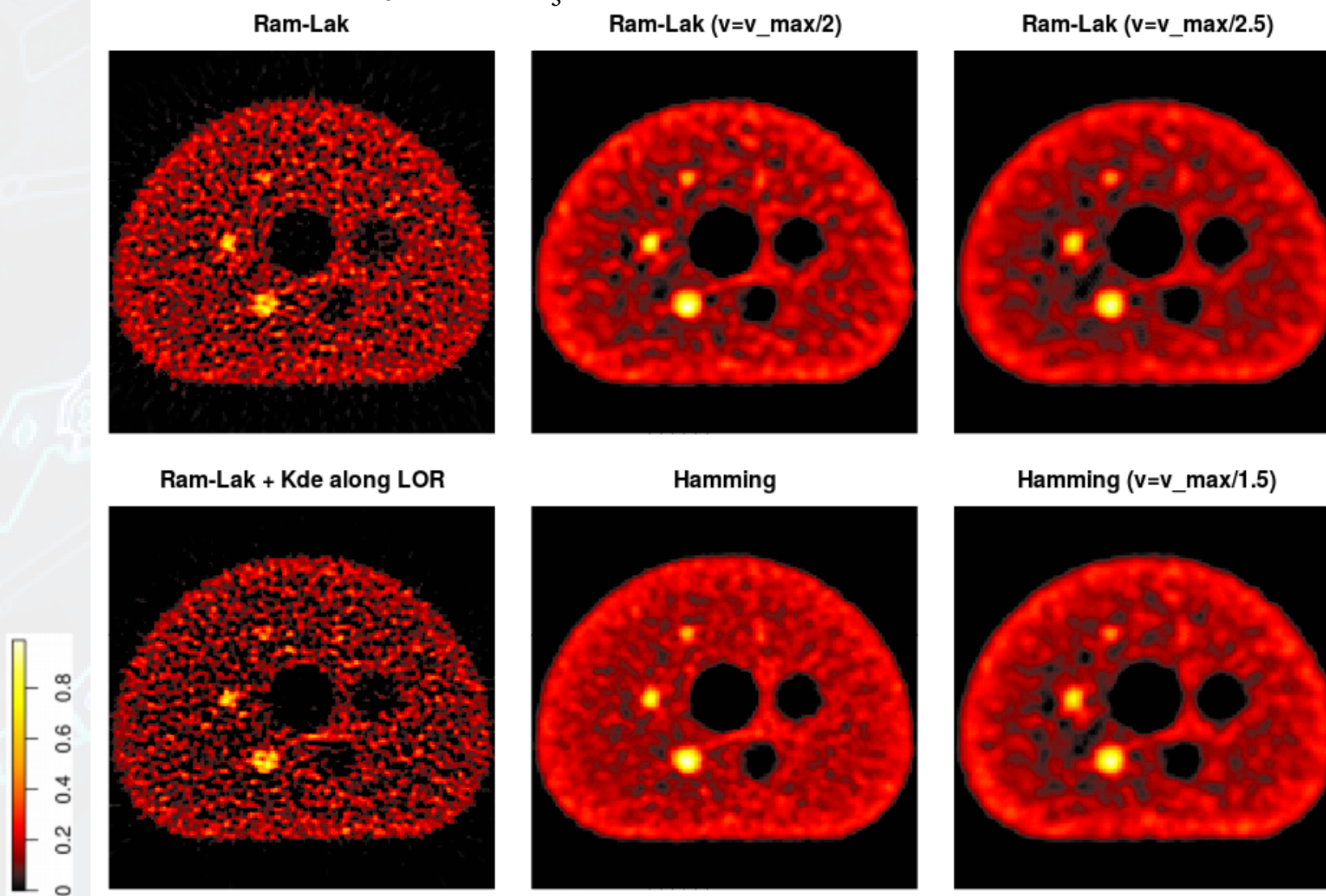
Results: reference reconstructions
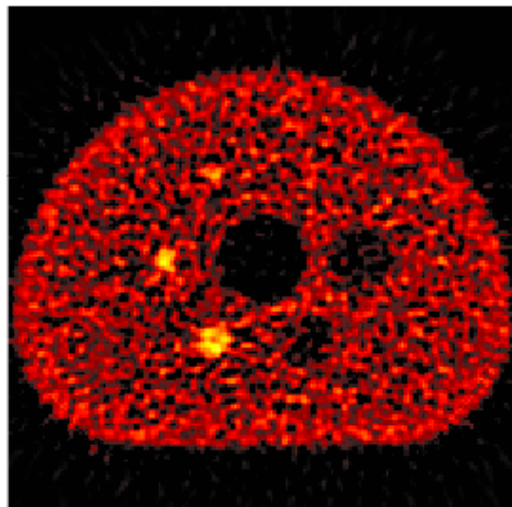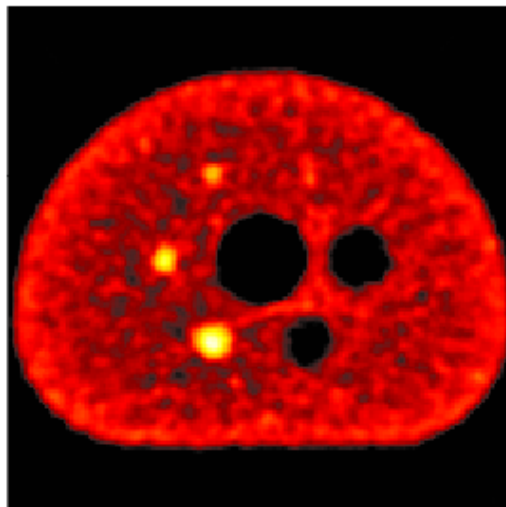