Study of deuteron-dark matter scattering within an effective field theory

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Motivations

- Since the '30s of last century, on galactic scales and over, a great number of gravitational anomalies has been detected.
  - The most popular explanation is the existence of a new kind of particles: the dark matter (DM) [Bertone *et al.*, 2005]

Our purpose is the study of the nuclear response to DM scattering, assumed to be composed by Weak Interacting Massive Particles (WIMPs)

- This response is needed to analyze the results of the various direct detection experiments, which are currently attempting to detect DM [Baudis, 2018]
- The WIMP's can be assumed to be nonrelativistic, since, in order to be gravitationally bound in the galaxy halos $|\frac{v_\chi}{c}| \sim 10^{-3}$
- The typical momentum and energy transfer is small and the nucleus does not break apart
- To describe this type of scattering, an effective field theory (EFT) approach to nuclear dynamics can be used
- We will use an EFT based on chiral symmetry (χEFT)

Quark-WIMP interactions

We start from the general dimension 6 effective Lagrangian for the interaction between quark and WIMP, the latter assumed to be a Dirac fermion. The Lagrangian can be cast in the form [Schwenk *et al.*, 2013]

$$\begin{aligned} \mathcal{L}_q &= \mathcal{L}_{\rm QCD}^{\mathcal{M}=0} + \bar{q}(x)\gamma^\mu \left(v_\mu(x) + \frac{1}{3} v_\mu^{(s)}(x) + \gamma^5 a_\mu(x) \right) q(x) \\ &\quad - \bar{q}(x)\left(s(x) - i\gamma^5 p(x)\right) q(x) + \bar{q}(x)\sigma^{\mu\nu} t_{\mu\nu}(x) q(x) \end{aligned}$$

$$\begin{aligned} s(x) &= -\frac{1}{\Lambda_S^2}\left(C_{S+} + C_{S-}\tau_z\right)\bar{\chi}\chi & p(x) &= \frac{1}{\Lambda_S^2}\left(C_{P+} + C_{P-}\tau_z\right)\bar{\chi} i\gamma_5\chi \\ \frac{1}{3} v^{\mu(s)}(x) &= \frac{1}{\Lambda_S^2} C_{V+}\bar{\chi}\gamma^\mu\chi & v^\mu(x) &= \frac{1}{\Lambda_S^2} C_{V-}\tau_z\bar{\chi}\gamma^\mu\chi \\ t^{\mu\nu}(x) &= \frac{1}{\Lambda_S^2}\left(C_{T+} + C_{T-}\tau_z\right)\bar{\chi}\sigma^{\mu\nu}\chi & a^\mu(x) &= \frac{1}{\Lambda_S^2} C_{A-}\tau_z\bar{\chi}\gamma^\mu\gamma_5\chi \end{aligned}$$

with $C_{X\pm}$ adimensional coupling constant to be determined by experimental data and $\Lambda_S = 1$ GeV inserted for dimensional reasons

Difference with respect to the standard approach

- Presence of an isoscalar part $a_\mu^{(s)}(x) \to SU(3)$
- Presence of tensor current $t^{\mu\nu} \to$ new terms in the χEFT Lagrangian

Nucleon-WIMP interactions

- Step 1: substituting the expression of s,p,v, ... in term of the WIMP field, determine the nucleon-pion-WIMP vertices
  - Example: Scalar interaction

$$\mathcal{L}_{int} = c_1 \bar{N}\langle\xi_+\rangle N + c_5 \bar{N}\hat{\xi}_+ N + \frac{f_\pi^2}{4}\left\langle \xi(x)U^\dagger(x) + U(x)\xi^\dagger(x)\right\rangle + \cdots$$

$$\begin{cases} U = e^{i\vec{\pi}\cdot\vec{\tau}/f_\pi} \quad u = \sqrt{U} \\ \xi_+ = u^\dagger \xi\, u^\dagger + u\,\xi^\dagger u \\ \xi(x) = 2B_c(s(x) + ip(x)) \\ c_1, c_5, B_c \text{ are LECs} \end{cases}$$

$$\mathcal{L}_{int} \approx -\frac{8B_c c_1}{\Lambda_S^2} C_{S+}\bar{N}N\bar{\chi}\chi - \frac{4B_c c_5}{\Lambda_S^2} C_{S-}\bar{N}\tau_z N\bar{\chi}\chi + \frac{C_{S+}}{\Lambda_S^2}\bar{\chi}\chi\pi^2 + O(\pi^3)$$

