

Microscopic Equations of State and Astrophysical Applications

(in view of GW170817)

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W. Zuo : **China**

(Not a review)

- BHF approach of (hyper)nuclear matter
- Neutron star properties
- Analysis of GW170817
- Quark matter and hybrid stars

PRC 61, 055801 (2000)
PRD 70, 043010 (2004)
PRC 73, 058801 (2006)
PRC 74, 047304 (2006)
PRD 76, 123015 (2007)
PRC 77, 034316 (2008)
PRC 78, 028801 (2008)
A&A 518, A17 (2010)
PRC 83, 025804 (2011)
PRC 84, 035801 (2011)
PRD 84, 105023 (2011)
EPJA 52, 21 (2016)
PRC 96, 044309 (2017)
PRC 98, 064322 (2017)
APJ 860, 139 (2018)
MNRAS 484, 5162 (2019)
JPG 46, 034001 (2019)

Project NEUMATT

NEUMATT NEUtron star MATter Theory

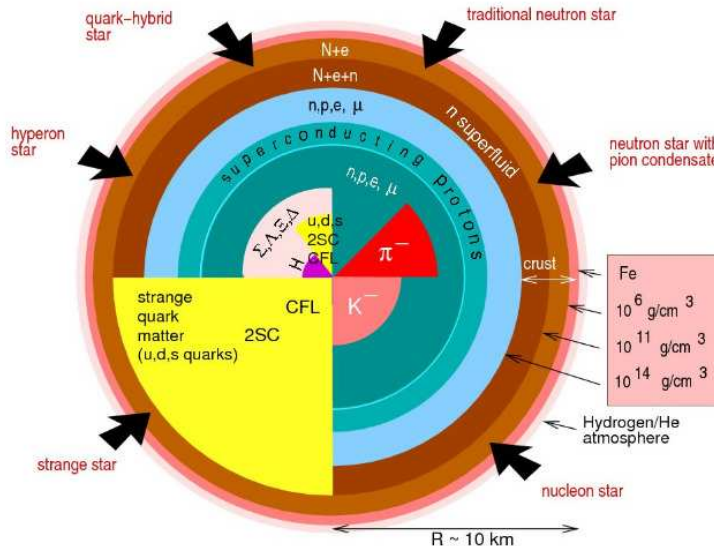
The Equation of State of Nuclear Matter and Neutron Star Structure

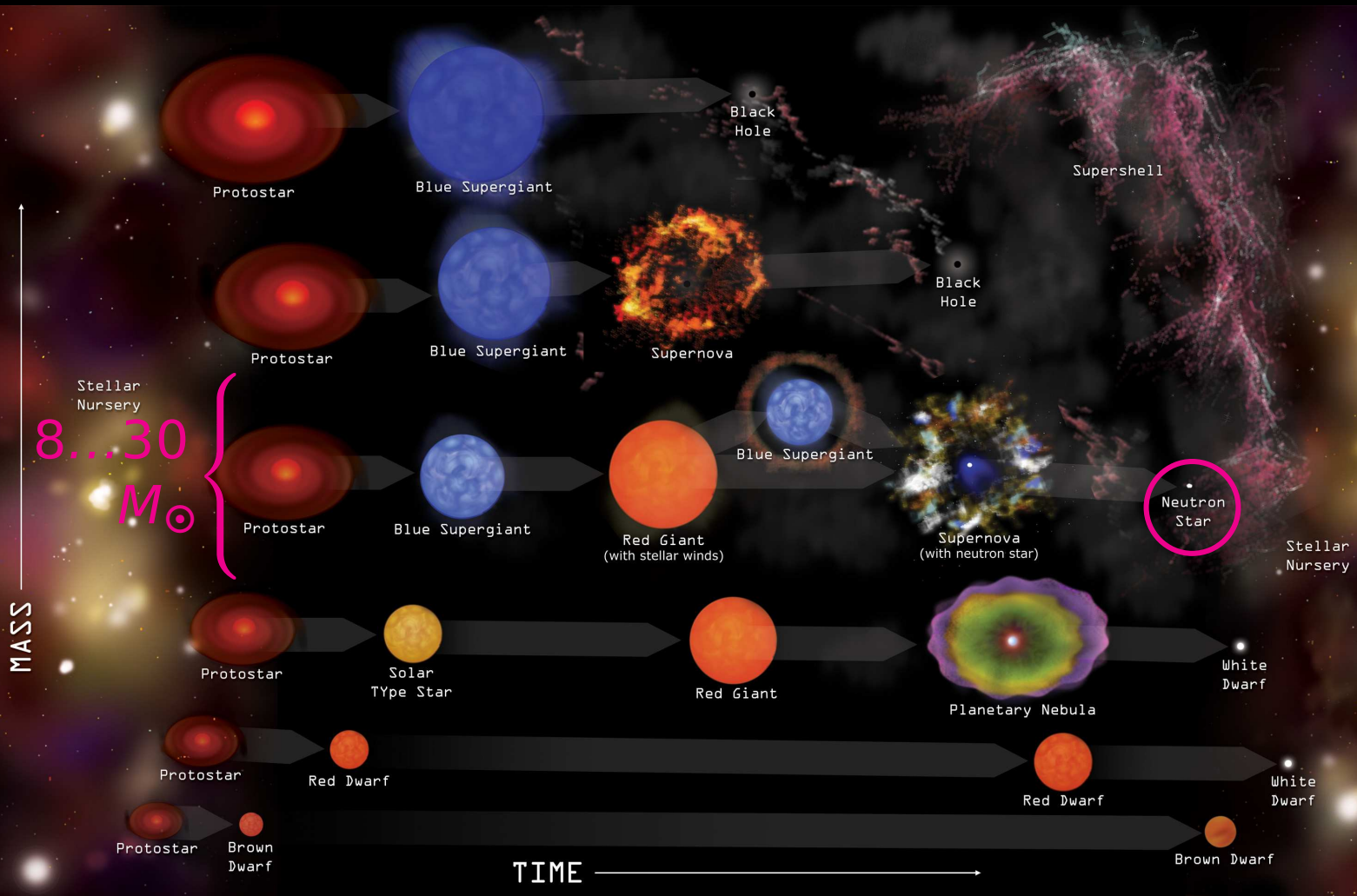


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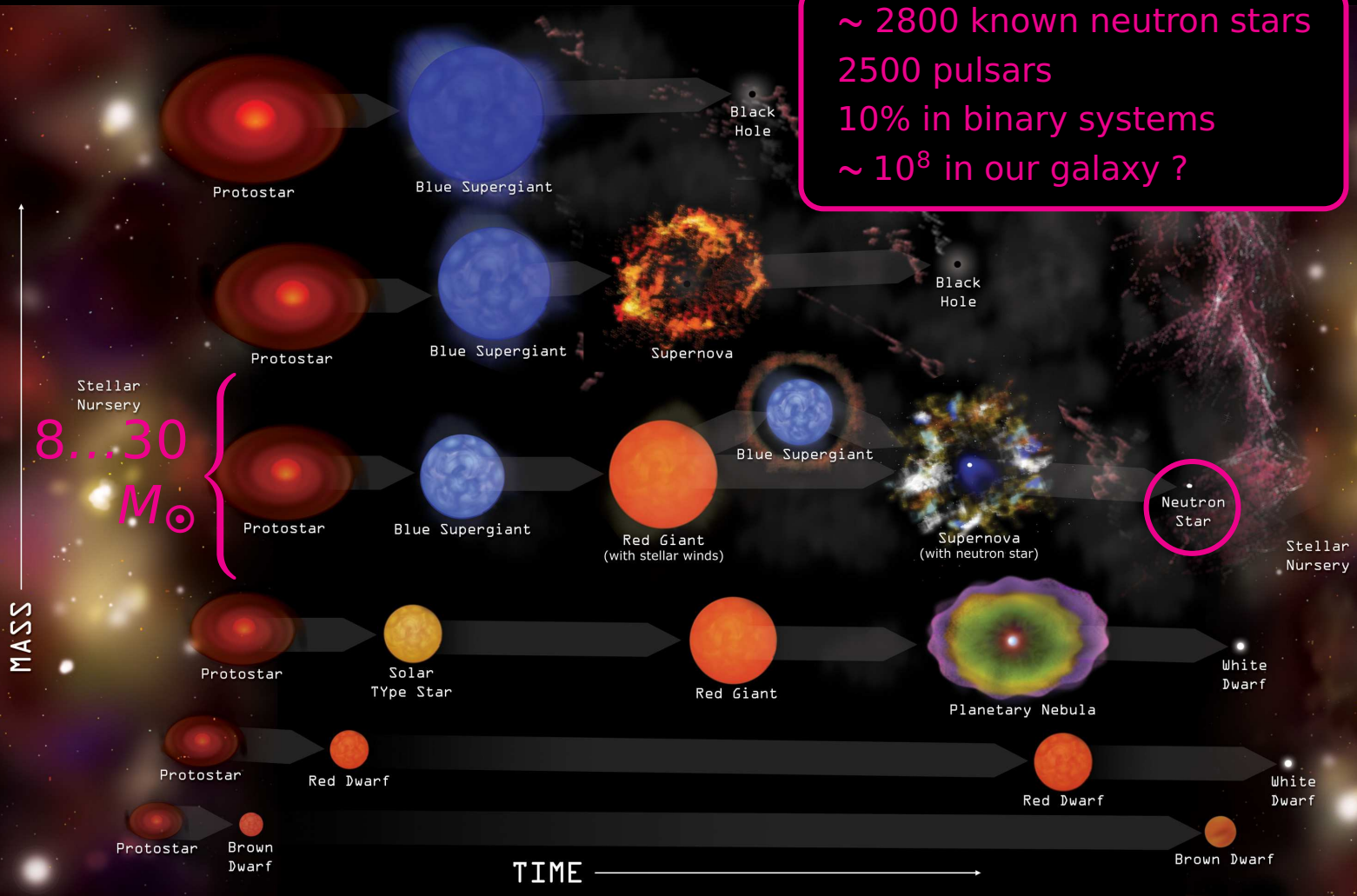
Abstract

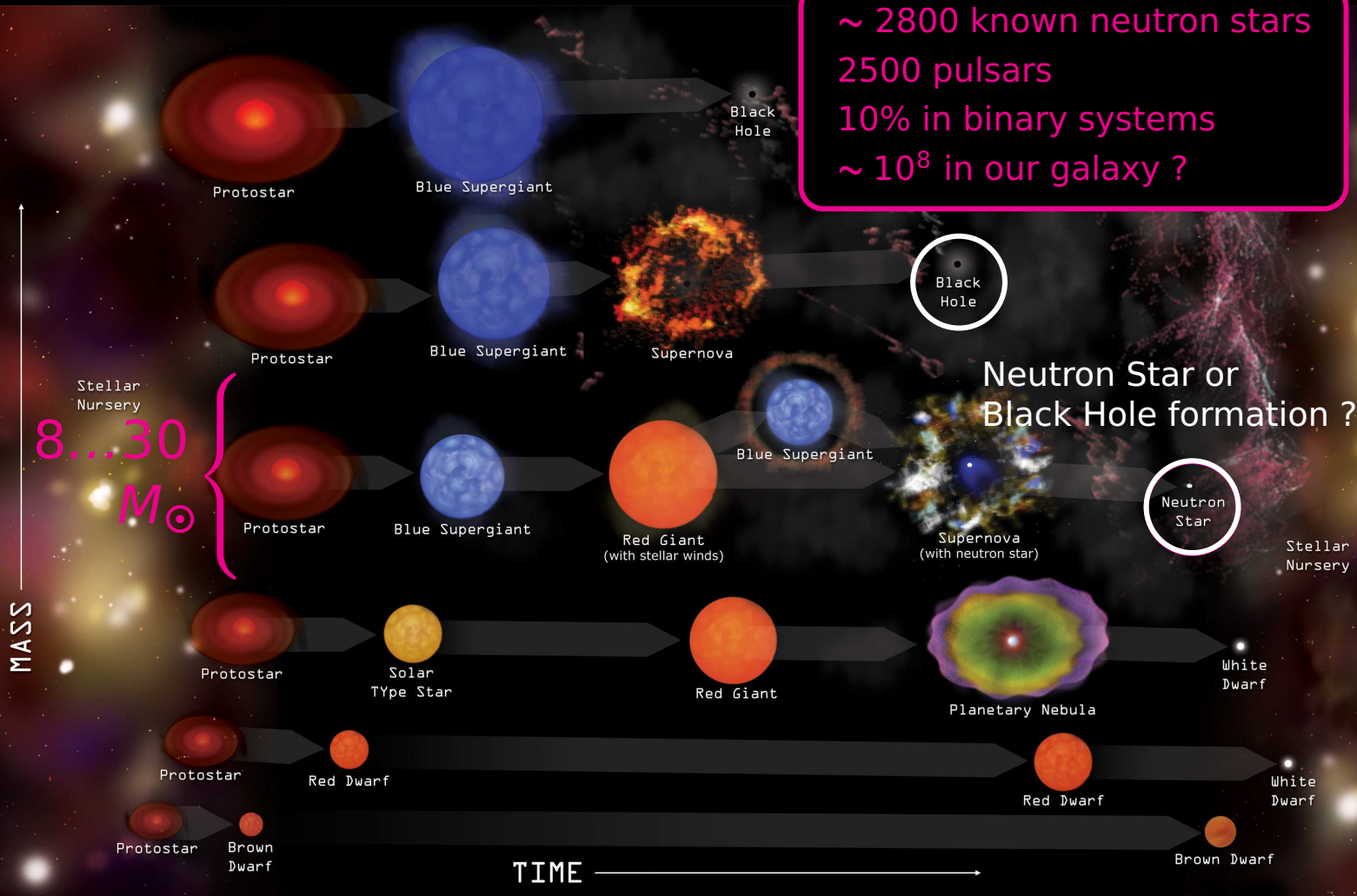
NETWORK





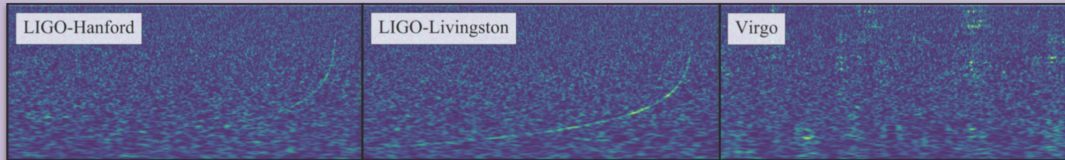
~ 2800 known neutron stars
2500 pulsars
10% in binary systems
~ 10⁸ in our galaxy ?





~ 2800 known neutron stars
2500 pulsars
10% in binary systems
~ 10^8 in our galaxy ?

