Looking at nuclear physics from the unitary window

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Outline

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Universal Window - Efimov Physics

- Universality
- Efimov Effect
- Discrete Scale Invariance
- Moving along the Universal Window

Finite-range Effect

- Path toward the unitary limit
- Universal function and Gaussian Level functions
- Universality in N-Body States

Spin-Isospin Universality

- Spin-Isospin Potential
- Nuclear cut
- Unitary Limit
- Physical point
- p-waves

Low Energy Physics

- Low energy $\Longleftrightarrow$ Low Temperature

$$\ell_{\text{de Broglie}} = \sqrt{\frac{\hbar^2}{2\pi m K_B T}} \gg \ell = \begin{array}{c}\text{typical}\\ \text{interaction}\\ \text{length}\end{array}$$

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$$\ell_{\text{de Broglie}} @ 100 \text{ nK} (\approx 10 \text{ peV}) = 938.6$$

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- **Nuclear systems**

$$\ell \sim 1/M_\pi \approx 1.5\text{ fm}$$
$$\ell_{\text{de Broglie}}@1\text{ MeV}(\approx 10^{10}\text{ K}) = 197\text{ fm}$$

Universality

@ Low Energy
$\ell \ll \ell_{\text{de Broglie}}$

$\Rightarrow$

Physics governed by
the scattering length
a

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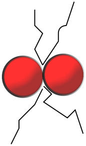
- $a \sim \ell$ Perturbative weak-coupling regime



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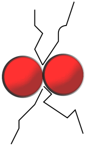

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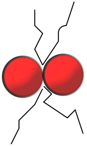

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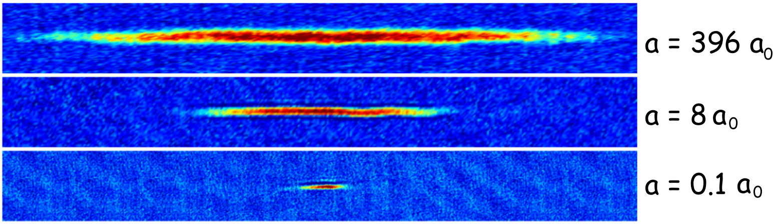
Rice University - R.G. Hulet - PRL 102, 090402 (2009)

100 µm



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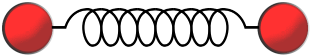

$$E_2 \approx -\frac{\hbar^2}{m a^2} \text{ for } a > 0$$

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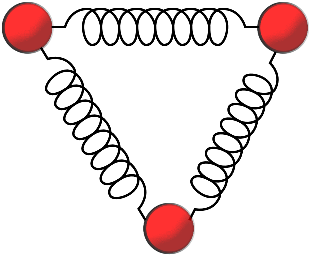

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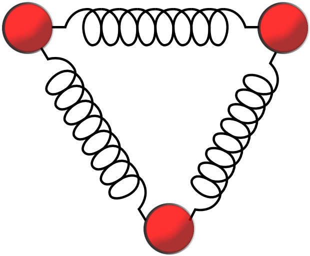

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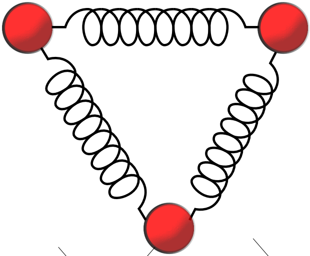

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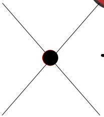
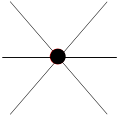
$V \propto a$  $+ \lambda_3$ 

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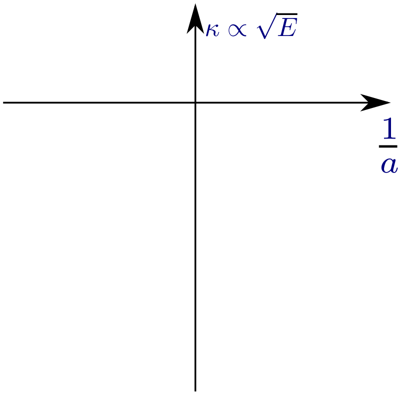

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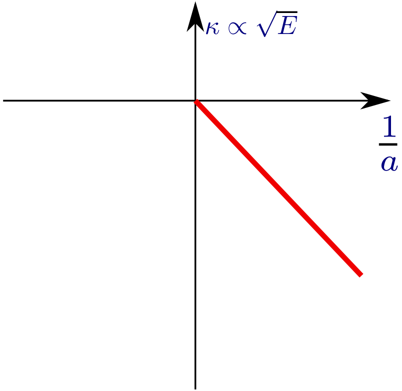

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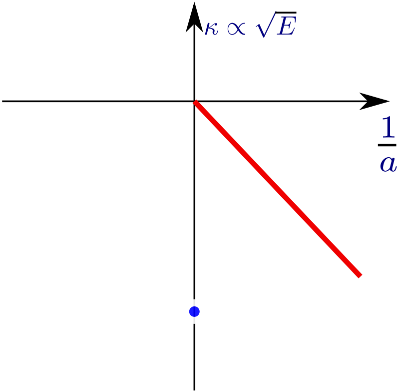

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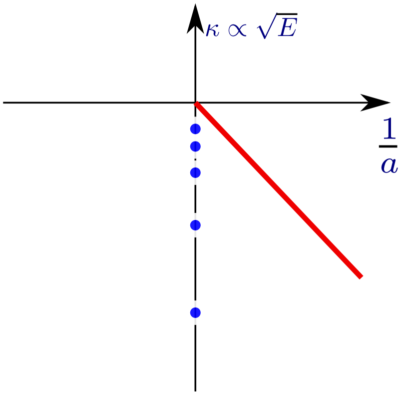

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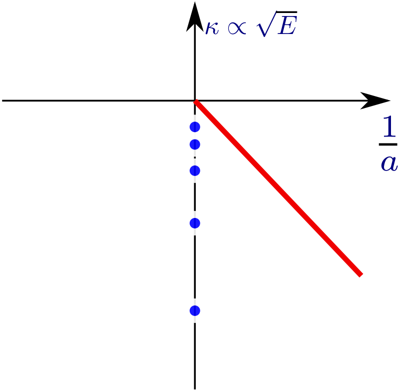

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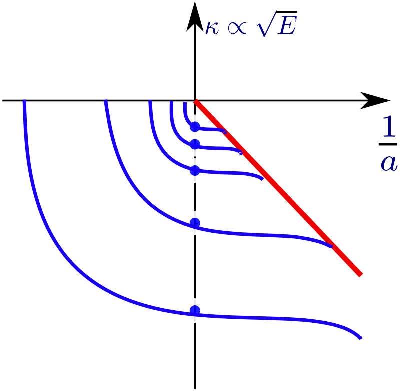

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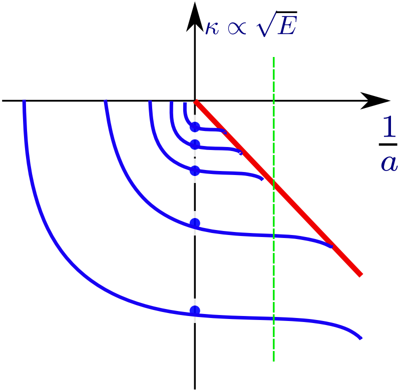

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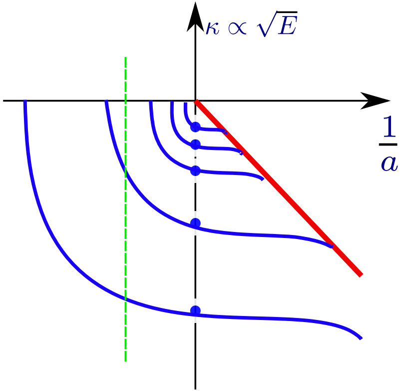

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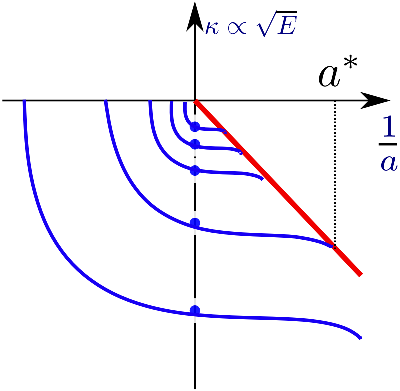

