Some remarks on total variation regularization based PET imaging

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Agenda:

- Analytic PET imaging:
  - TOF back-projection total variation (TV) regularization
- Ongoing works:
  - implementation of complete attenuation correction
  - introduction of shift-variance

1. TOF back-projection TV regularization (TOF-BPTV)

Subsequent steps of the proposed algorithm:

- List-mode data pre-correction

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- TOF back-projection

↓

- Reconstruction with regularization

1. TOF back-projection TV regularization (TOF-BPTV)

Subsequent steps of the proposed algorithm:

- List-mode data pre-correction

↓

- TOF back-projection $$\mathbf{b} = A\mathbf{f}$$

↓

- Reconstruction with regularization

1. TOF back-projection TV regularization (TOF-BPTV)

- Total Variation (TV) norm of an image can be defined as:

$$\mathrm{TV}(\mathbf{f}) = \sum_i |D_i \mathbf{f}|$$

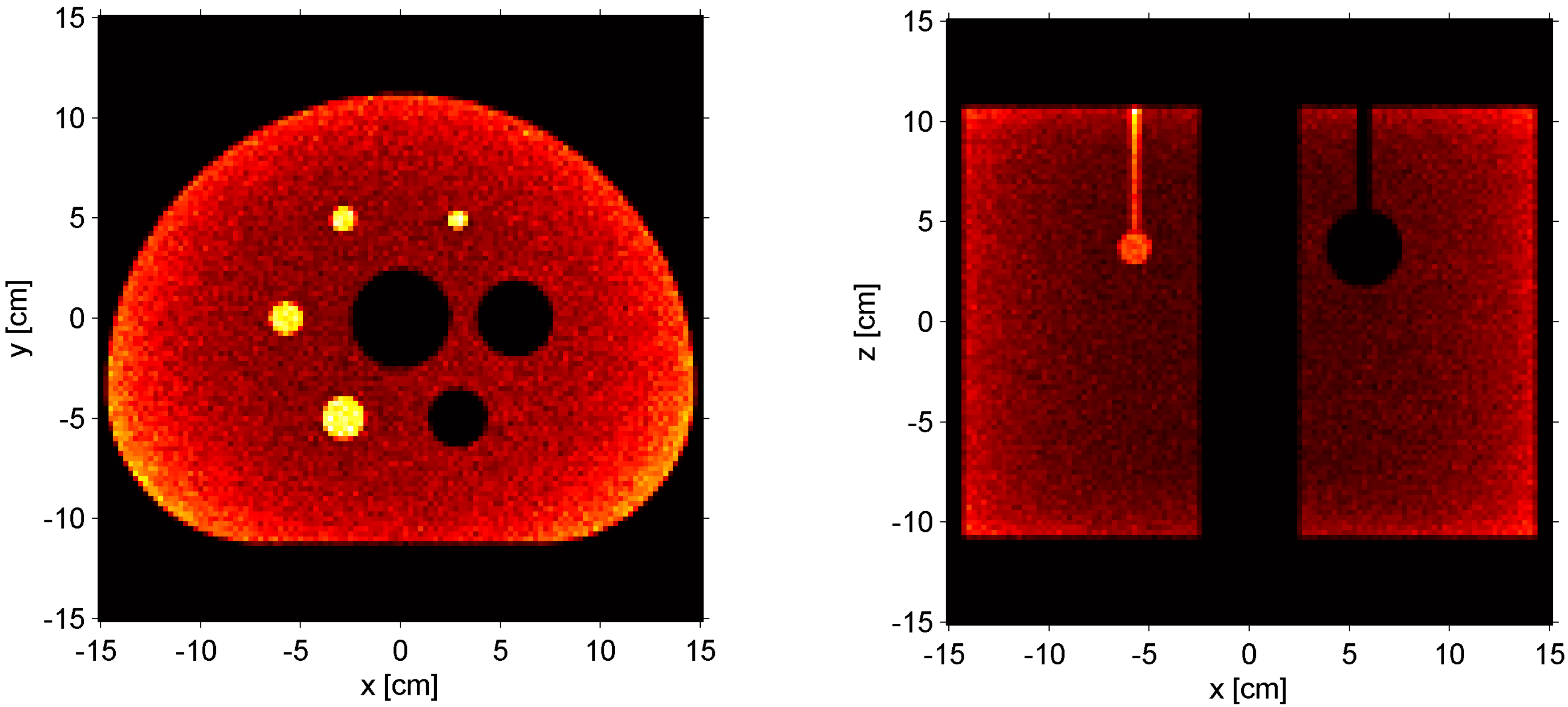
D : first-order forward finite-difference operator

- Image f is reconstructed by solving regularization problem:

$$\min_{\mathbf{f}} \ \mathrm{TV}(\mathbf{f}) + \frac{\mu}{2} \|A\mathbf{f} - \mathbf{b}\|_2^2$$

