A CONTACT MODEL FOR QCD AND ITS APPLICATIONS TO HEAVY-QUARKS PHYSICS

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TNPI 2017-CONFERENCE ON THEORETICAL NUCLEAR PHYSICS IN ITALY

CORTONA-OCTOBER 2017

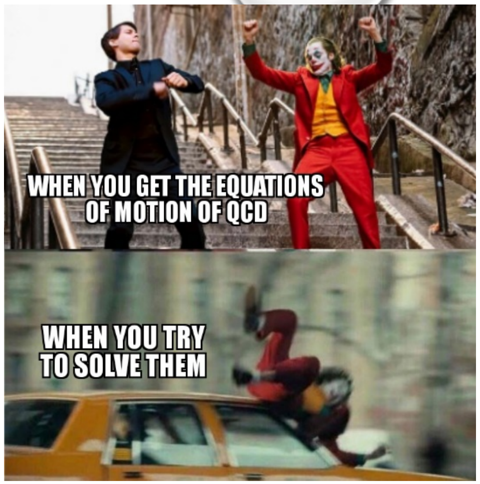
QCD and Hadron Physics
Schwinger-Dyson Equations

- Quantum chromodynamics (QCD) it the theory of quarks, gluons and their interactions.
- In particular, the way particles interact through strong interactions hides fascinating secrets, to be still found out in the next years.
- Due to the theory running coupling constant particular behavior, studies on QCD are done in three regimes:
- A light sector, where the interactions are mainly dominated by chiral symmetry breaking and a strong coupling.
- A heavy sector, where the coupling tends to approach asymptotic freedom and the systems can be treated in a non relativistic way.

QCD and Hadron Physics
Schwinger-Dyson Equations

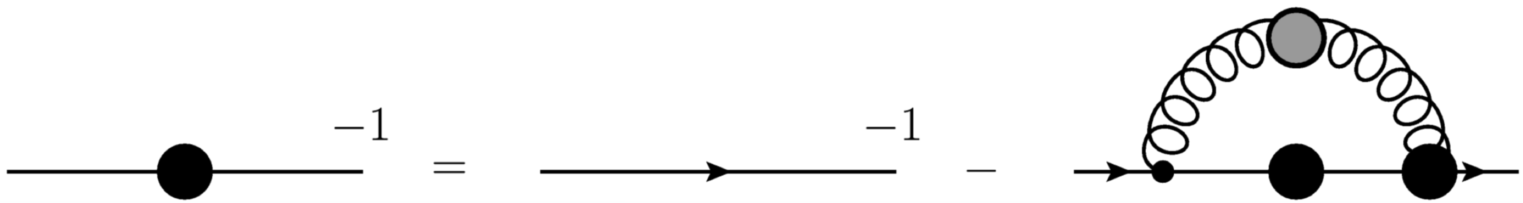
- A transition region, where the dynamics goes from the regime of chiral dynamics to the heavy sector symmetries.
- In order to study this bridge, we study the QCD simplest bound states: mesons.
- We use the Schwinger-Dyson equations (SDEs) of quantum chromodynamics. They provide a natural means to study the different QCD regions.
- Earlier studies of QCD through SDEs with refined truncations are a brute force numerical evaluation, which stops short of exploring the large momentum transfer region.
- We present a simple momentum independent contact interaction (CI) model, which provides a simple scheme to exploratory studies on QCD.

QCD and Hadron Physics
Schwinger-Dyson Equations



$$\int \mathcal{D}[\varphi]\frac{\delta}{\delta\varphi} = 0$$

Quark SDE:



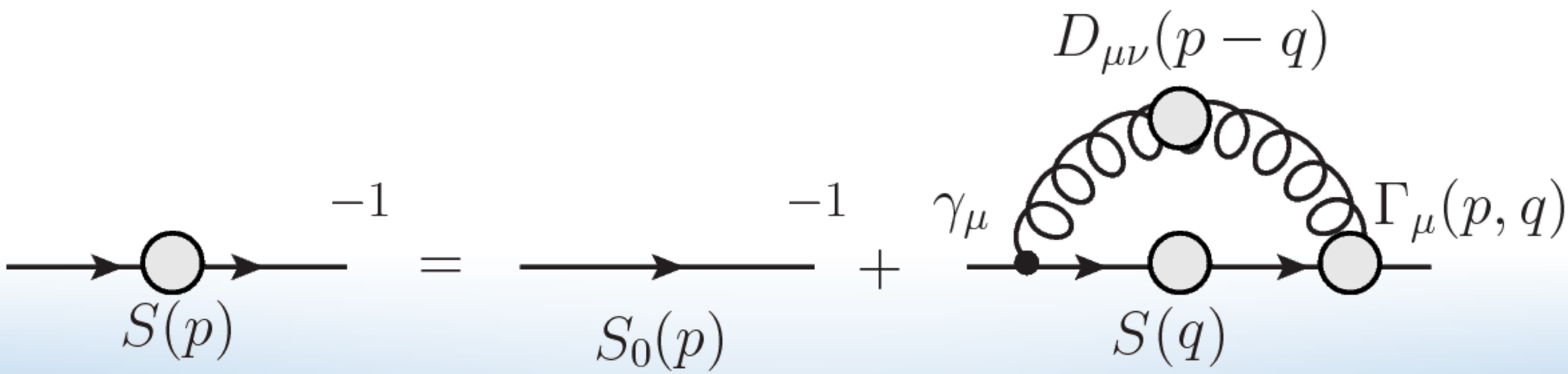
Mathematical expression:

$$S_f^{-1}(p) = \mathrm{i}\gamma\cdot p + m_f + \Sigma_f(p)$$

$$\Sigma_f(p) = \int \frac{d^4k}{(2\pi)^4} g^2 \gamma_\mu D_{\mu\nu}(p-k)\frac{\lambda^a}{2} S_f(k)\Gamma^a_\nu(p,k)$$