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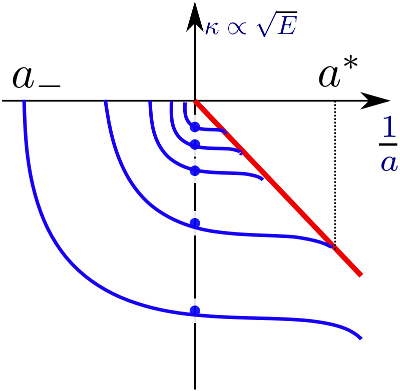

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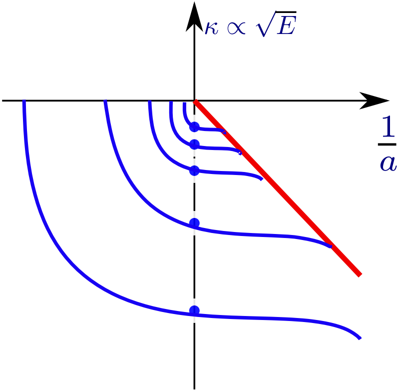

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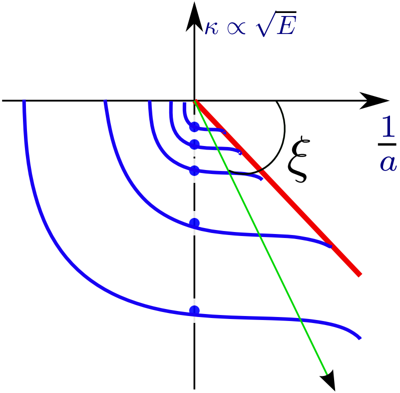

Polar coordinates

$$(H)^2 = (E_3 + E_2)/(\hbar^2/m)$$
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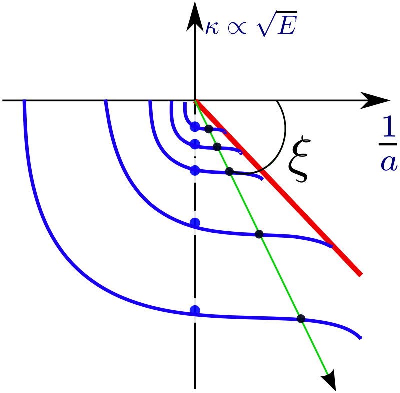

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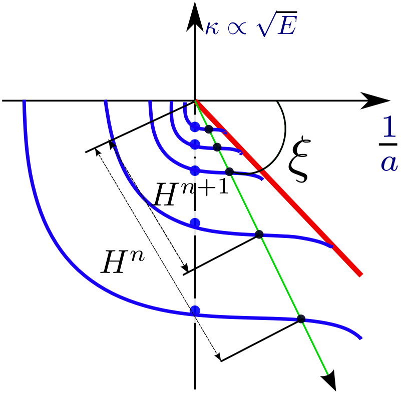

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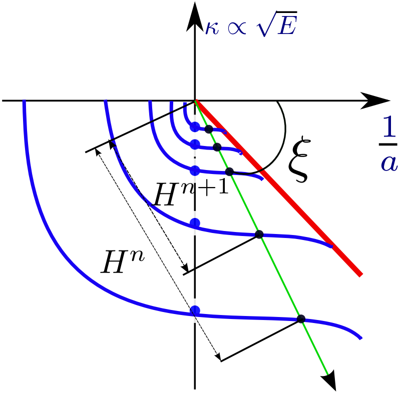

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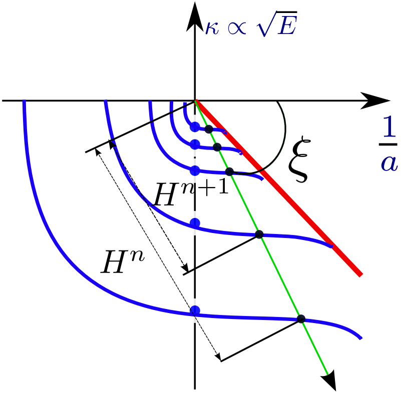

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Discrete Scale Invariance

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Particle-Dimer Scattering Length

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Recombination Rate at the threshold

$$K_3 = \frac{128\pi^2(4\pi - 3\sqrt{3})}{\sinh^2(\pi s_0) + \cosh^2(\pi s_0)\cot^2[s_0 \ln(\kappa_* a) + \gamma]} \frac{\hbar a^4}{m},$$

- γ Universal Constant

Changing $\kappa_* a \sim \xi$

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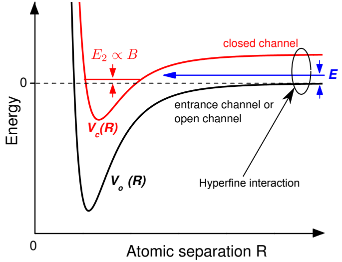
Atomic Physics



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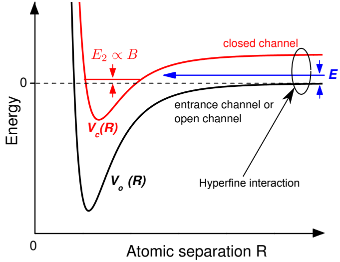
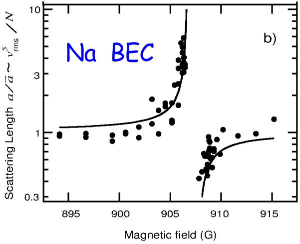


Ketterle's group
Nature 392, 151 (1998)



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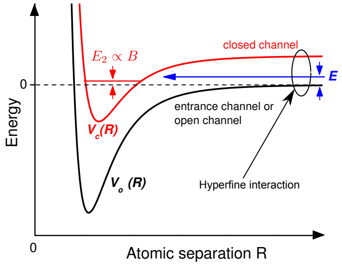

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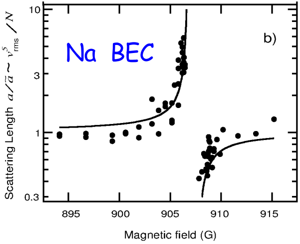

Nuclear physics

$$a^{^3s_1} = 5.42 \text{ fm}$$

$$(p\,n)$$

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$$(p\,p, n\,n, p\,n)$$

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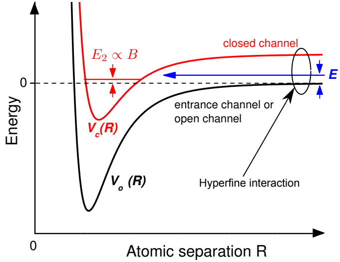

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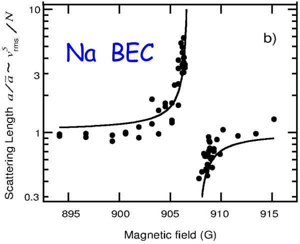

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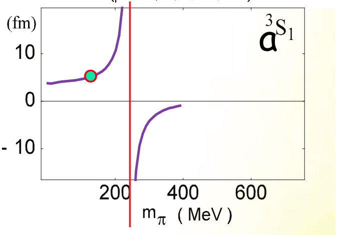
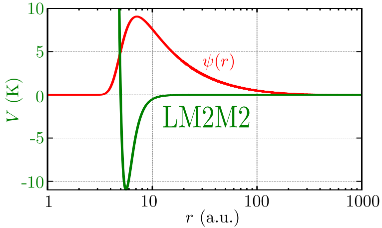
Beane et al.
Nucl. Phys. A 700, 377 (2002)
(picture from U. van Kolck)



Natural fine tuning

Atomic Physics - ^{4}He



$$\ell_{vdW} \approx 10a.u.$$
$$a_0 \approx 190 \text{ a.u.}$$
$$E_2 \approx -1.30 \text{ mK} \approx \hbar^2/ma_0^2$$
$$E_3^{(0)} \approx -126 \text{ mK and } E_3^{(1)} \simeq -2.3 \text{ mK}$$

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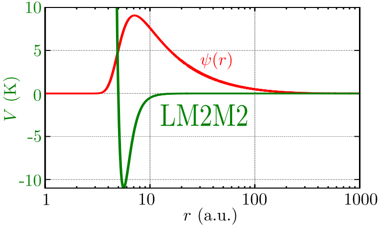