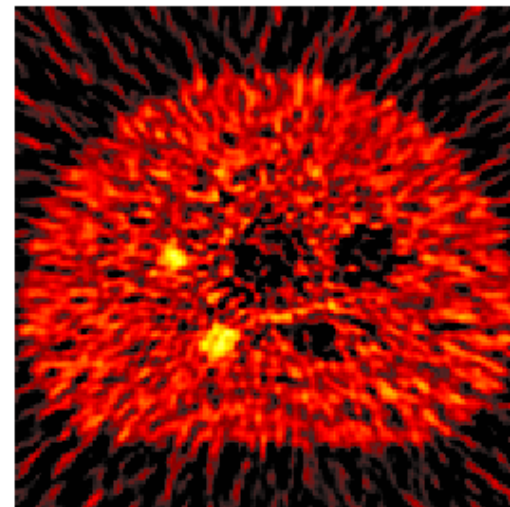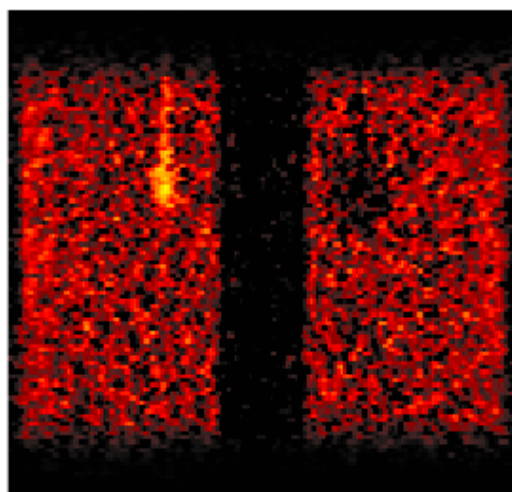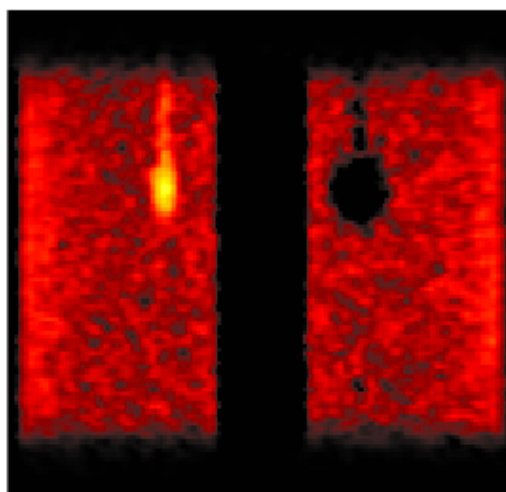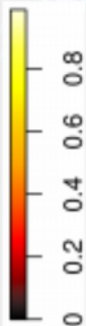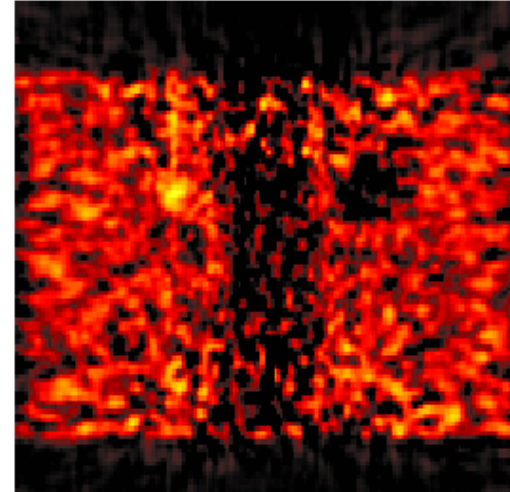
