
Advanced Series on Directions in High Energy Physics — Vol. 29

MULTIPLE PARTON INTERACTIONS AT THE LHC

Editors

Paolo Bartalini and Jonathan Richard Gaunt

World Scientific

Double parton correlations and transverse proton structure at the LHC

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DIPARTIMENTO DI FISICA E GEOLOGIA

Università degli Studi di Perugia, Italia

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- 3D structure of the proton
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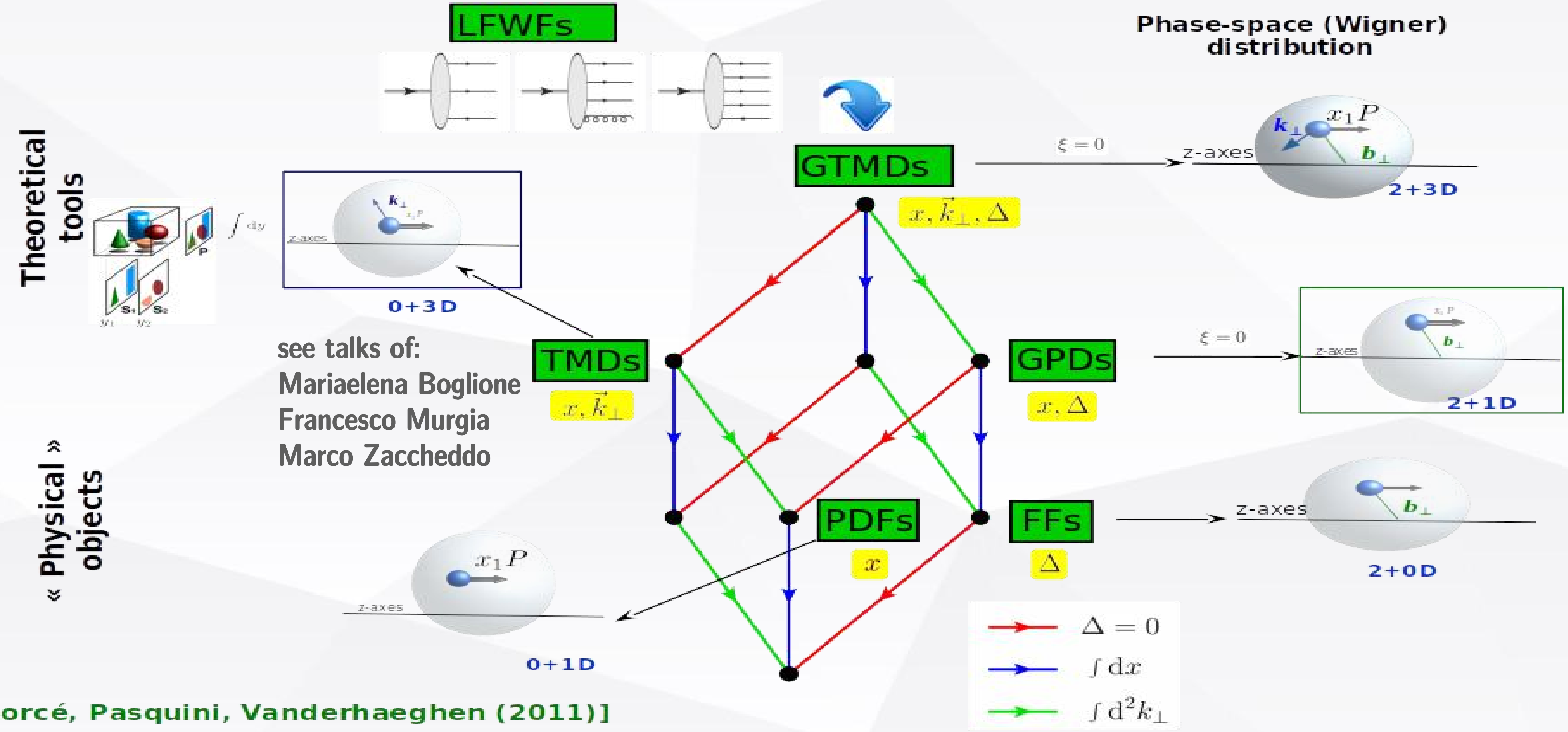
THE 3D STRUCTURE OF THE PROTON

The 3D structure of a strongly interacting system (e.g. nucleon, nucleus..) could be accessed through different processes (e.g. SIDIS, DVCS ...), measuring different kind of parton distributions, providing different kind of information. The parton distribution puzzle is:



see talks of:
Mariaelena Boglione
Francesco Murgia
Marco Zaccheddo

see Sara Fucini' talk for DVCS off ^{4}He

[Lorcé, Pasquini, Vanderhaeghen (2011)]

THE 3D STRUCTURE OF THE PROTON

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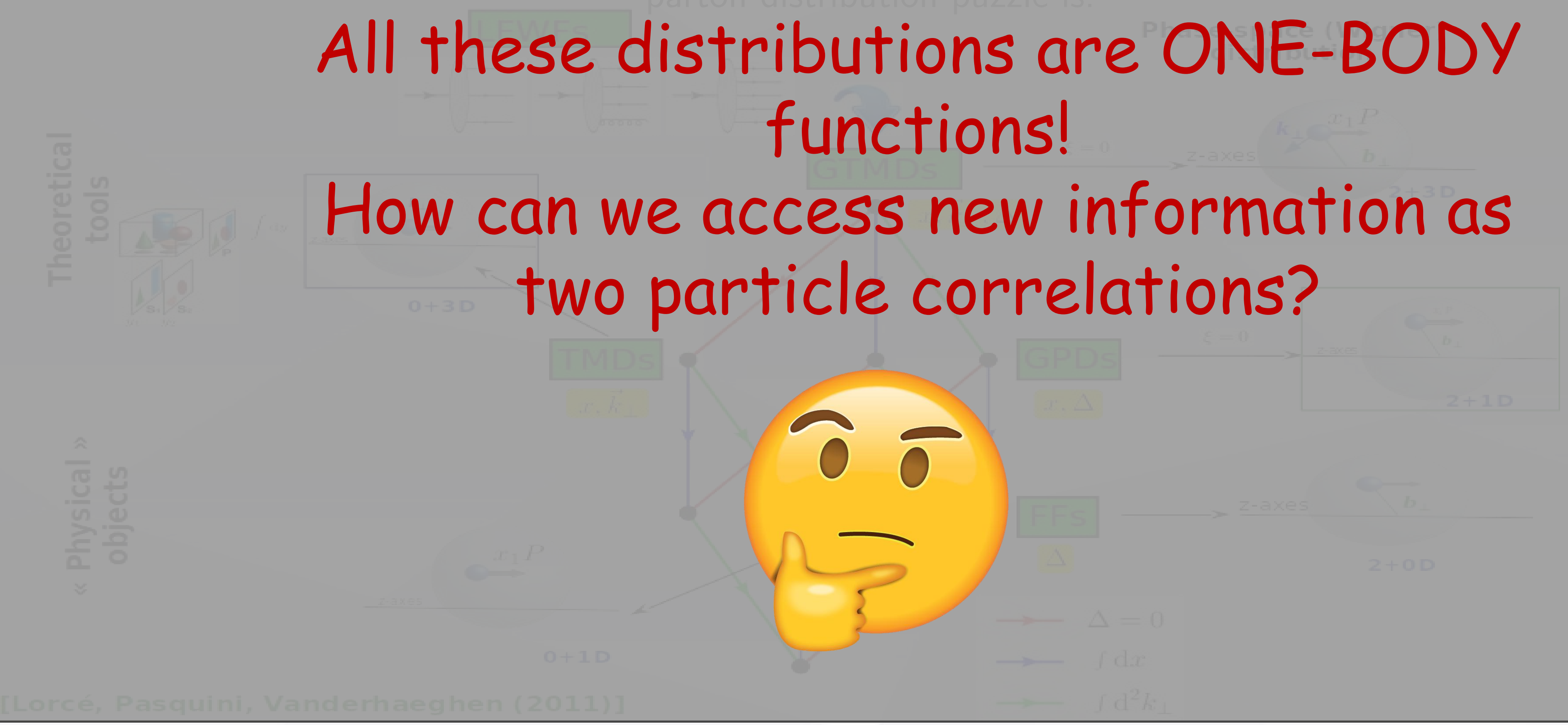
All these distributions are ONE-BODY functions!

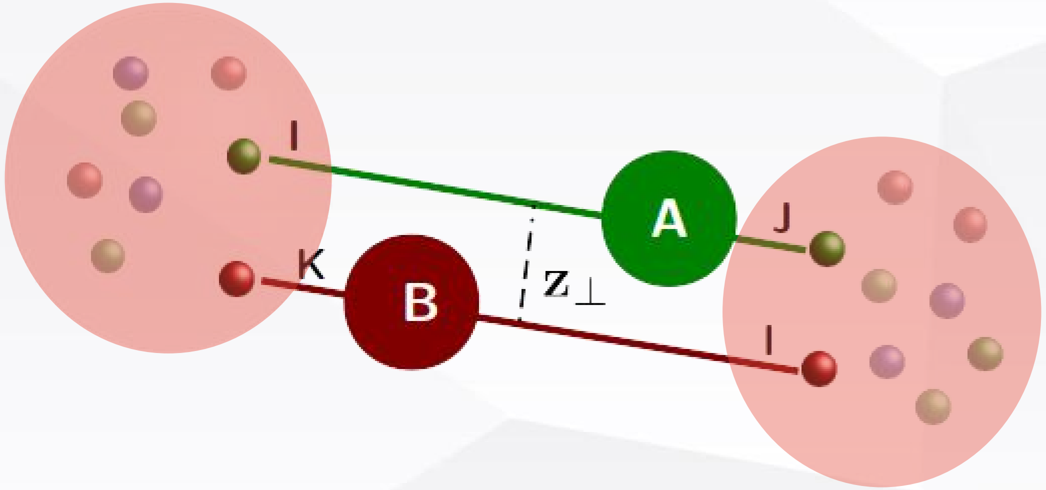
How can we access new information as two particle correlations?



[Lorcé, Pasquini, Vanderhaeghen (2011)]

Answer: MULTIPARTON INTERACTIONS

Multiparton interaction (MPI) can contribute to the, pp and pA, cross section @ the LHC:



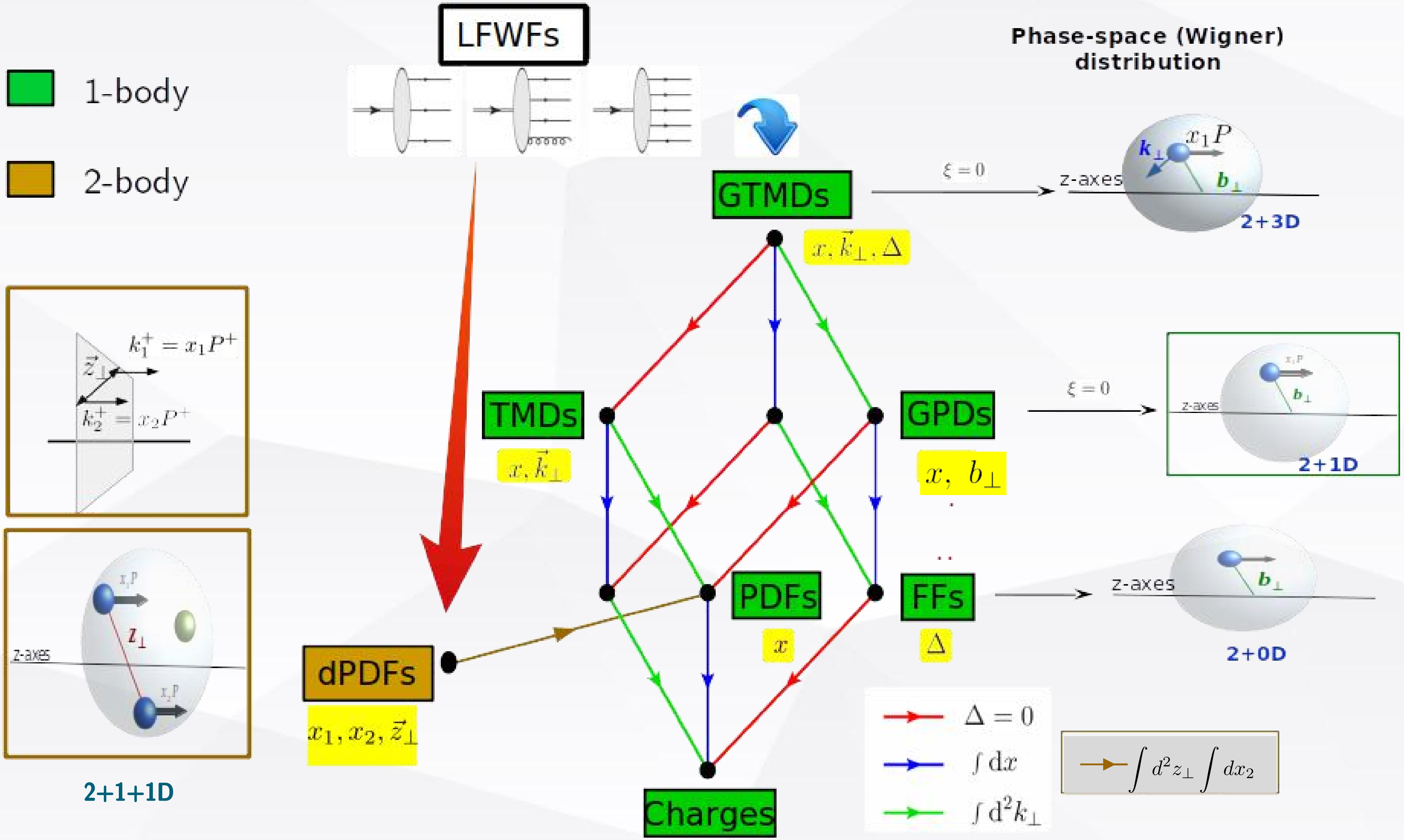
The cross section for a double parton scattering (DPS) event can be written in the following way:
N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982)