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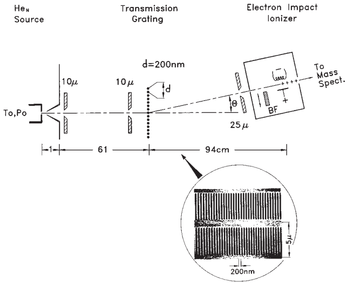
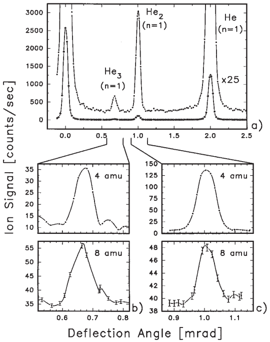
Schöllkopf and Toennies
J. Chem. Phys. 104, 1155 (1995)





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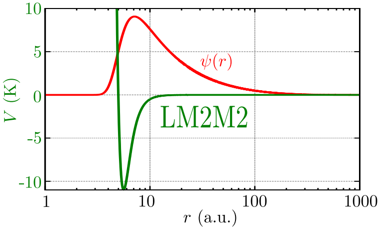
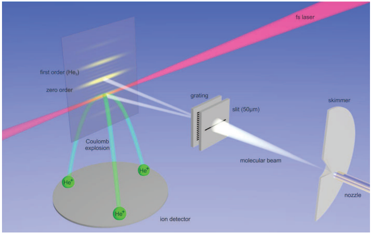
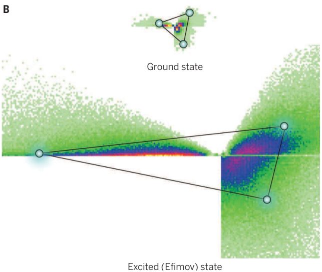
Atomic Physics - ^{4}He



$$\ell_{vdW} \approx 10a.u.$$
$$a_0 \approx 190 \text{ a.u.}$$
$$E_2 \approx -1.30 \text{ mK} \approx \hbar^2/ma_0^2$$
$$E_3^{(0)} \approx -126 \text{ mK and } E_3^{(1)} \simeq -2.3 \text{ mK}$$

M. Kunitski et al., Science **348**, 551 (2015)
Reinhard Doerner - University of Frankfur





Natural fine tuning

Nuclear Physics

$$\ell_\pi = 1.5 \text{ fm}$$
$$a^{{}^3s_1} = 5.42 \text{ fm} \qquad (p\,n)$$
$$a^{{}^1s_0} = -23.75 \text{ fm} \qquad (p\,p, n\,n, p\,n)$$

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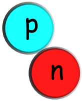
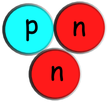
Deuterium



Binding Energy = 2.22 MeV

Tritium



Binding Energy = 8.48 MeV

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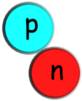

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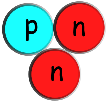
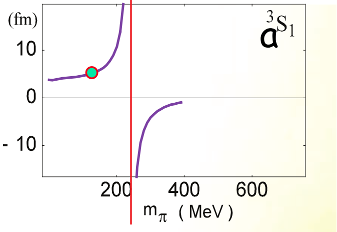


Binding Energy = 8.48 MeV

Beane et al.
Nucl. Phys. A 700, 377 (2002)
(picture from U. van Kolck)



Outline

Finite-range Calculations

- N-body calculation using Schrödinger Equation

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- Finite-range potential

$$V(r) = V_0 \; \mathrm{e}^{-r^2/r_0^2}$$

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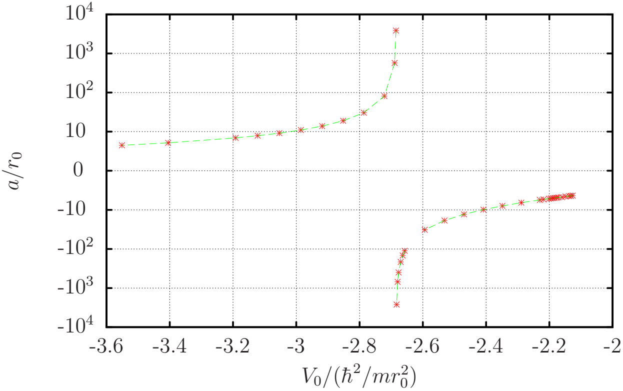
- Tuning of the Scattering Length



Path toward the Unitary limit

- Fine tuning

$$\lambda \longrightarrow \lambda V(r)$$

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$$k \cot \delta \approx -\frac{1}{a} + \frac{1}{2} r_e k^2$$

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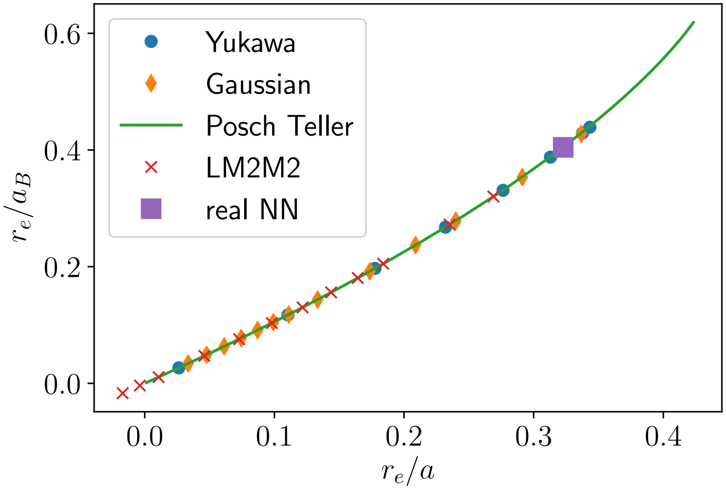
- Close to the Unitary limit

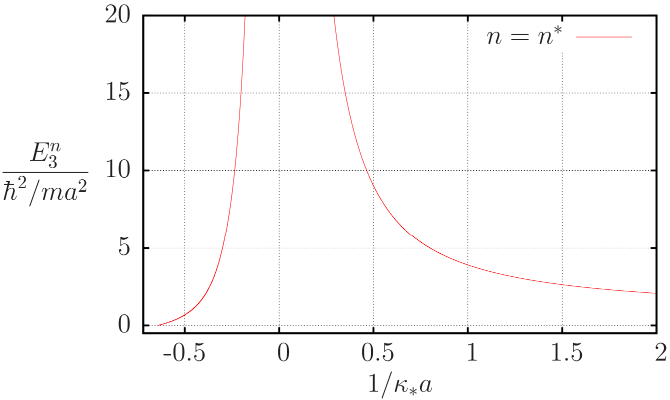
$$r_B = a - a_B \sim \text{constant}$$

$$a r_e \approx 2 a_B r_B$$

Path toward the Unitary limit



3-Body Bound States



$$\begin{cases} E_3^n/(\hbar^2/ma^2) = \tan^2\xi \\ \kappa_* a = e^{(n-n^*)\pi/s_0} \dfrac{e^{-\Delta(\xi)/2s_0}}{\cos\xi} \end{cases}$$

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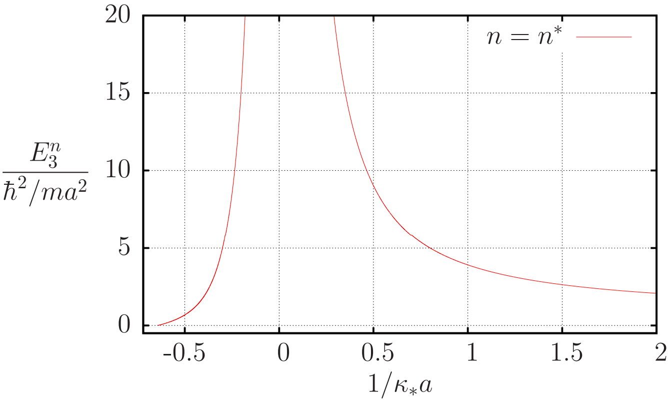

