Double charge exchange reactions and netrinoless double beta decay

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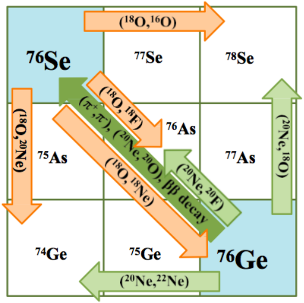
In heavy-ion DCE reactions two protons (neutrons) are converted into two neutrons (protons) in the target, and two neutrons (protons) are converted into two protons (neutrons) in the projectile, while the mass number of the target, A, and of the projectile, a, both remain unchanged.



Double Charge Exchange Experiment

NUMEN

- Canditates isotopes: ^{48}**Ca**, ^{82}**Se**, ^{100}**Mo**, ^{124}**Sn**, ^{128}**Te**, ^{130}**Te**, ^{136}**Xe**, ^{148}**Nd**, ^{150}**Nd**, ^{154}**Sm**, ^{160}**Gd**, ^{198}**Pt**.

The theory of the Heavy Ions Double Charge Exchange has never been developed before.

1. We provide a simple theoretical description of DCE processes,
2. We investigate the possibility of a factorization of the cross section into a reaction part and nuclear matrix element part, at least within some limits.
3. Without the factorization proof, the experimentalists cannot extract the nuclear matrix elements from the cross section.

The nucleon-nucleon charge-exchange effective potential we consider,

$$V_{\mathrm{CE}}(\vec{q}) = V_{\mathrm{OPE}}(\vec{q}) + V_{\mathrm{ZR}}$$

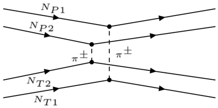
is the sum of a long- and medium-range one-pion-exchange (OPE) part and an effective zero-range (ZR) contact interaction. As from

Bertsch and H. Esbensen, Rep. Prog. Phys. 50, 607 (1987).

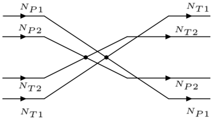
The latter, due to many body correlations, is written in coordinate space as follows

$$V_{\mathrm{ZR}}(\vec{r}) \quad = \left[c_{\mathrm{T}}(\vec{\tau}_1 \cdot \vec{\tau}_2) + c_{\mathrm{GT}}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)\right] \delta^3(\vec{r}) \ ,$$

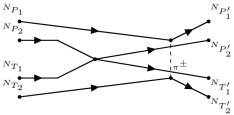
The OPE and ZR interactions provide the expressions for the vertices which we need in the computation of the following diagrams:



Double-pion-exchange interaction



Contact Term



One-pion-exchange plus contact term

We derived DCE effective potential that describes both long- and short-range interactions,

$$
\begin{aligned}
&V^{\mathrm{DCE}}(\mathbf{q_1},\mathbf{q_2})= \\
&\frac{4}{3}\left(\frac{f_\pi}{m_\pi}\right)^4\left(\frac{(\vec{\sigma}_{\mathrm{P1}}\cdot\mathbf{q_1})(\vec{\sigma}_{\mathrm{T1}}\cdot\mathbf{q_1})}{\tilde{m}_\pi^2+q_1^2}\;\vec{\tau}_{\mathrm{P1}}\cdot\vec{\tau}_{\mathrm{T1}}\right)\left(\frac{(\vec{\sigma}_{\mathrm{P2}}\cdot\mathbf{q_2})(\vec{\sigma}_{\mathrm{T2}}\cdot\mathbf{q_2})}{\tilde{m}_\pi^2+q_2^2}\;\vec{\tau}_{\mathrm{P2}}\cdot\vec{\tau}_{\mathrm{T2}}\right) \\
&+\frac{c_t^2}{\bar{E}_P^F+\bar{E}_T^F}(\vec{\tau}_{\mathrm{P1}}\cdot\vec{\tau}_{\mathrm{T1}})(\vec{\tau}_{\mathrm{P2}}\cdot\vec{\tau}_{\mathrm{T2}})+\Big[\frac{2c_{gt}^2}{\bar{E}_P^{GT}+\bar{E}_T^{GT}}(\vec{\sigma}_{\mathrm{P1}}\cdot\vec{\sigma}_{\mathrm{T1}})(\vec{\sigma}_{\mathrm{P2}}\cdot\vec{\sigma}_{\mathrm{T2}}) \\
&+\frac{2c_tc_{gt}}{\bar{E}_P^{GT}+\bar{E}_T^F}(\vec{\sigma}_{\mathrm{P2}}\cdot\vec{\sigma}_{\mathrm{T2}})+\frac{2c_tc_{gt}}{\bar{E}_P^F+\bar{E}_T^{GT}}(\vec{\sigma}_{\mathrm{P1}}\cdot\vec{\sigma}_{\mathrm{T1}})\Big](\vec{\tau}_{\mathrm{P1}}\cdot\vec{\tau}_{\mathrm{T1}})(\vec{\tau}_{\mathrm{P2}}\cdot\vec{\tau}_{\mathrm{T2}}) \\
&+\left(\frac{f_\pi}{m_\pi}\right)^2\left(\frac{(\vec{\sigma}_{\mathrm{P1}}\cdot\mathbf{q_1})(\vec{\sigma}_{\mathrm{T1}}\cdot\mathbf{q_1})}{\tilde{m}_\pi^2+q_2^2}\;\vec{\tau}_{\mathrm{P1}}\cdot\vec{\tau}_{\mathrm{T1}}\right) \\
&\times\Big(c_t(\vec{\tau}_{\mathrm{P2}}\cdot\vec{\tau}_{\mathrm{T2}})+c_{gt}(\vec{\sigma}_{\mathrm{P2}}\cdot\vec{\sigma}_2)(\vec{\tau}_{\mathrm{P2}}\cdot\vec{\tau}_{\mathrm{T2}})\Big)+1\leftrightarrow 2
\end{aligned}
$$