$$\mathbf{d}\sigma = \frac{1}{S}\sum_{i,j,k,l}\hat{\sigma}_{ij}(\underbrace{x_1, x_3}_{\text{Momentum fractions carried by the parton inside the proton}}, \mu_A)\hat{\sigma}_{kl}(x_2, x_4, \mu_B)\int d^2 z_\perp \overbrace{F_{ik}(x_1, x_2, \boxed{z_\perp}, \mu_A, \mu_B)}^{\text{dPDF}} F_{jl}(x_3, x_4, z_\perp, \overbrace{\mu_A, \mu_B}^{\text{Momentum scales}})$$

Transverse distance between partons

DPS processes are important for fundamental studies, e.g. the background for the research of new physics and to grasp information on the 3D PARTONIC STRUCTURE OF THE PROTON

Answer: MULTIPARTON INTERACTIONS



🟩 1-body

🟧 2-body

LFWFs

Phase-space (Wigner) distribution

GTMDs $x,\vec{k}_\perp,\Delta$

TMDs $x,\vec{k}_\perp$

GPDs $x,\ b_\perp$

PDFs x

FFs Δ

dPDFs $x_1,x_2,\vec{z}_\perp$

Charges

$\Delta = 0$

$\int \mathrm{d}x$

$\int \mathrm{d}^2k_\perp$

$\int d^2z_\perp \int dx_2$

$\xi = 0$

2+3D

2+1D

2+0D

2+1+1D

Parton correlations and dPDFs

@ LHC kinematics it is often used a factorized form of the dPDFs: $(\mathbf{x_1},\mathbf{x_2}) - \mathbf{z}_\perp$ factorization:

$$F_{ij}(x_1,x_2,\vec{z}_\perp,\mu) = F_{ij}(x_1,x_2,\mu)T(\vec{z}_\perp,\mu)$$

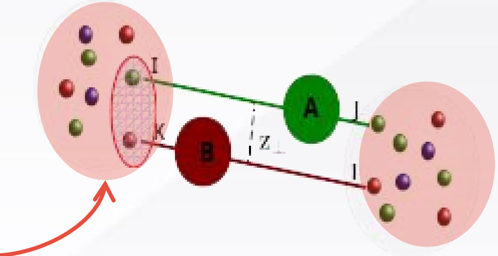
and $x_1,\ x_2$ factorization:

* Here and in the following: $\mu = \mu_A = \mu_B$

$$\overbrace{F_{ij}(x_1,x_2,\mu)}^{\text{dPDF (2-Body)}} = \underbrace{\overbrace{q_i(x_1,\mu)}^{\text{PDF (1-Body)}}\ q_j(x_2,\mu)}\,\theta(1-x_1-x_2)(1-x_1-x_2)^n$$

unknown

NO CORRELATION ANSATZ



In this scenario, parton correlations inside the proton are neglected ➡ NO NEW INFORMATION!

BUT:

- Correlations are present
- dPDFs are non perturbative in QCD and DPCs cannot be directly evaluated within QCD

HOW CAN WE BE SURE OF THE ACCURACY OF SUCH APPROXIMATION

WHAT CAN WE LEARN ABOUT dPDFs AND THE PROTON STRUCTURE?

DPCs in Constituent quark models (CQMs)

- Main features:
  - Effective potential
  - Effective particles strongly bound and correlated

- CQM are a proper framework to describe DPCs, but their predictions are reliable ONLY in the valence quark region at low energy scale

WHILE

- LHC data are available at small x .In this region, due to the large population of partons, the role of correlations could be less relevant BUT theoretical microscopic estimates are necessary!

i) dPDF evaluated at the initial scale of the model

pQCD evolution of dPDFs

ii) dPDF evaluated at high generic scale

CQM calculations are useful tools for the interpretation of data and for the planning of measurements of unknown quantities (e.g., TMDs in SiDIS, GPDs in DVCS...)

Similar expectations motivate the present investigation of dPDFs

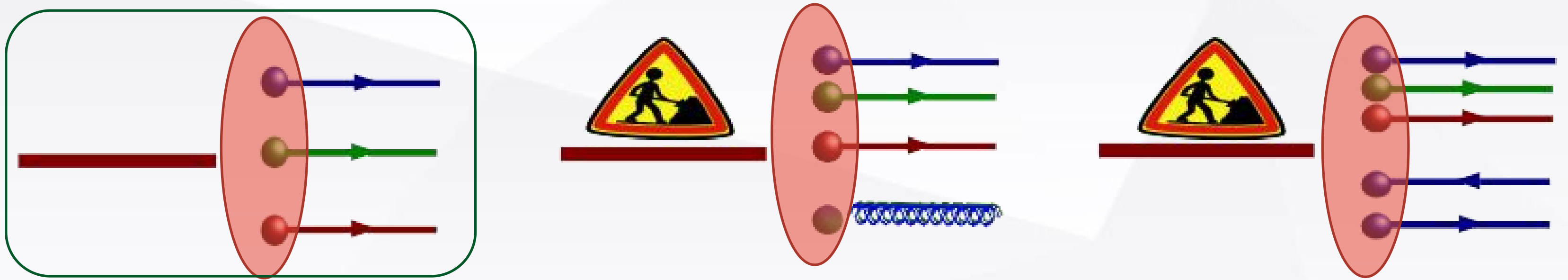
The Light-Front approach

Relativity can be implemented, for a CQM, by using a Light-Front (LF) approach. In the Relativistic Hamiltonian Dynamics (RHD) of an interacting system, introduced by Dirac (1949), one has:

- RHD
  - Instant Form: $t_0=0$
  - Evolution Operator: $P_0= E$
  - Front Form (LF): $x_+= t_0+z=0$
  - Evolution Operator: P_-
  - $a^\pm = a_0 \pm a_3$
- Fixed number of off-shell particles
- Full Poincare' covariance
- 7 Kinematical generators (maximum number): i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) $\mathbf{P}^+, \ \mathbf{P}_\perp$, iii) Rotation around z.
- The proton state can be represented in the following way:

see e.g.: **S. J. Brodsky, H. -C. Pauli, S. S. Pinsky, Phys.Rept. 301, 299 (1998)**

$$|p, P^+ \ \vec{P}_\perp\rangle = \boxed{\psi_{qqq}}|qqq\rangle + \psi_{qqq\ g}|qqq\ g\rangle + \psi_{qqq\ q\bar{q}}|qqq\ q\bar{q}\rangle$$



$\psi_n =$ **LF wave function**

Invariant under LF boosts

dPDFs in a Light-Front approach

M. R., S. Scopetta, M. Traini and V.Vento, JHEP 12, 028 (2014)

Extending the procedure developed in **S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. B 649, 243 (2003)** for GPDs, we obtained the following expression of the dPDF in momentum space, often called ${}_2$GPDs:

$$F_{ij}(x_1,x_2,k_\perp) = 3(\sqrt{3})^3 \int \prod_{i=1}^{3} d\vec{k}_i \delta\left(\sum_{i=1}^{3}\vec{k}_i\right) \Phi^*(\{\vec{k}_i\},k_\perp)\Phi(\{\vec{k}_i\},-k_\perp)$$

$$\times \quad \delta\left(x_1-\frac{k_1^+}{P_+}\right)\delta\left(x_2-\frac{k_2^+}{P_+}\right)$$

Conjugate to $z_\perp$

$$\Phi(\{\vec{k}_i\},\pm k_\perp) = \Phi\left(\vec{k}_1 \pm \frac{\vec{k}_\perp}{2}, \vec{k}_2 \mp \frac{\vec{k}_\perp}{2}, \vec{k}_3\right)$$

GOOD SUPPORT

$$\mathrm{x}_1 + x_2 > 1 \Rightarrow F_{ij}(x_1,x_2,k_\perp) = 0$$

$$\Phi(\vec{k}_1,\vec{k}_2,\vec{k}_3) = D^{\dagger 1/2}(R_{il}(\vec{k}_1))D^{\dagger 1/2}(R_{il}(\vec{k}_2))D^{\dagger 1/2}(R_{il}(\vec{k}_3))\boxed{\psi^{[i]}(\vec{k}_1,\vec{k}_2,\vec{k}_3)}$$

Melosh operator rotates canonical spin in LF one

Instant form proton w.f.

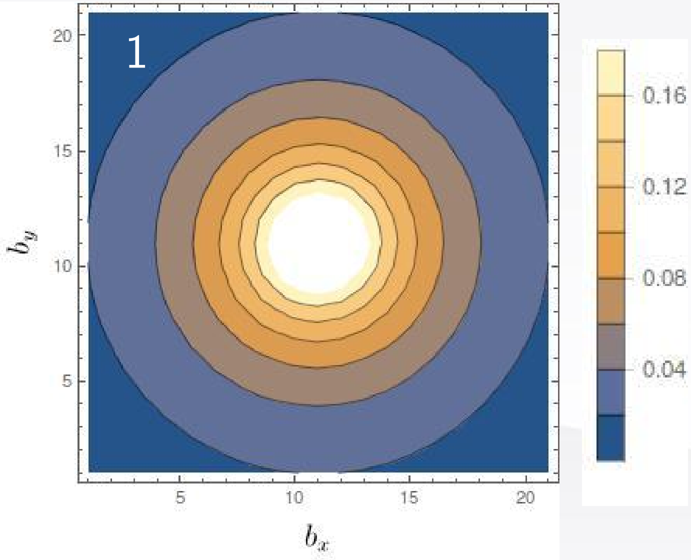
We need a CQM!

What we would like to learn: partonic mean distance

M. R. and F. A. Ceccopieri, arXiv: 1812.04286, JHEP accepted

Since, in coordinates space, dPDFs get a number density interpretation, in principle one can calculated the mean distance between partons!