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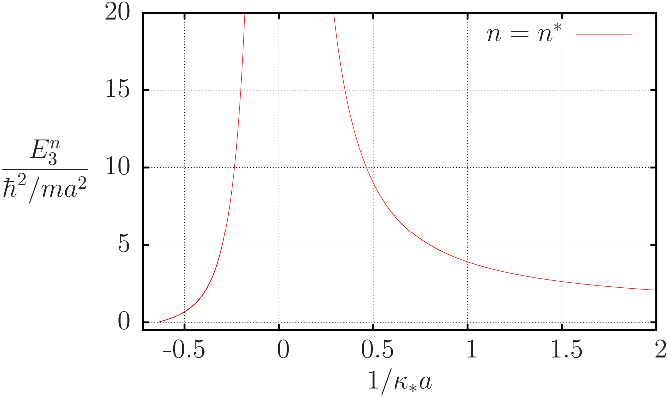

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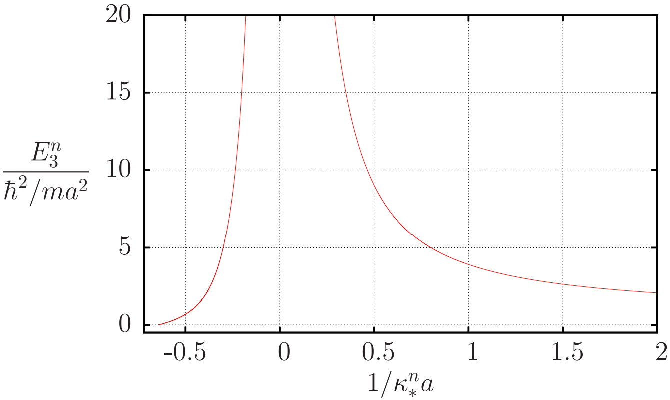

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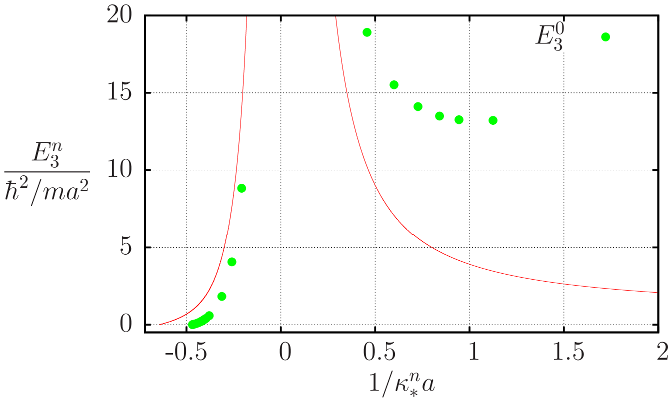

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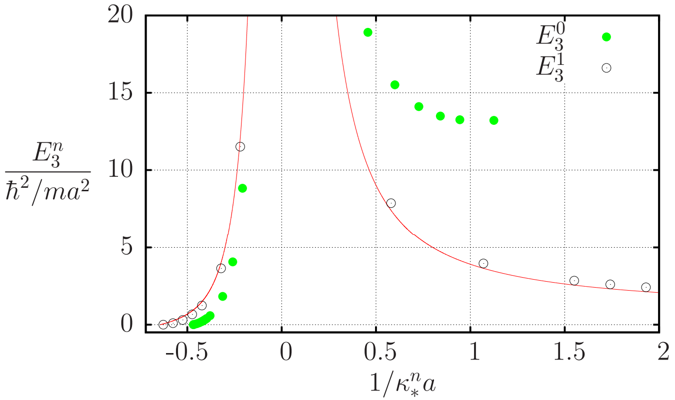

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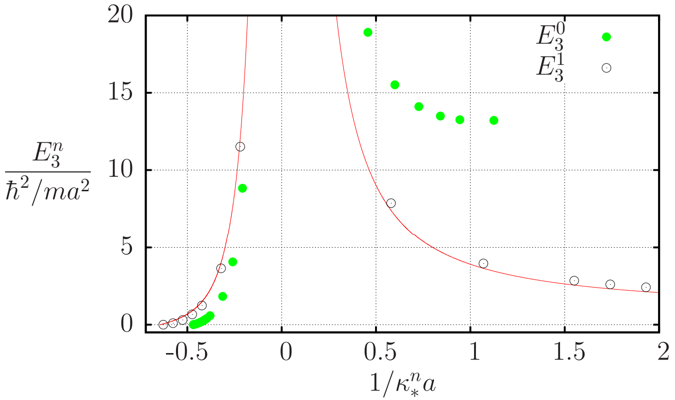

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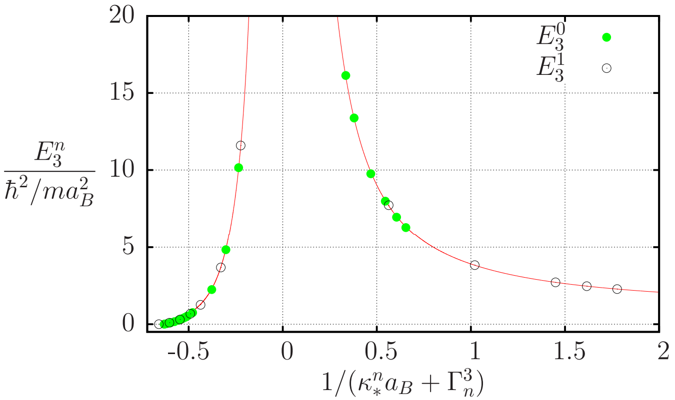

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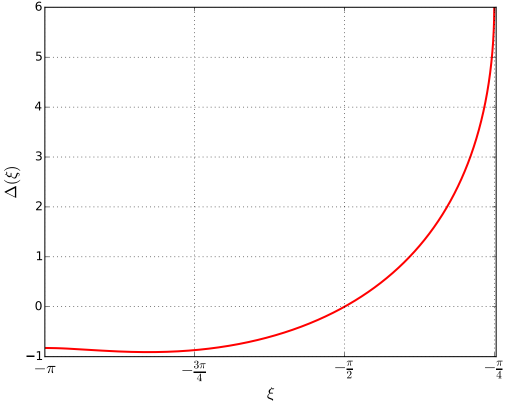
Gaussian Level Function - Universality

- Zero Range $\kappa_* a = \mathrm{e}^{-\Delta(\xi)/2s_0} / \cos\xi$



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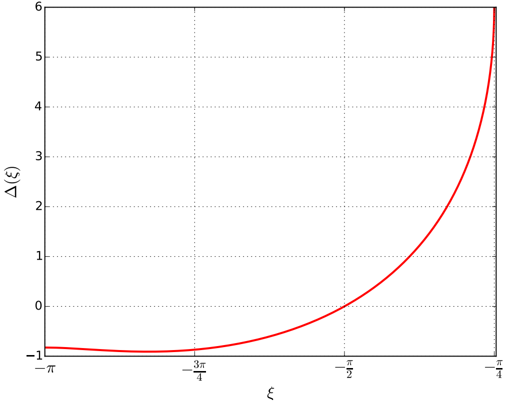

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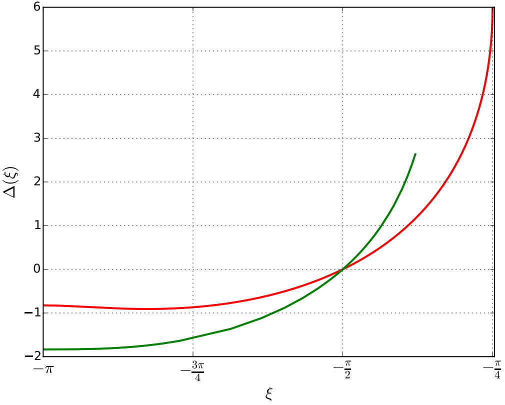
$$\kappa_* a_B = \mathrm{e}^{-\widetilde{\Delta}(\xi)/2s_0} / \cos\xi$$