Low-momentum-transfer limit.

We can extract a simple and more compact form for the transition amplitude

$$
\begin{aligned}
V^{\mathrm{DCE}} \;\xrightarrow[\vec{Q}\to 0]{}\; & 2\Big[\frac{c_{\mathrm{T}}^2}{\bar{E}_{\mathrm{P}}^{\mathrm{F}}+\bar{E}_{\mathrm{T}}^{\mathrm{F}}}+\frac{c_{\mathrm{GT}}^2(\vec{\sigma}_{\mathrm{P1}}\cdot\vec{\sigma}_{\mathrm{T1}})(\vec{\sigma}_{\mathrm{P2}}\cdot\vec{\sigma}_{\mathrm{T2}})}{\bar{E}_{\mathrm{P}}^{\mathrm{GT}}+\bar{E}_{\mathrm{T}}^{\mathrm{GT}}} \\
&+\frac{c_{\mathrm{T}}c_{\mathrm{GT}}(\vec{\sigma}_{\mathrm{P2}}\cdot\vec{\sigma}_{\mathrm{T2}})}{\bar{E}_{\mathrm{P}}^{\mathrm{GT}}+\bar{E}_{\mathrm{T}}^{\mathrm{F}}}+\frac{c_{\mathrm{T}}c_{\mathrm{GT}}(\vec{\sigma}_{\mathrm{P1}}\cdot\vec{\sigma}_{\mathrm{T1}})}{\bar{E}_{\mathrm{P}}^{\mathrm{F}}+\bar{E}_{\mathrm{T}}^{\mathrm{GT}}}\Big](\vec{\tau}_{\mathrm{P1}}\cdot\vec{\tau}_{\mathrm{T1}})(\vec{\tau}_{\mathrm{P2}}\cdot\vec{\tau}_{\mathrm{T2}})
\end{aligned}
$$

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Rapid Communications

Heavy-ion double-charge-exchange and its relation to neutrinoless double-β decay

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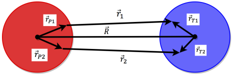
$$\frac{d\sigma}{d\Omega} = \frac{k}{k'}\left(\frac{\mu}{4\pi^2\hbar^2}\right)^2 |T_{\rm if}|^2$$



in the eikonal approximation:

$$T_{\rm if} = \left\langle \Psi^-_{\vec{k}'}\Phi_{\rm f} \middle| V \middle| \Psi^+_{\vec{k}}\Phi_{\rm i}\right\rangle = \frac{1}{(2\pi)^{3/2}} \int d\vec{R}\ {\rm e}^{i\left(\chi(b) - \vec{Q}\cdot\vec{R}\right)} M_{\rm if}(\vec{m}) \ ,$$

$$M_{\rm if}(\mathbf{m}) = \langle \Phi_{\rm f} | \, V^{\rm DCE} \, | \Phi_{\rm i} \rangle \ ,$$