Results: reference reconstructions
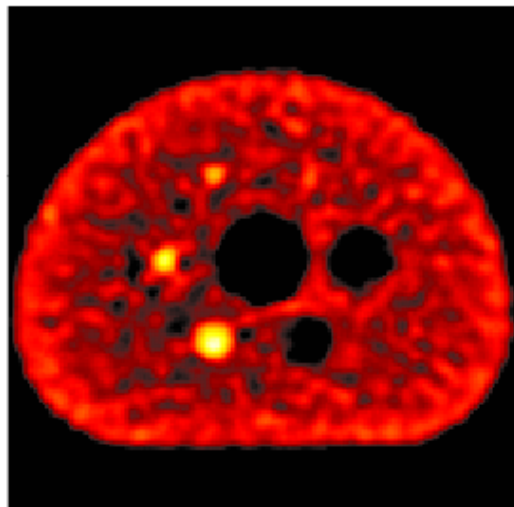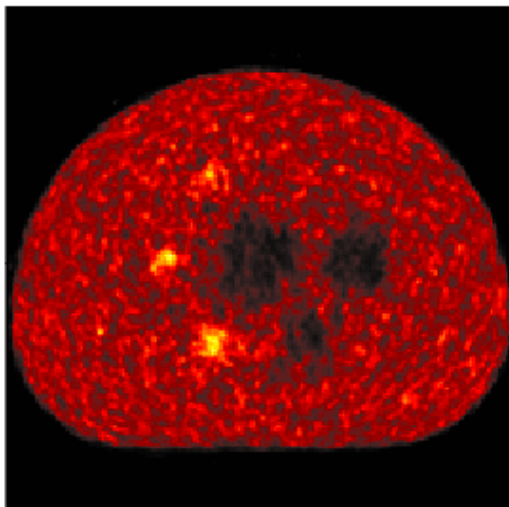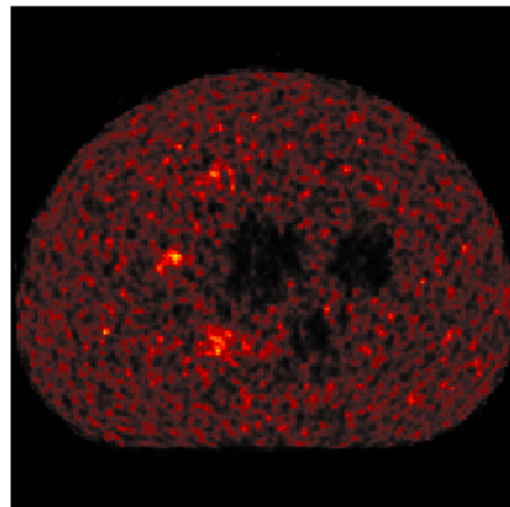