Gamow-Teller decay rates for pf-shell nuclei

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Nuclear weak-interaction processes in stars

Nuclear weak-interaction processes play a key role in many astrophysical scenarios.

- solar nuclear reaction network
- *r*- and *s*-process nucleosynthesis
- core-collpase (type-II) supernovae
- thermonuclear (type-Ia) supernovae

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weak interaction rates

$\Downarrow$

astrophysical simulations

Stellar weak interaction rates

- $(Z, A) + e^- \rightarrow (Z-1, A) + \nu$ electron capture
- $(Z, A) \rightarrow (Z-1, A) + e^+ + \nu$ β^+ decay
- $(Z, A) + e^+ \rightarrow (Z+1, A) + \bar{\nu}$ positron capture
- $(Z, A) \rightarrow (Z+1, A) + e^- + \bar{\nu}$ β^- decay

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Decay rate

$$\lambda^\alpha = \frac{\ln 2}{K} \sum_i \frac{(2J_i + 1) e^{-E_i/(kT)}}{G(Z, A, T)} \sum_j B_{ij} \Phi_{ij}^\alpha$$

$G(Z, A, T)$: partition function
Φ_{ij}^α: phase space integral $\Phi_{ij}^\alpha = \Phi_{ij}^\alpha(\rho Y_e, T)$
B_{ij}: reduced nuclear transition probability

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- Gamow-Teller

$$B_{ij} = B_{ij}(GT) = \frac{\langle j || \sum_k \sigma^k t_{\pm}^k || i \rangle^2}{2J_i + 1}$$

Nuclear models

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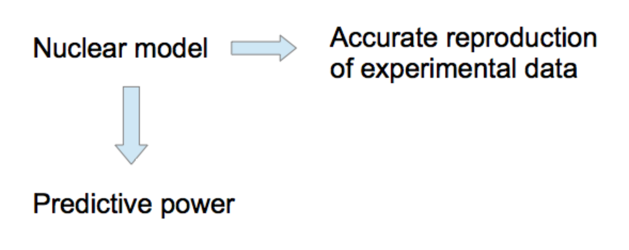

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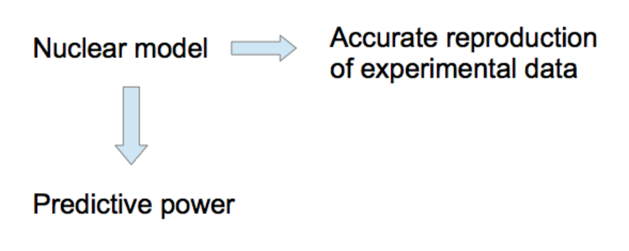

Fuller, Fowler and Newmann estimated stellar EC and β-decay rates $\Rightarrow$ independent particle model + experimental data

Nuclear models

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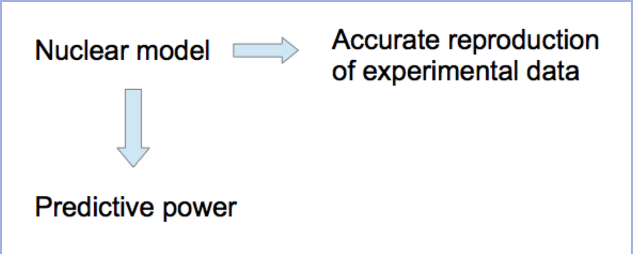

At the beginnning of the 90s Aufderheide, Mathews and collaborators pointed out that shell model is the method of choice for the calculation of stellar weak-interaction rates.