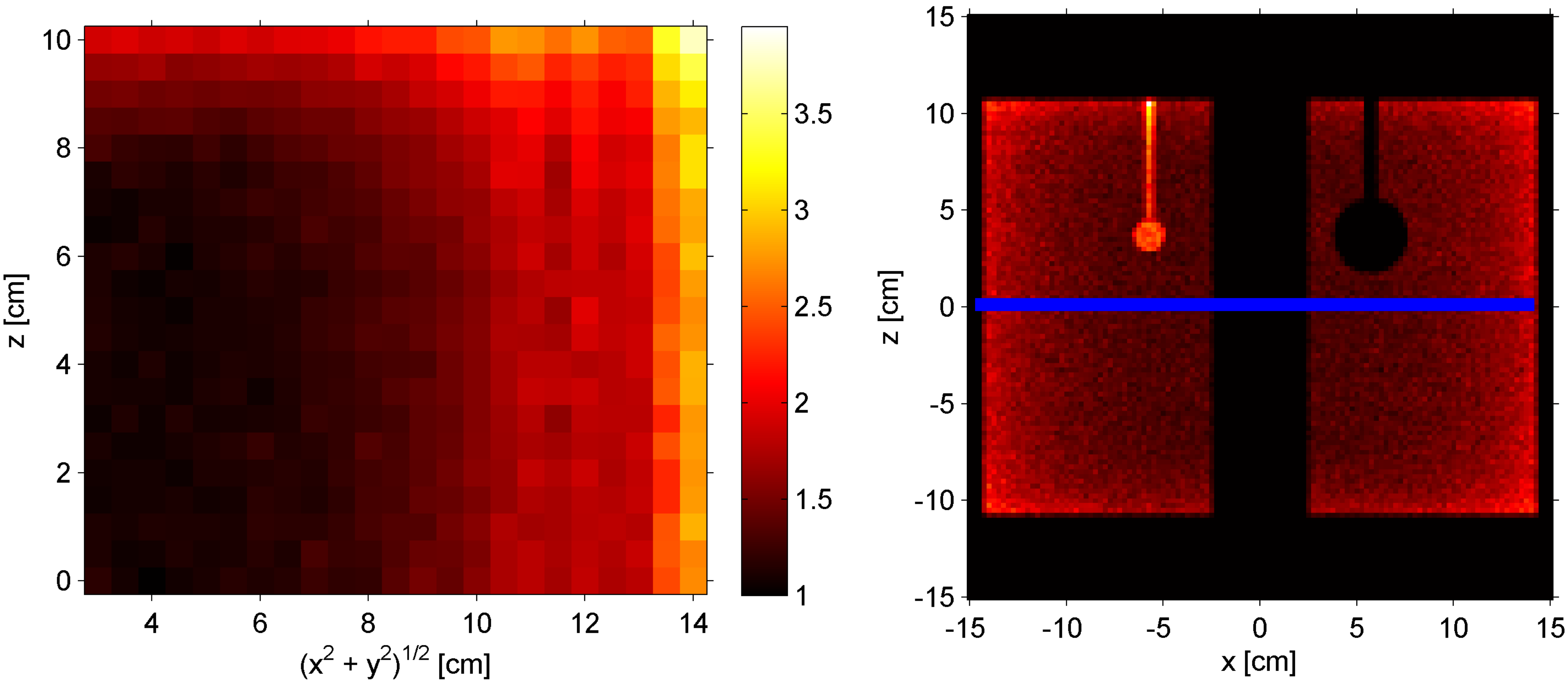
μ : regularization parameter

2. Attenuation correction



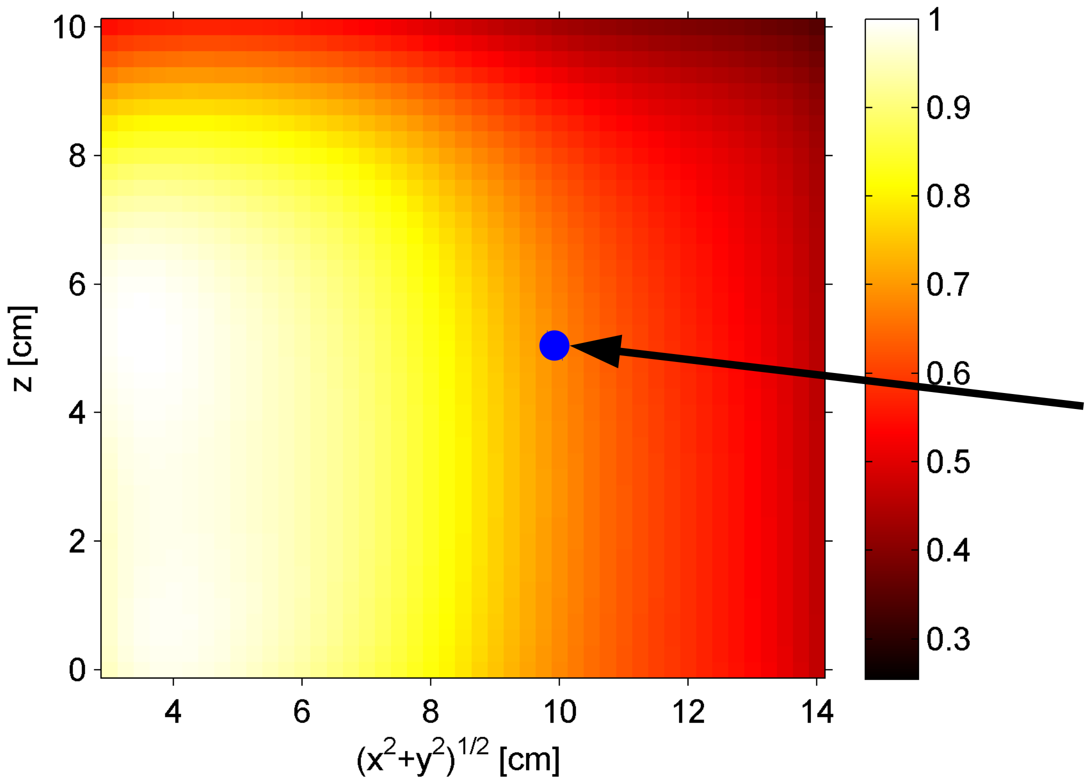
- Illustration of attenuation effect
- NEMA IEC Body phantom positioned in J-PET detector: (x,y,z=3.75cm) section on the left, (x,y=0cm, z) section on the right