GW170817 FACTSHEET

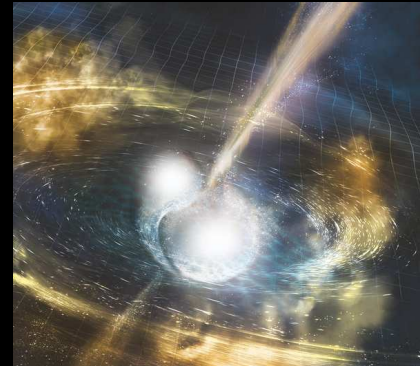


observed by	H, L, V	inferred duration from 30 Hz to 2048 Hz**	~ 60 s
source type	binary neutron star (NS)	inferred # of GW cycles from 30 Hz to 2048 Hz**	~ 3000
date	17 August 2017	initial astronomer alert latency*	27 min
time of merger	12:41:04 UTC	HLV sky map alert latency*	5 hrs 14 min
signal-to-noise ratio	32.4	HLV sky area†	28 deg ²
false alarm rate	< 1 in 80 000 years	# of EM observatories that followed the trigger	~ 70
distance	85 to 160 million light-years	also observed in	gamma-ray, X-ray, ultraviolet, optical, infrared, radio
total mass	2.73 to 3.29 M _⊙	host galaxy	NGC 4993
primary NS mass	1.36 to 2.26 M _⊙	source RA, Dec	13 ^h 09 ^m 48 ^s , -23°22'53"
secondary NS mass	0.86 to 1.36 M _⊙	sky location	in Hydra constellation
mass ratio	0.4 to 1.0	viewing angle (without and with host galaxy identification)	≤ 56° and ≤ 28°
radiated GW energy	> 0.025 M _⊙ c ²	Hubble constant inferred from host galaxy identification	62 to 107 km s ⁻¹ Mpc ⁻¹
radius of a 1.4 M _⊙ NS	likely ≈ 14 km		
effective spin parameter	-0.01 to 0.17		
effective precession spin parameter	unconstrained		
GW speed deviation from speed of light	< few parts in 10 ¹⁵		

Recent news:

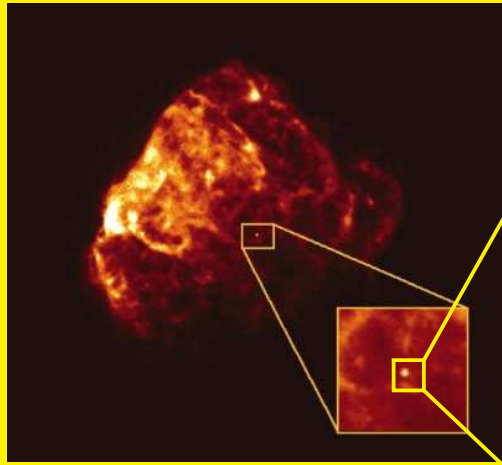
Gravitational waves from binary NS merger GW170817

Nobel prize 2017

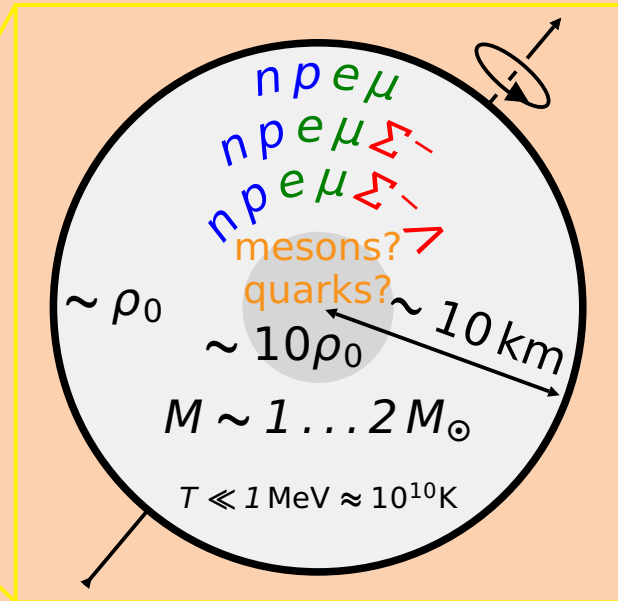


A Theorist's View of a Neutron Star:

A huge nucleus: $\sim 10^{57}$ nucleons :

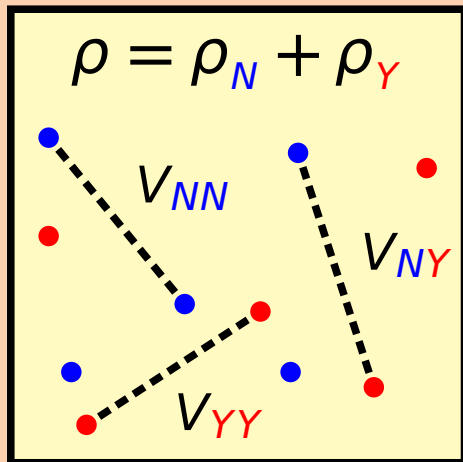


ROSAT image of *Puppis A*



↪ The only “laboratory” for $\rho_B \sim 10\rho_0$ in the Universe !
Need EOS of nuclear matter including hyperons and quarks

(Hyper)Nuclear Matter in the Neutron Star:



$$N = qq\bar{q}: \begin{matrix} n \\ p \end{matrix} \quad (939 \text{ MeV})$$

$$Y = qq\bar{s}: \begin{matrix} \Lambda^0 \\ \Sigma^{-0+} \end{matrix} \quad \begin{matrix} (1116 \text{ MeV}) \\ (1193 \text{ MeV}) \end{matrix}$$

$$qss: \Xi^{-0} \quad (1318 \text{ MeV})$$

V_{NN} : Argonne, Bonn, Paris, ... potential

V_{NY} : Nijmegen (NSC89, NSC97, ESC08...)

V_{YY} : ? (no scattering data)

In free space weak decay: $Y \rightarrow N + \pi$ etc. ($c\tau \approx 8 \text{ cm}$)

In dense nucleonic medium the decay is Pauli-blocked !

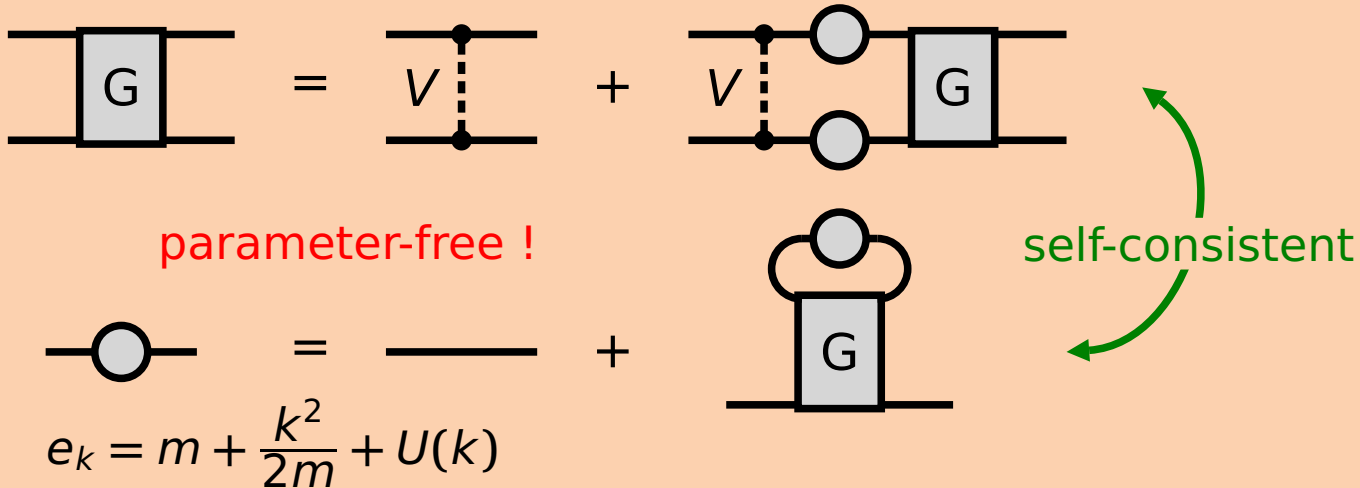
We need to compute the energy density of this system ...

Theoretical Methods for (Hyper)Nuclear Matter:

- “Phenomenological”: RMF, Skyrme, ...
(Contact) interactions fitted to nuclear matter properties around ρ_0 , uncontrolled extrapolation to much higher density, no predictive power.
Good for (hyper)nuclei, bad for neutron stars.
- “Microscopic”: (D)BHF, Monte-Carlo, Chiral Perturbation, ...
Based on phase-shift equivalent potentials + TBF + Many-body scheme.
Additional constraints from (hyper)nuclei.
- BHF is discussed in the following...

Brueckner Theory of (Hyper)Nuclear Matter:

- Effective in-medium interaction G from potential V :



Results: binding energy $\epsilon(\rho_n, \rho_p, \rho_\Lambda, \rho_\Sigma) = \sum_i \sum_{k < k_F^{(i)}} \left[e_k^{(i)} - \frac{U_i(k)}{2} \right]$
 s.p. properties, cross sections, ...

K.A. Brueckner and J.L. Gammel; PR 109, 1023 (1958) for nuclear matter

Extension to hypernuclear matter ...

- Framework: Brueckner-Bethe-Goldstone hole-line expansion

$$\frac{E}{A} = \frac{3 k_F^2}{52m} + \text{diagram 1} + \text{diagram 2} + \dots + \mathcal{O}(\kappa^4)$$

$$\approx [22 - 40 + 2 + ?] \text{ MeV}$$

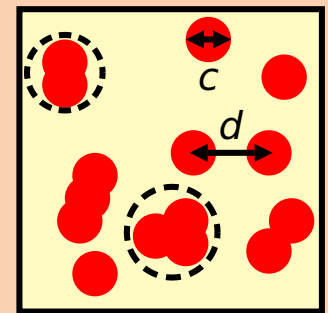
$\rho = \rho_0$, symmetric matter, V_{18} potential

- Expansion parameter $\kappa \sim \rho V_{\text{core}} \approx 0.2$:

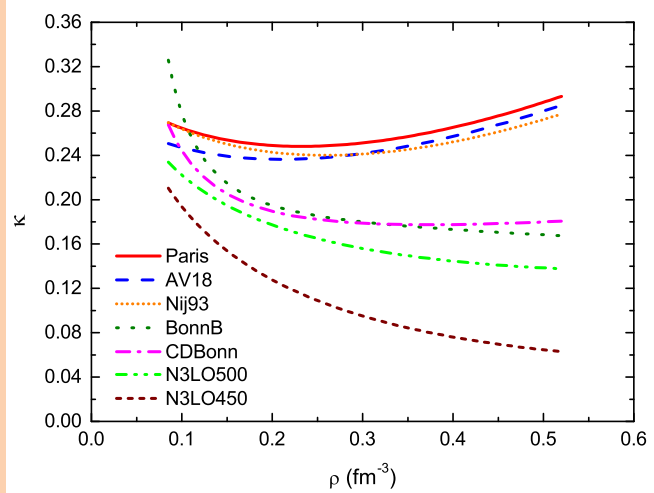
$$\kappa \equiv \frac{\sum_{k > k_F} n(k)}{\sum_{k < k_F} n(k)} = \rho \int d^3r \langle |\eta(\mathbf{r})|^2 \rangle_{S,T} = N \frac{V_{\text{core}}}{V} = \left(\frac{c}{d}\right)^3$$

$u - \phi$: calculated defect function

- Hierarchy of n-body correlations/clusters within hard-core range c , avg. distance d :
- Justified for hard-core potentials



- Correlation parameter for different NN potentials:

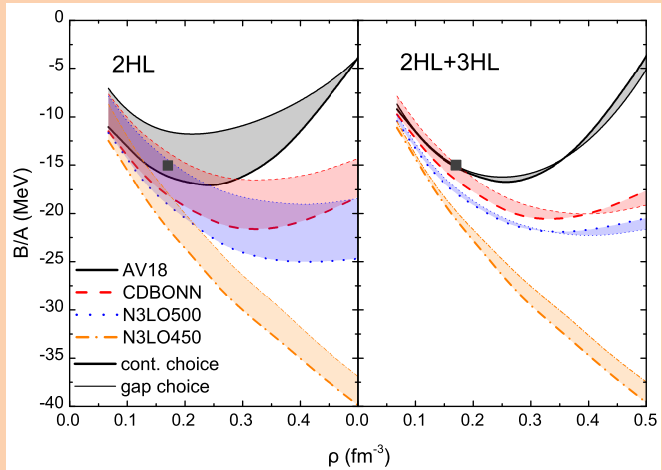


$$\kappa \approx \rho V_{\text{core}}(\rho)$$

Small up to large density

Hard vs. soft potentials

- Binding energy up to three hole lines:

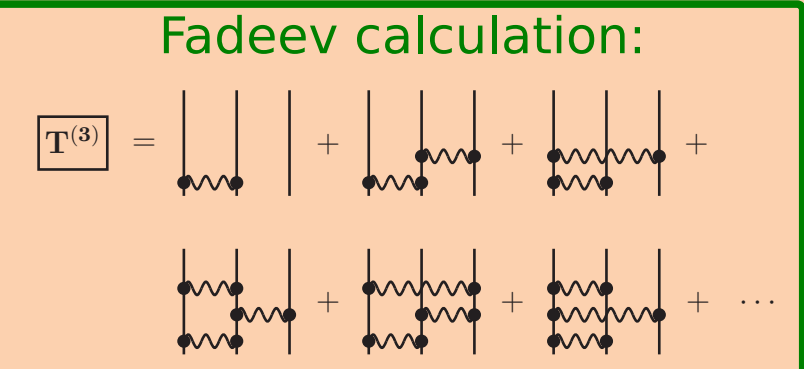
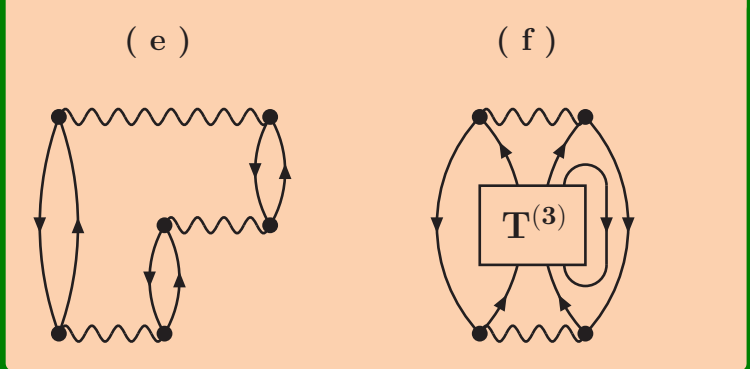
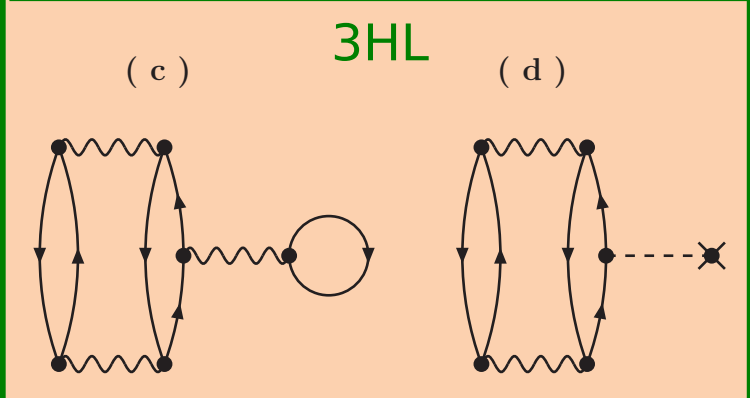
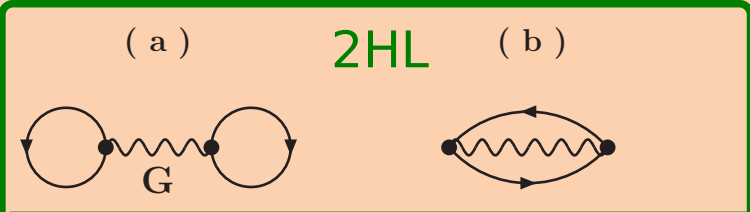


$$B/A = T + E_2 + E_3$$

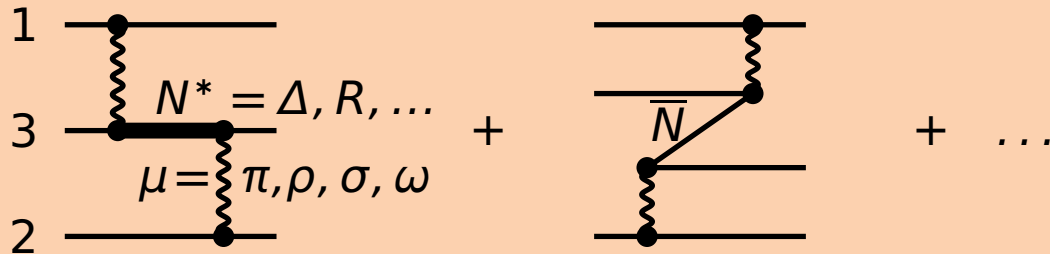
Hole-line expansion appears well converged, but misses slightly for AV18 the empirical saturation point of nuclear matter

