An *ab-initio* multiphonon approach to low- and high- energy nuclear spectroscopy

G. De Gregorio

- Università degli Studi della Campania "Luigi Vanvitelli"
- INFN Napoli

Università degli Studi della Campania *Luigi Vanvitelli*

INFN

In Collaboration with

F. Knapp	Charles University
N. Lo Iudice	Univ. Federico II & INFN Napoli
P. Veselý	NPI Řež
J. Pokorný	NPI Řež

Cortona 2019

Low-lying Nuclear spectroscopy approach: The Nuclear Shell Model

Infinite Space, **A** nucleons

$$H\psi_\alpha = E_\alpha\psi_\alpha$$

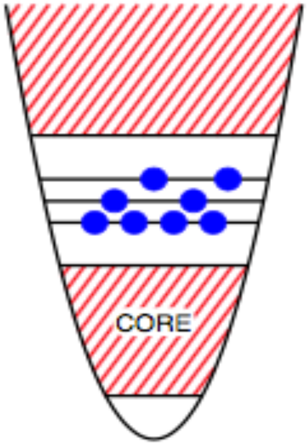
Model Space, v nucleons

$$H_{eff}\phi_\alpha = (T+V_{eff})\,\phi_\alpha = E_\alpha\phi_\alpha$$

The excitations of the core are absorbed by the effective hamiltonian

$$V_{eff} = V + V\frac{Q}{E-H_0}V_{eff}$$

Problem :size of the Hamiltonian matrix too large as v increases



SM describes only few low-lying states.
It cannot describe most of the collective states.

^{116}Sn, $N \sim 10^7$

^{130}Xe, $N \sim 10^9$

^{128}Xe, $N \sim 10^{10}$

Theoretical approaches to collective motions

Semiclassical

$\alpha \qquad \pi$

$$[H, O_\lambda^\dagger] = \hbar\omega O_\lambda^\dagger$$

mapping

Harmonic apprixmation

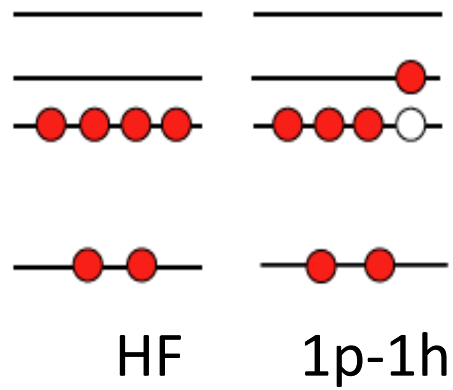
$$O_\lambda|0\rangle = 0$$

$$[H, O_\lambda^\dagger]|0\rangle = \omega_\lambda O_\lambda^\dagger|0\rangle = (E_\lambda - E_0) O_\lambda^\dagger|0\rangle$$

TDA

$$|\lambda\rangle = O_\lambda^\dagger |HF\rangle$$

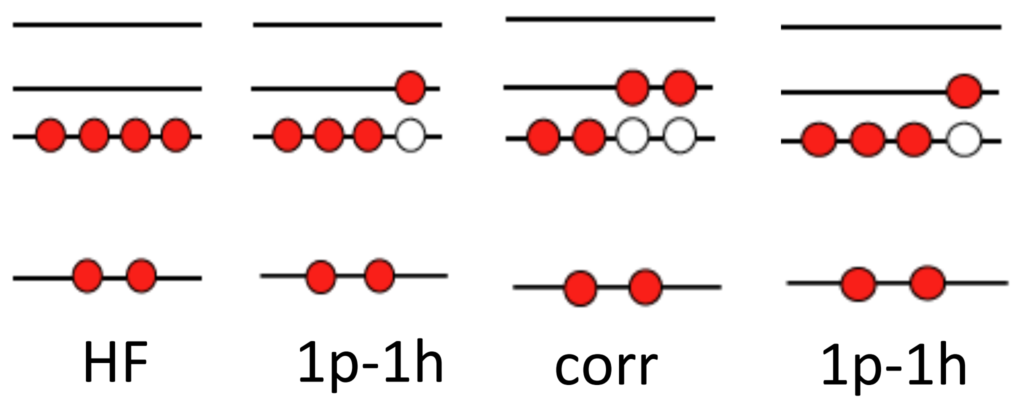
$$O_\lambda^\dagger = \Sigma_{ph}\, c_{ph}(\lambda)\, a^\dagger_p\, a_h$$



RPA

$$|\lambda\rangle = O_\lambda^\dagger |0_{corr}\rangle$$

$$O_\lambda^\dagger = \Sigma_{ph}\, [X_{ph}(\lambda)\, a^\dagger_p\, a_h - Y_{ph}(\lambda)\, a^\dagger_h\, a_p]$$



Beyond mean field: *Adopted Methods*

Non relativistic

- qp-Phonon

P. F. Bortignon et al. (Milano group)

- 2nd RPA

R. Roth et al.

D. Gambacurta et al.

Phenomenological

- QPM *(Soloviev School (Dubna))*

Relativistic: RTBA

E. Litvinova, P. Ring, D. Vretenar…..

Microscopic unified description: The Equation of Motion Phonon Method (EMPM)

Goal: Generate a multiphonon basis starting from TDA phonons

$$O_\lambda^\dagger = \Sigma_{ph}\, c_{ph}(\lambda)\, a^\dagger_p\, a_h$$

Microscopic unified description:
The Equation of Motion Phonon Method (EMPM)

Goal: Generate a multiphonon basis starting from TDA phonons

$$O_\lambda^\dagger = \Sigma_{ph}\, c_{ph}(\lambda)\, a^\dagger_p\, a_h$$

Eigenvalue problem

$$H \mid \Psi_\nu > = E_\nu \mid \Psi_\nu >$$

$| \Psi_\nu > \ni \mathcal{H} = \Sigma_n \oplus \mathcal{H}_n$ $\mathcal{H}_n \in |n; \beta> \equiv$ n-phonon **basis states**

Microscopic unified description: The Equation of Motion Phonon Method (EMPM)

Goal: Generate a multiphonon basis starting from TDA phonons

$$O_\lambda^\dagger = \Sigma_{ph}\, c_{ph}(\lambda)\, a^\dagger_p\, a_h$$

Eigenvalue problem

$$H \,|\, \Psi_\nu > = E_\nu \,|\, \Psi_\nu >$$

$| \Psi_\nu > \ni \mathcal{H} = \Sigma_n \oplus \mathcal{H}_n$

$\mathcal{H}_n \in |n; \beta> \equiv$ n-phonon **basis states**

An obvious, but unmanageable, multiphonon basis

$$|\lambda_1, \ldots, \lambda_i, \ldots \lambda_n> \;=\; O^\dagger_{\lambda_1} \ldots O^\dagger_{\lambda_i} \ldots O^\dagger_{\lambda_n} |0>$$

Microscopic unified description:
The Equation of Motion Phonon Method (EMPM)

Goal: Generate a multiphonon basis starting from TDA phonons

$$O_\lambda^\dagger = \Sigma_{ph}\, c_{ph}(\lambda)\, a^\dagger_p\, a_h$$

Eigenvalue problem

$$H \,|\, \Psi_v > = E_v \,|\, \Psi_v >$$

$| \Psi_v > \ni \mathcal{H} = \Sigma_n \oplus \mathcal{H}_n$ $\qquad \mathcal{H}_n \in |n; \beta> \equiv$ n-phonon **basis states**

An obvious, but unmanageable, multiphonon basis

$$|\lambda_1,\ldots,\lambda_i,\ldots\lambda_n> \;=\; O^\dagger_{\lambda_1} \ldots O^\dagger_{\lambda_i} \ldots O^\dagger_{\lambda_n} |0>$$

A viable route

$$|\beta> \;=\; \Sigma_{\lambda\alpha}\, C^\beta_{\lambda\alpha}\, O^\dagger_\lambda \,|\, \alpha >$$

Construction of |β> : EoM

Assuming |α> **known,** we solve the **Eq. of Motion**

$$\left\langle \beta \left| \left[H, O_\lambda^\dagger \right] \right| \alpha \right\rangle = \left(E_\beta - E_\alpha \right) \left\langle \beta \left| O_\lambda^\dagger \right| \alpha \right\rangle$$

X

Construction of |β> : EoM

Assuming |α> **known,** we solve the **Eq. of Motion**

$$\left\langle \beta \middle| \left[H, O_\lambda^\dagger \right] \middle| \alpha \right\rangle = \left(E_\beta - E_\alpha \right) \left\langle \beta \middle| O_\lambda^\dagger \middle| \alpha \right\rangle$$

⇓

$$\mathcal{A}\, X = \mathcal{E}\, X$$

Construction of |β> : EoM

Assuming $|\alpha>$ **known,** we solve the **Eq. of Motion**

$$\left\langle \beta \left| \left[H, O_\lambda^\dagger \right] \right| \alpha \right\rangle = \left(E_\beta - E_\alpha \right) \left\langle \beta \left| O_\lambda^\dagger \right| \alpha \right\rangle$$

$$\mathcal{A} \, X = \mathcal{E} \, X$$

X

Where

$$\mathcal{A}_{(\lambda\alpha)(\lambda'\gamma)} = (E_\lambda + E_\alpha)\delta_{\alpha\gamma}\delta_{\lambda\lambda'} + \mathcal{V}_{(\lambda\alpha)(\lambda'\gamma)}$$

Construction of |β> : EoM

Assuming |α> **known,** we solve the **Eq. of Motion**

$$\left\langle \beta \left| \left[H, O_\lambda^\dagger \right] \right| \alpha \right\rangle = \left(E_\beta - E_\alpha \right) \left\langle \beta \left| O_\lambda^\dagger \right| \alpha \right\rangle$$

$$\mathcal{A}\, X = \mathcal{E}\, X$$

X

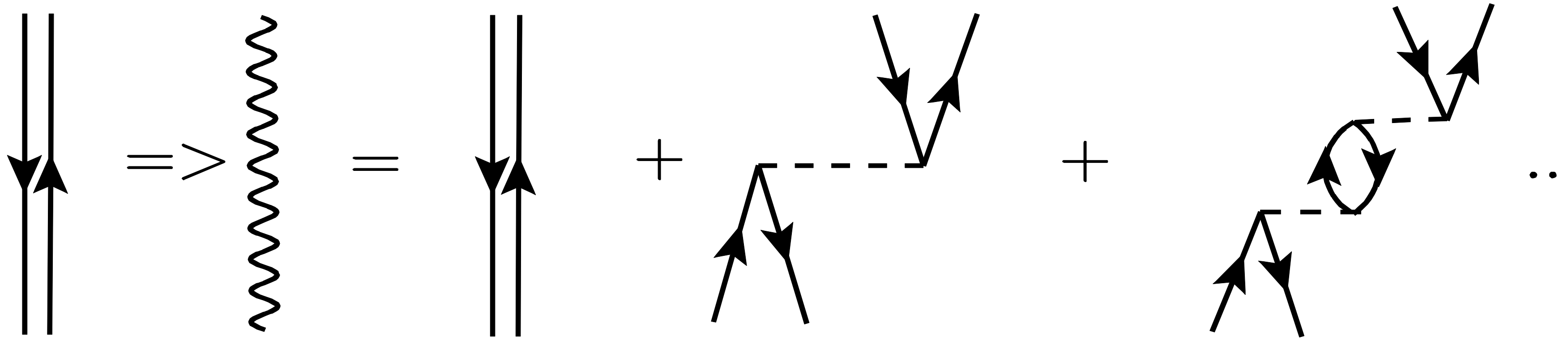
Where

$$\mathcal{A}_{(\lambda\alpha)(\lambda'\gamma)} = (E_\lambda + E_\alpha)\delta_{\alpha\gamma}\delta_{\lambda\lambda'} + \mathcal{V}_{(\lambda\alpha)(\lambda'\gamma)}$$

TDA matrix

$$A_{(ph)(p'h')} = (\varepsilon_p - \varepsilon_h)\,\delta_{pp'}\,\delta_{hh'} + V_{ph'hp'}$$

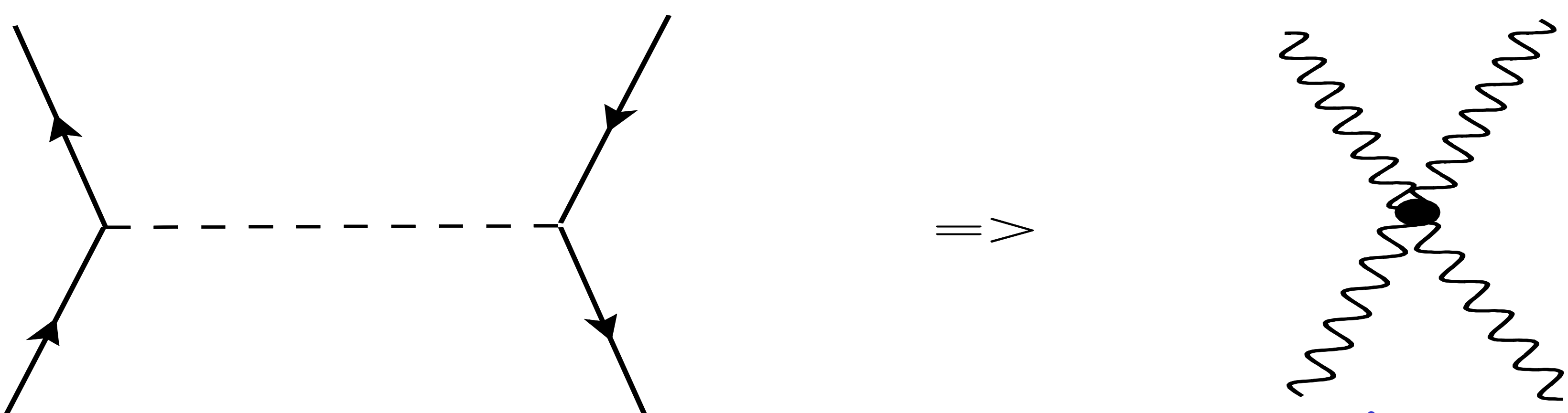
From p-h to TDA



$\varepsilon_p - \varepsilon_h \Longrightarrow E_\beta$

$|p\,h\rangle \Rightarrow |\beta\rangle = \Sigma_{ph}\, c^{\beta}_{ph}\, a^{\dagger}_{p}\, a_{h}\, |\rangle$

From TDA to EMPM



$V_{ph'hp'} \Longrightarrow \mathcal{V}_{(\lambda\alpha)(\lambda'\gamma)}$

Construction of $|\beta\rangle$: EoM

Problem

$$\mathcal{A}\,X = \mathcal{E}\,X$$

is **not** a true Eigenvalue Eq.!

Construction of |β> : EoM

Problem

$$\mathcal{A}\,X = \mathcal{E}\,X$$

is **not** a true Eigenvalue Eq.!

$\{O^\dagger_\lambda\ |\alpha>\}$ form a **non-orthogonal redundant** basis

$$\Downarrow$$

$X = \langle\beta\,|\,O^\dagger_\lambda\,|\alpha\rangle$ is **not** a **true** expansion coefficient

Recipe for solving the problem

1° step

$$\mathcal{A}\,\mathcal{X} = \mathcal{E}\,\mathcal{X}$$

$$\mathcal{X} = <\beta|\mathbf{O}^{\dagger}_{\lambda}\;|\alpha> = <\beta\,|\,\lambda\,\alpha>$$

Recipe for solving the problem

1° step

$$\mathcal{A}\,\mathcal{X} = \mathcal{E}\,\mathcal{X}$$

$$\mathcal{X} = \langle \beta | O^{\dagger}_{\lambda} | \alpha \rangle = \langle \beta | \lambda\, \alpha \rangle$$

Expressing $\mathcal{X}$ in terms of $\mathcal{C}$

$$\mathcal{X} = \mathcal{D}\,\mathcal{C}$$

Where $\mathcal{D}$ is the metric matrix

$$\mathcal{D} \equiv \{ \langle \lambda' \alpha' | \lambda \alpha \rangle \}$$

Recipe for solving the problem

1° step

$$\mathcal{A}\,\mathcal{X} = \mathcal{E}\,\mathcal{X}$$

$$\mathcal{X} = <\beta|\mathbf{O}^{\dagger}_{\lambda}\,|\alpha> = <\beta\,|\,\lambda\,\alpha>$$

Expressing $\mathcal{X}$ in terms of $\mathcal{C}$

$$\mathcal{X} = \mathcal{D}\,\mathcal{C}$$

Where $\mathcal{D}$ is the metric matrix

$$\mathcal{D} \equiv \{<\lambda'\alpha'\,|\,\lambda\alpha>\}$$

$$[\mathcal{H} - \mathcal{E}\mathcal{D}]\,\mathbf{C} = 0$$

where

$$\mathcal{H} = \mathcal{A}\mathcal{D}$$

Recipe for solving the problem

1° step

$$\mathcal{A}\,\mathcal{X} = \mathcal{E}\,\mathcal{X}$$

$$\mathcal{X} = \langle \beta | \mathbf{O}^{\dagger}_{\lambda} | \alpha \rangle = \langle \beta | \lambda\, \alpha \rangle$$

Expressing $\mathcal{X}$ in terms of $\mathcal{C}$

$$\mathcal{X} = \mathcal{D}\,\mathcal{C}$$

Where $\mathcal{D}$ is the metric matrix

$$\mathcal{D} \equiv \{\langle \lambda' \alpha' | \lambda\, \alpha \rangle\}$$

$$[\mathcal{H} - \mathcal{E}\mathcal{D}]\,\mathbf{C} = 0$$

where $\mathcal{H} = \mathcal{A}\mathcal{D}$ *But* $\mathcal{D}$ is **singular !**

2° conclusive step: Choleski

$$[\mathcal{H} - \mathcal{E}\mathcal{D}]\,\mathbf{C} = 0$$

$$\mathcal{H} = \mathcal{A}\mathcal{D}$$

$$\mathcal{D} \equiv \{< \lambda'\alpha' \mid \lambda\alpha >\}$$

2° conclusive step: Choleski

$$[\mathcal{H} - \mathcal{E}\mathcal{D}]\,\mathbf{C} = 0$$

$$\mathcal{H} = \mathcal{A}\mathcal{D}$$

Cholesky

$$\mathcal{D} \rightarrow \mathbf{D}$$

$$\mathcal{D} \equiv \{<\lambda'\alpha' \mid \lambda\alpha>\}$$

2° conclusive step: Choleski

$$[\mathcal{H} - \mathcal{E}\mathcal{D}]\,\mathbf{C} = 0$$

$$\mathcal{H} = \mathcal{A}\mathcal{D}$$

Cholesky ⇓

$$\mathcal{D} \rightarrow \mathbf{D}$$

$$\mathcal{D} \equiv \{<\lambda'\alpha' \mid \lambda\alpha>\}$$

$$[\mathbf{H}^n - \mathbf{E}^n]\,\mathbf{C}^n = 0$$

$$\mathbf{H} = \mathbf{D}^{-1}\mathcal{A}\mathcal{D}$$

2° conclusive step: Choleski

$$[\mathcal{H} - \mathcal{E}\mathcal{D}]\, \mathbf{C} = 0$$

$$\mathcal{H} = \mathcal{A}\mathcal{D}$$

Cholesky

$$\mathcal{D} \rightarrow \mathbf{D}$$

$$\mathcal{D} \equiv \{ <\lambda'\alpha' \mid \lambda\alpha> \}$$

$$[\mathbf{H}^n - \mathbf{E}^n]\, \mathbf{C}^n = 0$$

$$\mathbf{H} = \mathbf{D}^{-1}\mathcal{A}\mathcal{D}$$

$$|\beta\rangle = \sum_{\lambda\alpha} C^{\beta}_{\lambda\alpha} \left| (\lambda \times \alpha)^{\beta} \right\rangle = \sum_{\lambda\alpha} C^{\beta}_{\lambda\alpha} \left(O^{\dagger}_{\lambda} \, |\alpha\rangle \right)^{\beta}$$

$$<\alpha_n \mid \beta_{n'}> = \delta_{nn'}\ \delta_{\alpha\beta}$$

Eigenvalue problem in the Multiphonon basis $\{|\alpha_n>\}$

$$\sum_{n'\beta_{n'}} \mathrm{H}_{\alpha_n\beta_{n'}} C^{\nu}_{\beta_{n'}} = \left[\left(E_{\alpha_n} - \varepsilon_\nu\right)\delta_{nn'}\delta_{\alpha_n\beta_{n'}} + V_{\alpha_n\beta_{n'}}\right] C^{\nu}_{\beta_{n'}} = 0$$

$$\mathrm{H}_{\alpha_n\beta_{n'}} = \begin{pmatrix} E^{(0)} & H_{01} & H_{02} & 0 \\ H_{01} & \begin{matrix} E_1^{(1)} & & 0 \\ & E_2^{(1)} & \\ & & \ldots \\ 0 & & E_{n1}^{(1)} \end{matrix} & H_{12} & H_{13} \\ H_{20} & H_{21} & \begin{matrix} E_1^{(2)} & & 0 \\ & E_2^{(2)} & \\ & & \ldots \\ 0 & & E_{n2}^{(2)} \end{matrix} & H_{23} \\ 0 & H_{31} & H_{32} & \begin{matrix} E_1^{(3)} \\ \ldots \end{matrix} \end{pmatrix}$$

Final solution

$$\left|\Psi_\nu\right\rangle = \sum_{\alpha_n} C^{\nu}_{\alpha_n} \left|\alpha_n\right\rangle$$

$$\left|\alpha_n\right\rangle = \sum_{\lambda\alpha_{n-1}} C^{\alpha_n}_{\lambda\alpha_{n-1}} \left|\left(\lambda \times \alpha_{n-1}\right)^{\alpha_n}\right\rangle$$

Implementation for even nuclei

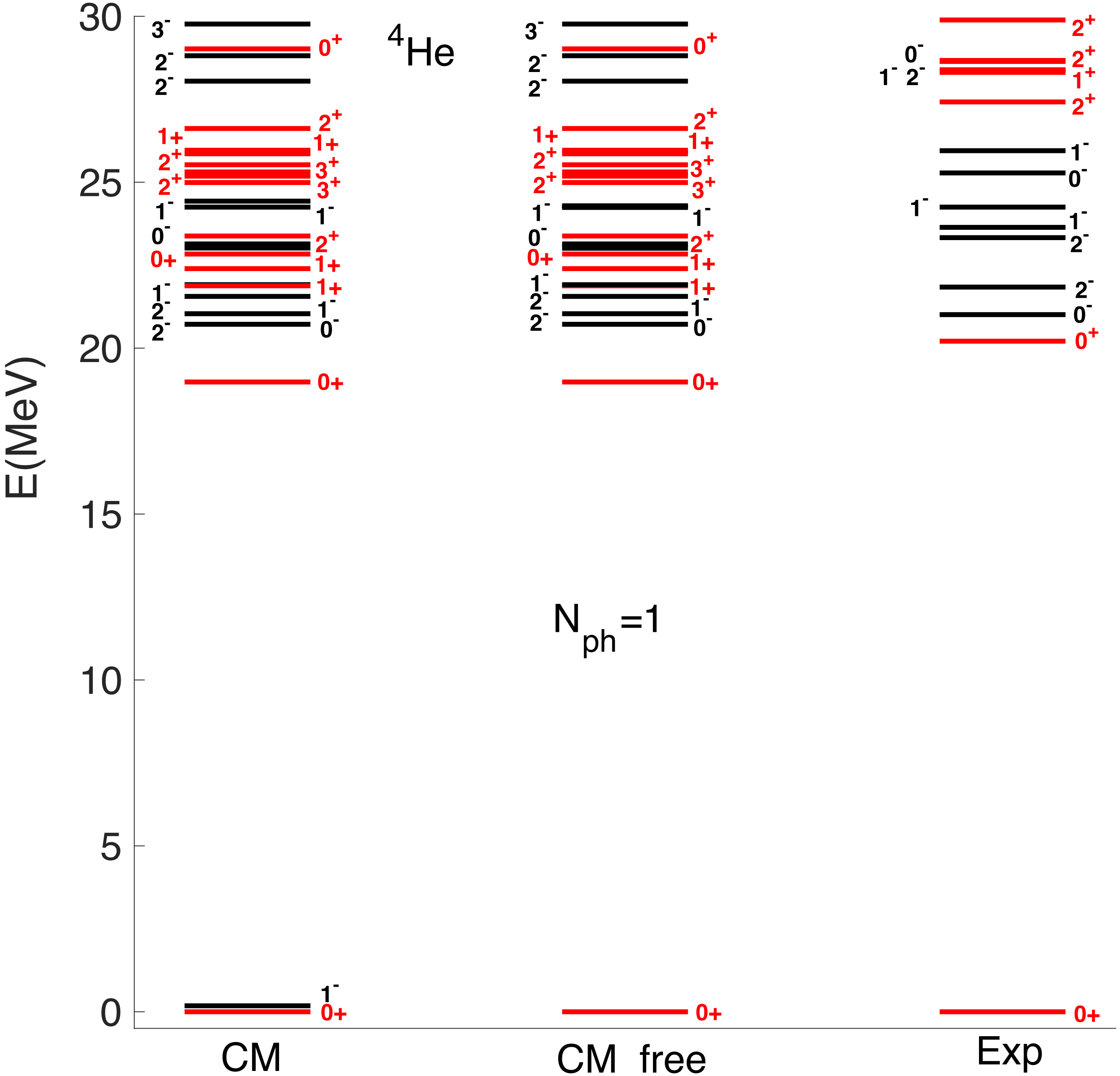
Hamiltonian

$$H = T_{int} + V \qquad V = NNLO_{sat}$$

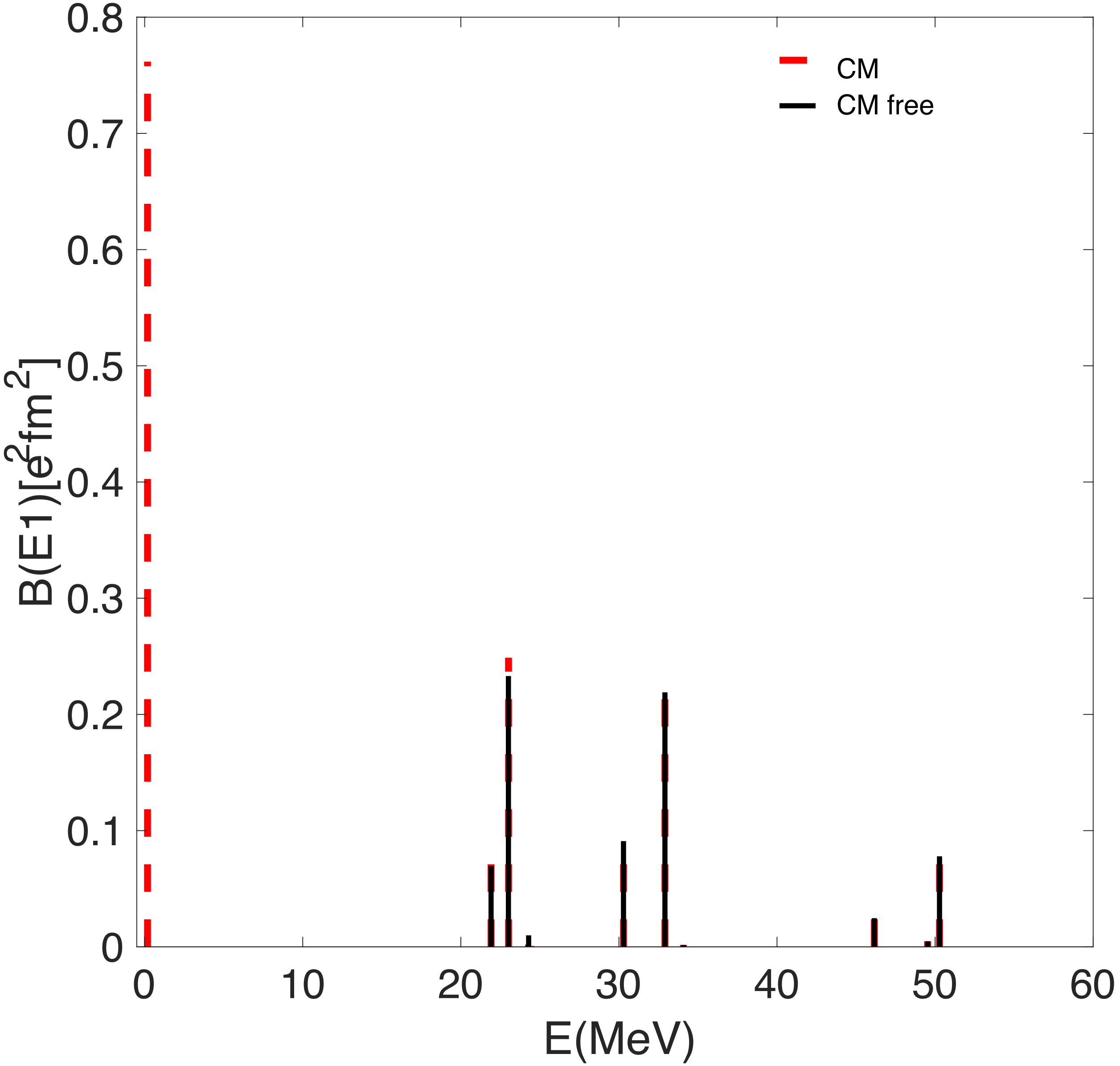
A. Ekstrom *et al*., PRL **110**, 192502 (2015)