- Step 2: Using the Legendre transformation, write the H_{int} in the Schrödinger picture

$$H_{int} = H^{NN\chi\chi,00} + H^{\pi\pi\chi\chi,02} + H^{\pi\pi\chi\chi,11} + H^{\pi\pi\chi\chi,20} + H^{\pi NN,01} + H^{\pi NN,10} + \cdots$$

$$H^{NN\chi\chi,00} = \frac{1}{\Omega}\sum_{\mathbf{p}'s't',\mathbf{p}st,\mathbf{k}'r',\mathbf{k},r} b^\dagger_{\mathbf{p}',s',t'} b_{\mathbf{p},s,t} B^\dagger_{\mathbf{k}',r'} B_{\mathbf{k},r}\, M^{NN\chi\chi,00}_{\mathbf{p}'s't'\mathbf{p}st\mathbf{k}'r'\mathbf{k}r}\,\delta_{\mathbf{p}'+\mathbf{k}',\mathbf{p}+\mathbf{k}}$$

$b^\dagger$, b, $B^\dagger$, B= creation and annihilation operators, $\mathbf{p}$, s, t, $\mathbf{p}'$, s', t'= nucleon states, $\mathbf{k}$, s, $\mathbf{k}'$, s'= WIMP states

- Example: NR expansion of vertex function M for the scalar interaction up to $O(1/M^2)$

$$M^{NN\chi\chi,00}_{\alpha'\alpha\mathbf{k}'r'\mathbf{k}r} \approx \left(\frac{8B_c c_1 C_{S+}}{\Lambda_S^2} + \frac{4B_c c_5 C_{S-}}{\Lambda_S^2}\tau_z\right)_{t't}\left(1 - \frac{(\mathbf{p}+\mathbf{p}')^2}{8M^2} - \frac{i(\mathbf{p}'\times\mathbf{p})\cdot\boldsymbol{\sigma}}{4M^2}\right)_{s's}\times\left(1 - \frac{(\mathbf{k}+\mathbf{k}')^2}{8M_\chi^2} - \frac{i(\mathbf{k}'\times\mathbf{k})\cdot\boldsymbol{\sigma}}{4M_\chi^2}\right)_{r'r}$$

$(\tau_i)_{t't}$, $(\boldsymbol{\sigma})_{s's}$= matrix elements of spin isospin Pauli matrices

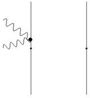
- Step 3: Using the perturbation theory evaluate the T-matrix

$$T = H_{int} + H_{int}\frac{1}{E_i - H_0}H_{int} + H_{int}\frac{1}{E_i - H_0}H_{int}\frac{1}{E_i - H_0}H_{int} + \cdots$$

- Step 4: Each term contributing to the T-matrix can be visualised as a time ordered diagram

- Step 5: Associate to each diagram a chiral order ν: its contribution is of order $(Q/\Lambda_\chi)^\nu$
  ($Q \sim m_\pi =$ nucleon momentum, $\Lambda_\chi = 4\pi f_\pi \sim 1$ GeV)

Example: Scalar interaction

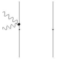
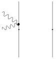
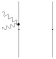
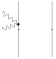
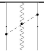
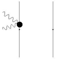
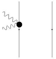
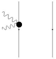
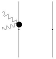
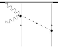


$$= M^{NN\chi\chi}_{\mathbf{p}_1'\mathbf{p}_1\mathbf{k}'\mathbf{k}}\,\delta_{\mathbf{p}_1'+\mathbf{k}',\mathbf{p}_1+\mathbf{k}}\,\delta_{\mathbf{p}_2',\mathbf{p}_2} \sim Q^{-3}$$

- Chiral order of each diagrams
  - NR expansion of vertex functions $M^{NN\chi\chi}$, . . . in powers of Q/Λ_χ
  - Energy denominators $\sim Q^{-1}$
  - δ in addition to the total momentum conservation $\sim Q^{-3}$

For the deuteron (T=0) only the diagrams giving isoscalar operators will contribute

Chiral order of some diagrams that we have considered in our work

Chiral order	S	P	V	A	T
Q^{-3}		—			
Q^{-2}		—	—	—	—
Q^{-1}					*n.c*
Q^{0}	*n.c*	*n.c*	*n.c*		*n.c*

n.c=not considered

Deuteron-WIMP transition amplitude

$T_{f,i}$ between the initial deuteron+WIMP state $|\mathbf{P}_i, M_i, \mathbf{k}, r\rangle$ and deuteron+WIMP final state $|\mathbf{P}_f, M_f, \mathbf{k}', r'\rangle$ is

$$\langle \mathbf{P}_f \mathbf{k}' | T | \mathbf{P}_i \mathbf{k} \rangle = \frac{C_{X+}}{\Lambda_S^2} \frac{1}{\Omega} \delta_{\mathbf{q}+\mathbf{P}_i,\mathbf{P}_f} \left\{ \int d\mathbf{r} \psi_f^*(\mathbf{r}) \int e^{i\mathbf{q}\cdot\mathbf{x}} J_\mu(x) \cdot L^\mu d\mathbf{x} \psi_i(\mathbf{r}) \right\}$$

where $J_\mu(x)$ contains the two body and one body nuclear currents and L^μ is the WIMP current