2. Attenuation correction – simplified model



- Experimental measurement of attenuation effect using 2D map (on the left)

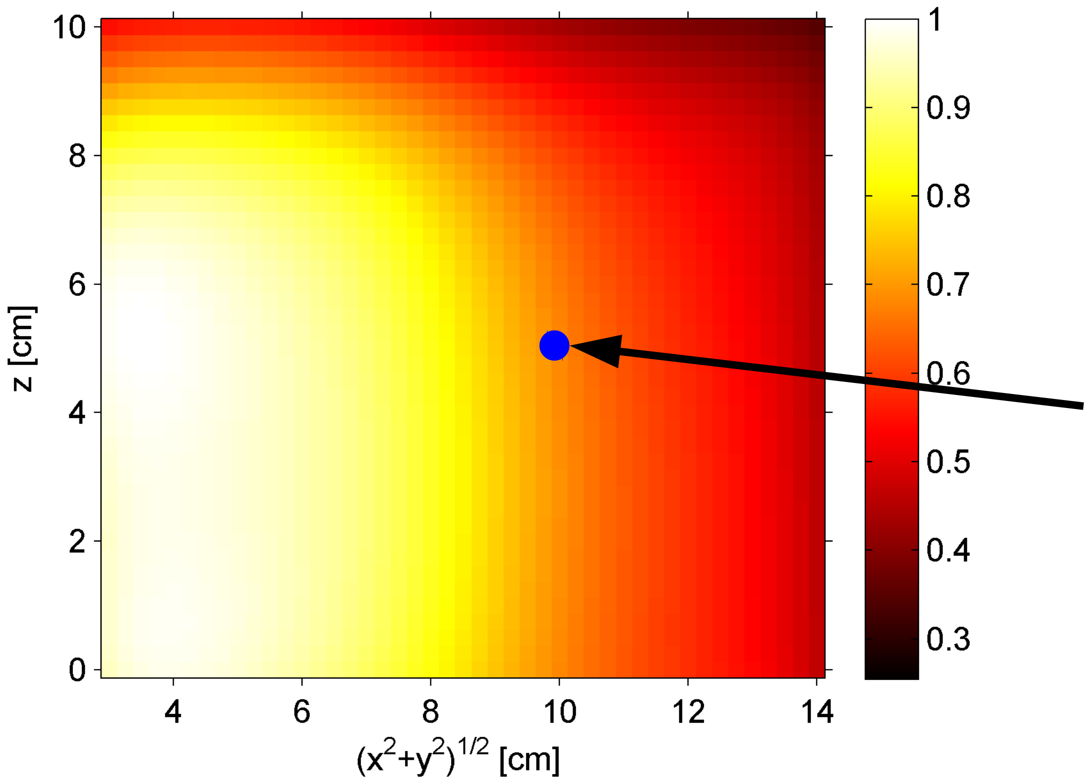
2. Attenuation correction – simplified model



Data pre-correction algorithm:

1) Take an event from the list
2) Estimate its position on the map

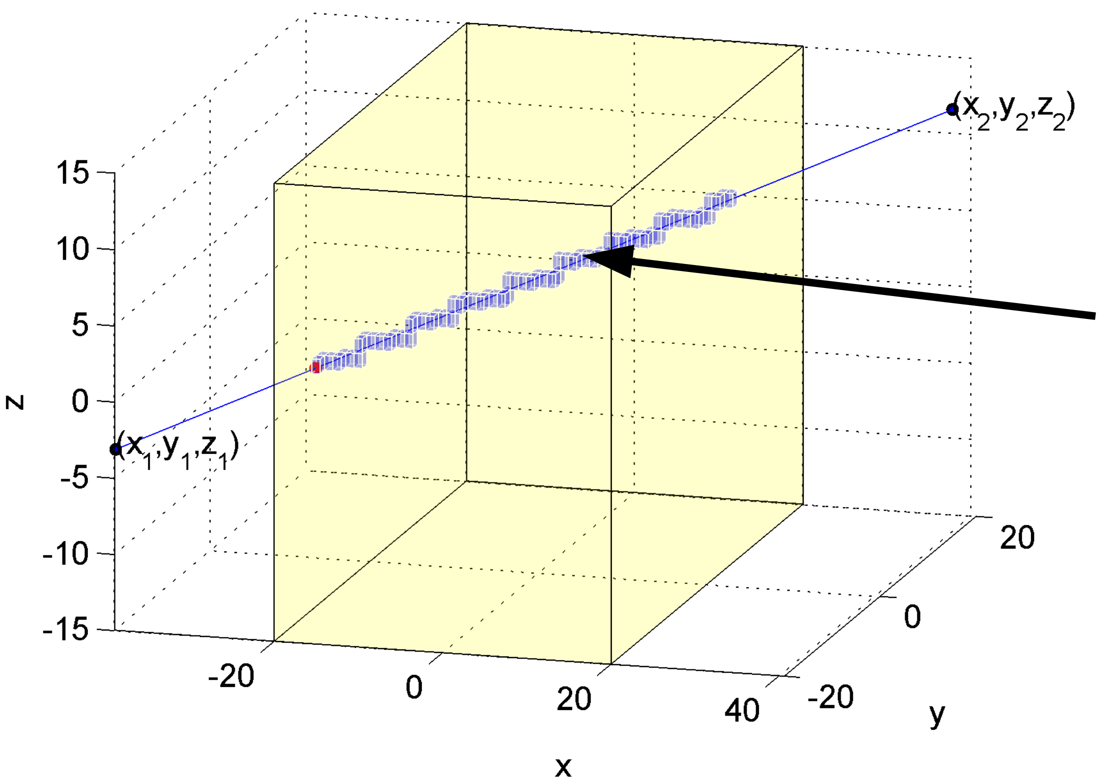
2. Attenuation correction – simplified model