• Diagrams up to 3HL:

B.D. Day, PRC 24, 1203 (1981)

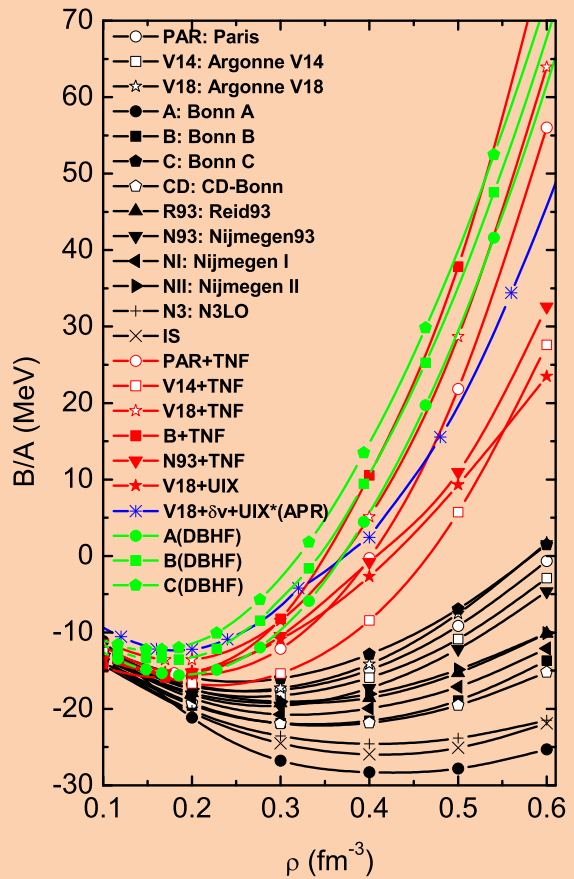


Three-Nucleon Forces:



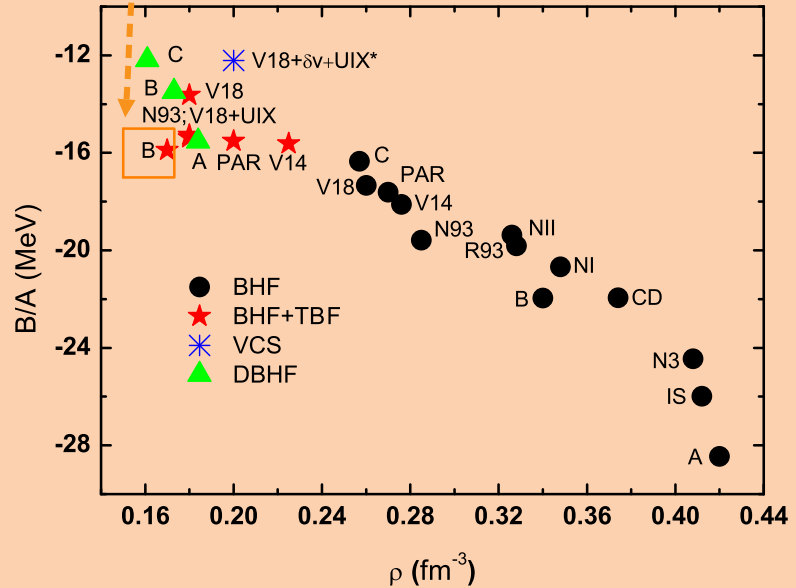
- Only small effect required [$\delta(B/A) \approx 1$ MeV at ρ_0]
- Model dependent, no final theory yet
- Use and compare microscopic and phenomenological TBF...
 - Microscopic TBF of P. Grangé et al., PRC 40, 1040 (1989):
Exchange of $\pi, \rho, \sigma, \omega$ via $\Delta(1232), R(1440), N\bar{N}$
Parameters compatible with two-nucleon potential (Bonn, V_{18}, \dots)
 - Urbana IX phenomenological TBF:
Only 2π -TBF + phenomenological repulsion
Fit saturation point

● BHF binding energy and saturation point of nuclear matter:



Nuclear mass formula

Coester band:

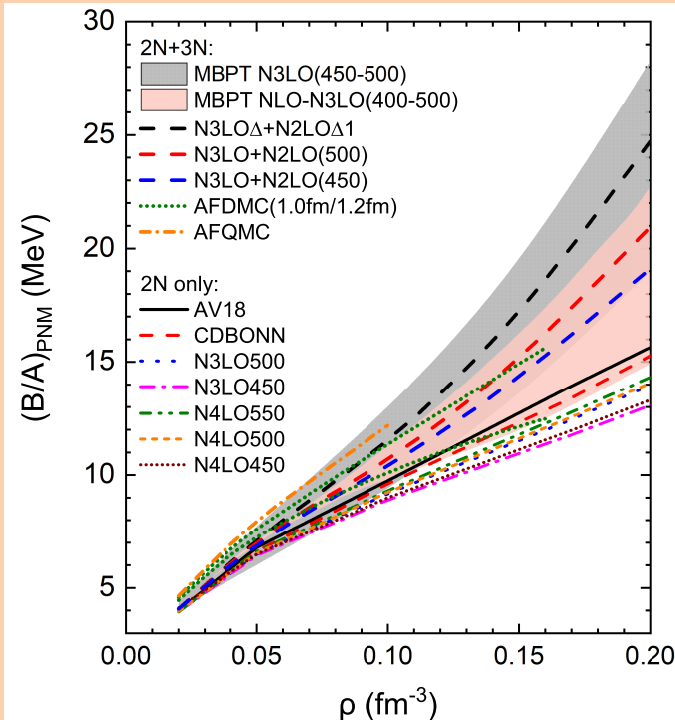


● Dependence on NN potential

● TBF needed to improve saturation properties

Regularization Dependence of CPT:

- Expansion for low momenta $\leftrightarrow E_{\text{lab}}^{NN} \lesssim 200 \text{ MeV}$, $\rho \lesssim \rho_0$



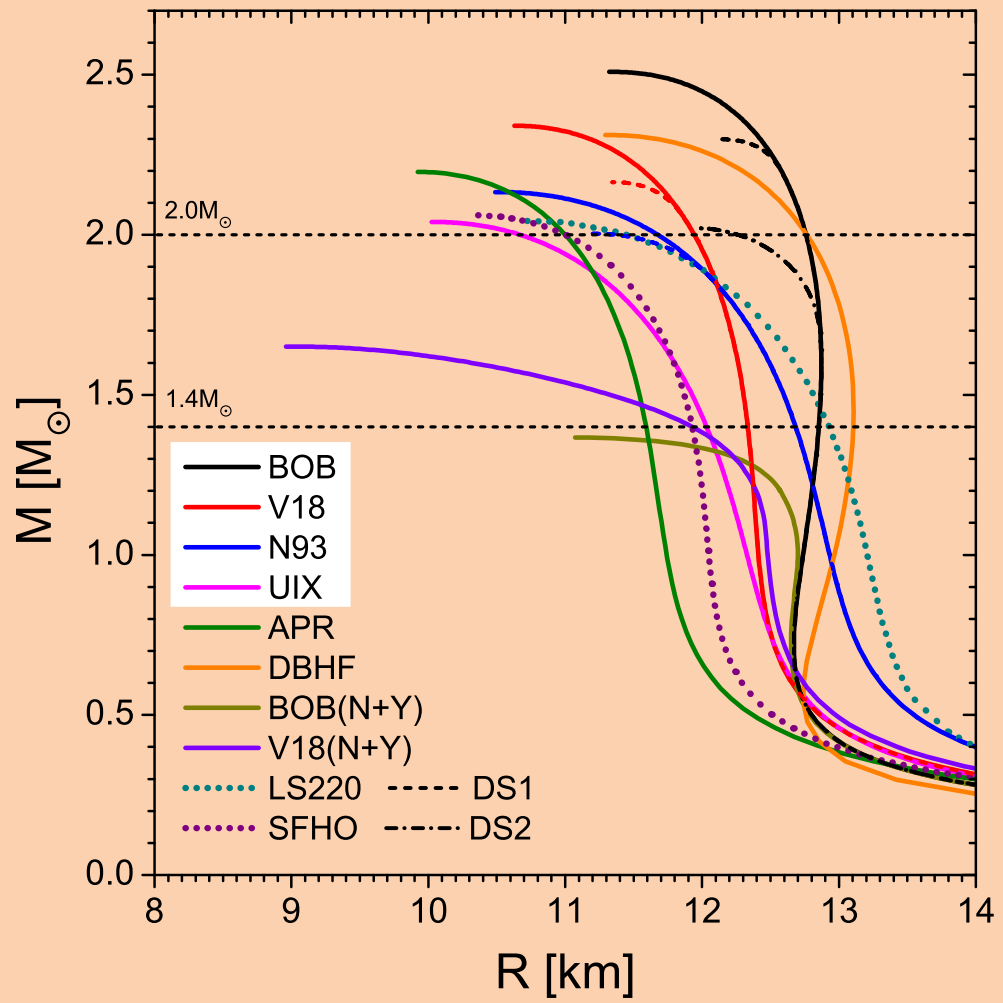
PRC94, 054307 (2016)

PRC95, 034326 (2017)

PRC94, 064001 (2016)

- $\sim 50\%$ uncertainty at ρ_0 at 4th order (N3LO 2BF+3BF) !

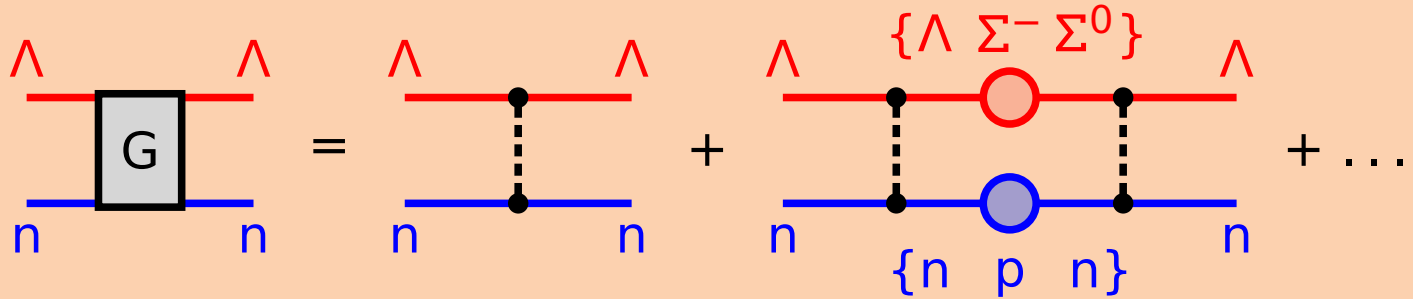
● Mass-radius relations with microscopic BHF EOSs:



Variation due to uncertain high-density TBF

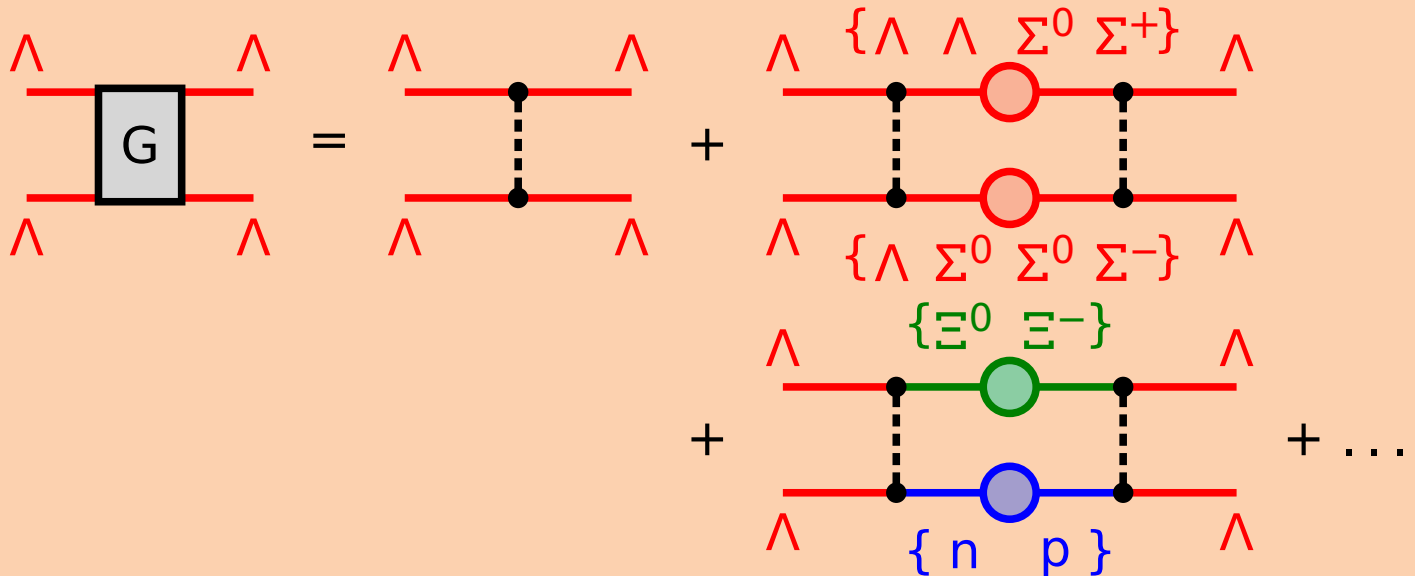
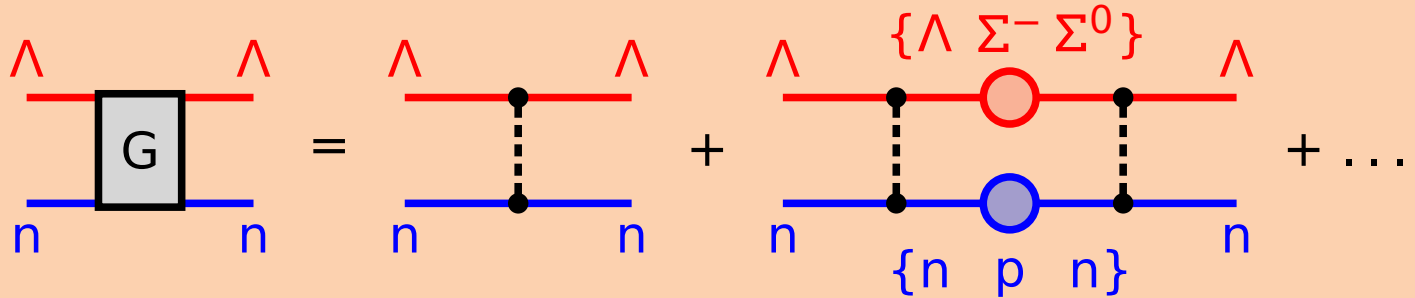
Include Hyperons:

- Technical difficulty: coupled channels:



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- Technical difficulty: coupled channels:



Neutron Stars



Vela pulsar

«Recipe» for Neutron Star Structure Calculation:

Brueckner results: $\epsilon(\{\rho_i\}) ; i = n, p, e, \mu, \Lambda, \Sigma, u, d, s, \dots$

Chemical potentials: $\mu_i = \frac{\partial \epsilon}{\partial \rho_i}$

Beta-equilibrium: $\mu_i = b_i \mu_n - q_i \mu_e$

Charge neutrality: $\sum_i x_i q_i = 0$

Composition: $x_i(\rho)$

Equation of state: $\mathbf{p}(\rho) = \rho^2 \frac{d(\epsilon/\rho)}{d\rho}(\rho, x_i(\rho))$

TOV equations: $\frac{dp}{dr} = -\frac{Gm\epsilon}{r^2} \frac{(1 + p/\epsilon)(1 + 4\pi r^3 \rho/m)}{1 - 2Gm/r}$
 $\frac{dm}{dr} = 4\pi r^2 \epsilon$

Structure of the star: $\rho(r), \mathbf{M}(\mathbf{R})$ etc.