Performing the multipole expansion of the current

$$= \frac{C_{X+}}{\Lambda_S^2} \frac{1}{\Omega} \delta_{\mathbf{q}+\mathbf{P}_i,\mathbf{P}_f} (-1)^{J-J_z} \Bigg(\sum_{l\geq 0}^{\infty} \sum_{m=-l}^{l} i^l \mathcal{D}^l_{m,0}(\varphi,\theta,-\varphi) \sqrt{4\pi} (J' J'_z J - J_z | lm) \{ L_0 X_l^C(q,r) + L_z X_l^L(q,r) \}$$

$$+ \sum_{l=1}^{\infty} \sum_{m=-l}^{l} i^l \mathcal{D}^l_{m,\lambda}(\varphi,\theta,-\varphi) \sqrt{2\pi} (J' J'_z J - J_z | lm) \times L_{-\lambda} \left\{ -\lambda X_l^M(q,r) - X_l^E(q,r) \right\} \Bigg)$$

$$\mathbf{q} = \mathbf{k} - \mathbf{k}'$$

$\mathcal{D}^l_{m,m'}(\alpha,\beta,\gamma)$ standard rotation matrices, θ, φ angles of $\mathbf{q}$, X_l^Y reduced matrix elements (RMEs)

- $\mathbf{L} = \mathbf{L} \cdot \hat{\epsilon}_{\mathbf{q},\lambda}$, $\hat{\epsilon}_{\mathbf{q},\lambda}$ = set of 3 versor forming a destrorse tern
- For the deuteron $J = J' = 1 \to 0 \leq l \leq 2$, other selection rules follow from the parity
- We first evaluate the matrix element integrating over the $d\mathbf{r}$ (N_r, N_θ, N_φ integration points) and invert the equation to determine the RMEs

Deuteron wave function

The deuteron wave function is written as

$$\psi^d_{J_z}(\mathbf{r}) = \sum_{L=0,2} u_L(r)\,[Y_L(\hat{\mathbf{r}})\chi_S]_{JJ_z}\,\xi_{T,0} \qquad S=1, J=1, T=0$$

$u_L(r)$ are expanded over an orthonormal basis of functions f_n

$$u_L(r) = \sum_{n=0}^{N_L-1} a_{L,n} f_n(r) \qquad f_n(r) = N_n e^{-\gamma r/2} L_n^{(2)}(\gamma r) \qquad N_n = \sqrt{\frac{n!}{(n+2)!}\gamma^3}$$

$L_n^{(2)}(\gamma r)$ = the generalized Laguerre orthogonal polynomials of order two [Abramowitz, 1970].

For fixed values of γ and N_L, $a_{L,n}$ are obtained through the application of the variational principle:

$$\sum_{L',n'} \langle \psi_{L,n} | H | \psi_{L',n'} \rangle \, a_{L',n'} = E a_{L,n}$$

We solve the eigenvalue problem using standard methods (LAPACK subroutines).

E in function of γ, N_L and N_I = # integration points, calculated using AV18 [Wiringa *et al.* 1995] and N4LO500 [D. R. Entem *et al.*, 2017] potentials

γ [fm^{-1}]	N_L	N_I	AV18 [MeV]	N4LO500 [MeV]
3.0	20	80	-2.221225	-2.222311
3.0	30	80	-2.224293	-2.224534
3.5	30	80	-2.224536	-2.224569
4.0	30	80	-2.224537	-2.224568
4.0	40	80	-2.224572	-2.224586
4.0	60	80	-2.224574	-2.224587
4.0	60	100	-2.224574	-2.224587

The deuteron energy values agree with the experimental value $E = -B_{\rm exp} = -2.224579(9)$ MeV

Cross-section

The non-polarized cross section for this process is calculated from Fermi golden rule, by mediating over the initial polarizations and summing over the final ones,

$$\sigma_{fi} = \frac{1}{6} 2\pi \sum_{r' r} \sum_{s'_d s_d} \sum_{\mathbf{k}'} \sum_{\mathbf{P}'_d} \delta\left(\frac{k^2}{2M_\chi} - \frac{P'^2_d}{2M_d} - \frac{k'^2}{2M_\chi}\right) \frac{\Omega}{k/M_\chi} \left| \frac{\delta_{\mathbf{k},\mathbf{P}'_d+\mathbf{k}'}}{\Omega} \frac{C_{\chi+}}{\Lambda_S^2} \langle \psi^d | \int e^{i\mathbf{q}\cdot\mathbf{x}} J_\mu(x) \cdot L^\mu \, d\mathbf{x} | \psi^d \rangle \right|^2$$