- Perform **HF**
- Construct **TDA** phonons (**free** of **CM** spurious admixtures)
- Generate the multiphonon core basis $\{|\alpha_n>\}$ (**free** of **CM** spurious admixtures)
- Full diagonalization

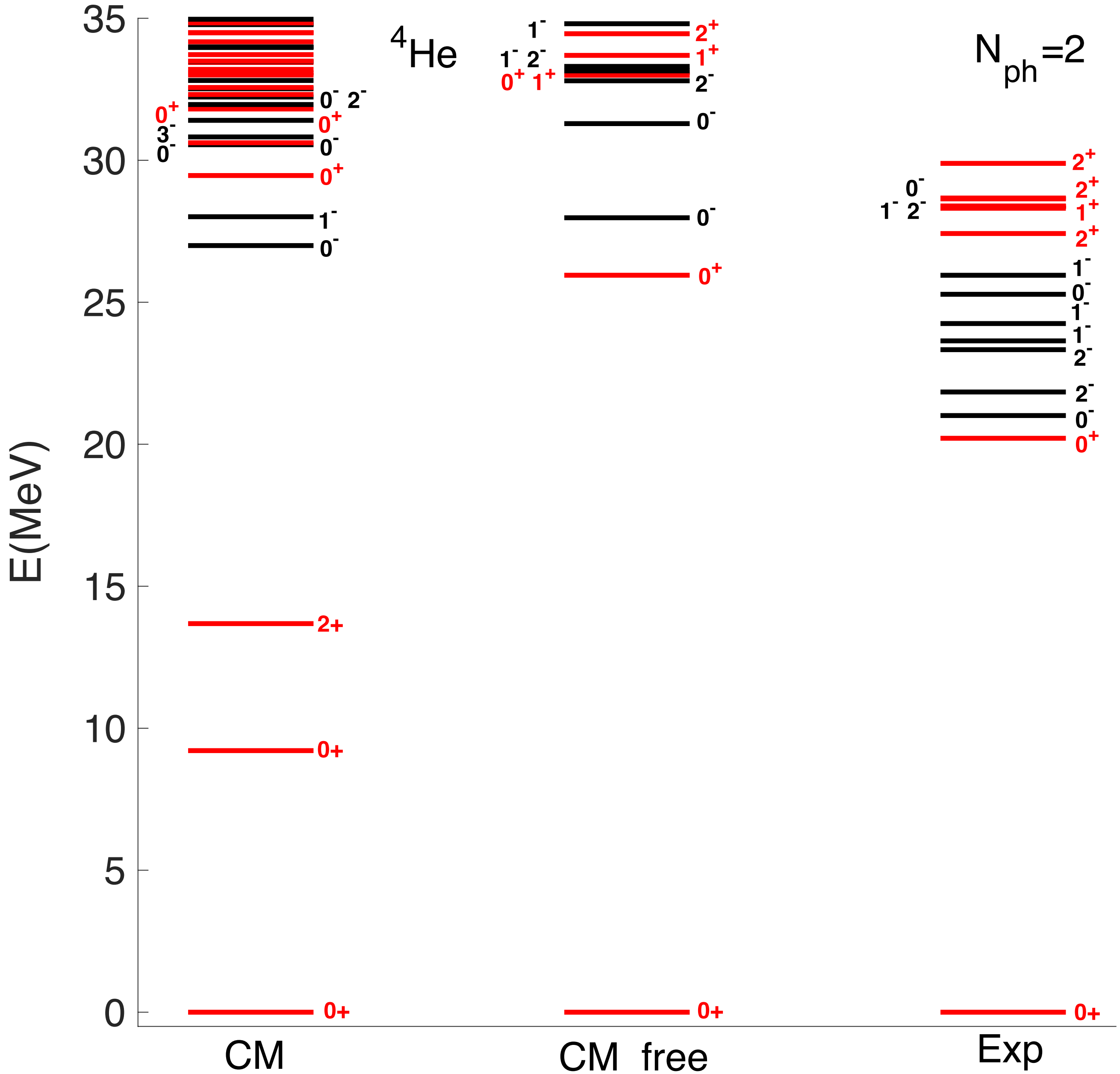
CM problem: Application to ^4He



CM problem: Application to ^{4}He



CM problem: Application to ^{4}He



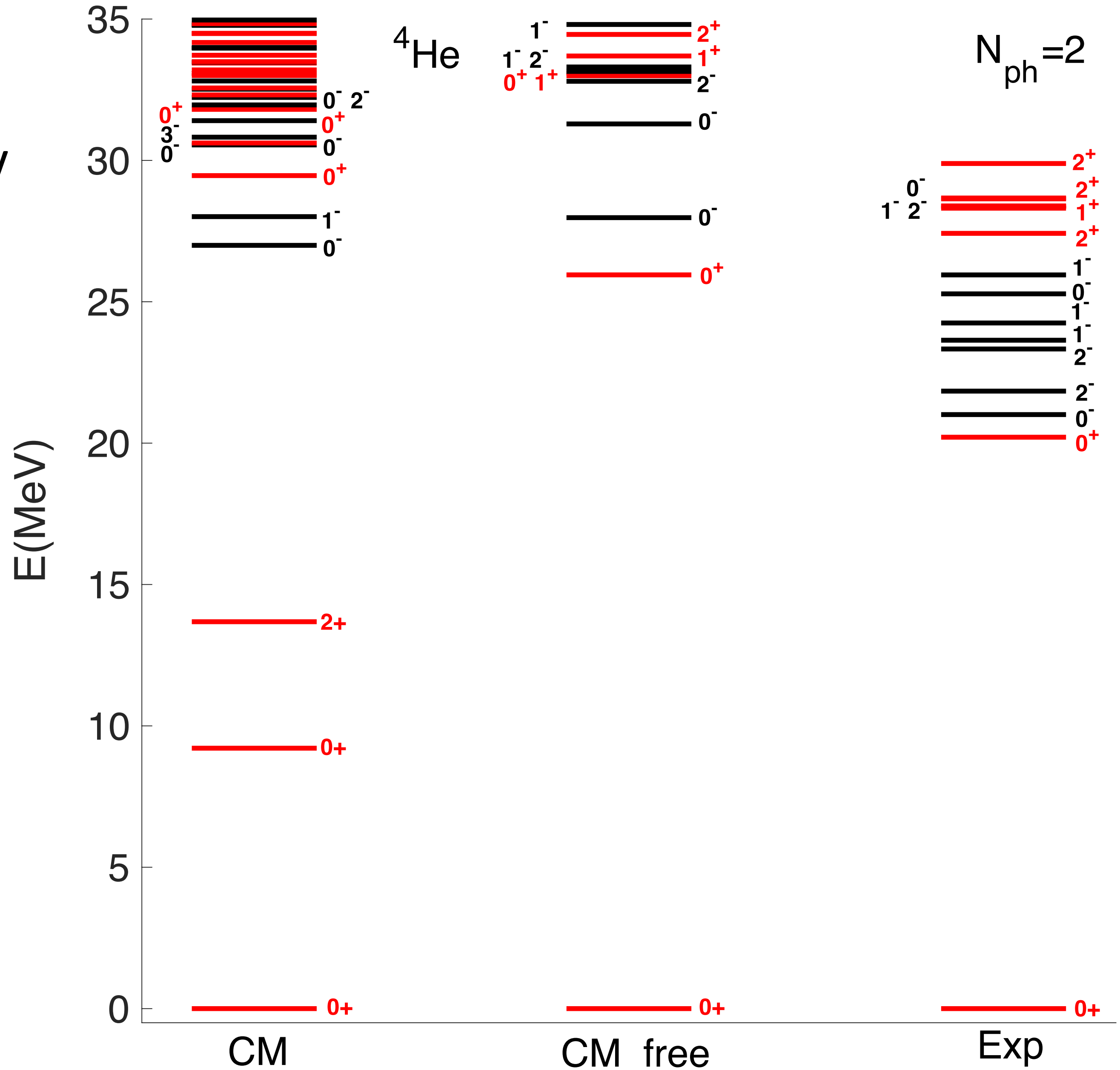
CM problem: Application to ^4He

E_{HF}=-14.3712 MeV

E_{corr}=-12.4943 MeV

B_{th}/A=6.7164 MeV

B_{exp}/A=7.073 MeV



Numerical implementation

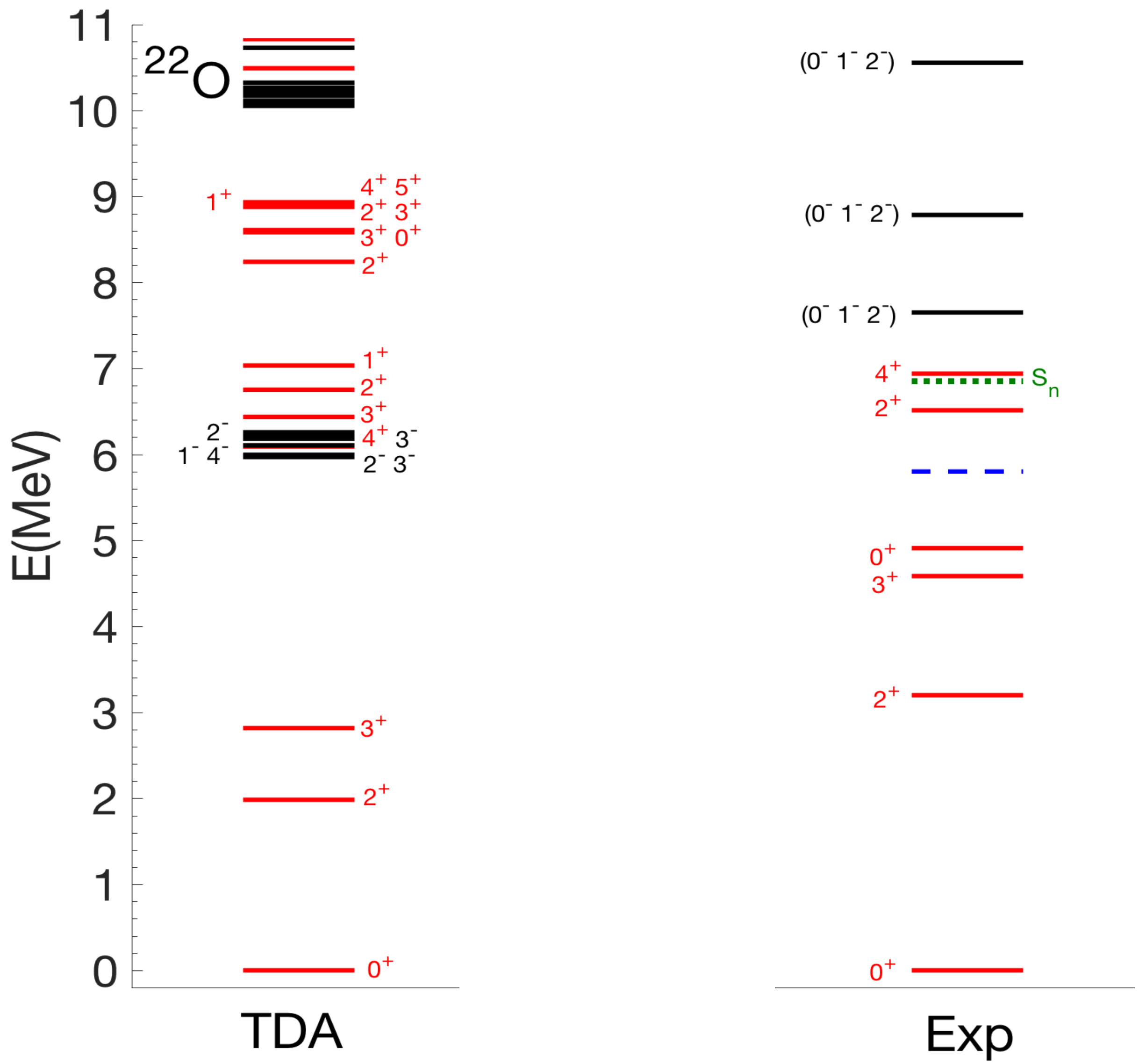
Hamiltonian

$$H = T_{int} + V_2 \qquad V_2 = \mathrm{NNLO}_{opt}$$

A. Ekstrom *et al*., PRL **110**, 192502 (2013)

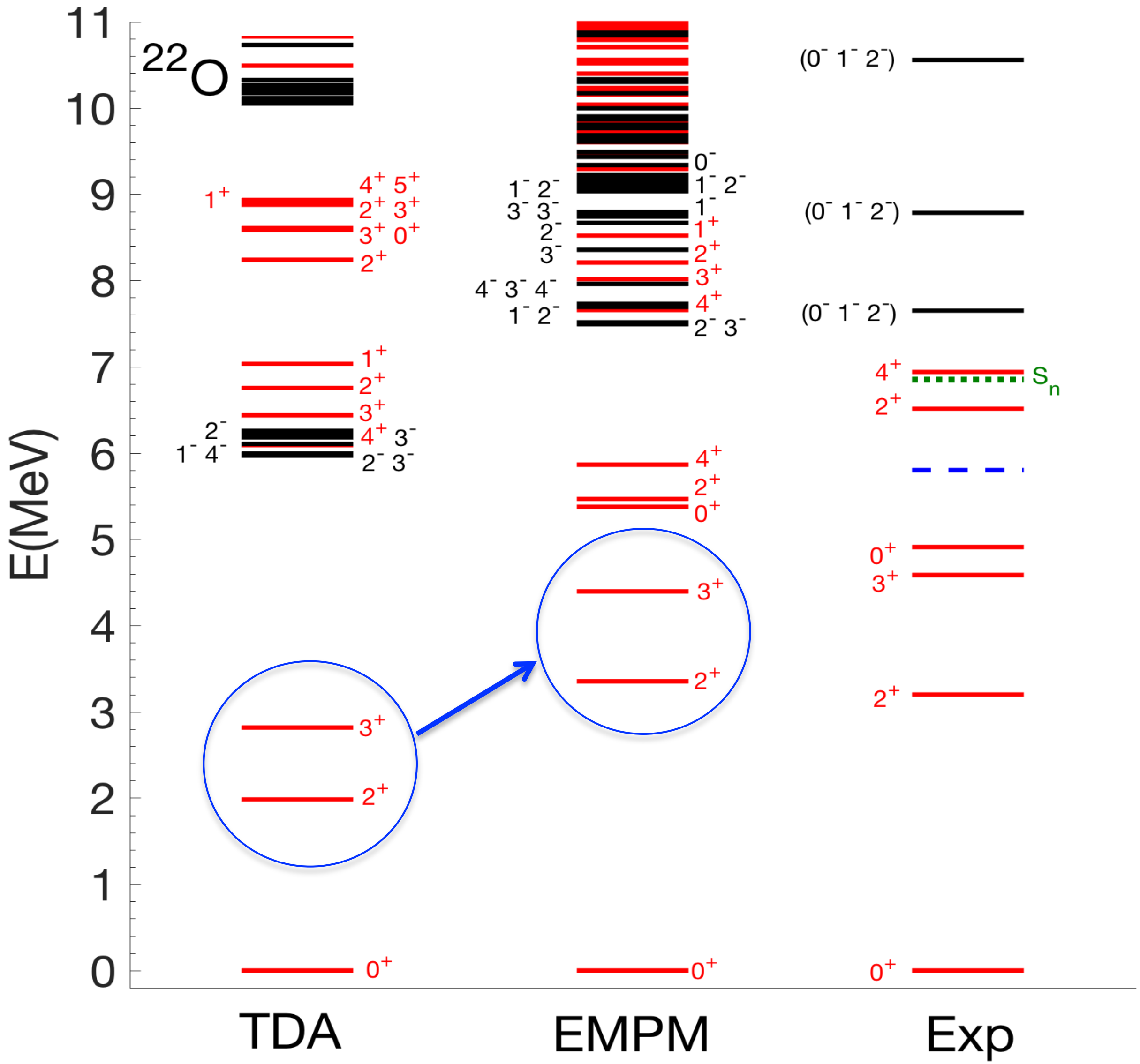
- Perform **HF**
- Construct **TDA** phonons (**free** of **CM** spurious admixtures)
- Generate the multiphonon basis
- Full diagonalization

Application to the neutron rich ^{22}O



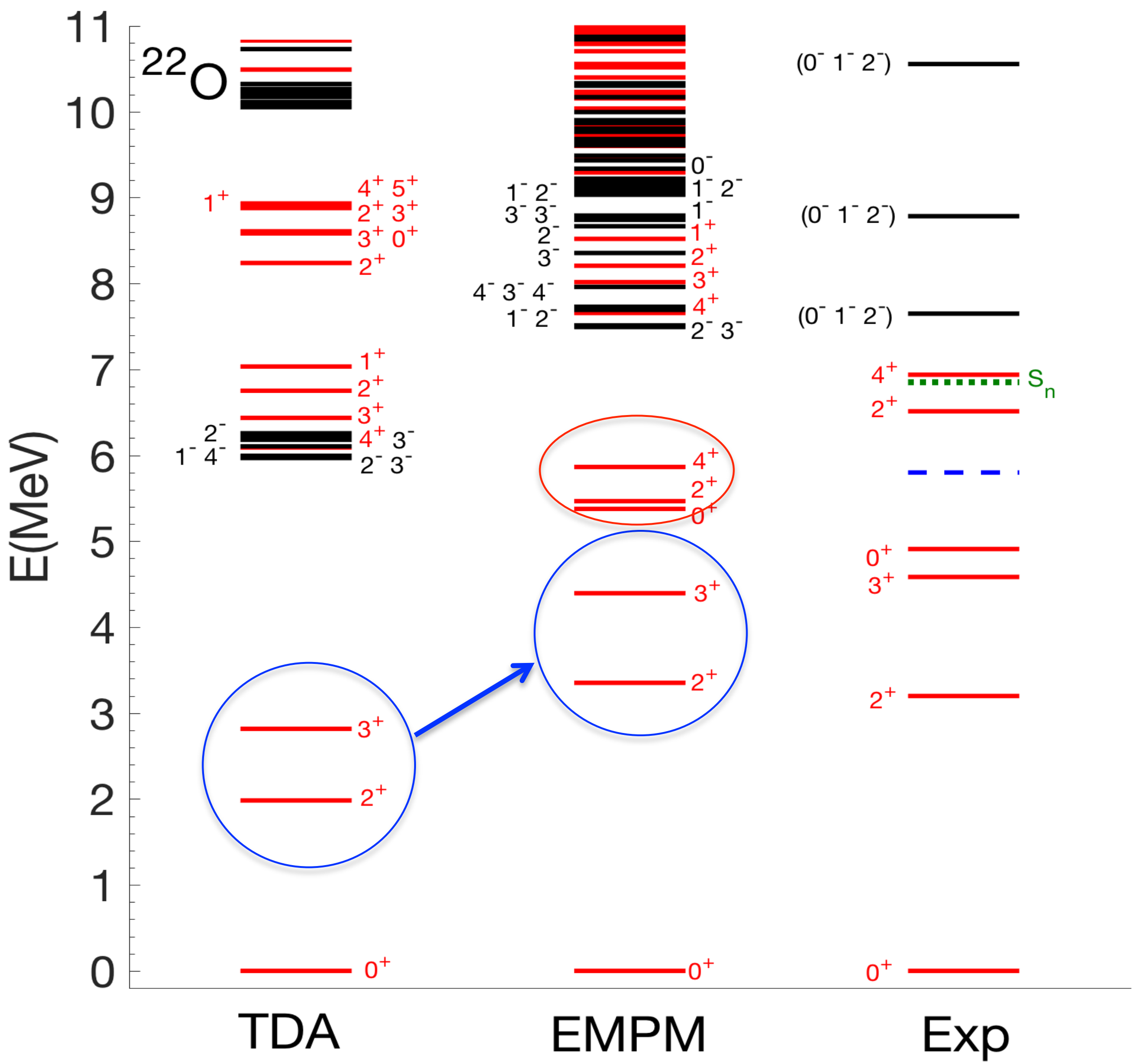
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 34311 (2018)

Application to the neutron rich ^{22}O



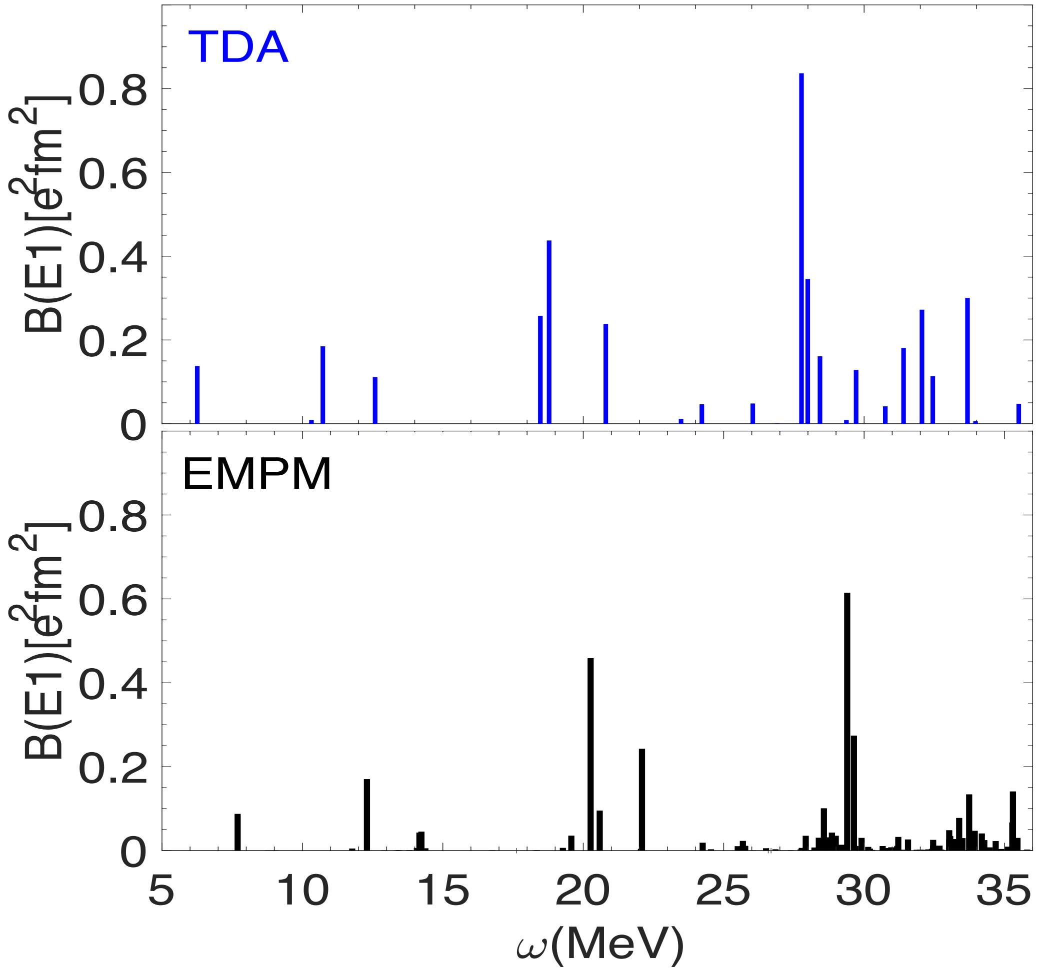
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 34311 (2018)

Application to the neutron rich ^{22}O



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 34311 (2018)