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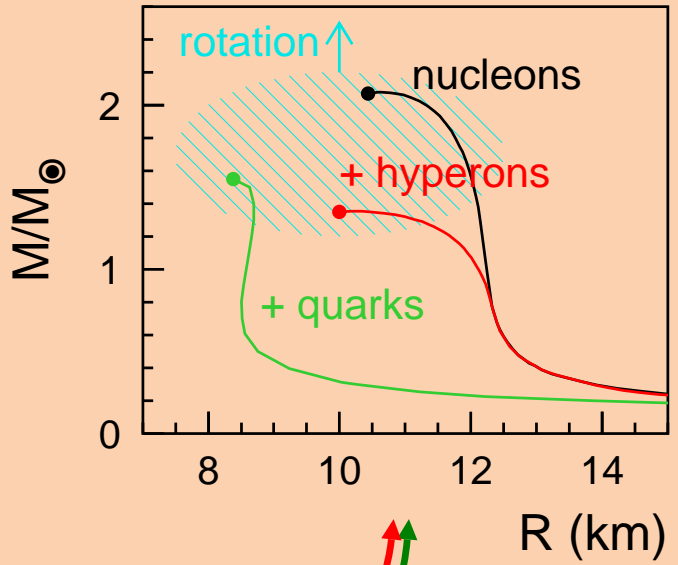
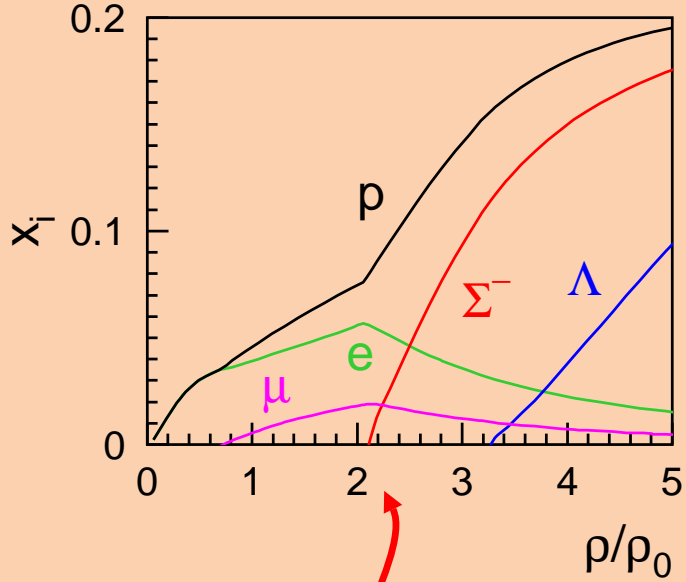
$$\mu_e = \mu_\mu = \mu_n - \mu_p$$

$$\mu_{\Sigma^-} = 2\mu_n - \mu_p$$

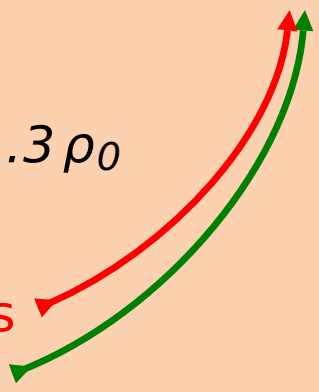
$$\mu_{\Sigma^0} = \mu_\Lambda = \mu_n$$

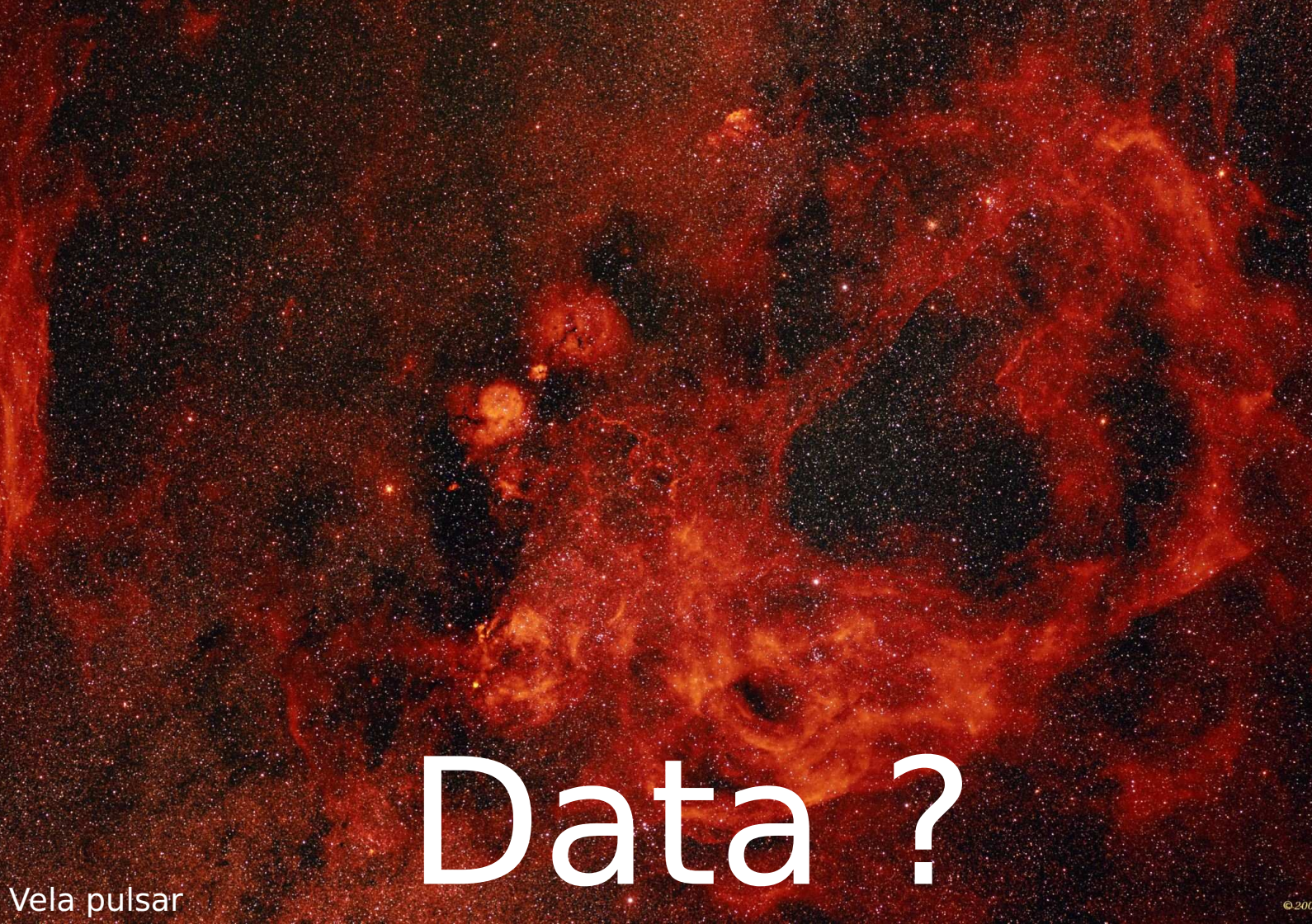
$$\mu_{\Sigma^+} = \mu_p$$

• Generic implications for EOS and stellar structure:



- Hyperon onset occurs at $\rho \sim 2...3 \rho_0$
- Softer EOS
- NS structure including hyperons
... and including quark matter

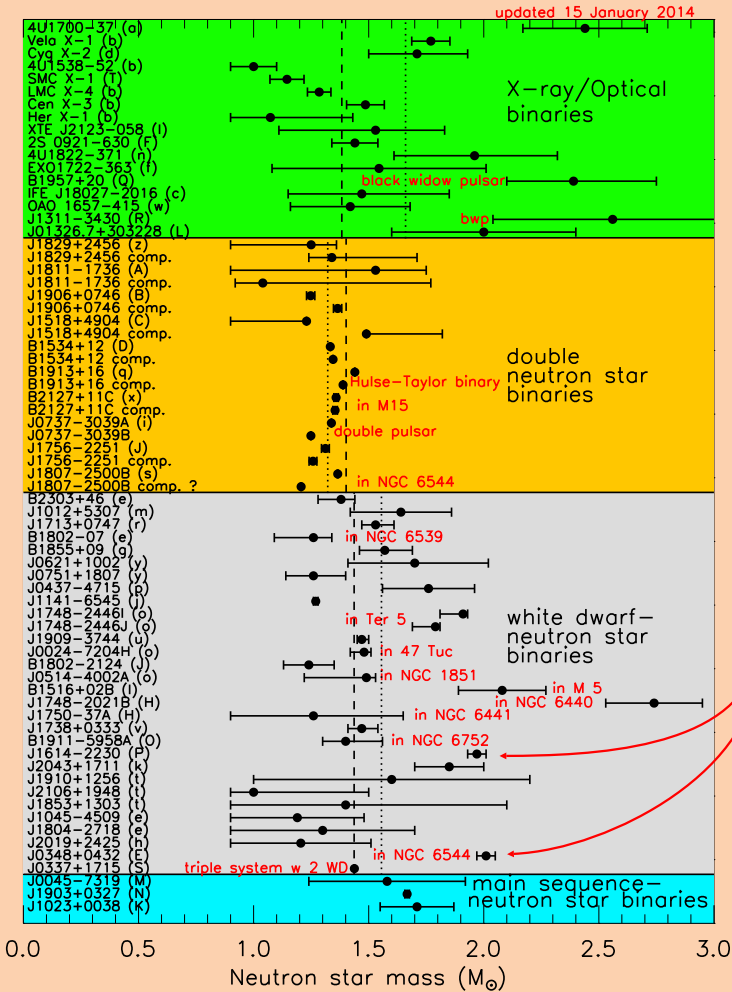




Data ?

Observational Data: Masses

Courtesy of J. Lattimer



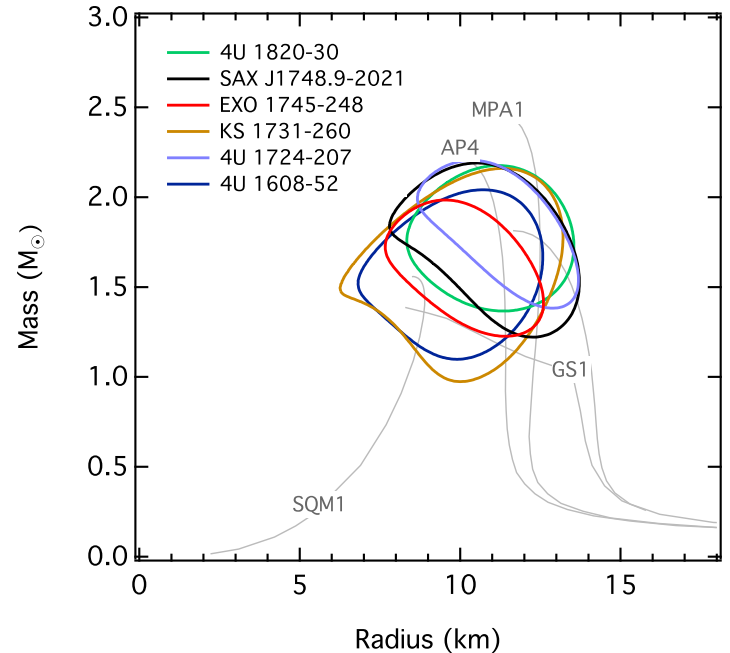
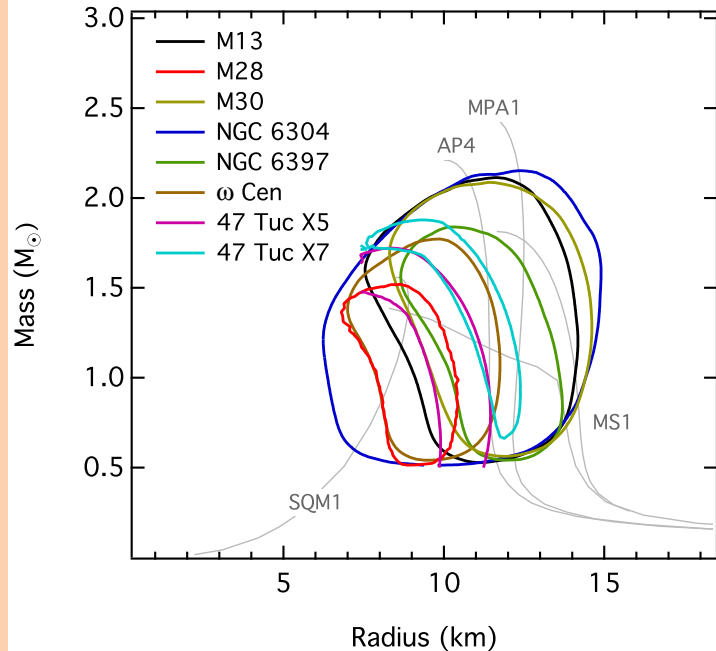
The heaviest neutron stars:

- Recent: $\sim 1.97M_{\odot}$ (Nature 09466)
- $\sim 2.01M_{\odot}$ (Science 340)
- $\sim 2.14 \pm 0.1M_{\odot}$ (1904.06759)

No combined (M, R) measurements!
(Would practically fix the EOS)

Observational Data: Radii

F. Özel et al., APJ820, 28 (2016)

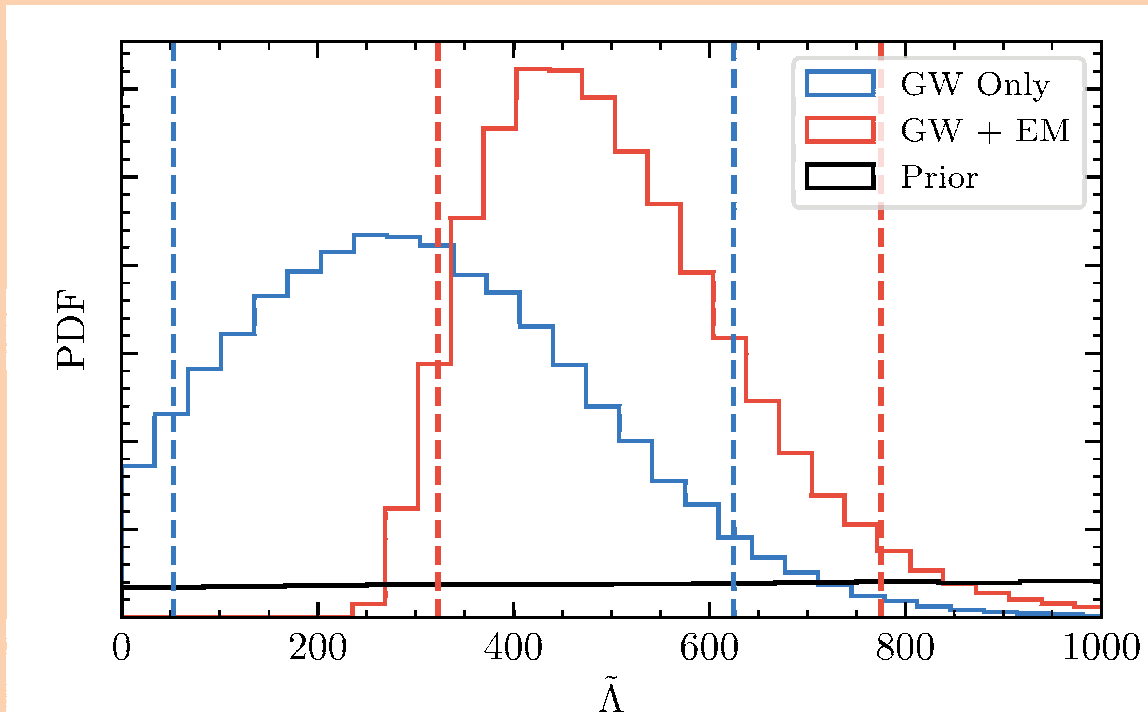


↪ The measurement is difficult: currently no accurate results

Awaiting NICER results ...