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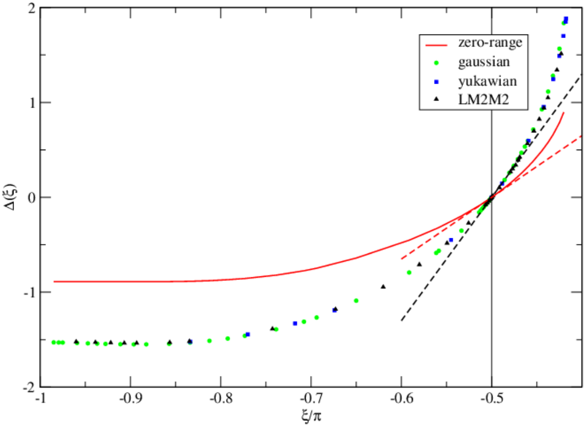
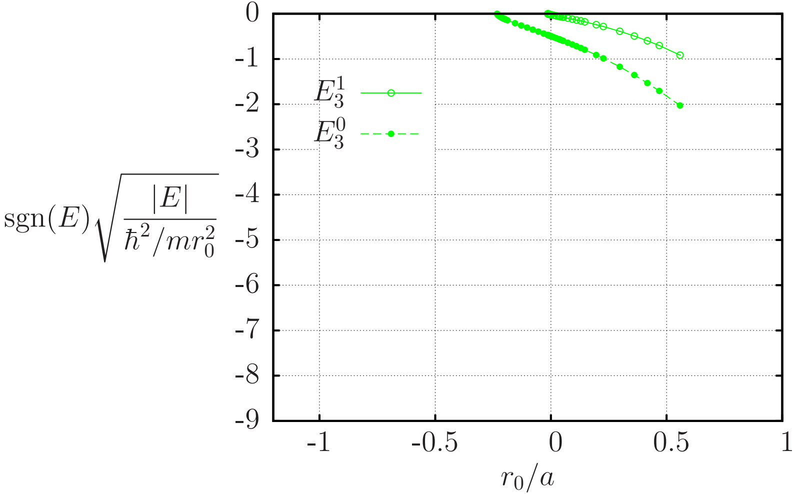


N-body Efimov Plot



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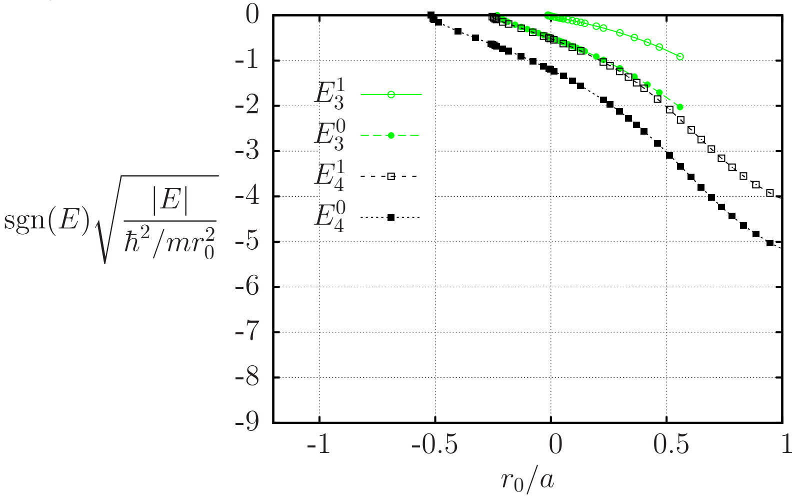
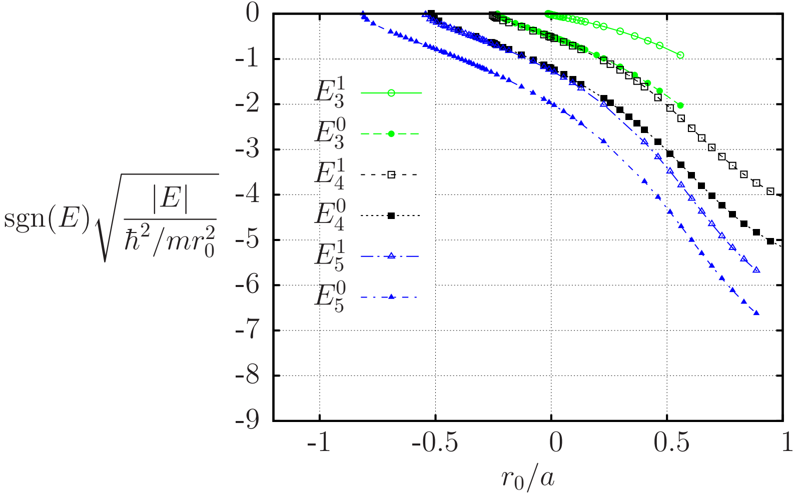


- Two four-body states for each three-body state

N-body Efimov Plot



- Two four-body states for each three-body state
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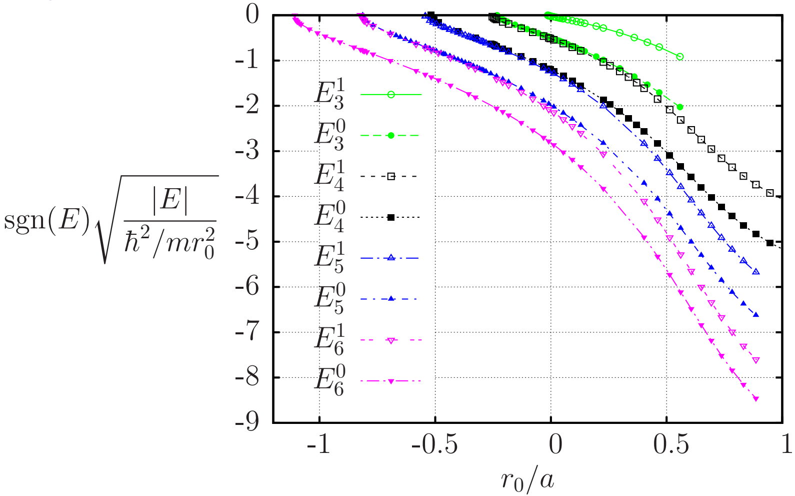
N-body Efimov Plot



- Two four-body states for each three-body state
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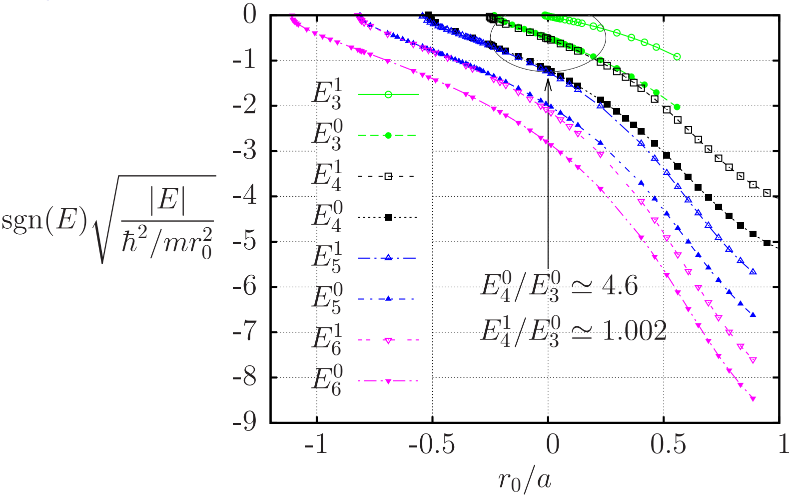

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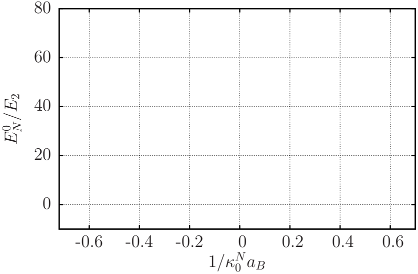
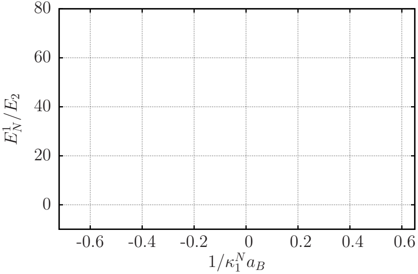
Universality