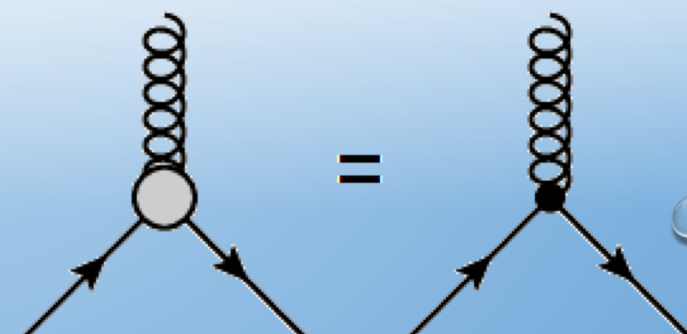
QCD and Hadron Physics
Schwinger-Dyson Equations

In order to solve the SDE, we employ a truncation scheme:



Rainbow truncation:

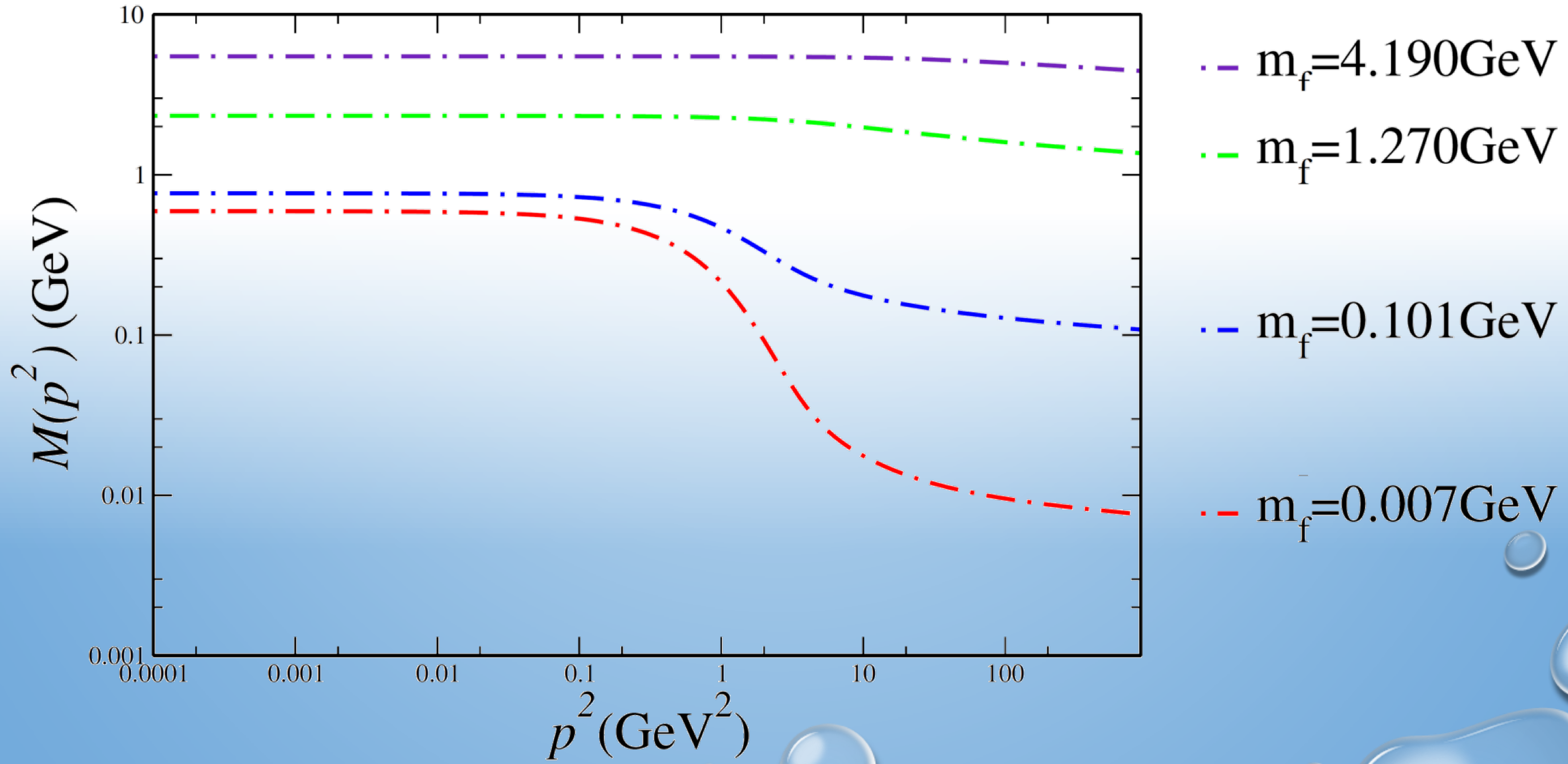
x **effective coupling**



QCD and Hadron Physics
Schwinger-Dyson Equations

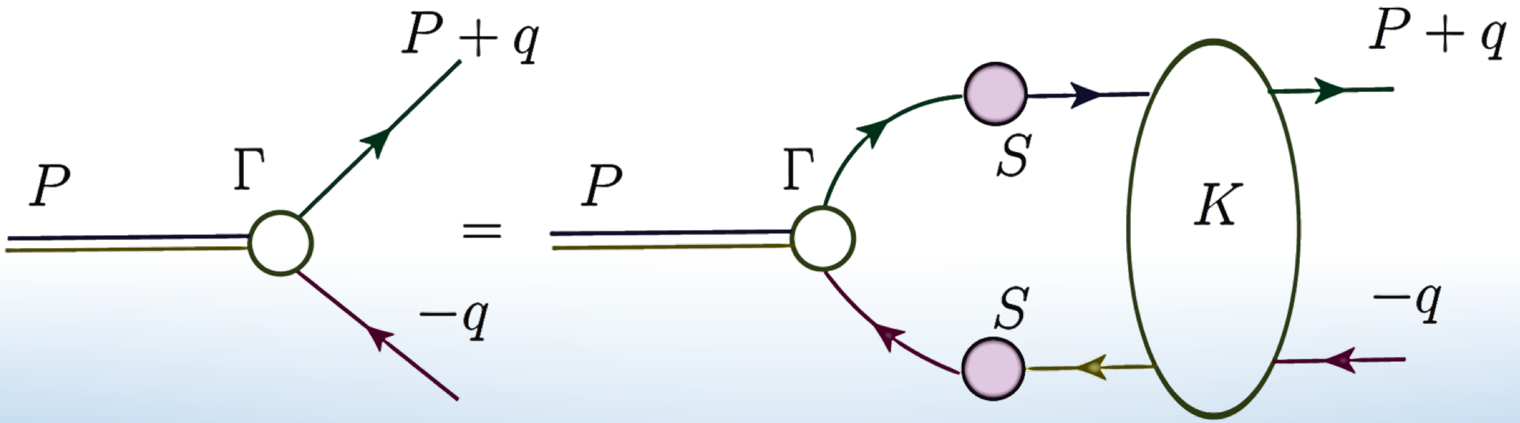
- Mass function for different quark masses.

P. Maris and P. C. Tandy Phys. Rev. C60, 055214 (1999)



Bethe-Salpeter Equation

A meson appears as a pole in the quark-antiquark (quark-quark) Green function → Bethe-Salpeter Equation. In order to compute diquark mass, there is a factor a ½ due to the color factors.