Data pre-correction algorithm:

1) Take an event from the list
2) Estimate its position on the map
3) Estimate probability (***p***) of event leaving on the list (here ***p*** = 0.67)
4) Draw a number (***q***) from uniform distribution (0-1)
5) Reject an event if ***p*** < ***q***

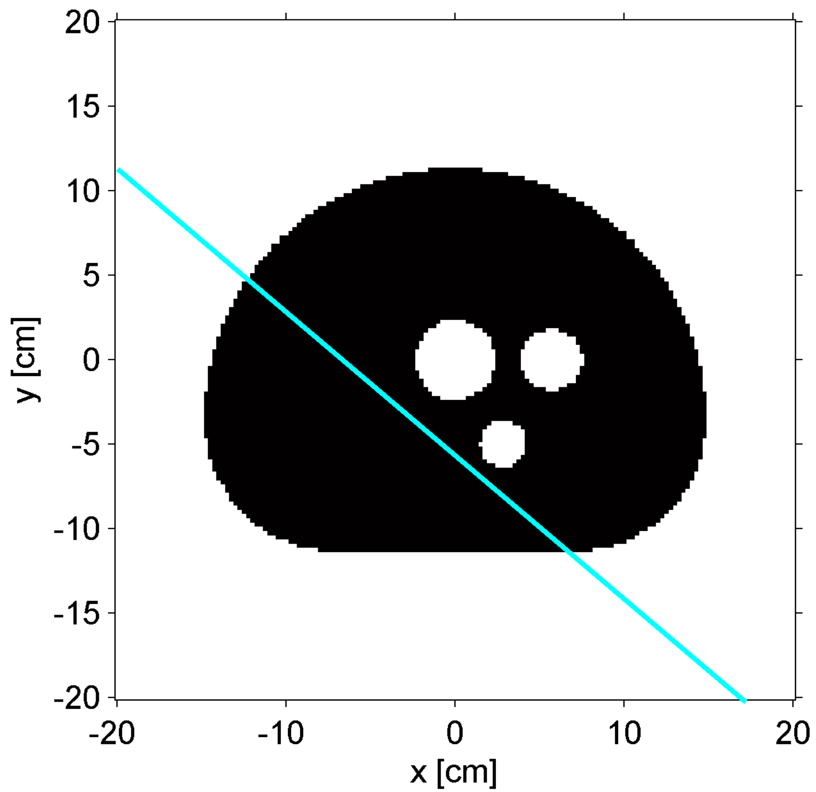
2. Attenuation correction – complete model



Data pre-correction algorithm:

1) Take an event from the list
2) Estimate position of line of response (LOR) inside the 3D volume of reconstructed image (yellow area)
3) assign attenuation coefficient for each voxel

2. Attenuation correction – complete model



3) assign attenuation coefficient for each voxel:

$$\mu = 0 \ \mathrm{cm}^{-1} \quad \text{for air}$$

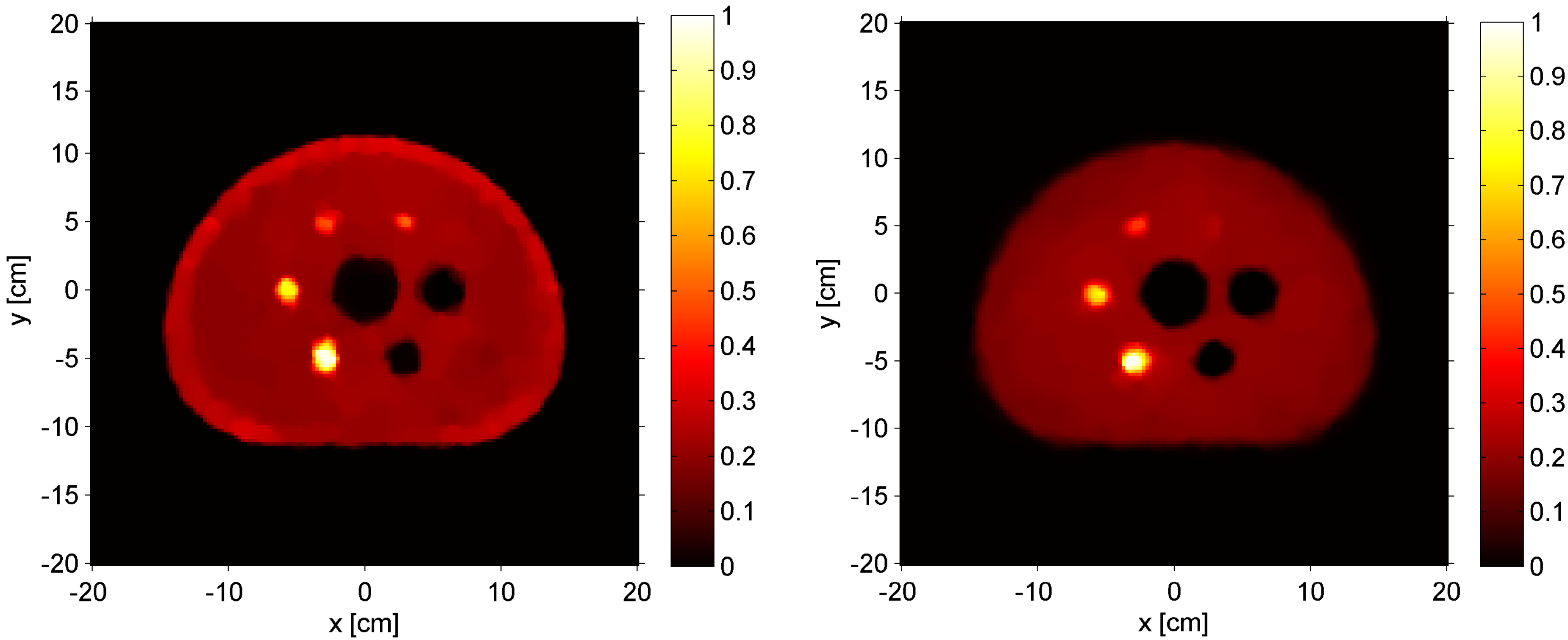
$$\mu = 0.096 \ \mathrm{cm}^{-1} \quad \text{for water}$$