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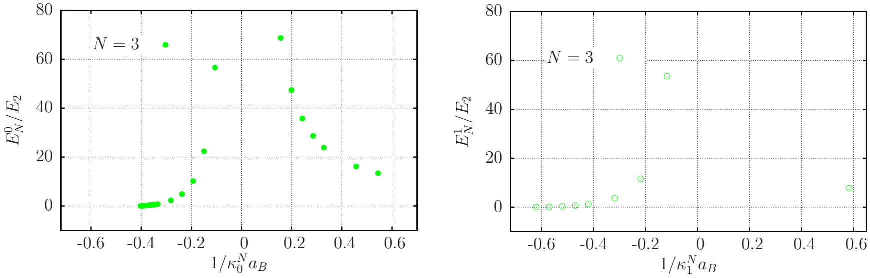

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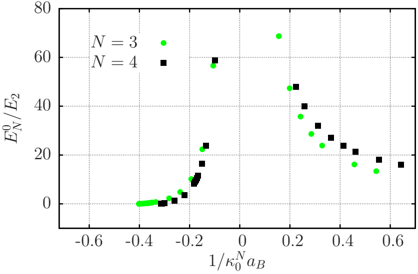

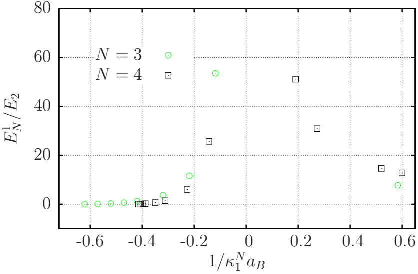

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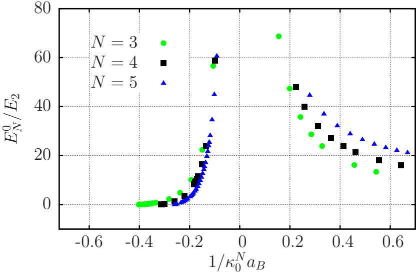

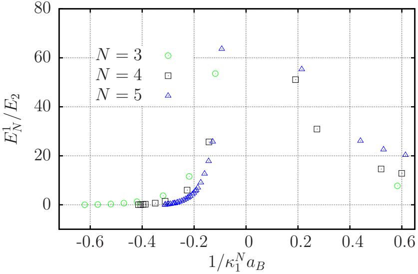

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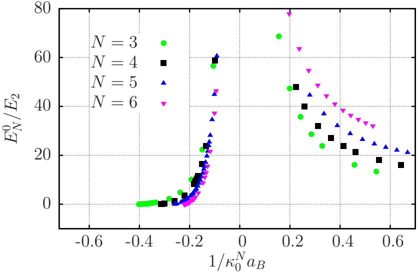

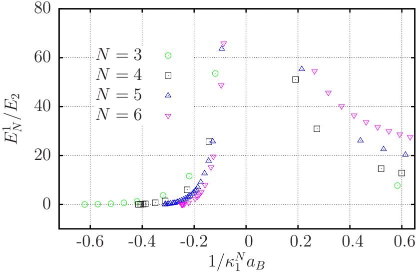

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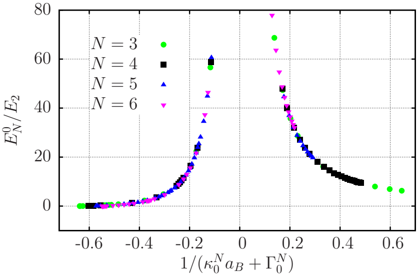

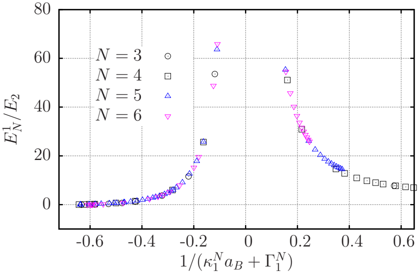

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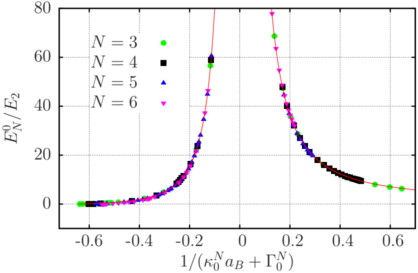
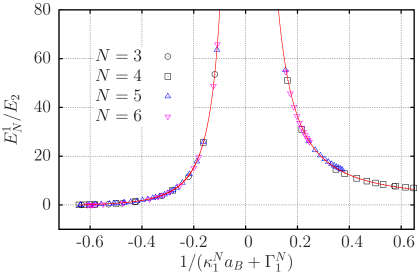


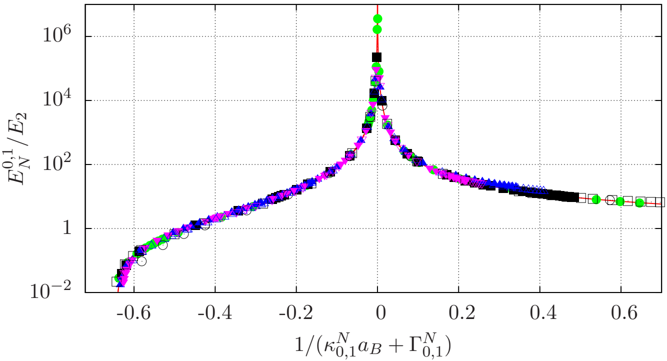


Universal Formula

$$E_N^n/E_2 = \tan^2 \xi$$

$$\kappa_n^N a_B + \Gamma_n^N = \frac{\mathrm{e}^{-\Delta(\xi)/2s_0}}{\cos \xi}$$

Universality



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Efimov Straighteners

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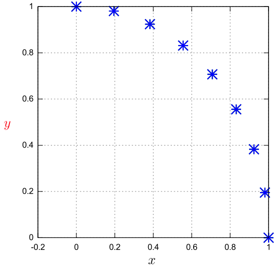
Data on a Circle



Efimov Straighteners

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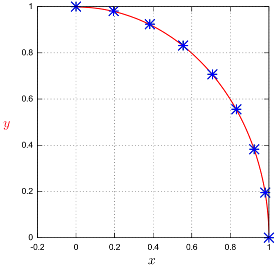
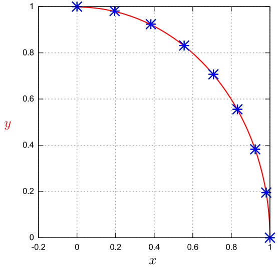


$$y = \sin \xi$$

$$x = \cos \xi$$

Efimov Straighteners

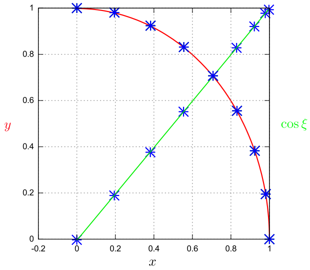
Data on a Circle



$$\begin{matrix} y = \sin\xi \\ x = \cos\xi \end{matrix} \Leftrightarrow \begin{matrix} y/x = \tan\xi \\ x = \cos\xi(x,y) \end{matrix}$$

Efimov Straighteners

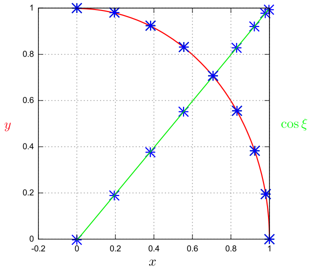
Data on a Circle



$$y = \sin\xi \quad \Leftrightarrow \quad y/x = \tan\xi$$
$$x = \cos\xi \qquad\quad x = \cos\xi(x,y)$$

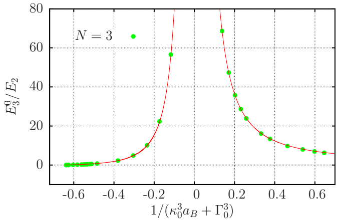
Efimov Straighteners

Data on a Circle



$$y = \sin\xi \qquad y/x = \tan\xi$$
$$\Leftrightarrow$$
$$x = \cos\xi \qquad x = \cos\xi(x,y)$$

Data on Efimov curve



$$E_3^0/E_2 = \tan^2\xi$$

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Efimov Straighteners

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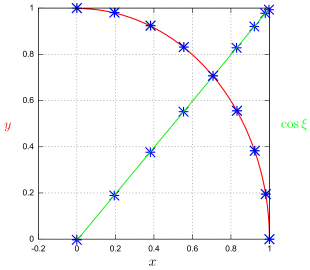