Using the multipole expansion $d^2\sigma_{fi}$ can be written as

$$\frac{d^2\sigma_{fi}}{dE'_d d\hat{\mathbf{P}}'_d} = \frac{\pi}{3(2\pi)^3} \frac{C^2_{\chi+}}{\Lambda_S^4} \sum_{r' r} M_d \, \delta\left(\mathbf{v} \cdot \frac{\mathbf{P}'_d}{P'_d} - \frac{P'_d}{2\mu}\right) \frac{1}{v}$$

$$\cdot \left\{ (4\pi) \sum_{l\geq 0}^{2} \left[\left(L_0 L_0{}^* |X_l^C|^2 + L_z L_z{}^* |X_l^L|^2 \right) + 2 L_0 L_z{}^* Re\left(X_l^C X_l^{L*} \right) \right] \right.$$

$$\left. + (2\pi) \sum_{l\geq 1}^{2} \sum_{\lambda} L_{-\lambda} L^*_{-\lambda} \left(|\lambda X_l^M|^2 + |X_l^E|^2 \right) \right\} \qquad \mathbf{v} = \frac{\mathbf{k}}{M_\chi}$$

Interaction rate

The double-differential interation rate is given by,

$$\frac{d^2R}{dE'_d\hat{\mathbf{P}}'_d} = N_\chi N_T \int d^3\mathbf{v}\, v\, f(\mathbf{v}) \frac{d^2\sigma}{dE'_d d\hat{\mathbf{P}}'_d}$$

$$N_T = \frac{100\ \text{ton}}{\text{Deuteron Mass}}\ \text{(arbitarily chosen)}$$

$$N_\chi = \frac{0.6\ \text{GeV/cm}^3}{\text{WIMP Mass}}$$ [*Cadeddu et al.*, 2017]

We assume the Standard Halo Model (SHM), i.e. a Maxwell-Boltzmann WIMP velocity distribution of width σ_v [Cadeddu *et al.*, 2017]

$$f(\mathbf{v}) = \frac{1}{\sqrt{(2\pi\sigma_v^2)^3}} e^{-\frac{1}{2}\left(\frac{\mathbf{v}+\mathbf{V}}{\sigma_v}\right)^2} \qquad \sigma_V = \frac{V}{\sqrt{2}} \qquad V \approx 220\,\text{km/s (Sun velocity)}$$

Sum over the WIMP spin

$$\sum_{r'r} L_i L_j^* = a + \mathbf{b}\cdot\mathbf{u} + cu^2 + (\mathbf{d}\cdot\mathbf{u})^2 + O(\mathbf{u})^3 \qquad a, \mathbf{b}, c, \mathbf{d} = \text{constant terms} \qquad \mathbf{u} = \mathbf{v} + \mathbf{V}$$

After integrating over $\mathbf{u}$

$$\frac{d^2R}{dE'_d d\hat{\mathbf{P}}'_d} = \frac{1}{6\pi}\frac{C_{X+}^2}{\Lambda_S^4} N_T N_\chi M_d \frac{e^{-\frac{A^2}{2\sigma_v^2}}}{\sqrt{2\pi\sigma_v^2}} \left(a + \mathbf{b}\cdot\hat{\mathbf{q}}A + 2c\sigma_v^2 + cA^2 + d^2\sigma_v^2 - (\mathbf{d}\cdot\hat{\mathbf{q}})^2(\sigma_v^2 - A^2)\right) \sum_l |X_l^Y|^2$$

$A = \mathbf{V}\cdot\hat{\mathbf{q}} + \frac{q}{2\mu}$, μ=reduced mass

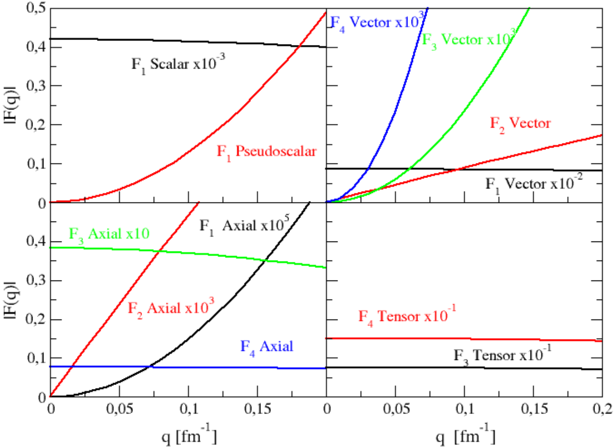
Structure Functions

$$F_1 = \sum_l |X_l^C|^2$$

$$F_2 = \sum_l 2Re\left(X_l^C X_l^{L*}\right)$$

$$F_3 = \sum_l |X_l^L|^2$$

$$F_4 = \sum_l |X_l^M|^2 + |X_l^E|^2$$



- Scalar interaction dominant compared to other types of interactions

Contribution of the *NLO* and *N2LO* currents

F_1 structure function for the scalar interaction case as a function of **q**

q [fm^{-1}]	LO	NLO	N2LO
0	425.95	426.01	419.73
0.05	424.57	424.64	418.36
0.10	420.48	420.55	414.30
0.15	404.74	404.80	398.67