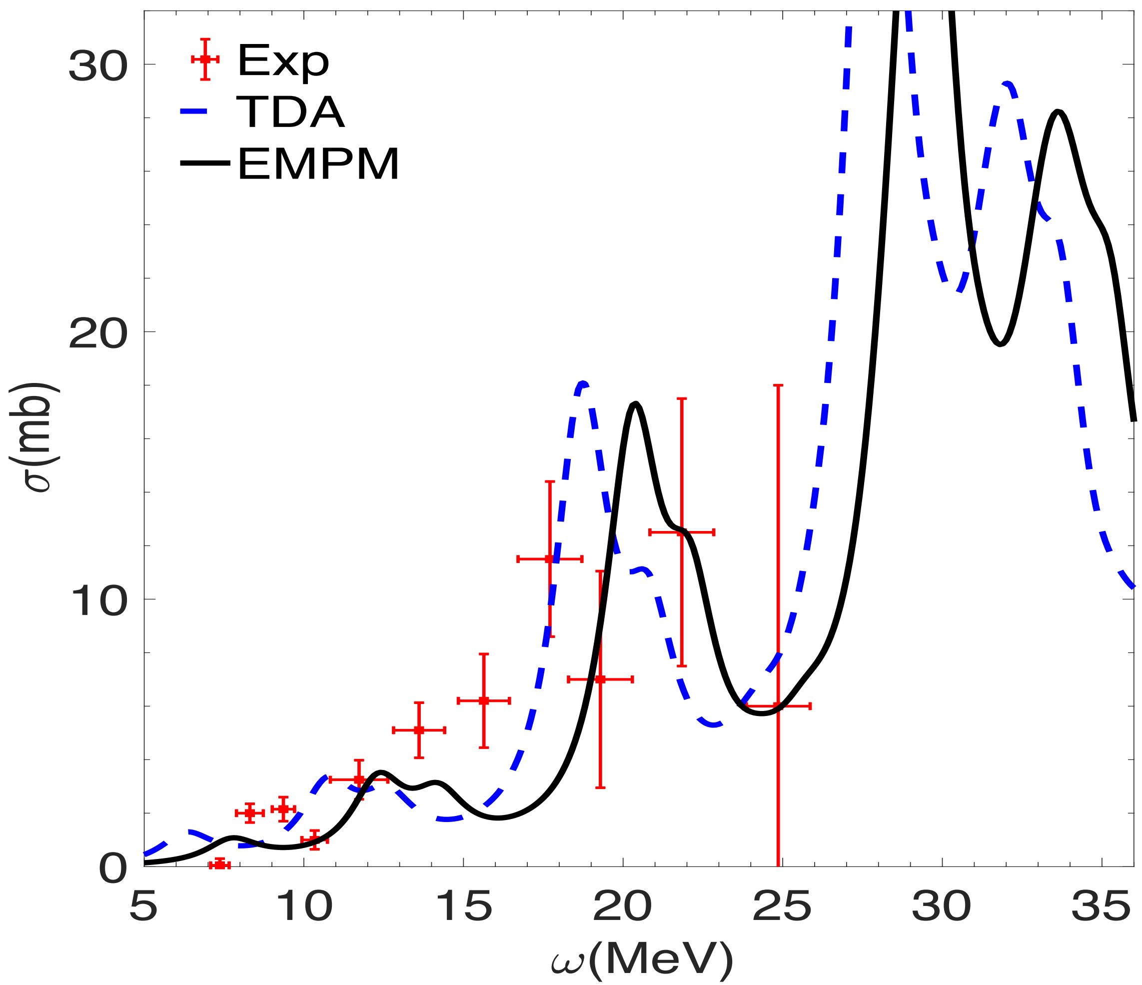
Dipole response in ^{22}O



$$B_\nu[E1](0_1^+ \rightarrow 1_\nu^-) = \left| \left\langle \Psi_{1^-}^\nu \middle\| M(E1) \middle\| \Psi_{0^+} \right\rangle \right|^2$$

G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 34311 (2018)
A. Leistenschneider, et al., Phys. Rev. Lett. 86, 5442 (2001)

Dipole response in ^{22}O

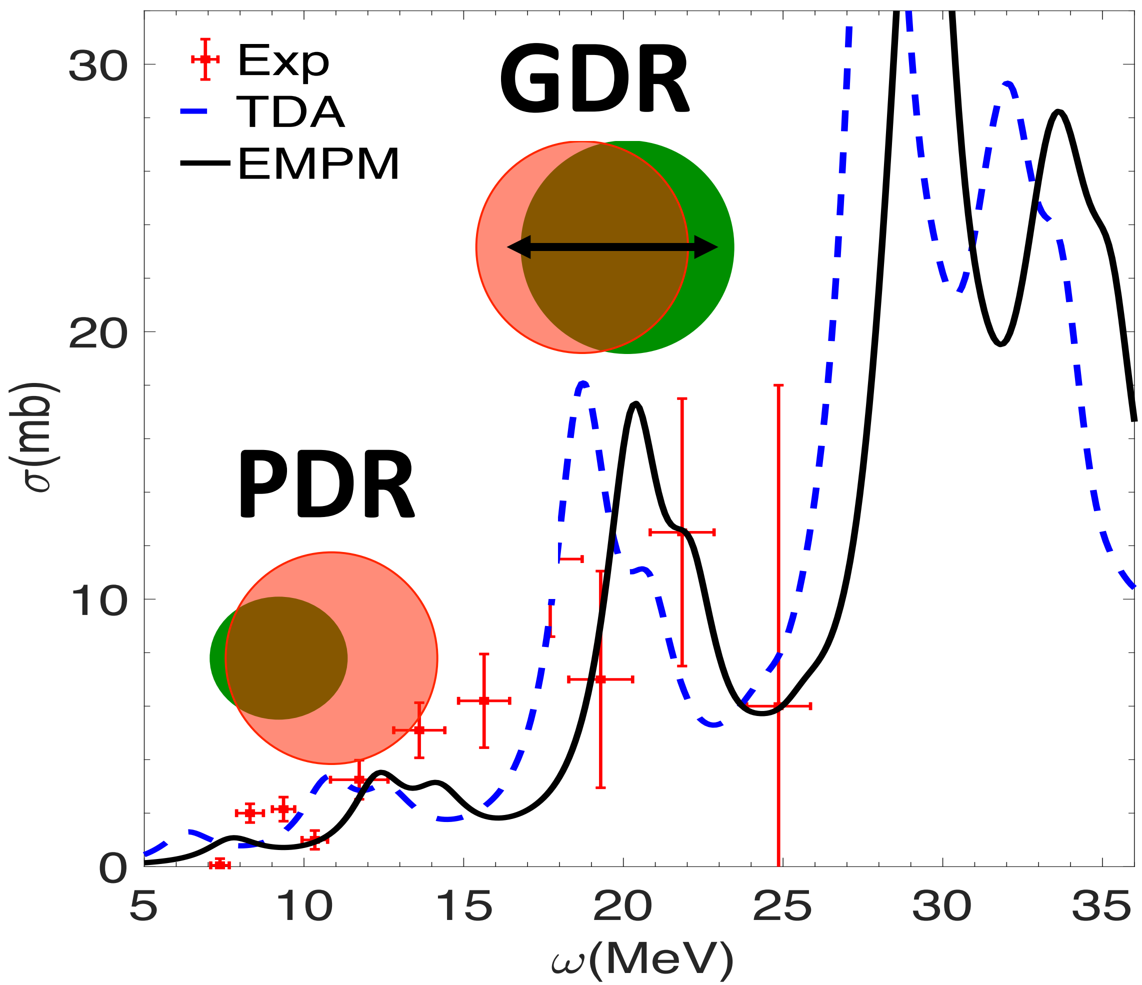


$$S(E\lambda,\omega)=\sum_{\nu}B_{\nu}(E\lambda)\delta(\omega-\omega_{\nu})\approx\sum_{\nu}B_{\nu}(E\lambda)\rho_{\Delta}(\omega-\omega_{\nu})$$

$$\sigma=\int_{0}^{\infty}\sigma(\omega)d\omega=\frac{16\pi^{3}}{9\hbar c}\int_{0}^{\infty}\omega S(E1,\omega)d\omega$$

G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 34311 (2018)
A. Leistenschneider, et al., Phys. Rev. Lett. 86, 5442 (2001)

Dipole response in ^{22}O

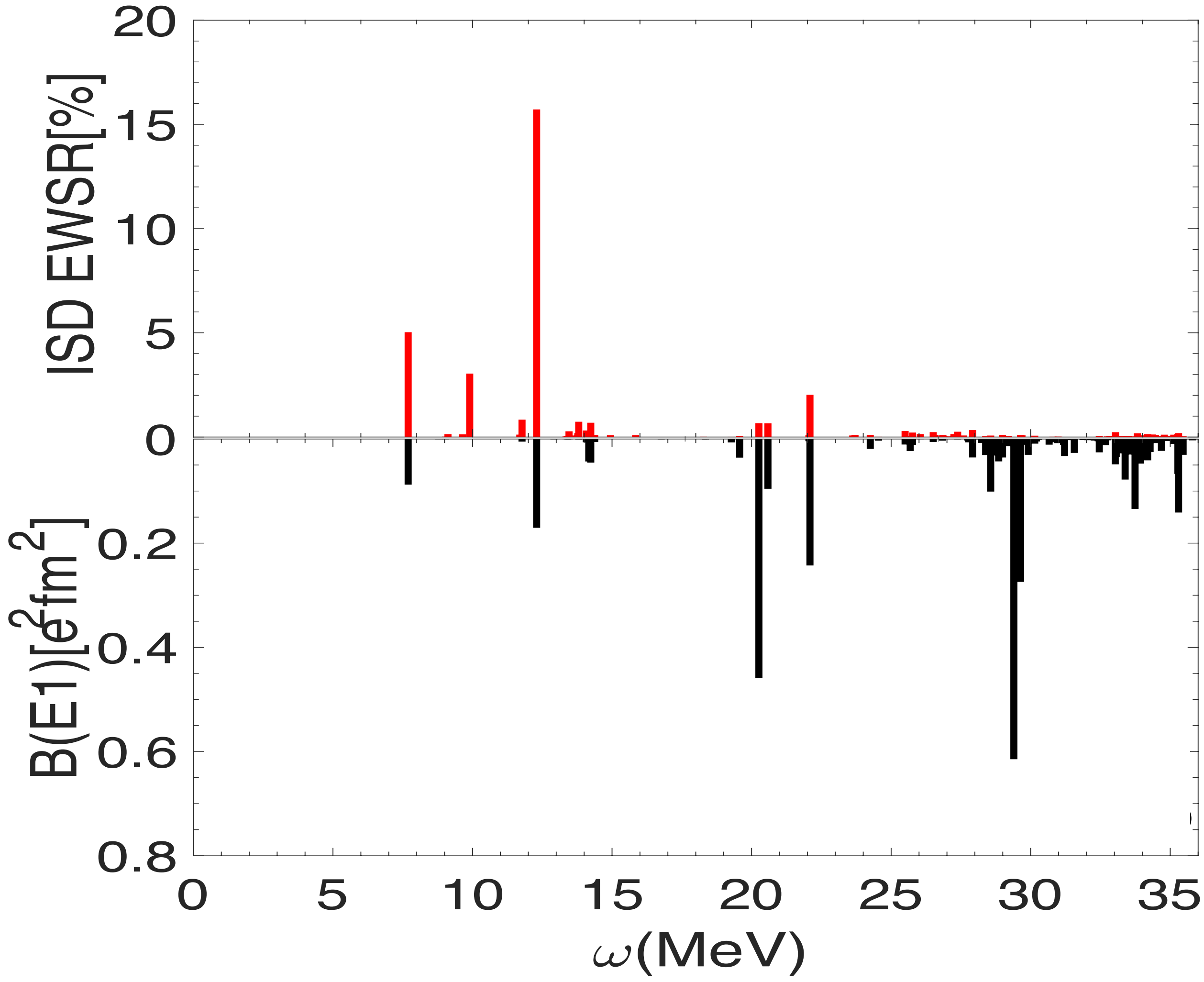


$$S(E\lambda,\omega) = \sum_{\nu} B_{\nu}(E\lambda)\delta(\omega-\omega_{\nu}) \approx \sum_{\nu} B_{\nu}(E\lambda)\rho_{\Delta}(\omega-\omega_{\nu})$$

$$\sigma = \int_0^{\infty} \sigma(\omega)d\omega = \frac{16\pi^3}{9\hbar c}\int_0^{\infty} \omega S(E1,\omega)d\omega$$

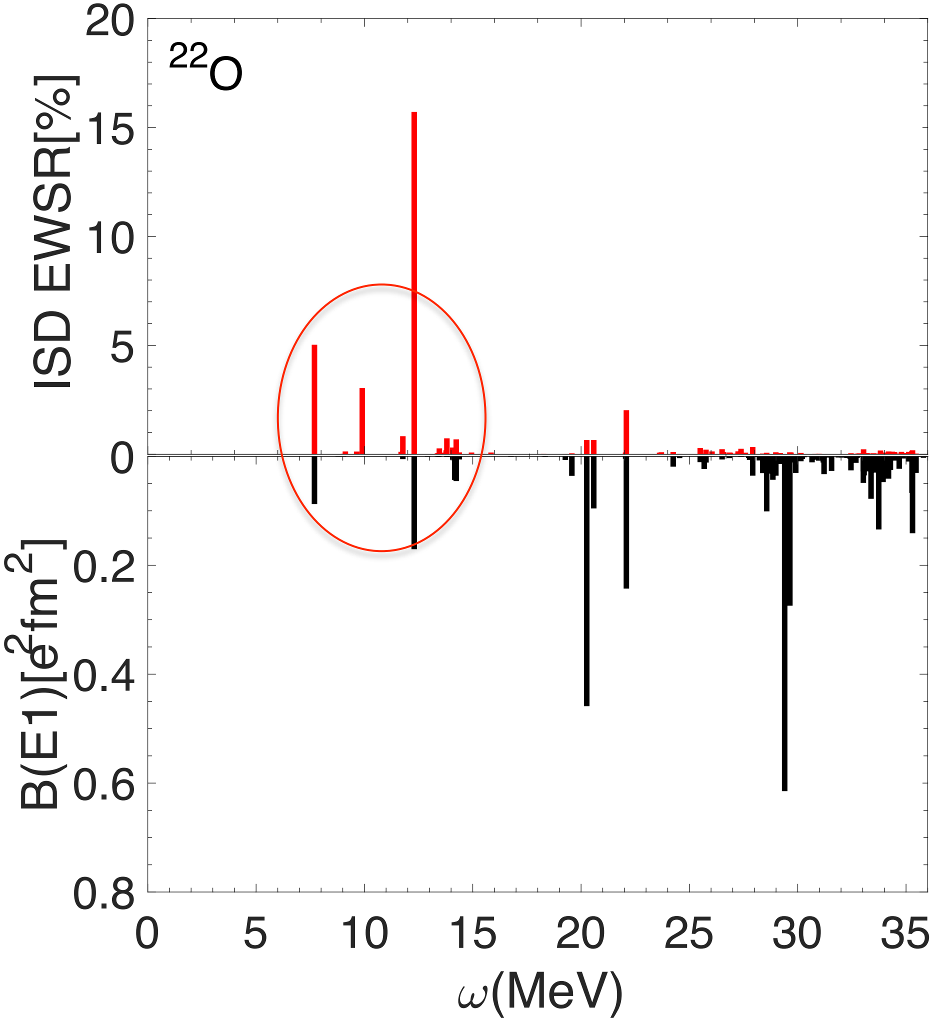
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 34311 (2018)
A. Leistenschneider, et al., Phys. Rev. Lett. 86, 5442 (2001)

Dipole response in ^{22}O



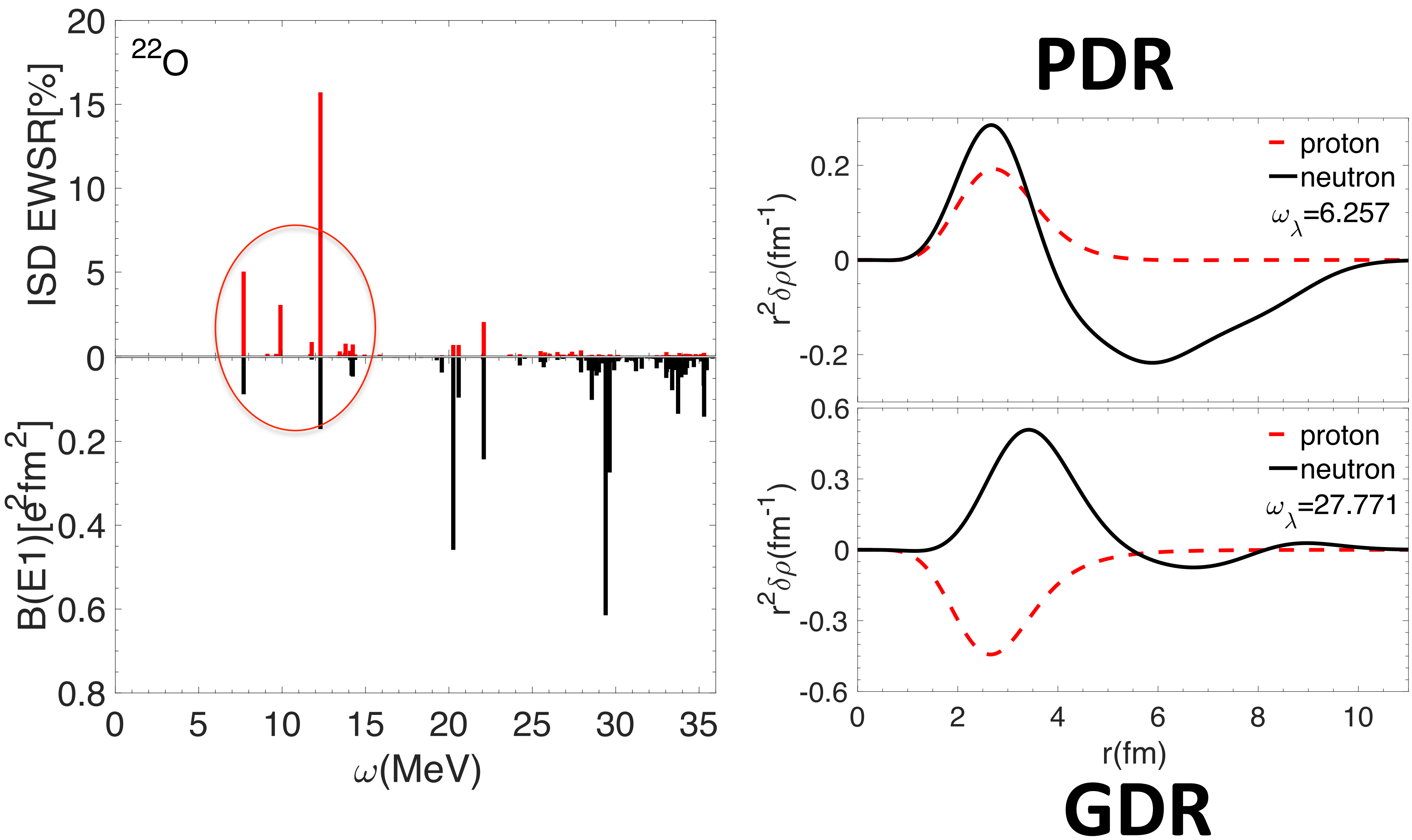
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 34311 (2018)
A. Leistenschneider, et al., Phys. Rev. Lett. 86, 5442 (2001)

Dipole response in ^{22}O



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 34311 (2018)
A. Leistenschneider, et al., Phys. Rev. Lett. 86, 5442 (2001)

Dipole response in ^{22}O



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 34311 (2018)

A. Leistenschneider, et al., Phys. Rev. Lett. 86, 5442 (2001)

EMPM for Odd nuclei: p (h) multiphonon scheme

$$|\lambda\, \alpha_{n-1}\rangle = O^{\dagger}_{\lambda}\, |\alpha_{n-1}\rangle \quad \Rightarrow \quad |p\, \alpha_n\rangle = a^{\dagger}_{p}\, |\alpha_n\rangle$$

EMPM for Odd nuclei: p (h) multiphonon scheme

$| \lambda \, \alpha_{n-1} > = O^{\dagger}_{\lambda} \, | \alpha_{n-1} >$ ⇒ $| p \, \alpha_n > = a^{\dagger}_{p} \, | \alpha_n >$

We solve the **Eq. of Motion**

$$< \nu_n \, || [H, a^{\dagger}_{p}] || \alpha_n > = (E_{\nu_n} - E_{\alpha_n}) < \nu_n \, | a^{\dagger}_{p} \, | \alpha_n >$$

χ

EMPM for Odd nuclei: p (h) multiphonon scheme

$|\lambda\,\alpha_{n-1}\rangle = O^{\dagger}_{\lambda}\,|\alpha_{n-1}\rangle \;\Rightarrow\; |p\,\alpha_n\rangle = a^{\dagger}_{p}\,|\alpha_n\rangle$

We solve the **Eq. of Motion**

$$\langle \nu_n \| [H, a^{\dagger}_{p}] \| \alpha_n \rangle = (E_{\nu_n} - E_{\alpha_n})\,\langle \nu_n | a^{\dagger}_{p} | \alpha_n \rangle$$

$\mathcal{X}$

Expressing $\mathcal{X}$ in terms of $\mathcal{C}$

$$\mathcal{X} = \mathcal{D}\,\mathcal{C}$$

$$\mathcal{A}\,\mathcal{X} = \mathcal{E}\,\mathcal{X} \;\Rightarrow\; [\mathcal{H} - \mathcal{E}\mathcal{D}]\,\mathbf{C} = 0$$

$$\mathcal{D} \equiv \{\langle p\,\alpha \,|\, p'\,\alpha' \rangle\}$$

EMPM for Odd nuclei: p (h) multiphonon scheme

$| \lambda\, \alpha_{n-1} > = O^{\dagger}_{\lambda}\ | \alpha_{n-1} >$ ⟶ $| p\, \alpha_n > = a^{\dagger}_{p}\ | \alpha_n >$

We solve the **Eq. of Motion**

$$< \nu_n \,|[H, a^{\dagger}_{p}]|\alpha_n > = (E_{\nu_n} - E_{\alpha_n}) < \nu_n |a^{\dagger}_{p} |\alpha_n >$$

Expressing $\mathcal{X}$ in terms of $\mathcal{C}$

$$\mathcal{X} = \mathcal{D}\,\mathcal{C}$$

$\mathcal{X}$

$$\mathcal{A}\,\mathcal{X} = \mathcal{E}\,\mathcal{X}$$ ⟶ $$[\mathcal{H} - \mathcal{E}\mathcal{D}]\,\mathbf{C} = 0$$

$$\mathcal{D} \equiv \{< p\, \alpha \,|\, p'\, \alpha' >\}$$

Cholesky

$$\mathcal{D} \rightarrow \mathbf{D}$$

EMPM for Odd nuclei: p (h) multiphonon scheme

$$[\mathbf{H} - \mathbf{E}]\,\mathbf{C} = 0 \qquad \mathbf{H} = \mathbf{D}^{-1}\mathcal{A}\mathcal{D}$$

EMPM for Odd nuclei: p (h) multiphonon scheme

$$[H - E]\, C = 0 \qquad H = D^{-1} \mathcal{A} \mathcal{D}$$

$$|\nu_n> = \Sigma_{p\alpha}\; C^{\nu}_{p\alpha}\; a^{\dagger}_{p}\, |\alpha>$$

$$<\nu'_{n'}|\nu_n> = \delta_{\nu\nu'}\,\delta_{nn'}$$

EMPM for Odd nuclei: p (h) multiphonon scheme

$$[H - E] C = 0 \qquad H = D^{-1} \mathcal{A}\mathcal{D}$$

$$|\nu_n> = \Sigma_{p\alpha} C^{\nu}_{p\alpha} a^{\dagger}_{p} |\alpha >$$

$$<\nu'_{n'}|\nu_n> = \delta_{\nu\nu'} \delta_{nn'}$$

$$|\Psi_{\Omega}> = \Sigma_{n\nu_n} C^{\Omega}_{\nu_n} |\nu_n>$$

EMPM for Odd nuclei: p (h) multiphonon scheme

$$[H - E]\, C = 0 \qquad H = D^{-1} \mathcal{A} \mathcal{D}$$

$$|\nu_n> = \Sigma_{p\alpha}\, C^{\nu}_{p\alpha}\, a^{\dagger}_{p}\, |\alpha>$$

$$<\nu'_{n'}|\nu_n> = \delta_{\nu\nu'}\, \delta_{nn'}$$

$$|\Psi_{\Omega}> = \Sigma_{n\nu_n}\, C^{\Omega}_{\nu_n}\, |\nu_n>$$

- **No approximations except for truncation !**
- **Pauli** principle fully accounted for
- **No redundant** states**!**
- $|\nu_n>$ form an **orthonormal** basis