For example, for 2 gluons perturbatively generated:

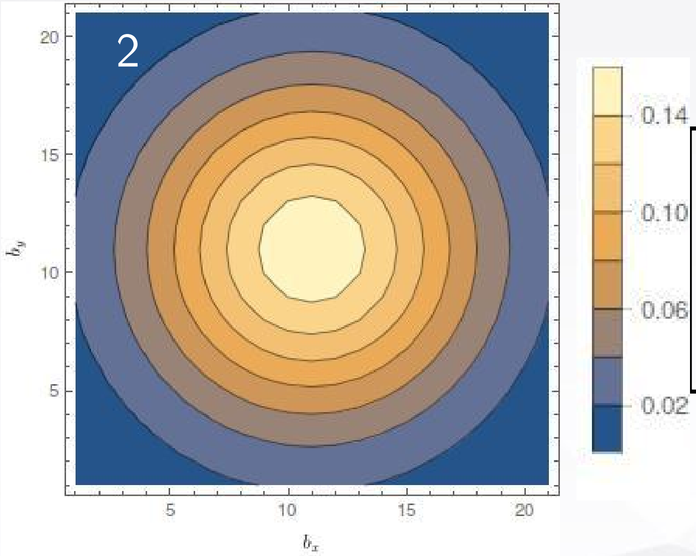


1) HP model

2) HO model

$x_1 = 10^{-4}$ and $x_2 = 10^{-2}$

$$\vec{d}_\perp = \vec{b}_\perp = \vec{z}_\perp$$



M. Traini *et al*, Nucl. Phys. A 656, 400-420 (1999), non relativistic Hyper-Central CQM (potential by M. Ferraris *et al,* PLB 364 (1995)) (HP)

The harmonic oscillator (HO)

One can also define the mean transverse distance $(x_1 - x_2)$ distribution as follows:

$$\boxed{\langle d_\perp^2 \rangle^{ij}_{x_1,x_2} = \frac{\int d^2 b_\perp \; b_\perp^2 F_{ij}(x_1,x_2,b_\perp,Q^2 = M_W^2)}{\int d^2 b_\perp \; F_{ij}(x_1,x_2,b_\perp,Q^2 = M_W^2)}}$$

For example, for 2 gluons and two different models, one gets:

$$\nearrow \sqrt{\langle d_\perp^2 \rangle_{10^{-2},10^{-2}}} = \begin{array}{l} \nearrow 0.404 \text{ fm} \quad \text{HP} \\ \searrow 0.365 \text{ fm} \quad \text{HO} \end{array}$$

$$\searrow \sqrt{\langle d_\perp^2 \rangle_{10^{-4},10^{-4}}} = \begin{array}{l} \nearrow 0.391 \text{ fm} \quad \text{HP} \\ \searrow 0.393 \text{ fm} \quad \text{HO} \end{array}$$

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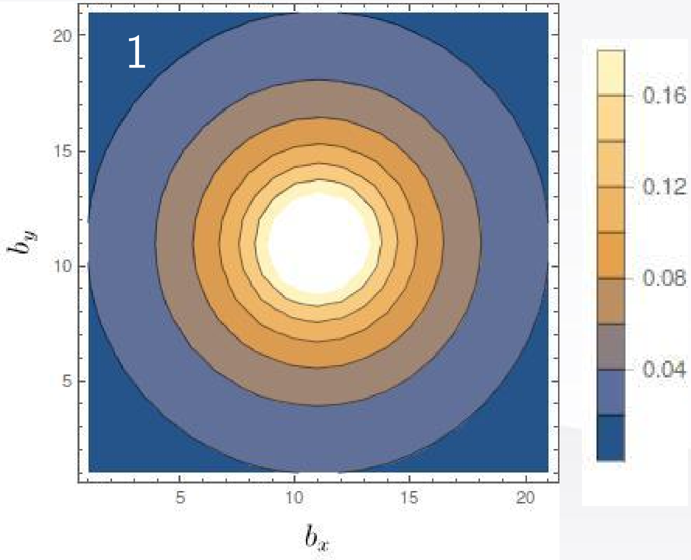

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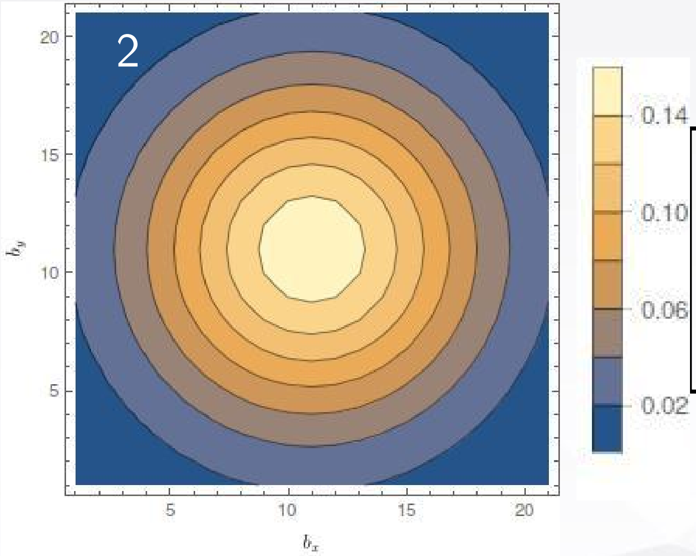

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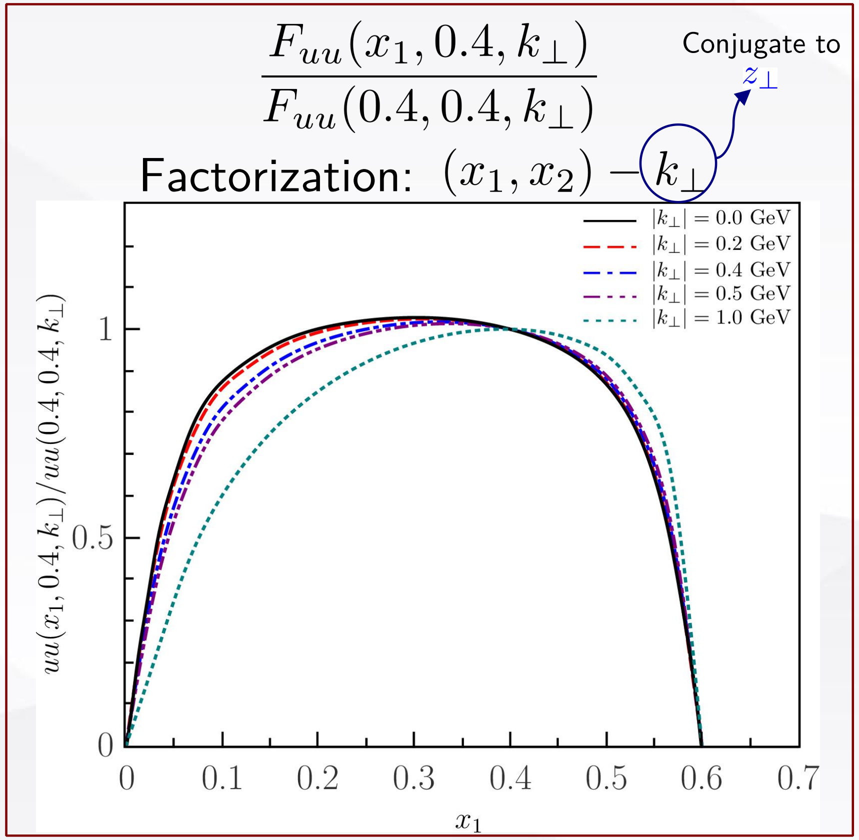
Are two slow partons closer (in $\perp$ plane) then two fast partons?

What we learned:

M. R., S. Scopetta, M. Traini and V.Vento, JHEP 12, 028 (2014)

Ratios, sensitive to correlations, are shown in order to test the factorization ansatz! Use has been made of relativistic HP CQM.

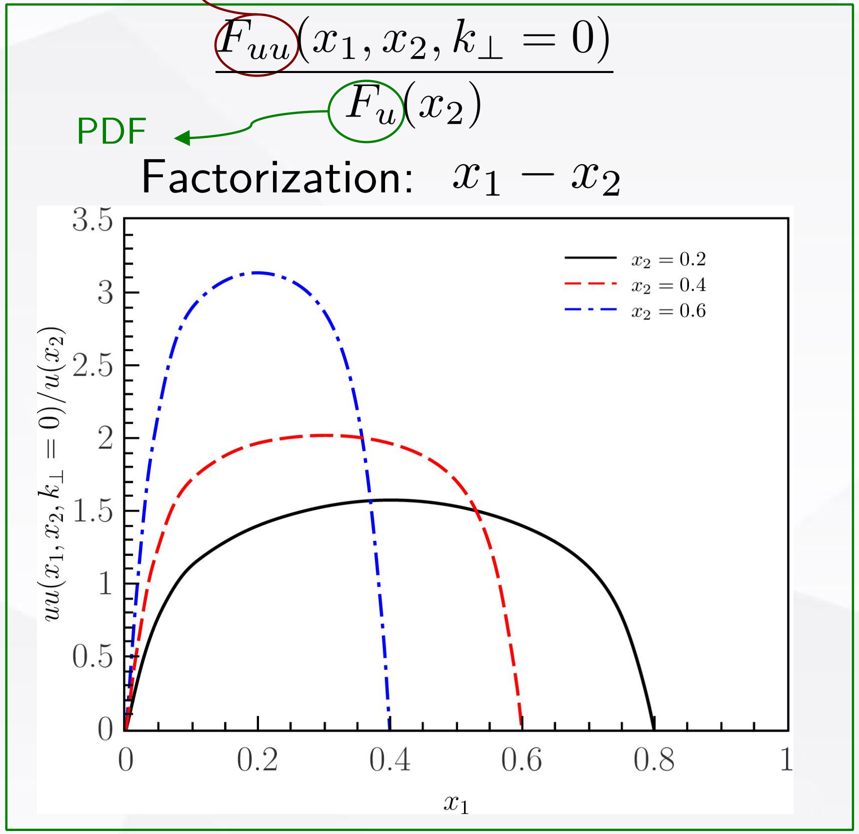


$$\frac{F_{uu}(x_1, 0.4, k_\perp)}{F_{uu}(0.4, 0.4, k_\perp)}$$

Conjugate to $z_\perp$

Factorization: $(x_1, x_2) - k_\perp$

$_2$GPD



$$\frac{F_{uu}(x_1, x_2, k_\perp = 0)}{F_u(x_2)}$$

PDF

Factorization: $x_1 - x_2$

The $(x_1, x_2) - k_\perp$ and $x_1 - x_2$ factorizations are violated in **all quark model analyses**!

M.R., S. Scopetta and V. Vento, PRD 87, 114021 (2013), H.-M. Chang, A.V. Manohar, and W.J. Waalewijn, PRD 87, 034009 (2013)

The Effective X-section

A fundamental tool for the comprehension of the role of DPS in hadron-hadron collisions is the so called "effective X-section".

This object can be defined through a "pocket formula":

Sensitive to correlations

$$\sigma_{eff} = \boxed{\frac{m}{2}} \frac{\sigma_A^{pp'} \boxed{\sigma_B^{pp'}}}{\sigma_{double}^{pp}}$$

Differential cross section for the process: $\text{pp'} \to A(B) + X$

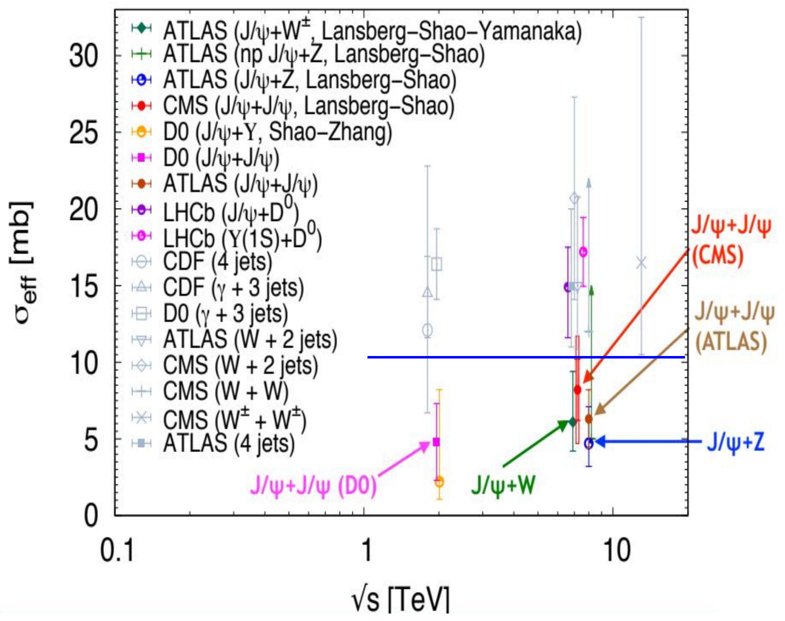
Combinatorial factor

Differential cross section for a DPS event: $\text{pp'} \to A + B + X$

....EXPERIMENTAL STATUS:

- Difficult extraction, approved analysis for the same
- sign *W's* production @LHC (RUN 2)
- the model dependent extraction of σ_{eff} from data is almost consistent with a "constant" (within errors) (uncorrelated ansatz usually assumed!)
- different ranges in x_i accessed in different

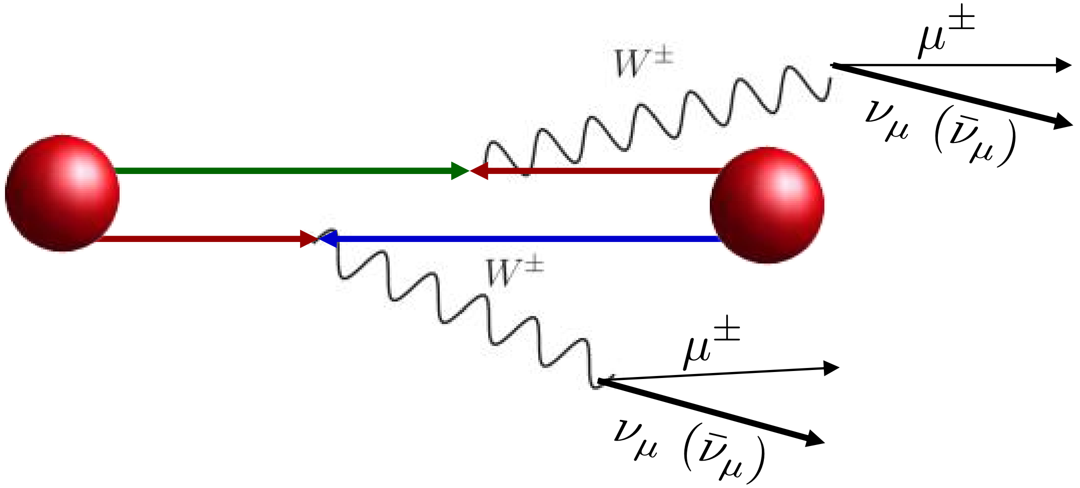
Within our CQM framework, we can calculate σ_{eff} without any approximations!



Yamanaka's talk mpi@LHC (2018)

Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta, Phys.Rev. D95 (2017) no.11, 114030



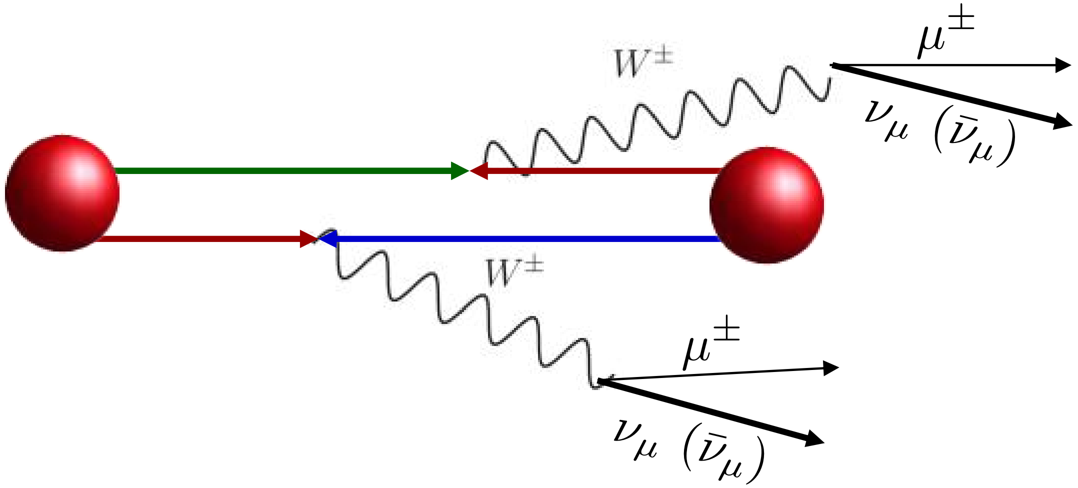
In this channel, the single parton scattering (usually dominant w.r.t to the double one) starts to contribute to higher order in strong coupling constant.

⬇

"Same-sign W boson pairs production is a promising channel to look for signature of double Parton interactions at the LHC."

Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta, Phys.Rev. D95 (2017) no.11, 114030



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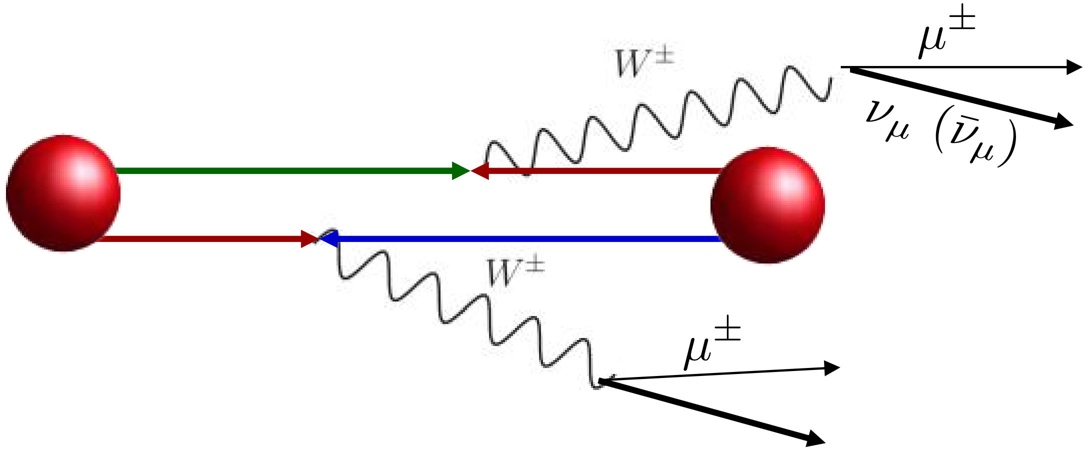
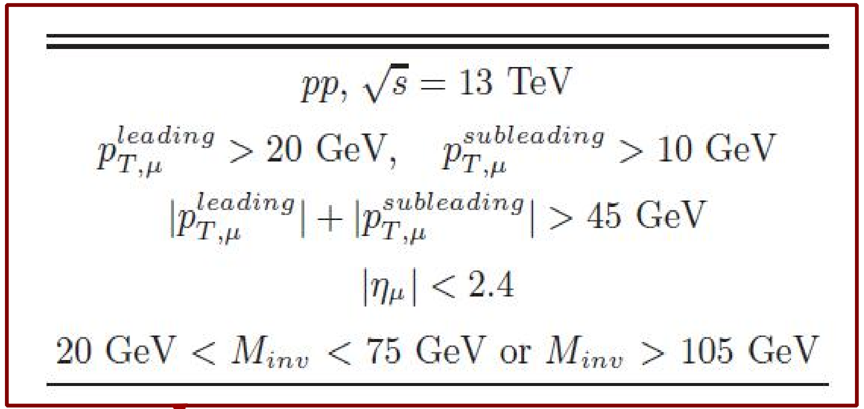
Can double parton correlations be observed for the first time in the next LHC run ?

Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta, Phys.Rev. D95 (2017) no.11, 114030



Kinematical cuts



pp, $\sqrt{s} = 13$ TeV

$p_{T,\mu}^{leading} > 20$ GeV, $p_{T,\mu}^{subleading} > 10$ GeV

$|p_{T,\mu}^{leading}| + |p_{T,\mu}^{subleading}| > 45$ GeV

$|\eta_\mu| < 2.4$

$20 \text{ GeV} < M_{inv} < 75 \text{ GeV or } M_{inv} > 105 \text{ GeV}$

DPS cross section:

$$\frac{d^4\sigma^{pp\to\mu^\pm\mu^\pm X}}{d\eta_1 dp_{T,1} d\eta_2 dp_{T,2}} = \sum_{i,k,j,l} \frac{1}{2} \int d^2\vec{b}_\perp \boxed{F_{ij}(x_1, x_2, \vec{b}_\perp, M_W) F_{kl}(x_3, x_4, \vec{b}_\perp, M_W)} \frac{d^2\sigma_{ik}^{pp\to\mu^\pm X}}{d\eta_1 dp_{T,1}} \frac{d^2\sigma_{jl}^{pp\to\mu^\pm X}}{d\eta_2 dp_{T,2}} \boxed{\mathcal{I}(\eta_i, p_{T,i})}$$

$M_W \longrightarrow$ Momentum scale

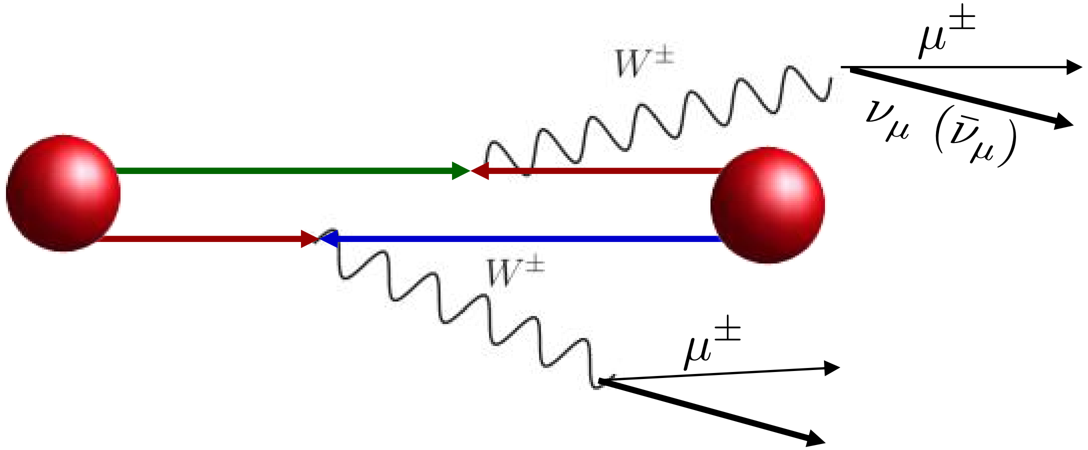
In order to estimate the role of double parton correlations we have used as input of dPDFs:

1) Longitudinal and transverse correlations arise from the relativistic CQM model describing three valence quarks

2) These correlations propagate to sea quarks and gluons through pQCD evolution

Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta, Phys.Rev. D95 (2017) no.11, 114030



Kinematical cuts

pp, $\sqrt{s} = 13$ TeV

$p_{T,\mu}^{leading} > 20$ GeV, $p_{T,\mu}^{subleading} > 10$ GeV

$|p_{T,\mu}^{leading}| + |p_{T,\mu}^{subleading}| > 45$ GeV

$|\eta_\mu| < 2.4$

20 GeV $< M_{inv} <$ 75 GeV or $M_{inv} >$ 105 GeV

DPS cross section:

$$\frac{d^4\sigma^{pp\to\mu^\pm\mu^\pm X}}{d\eta_1 dp_{T,1} d\eta_2 dp_{T,2}} = \sum_{i,k,j,l} \frac{1}{2} \int d^2\vec{b}_\perp \boxed{F_{ij}(x_1, x_2, \vec{b}_\perp, M_W) F_{kl}(x_3, x_4, \vec{b}_\perp, M_W)} \frac{d^2\sigma_{ik}^{pp\to\mu^\pm X}}{d\eta_1 dp_{T,1}} \frac{d^2\sigma_{jl}^{pp\to\mu^\pm X}}{d\eta_2 dp_{T,2}} \boxed{\mathcal{I}(\eta_i, p_{T,i})}$$

$M_W \longrightarrow$ Momentum scale

In order to estimate the role of double parton correlations we have used as input of dPDFs:

Relativistic model: **QM** M. R., S. Scopetta, M. Traini and V.Vento, JHEP 12, 028 (2014)

Final Results: $\sigma^{++} + \sigma^{--}[\text{fb}] \sim 0.69 \pm 0.18(\delta\mu_F)^{+0.12}_{-0.16}(\delta Q_0)^*$