- Contributions of the order NLO and N2LO are very small compared to the lowest order
- We note that the NLO contribution is very tiny, and also less important than the N2LO one
- N2LO scalar current is proportional to the factor $|8B_c c_1| \approx 20$
- NLO current (the pion-in-flight diagram) has not that factor $\rightarrow$ the chiral counting is alterated
- LO contribution is dominant and we do not expect that further orders will be sizeable

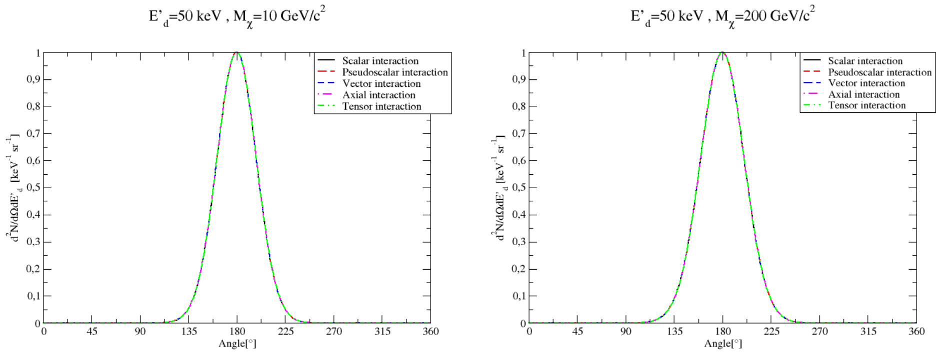
F_1 structure function for the vector interaction case

q [fm^{-1}]	LO	N2LO
0	8.5943	8.5943
0.05	8.5666	8.5664
0.10	8.4841	8.4834
0.15	8.1663	8.1637

Events per day

$$\frac{d^2N}{d\hat{\mathbf{P}'}_d\, dE'_d} \equiv \frac{d^2R}{d\hat{\mathbf{P}'}_d\, dE'_d} \times 1 \;\; \text{day} = \text{events per day per unit of deuteron recoil energy and per steradiant}$$

(indipendent of φ)



Event per day as function of the angle between $\mathbf{P}'_d$ and $\mathbf{V}$ for various interaction types calculated using the N4LO potential.

- No sensitivity to the interaction type
- $M_\chi \Rightarrow$Width

Dependence of results on deuteron wave function

Dependence on integration points, N_L and γ

AV18 E_d=50keV M_χ=10 GeV

γ	N_L	N_l	N_r	N_θ	N_ϕ	$\frac{d^2N}{d\hat{\mathbf{P}}'_d dE'_d}\vert_{\theta=90^\circ}$	$\frac{d^2N}{d\hat{\mathbf{P}}'_d dE'_d}\vert_{\theta=180^\circ}$
3.0	50	80	40	10	10	$7.5844 \cdot 10$	$3.8101 \cdot 10^6$
4.0	60	100	50	20	20	$7.5832 \cdot 10$	$3.8097 \cdot 10^6$
3.0	40	80	60	30	30	$7.5815 \cdot 10$	$3.8089 \cdot 10^6$

γ is in units of fm^{-1}, N_L=number of Laguerre polynomials, N_l, N_r, N_θ, N_ϕ=Integration points