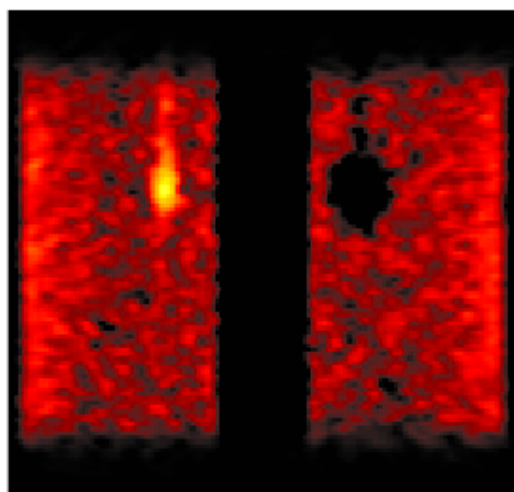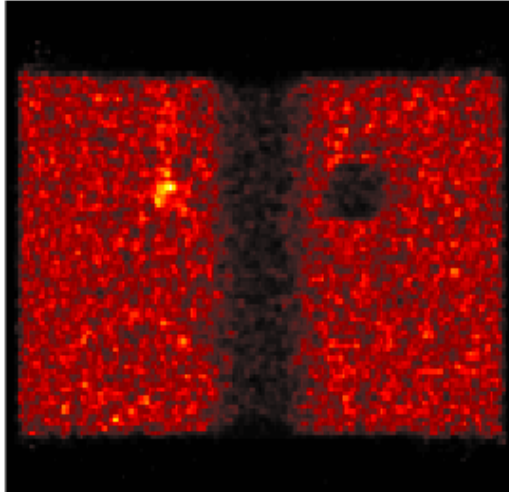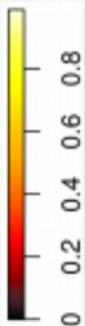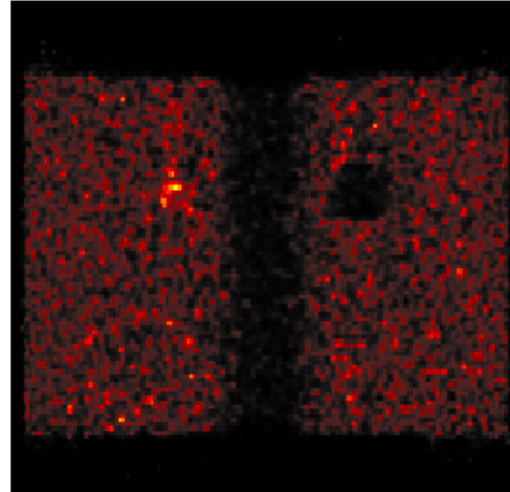
Results: image quality