$$[\Gamma_H(k;P)]_{tu} = \int dq \chi(q;P)_{sr} K^{rs}_{tu}(q,k;P)$$

$$\chi(q;P) = S_f(q_+)\Gamma_H(q;P)S_g(q_-)$$

Remember G. Salmè Talk

E. E. Salpeter **Phys. Rev. 84, 1226 (1951)**

E. E. Salpeter and H. A. Bethe **Phys. Rev. 84, 1232 (1951)**

Contact Interaction

- We use a contact interaction model mediated by a vector-vector interaction employed in:

  L. Xiomara Gutiérrez, et. al., Phys. Rev. C81, 065202 (2010);
  H.L.L. Roberts, et. al. Phys. Rev. C82, 065202 (2010);
  Phys. Rev. C83, 065206 (2011).

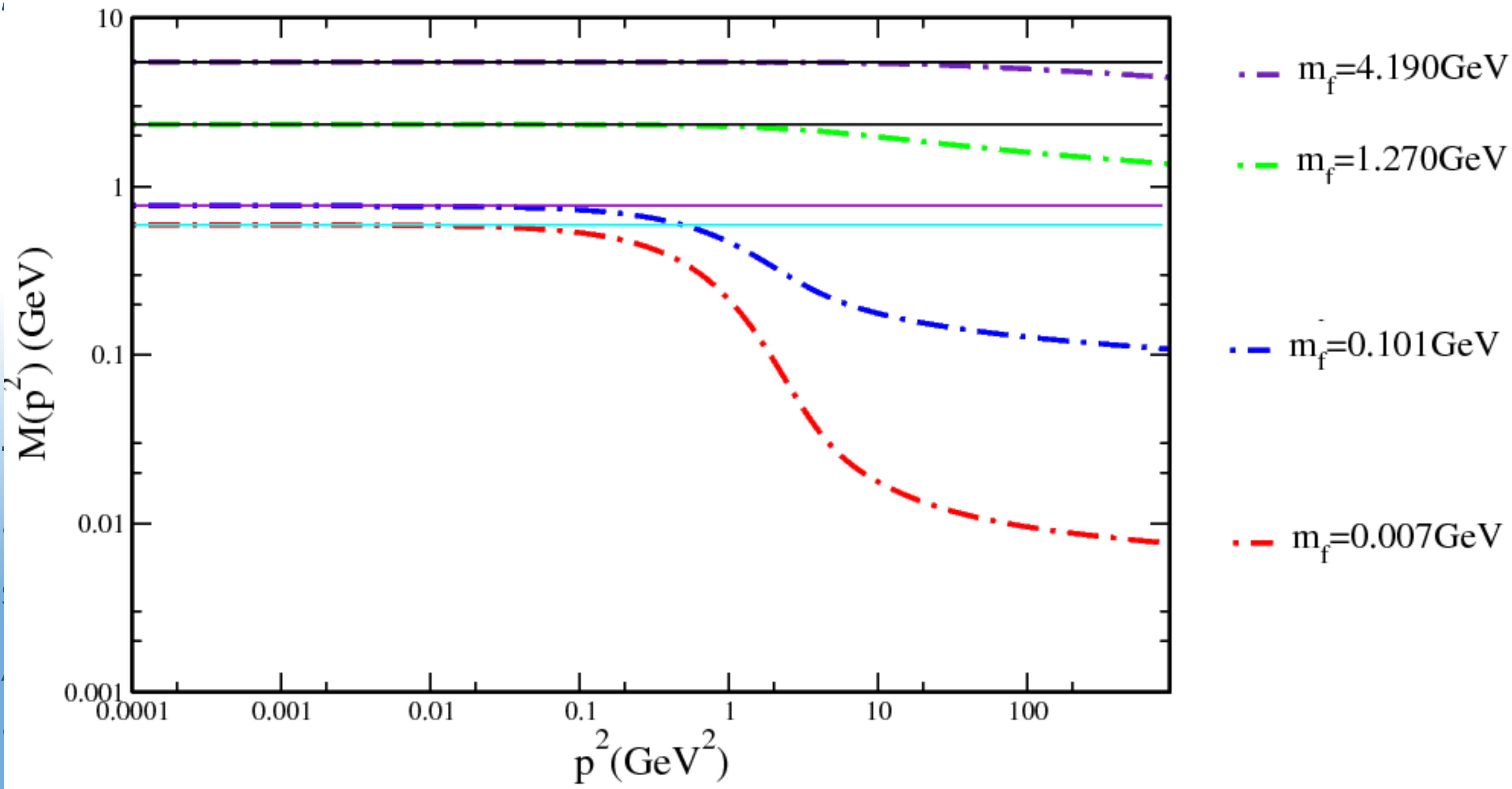
$$g^2 D_{\mu\nu}(k) = \frac{4\pi\alpha_{\mathrm{IR}}}{m_g^2}\delta_{\mu\nu} \equiv \frac{1}{m_G^2}\delta_{\mu\nu}$$