Implementation for odd nuclei

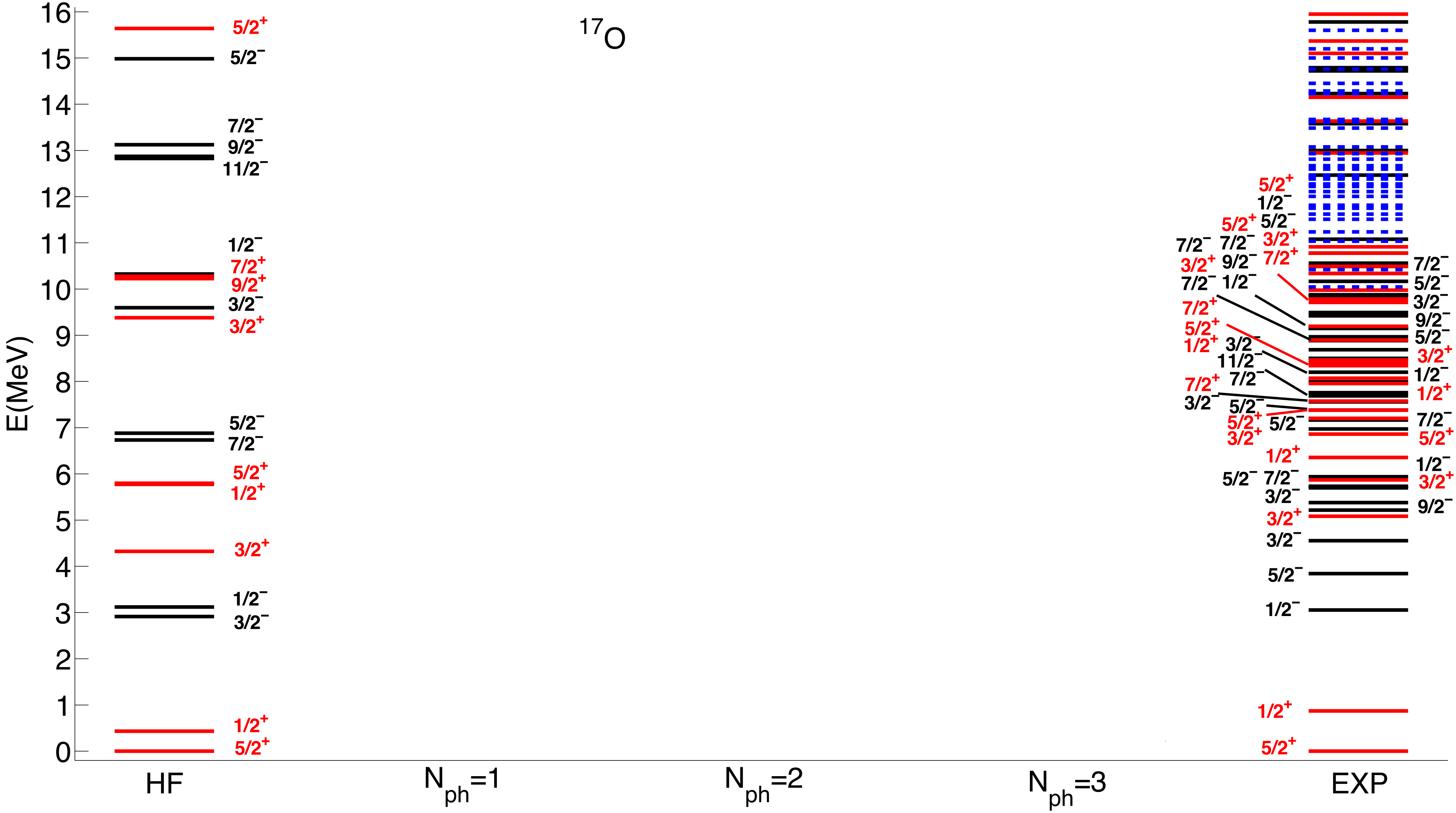
Hamiltonian

$$H = T_{int} + V_2 \qquad V_2 = NNLO_{opt}$$

A. Ekstrom *et al*., PRL **110**, 192502 (2013)

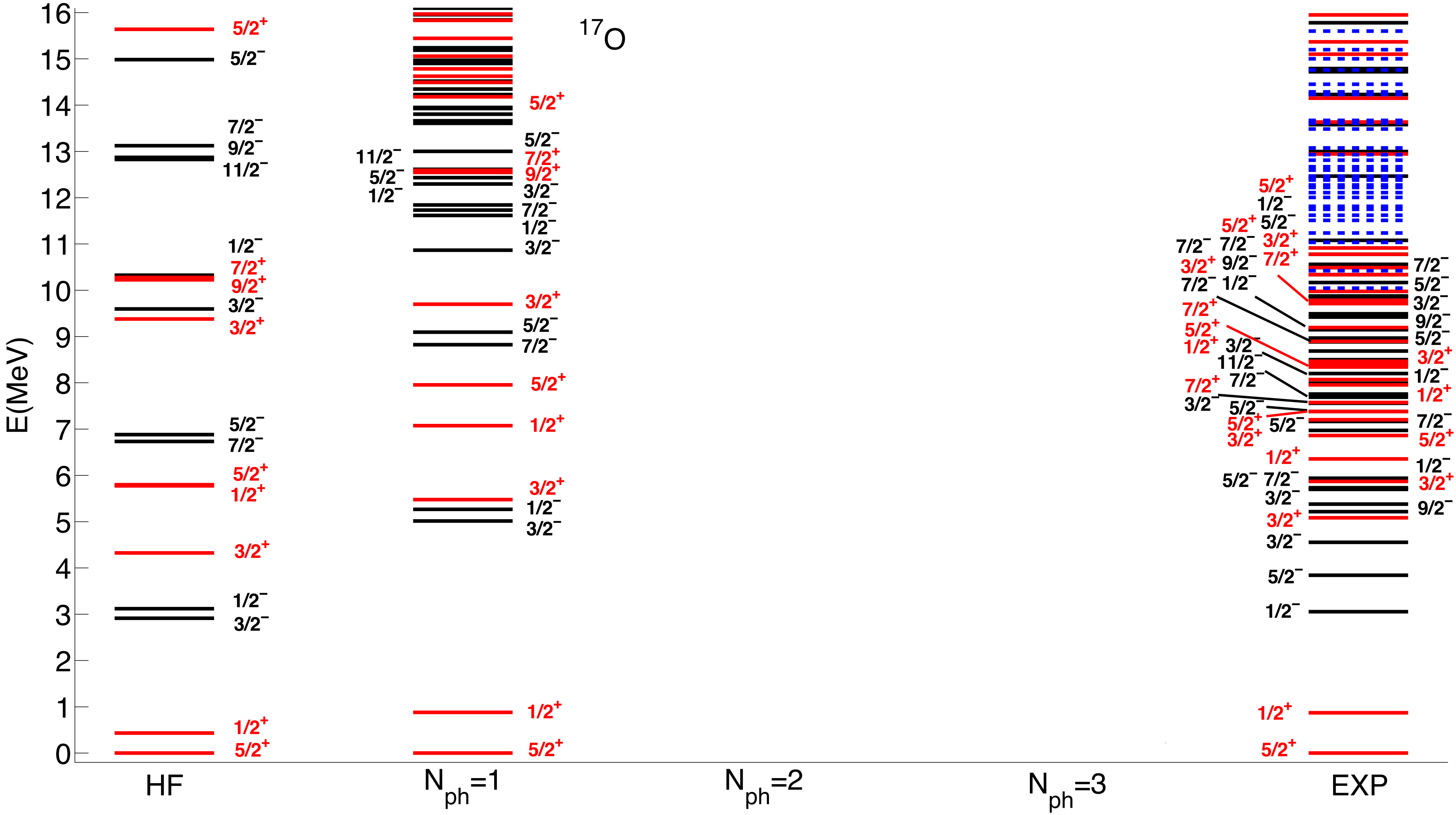
- Perform **HF**
- Construct **TDA** phonons (**free** of **CM** spurious admixtures)
- Generate the multiphonon core basis $\{|\alpha_n>\}$ (**free** of **CM** spurious admixtures)
- Generate the particle-phonon basis $\{|\nu_n>\}$
- Full diagonalization

Application: ^{17}O spectra



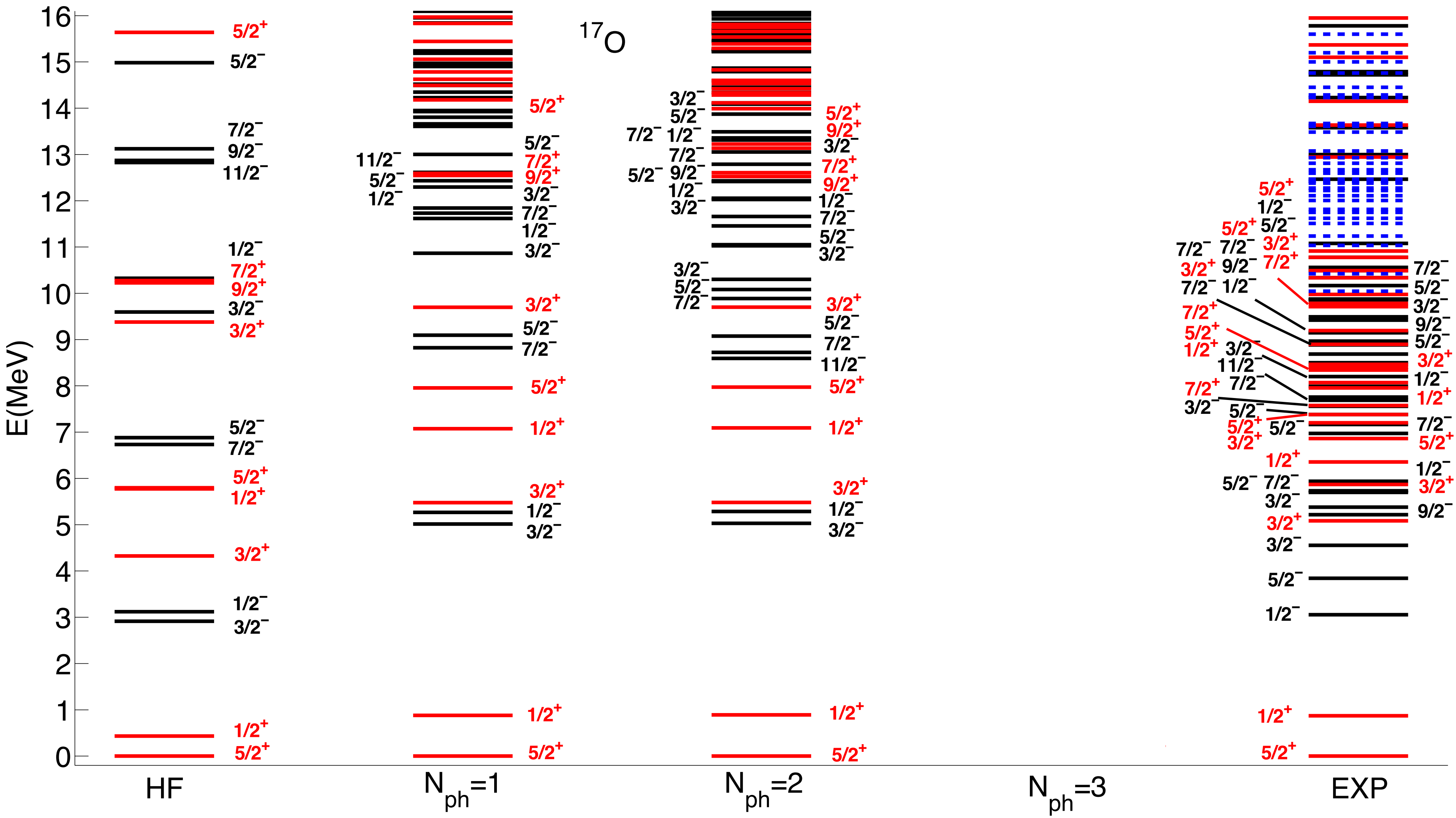
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **94**, 061301(R) (2016)

Application: ^{17}O spectra



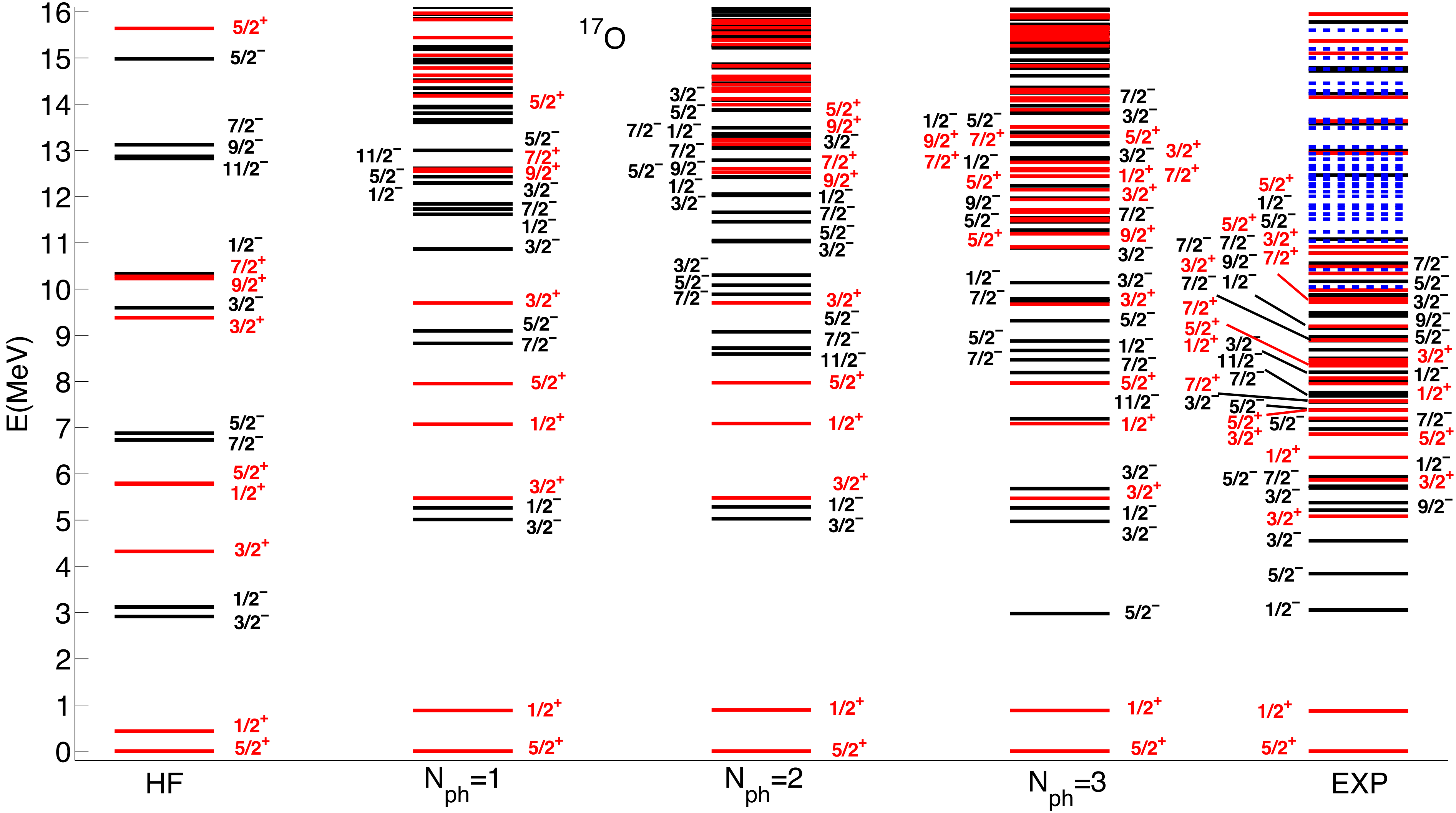
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **94**, 061301(R) (2016)

Application: ^{17}O spectra



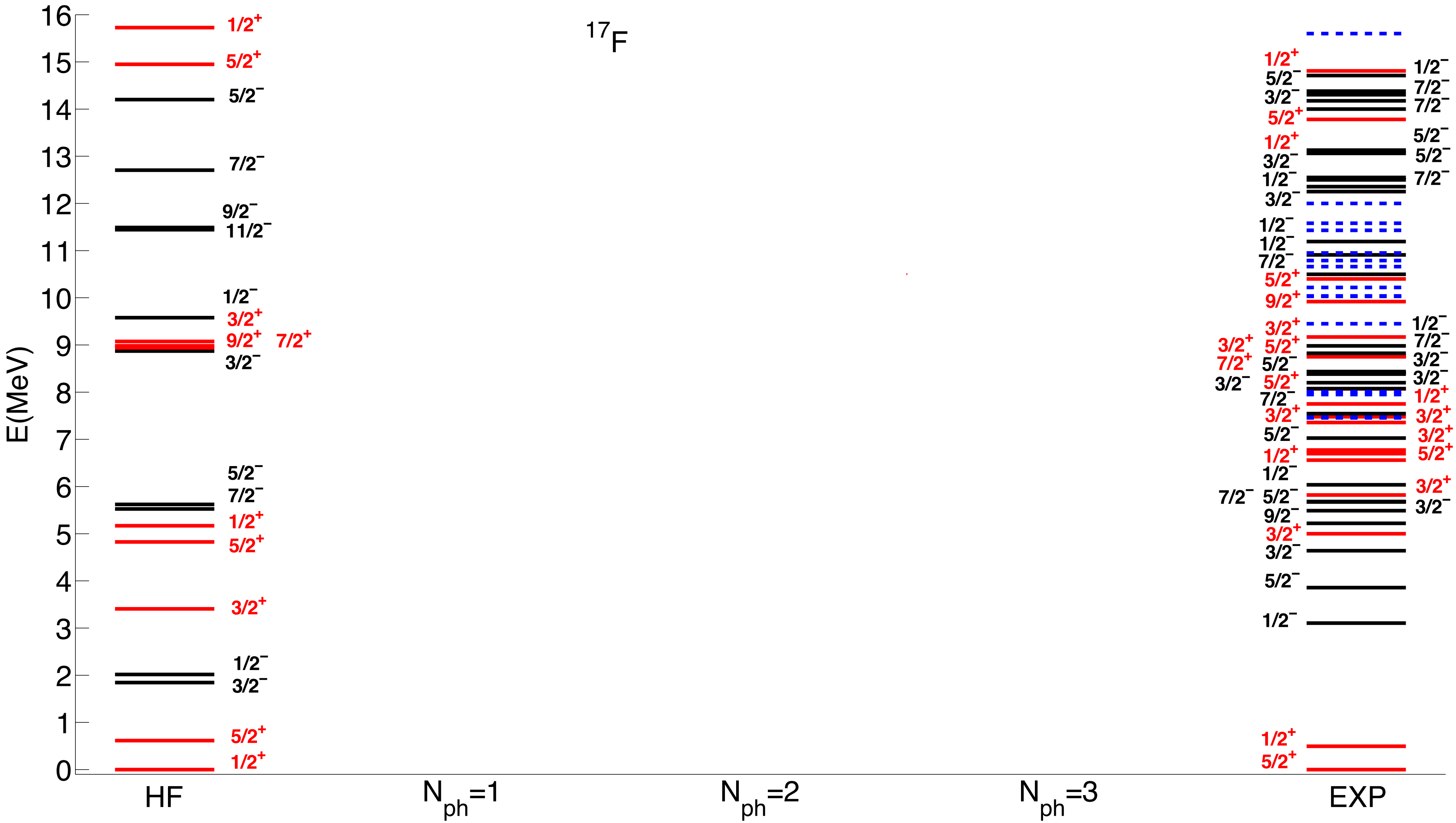
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **94**, 061301(R) (2016)

Application: ^{17}O spectra



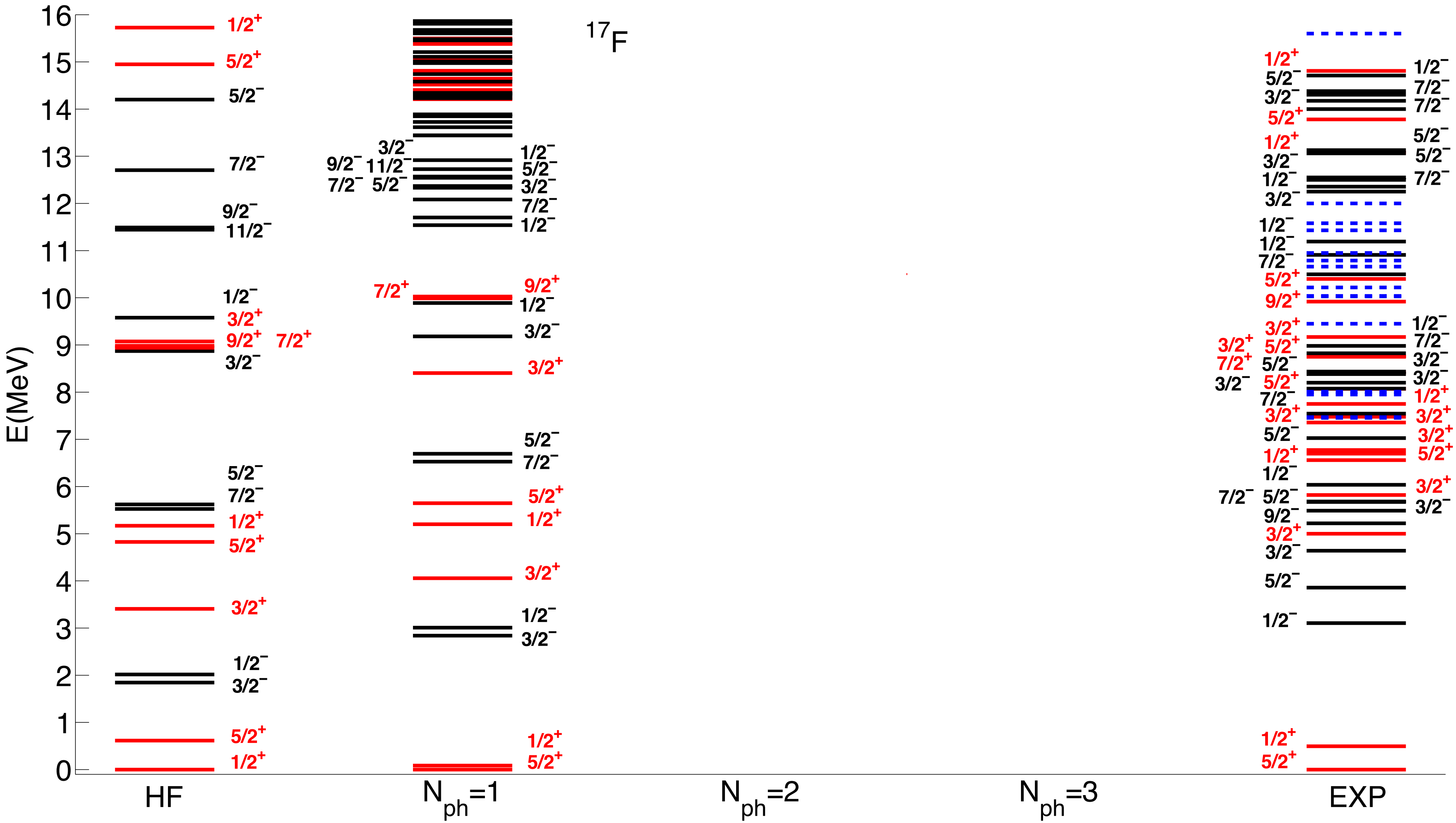
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **94**, 061301(R) (2016)

Application: ^{17}F spectra



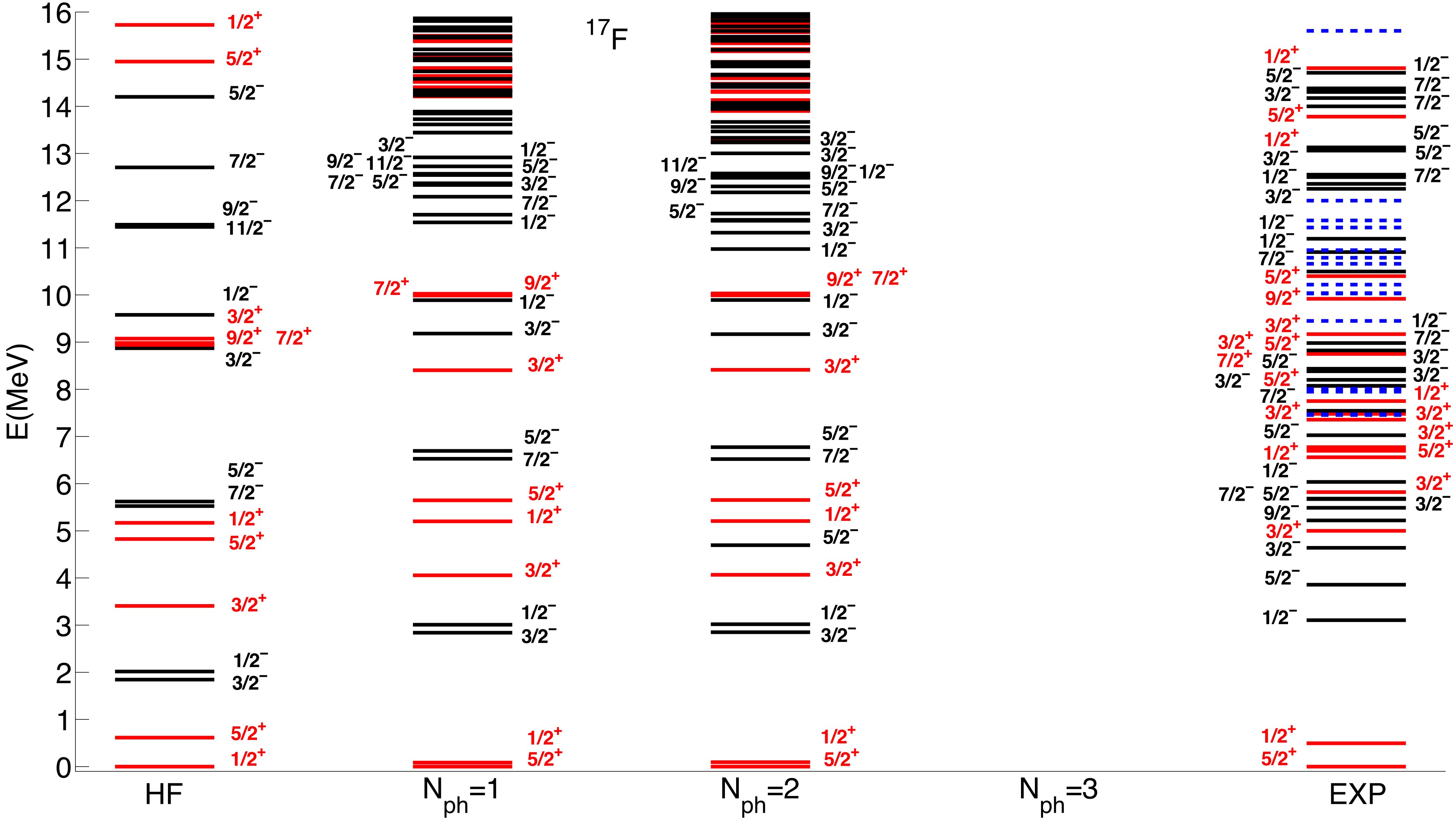
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 034327 (2017)