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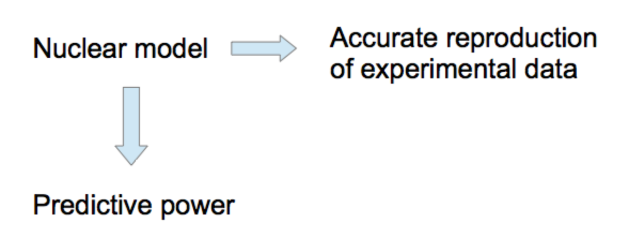
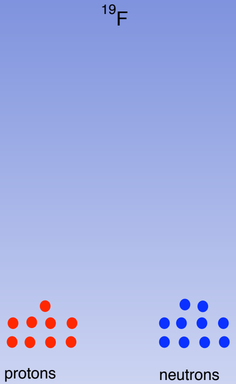


Langanke & Martinez Pinedo, 2000 $\Rightarrow$ *pf*-shell nuclei

An example: ^{19}F



- 9 protons & 10 neutrons interacting
- spherically symmetric mean field (e.g. harmonic oscillator)
- 1 valence proton & 2 valence neutrons interacting in a truncated model space

The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.

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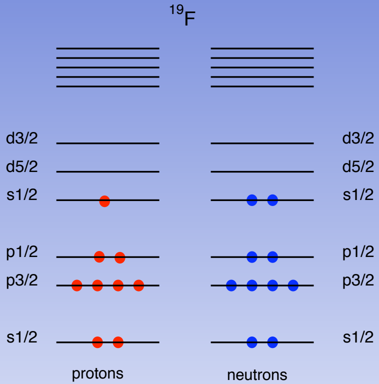

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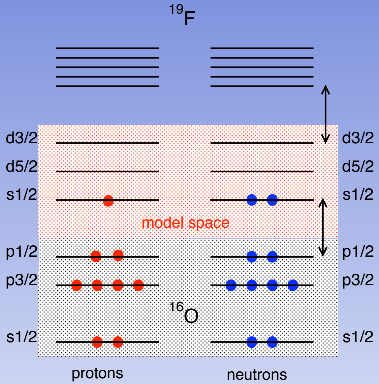

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Beta decay in the stellar interior

The rate of weak decay from the *i*th state of the parent to the *j*th state of the daughter nucleus is given by

$$\lambda_{ij} = \ln 2 \frac{\Phi_{ij}(\rho, U_F, T)}{(ft)_{ij}}$$

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Comparative half-life

$$(\log ft)_{GT} = 3.596 - |M_{GT}|^2$$

$$(\log ft)_F = 3.791 - |M_F|^2$$

Brown, Chung, and Wildenthal 1978

Phase space integral

The rate of weak decay from the *i*th state of the parent to the *j*th state of the daughter nucleus is given by

$$\Phi_{ij} = \int_1^{q_n} w^2 (q_n - w)^2 G(\pm Z, w)(1 - S_\mp)(1 - S_\nu) dw$$

for electron (*upper signs*) or positron (*lower signs*) emission or

$$\Phi_{ij} = \int_{w_l}^{\infty} w^2 (q_n + w)^2 G(\pm Z, w) S_\mp (1 - S_\nu) dw$$

for continuum positron (*upper signs*) or electron (*lower signs*) capture

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Phase space integral

$$q_n = \frac{Q_n}{m_e c^2} = \frac{(M_p - M_d + E_i - E_j)}{m_e c^2}$$

For the distribution function a first approximation is to neglect correction due to bound electrons and ions

$$S_- = \left(\exp\left(\frac{U - U_F}{kT}\right) + 1\right)^{-1}$$

$$S_+ = \left(\exp\left(\frac{U + U_F + 2m_e c^2}{kT}\right) + 1\right)^{-1}$$

where $U = (w - 1)m_e c^2$ and $p = (w^2 - 1)^{1/2}$

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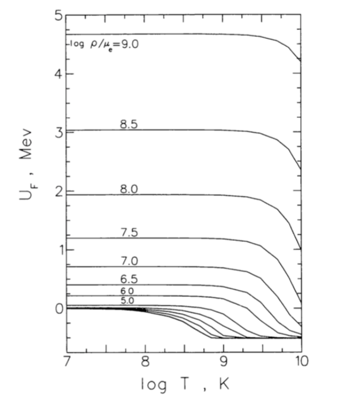
Electron chemical potential

$$\frac{\rho}{\mu_e} = \frac{1}{\pi^2 N_A}\left(\frac{m_e c}{\hbar}\right)^3 \int_0^\infty (S_- - S_+)p^2 dp$$