$$m_g = 800\,\mathrm{MeV}$$

$$\alpha_{\mathrm{IR}} = 0.93\pi$$

- This model provides a simple scheme to exploratory studies of the spontaneous chiral symmetry breaking and its consequences like:
  - Dynamical mass generation.
  - Quark condensate.
  - Goldstone bosons in chiral limit.
  - Confinement.

Contact Interaction



Light mesons: masses and decay constants

C. Chen, et. al. Few Body Syst. 53, 293 (2012).
Few Body Syst. 51, 1 (2011).

M_u=367	Λ_{UV}=905	Λ_{IR}=240	$m_{u,d}$=7	α_{IR}=0.93π
Masses in MeV	$m_\pi(1S)$	$m_\rho(1S)$	$m_\sigma(1P)$	$m_{a1}(1P)$
PDG(2016)	140	780	1000-1200	1230
Contact Interaction	140	930	1290	1380
Decay Constants (GeV) (g_{so}=0.24)				
PDG(2016)	130	152		
Contact Interaction	101	129		
Bethe-Salpeter Amplitudes				
E_H	3.593	1.530	0.472	0.309
F_H	0.474			
Charge-Radii (fm)				
PDG(2016)	0.672			
Contact Interaction	0.450			

Charm mesons: masses and decay constants

M.A. Bedolla, et. al — Phys. Rev. D 92, 054031 (2015).
M.A Bedolla, et. al. — Phys. Rev. D 93, 094025 (2016).

M_u=1600	Λ_{UV}=905	Λ_{IR}=240	$m_{u,d}$=1578	α_{IR}=0.93π
Masses in MeV	$m_{\eta c}(1S)$	$m_{J/\Psi}(1S)$	$m_{\chi c0}(1P)$	$m_{\chi c1}(1P)$
PDG(2016)	2983	3096	3414	3510
Contact Interaction	298	2994	3419	3442
Decay Constants (MeV) (g_{so}=0.24)				
PDG(2016)	361	416		
Contact Interaction	8.38	7.96		
Bethe-Salpeter Amplitudes				
E_H	6.026	3.023	0.437	0.297
F_H	1.711			
Charge-Radii (fm)				
SDE(2017)	0.219			
Contact Interaction	0.186			

Contact Interaction

M.A. Bedolla, et. al Phys. Rev. D 92, 054031 (2015).
M.A Bedolla, et. al. Phys. Rev. D 93, 094025 (2016).

- The leptonic decay constant is highly influenced by the high momentum tail of the quark mass function.
- This high momentum region probes the wave function at the origin.
- The CI yields constant dressing functions with no perturbative tail.
- By increasing the mass of heavy quarks, quarkonuim becomes increasingly point like—and the closer the quarks, the smaller the interaction between them.
- We need to reduce the effective interaction strength for the ci to extend to the heavy quarks sector.
- A reduction in the strength of the kernel has to be compensated by an increased ultraviolet cutoff.

Charm mesons: masses and decay constants

M.A. Bedolla, et. al Phys. Rev. D 92, 054031 (2015).
M.A Bedolla, et. al. Phys. Rev. D 93, 094025 (2016).

M_c=1482	Λ_{UV}=2400	Λ_{IR}=240	m_c=1090	α_{IR}=0.172
Masses in MeV	$m_{\eta c}(1S)$	$m_{J/\Psi}$ (1S)	$m_{\chi c0}(1P)$	$m_{\chi c1}(1P)$
PDG(2016)	2983	3096	3414	3510
Contact Interaction	2976	3090	3374	3400
Decay Constants (MeV) (g_{so}=0.24)				
PDG(2016)	238	294		
Contact Interaction	255	206		
Bethe-Salpeter Amplitudes				
E_H	2.156	0.612	0.051	0.028
F_H	0.406			
Charge-Radii (fm)				
SDE(2017)	0.219			
Contact Interaction	0.250			

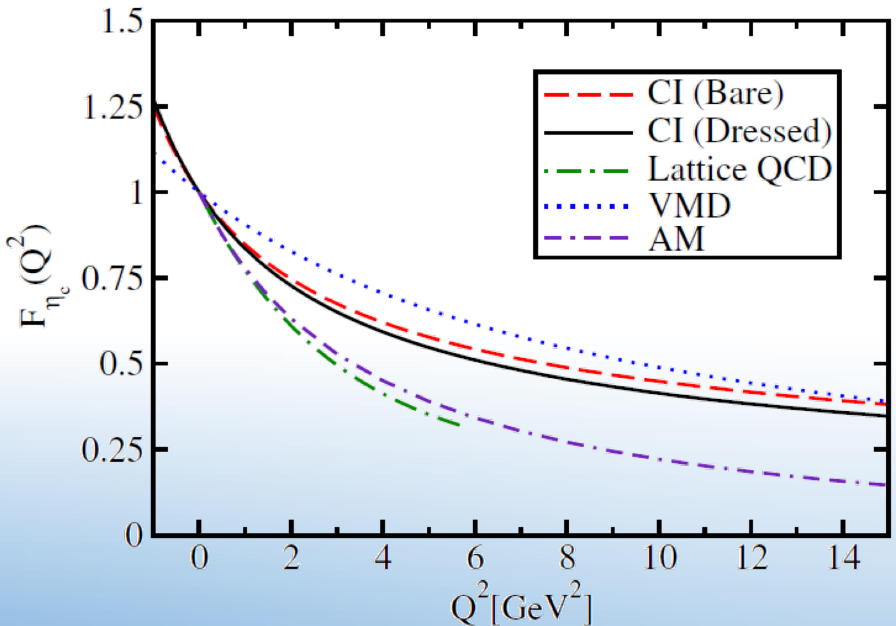
Bottom mesons: masses and decay constants