Application: ^{17}F spectra



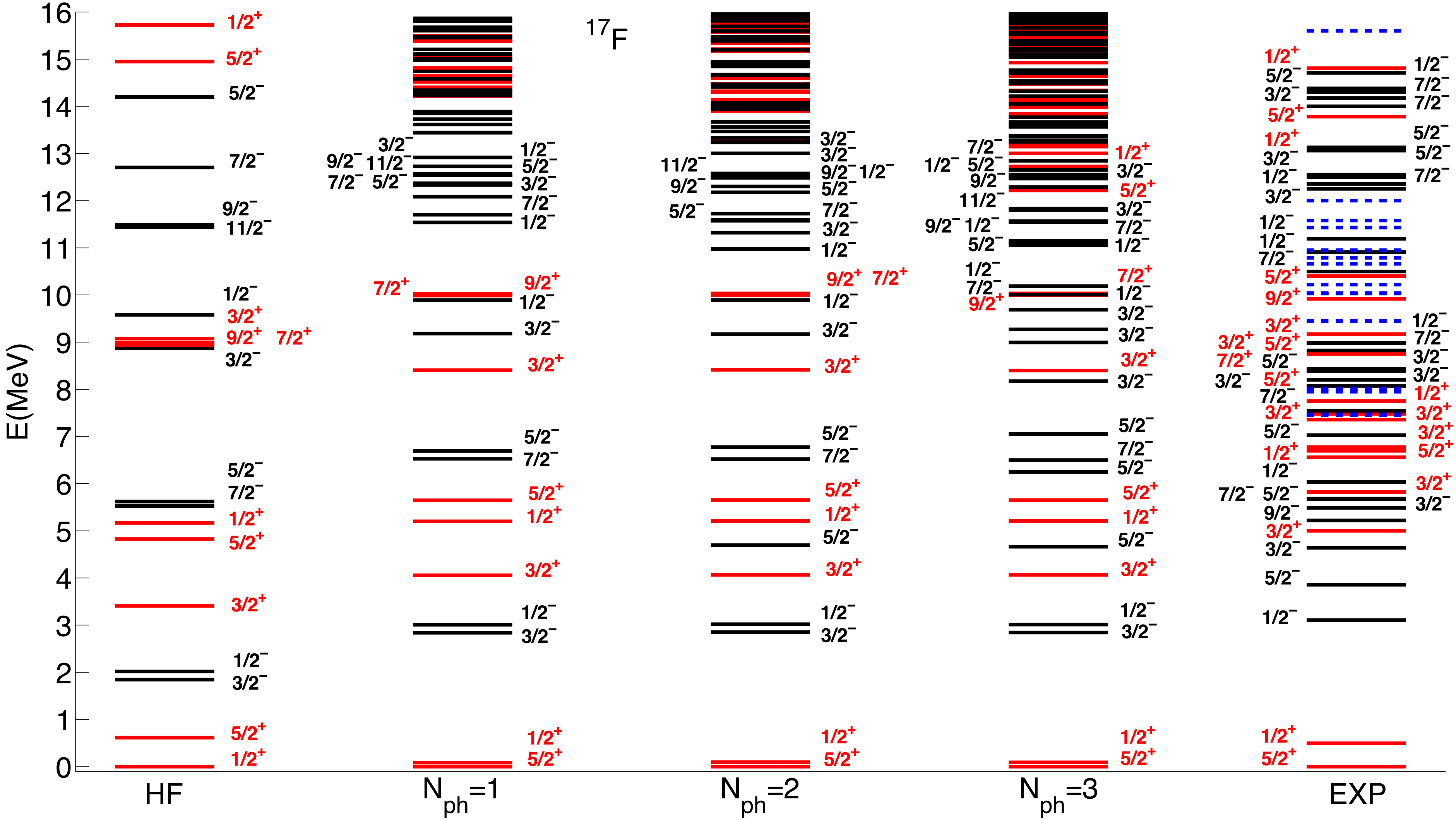
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 034327 (2017)

Application: ^{17}F spectra



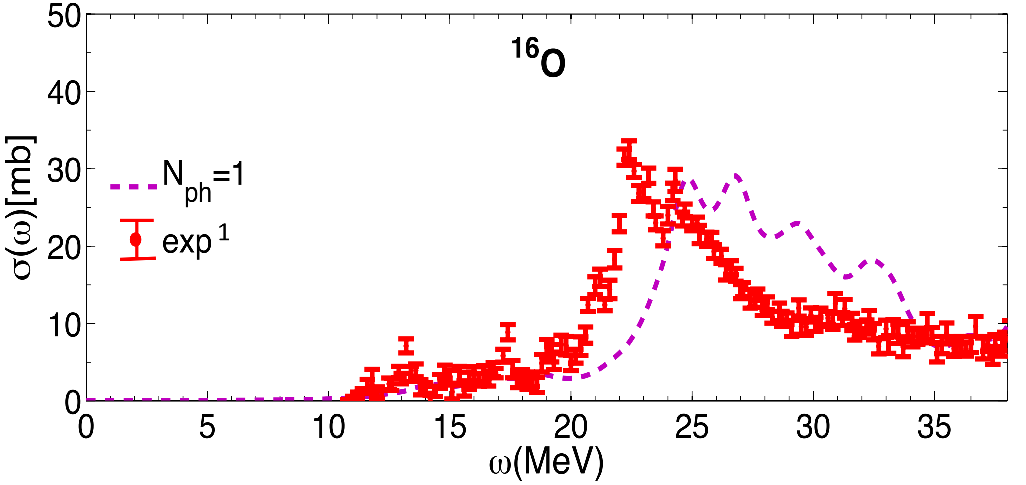
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 034327 (2017)

Application: ^{17}F spectra



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 034327 (2017)

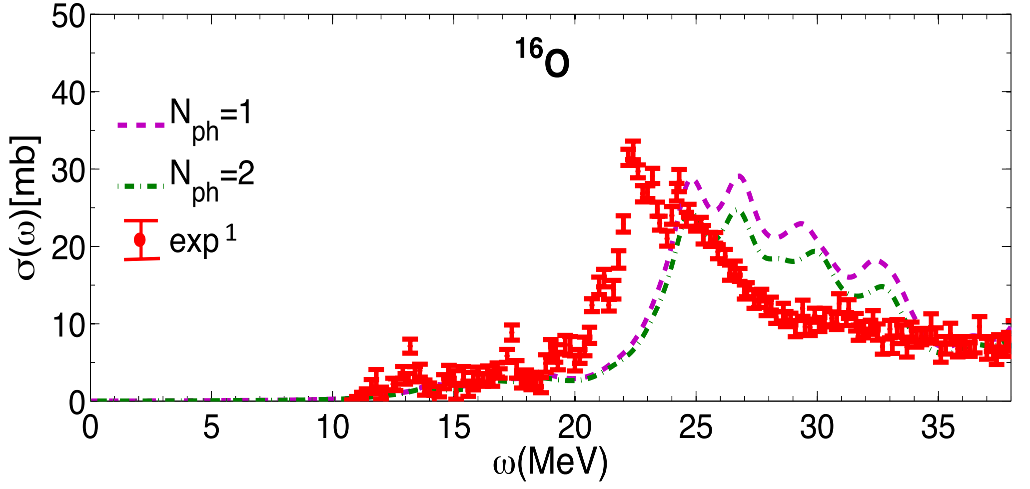
Cross section



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 034327 (2017)

[1]Ahrens J et al Nucl. Phys. A **251** 479 (1975)

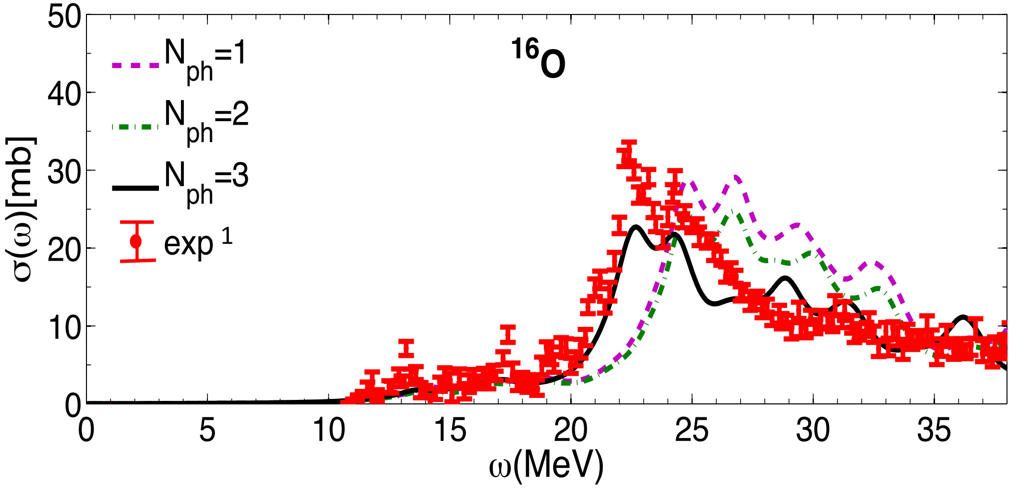
Cross section



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely
Phys. Rev. C **95**, 034327 (2017)

[1]Ahrens J et al Nucl. Phys. A **251** 479 (1975)

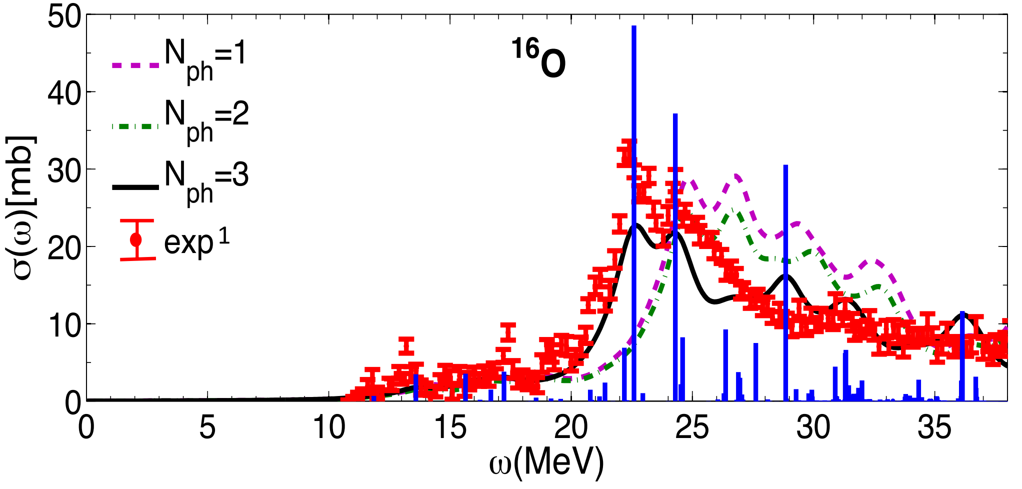
Cross section



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely
Phys. Rev. C **95**, 034327 (2017)

[1]Ahrens J et al Nucl. Phys. A **251** 479 (1975)

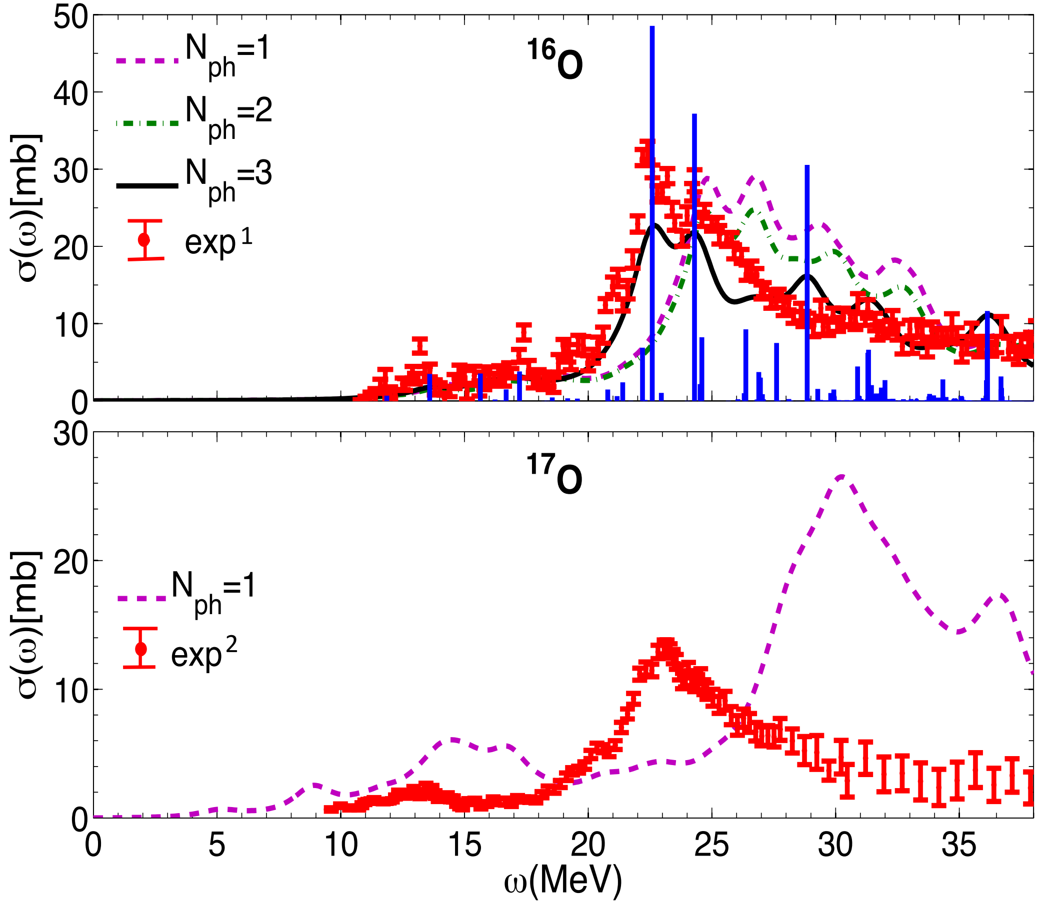
Cross section



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 034327 (2017)

[1]Ahrens J et al Nucl. Phys. A **251** 479 (1975)

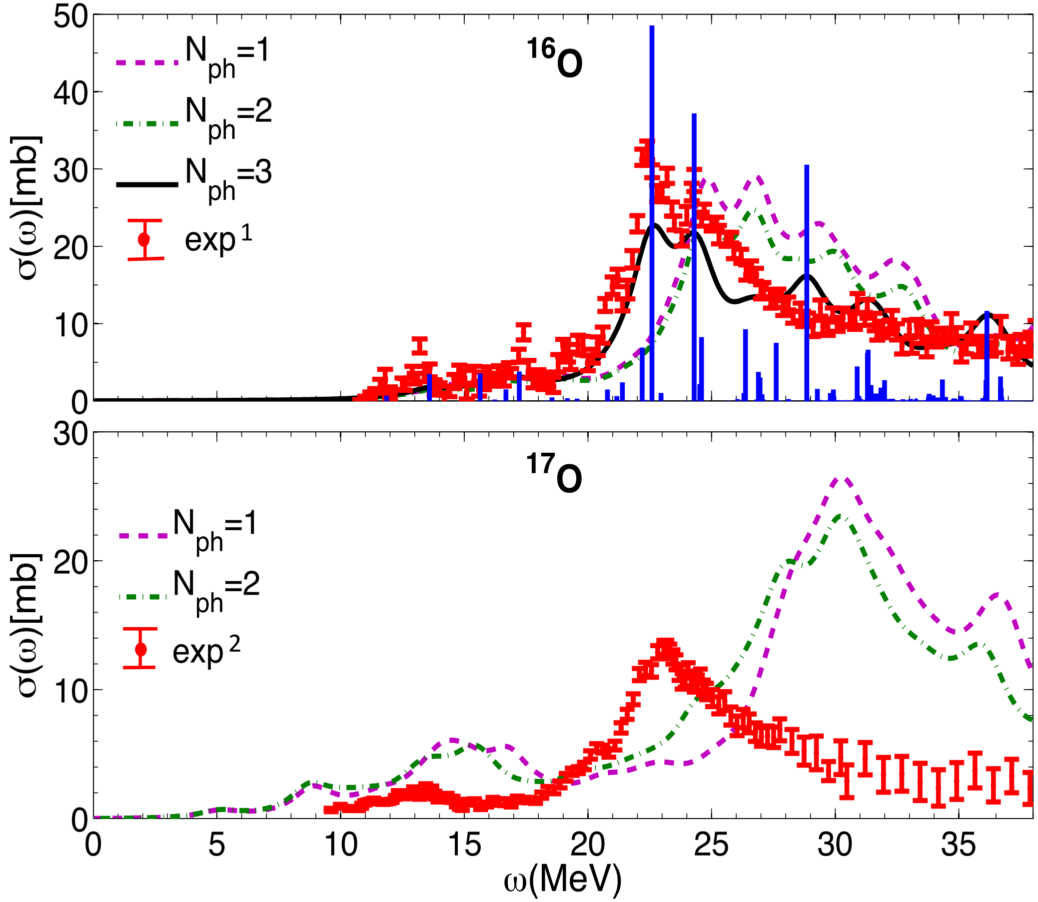
Cross section



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely
Phys. Rev. C **95**, 034327 (2017)

[1] Ahrens J et al Nucl. Phys. A **251** 479 (1975)
[2] J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

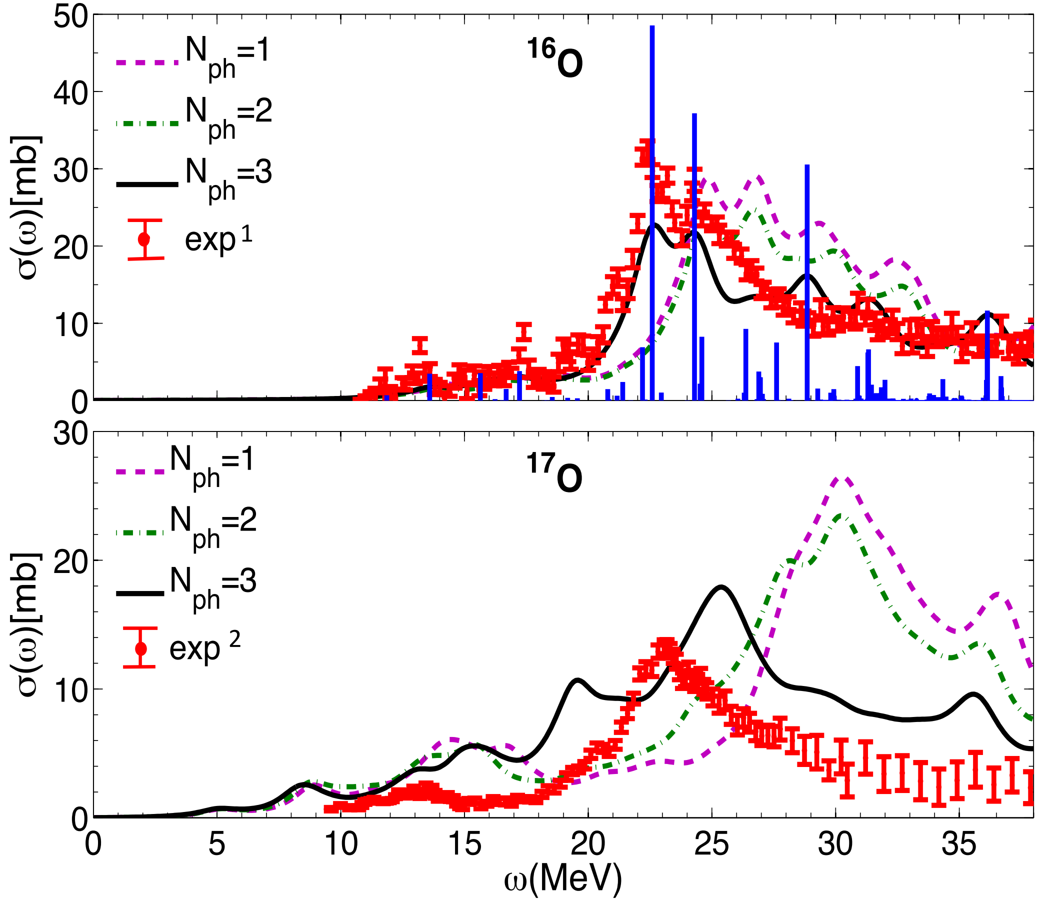
Cross section



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely
Phys. Rev. C **95**, 034327 (2017)

[1]Ahrens J et al Nucl. Phys. A **251** 479 (1975)
[2] J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

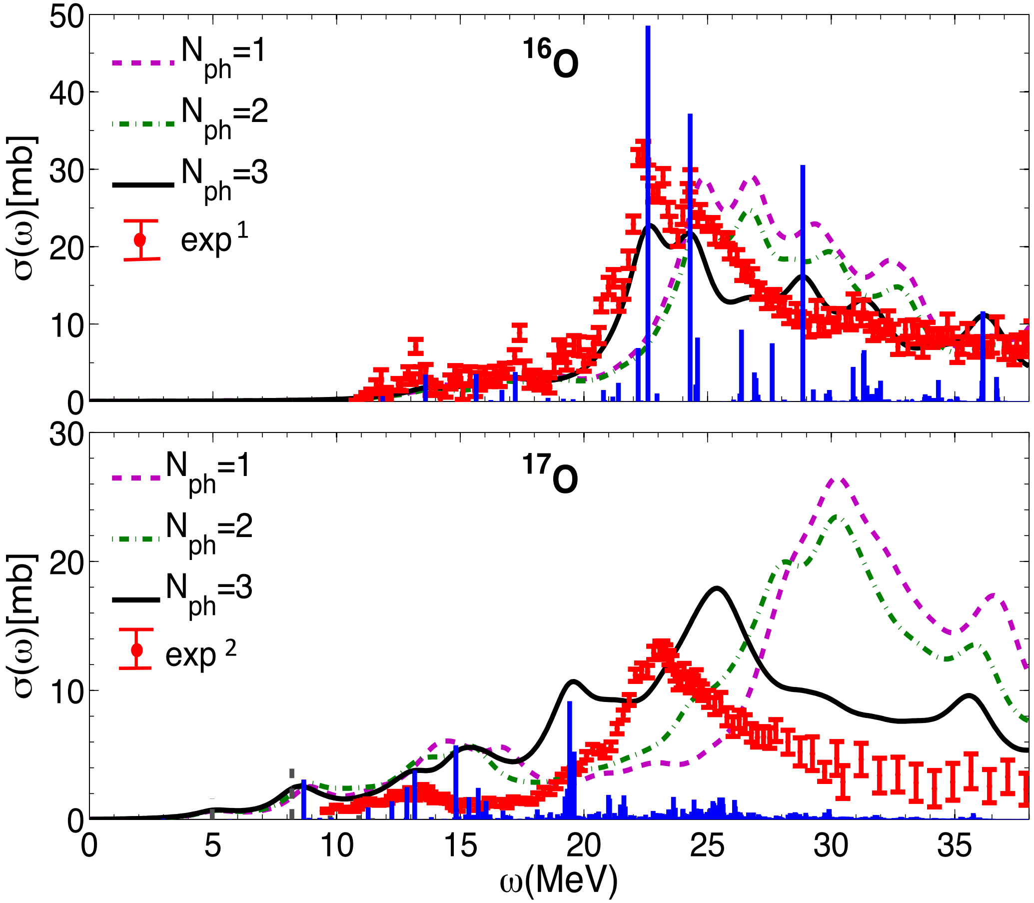
Cross section



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely
Phys. Rev. C **95**, 034327 (2017)

[1]Ahrens J et al Nucl. Phys. A **251** 479 (1975)
[2] J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

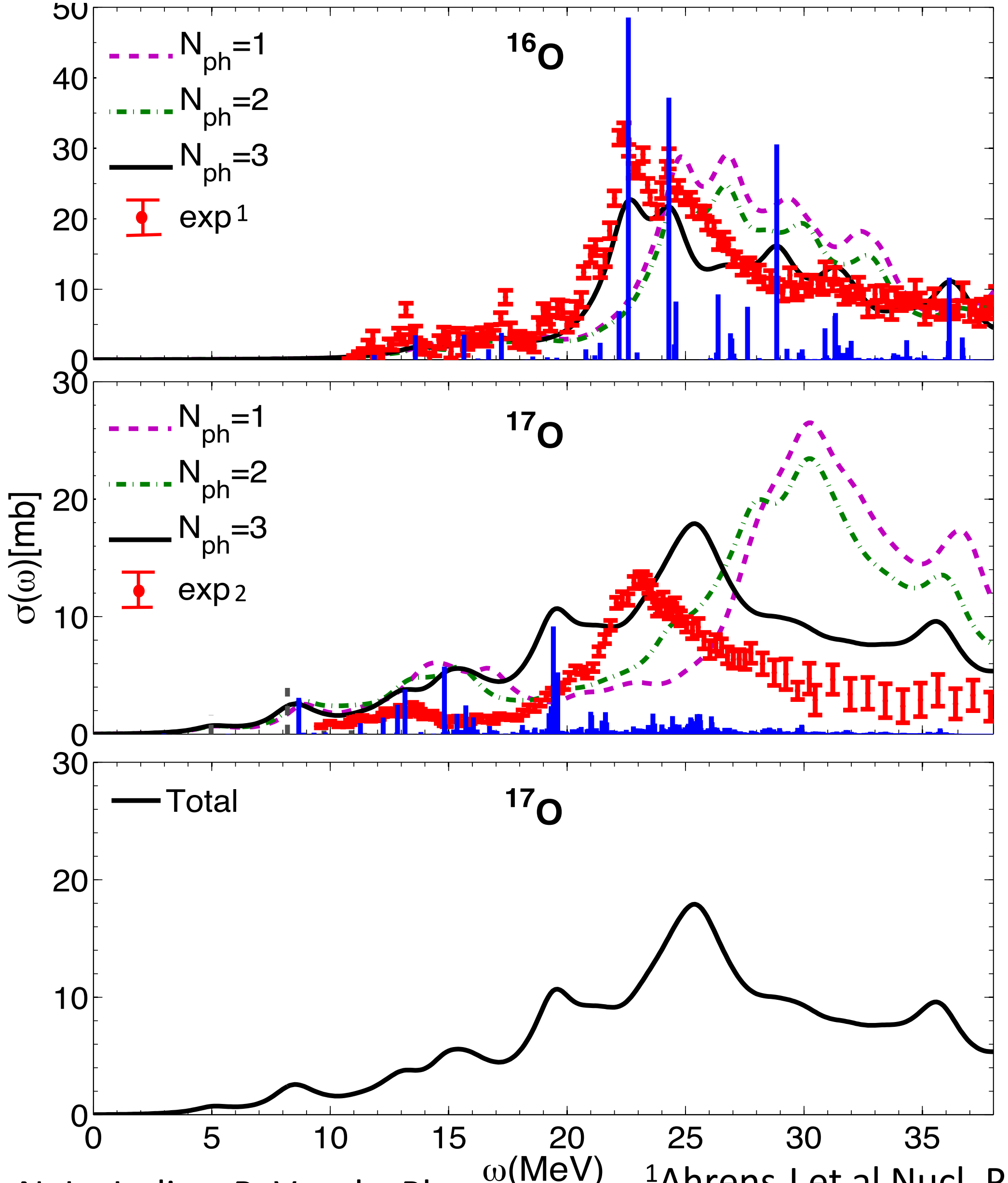
Cross section



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely
Phys. Rev. C **95**, 034327 (2017)