Dependence on the NN potential

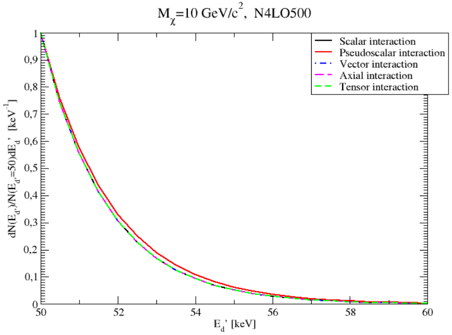
$E'_d = 50$keV $M_\chi = 10$GeV

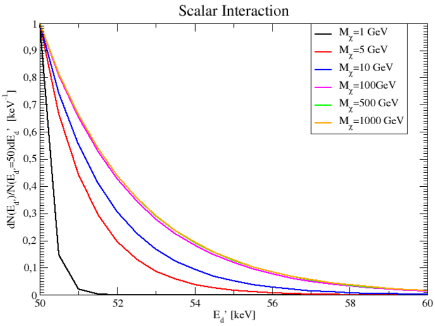
θ[deg]	AV18	LO500	NLO500	N2LO500	N3LO500	N4LO500
0	$2.043 \cdot 10^{-4}$	$2.045 \cdot 10^{-4}$	$2.058 \cdot 10^{-4}$	$2.054 \cdot 10^{-4}$	$2.055 \cdot 10^{-4}$	$2.054 \cdot 10^{-4}$
45	$1.075 \cdot 10^{-2}$	$1.077 \cdot 10^{-2}$	$1.083 \cdot 10^{-2}$	$1.081 \cdot 10^{-2}$	$1.081 \cdot 10^{-2}$	$1.081 \cdot 10^{-2}$
90	$7.582 \cdot 10$	$7.593 \cdot 10$	$7.639 \cdot 10$	$7.627 \cdot 10$	$7.627 \cdot 10$	$7.627 \cdot 10$
135	$1.968 \cdot 10^{5}$	$1.970 \cdot 10^{5}$	$1.982 \cdot 10^{5}$	$1.978 \cdot 10^{5}$	$1.979 \cdot 10^{5}$	$1.978 \cdot 10^{5}$
180	$3.810 \cdot 10^{6}$	$3.814 \cdot 10^{6}$	$3.837 \cdot 10^{6}$	$3.830 \cdot 10^{6}$	$3.832 \cdot 10^{6}$	$3.831 \cdot 10^{6}$

with $C^2_{S+} = 1$

To study the dependence of the rate from the deuteron energy we have integrated d^2N over the solid angle

$$\frac{dN}{dE'_d}(E'_d) = 2\pi \int_0^{\pi} \frac{d^2N}{d\hat{\mathbf{P}}_d dE'_d} \sin\theta d\theta$$





- Rapid decay of the rate as E'_d increases, due mainly by the factor $e^{-\frac{A^2}{\sigma_v^2}}$
- Dependence on the WIMP mass

Conclusion & Future perspectives

With respect to other calculations in the literature

- Inclusion of two body currents, and treated $a_\mu^{(s)}$ and $t^{\mu\nu}$ currents
- Systematically study of the interation rate for each interactione type
- Construction of codes for the calculation of currents on the nuclear wave function
  - we used the deuteron wave function but the code is general

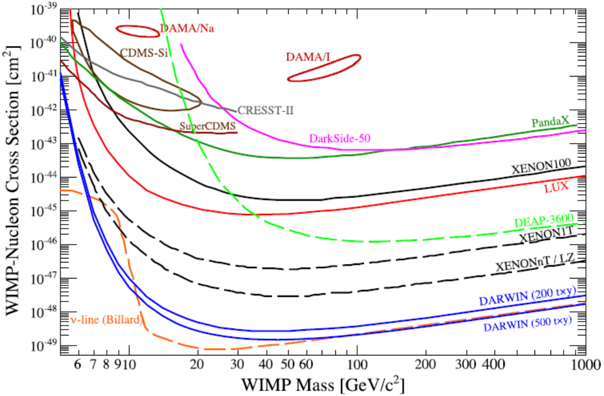
Future goals

- ${}^4\mathrm{He}\,\chi \to {}^4\mathrm{He}\,\chi$
  - Under study since more sensitive to light WIMP [W. Guo *et al*, 2013]
- Extend the theory to treat also the case of scalar, vector or tensorial DM particles

Backup slides

Experimental situation

WIMP-nucleon cross section limits



[Baudis, 2018]

The chiral symmetry in QCD

Considering only the lightest flavour quarks, u and d [Peskin & Schroeder, 2005]

$$\mathcal{L}_{\rm QCD} = \bar{q}(x)\big(i\gamma^\mu D_\mu - \mathcal{M}\big)q(x) - \frac{1}{4}\mathcal{G}_{\mu\nu,a}(x)\mathcal{G}_a^{\mu\nu}(x) \qquad q = \begin{pmatrix} u \\ d \end{pmatrix},\ \mathcal{M} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

If we neglet the mass term, the so-called chiral limit, $\mathcal{L}_{\rm QCD}$ is invariant under the trasformation of the group $G_\chi \equiv U(1)_R \otimes U(1)_L \otimes SU(2)_R \otimes SU(2)_L$ isomorphic to $U(1)_V \otimes U(1)_A \otimes SU(2)_V \otimes SU(2)_A$