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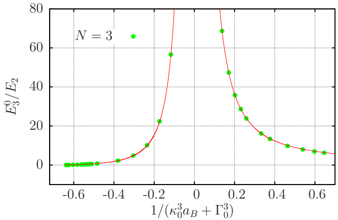

$$E_3^0/E_2 = \tan^2\xi$$

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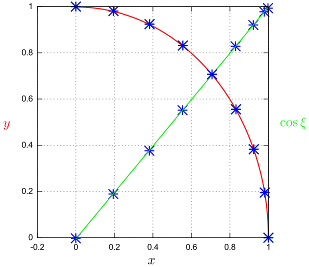
$$y(\xi) \stackrel{\text{def}}{=} \frac{\mathrm{e}^{-\Delta(\xi)/2s_0}}{\cos\xi}$$

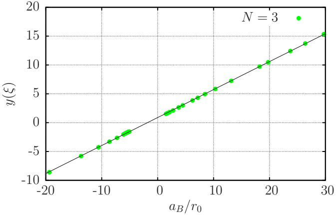
Efimov Straighteners

Data on a Circle



$$y = \sin\xi \qquad y/x = \tan\xi$$
$$\Leftrightarrow$$
$$x = \cos\xi \qquad x = \cos\xi(x,y)$$

Data on Efimov curve



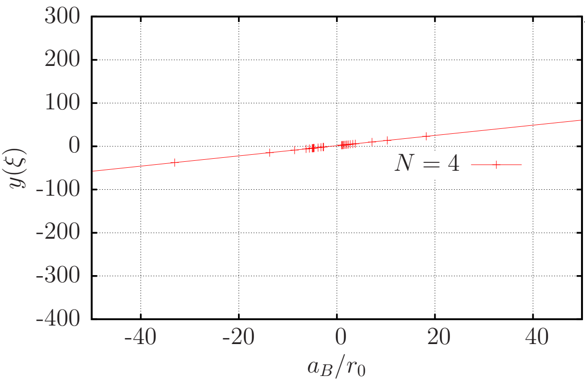
$$E_3^0/E_2 = \tan^2\xi$$

$$\kappa_0^3 a_B + \Gamma_0^3 = \frac{e^{-\Delta(\xi)/2s_0}}{\cos\xi}$$

$$y(\xi) \stackrel{\text{def}}{=} \frac{e^{-\Delta(\xi)/2s_0}}{\cos\xi}$$

Universality up to $N = 16$

$$y(\xi) = \kappa_N a_B + \Gamma_N$$



N	$\kappa_N r_0$	Γ_N
4	1.185	1.475

$$V(r) = V_0\ \mathrm{e}^{-r^2/r_0^2}$$

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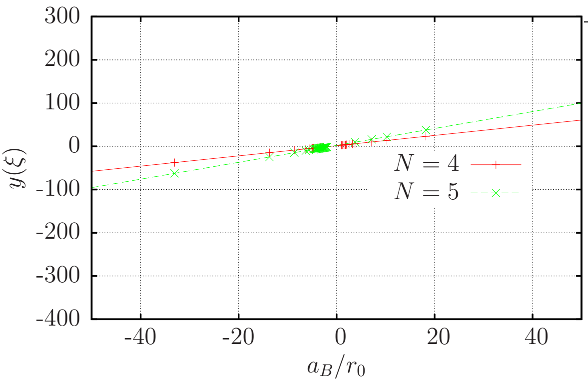

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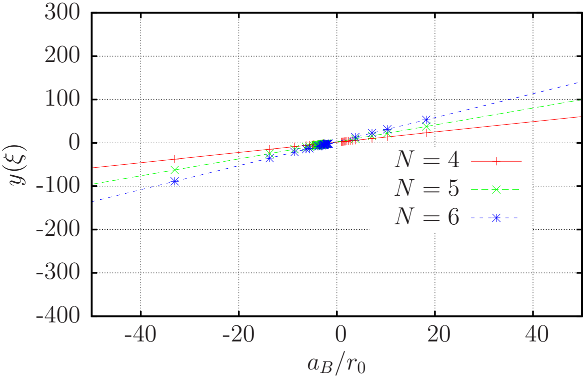

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6	2.770	2.752

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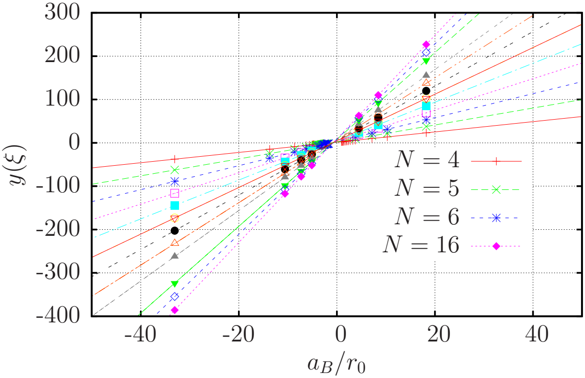

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6	2.770	2.752
7	3.617	3.344
8	4.487	3.983
9	5.377	4.625
10	6.282	5.268
11	7.201	5.912
12	8.131	6.557
13	9.071	7.202
14	10.02	7.848
15	10.98	8.494
16	11.94	9.141

Outline

Spin-Isospin Potential

Efimov ”ingredients”

- Efimov physics only s-wave, $L = 0$
- Symmetric spatial wave function
- Spin+Isospin = 4 internal degree of freedom

$$\Rightarrow$$

Possible Efimov scenario up to $N = 4$

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Gaussian Potential

$$V(r) = \sum_{S,T=\{0,1\}} V_{ST}\, e^{-(r/r_{ST})^2} \mathcal{P}_{ST}$$

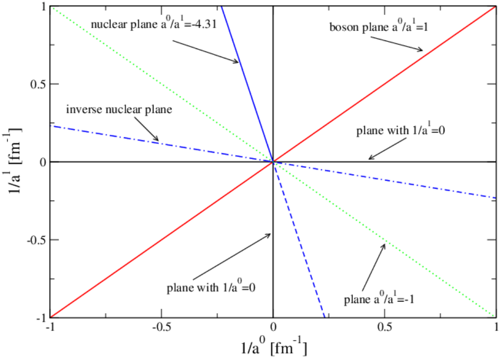
- Antisymmetry $\Rightarrow L + T + S = \text{odd}$
- $V_{00} = V_{11} = 0$
- $r_{01} = r_{10} = r_0 = 1.65$ fm

Spin-Isospin Potential

$$V(r) = V_{01}\, e^{-(r/r_0)^2} \mathcal{P}_{01} + V_{10}\, e^{-(r/r_0)^2} \mathcal{P}_{10}$$

Two control parameters

- $V_{01} \longleftrightarrow a_0$
- $V_{10} \longleftrightarrow a_1$



Nuclear cut $a_0/a_1 = -4.31$

$N = 3 \to T = 1/2$ and $S = 1/2 \Rightarrow$ ^{3}H and ^{3}He

$N = 4 \to T = 0$ and $S = 0 \Rightarrow$ ^{4}He and ^{4}He*

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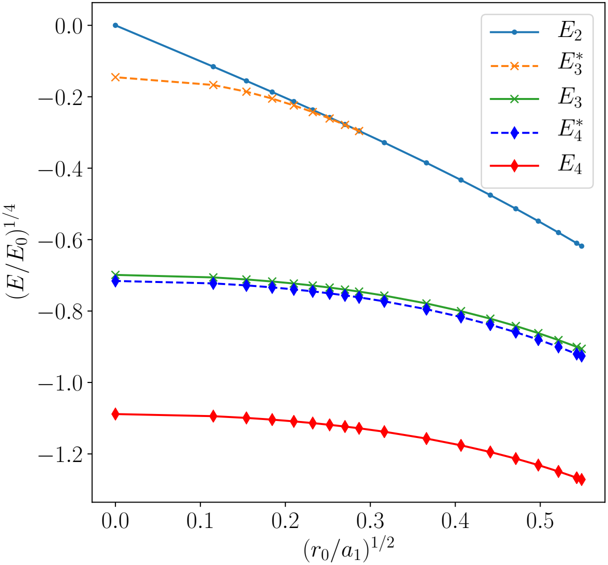