The transition amplitude within the low-momentum-transfer limit,

$$M_{\rm if}(\mathbf{m}) \underset{\vec{Q}\to 0}{\longrightarrow} 6\sum_J \left\{ \begin{array}{ccc} 1 & 1 & J \\ 1 & 1 & J \\ 0 & 0 & 0 \end{array} \right\} \langle \phi_{\rm f}^{\rm T1}\phi_{\rm f}^{\rm P1}\phi_{\rm f}^{\rm T2}\phi_{\rm f}^{\rm P2}| \left\{ \frac{c_{\rm GT}^2(2J+1)}{\bar{E}_{\rm P}^{\rm GT}+\bar{E}_{\rm T}^{\rm GT}} \left[[\vec{\sigma}_{\rm P1}\times\vec{\sigma}_{\rm P2}]^{(J)}\ [\vec{\sigma}_{\rm T1}\times\vec{\sigma}_{\rm T2}]^{(J)} \right]^{(0)} \right.$$
$$\left. + \frac{c_{\rm T}^2\delta_{J,0}}{\bar{E}_{\rm P}^{\rm F}+\bar{E}_{\rm T}^{\rm F}} + \sqrt{3}c_{\rm T}c_{\rm GT}\delta_{J,0}\left(\frac{[\vec{\sigma}_{\rm P2}\times\vec{\sigma}_{\rm T2}]^{(0)}}{\bar{E}_{\rm P}^{\rm GT}+\bar{E}_{\rm T}^{\rm F}} + \frac{[\vec{\sigma}_{\rm P1}\times\vec{\sigma}_{\rm T1}]^{(0)}}{\bar{E}_{\rm P}^{\rm F}+\bar{E}_{\rm T}^{\rm GT}} \right) \right\} (\tau_{\rm T1}^{+}\tau_{\rm T2}^{+}\tau_{\rm P1}^{-}\tau_{\rm P2}^{-}) |\phi_{\rm i}^{\rm T1}\phi_{\rm i}^{\rm P1}\phi_{\rm i}^{\rm T2}\phi^{\rm P2}\rangle ,$$

We study the case of a target $0^+_{\rm i,T} \to 0^+_{\rm f,T}$ transitions,

$$M_{\rm if}(\mathbf{m}) \underset{\vec{Q}\to \mathbf{0}}{\longrightarrow} 2\left[\left(\frac{\mathcal{M}^{\rm DGT}_{\rm T\to T'}\mathcal{M}^{\rm DGT}_{\rm P\to P'}}{\bar{E}_{\rm P}^{\rm GT}+\bar{E}_{\rm T}^{\rm GT}} \right) + \left(\frac{\mathcal{M}^{\rm DF}_{\rm T\to T'}\ \mathcal{M}^{\rm DF}_{\rm P\to P'}}{\bar{E}_{\rm P}^{\rm F}+\bar{E}_{\rm T}^{\rm F}} \right) \right]$$

DCE-Double-Gamow-Teller (DGT) and DCE-Double-Fermi (DF) matrix elements, respectively, for a given nuclear transition of the projectile/target (A = P, T), defined as

$$\mathcal{M}^{\mathrm{DGT}}_{\mathrm{A}\to\mathrm{A}'} = c_{\mathrm{GT}} \left\langle \Phi^{(\mathrm{A}')}_{J'} \middle| \sum_{n,n'} [\vec{\sigma}_n \times \vec{\sigma}_{n'}]^{(0)} \tau^{\dagger}_{n} \tau^{\dagger}_{n'} \middle| \Phi^{(\mathrm{A})}_{J} \right\rangle \;,$$

and

$$\mathcal{M}^{\mathrm{DF}}_{\mathrm{A}\to\mathrm{A}'} = c_{\mathrm{T}} \left\langle \Phi^{(\mathrm{A}')}_{J'} \middle| \sum_{n,n'} \tau^{\dagger}_{n} \tau^{\dagger}_{n'} \middle| \Phi^{(\mathrm{A})}_{J} \right\rangle \;,$$

where the sum is over the nucleons (n, n') involved in the process.

The transition operator for $0\nu\beta\beta$-decay is divided into three contributions:

$$T^{0\nu} = T_F^{(0\nu)} + T_{GT}^{(0\nu)} + T_T^{(0\nu)},$$

where each contribution can be written in compact form as:

$$\begin{gathered} T_{s_1,s_2}^{(\lambda)} = -\tfrac{1}{4}\sigma_{j_1 j_2}\sigma_{j_1' j_2'}\sigma_J(-1)^J\sqrt{1+(-1)^J\delta_{j_1 j_2}} \\ \times\sqrt{1+(-1)^J\delta_{j_1' j_2'}}\,G_{s_1 s_2}^{(\lambda)}(j_1 j_2 j_1' j_2; J) \\ \times(\pi_{j1}^\dagger\times\pi_{j2}^\dagger)^{(J)}\cdot(\tilde{v}_{j_1'}\times\tilde{v}_{j2}')^{(J)} \end{gathered}$$