QCD Lagrangian for massless quarks invariant under the local G_χ transformation with external currents [Gasser & Leutwyler, 1984]

$$\mathcal{L}_q = \mathcal{L}_{\rm QCD}^{\mathcal{M}=0} + \bar{q}(x)\gamma^\mu\left(v_\mu(x) + \frac{1}{3}v_\mu^{(s)}(x) + \gamma^5 a_\mu(x)\right)q(x) - \bar{q}(x)\big(s(x) - i\gamma^5 p(x)\big)q(x)$$

$$\begin{cases} v_\mu(x) = \sum_{a=x,y,z}\frac{\tau_a}{2}v_\mu^a(x) \quad a_\mu(x) = \sum_{a=x,y,z}\frac{\tau_a}{2}a_\mu^a(x) \\ s(x) = \sum_{a=0}^{3}\tau_a s^a(x) \quad p(x) = \sum_{a=0}^{3}\tau_a p^a(x) \\ \tau_0 = \mathbf{1},\ \tau_{i=1,2,3} = \text{Pauli matrices} \end{cases}$$

- no $a_\mu^{(s)}$ current due to $U_A(1)$ anomaly
- coupling with EM field: $v_\mu(x) = -e\frac{\tau_z}{2}\mathcal{A}_\mu(x)$, $v_\mu^{(s)}(x) = -\frac{e}{2}\mathcal{A}_\mu(x)$
- G_χ is spontaneusly broken, pions are Goldstone boson: $s = \mathcal{M}$ the breaking is taking into account

χEFT for hadrons

- Description of hadrons using QCD is very complicated$\rightarrow$ χEFT [Weinberg, 1969]
- Expansion in a power $(Q/\Lambda_\chi)^\nu$ [Weinberg, 1990]
  - ν =chiral order, $Q \sim m_\pi$ = typical value of nucleon momentum inside a nucleus, $\Lambda_\chi = 4\pi f_\pi \sim 1$ GeV= typical energy scale of the strong interaction
- The symmetries used to build the effective Lagrangian are
  - chiral symmetry
  - Lorentz invariance
  - C, P
- $\mathcal{L} = \sum_\nu \mathcal{L}^{(\nu)} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(2)}_{\pi\pi} + \dots$

- Nucleon and pion fields $N = \begin{pmatrix} p \\ n \end{pmatrix}$ $\quad \vec{\pi} = (\pi_x, \pi_y, \pi_z)$
- Basic quantities $U = e^{i\vec{\pi}\cdot\vec{\tau}/f_\pi}$ $\quad U' = RUL^\dagger \quad u = \sqrt{U} \quad u' = Ruh^\dagger \quad h \equiv h(L, R, \pi)$
- N under $G_\chi \rightarrow N' = hN$
- The unknown quark dynamics is parametrized via the so called low-energy constants (LECs) which can be fixed in general from experiments

Used to describe $\pi\pi$, $N\pi$, NN and $3N$ interactions

- NN interactions derived up to order $(Q/\Lambda_\chi)^\nu$ $\nu = 5$ [Machleidt *et al*, 2017]
- 3N interactions up to $(Q/\Lambda_\chi)^2$ [Epelbaum *et al*, 2002]

Electromagnetic & Weak interactions $\rightarrow$ $v_\mu \& a_\mu$ currents [Pastore *et al*, 2009] [Baroni *et al*,2013]

Invariant terms under local G_χ [Fettes, 2000]:

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4}\left\langle D_\mu U(x)(D^\mu U(x))^\dagger\right\rangle + \frac{f_\pi^2}{4}\left\langle \xi(x)U^\dagger(x) + U(x)\xi^\dagger(x)\right\rangle$$

$$r_\mu = v_\mu + a_\mu\,, \quad l_\mu = v_\mu - a_\mu$$

$$D_\mu U(x) = \partial_\mu U(x) - ir_\mu(x)U(x) + il_\mu(x)U(x)$$

$$\xi(x) = 2B_c\left(s(x) + ip(x)\right)$$

$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}\left(i\not{D} - M + \frac{g_A}{2}\gamma^\mu\gamma^5 u_\mu\right)N$$

$$\begin{aligned}\mathcal{L}^{(2)}_{\pi N} = {} & c_1 \bar{N}\langle\xi_+\rangle N \\ & - \frac{c_2}{8M^2}\left[\bar{N}\langle u_\mu u_\nu\rangle\{D^\mu, D^\nu\}N + \text{h.c.}\right] \\ & + \frac{c_3}{2}\bar{N}\langle u_\mu u^\mu\rangle N \\ & + \frac{i c_4}{4}\bar{N}[u_\mu, u_\nu]\sigma^{\mu\nu}N \\ & + c_5 \bar{N}\hat{\xi}_+ N \\ & + \frac{c_6}{8M}\bar{N}\sigma^{\mu\nu}F^+_{\mu\nu}N \\ & + \frac{c_7}{4M}\bar{N}\sigma^{\mu\nu}F^{(s)}_{\mu\nu}N\end{aligned}$$