Optimal parameters are yet to find, but in general ***TOF FBP is superior to non-TOF*** STIR reconstructions. *Ideal: CRC = 1.0, BV = 0, SNR = Inf.*

Three spheres chosen –
two hot and one cold:
13 mm, 22 mm and 28 mm

Iterative OSMAPOSL (lines)
comprises three points for
number of sub-iterations:
24, 36 and 48.

*Hamming filter and
Ram-Lak with 2x cutoff*
frequency appear to be
the best solution (*triangles*).

Image quality comparison

Legend:
- 13 mm (red ●)
- 22 mm (yellow ●)
- 28 mm (grey ●)
- KDE (samse) (○)
- FBP3DRP (●)
- OSMAPOSL (—●—)
- -- TOF FBP --
- +KDE(full) (×)
- +KDE(along LOR) (✱)
- Ram-Lak (○)
- Ram-Lak(cut by half) (▽)
- Hamming (△)

Axes: CRC (y), BV (x)
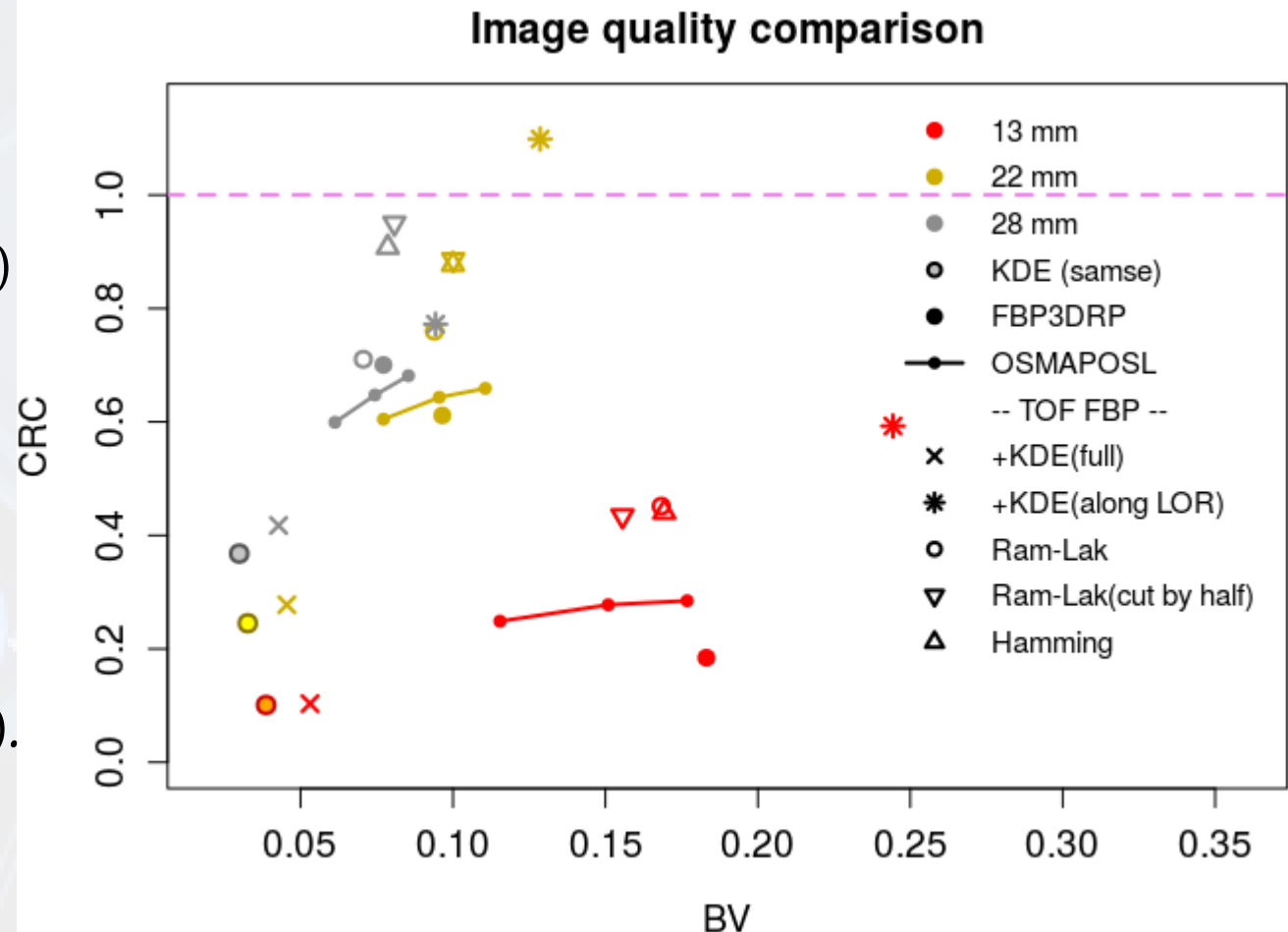
Results: image quality