The $G_{s1,s2}^{(\lambda)}(j_1, j_2, j_{1'}, j_{2'}; J)$ are the two-body matrix elements of $T_{s_1 s_2}^{(\lambda)}$ between two fermion states.
Iachello Phys. Rev. C 91 (2015)

Two body matrix element in $0\nu\beta\beta$

The general two-body matrix element for $0\nu\beta\beta$ in the closure approximation is given by :

$$
\begin{aligned}
&\langle j_1, j_2, J \| [[\Sigma^{(s_1)} \times \Sigma^{(s_2)}]^{(\lambda_1)} \times V(r) C^{(\lambda_2)}]^{(\lambda)} \| j_1', j_2' J' \rangle \\
&= \sum_{kk'} \sum_{k_1 k_2} i^{k_1 - k_2 + \lambda_2} \frac{(2k_1+1)(2k_2+1)}{2\lambda_2+1} \\
&\langle k_1 0 k_2 0 \lambda_2 0 \rangle v^{k_1, k_2; \lambda_2}(r_1, r_2) \\
&\hat{k}\hat{k}'\hat{\lambda}_1\hat{\lambda}_2 \begin{Bmatrix} s_1 & k_1 & k \\ s_2 & k_2 & k' \\ \lambda_1 & \lambda_2 & \lambda \end{Bmatrix} \hat{J}\hat{\lambda}\hat{J}' \begin{Bmatrix} j_1 & j_2 & J \\ j_1' & j_2' & J' \\ k & k' & \lambda \end{Bmatrix} \\
&\hat{j}_1 \hat{k} \hat{j}_1' \begin{Bmatrix} \frac{1}{2} & l_1 & j_1 \\ \frac{1}{2} & l_1' & j_1' \\ s_1 & k_1 & k \end{Bmatrix} \langle \tfrac{1}{2} \| \Sigma^{(s_1)} \| \tfrac{1}{2} \rangle (-1)^{-k_1} \hat{l}_1 \langle l_1 0 k_1 |_1' 0 \rangle \\
&\hat{j}_2 \hat{k}' \hat{j}_2' \begin{Bmatrix} \frac{1}{2} & l_2 & j_2 \\ \frac{1}{2} & l_2' & j_2' \\ s_2 & k_2 & k \end{Bmatrix} \langle \tfrac{1}{2} \| \Sigma^{(s_2)} \| \tfrac{1}{2} \rangle (-1)^{-k_2} \hat{l}_2 \langle l_2 0 k_2 |_2' 0 \rangle \\
&R^{(k_1 k_2 \lambda_2)}(n_1, l_1, n_2, l_2, n_1', l_1', n_2', l_2')
\end{aligned}
$$

Ruslan Magana PhD Thesis 2018

DCE-DGT and DCE-DF matrix elements MDCE calculated in the Microscopic IBM2

Table: Our calculated DCE-DGT (second column) and DCE-DF (third column) matrix elements for the target are compared with the $0\nu\beta\beta$-DGT (fourth column), $0\nu\beta\beta$-DF (fifth column) and $0\nu\beta\beta$-total (sixth column) (with $g_A = 1$). The matrix elements are in fm^{-1}.

Reaction	$\mathcal{M}^{T,DGT}_{DCE}$	$\mathcal{M}^{T,DF}_{DCE}$	$\mathcal{M}^{T,DGT}_{0\nu\beta\beta}$	$\mathcal{M}^{T,DF}_{0\nu\beta\beta}$	$\mathcal{M}^{TOT}_{0\nu\beta\beta}$
^{116}Cd $\rightarrow$ ^{116}Sn	0.20	0.05	0.21	−0.02	0.25
^{82}Se $\rightarrow$ ^{82}Kr	0.28	0.08	0.31	−0.21	0.50
^{128}Te $\rightarrow$ ^{128}Xe	0.27	0.07	0.28	−0.16	0.43
^{76}Ge $\rightarrow$ ^{76}Se	0.34	0.10	0.40	−0.25	0.63

With the value of the constants ct = 151 MeV fm3 and cgt = 217 MeV fm3 from G.F Bertsch et al, Rep. Prog. Phys. 50 607 (1987). We can observe that the dominant contribution is the Double Gamow-Teller.