[1]Ahrens J et al Nucl. Phys. A **251** 479 (1975)
[2] J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

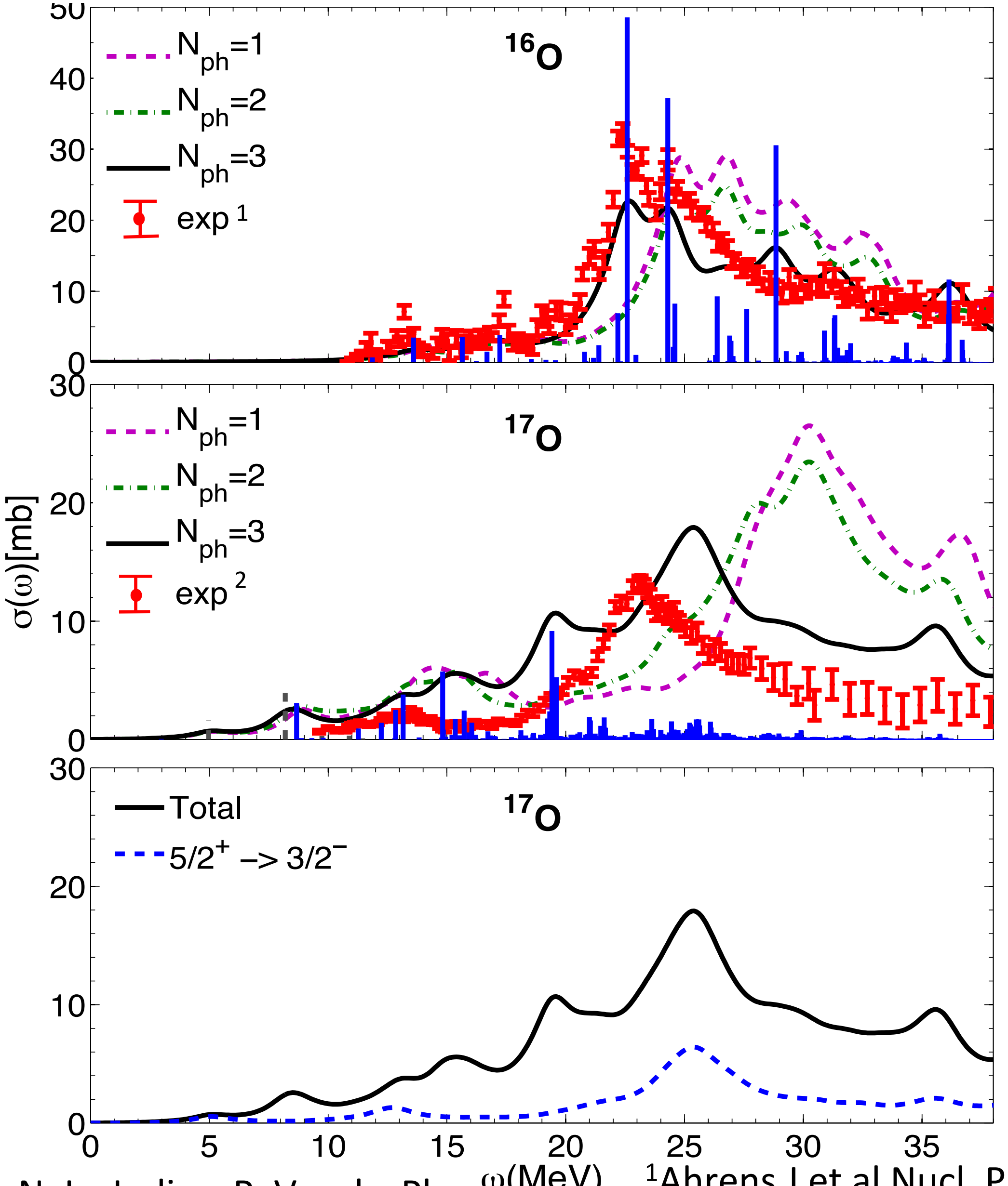
Cross section



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 034327 (2017)

[1]Ahrens J et al Nucl. Phys. A **251** 479 (1975)
[2] J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

Cross section

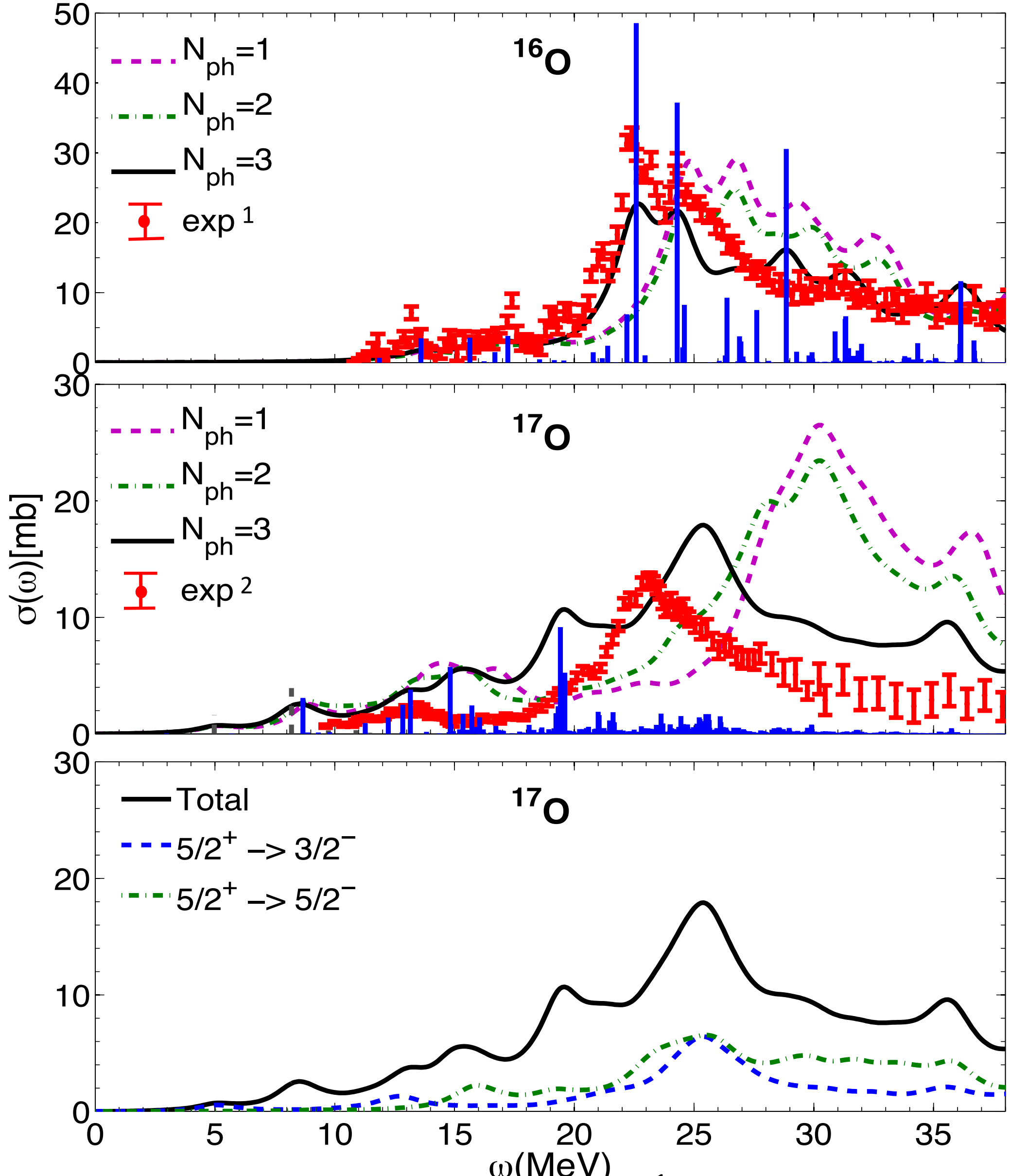


G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 034327 (2017)

[1]Ahrens J et al Nucl. Phys. A **251** 479 (1975)

[2] J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

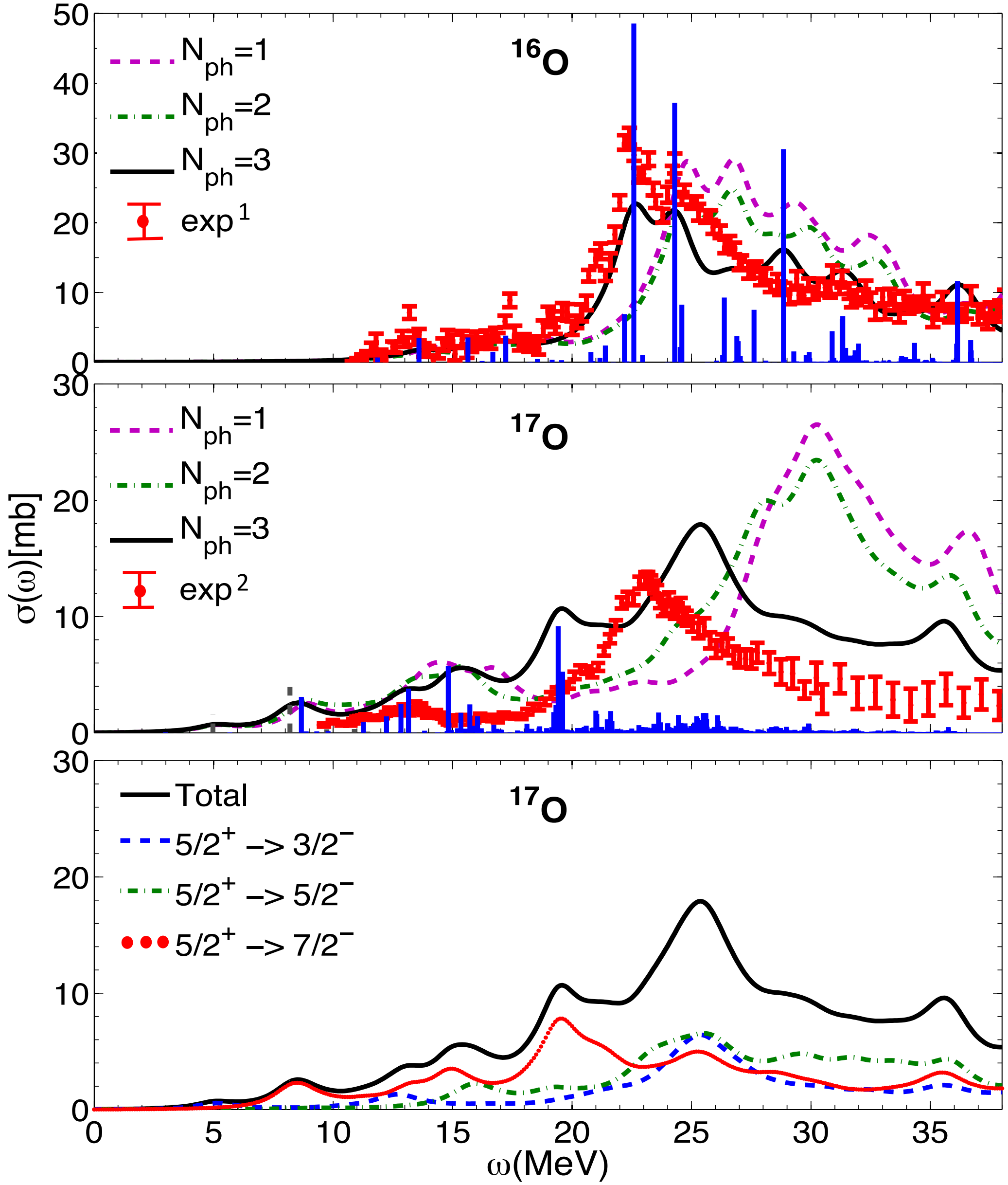
Cross section



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 034327 (2017)

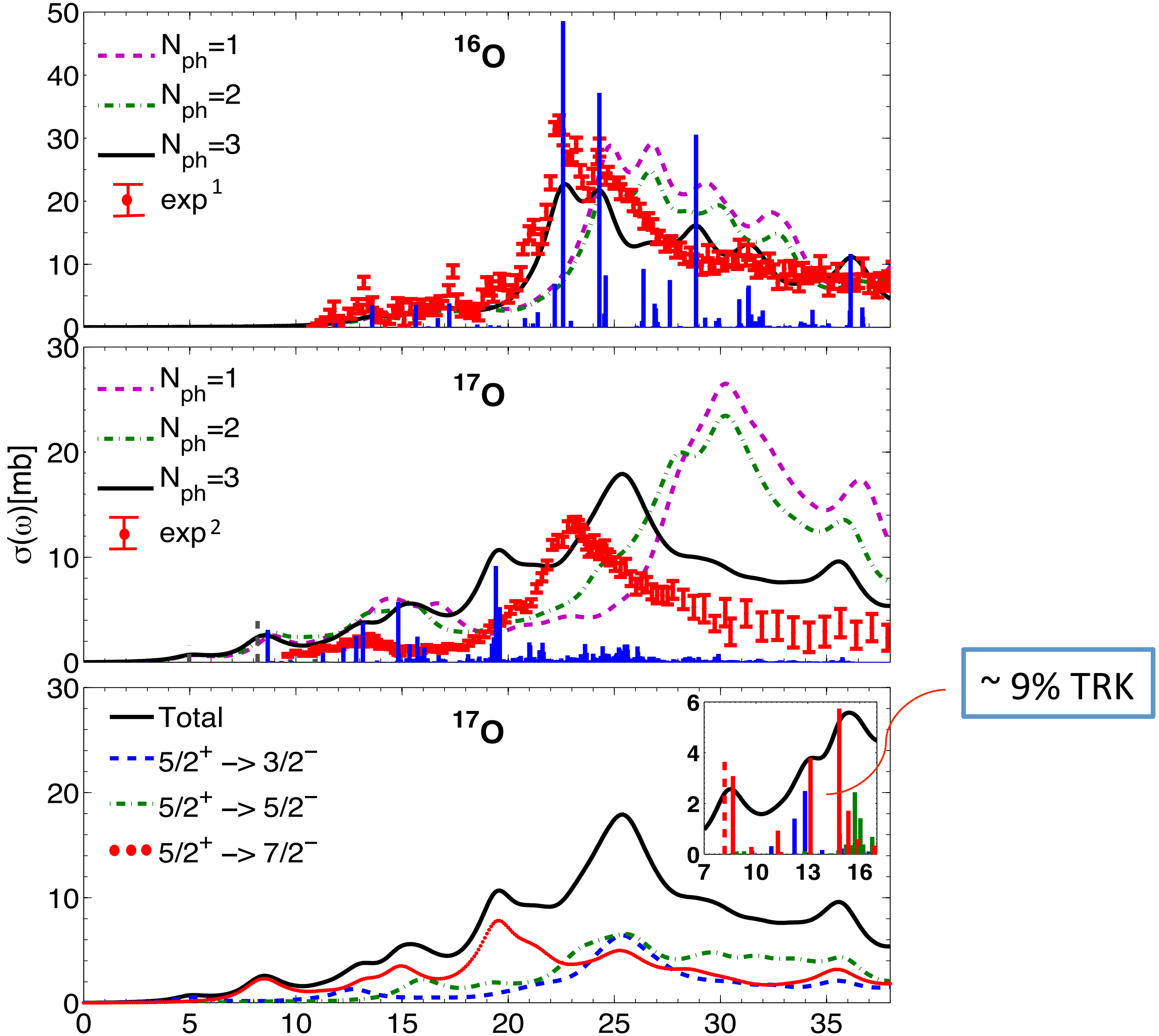
[1]Ahrens J et al Nucl. Phys. A **251** 479 (1975)
[2] J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

Cross section



[1]Ahrens J et al Nucl. Phys. A **251** 479 (1975) [2] J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

Cross section

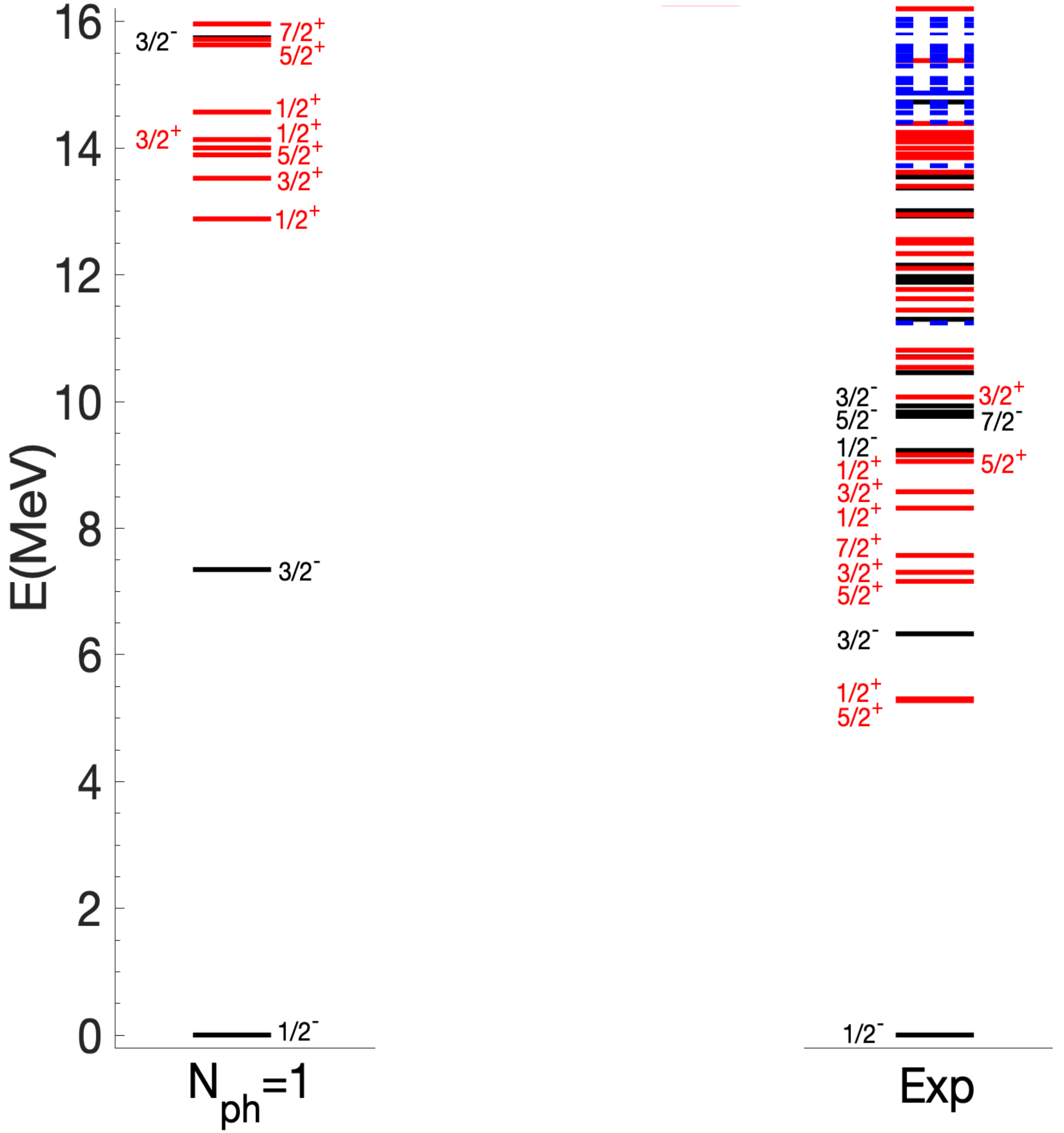


G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 034327 (2017)

[1] Ahrens J et al Nucl. Phys. A **251** 479 (1975)

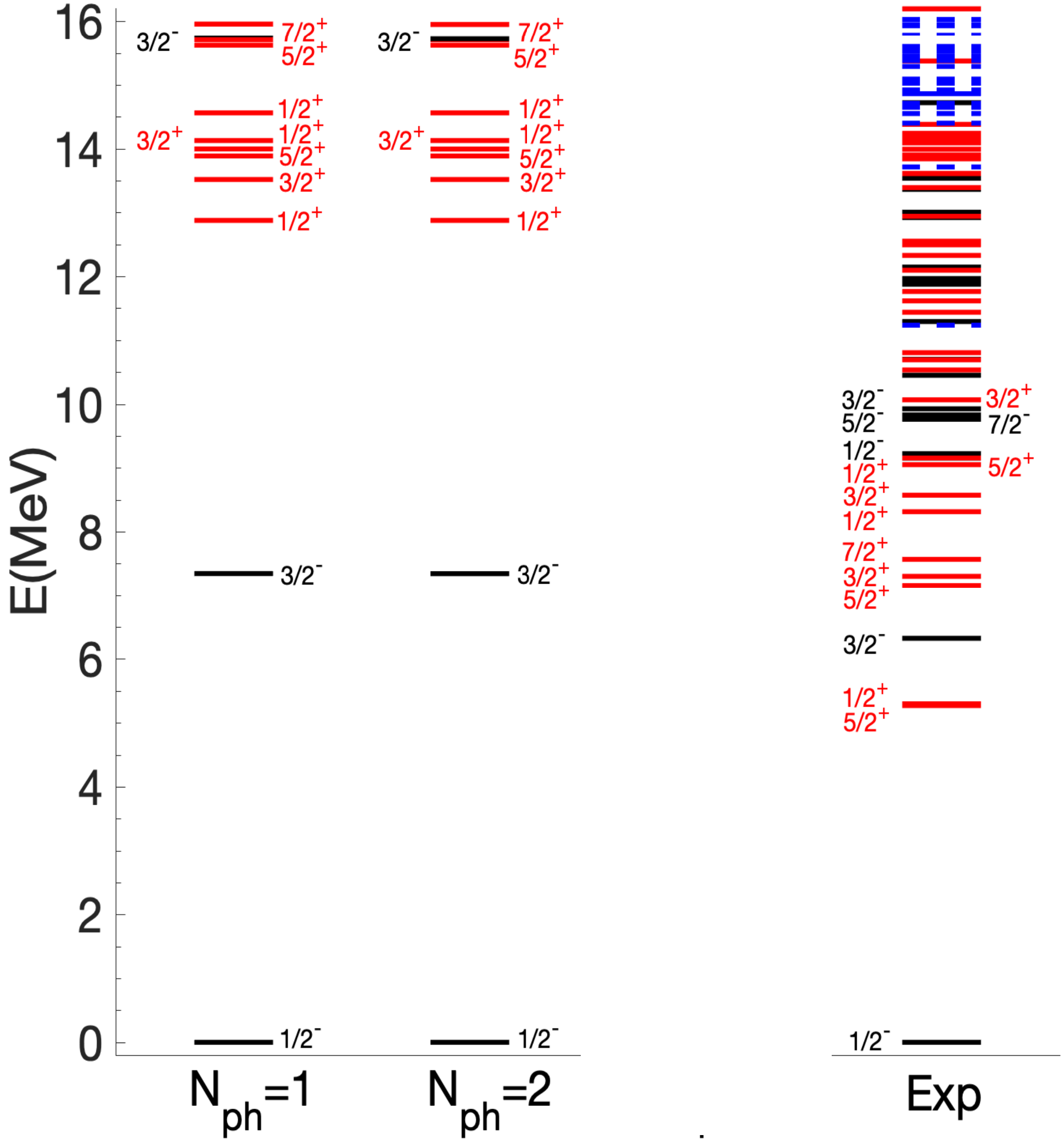
[2] J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

Application: ^{15}N spectra



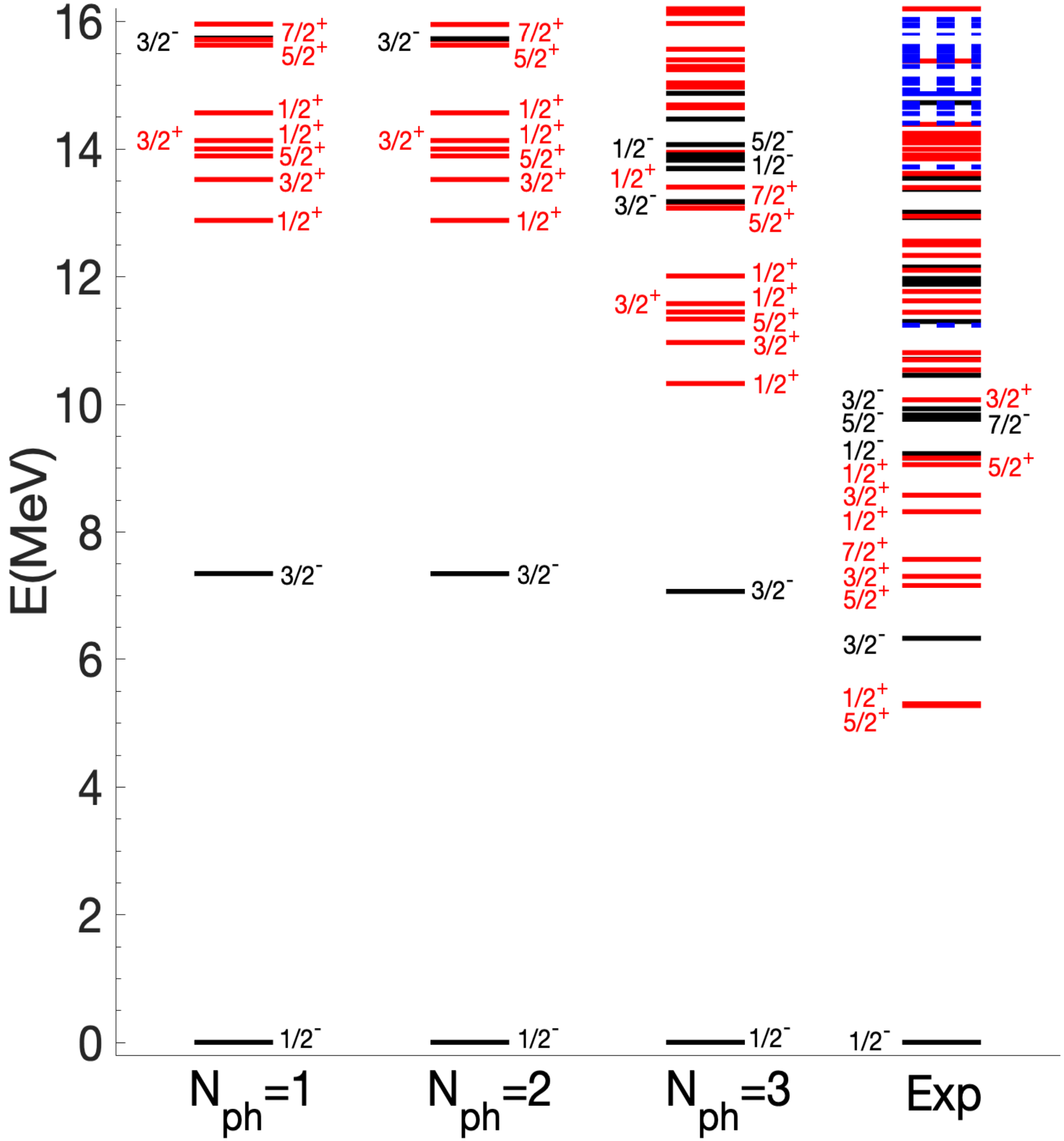
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **99**, 014316 (2019)