Signal-to-noise ratio is better for Hamming filter than for the apodised Ram-Lak, even if the variance taken into account (not shown)
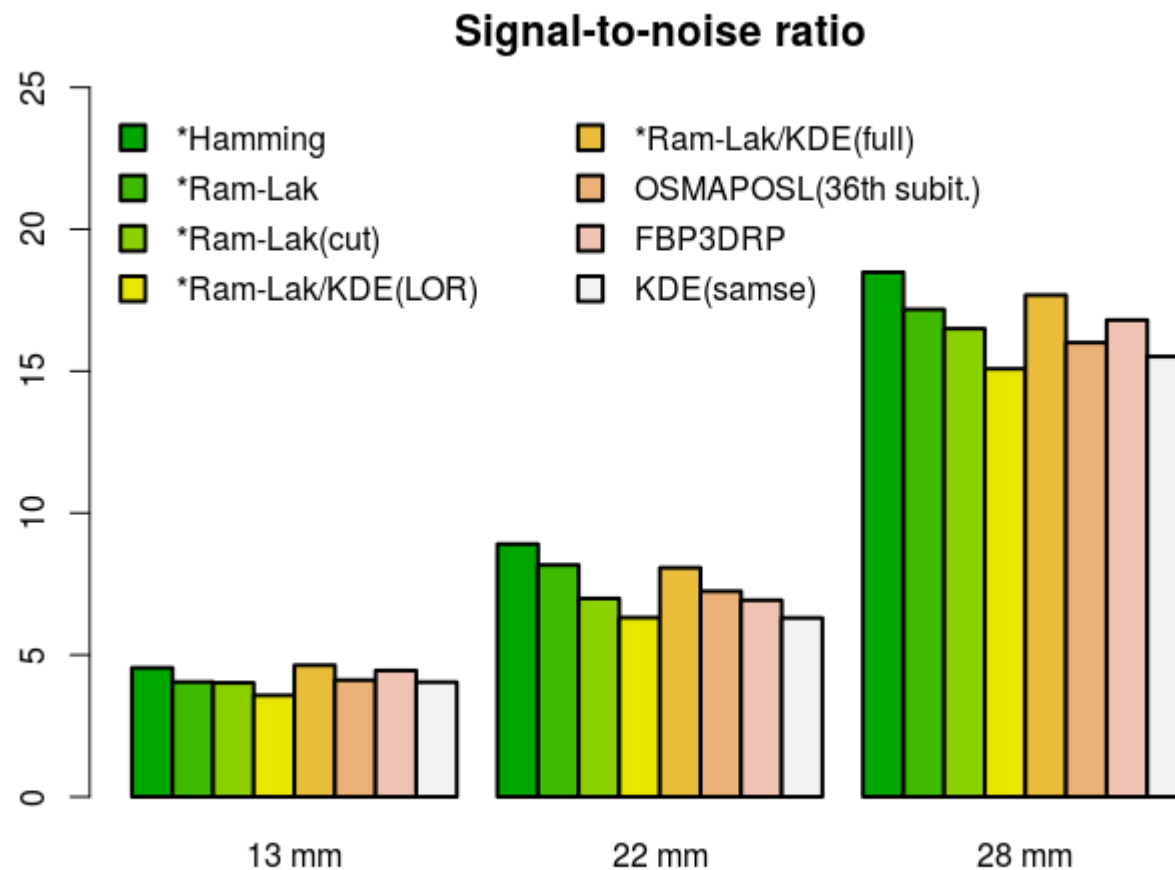
Conclusions

– TOF based FBP is a promising solution for J-PET and could be employed using filters defined in image space and applied to each LOR as three separate kernels in event-by-event way. Since the intensity may be updated only for the small ROR instead of the full FOV, there are distinct prospects for real time imaging.

– Imposing CRT-defined Gaussian kernels along LOR and Z-axis would blur the image and affect spatial resolution. Reducing standard deviation σ_{TOF} would increase the noise instead.

– Optimal parameters could not be found using MISE algorithms approved for KDE, because 3D kernel is not symmetric. Mixing Ram-Lak filter with the solutions for KDE bandwidth matrix, estimated by SAMSE method, result in worse outcomes than CRT-defined model.

– Single-event based TOF FBP achieve better results for image quality analysis of NEMA IEC phantom, if compared to non-TOF reconstructions from STIR and non-filtered TOF KDE. Further adjustments of filter parameters improve the outcome, with the best CRT/BV/SNR-combination obtained for Hamming filter.

Unresolved:
– Is it possible to find optimal parametisation for asymmetric TOF FBP kernel analytically?
– Compare the results for TOF FBP with other TOF based algorithms (MLEM, TV etc).
– Employ the model for FPGA.

Grazie mille per la vostra attenzione!