DCE in the low-momentum transfer limit

- The cross-section in the eikonal approximation and low-momentum transfer limit is given as follows

$$\frac{d\sigma}{d\Omega}\Big|_{[\vec{Q}\to 0]} \quad\to\quad \frac{k}{k'}\left(\frac{\mu}{4\pi^2\hbar^2}\right)^2 \Big|2F(\theta)\Big(\frac{\mathcal{M}^{\mathrm{DGT}}_{\mathrm{T}\to\mathrm{T}'}\,\mathcal{M}^{\mathrm{DGT}}_{\mathrm{P}\to\mathrm{P}'}}{\bar{E}^{GT}_P+\bar{E}^{GT}_T} + \frac{\mathcal{M}^{\mathrm{DF}}_{\mathrm{T}\to\mathrm{T}'}\,\mathcal{M}^{\mathrm{DF}}_{\mathrm{P}\to\mathrm{P}'}}{\bar{E}^{F}_P+\bar{E}^{F}_T}\Big)\Big|^2,$$

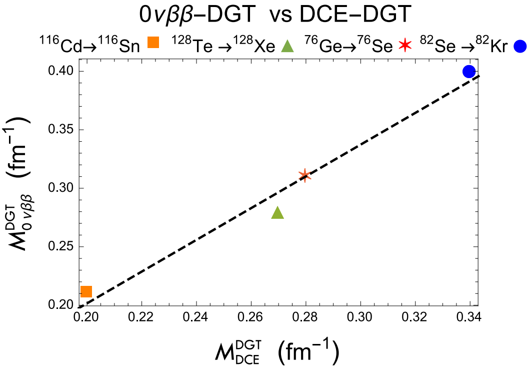
  where

$$F(\theta)\xrightarrow[Q_z\to 0]{} 2\pi\int_{-\infty}^{\infty}dz\int_0^{\infty}db\; e^{-izQ_z}\; bJ_0(kb\sin\theta)\; e^{i\chi(b)}\;.$$

  with $\vec{Q}=(\vec{Q}_t,Q_z)$ in cylindrical coordinates, and $|\vec{Q}_t|\simeq k\,\sin\theta$ and $F(\theta)$ is evaluated in the sharp-cutoff limit.

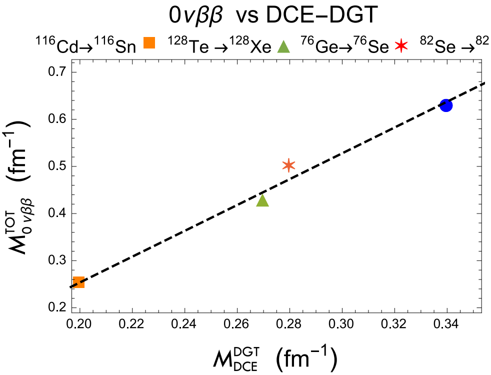
- The nuclear part of the differential cross-section is the sum of DGT and DF amplitudes, which are both factorized in terms of target and projectile NMEs.
- This will open the possibility of extracting neatly DGT and DF NMEs from DCE experimental data at $\theta = 0^\circ$.

There is linear correlation between DGT nuclear matrix elements of DCX and nuclear matrix elements of the DGT for $0\nu\beta\beta$-decay.



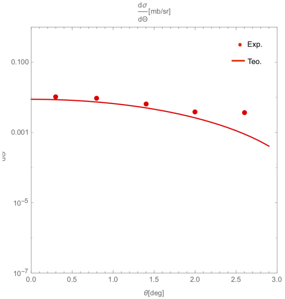
Gamow Teller Linear Correlation

There is linear correlation between total nuclear matrix elements of $0\nu\beta\beta$-decay and Double Gamow Teller in DCX.



Results for $^{40}Ca(^{18}O, ^{18}Ne)^{40}Ar$

- We computed the 40Ca(18O, 18Ne)40Ar DCE cross-section at $\theta = 0$.
- The results for θ different from zero are preliminary. at $\theta = 0$ we obtain a value of 8.9μb/sr inside the experimental range, (8.0 - 10.5)μb/sr (Eur. Phys J.A (2015) 51 145).



Conclusions and outlooks

- We have presented the formalism for calculating the differential heavy-ion DCE cross-sections in the eikonal approximation at very forward angles.
- The DCE differential cross-section can be factorized into a nuclear part and a reaction factor, where the latter is computed by means low momentum transfer.
- The factorizing at very forward differential DCE-cross-section, and the existence of a linear correlation between the DCE-DGT and $0\nu\beta\beta$ NMEs, opens the possibility to place constraints on neutrinoless double-beta-decay NMEs in terms of the DCE experimental data at $\theta = 0^\circ$.

Current progress

- The nuclear reaction part can be improved by using a more general DWBA for double charge exchange without eikonal approximation developed by Colonna-Lenske.
- Currently we are collaborating with them on transfer reactions and form factors.