Application: ^{15}N spectra



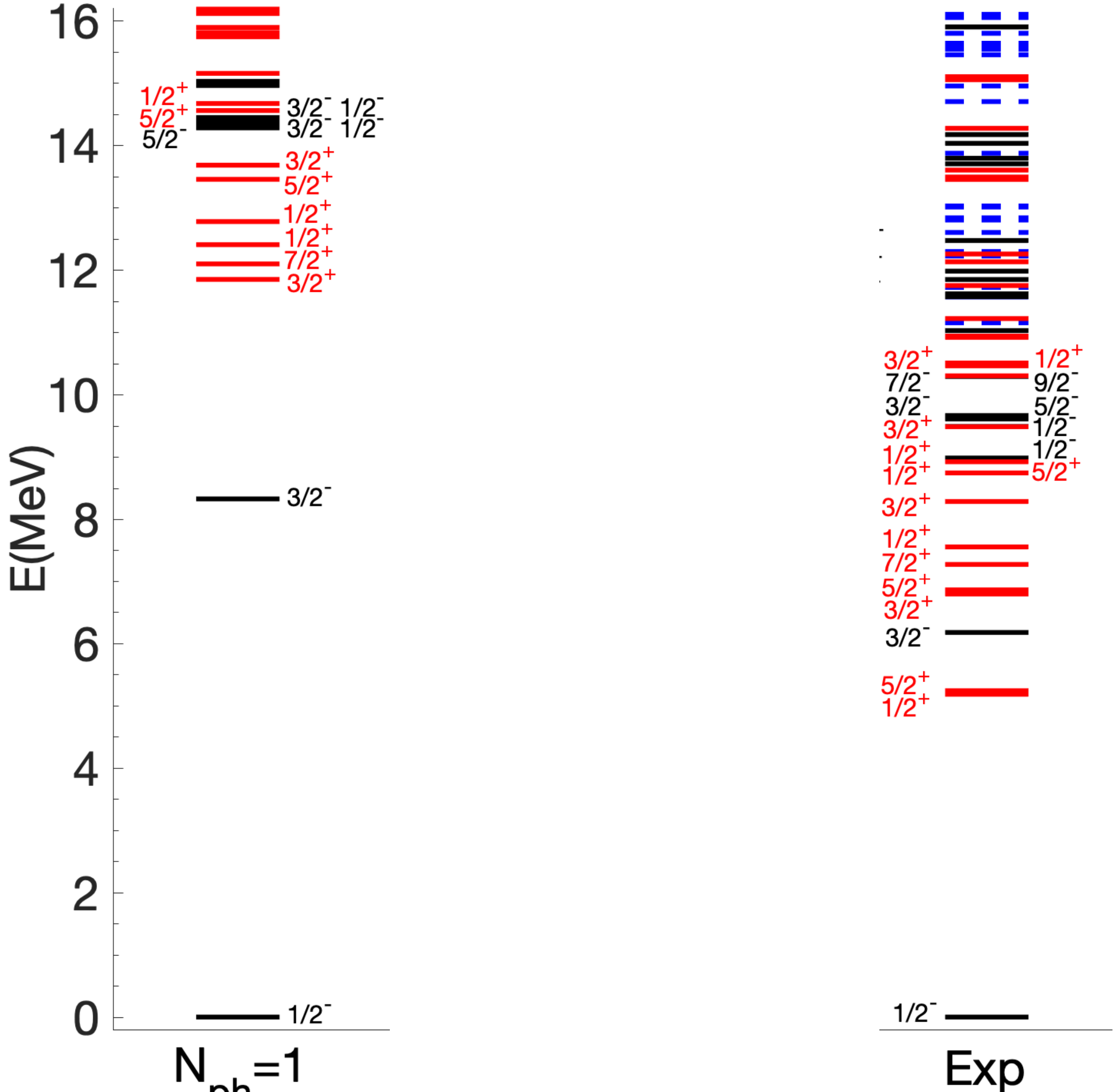
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **99**, 014316 (2019)

Application: ^{15}N spectra



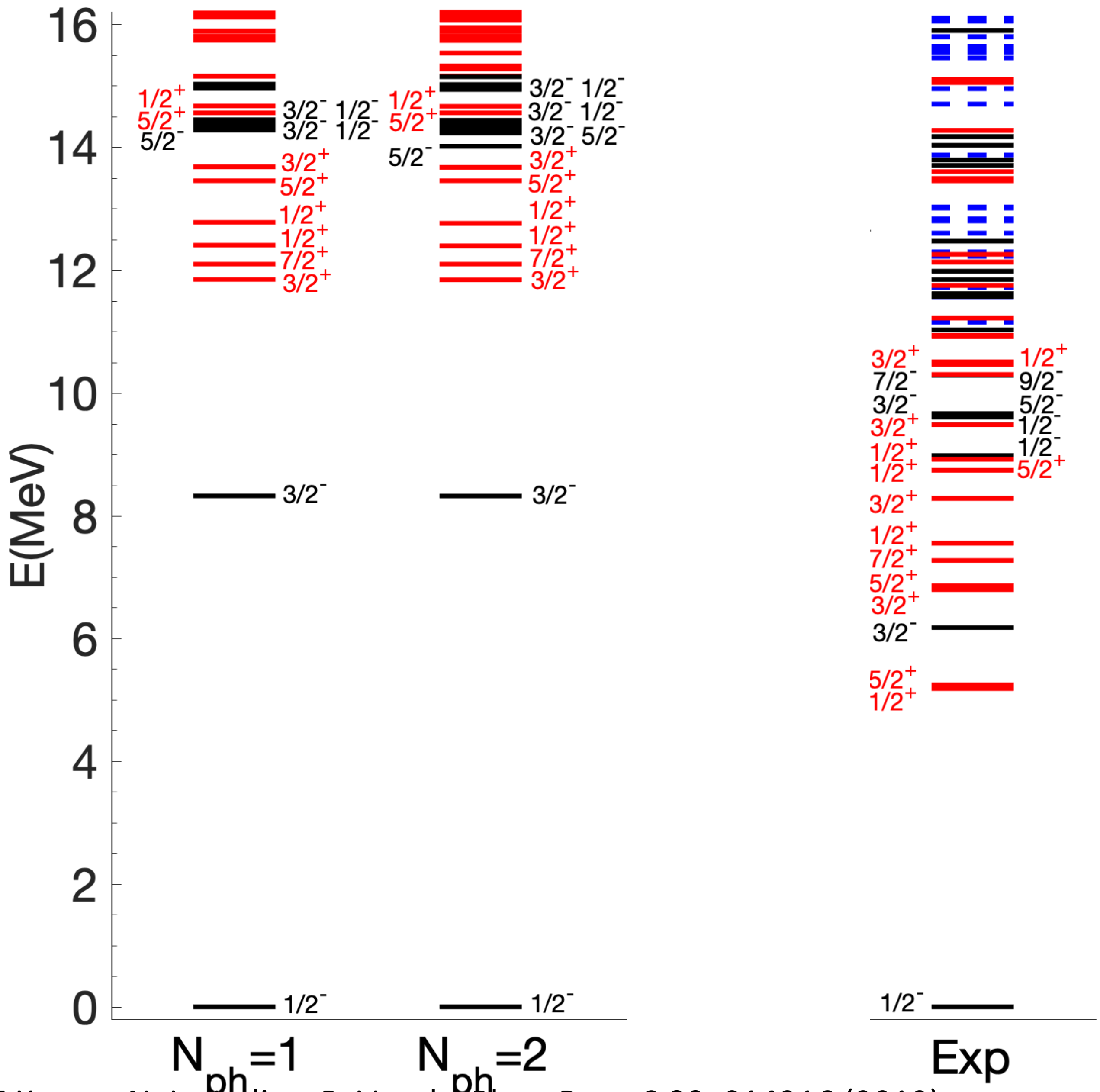
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **99**, 014316 (2019)

Application: ^{15}O spectra



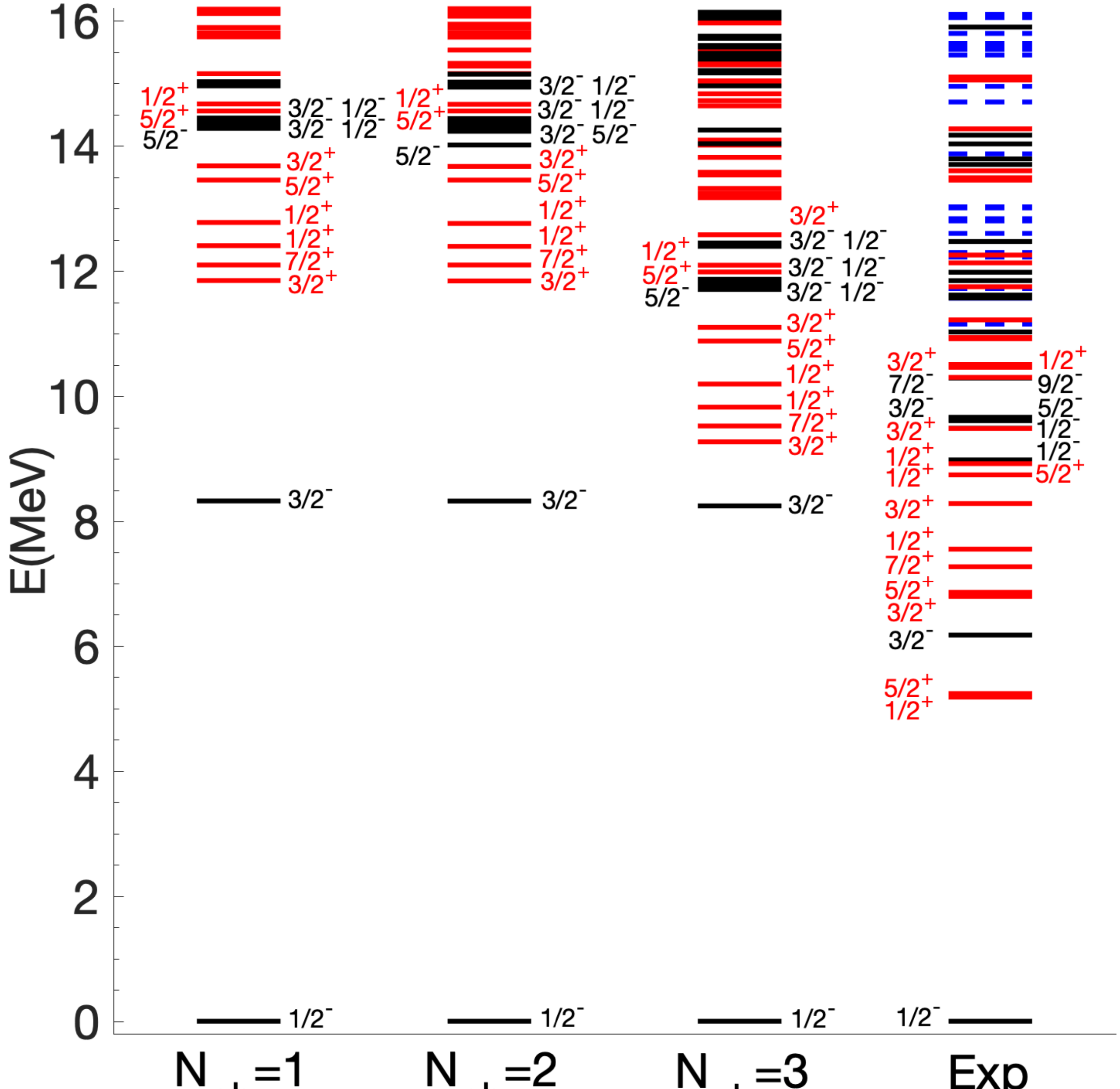
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **99**, 014316 (2019)

Application: ^{15}O spectra



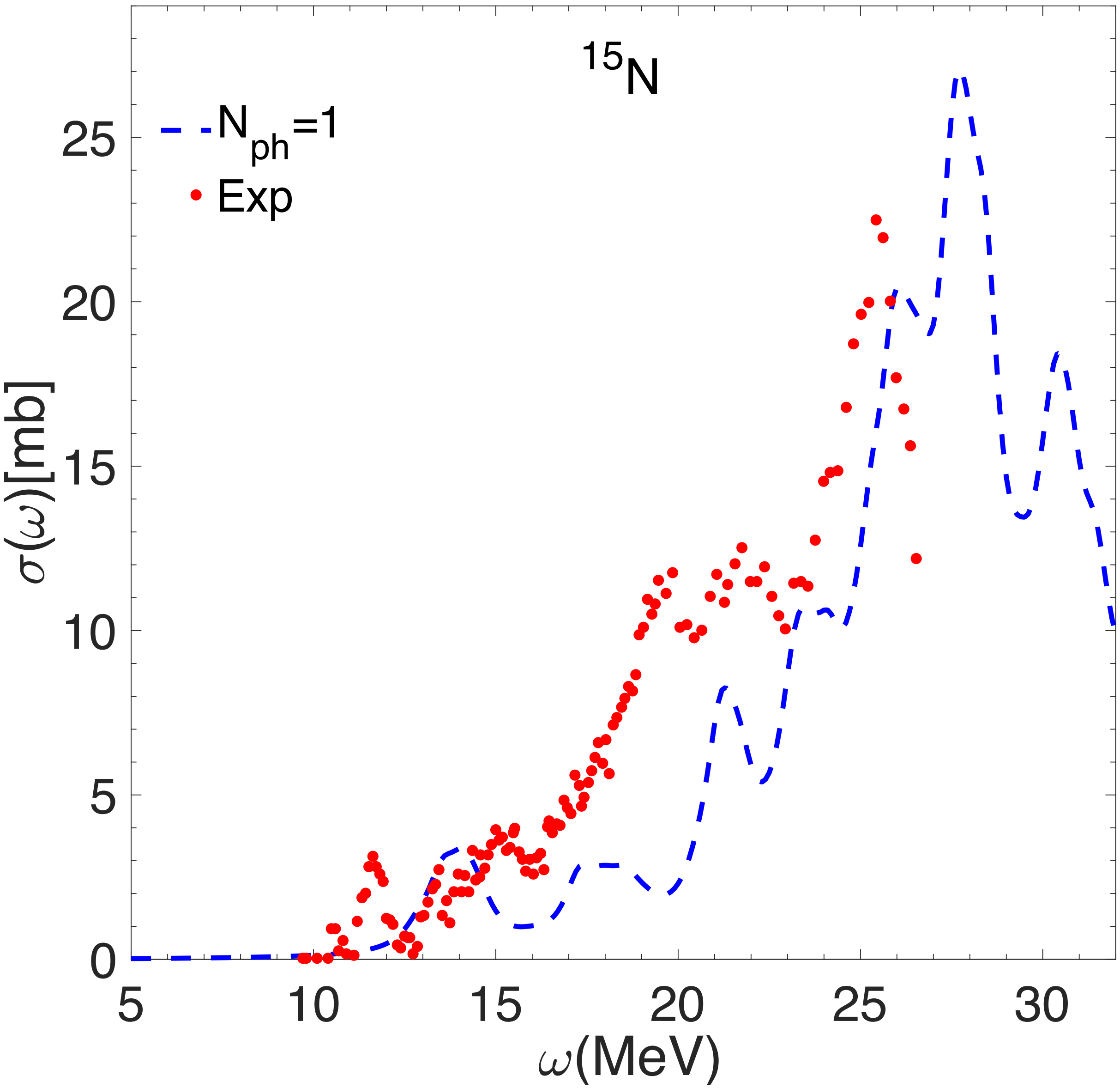
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **99**, 014316 (2019)

Application: ^{15}O spectra



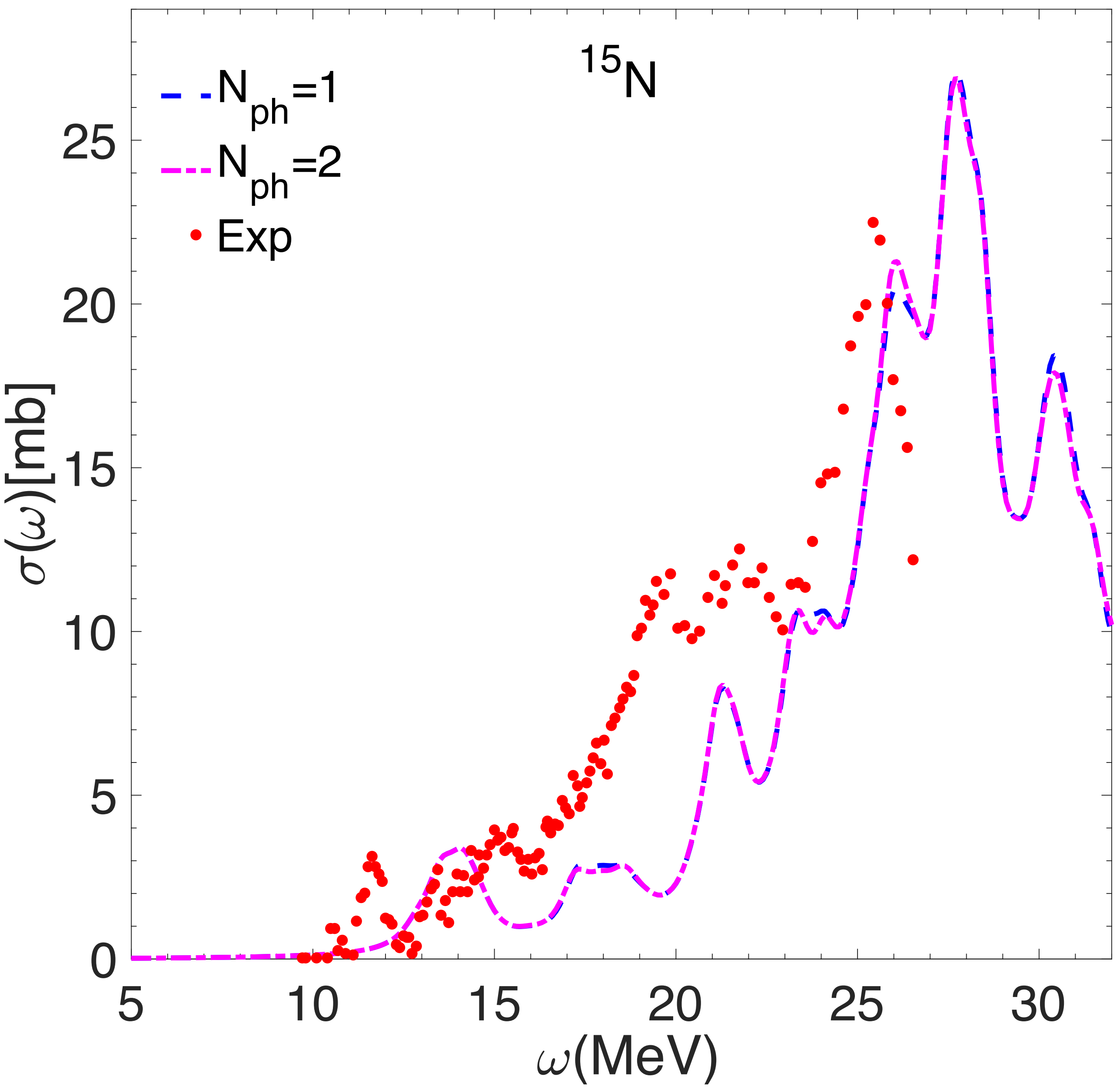
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **99**, 014316 (2019)

E1 response in ^{15}N



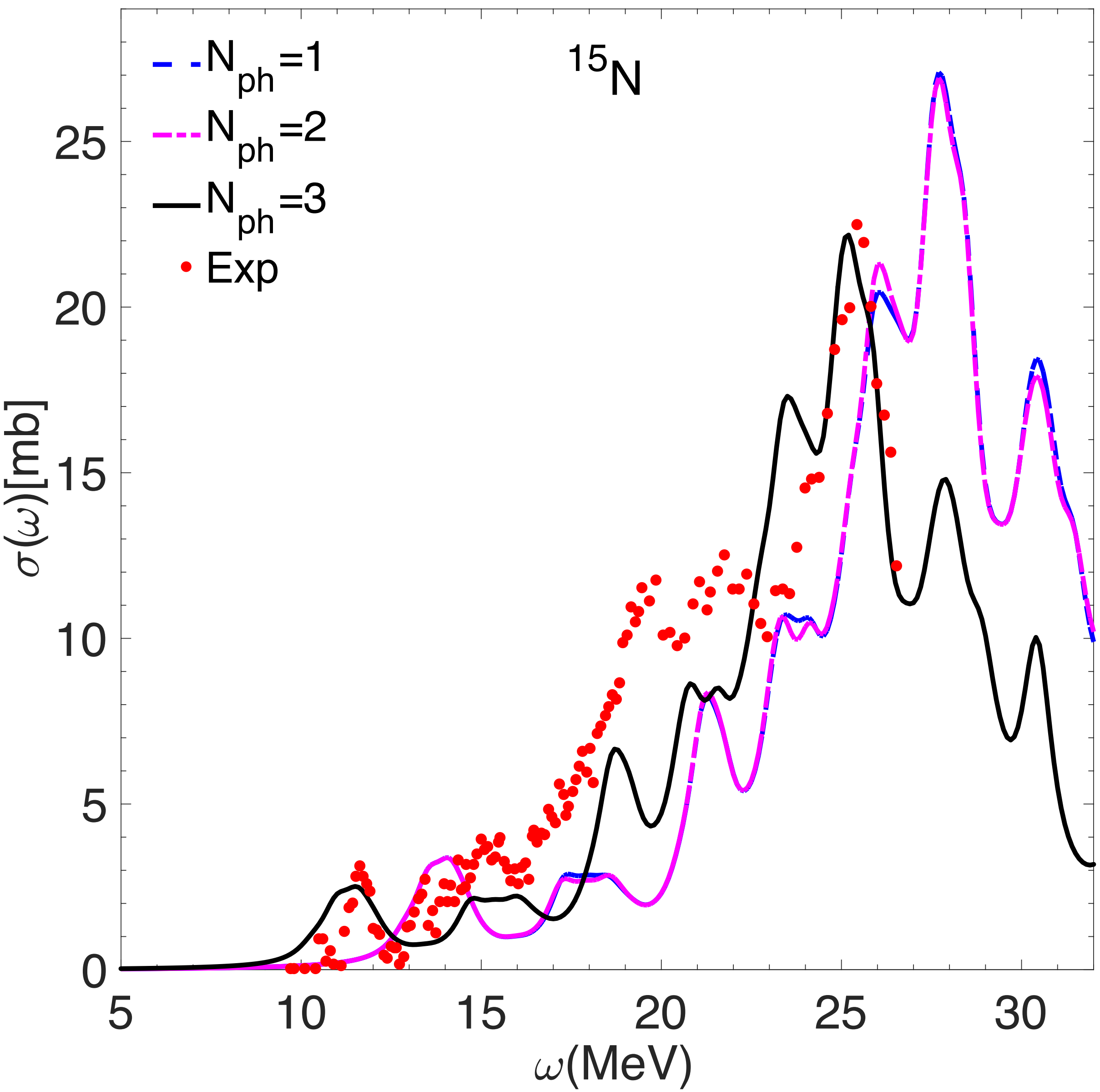
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **99**, 014316 (2019)

E1 response in ^{15}N



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **99**, 014316 (2019)

E1 response in ^{15}N



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **99**, 014316 (2019)

Ground state moments and transitions in A=15

^{15}O

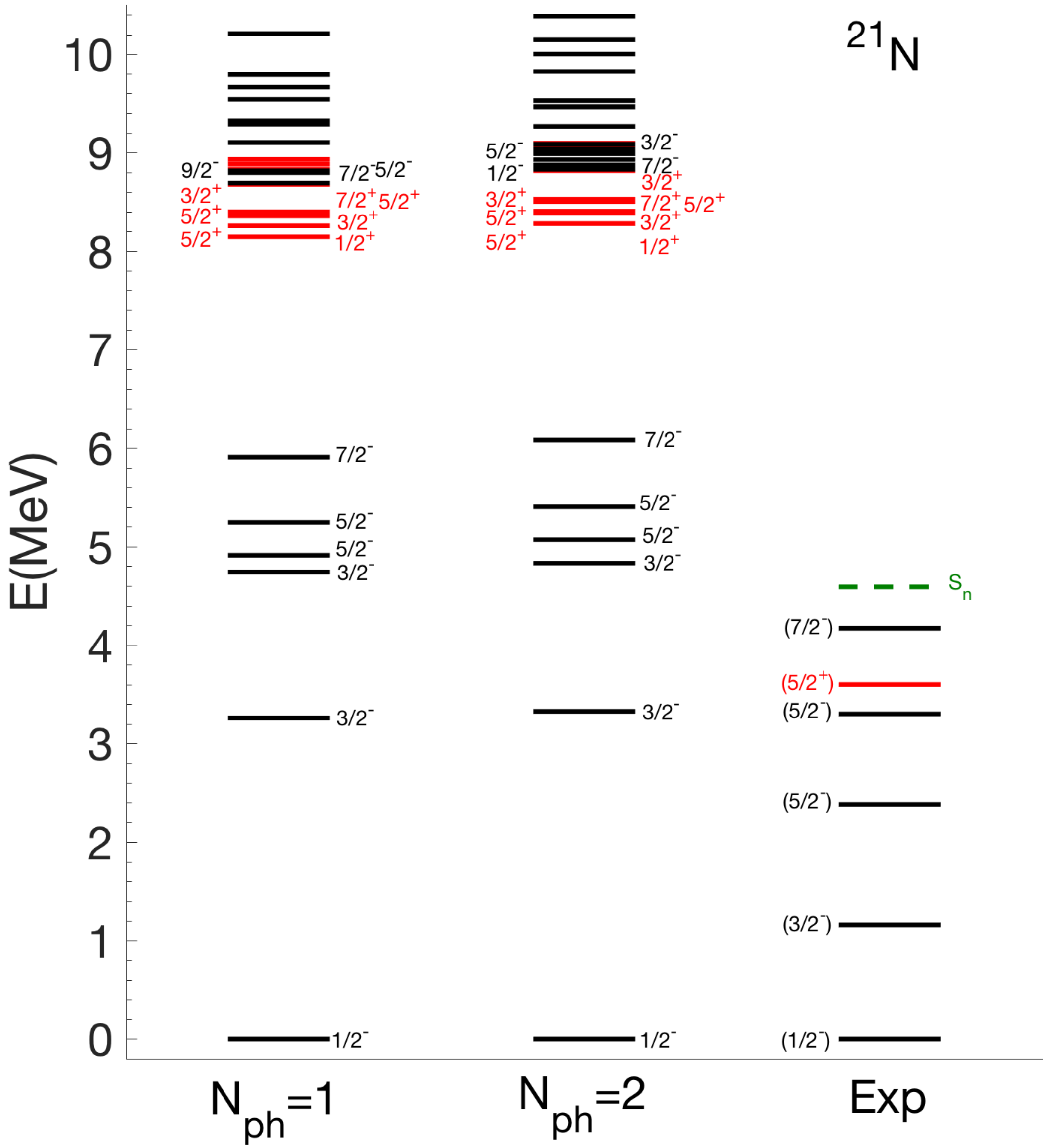
ν_f	Theory	Exp
μ	+0.5986	±0.7189(8)
log*ft*	3.650	3.637
$B(E1; 1/2^+_1 \rightarrow 1/2_1^-)$	0.06	$(1.4 \pm 0.2) \times 10^{-3}$
$B(E2; 3/2^-_1 \rightarrow 1/2_1^-)$	0.03	>0.28
$B(E3; 5/2^+_1 \rightarrow 1/2_1^-)$	3.95	4±2
$B(M1; 3/2^-_1 \rightarrow 1/2_1^-)$	0.56	$>5.3 \times 10^{-2}$