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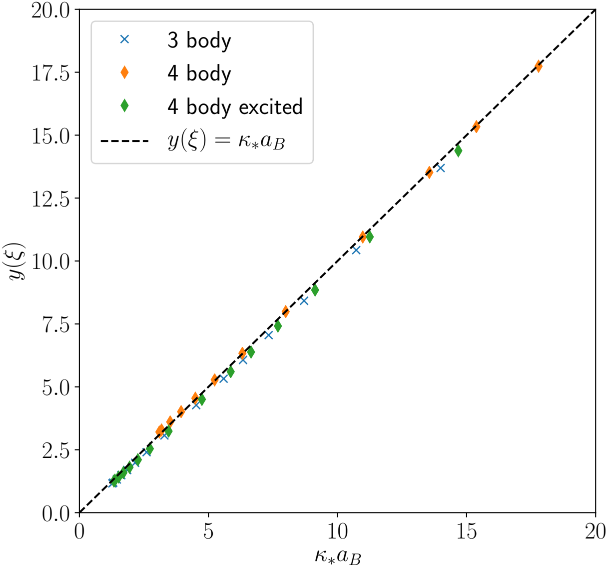

Unitary limit

- Scale Invariance

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A	m	$r_0 \kappa_A^m \vert_G$	$\tan^2 \xi \vert_{\text{exp}}$	$a_B/r_0 \vert_G$	$\kappa_A^m \vert_{\text{exp}}(\text{fm}^{-1})$	$E_A^m \vert_{\text{exp}}(\text{MeV})$
3	0	0.4883	3.81	2.1866	0.2473	2.536
3	1	0.0211				
4	0	1.1847	13.13	2.0774	0.570	13.474
4	1	0.5124				

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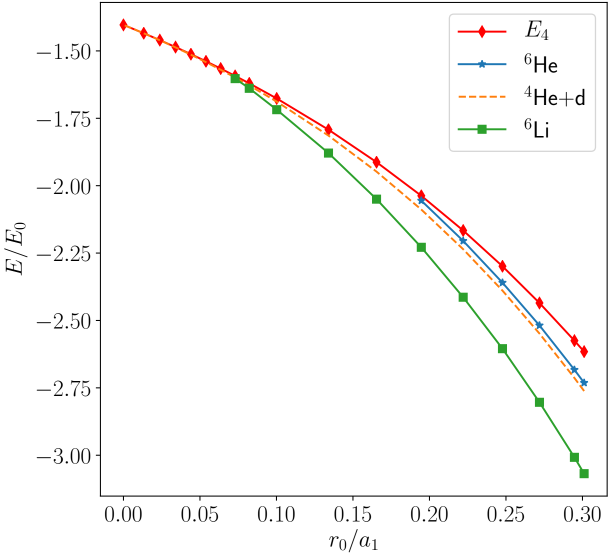

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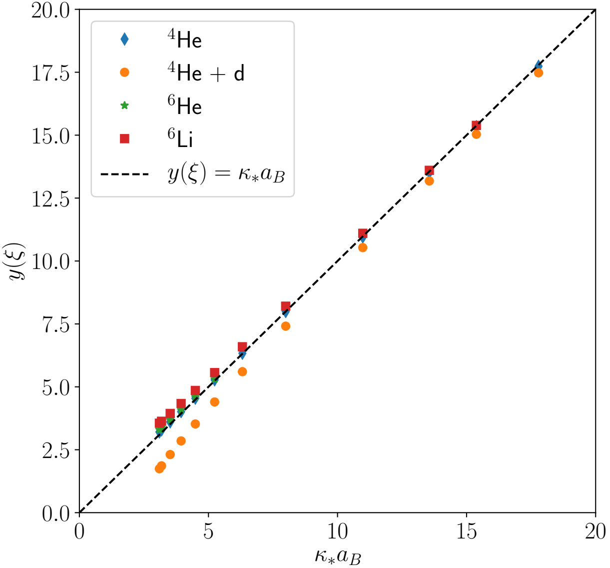
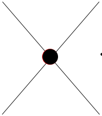
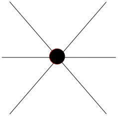


Physical point

Physics close the unitary limit - Efimov physics

$V \propto a$  $+ \lambda_3$ 

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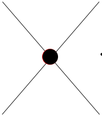
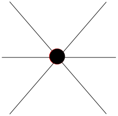
$V \propto a$  $+ \lambda_3$ 

$$W(\rho) = W_0 \, e^{-(r_{12}^2 + r_{13}^2 + r_{23}^2)/R_3^2}$$

W_0(MeV)	R_3(fm)	E_3(MeV)	E_4(MeV)	E_4^*(MeV)	^{3}He(MeV)	^{4}He(MeV)	^{4}He*(MeV)
0	-	-10.2455	-39.843	-11.193	-9.426	-38.789	-10.655
11.922	2.5	-8.48	-28.670	-8.75	-7.722	-27.754	
9.072	2.8	-8.48	-29.014	-8.79	-7.718	-28.060	
7.8	3.0	-8.48	-29.223	-8.80	-7.715	-28.258	
7.638	3.03	-8.48	-29.255	-8.80	-7.714	-28.290	
7.612	3.035	-8.48	-29.260	-8.80	-7.714	-28.295	
7.6044	3.035	-8.482	-29.269	-8.80	-7.716	-28.305	
Experimental Values		-8.482			-7.718	-28.296	

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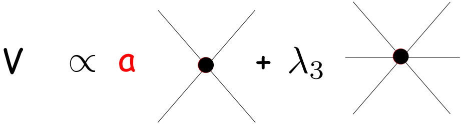

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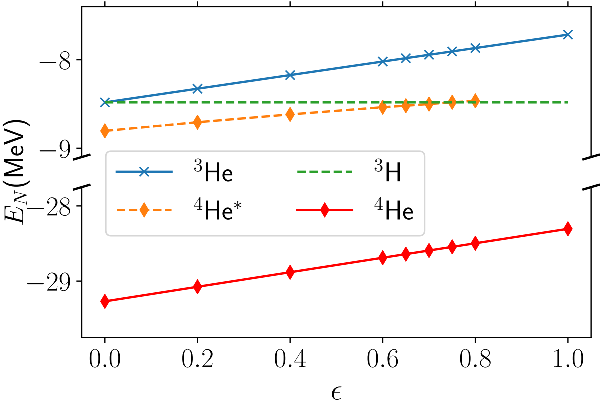
- No ^{4}He* state
- ^{6}He and ^{6}Li go to their thresholds

Fate of ^{4}He*



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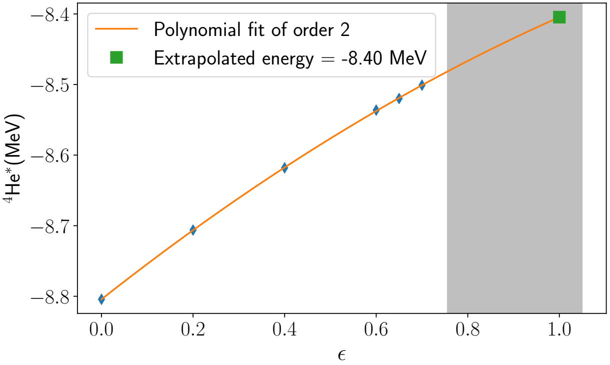

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Rôle of p-waves

- We fix V_{10} and V_{01} with the scattering lengths
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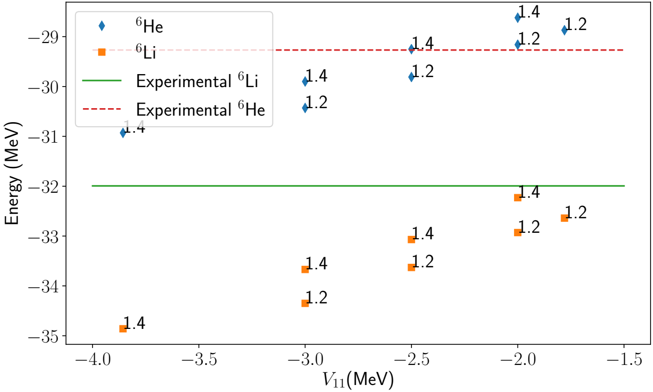

V_{11} can bind the $N = 6$ system !

Thanks!