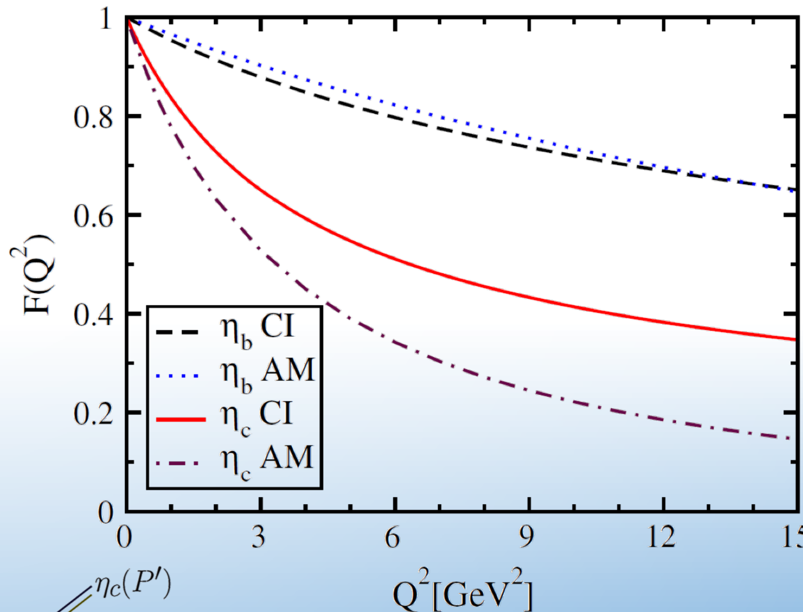
K. Raya, et. al   Few Body Syst. 59 (2018) no.6, 133

M_b=4710	Λ_{UV}=6400	Λ_{IR}=240	m_b=3800	α_{IR}=0.023
Masses in MeV	$m_{\eta b}(1S)$	$m_{\Upsilon}(1S)$	$m_{\chi b0}(1P)$	$m_{\chi b1}(1P)$
PDG(2016)	9400	9460	9860	9892
Contact Interaction	9407	9547	9671	9680
Decay Constants (MeV) (g_{so}=0.24)				
PDG(2016)	----	506		
Contact Interaction	553	219		
Bethe-Salpeter Amplitudes				
E_H	0.851	0.184	0.001	0.0008
F_H	0.173			
Charge-Radii (fm)				
SDE(2017)	0.086			
Contact Interaction	0.109			

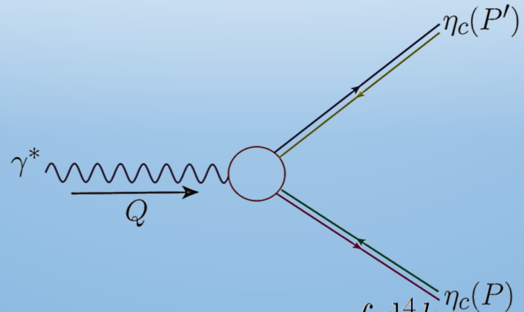
Elastic Form Factor

M.A Bedolla, et. al Phys. Rev. D 93, 094025 (2016).
K. Raya, et. al Few Body Syst. 59 (2018) no.6, 133





Charge Radii	
Dyson-Schwinger Equations	0.219fm
Lattice	0.25fm
Contact Interaction	0.21fm
VMD	0.156fm
Algebraic Model	0.256fm