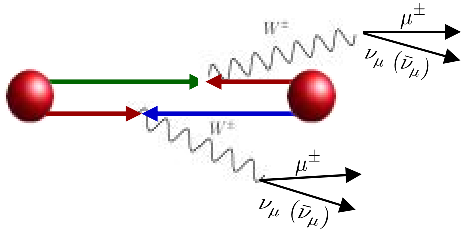
Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta, Phys.Rev. D95 (2017) no.11, 114030

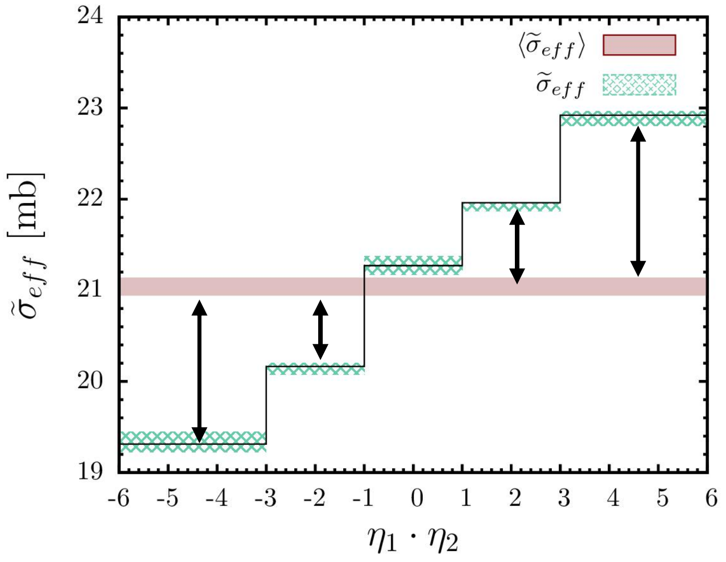
In order to understand whether correlations can be accessed in experimental observations, using dPDF evaluated within the QM model, the effective cross section has been calculated for this process and compared with its mean value:

$$\langle\tilde{\sigma}_{eff}\rangle = 21.04\,^{+0.07}_{-0.07}\,(\delta Q_0)\,^{+0.06}_{-0.07}(\delta\mu_F)\ \text{mb}\ .$$

$$\tilde{\sigma}_{eff} = \frac{m}{2}\frac{\sigma_A^{pp'}\ \sigma_B^{pp'}}{\sigma_{double}^{pp}}$$



$$\eta_1 \cdot \eta_2 \simeq \frac{1}{4} ln\frac{x_1}{x_3} ln\frac{x_2}{x_4}$$



Difference ↕ between green and red line is due to correlations effects

x- dependence of effective x-section consistent with analyses:

M.Rinaldi et al PLB 752,40 (2016)

M. Traini, M. R. et al, PLB 768, 270 (2017)

"Assuming that the results of the first and the last bins can be distinguished if they differ by 1 sigma, we estimated that

$$\boxed{\mathcal{L} = 1000\ \text{fb}^{-1}}$$

is necessary to observe correlations"

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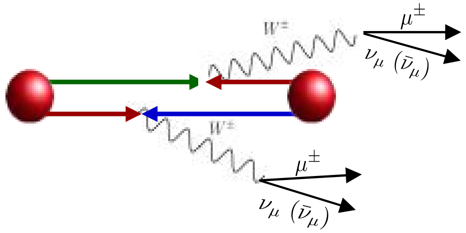

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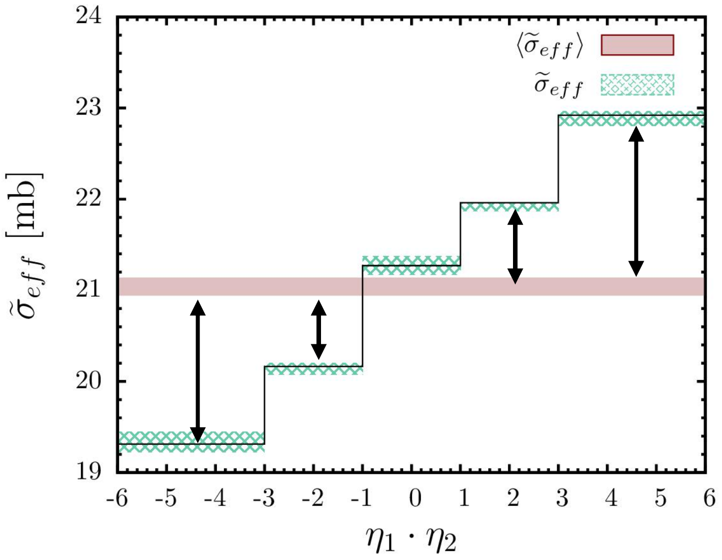

Difference [↕] between green and red line is due to correlations effects

To observe correlations, $\boxed{\mathcal{L} = 1000\ \text{fb}^{-1}}$ is needed! ➡ REACHABLE IN THE PLANNED LHC RUN

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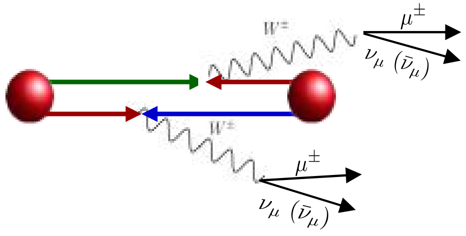

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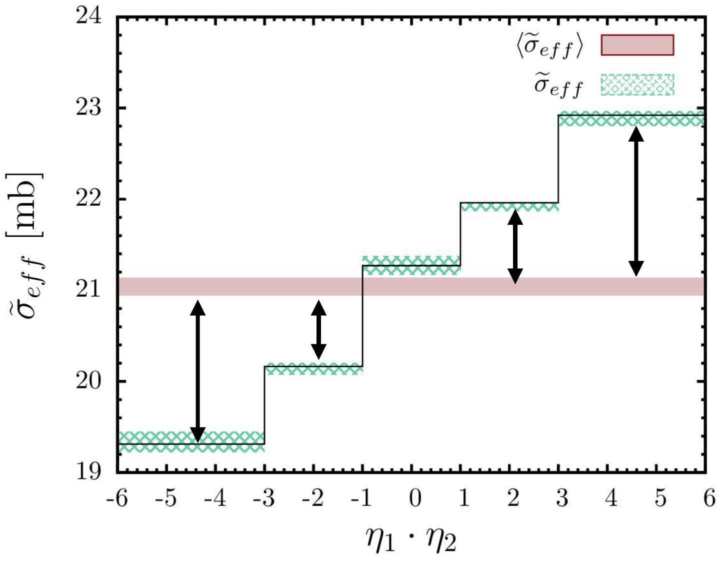

Difference [↕] between green and red line is due to correlations effects

IN THIS CHANNEL, WE ESTABLISHED THE POSSIBILITY TO OBSERVE, FOR THE FIRST TIME, TWO-PARTON CORRELATIONS IN THE NEXT LHC RUN!

A clue from data?

M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018) rapid communication

Considering the factorization ansatz, for which some estimates of σ_{eff} are available, one has:

$$\longrightarrow \sigma_{eff} = \left[\int \frac{d\vec{k}_\perp}{(2\pi)^2} \tilde{T}(\vec{k}_\perp) \boxed{\tilde{T}(-\vec{k}_\perp)} \right]^{-1} \longrightarrow \text{Effective form factor (Eff)}$$

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Effective form factor (Eff)

Eff can be formally defined as **FIRST MOMENT** of dPDF (like for GPDs) through the proton wave function:

$$\tilde{T}(k_\perp) = \frac{1}{2} \int dx_1 dx_2 F(x_1, x_2, k_\perp) = \int d\vec{k}_1 d\vec{k}_2 \Psi(\vec{k}_1 + \vec{k}_\perp, \vec{k}_2) \Psi^\dagger(\vec{k}_1, \vec{k}_2 + \vec{k}_\perp)$$

From the above quantity the mean distance in the transverse plane between two partons can be defined:

$$\langle b^2 \rangle \sim -2 \frac{d}{k_\perp dk_\perp} \tilde{T}(k_\perp) \Big|_{k_\perp = 0}$$

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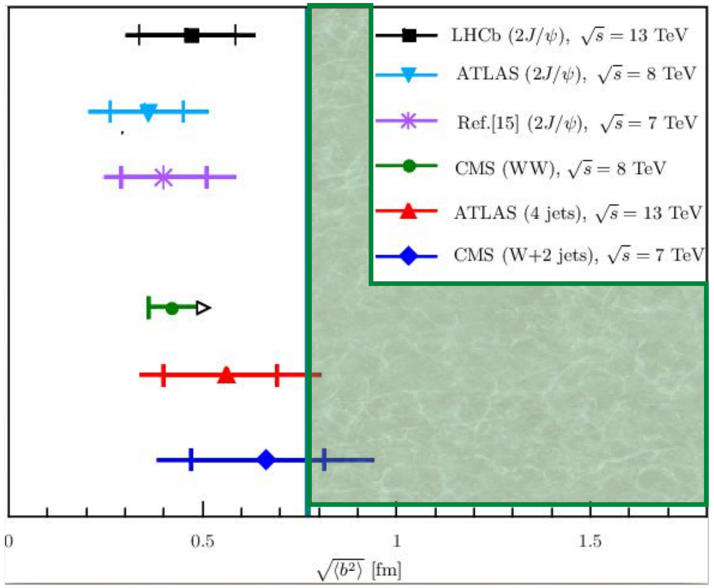
Eff is unknown but using general model independent properties and comparing Eff with standard proton ff, we found:

$$\frac{\sigma_{eff}}{3\pi} \leq \langle b^2 \rangle \leq \frac{\sigma_{eff}}{\pi}$$

DPS processes:

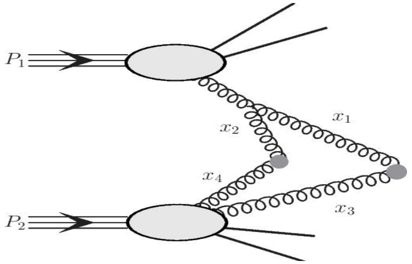
The vertical line stands for the transverse proton radius



We also:

M. R. and F. A. Ceccopieri, JHEP 1909, 097 (2019)

Extended the approach including splitting term



- Extended the approach to the most general unfactorized case

What next: pion double PDF

M. R., S. Scopetta, M. Traini and V.Vento, EPJC 78, no. 9,782 (2018)

The dPDF expression, at the hadronic scale, evaluated in the intrinsic frame, in term of meson wave function:

$$f_2(x,k_\perp) = \frac{1}{2}\sum_{h,h'}\int \frac{d^2k_{1\perp}}{2(2\pi)^3}\psi_{h,h'}(x,\vec{k}_{1\perp})\underbrace{\psi^*_{h,h'}(x,\vec{k}_{1\perp}+\vec{k}_\perp)}_{\text{Meson wave function}}$$

Parton helicities

Intrinsic parton momentum

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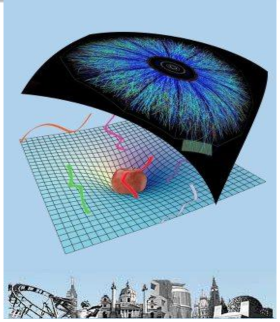

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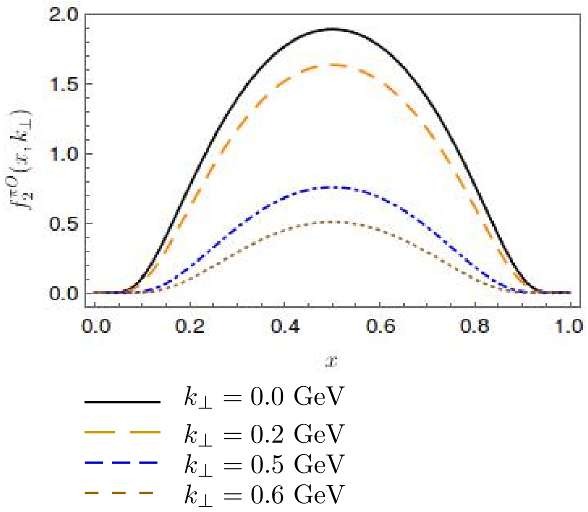
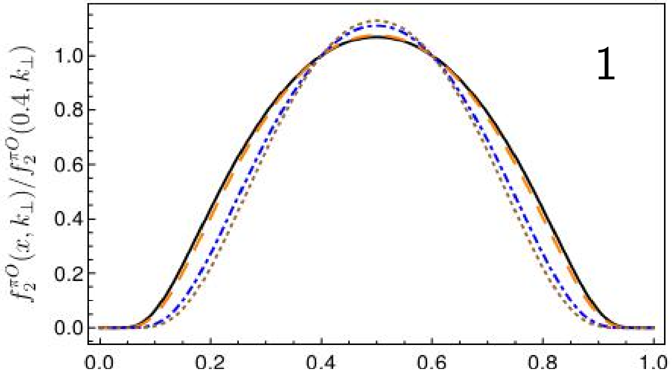