Relativistic Coulomb barrier

The last factor appearing in the phase space integral Φ_{ij} is

$$G(\pm Z, w) = \frac{p}{w} F(\pm Z, w)$$

where $F(\pm Z, w)$ is the relativistic Coulomb barrier factor given by the approximation

$$F(\pm Z, w) \approx 2(1+s)(2pR)^{2(s-1)} e^{\pi\eta} \left| \frac{\Gamma(s+i\eta)}{\Gamma(2s+1)} \right|^2$$

where $s = [1-(\alpha Z)^2]^{1/2}$ and $\eta = \pm \alpha Z w / p$

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pf-shell nuclei

Astrophysical interest

- electron capture rates relevant for late stellar evolution
- GT-strength distributions
  - β-decay experiments
  - charge-exchange reactions: (n,p), (d,^{2}He), (t,^{3}He)

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Realistic shell-model calculation

- Realistic potential $V_{NN} \Rightarrow$ high-precision *NN* CD-Bonn potential
- $\Rightarrow H_{\rm eff}$ & $GT_{\rm eff}$ consistently derived in the MBPT frame

Conclusions

- RSM calculations provide a satisfactory description of observed GT-strength distributions in *pf* nuclei without resorting to any “ad hoc” GT-operator quenching

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- RSM calculations provide a satisfactory description of observed GT-strength distributions in *pf* nuclei without resorting to any "ad hoc" GT-operator quenching
- investigate other nuclei of astrophysical interest (e.g. *sd* nuclei)
- calculation of *EC* rates at relevant stellar temperatures and densities ($T \simeq 1 - 10 \times 10^9 K$, $\rho Y_e \simeq 10^7 - 10^9 g/cm^3$)

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Thank you for your kind attention

Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} \ (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

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The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian

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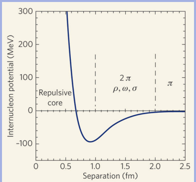
Workflow for a realistic shell-model calculation

1. Choose a realistic *NN* potential (*NNN*)
2. Determine the model space better tailored to study the system under investigation
3. Derive the effective shell-model hamiltonian and operators by way of a many-body theory
4. Calculate the physical observables (energies, e.m. transition probabilities, ...)

Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion



How to handle the short-range repulsion ?

- Brueckner G matrix
- low-momentum NN potentials

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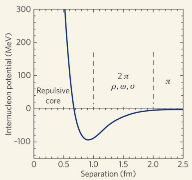

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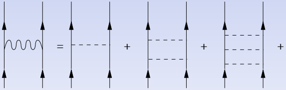

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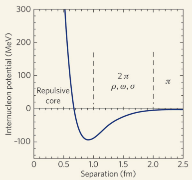

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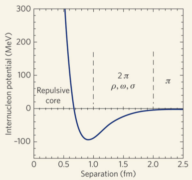

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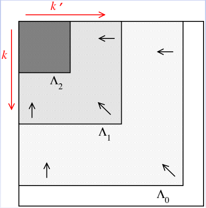

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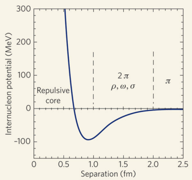

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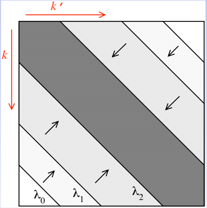

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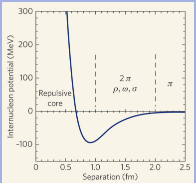
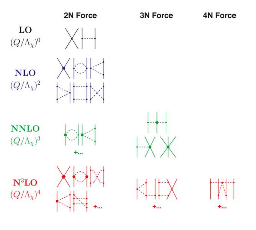