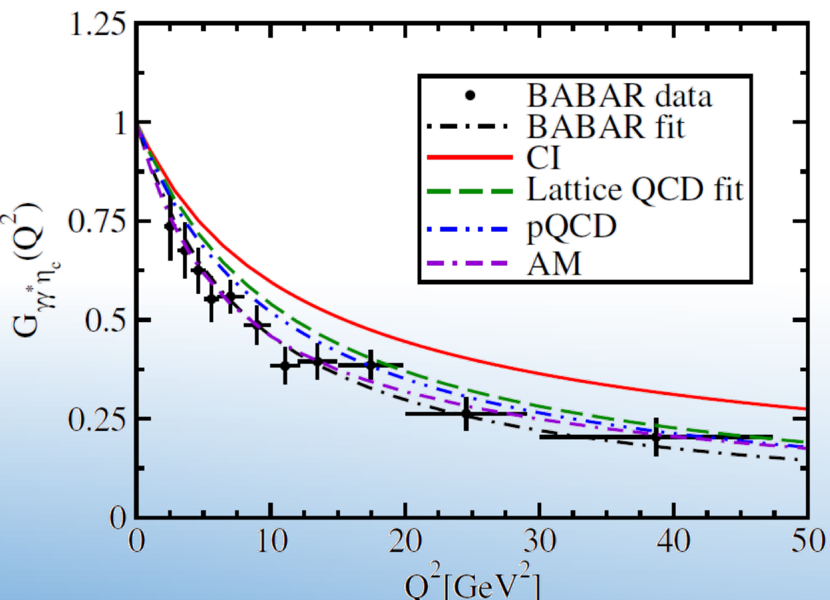


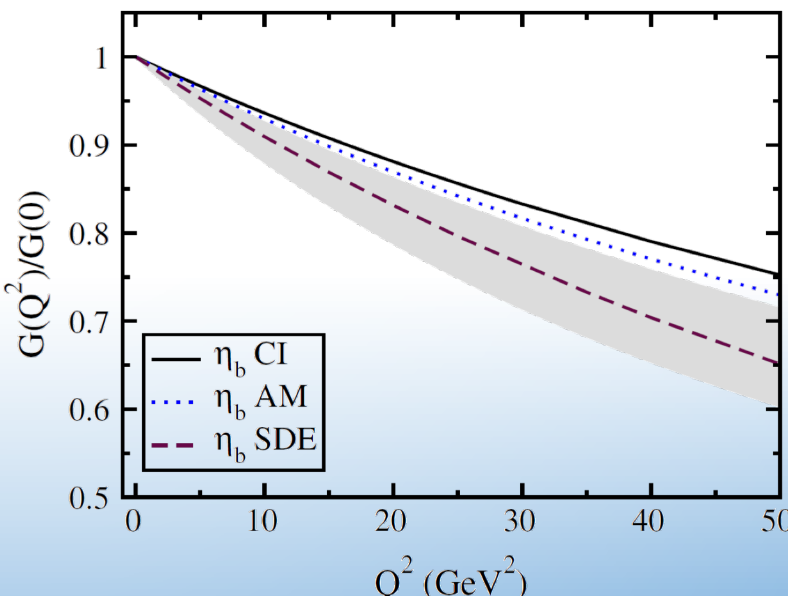
Charge Radii	
η_b	0.107fm
η_c	0.210fm

$$\Lambda_\mu^{\gamma^* f/H}(P,Q) = 2N_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\,[i\Gamma_H(-P_o)S_f(k_2) \\ i\Gamma_\mu(Q)S_f(k_1)i\Gamma_H(P_i)S_g(k)]\,,$$

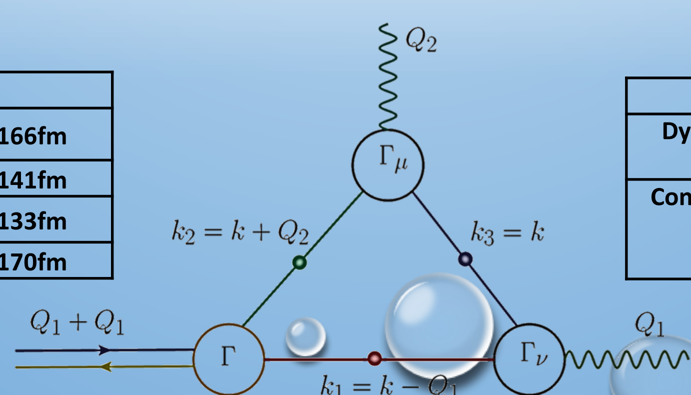
Transition Form Factor

M.A Bedolla, et. al. — Phys. Rev. D 93, 094025 (2016).
K. Raya, et. al — Few Body Syst. 59 (2018) no.6, 133
K. Raya, M. Ding et. al. — Phys. Rev. D 95, 074014.





Interaction Radii	
BABAR	0.166fm
Lattice	0.141fm
CI model	0.133fm
Algebraic Model	0.170fm



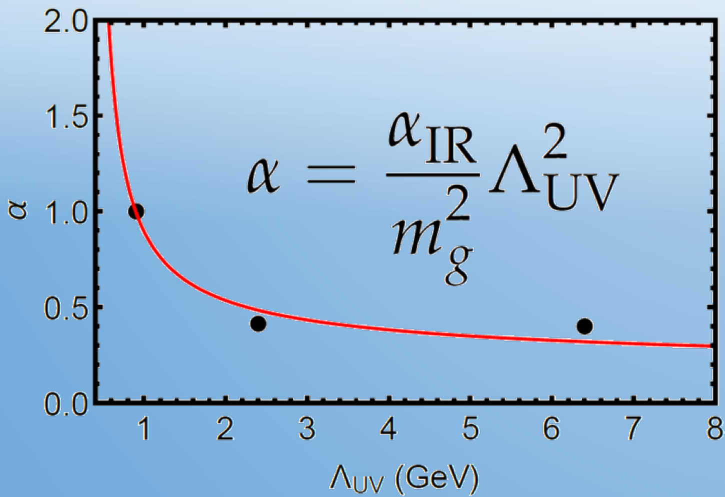
Interaction Radii	
Dyson-Schwinger Equations	0.041fm
Contact Interaction	0.043fm

Contact Interaction

K. Raya, et. al Few Body Syst. 59 (2018) no.6, 133

quark	α_{IR}	Λ_{UV} [GeV]	α	Normalized
u, d, s	2.922	0.905	3.739	1
c	0.172	2.4	1.547	0.414
b	0.023	6.4	1.496	0.400