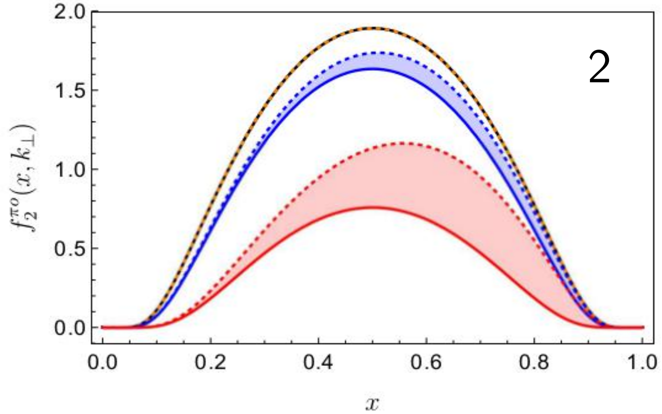
Pion: w.f. calculated within the AdS/QCD soft-wall model

S. J. Brodsky et al, PRD 77, 056007 (2008)

1) Also for pion, model calculations indicate that factorization on $x - k_\perp$ does not work!



2) Also for pion, model calculations indicate that dPDF can not be described in terms of GPDs (Dotted line=dPDF approximated).



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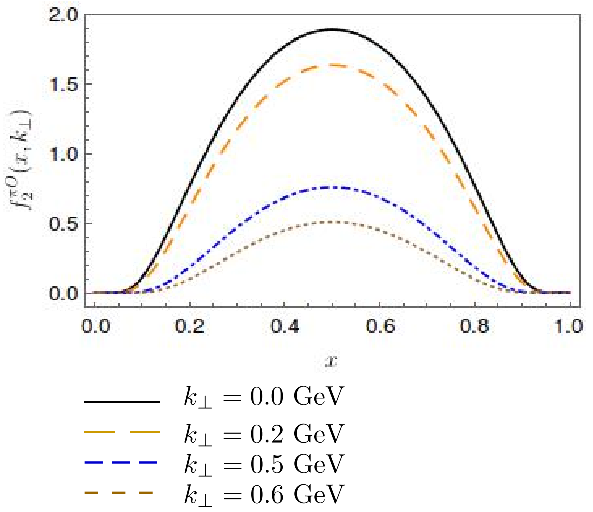
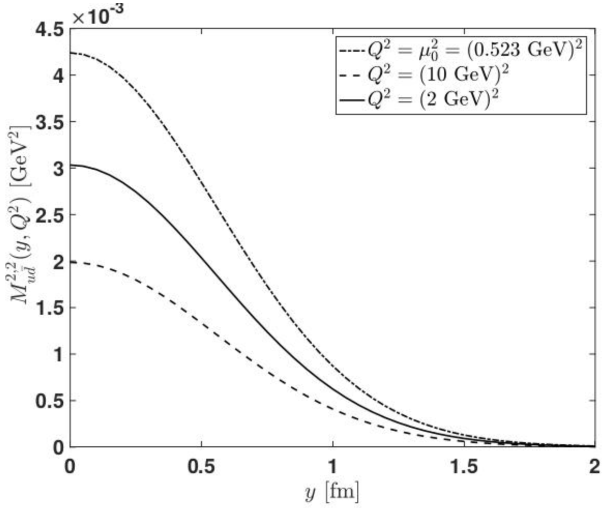
Meson wave function



Pion: w.f. calculated within the AdS/QCD soft-wall model

S. J. Brodsky et al, PRD 77, 056007 (2008)

$$M^{22}_{u\bar{d}}(y,Q^2) = \int_0^1 dx_1 \int_0^{1-x_1} dx_2\, x_1 x_2 \bar{F}_{u\bar{d}}(x_1,x_2,y,Q^2) \xrightarrow{\qquad\qquad}$$



The latter is a quantity close to those evaluated in “new” lattice studies of DPS. Future comparison are in principle possible to obtain new information on dPDF from lattice QCD.

What next: pion double PDF

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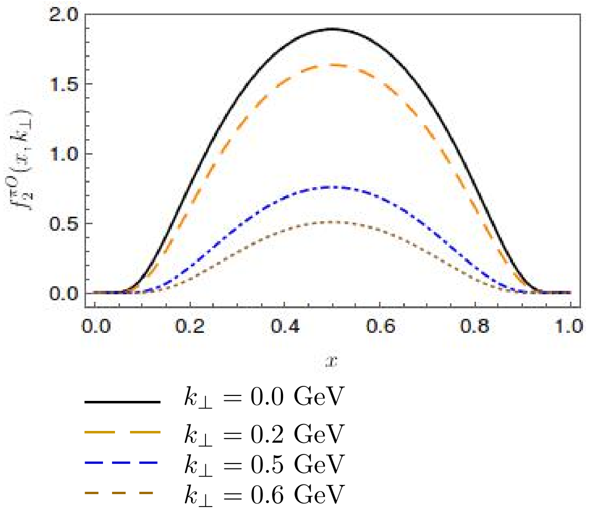

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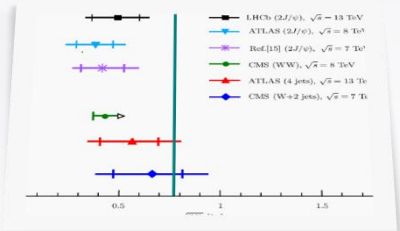
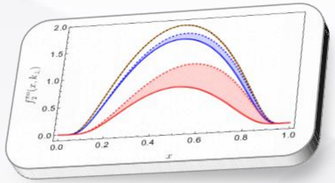
We also computed the ππ mean σ_{eff}:

$$\overline{\sigma}_{eff} = 41 \text{ mb}$$

This result has been used in experimental analysis for DPS at COMPASS: **arXiv: 1909.06195**

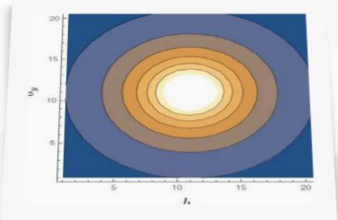
Conclusions

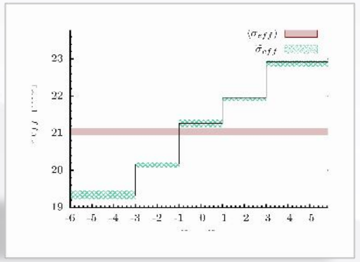




A CQM calculation of the dPDFs with a Poincare' invariant approach

- ✓ **longitudinal and transverse correlations are found**;
- ✓ deep study on relativistic effects: **transverse and longitudinal model independent**
- ✓ **correlations have been found**;
- ✓ pQCD evolution of dPDFs, including non perturbative degrees of freedom into the
- ✓ scheme: **correlations are present at high energy scales and in the low *x* region**;
- ✓ calculation of the effective X-section within different models in the valence region:
- ✓ **x-dependent quantity obtained!**
- ✓ calculation of mean partonic distance from present experimental analyses
- ✓ calculation of pion dPDF



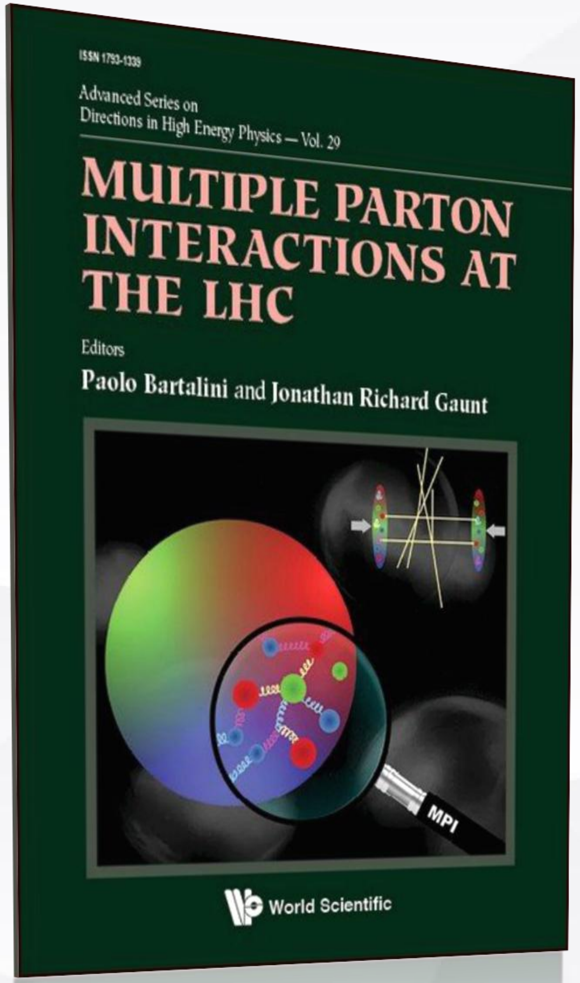


Study of DPS in same sign WW production at the LHC

- ✓ Calculations of the DPS cross section of same sign WW production
- ✓ **dynamical correlations are found to be measurable in the next run at the LHC**

A proton imagining (complementary to that investigated by means of electromagnetic probes) can/will be obtained in the next LHC runs!

Further Information on:



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Advanced Series on
Directions in High Energy Physics — Vol. 29

MULTIPLE PARTON INTERACTIONS AT THE LHC

Editors

Paolo Bartalini and Jonathan Richard Gaunt

World Scientific

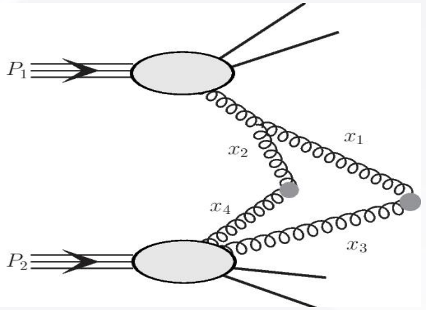
Thanks

Some extensions of the relation : $\frac{\sigma_{\text{eff}}}{3\pi} \le \langle \mathrm{b}^2 \rangle \le \frac{\sigma_{\text{eff}}}{\pi}$ I

M. R. and F. A. Ceccopieri, arXiv: 1812.04286. JHEP accepted.

Relevant for processes involving heavy particles is the splitting term (1v2 mechanism) i.e.:

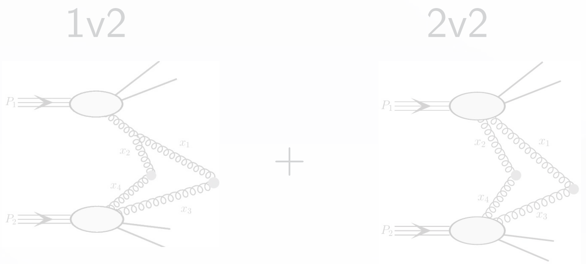
* $D_{j_1,j_2}(x_1,x_2) = \int d^2b_\perp \, \tilde{F}_{j_1,j_2}(x_{1,2}\,,b_\perp)$



In pQCD evolution:

$$\frac{dD_{j_1j_2}(x{,}x_2;t)}{dt} = \begin{cases} \text{Homogeneous term (double DGLAP)} \\ + \\ \underbrace{\sum_{j'} F_{j'}(x_1+x_2;t)\frac{1}{x_1+x_2}P_{j'\to j_1j_2}}_{\text{SPLITTING TERM}}\left(\frac{x_1}{x_1+x_2}\right) \end{cases}$$