Observational Data: Tidal Deformability $\tilde{\Lambda} \sim Q_{ij}/E_{ij}$

Multimessenger Parameter Estimation of GW170817; D. Radice and L. Dai; EPJA 55, 50 (2019)
Analysis of accretion disk mass; assumes a one-family scenario

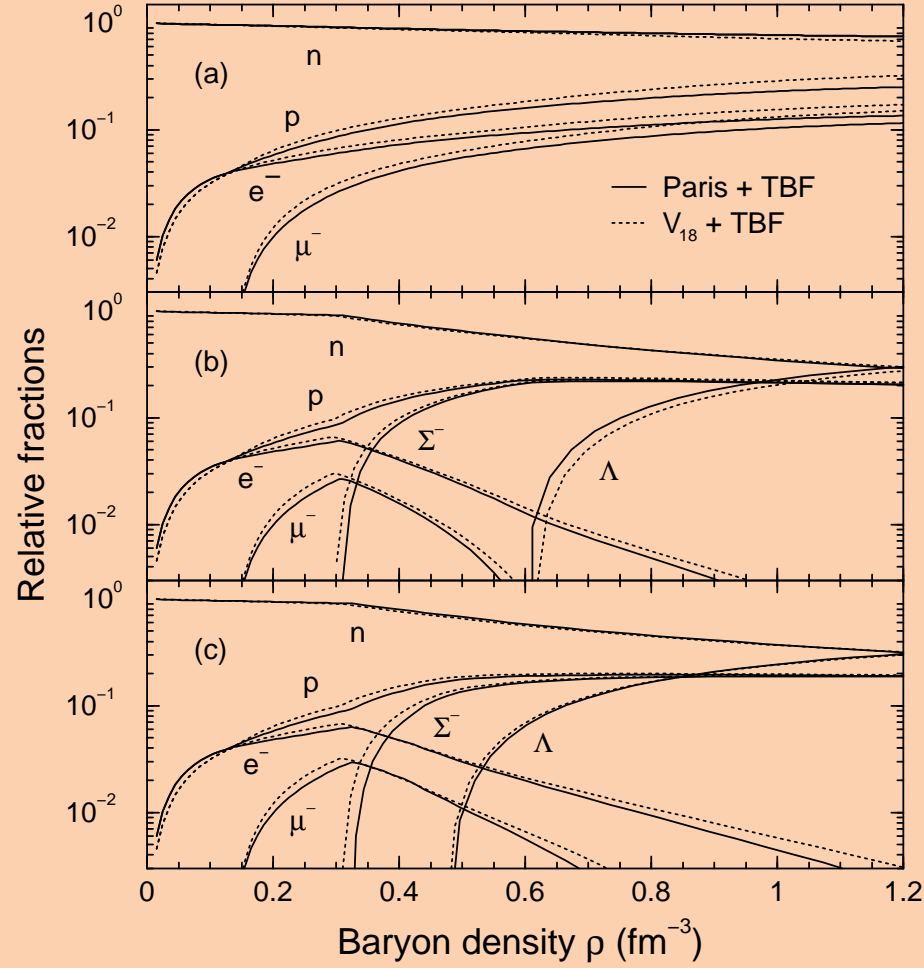


$300 \lesssim \tilde{\Lambda} \lesssim 800 \implies 11.2 \lesssim R \lesssim 13.4 \text{ km}$
(previous result was $400 \lesssim \tilde{\Lambda} \lesssim 800$!)



BHF Results ...

● Composition of neutron star matter:

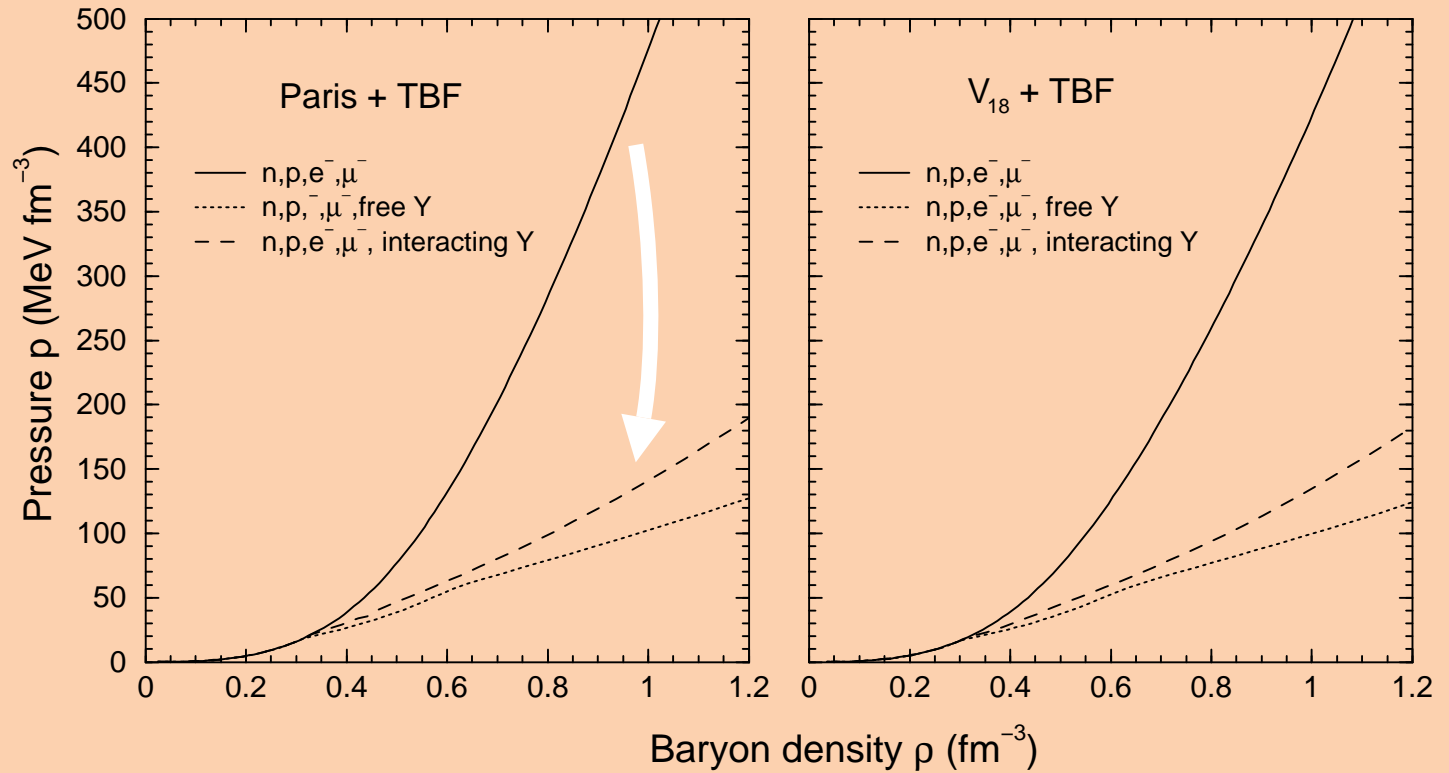


No hyperons

Free hyperons

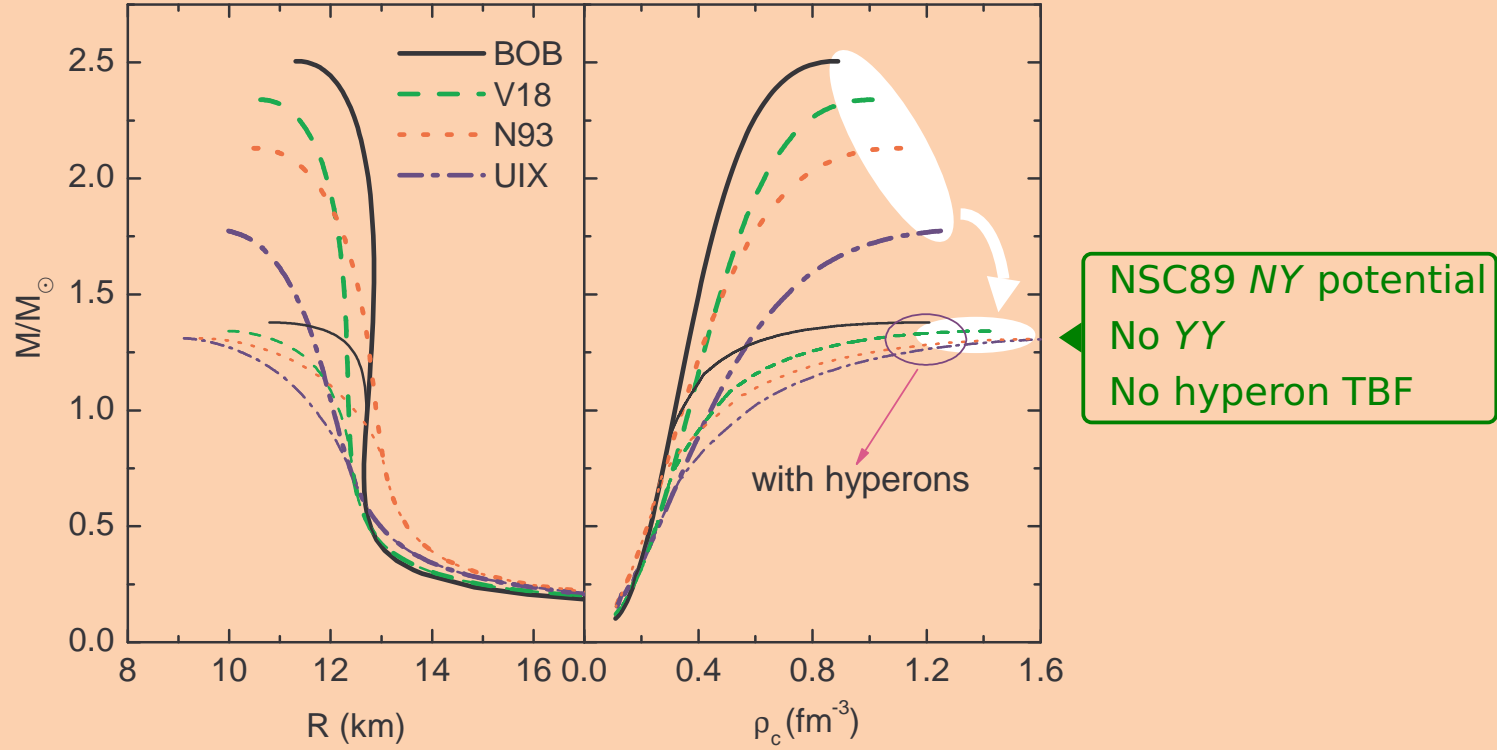
Interacting hyperons
(Σ^- repulsive, Λ attractive)
NY interaction determines
Y onset

• EOS of neutron star matter:



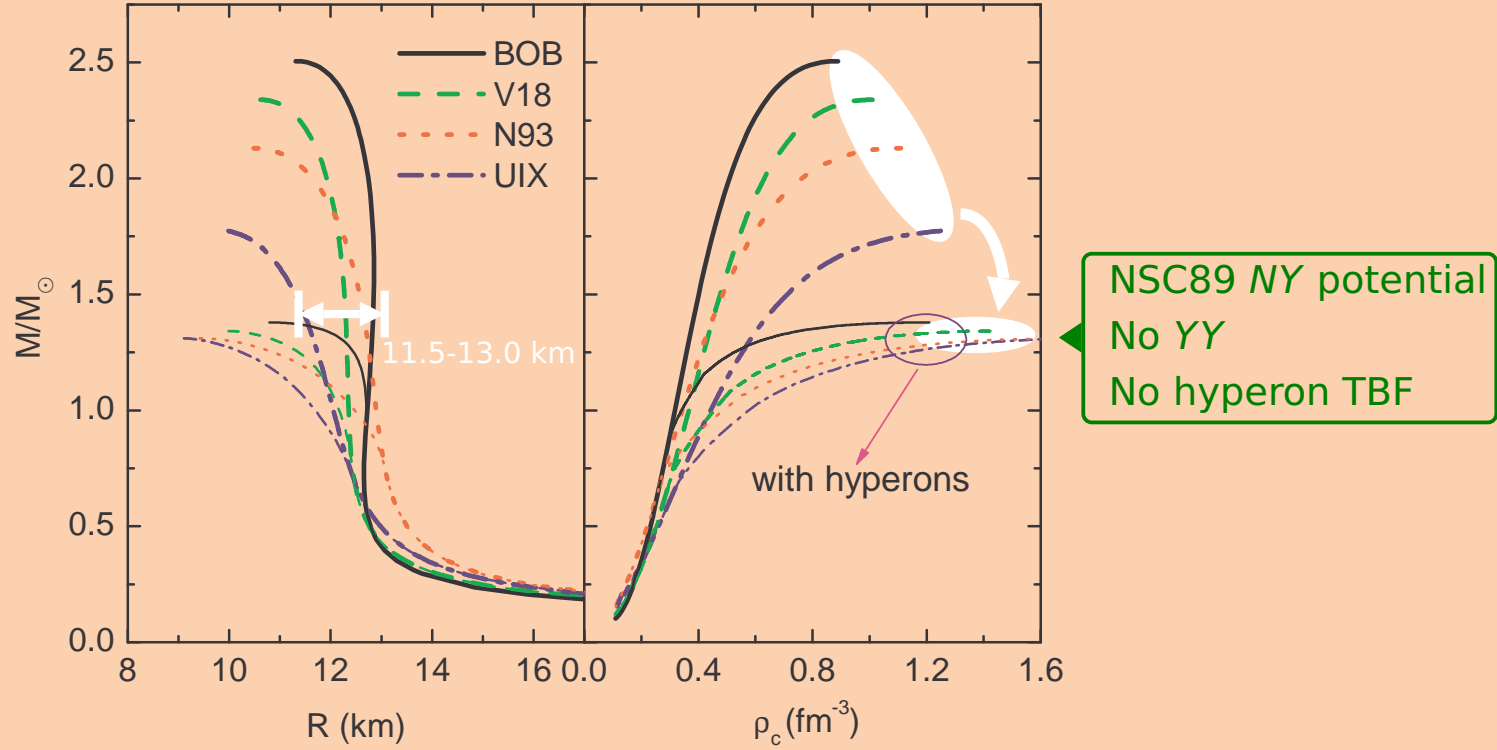
↪ Strong softening due to hyperons !
(More Fermi seas available)