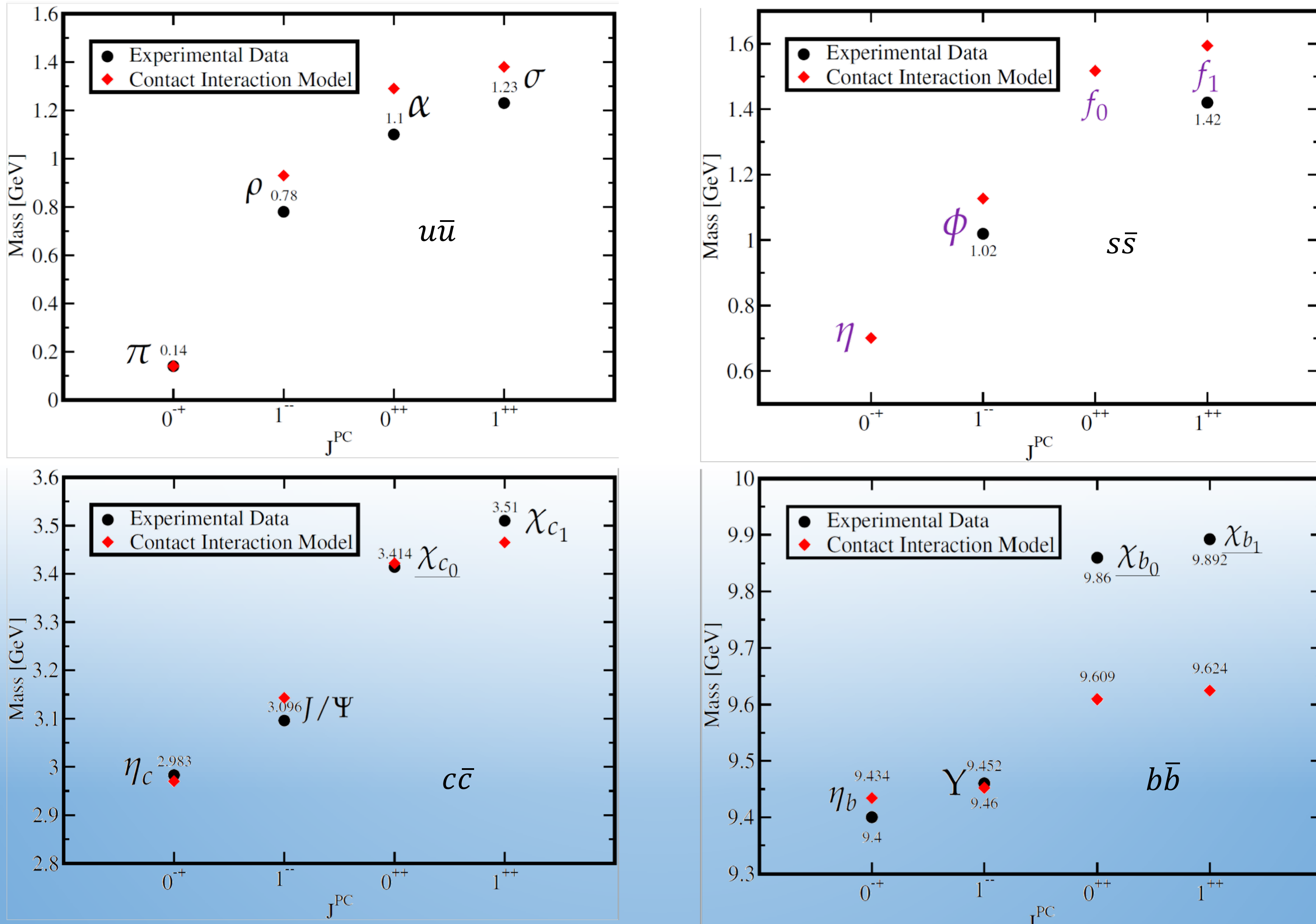
Let's define a dimensionless coupling constant.