^{15}N

ν_f	Theory	Exp
μ	-0.2499	-0.2832
$B(E1; 1/2^+_1 \rightarrow 1/2_1^-)$	0.03	$(4.3 \pm 1.1) \times 10^{-4}$
$B(E2; 3/2^-_1 \rightarrow 1/2_1^-)$	1.20	2.91±0.24
$B(E3; 5/2^+_1 \rightarrow 1/2_1^-)$	3.11	7±2
$B(M1; 3/2^-_1 \rightarrow 1/2_1^-)$	0.687	0.578±0.015

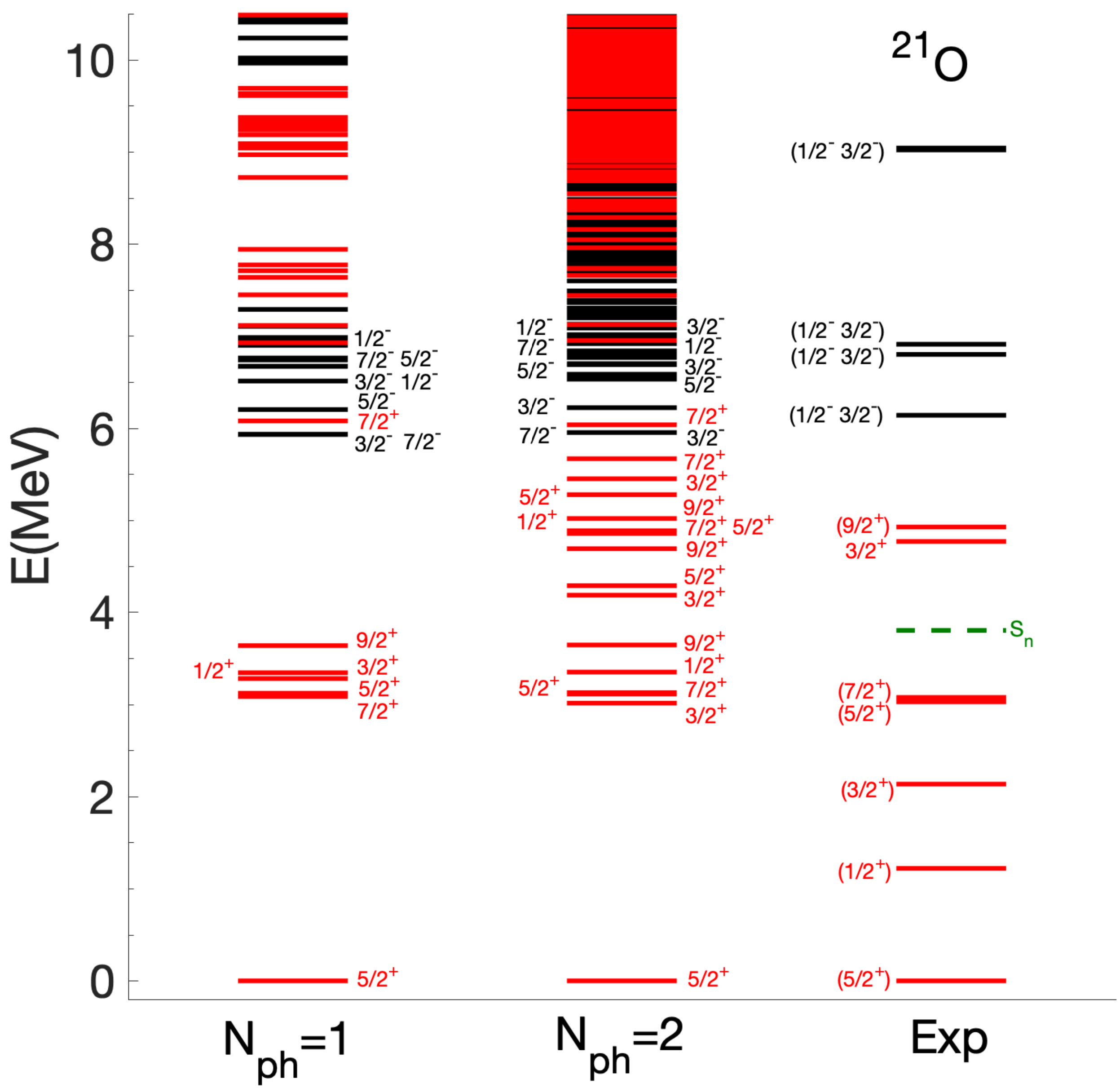
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **99**, 014316 (2019)

Application to neutron rich nuclei: ^{21}N spectra



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 034311 (2018)

Application to neutron rich nuclei: ^{21}O spectra



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 034311 (2018)

Ground state β decay of $^{21}N \longrightarrow {}^{21}O$

ν_f	ω_f (MeV)	log(*ft*)	B(GT)
$3/2^-$	5.95	7.14	0.00044
$3/2^-$	6.53	7.62	0.00015
1/2-	6.69	8.29	0.00003
$1/2^-$	7.30	7.55	0.00016
$3/2^-$	10.02	5.60	0.01513
$3/2^-$	10.43	6.30	0.00306
$3/2^-$	13.05	5.70	0.00004
$3/2^+$	14.54	5.32	0.01221

ν_f	ω_f (MeV) $_{EXP}$	log(*ft*)$_{EXP}$	B(GT) $_{EXP}$
$(1/2^-, 3/2^-)$	6.14	5.44 ± 0.06	0.0224 ± 0.0032
$(1/2^-, 3/2^-)$	6.80	5.19 ± 0.06	0.0399 ± 0.0056
$(1/2^-, 3/2^-)$	6.91	5.44 ± 0.07	0.0224 ± 0.0035
$(1/2^-, 3/2^-)$	9.02	4.78 ± 0.06	0.1015 ± 0.0145
$(1/2^-, 3/2^-)$	9.04	4.62 ± 0.06	0.1462 ± 0.0206

G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **97**, 034311 (2018)

Concluding remarks

- We have seen that the multiphonon play an important role.

- Expecially the three-phonon which through their coupling to one-phonon states enhance the density of states at low-energy in agreement with experiments.

- The *E*1 cross sections of **^{17}O and ^{15}N and ^{22}O** are in fair agreement with the experimental data.

Serious discrepancies:

- **The positive parity states of A=17 remain at high energy.**
- **The low-lying states of A=15 are too high in energy.**

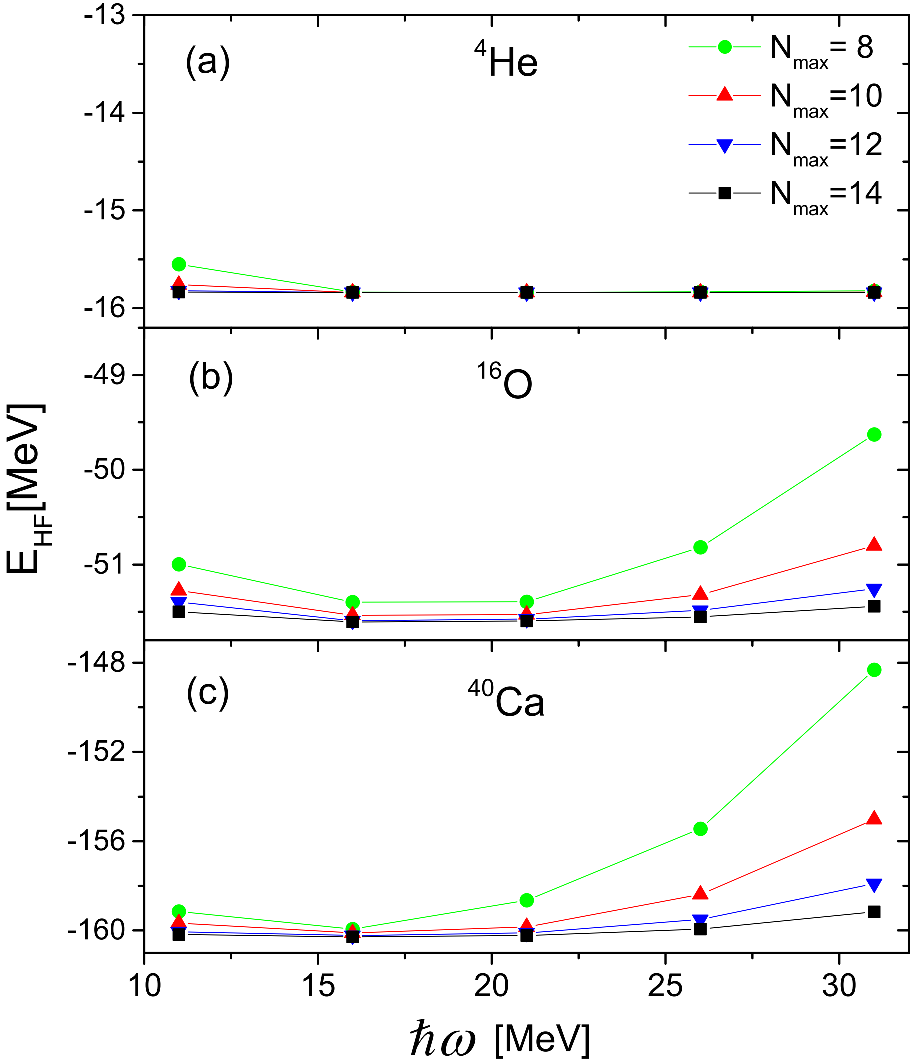
- **Recipies:**

1) Use of a better potential: Preliminary results using $NNLO_{sat}$ potential, which includes explicitly the three-body force, are promising.

2) Need of four phonon states

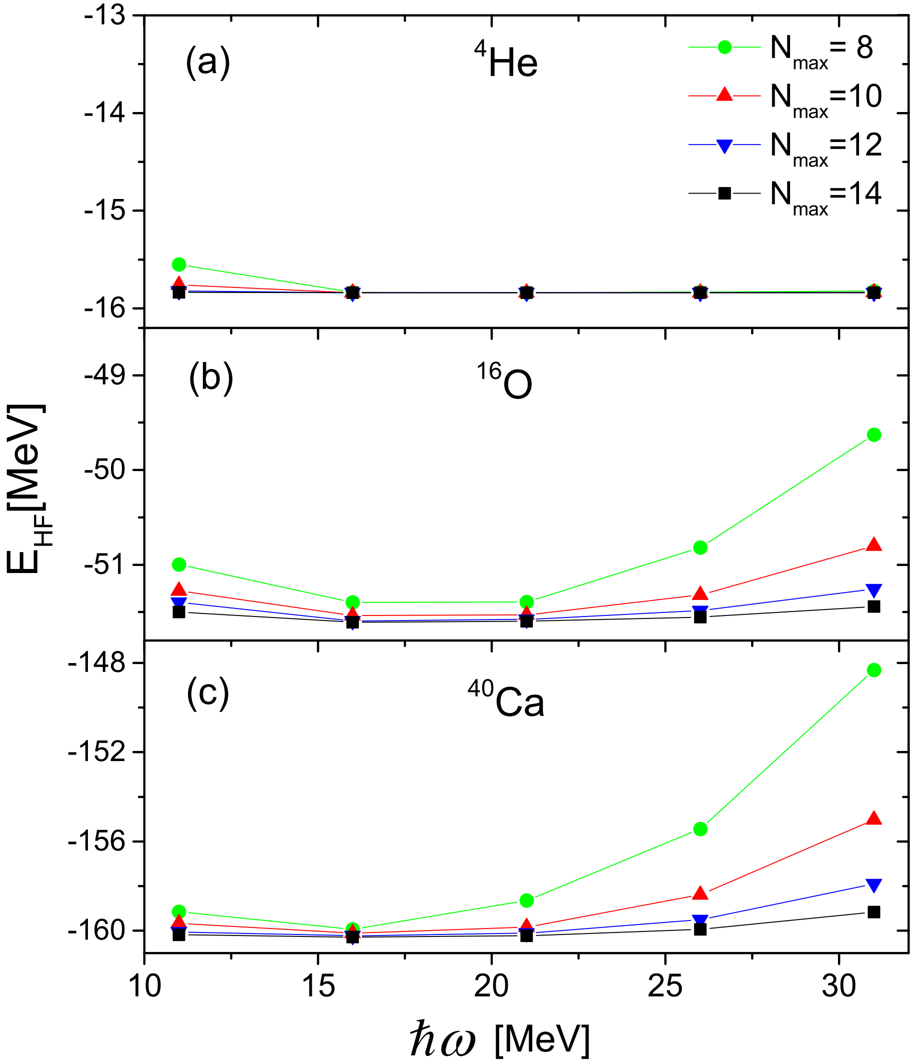
Thank you

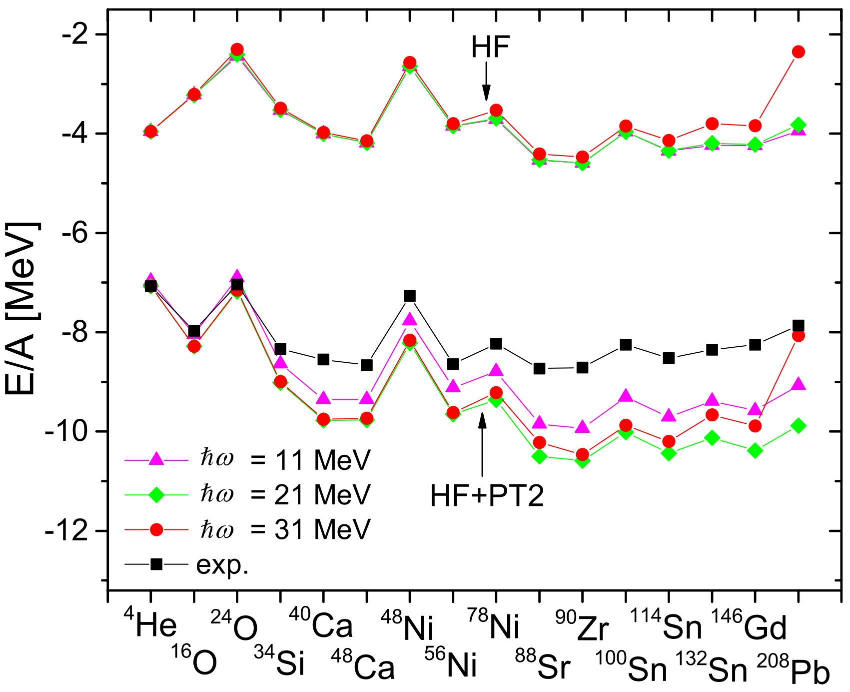
Ground state correlations



G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 024306 (2017)

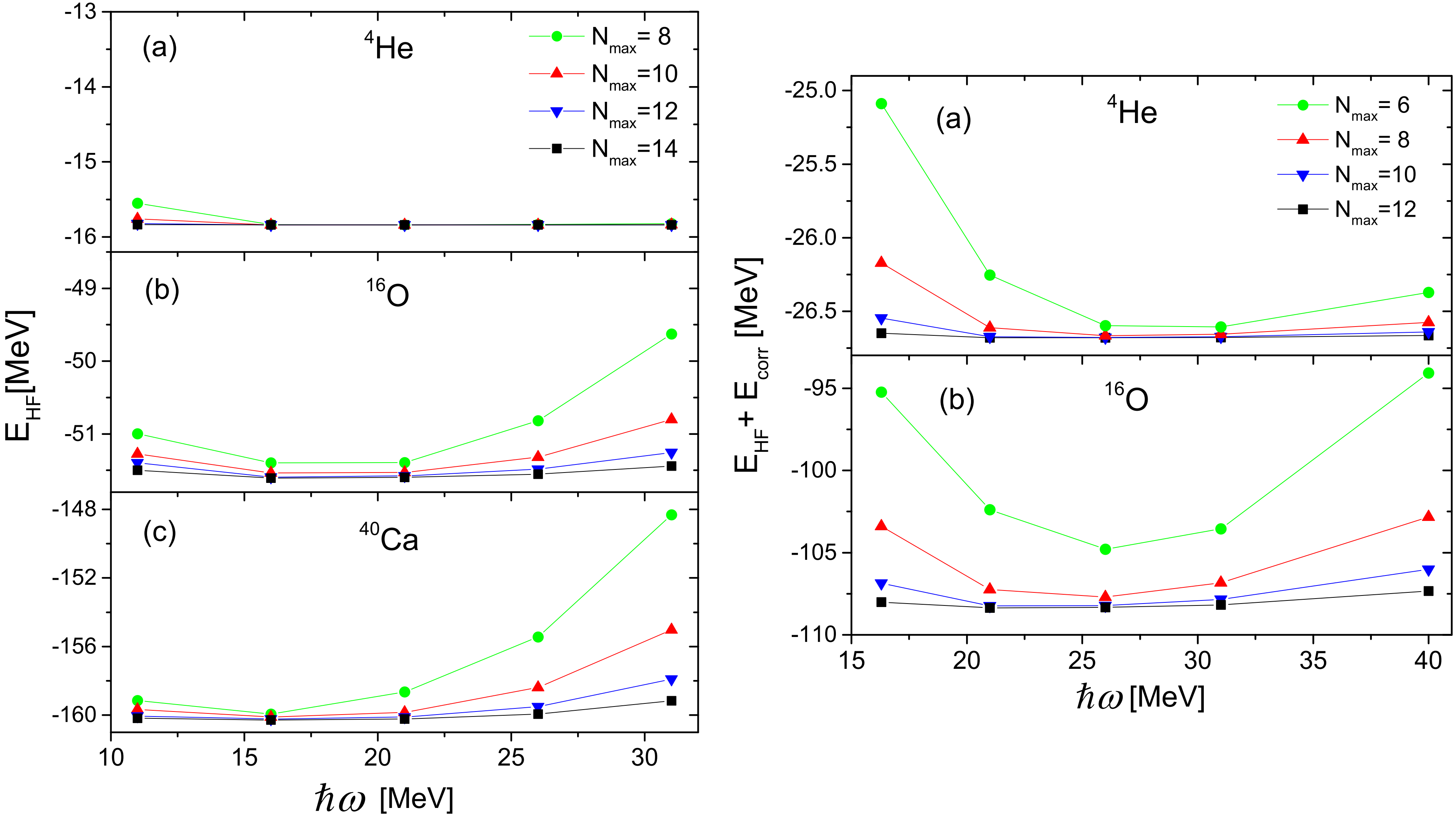
Ground state correlations





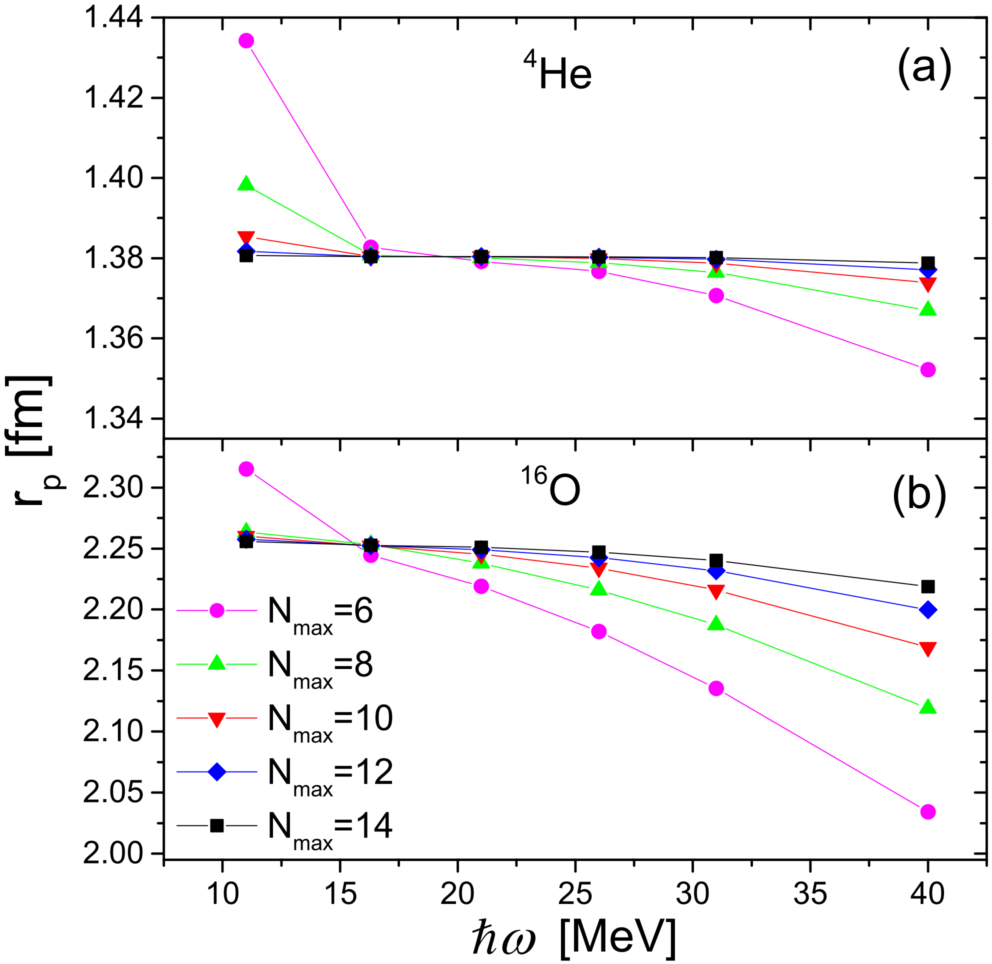
G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 024306 (2017)

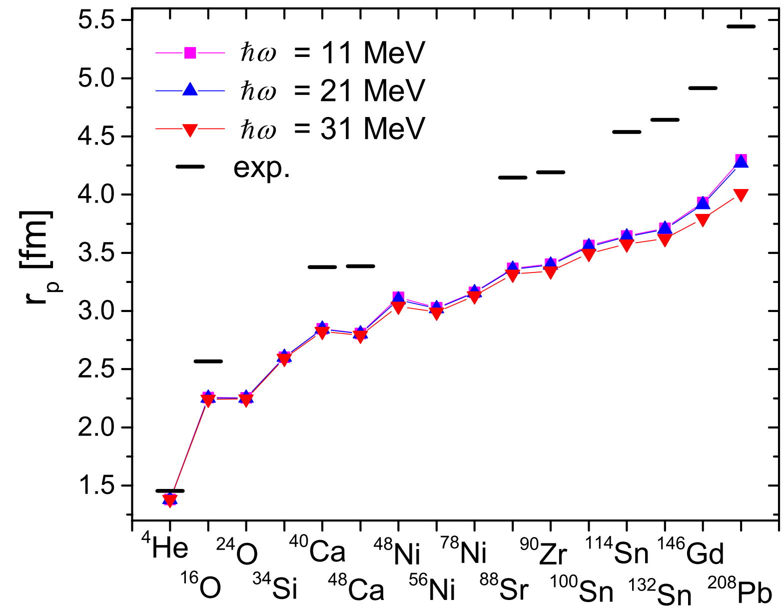
Ground state correlations

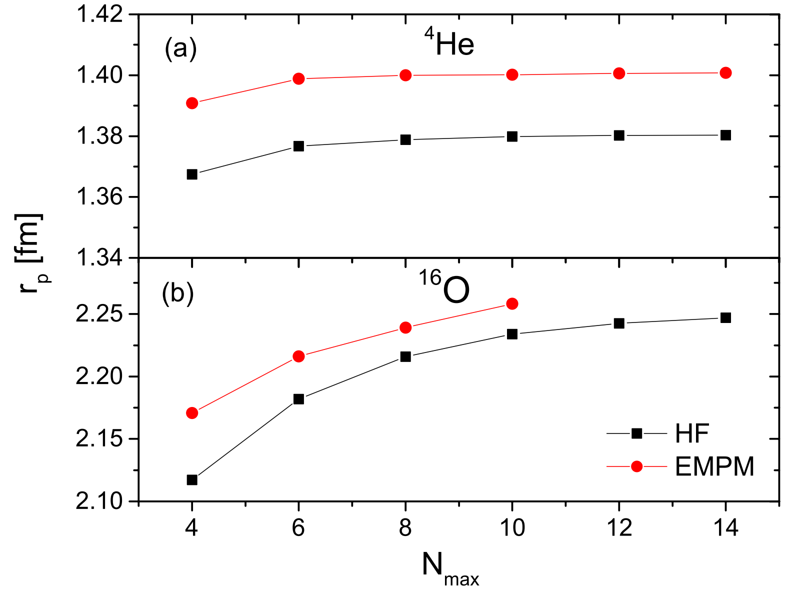


G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 024306 (2017)

Ground state correlations







G. De Gregorio, F Knapp, N. Lo Iudice, P. Vesely Phys. Rev. C **95**, 024306 (2017)