• Mass-radius relations with different nucleonic TBF:



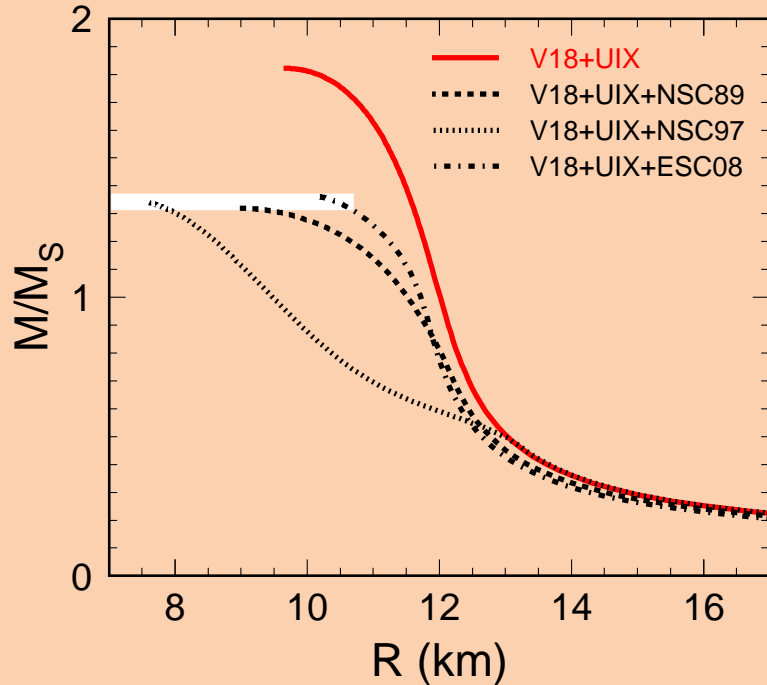
↪ Large variation of M_{max} with nucleonic TBF
Self-regulating softening due to hyperon appearance
(stiffer nucleonic EOS \rightarrow earlier hyperon onset)

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- Mass-radius relations using different NY potentials:

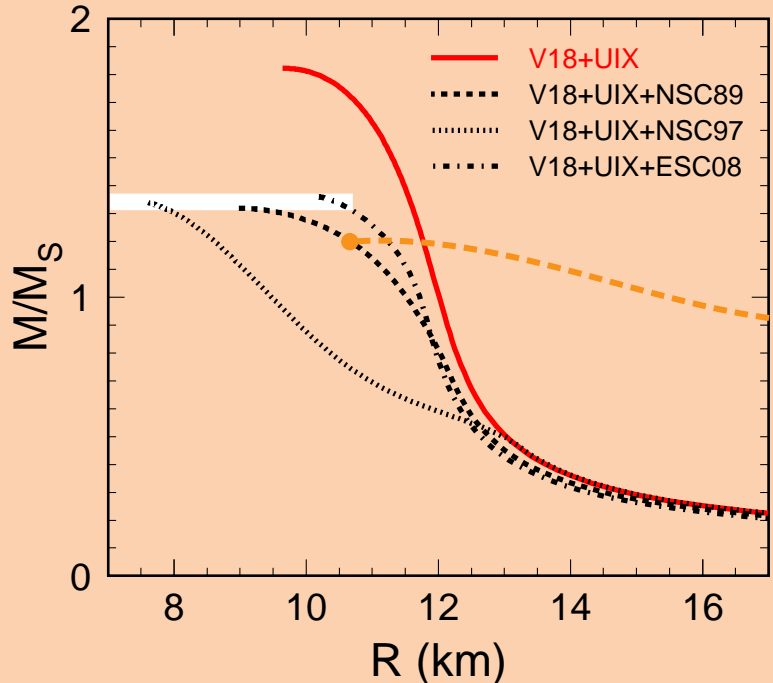


Maximum mass independent of potentials !

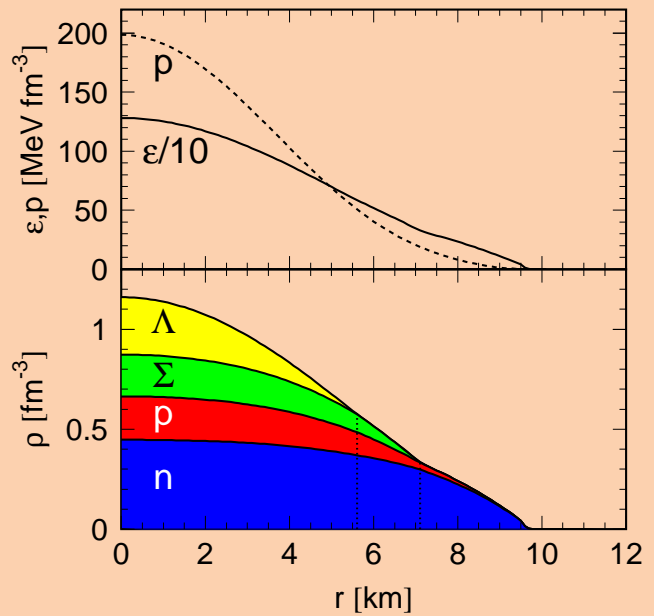
Maximum mass too low ($< 1.4 M_{\odot}$) !

Proof for “quark” matter inside neutron stars ?

● Mass-radius relations using different NY potentials:



BHF(AV₁₈+UIX+NSC89), $M/M_{\odot} = 1.20$

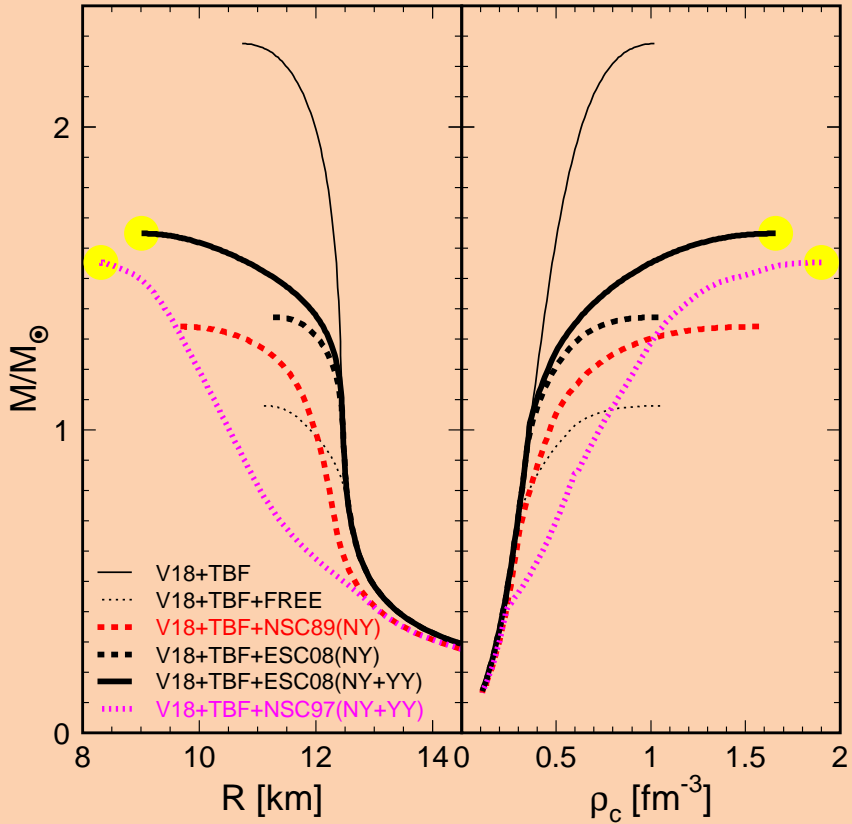


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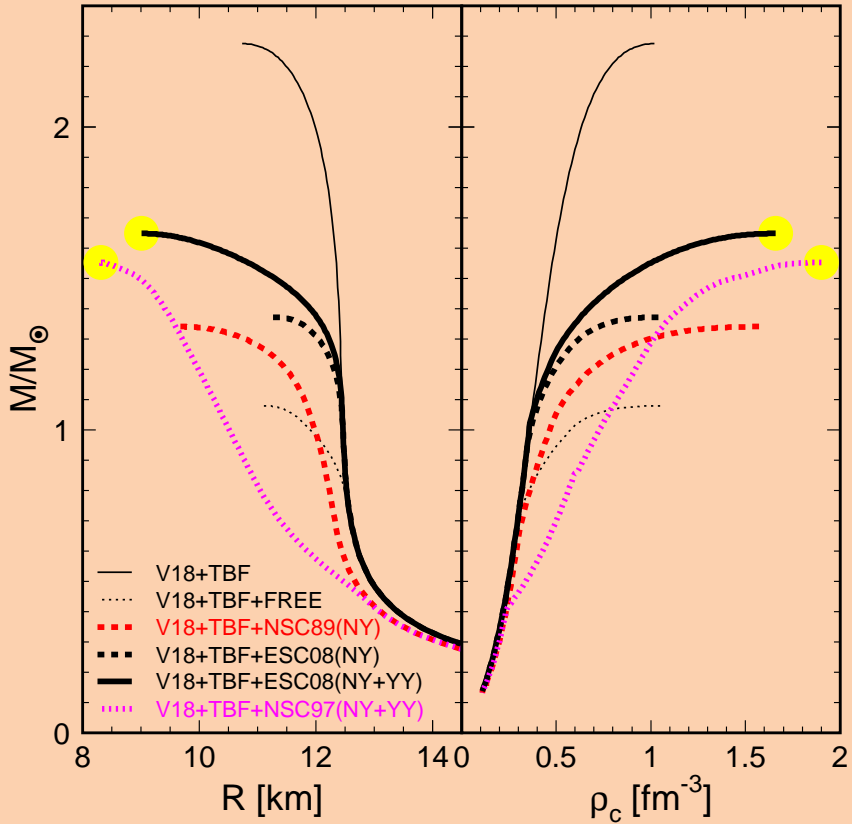
Proof for "quark" matter inside neutron stars ?

● Effect of YY Interactions:

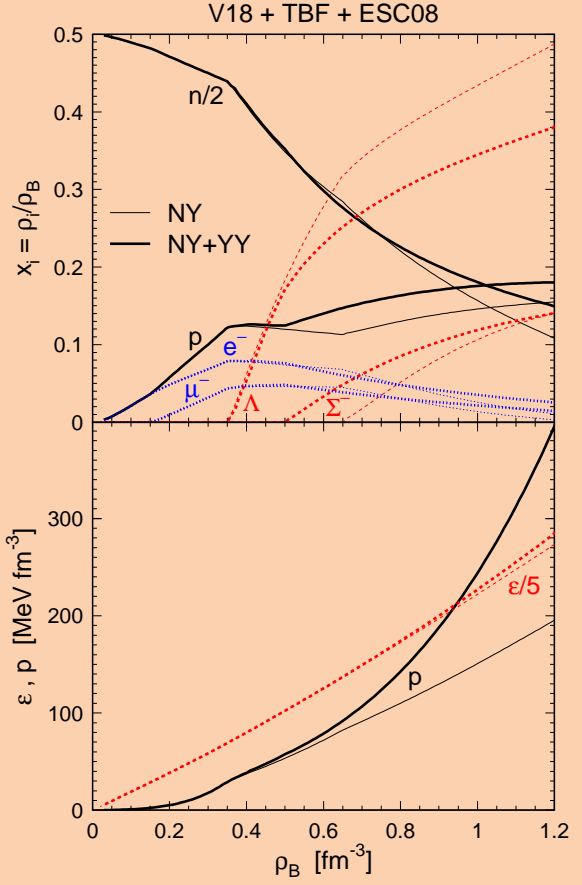


Mass increase to $\lesssim 1.7M_{\odot}$

● Effect of YY Interactions:

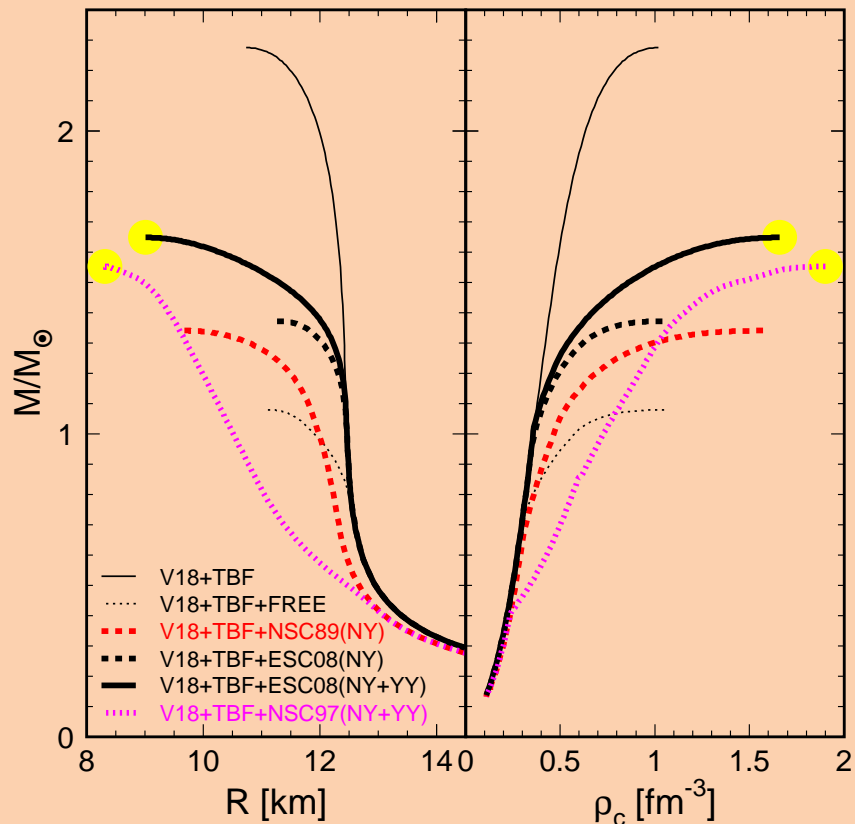


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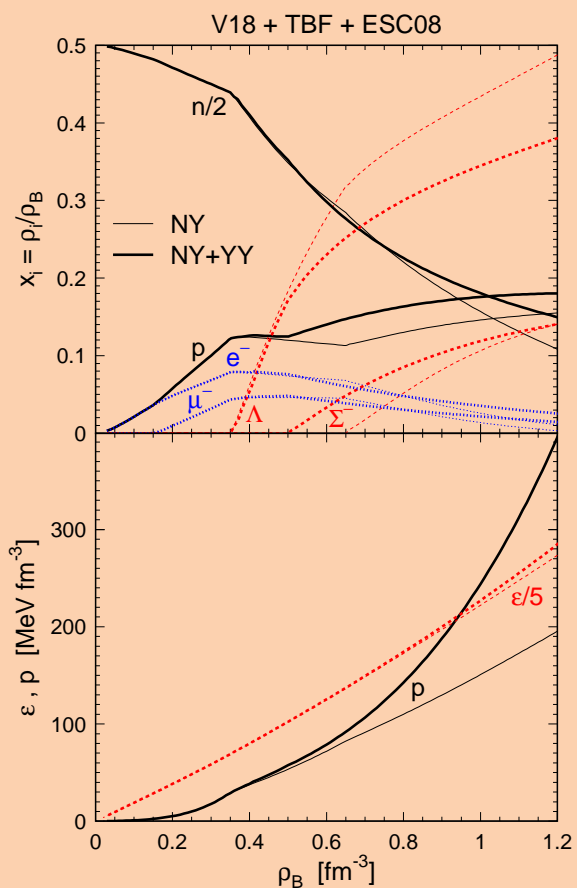


$\Lambda\Lambda, \Sigma^-\Sigma^-$ repulsive
 $\Lambda\Sigma^-$ attractive !