$$\alpha(\Lambda_{UV}) = a \ln^{-1}(\Lambda_{UV}/\Lambda_0)$$

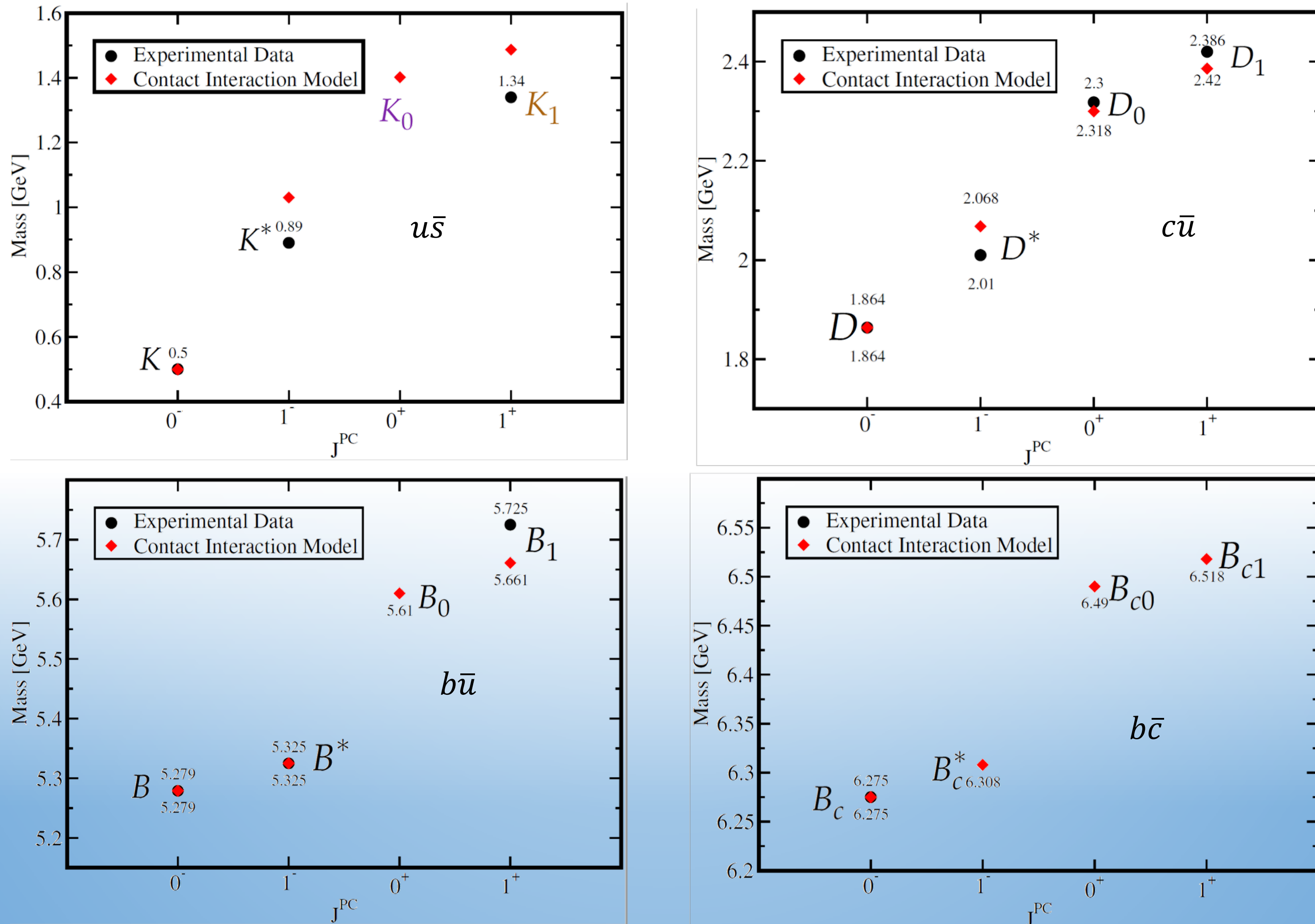
$$a = 0.923 \quad \Lambda_0 = 0.357$$

Meson Spectra



M. A. Bedolla, and E. Santopinto EPJ Web Conf. 192 (2018) 00039

Meson Spectra

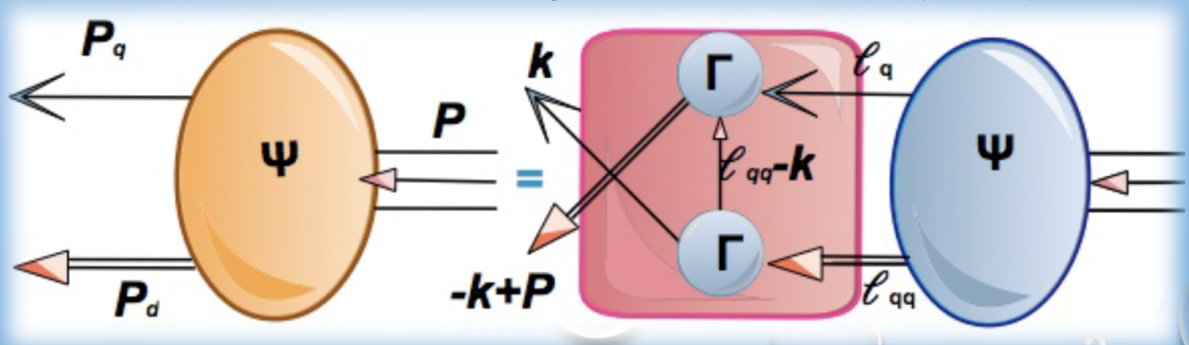


M. A. Bedolla, and E. Santopinto EPJ Web Conf. 192 (2018) 00039

How to study baryons in the Schwinger-Dyson Equations approach

- The single heavy baryons are not all discovered.
- In 2002, the observation of the double charm Ξ_{cc}^{++} baryon with a mass of 3460. However, recent observations by LHCb put it in the range of 3621. This produced a new interest in restudy baryons with heavy-quarks.
- The Schwinger-Dyson equations and the contact interaction have a baryon description in a quark-diquark interaction kernel with a quark exchange interaction.
- These studies have been performed in both light-sector and heavy-sector. Studies on tetraquarks using this scheme produces mainly meson-molecules

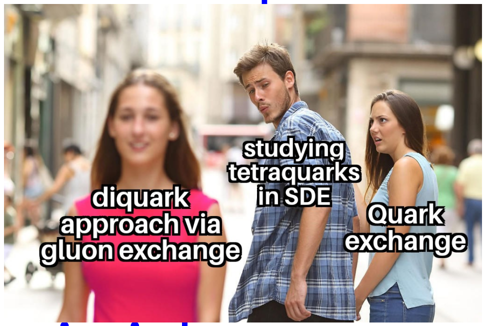
Chen Chen, et. al — Few Body Syst. 53 (2012) 293-326
Pei-Li Yin, et. al — Phys. Rev. D100 (2019) no.3, 034008
P.C. Wallbott, et. al — J. Phys. Conf. Ser. 1024 (2018) no.1, 01205



Tetraquarks in the Diquark Model

M.A. Bedolla, Few Body Syst. 60 (2019) no.2, 24.

- In the diquark model, a tetraquark is seen as a pointlike diquark embedded with a pointlike antidiquark:



$$t = D\bar{D}$$

- Diquarks are colored, so they should be confined.
- Two quarks can couple in an antisymmetric color anti triplet or a symmetric color sextet: $3 \otimes 3 = \bar{3} \oplus 6$
- The color sextet leads to repulsive interactions, so diquarks in the anti triplet are the only ones that can couple to form bound-states.
- The ground-state diquark is in S-wave, so its spin is 0 or 1 .
- To study scalar tetraquarks, Fermi-Dirac statistics only allows spin 1 diquarks for tetraquarks with the same flavor.
- We compute the scalar *bbbb* tetraquark mass an get m=18.45

Final Remarks

The contact interaction is a simple model in such a way that the calculation of static and dynamic observables are performed in an analytical or semi-analytical way. These results can be compared and contrasted with "full QCD" and experiments.

The contact interaction includes important features of QCD, for example:

- Confinement.
- Ward identities and its consequences: Goldberger-Triemann relations.
- Dynamical chiral symmetry breaking.
- Gell-Mann-Oakes-Renner relations.

Final Remarks

We used the contact interaction in order to describe the spectra of light , charm, bottom and heavy-light mesons successfully.

Generally, the coupling is a function of the larger mass scale (which is related to the quarks masses involved). Therefore, we can utilize the contact interaction to study mesons composed of light and heavy quarks.

We also calculated the weak decay constants of pseudoscalar and vector mesons. And the elastic form factors of pseudoscalars.

The form factors are harder than those provided by "full QCD" and experimental results.

Final Remarks

When it is possible to make a comparison, our results are in a good of agreement with experimental data and with models employing the SDE-BSE formalism that employ more sophisticated interaction kernels.

In order to calculate other observables like the decay constants, the parameters of heavy-sector needs to be different from light-sector.

We introduce a new scheme to study tetraquarks by pairing the Schwinger-Dyson equation formalism with quark models.

With our results for mesons, we calculate the mass of a *bbbb* tetraquark in a contact interaction under diquark-antidiquark picture. We computed m=18.45, a value below the $\eta_b\eta_b$ threshold.


THANK
YOU!