4) calculate attenuation correction factor lor LOR:

$$a = \int_{-\infty}^{+\infty} \exp(-\mu(x)x) \, \mathrm{d}x,$$

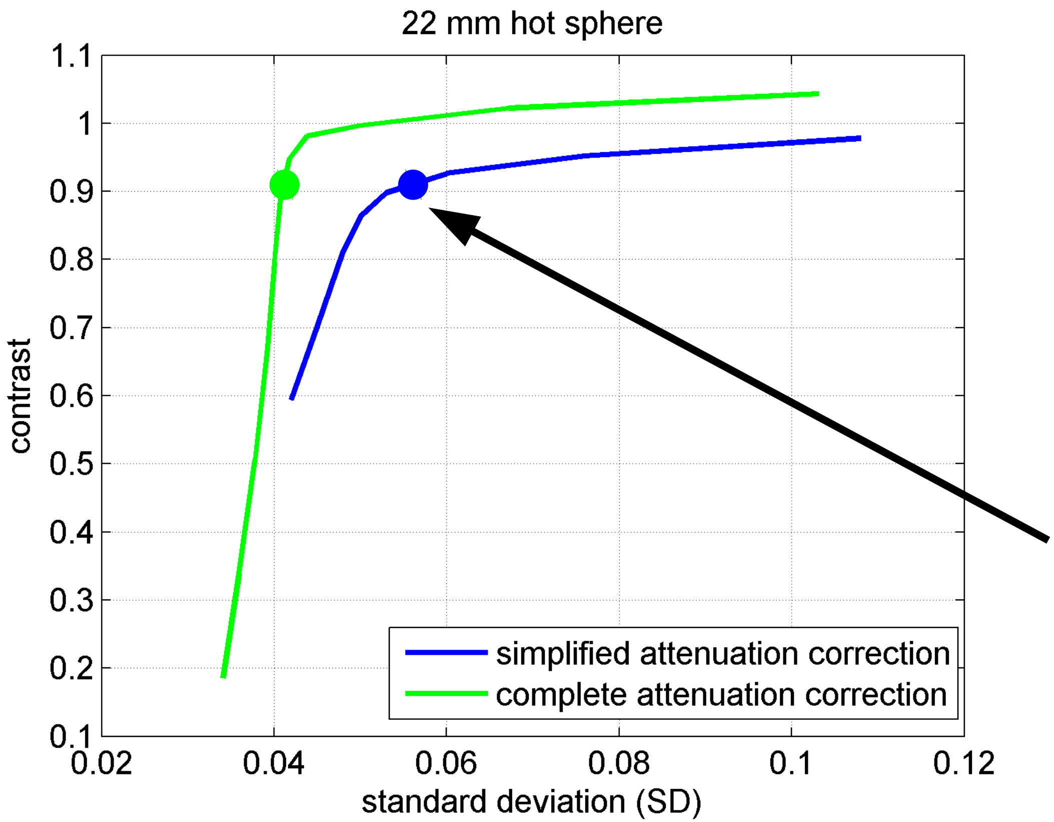
where x is the integration variable along the LOR

2. Attenuation correction – model comparison



- Image reconstruction with TOF-BPTV method with simplified (left) and complete (right) attenuation correction.
- For simplified approach a ring at the boundary of the phantom is observed.

2. Attenuation correction – model comparison



- Trade-off between contrast and standard deviation in reconstructed images for two approaches.
- The optimal point is: (SD=0.0, contrast=1.0)
- Circles denote the values of SD and contrast from presented example (optimal points on the curves)

3. Shift variance of kernel operator

- Consider filtration problem:

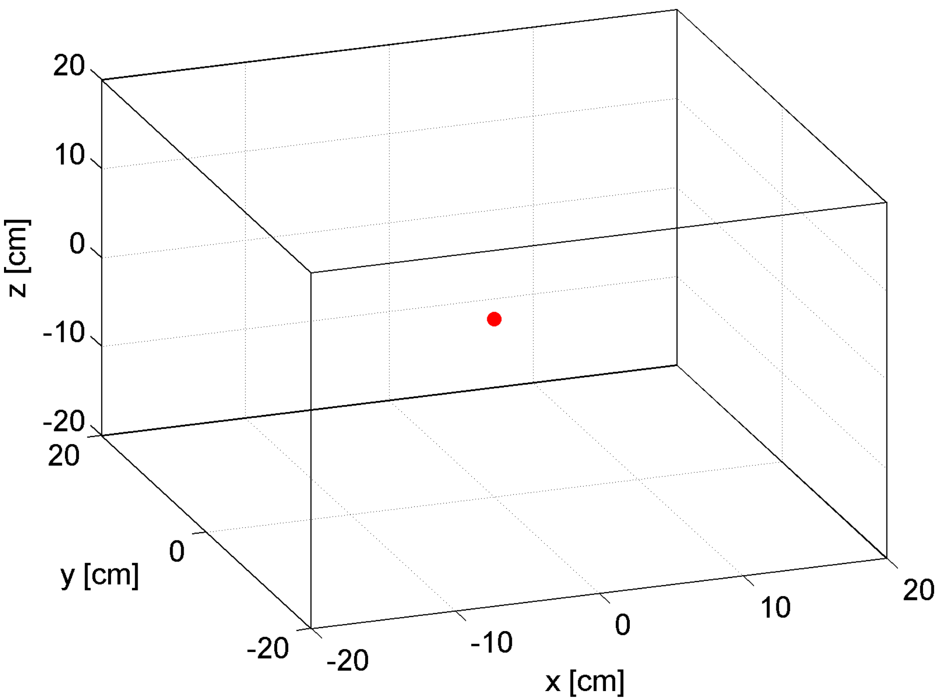
$$\mathbf{b} = A\mathbf{f}$$

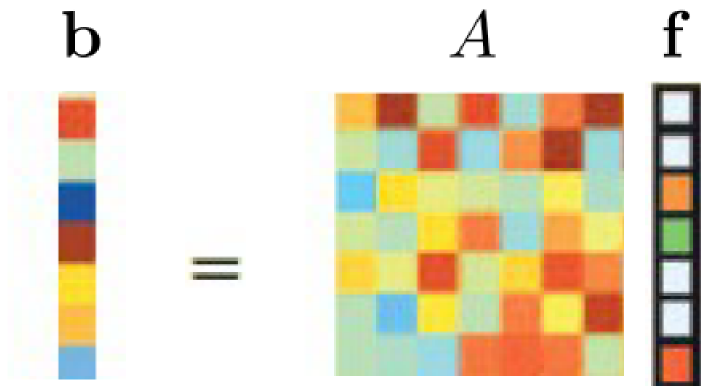
where **b, f** are images represented by vectors,
A is overall TOF projection operator (matrix)

- For 3D image with 200 x 200 x 200 voxels:
  b, f are vectors with 8 000 000 elements
  A is a matrix with 64 000 000 000 000 elements !!!
- Some simplifications / approximations are required !!!