Gaunt J.R. and Stirling W. J., JHEP 03 (2010)



$$\frac{\sigma_{eff}}{3\pi}\left(1+\frac{3}{2}r_v\right) \le \langle b^2 \rangle \le \frac{\sigma_{eff}}{\pi}\left(1+2\boxed{r_v}\right)$$

$$r_v \sim \frac{F^{splitting}_{j_1j_2}(x_1,x_2,k_\perp=0;t)}{F_{j_1j_2}(x_1,x_2,k_\perp=0;t)}$$

with:
$0 \le r_v \le 1$

Due to the difficulty in the estimate of the 2 contributions:

Absolute minimum $r_v = 0$ $\boxed{\frac{\sigma_{eff}}{3\pi}} \le \langle b^2 \rangle \le \boxed{\frac{3\ \sigma_{eff}}{\pi}}$ Absolute maximum $\mathbf{r_v = 1}$

Some extensions of the relation : $\frac{\sigma_{\rm eff}}{3\pi} \leq \langle b^2 \rangle \leq \frac{\sigma_{\rm eff}}{\pi}$ I

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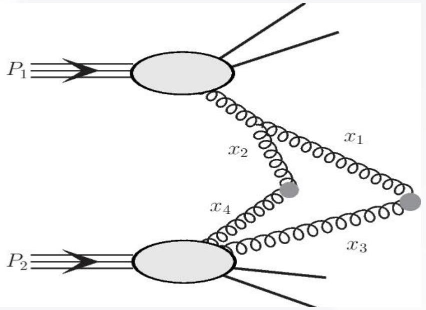

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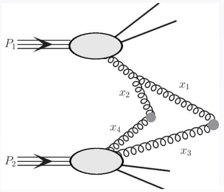
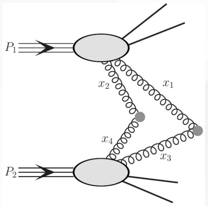
1v2



$+$

2v2



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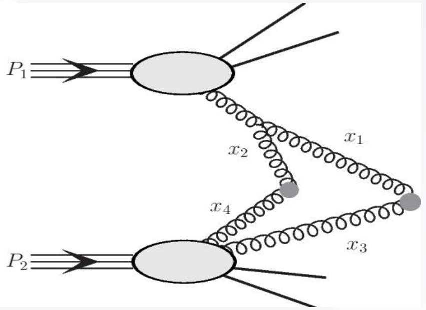

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SPLITTING TERM

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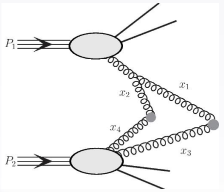
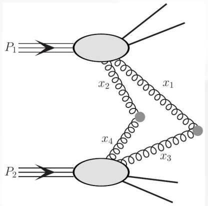
1v2



+

2v2



$$\frac{\sigma_{eff}}{3\pi}\left(1+\frac{3}{2}r_v\right) \le \langle b^2 \rangle \le \frac{\sigma_{eff}}{\pi}\left(1+2\boxed{r_v}\right)$$

$$r_v \sim \frac{F^{splitting}_{j_1 j_2}(x_1,x_2,k_\perp=0;t)}{F_{j_1 j_2}(x_1,x_2,k_\perp=0;t)}$$

with:

$0 \le r_v \le 1$

Due to the difficulty in the estimate of the 2 contributions:

Absolute minimum $r_v = 0$

$$\frac{\sigma_{eff}}{3\pi} \le \langle b^2 \rangle \le \frac{3\ \sigma_{eff}}{\pi}$$

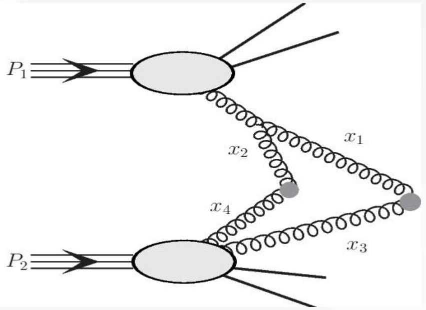
Absolute maximum $\mathbf{r_v = 1}$

Some extensions of the relation : $\frac{\sigma_{\text{eff}}}{3\pi} \leq \langle b^2 \rangle \leq \frac{\sigma_{\text{eff}}}{\pi}$ I

M. R. and F. A. Ceccopieri, arXiv: 1812.04286. JHEP accepted.

Relevant for processes involving heavy particles is the splitting term (1v2 mechanism) i.e.:

$^*\ D_{j_1,j_2}(x_1,x_2) = \int d^2b_\perp\ \tilde{F}_{j_1,j_2}(x_{1,2}\,, b_\perp)$



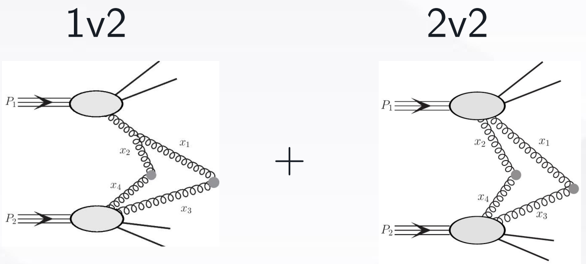
In pQCD evolution: $\frac{dD_{j_1j_2}(x_1,x_2;t)}{\ } = \Big\{$ Homogeneous term (double DGLAP)

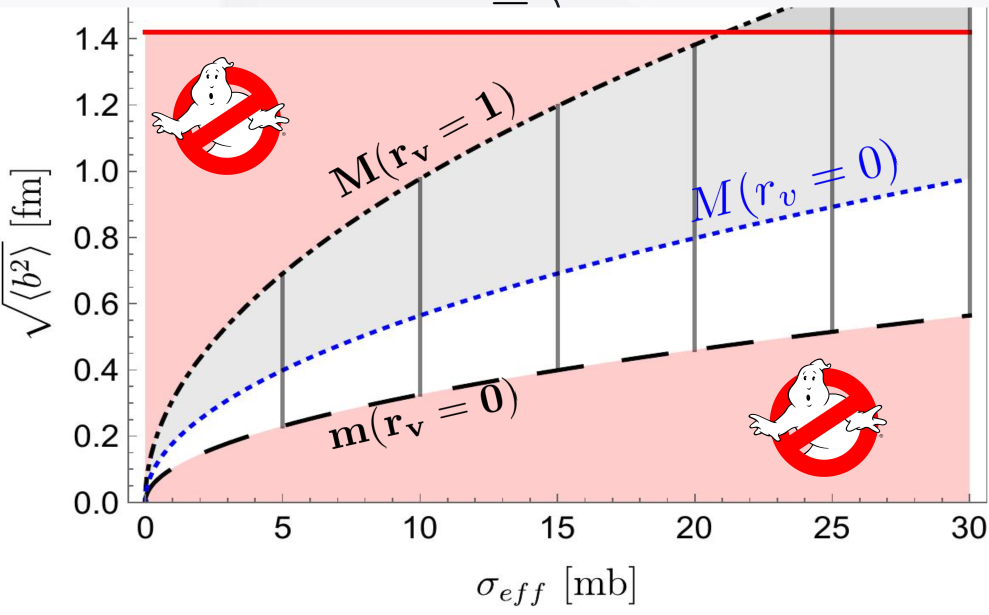
$+$

$\underbrace{\frac{1}{1+x_2} P_{j'\to j_1j_2}\left(\frac{x_1}{x_1+x_2}\right)}$

NG TERM

Gaunt J.R. and Stirling W





$$r_v \sim \frac{F^{splitting}_{j_1j_2}(x_1,x_2,k_\perp=0;t)}{F_{j_1j_2}(x_1,x_2,k_\perp=0;t)}$$

with:

$$0 \leq r_v \leq 1$$

1) Minimum as function of r_v

$m(r_v)$

2) Maximum as function of r_v

$M(r_v)$

Absolute minimum $r_v = 0$ $\boxed{\frac{\sigma_{eff}}{3\pi}} \leq \langle b^2 \rangle \leq \boxed{\frac{3\ \sigma_{eff}}{\pi}}$ Absolute maximum $\mathbf{r_v = 1}$

Some extensions of the relation : $\frac{\sigma_{\text{eff}}}{3\pi} \le \langle \mathrm{b}^2 \rangle \le \frac{\sigma_{\text{eff}}}{\pi}$ II

M. R. and F. A. Ceccopieri, arXiv: 1812.04286. JHEP accepted.

IF WE DO NOT CONSIDER ANY FACTORIZATION ANSATZ IN DOUBLE PDFs:

$$\frac{\sigma_{eff}(x_1,x_2)}{3\pi}\left[r^{2v2}(x_1,x_2)^2+\frac{3}{2}r^{2v1}(x_1,x_2)^2\ r_v\right] \le \langle b^2\rangle_{x_1,x_2} \le \frac{\sigma_{eff}(x_1,x_2)}{\pi}\left[r^{2v2}(x_1,x_2)^2+2r^{2v1}(x_1,x_2)^2\ r_v\right]$$

$$r^{2v2}(x_1,x_2)=\frac{F(x_1,x_2,k_\perp=0;t)}{F(x_1;t)F(x_2;t)}$$

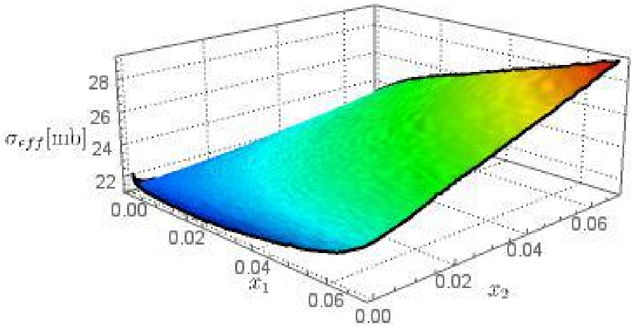
$$r^{2v1}(x_1,x_2)=\frac{F^{splitting}(x_1,x_2,k_\perp=0;t)}{F(x_1;t)F(x_2;t)}$$

The Effective X-section calculation

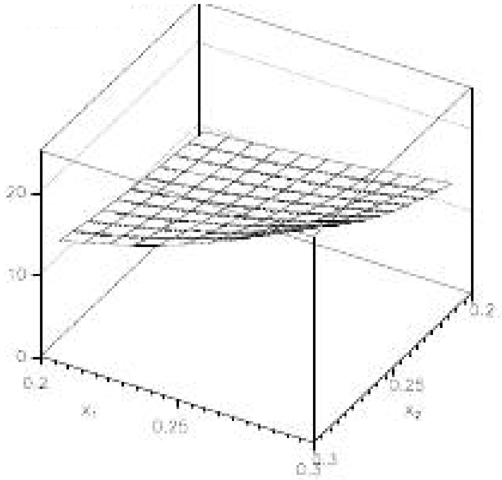
M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

Our predictions of σ_{eff} , **without any approximation**, in the valence region at different energy scales:

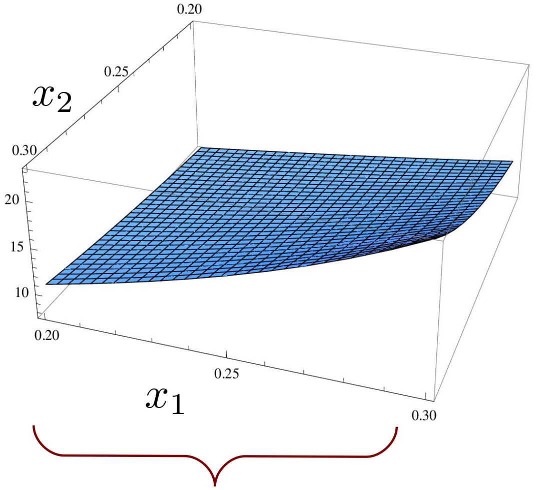
$$\sigma_{\text{eff}}(x_1, x_2, \mu_0^2) \xrightarrow[\text{pQCD evolution of dPDFs}]{\text{pQCD evolution of PDFs}} \sigma_{\text{eff}}(x_1, x_2, Q^2)$$



Gluons ⊗ Gluons



Valence ⊗ Valence quarks



Valence quark ⊗ Sea quark
Partons involved in, e.g., same sign WW production.