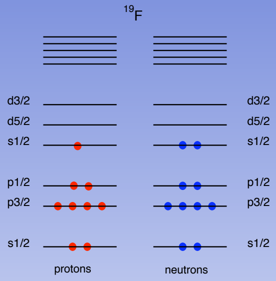


Effective shell-model hamiltonian

A-nucleon system Schrödinger equation

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

$$H = H_0 + H_1 = \sum_{i=1}^{A}(T_i + U_i) + \sum_{i<j}(V_{ij}^{NN} - U_i)$$



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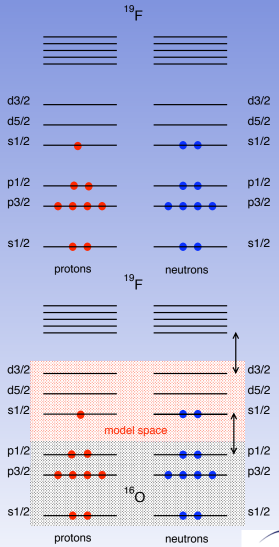
Model space

$$|\Phi_i\rangle = [a_1^\dagger a_2^\dagger \ldots a_n^\dagger]_i|c\rangle \Rightarrow P = \sum_{i=1}^{d}|\Phi_i\rangle\langle\Phi_i|$$

Model-space eigenvalue problem

$$H_{\rm eff}P|\Psi_\alpha\rangle = E_\alpha P|\Psi_\alpha\rangle$$



The shell-model effective hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array}\right) \quad \xRightarrow{\mathcal{H} = X^{-1}HX} \quad \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array}\right)$$

$$H_{\mathrm{eff}} = P\mathcal{H}P$$

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$$H_{\rm eff} = P\mathcal{H}P$$

The shell-model effective hamiltonian

Folded-diagram expansion

$\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

$\Rightarrow$ Recursive equation for $H_{\rm eff}$ $\Rightarrow$ iterative techniques (Krenciglowa-Kuo, Lee-Suzuki, ...)

$$H_{\rm eff} = \hat{Q} - \hat{Q}'\int\hat{Q} + \hat{Q}'\int\hat{Q}\int\hat{Q} - \hat{Q}'\int\hat{Q}\int\hat{Q}\int\hat{Q}\cdots ,$$

generalized folding

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

The $\hat{Q}$-box can be calculated perturbatively

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty}\frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

The diagrammatic expansion of the $\hat{Q}$-box

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The perturbative approach to the shell-model $H^{\rm eff}$

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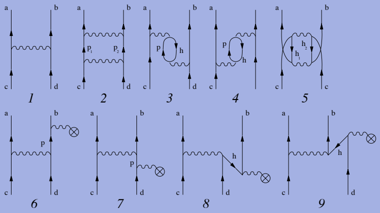

Effective operators

$\Phi_\alpha =$ eigenvectors obtained diagonalizing H_{eff} in the reduced model space $\Rightarrow |\Phi_\alpha\rangle = P|\Psi_\alpha\rangle$

$$\langle\Phi_\alpha|\hat{O}|\Phi_\beta\rangle \neq \langle\Psi_\alpha|\hat{O}|\Psi_\beta\rangle$$

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Effective operator $\hat{O}_{\text{eff}}$: definition

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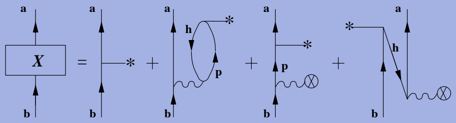
$\hat{O}_{\rm eff}$ can be derived consistently in the MBPT framework

One-body operator



Realistic shell-model calculations

In the last 20 years realistic shell-model calculations have been widely employed with success to explore various regions of the nuclear landscape

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- $H^{\rm eff}$ for systems with one and two valence nucleons
- $\hat{Q}$-box $\Rightarrow$ Goldstone diagrams up to third order in V_{NN} (up to 2p-2h core excitations)
- Effective operators consistently derived by way of MBPT