3. Shift variance of kernel operator

$$\mathrm{b} = A\mathrm{f}$$

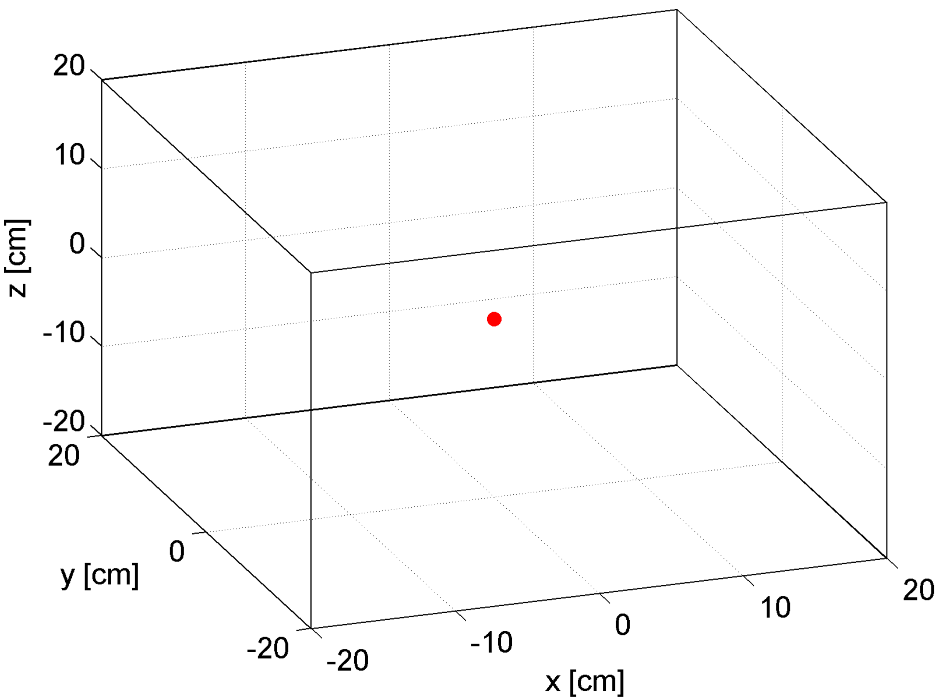


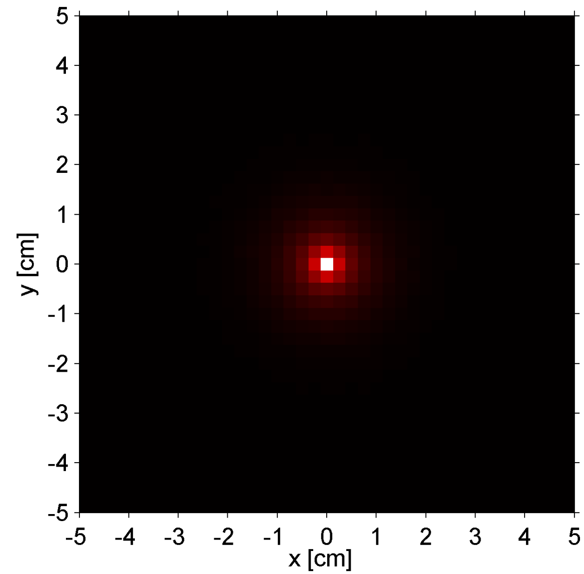


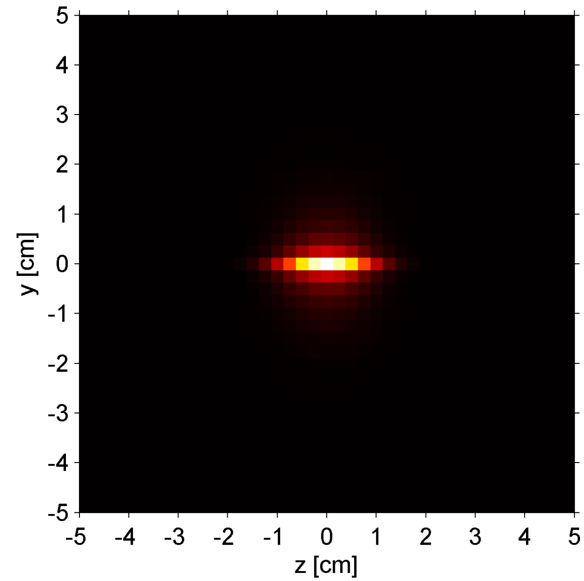
- Each column of matrix A stores the filter response for each voxel of image f.
- We may assume that filter response is the same in all the volume; this leads to shift-invariant model

3. Shift variance of kernel operator

- Filter operator (kernel) for center position (0,0,0) in x,y (top) and y,z (bottom) cross-sections.