● Effect of YY Interactions:



Mass increase to $\lesssim 1.7M_{\odot}$



$\Lambda\Lambda$, $\Sigma^-\Sigma^-$ repulsive
 $\Lambda\Sigma^-$ attractive !

● Hyperon TBF (YNN , YYN , YYY) unknown (exp. and theor.) !

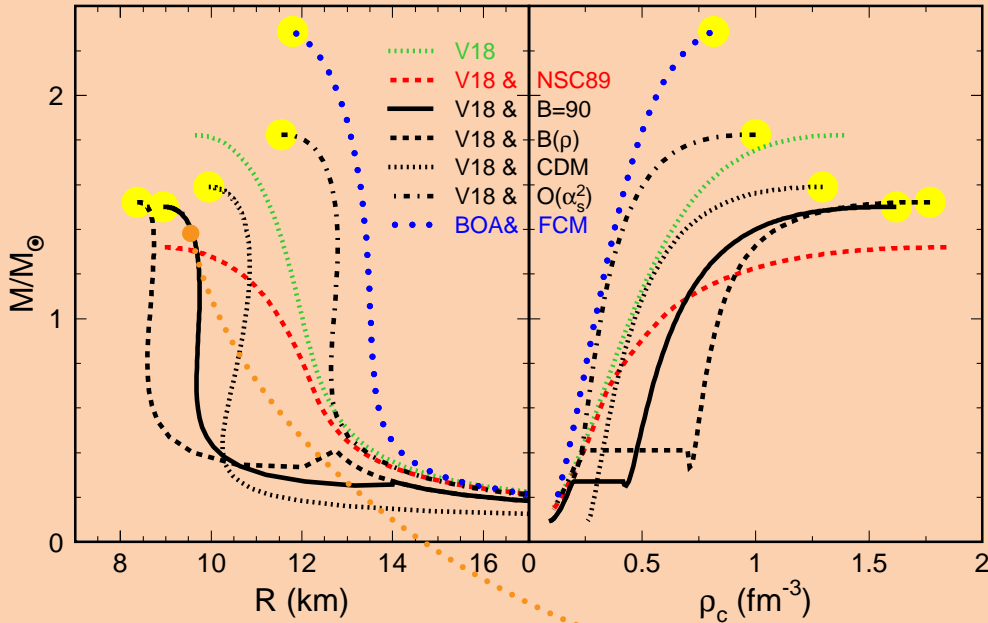
Quark Matter EOS of Dense Matter:

- Problem: No “exact” results from QCD:
Large theoretical uncertainties, limited predictive power
- Current strategy:
Use available eff. quark models (MIT, NJL, CDM, DSM, ...) in combination with the hadronic EOS
- An important constraint (from heavy ion collisions):
In symmetric matter phase transition not below $\approx 3\rho_0$
- ➡ E.g., the simplest (MIT) quark model requires a density-dependent bag “constant”:

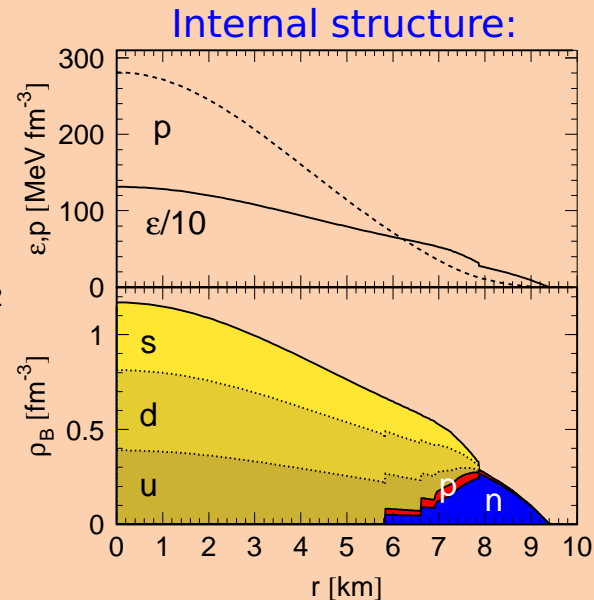
$$\epsilon_Q = B + \epsilon_{\text{kin}} + \alpha_s \times \dots$$

$$B(\rho) = B_\infty + (B_0 - B_\infty) \exp\left[-\beta\left(\rho/\rho_0\right)^2\right]$$

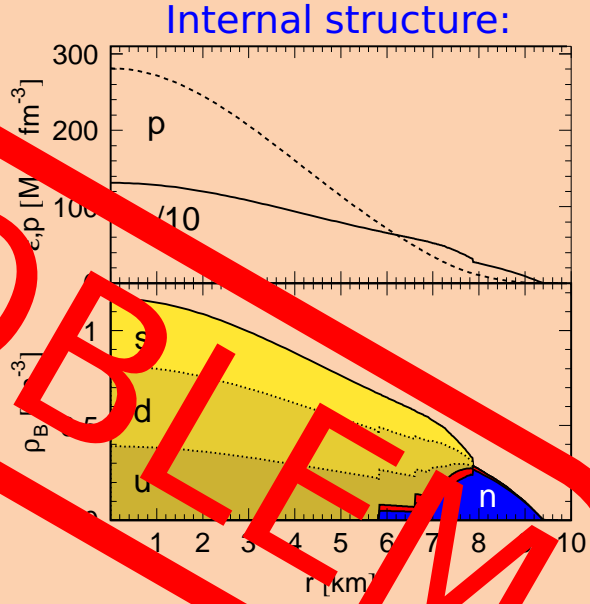
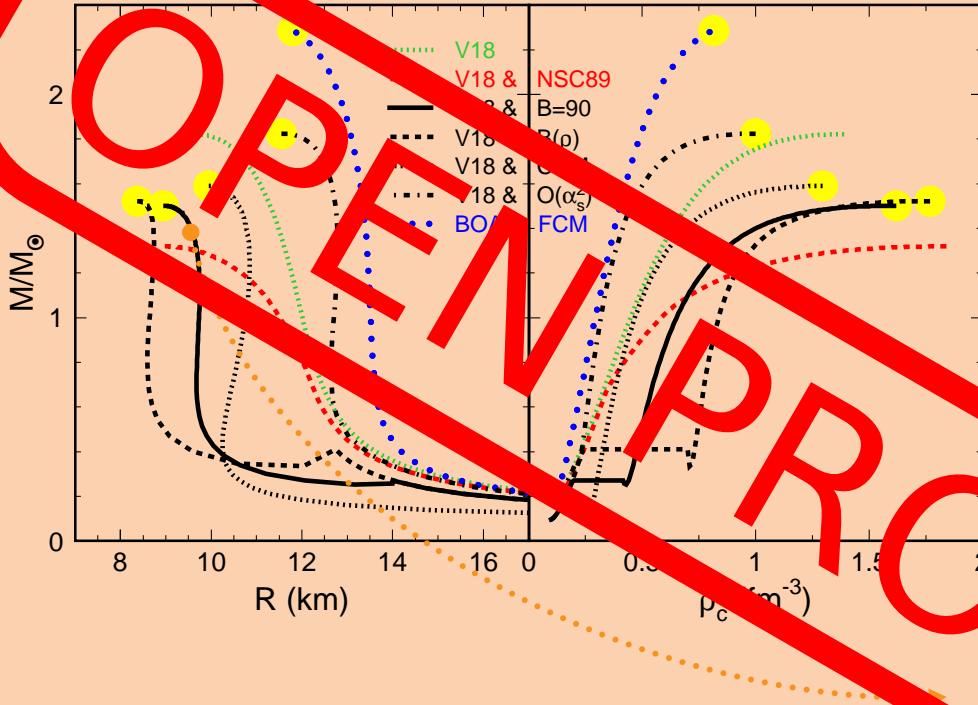
Hybrid (quark-matter core) Stars:



NJL, FCM, Dyson-Schwinger models:
 Hyperons prevent phase transition !

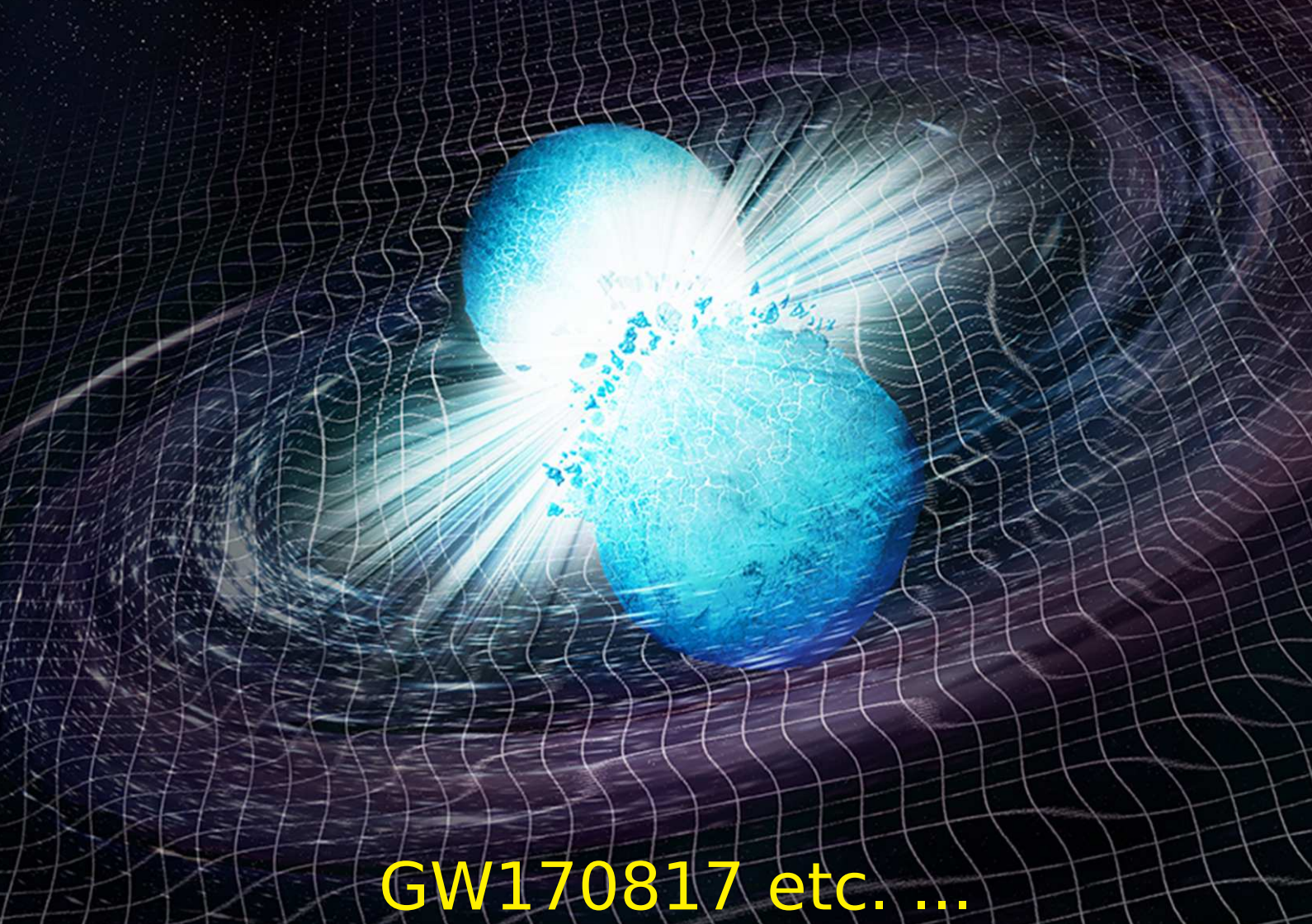


Hybrid (quark-matter core) Stars:



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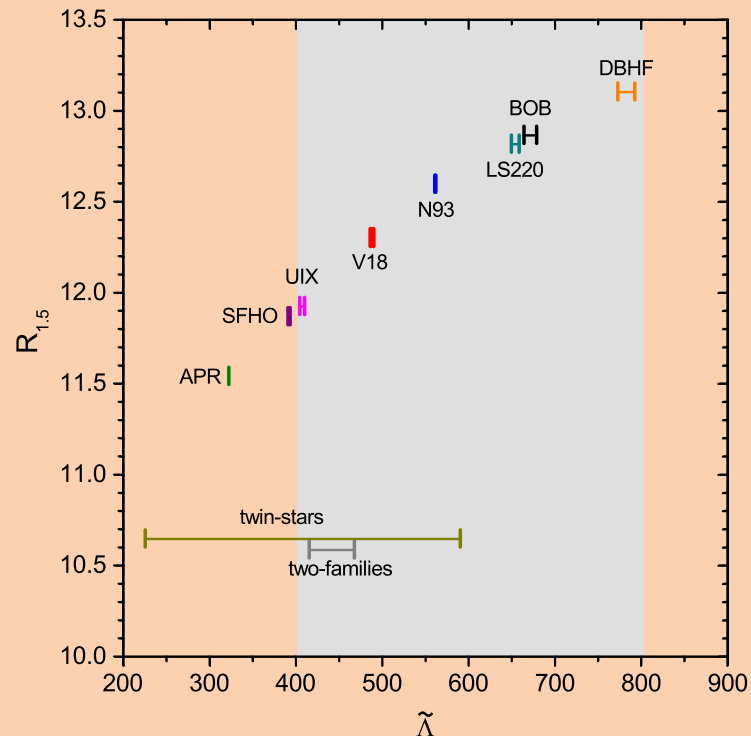
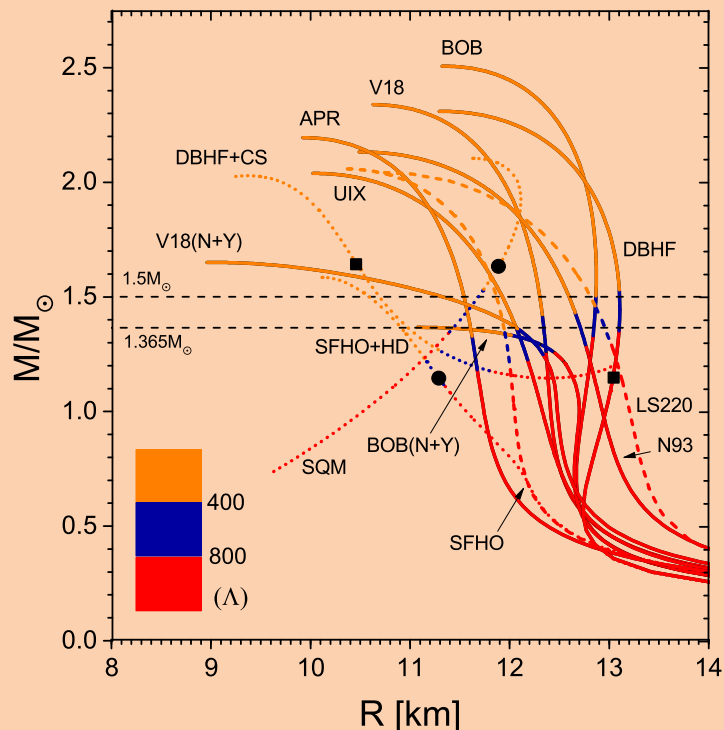
OPEN PROBLEM



GW170817 etc. ...