The old data lie in the obtained range of σ_{eff}

$$\bar{\sigma}_{\text{eff}} \sim 21 \text{ mb}$$

Similar results obtained with dPDFs calculated within AdS/QCD soft-wall model
M. Traini, M. R., S. Scopetta and V.Vento, PLB *768, 270 (2017)*

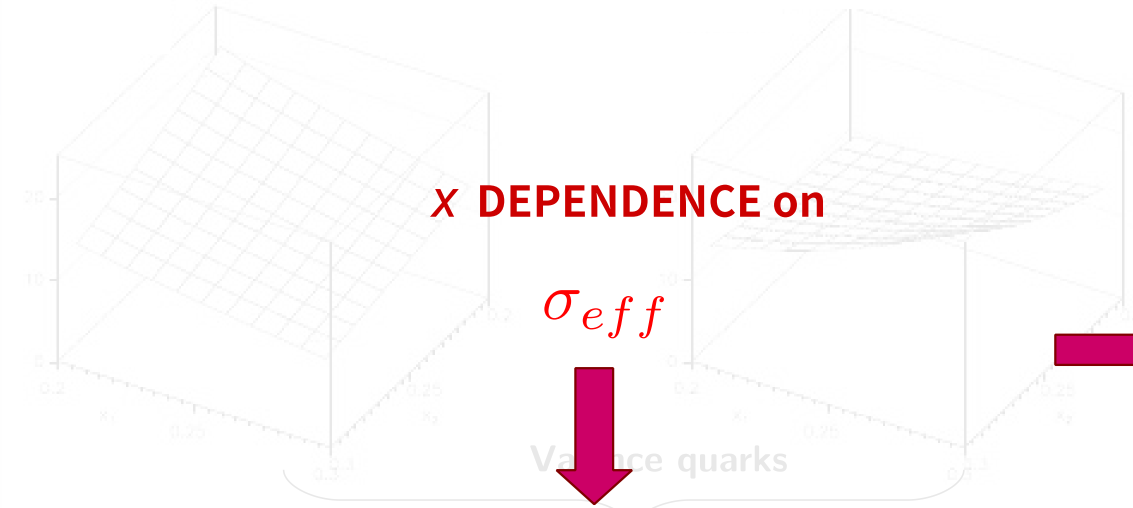
- x_i **dependence of σ_{eff} may be model independent feature**
- **Absolute value of σ_{eff} is a model dependent result**

The Effective X-section calculation

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

Our predictions of σ_{eff}, **without any approximation**, in the valence region at different energy scales:

$$\sigma_{eff}(x_1, x_2, \mu_0^2) \xrightarrow[\text{pQCD evolution of dPDFs}]{\text{pQCD evolution of PDFs}} \sigma_{eff}(x_1, x_2, Q^2 = 250 \text{ GeV}^2)$$

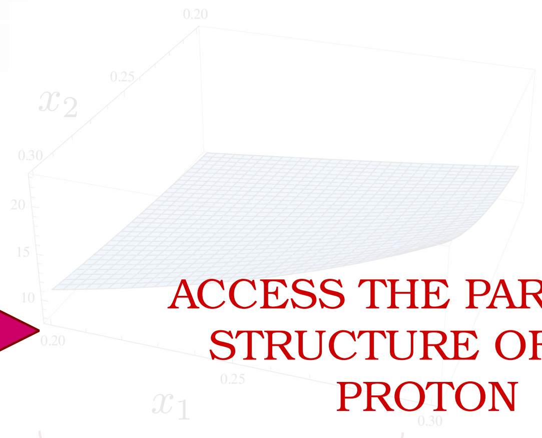


x **DEPENDENCE on**

σ_{eff}

ACCESS THE DOUBLE PARTON CORRELATIONS

$\sigma_{eff} \sim$ [illegible] mb



ACCESS THE PARTONIC STRUCTURE OF THE PROTON

Valence quark ⊗ Sea quark

Partons involved in, e.g., same sign WW production.

The old data lie in the obtained range of

σ_{eff}

Similar results obtained with dPDFs calculated within AdS/QCD soft-wall model

M. Traini, M. R., S. Scopetta and V.Vento, PLB *768, 270 (2017)*

- **x_i dependence of σ_{eff} may be model independent feature**
- **Absolute value of σ_{eff} is a model dependent result**

The Effective X-section calculation

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

$$\boxed{\sigma_{eff} = \frac{m}{2}\frac{\sigma_A^{pp'}\,\sigma_B^{pp'}}{\sigma_{double}^{pp}}}$$

This quantity can be written in terms of PDFs and dPDFs ($_2$GPDs)

Here the scale is omitted

Colour coefficient

$$\boxed{\sigma_{\mathrm{eff}}(x_1,x_1',x_2,x_2') = \frac{\sum_{i,k,j,l} F_i(x_1)F_k(x_1')F_j(x_2)F_l(x_2')C_{ik}\boxed{C_{jl}}}{\sum_{i,j,k,l} C_{ik}C_{jl}\int F_{ij}(x_1,x_2;k_\perp)F_{kl}(x_1',x_2';-k_\perp)\frac{dk_\perp}{(2\pi)^2}}}$$

Non trivial x-dependence

If factorization between dPDF and PDFs held:

$$F_{ab}(x_1,x_2,\vec{k}_\perp) = F_a(x_1)F_b(x_2)\tilde{T}(\vec{k}_\perp)$$

"EFFECTIVE FORM FACTOR"

Conjugated variable to $\vec{k}_\perp$

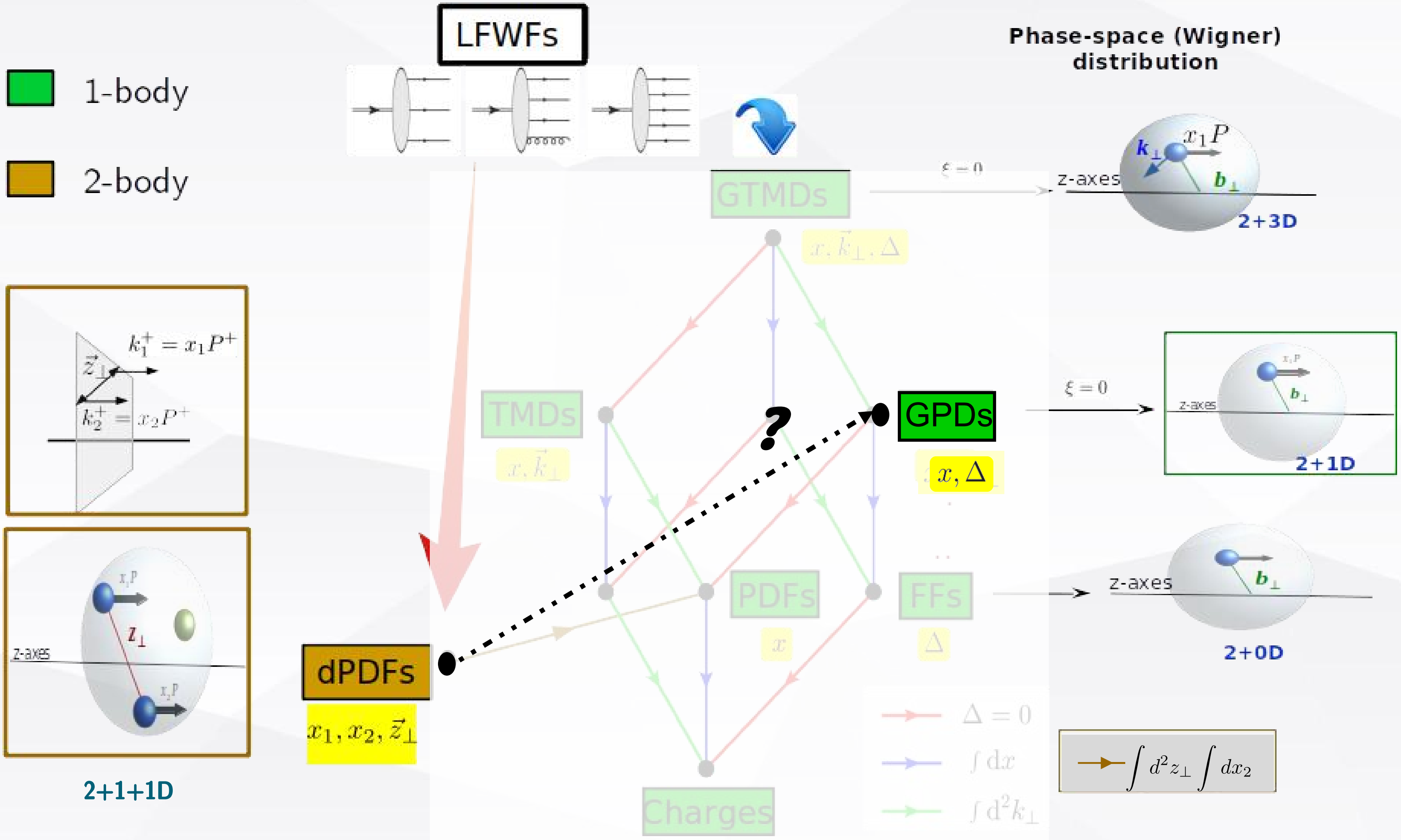
$$\sigma_{eff}(x_1,x_1',x_2,x_2') \to \sigma_{eff} = \left[\int \frac{d\vec{k}_\perp}{(2\pi)^2}\tilde{T}(\vec{k}_\perp)T(-\vec{k}_\perp)\right]^{-1} = \left[\int d\vec{b}_\perp T(\vec{b}_\perp)^2\right]^{-1}$$

Constant value w.r.t. x_i ➡ **NO CORRELATIONS!**

Answer: MULTIPARTON INTERACTIONS

🟩 1-body

🟧 2-body



What we learned: a link between dPDFs and GPDs?

The dPDF is formally defined through the Light-cone correlator:

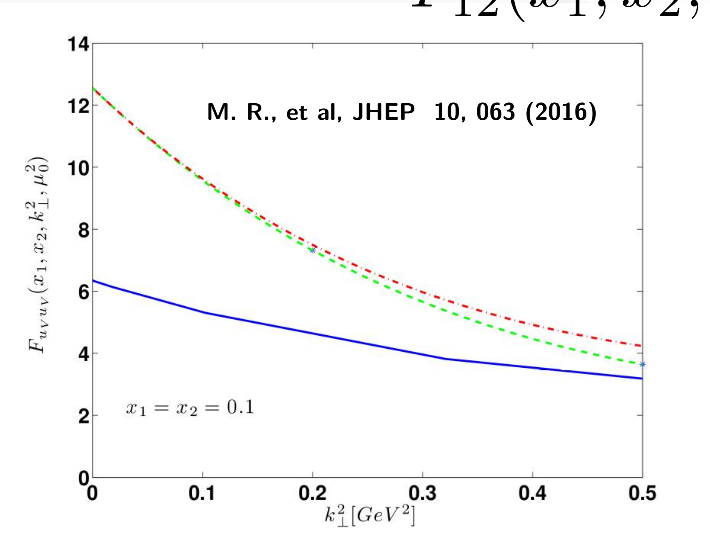
$$F_{12}(x_1,x_2,\vec{z}_\perp) \propto \sum_X \int dz^- \left[\prod_{i=1}^{2} dl_i^- \, e^{i x_i l_i^- p^+}\right] \langle p|O(z,l_1)|X\rangle\langle X|O(0,l_2)|p\rangle \Big|_{l_1^+=l_2^+=z^+=0}^{\vec{l}_{1\perp}=\vec{l}_{2\perp}=0}$$

Approximated by the proton state!

$$\int \frac{dp'^+ d\vec{p}'_\perp}{p'^+} |p'\rangle\langle p'|$$

GPDs

$$F_{12}(x_1,x_2,\vec{k}_\perp) \sim \underbrace{f(x_1,0,\vec{k}_\perp) f(x_2,0,\vec{k}_\perp)}$$



M. R., et al, JHEP 10, 063 (2016)

┈┈ dPDF = GPD x GPD

— dPDF

In GPDs, the variables $k_\perp$ and x are correlated!

⇩

Correlations between $\vec{z}_\perp$ and $x_1,\ x_2$ could be present in dPDFs !