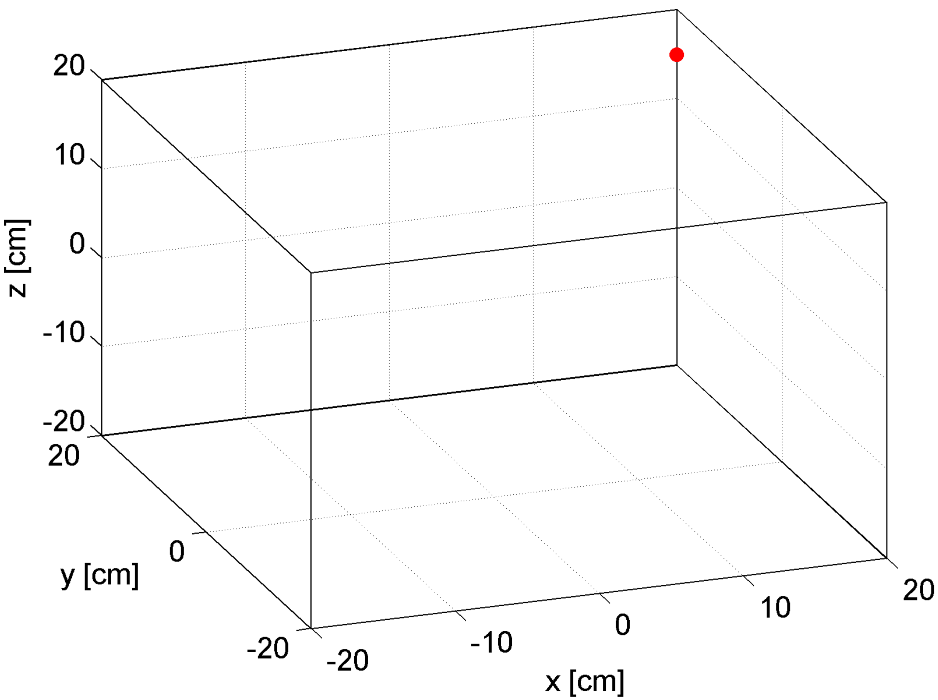


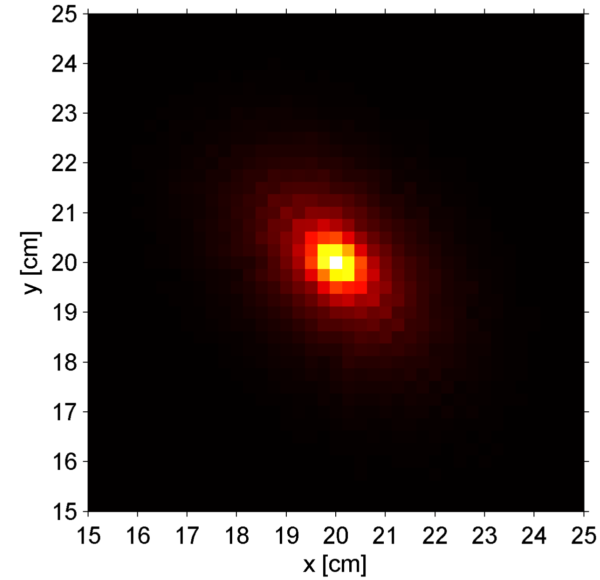


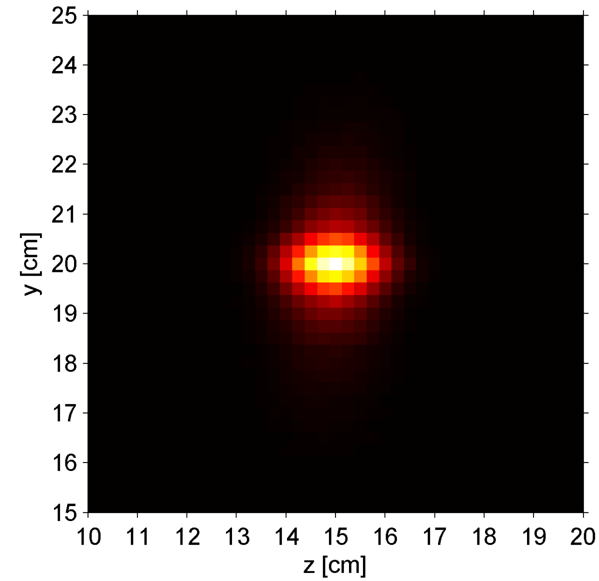


3. Shift variance of kernel operator

- Filter operator (kernel) close to edge position (20,20,15) in x,y (top) and y,z (bottom) cross-sections.

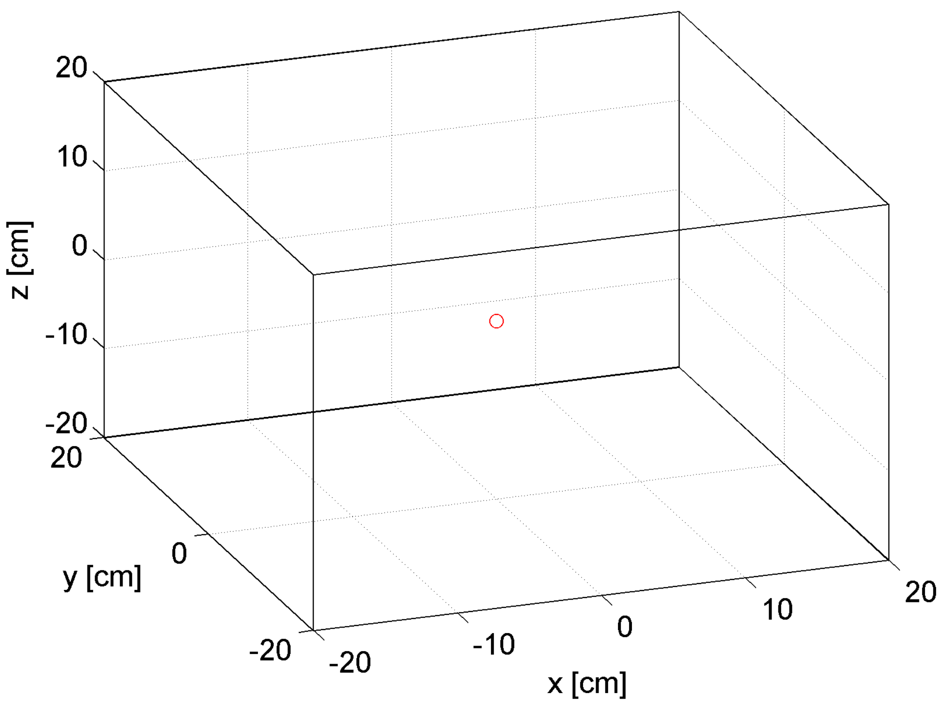


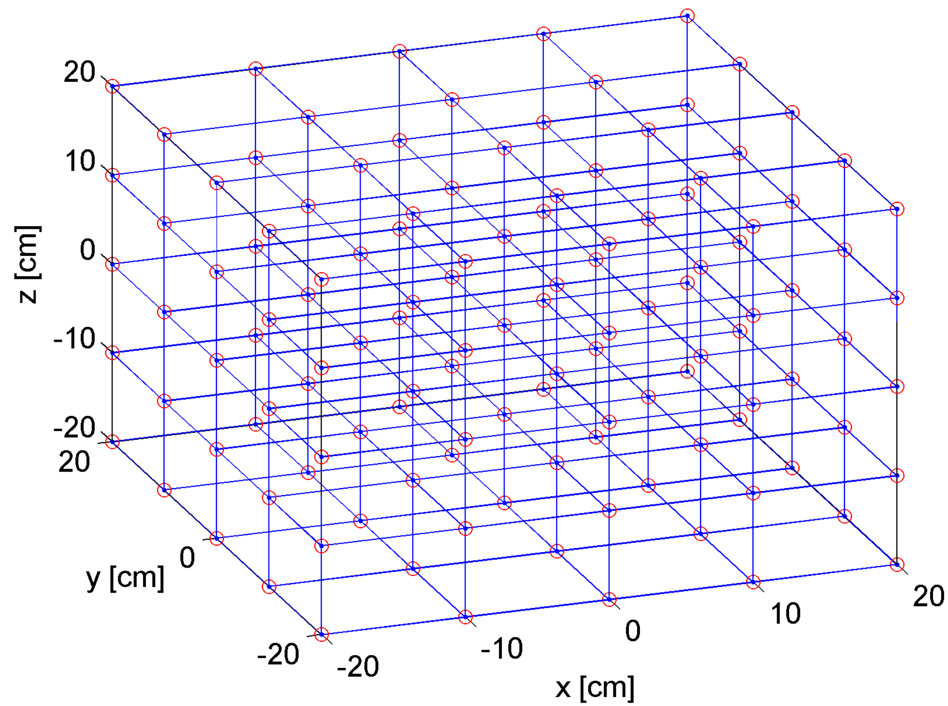




Computing Centre

3. Shift variance of kernel operator





- Incorporation of the shift variance by evaluation of a set of operators for set of point sources placed inside the detector volume
- For each kernel a small sub-image is calculated independently
- Final image is evaluated as weighted sum of the sub-images