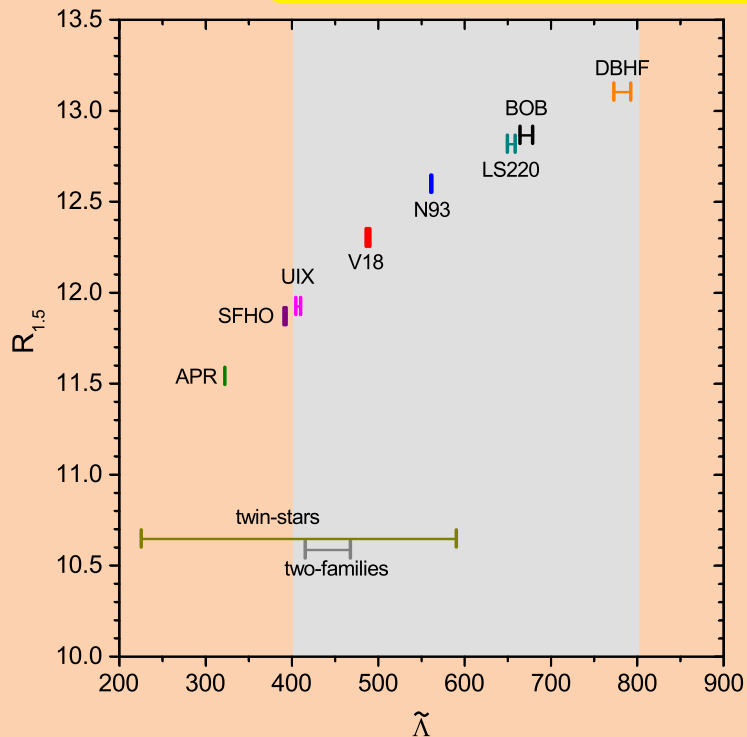
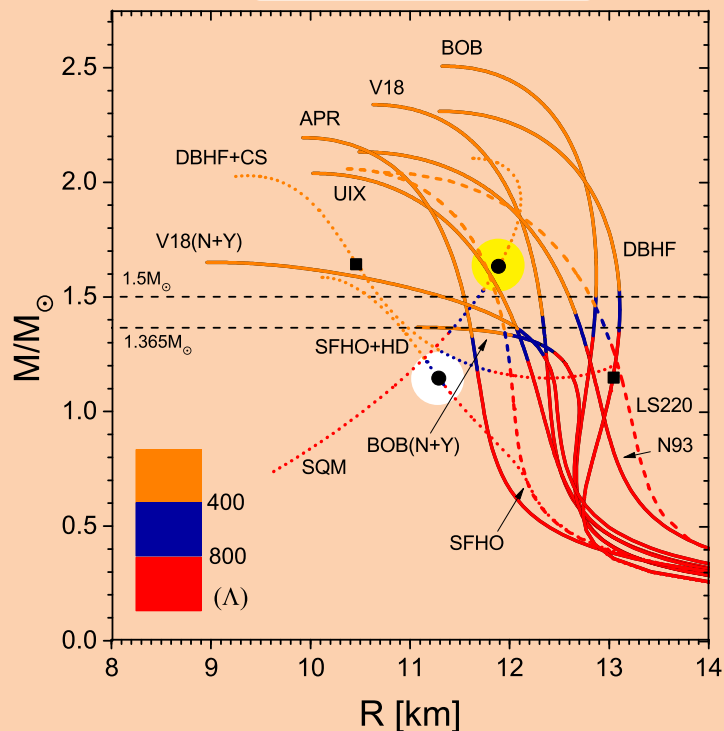
Some recent results with BHF (hyperonic) EOs:

- In the *two-families* scenario could coexist APJ 860, 139 (2018)
low-mass hyperon stars and high-mass strange quark stars:



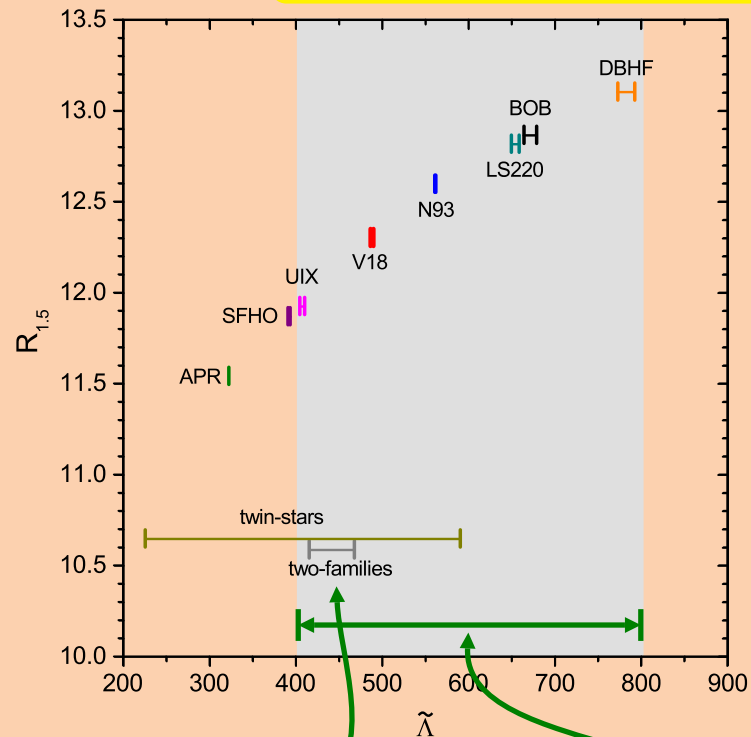
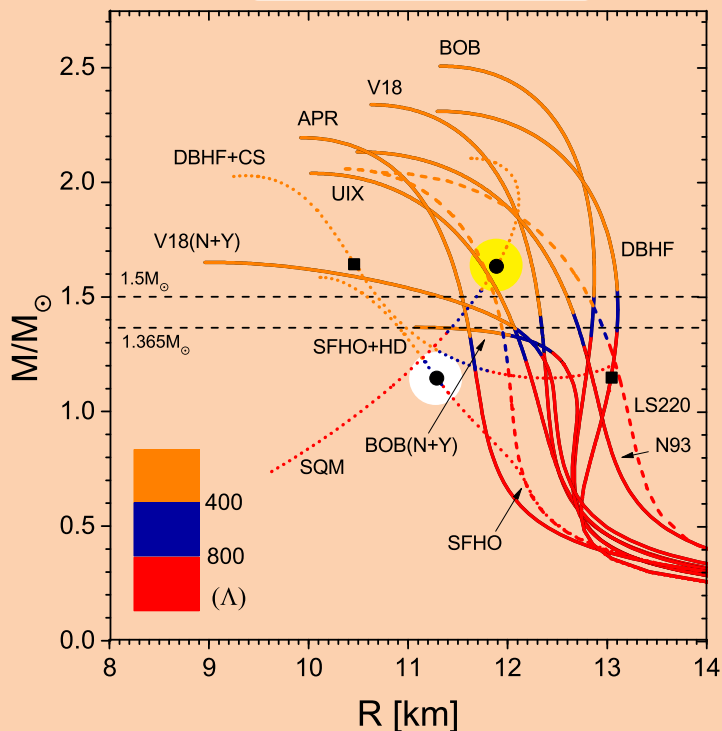
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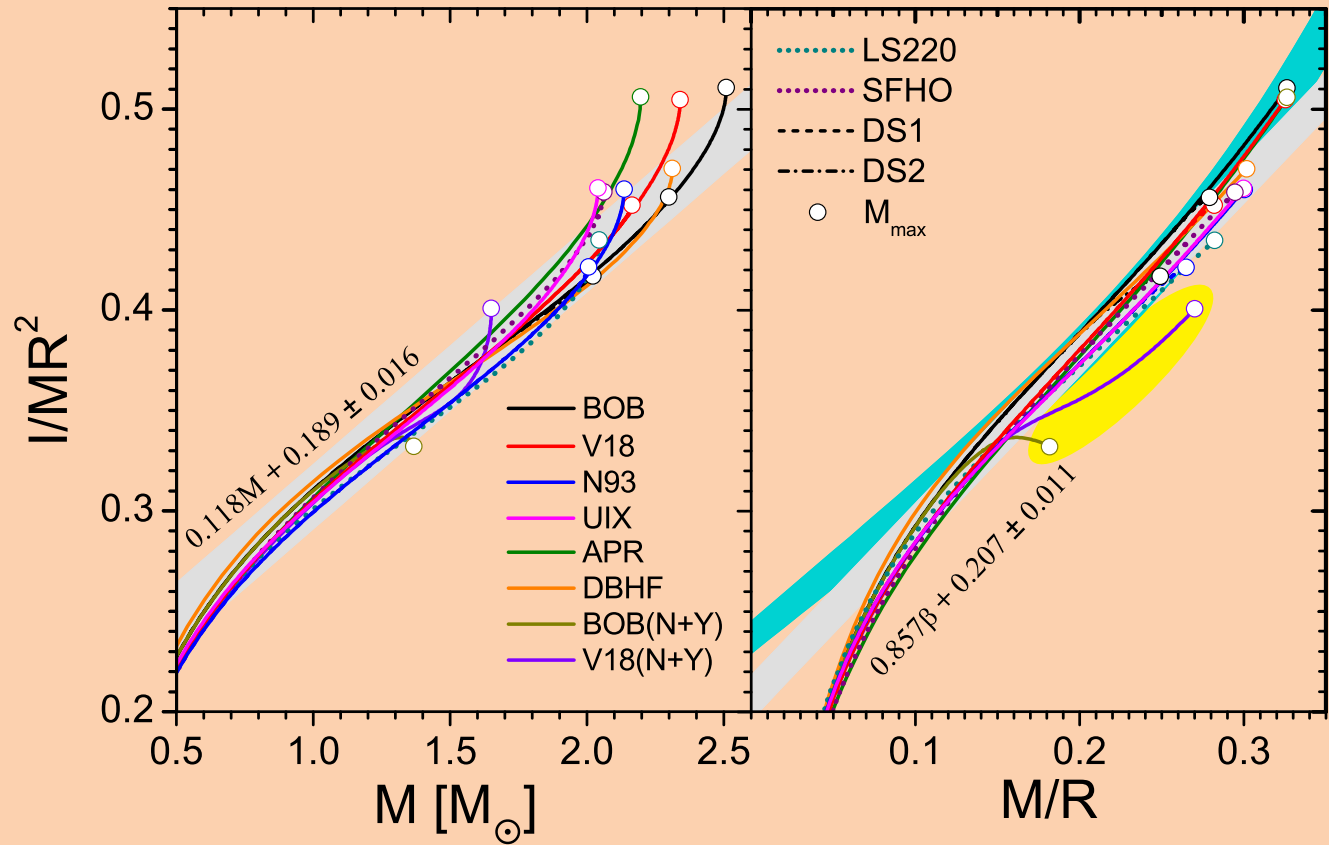


Variation due to allowed mass asymmetry $q = M_2/M_1$

Constrained by GW170817 analyses

● *Universal Relations* might be violated by hyperonic EOSs:

e.g., for moment of inertia I : [JPG 46, 034001 \(2019\)](#) :

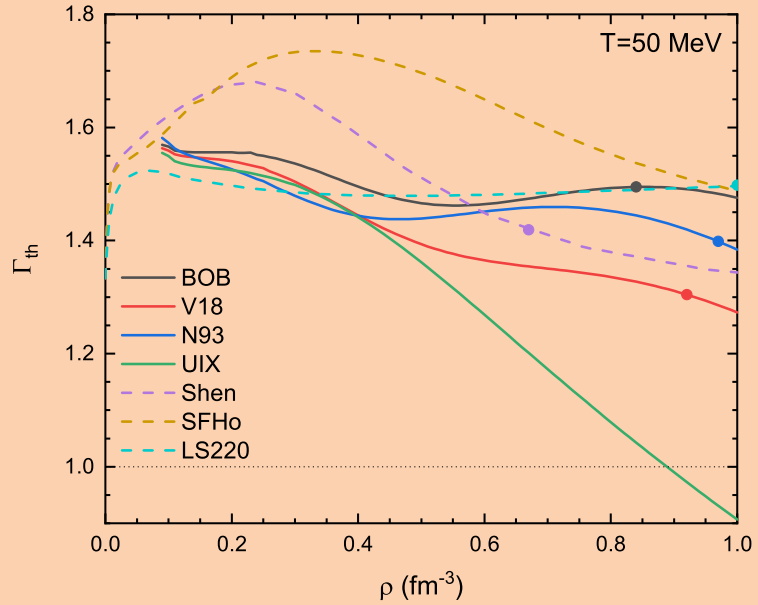


Deviation from univ. relation due to atypically small radii

● BHF EOS at finite temperature:

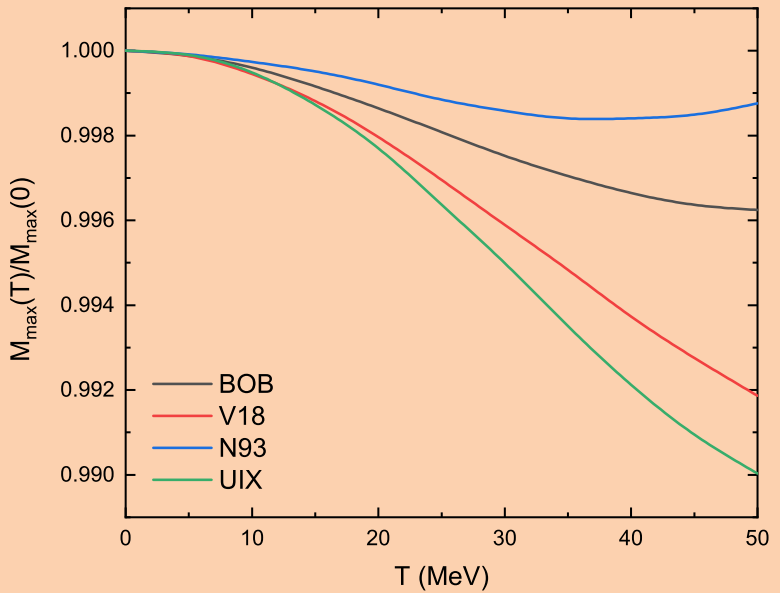
adiabatic index

$$\Gamma_{\text{th}}(\rho, T) \equiv 1 + \frac{p(\rho, T) - p(\rho, 0)}{\epsilon(\rho, T) - \epsilon(\rho, 0)}$$



maximum NS mass

$$M_{\text{max}}(T)$$



important for numerical merger simulations
(estimate of NS M_{max})

Summary:

- Neutron star physics probes the 4 fundamental interactions:
 - Gravitation: Densest object in the Universe
 - Strong: Nuclear EOS
 - Weak: Beta-equilibrium of matter, Neutrino physics
 - EM: Charge-neutrality, Mixed-phase structures, Crust
- BHF+TBF approach with meson-exchange NN forces provides reasonable nuclear EOS up to high density:
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However:

- Hyperons cannot be ignored !
- BHF EOS with hyperons predicts M_{max} not above $\sim 1.7 M_{\odot}$
- Need “quark matter” to reach higher masses of hybrid stars