$$u_\mu = i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu u^\dagger)\}$$

$$\xi_\pm = u^\dagger \xi\, u^\dagger \pm u\,\xi^\dagger u$$

$$F^{(s)}_{\mu\nu} = \partial_\mu v^{(s)}_\nu - \partial_\nu v^{(s)}_\mu$$

$$F^\pm_{\mu\nu} = u^\dagger F^R_{\mu\nu} u \pm u\, F^L_{\mu\nu} u^\dagger$$

where $\langle\ldots\rangle$ indicates the trace of the matrices and $\hat{\xi}_+ = \xi_+ - \frac{1}{2}\langle\xi_+\rangle$

- The unknown quark dynamics is parametrized via the so called low-energy constants (LECs) B_c, c_1, c_2,..which can be fixed in general from experiments

LECs

- B_c related to the quarks condensate and to the pion mass. From $\mathcal{L}_\pi^{(2)}$ $m_\pi^2 = 2m_q B_c \rightarrow B_c \approx 2.78\,\mathrm{GeV}$ higher order corrections from $\mathcal{L}_\pi^{(4)}$ etc. We adopted the more precise estimate from the lattice calculations [S. Aoki *et al.*, 2017]
- $c_1 - c_5$ extracted from an accurate analysis of πN scattering data [M. Hoferichter *et al.*, 2015]
- c_6 and c_7 related to the anomalous magnetic moment of the nucleons [J. Carlson and R. Schiavilla, 1988]
  $c_6 = \kappa_p - \kappa_n \;, \qquad c_7 = \kappa_p + \kappa_n \qquad \kappa_p = 1.793\,, \kappa_n = -1.913$
- $F + D \sim 1.26 \sim g_A$, $3F - D = 0.31$ [A. Hosaka *et al.*, 2009]
- $\tilde{c}_1$ and $\tilde{c}_2$ from the results of a recent lattice calculation on the tensor charges of the nucleons [C. Alexandrou *et al.*,2017]
- Other LECs fixed in similar way

LEC	Value
B_c	$2.40\,\mathrm{GeV}$
c_1	$-1.10\,\mathrm{GeV}^{-1}$
c_6	3.71
c_7	-0.12
D	0.86
F	0.39
$4\tilde{c}_1$	0.58

Incorporation of the isoscalar axial current

Due to the $U_A(1)$ anomaly, it is not possible to introduce isoscalar axial current in $SU(2)$ space$\rightarrow$ $SU(3)$ [Scherer & Schindler, 2004]

$$\langle N|\bar{u}\gamma_\mu\gamma^5 u + \bar{d}\gamma_\mu\gamma^5 d|N\rangle \rightarrow \langle N|A_\mu^{(8)}|N\rangle$$

where

$$A_\mu^{(8)} = \bar{u}\gamma_\mu\gamma^5 u + \bar{d}\gamma_\mu\gamma^5 d - 2\bar{s}\gamma_\mu\gamma^5 s = \sqrt{3}\bar{q}\gamma_\mu\gamma^5\lambda_8 q \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix} \; q(x) = \begin{pmatrix}u(x)\\ d(x)\\ s(x)\end{pmatrix}$$

Valid in the hypothesis that the content of the strange quark in the nucleon vanishes [Ellis, Nagata & Olive, 2018]

The axial current part of the quark Lagrangian in the $SU(3)$ space is

$$\mathcal{L}_q^{axial} = \sum_i \alpha_i \bar{q}\gamma_\mu\gamma^5\lambda_i q\bar{\chi}\gamma^\mu\gamma^5\chi \qquad a_\mu = a_{\mu i}\lambda_i = \alpha_i\bar{\chi}\gamma_\mu\gamma^5\chi\lambda_i$$

where the costants α_i are zero except

$$\alpha_3 = \frac{C_{A-}}{\Lambda_S^2} \qquad \alpha_8 = \frac{C_{A+}}{\Lambda_S^2} \qquad \lambda_3 = \begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}$$

- Since now $\langle\lambda_i\rangle = 0 \rightarrow$ no anomaly

Tensor current

Not treated in literature

$$t^{\mu\nu} = t_0^{\mu\nu}\mathbf{1} + t_a^{\mu\nu}\tau_a$$

Assuming

$$t^{\mu\nu} \to t^{\mu\nu\,\prime} = L t^{\mu\nu} R^\dagger$$

$$\mathcal{L}_q^{tens} = \bar{q}\sigma_{\mu\nu}t^{\mu\nu}q = \bar{q}_R\sigma_{\mu\nu}t^{\mu\nu}q_L + \bar{q}_L\sigma_{\mu\nu}(t^{\mu\nu})^\dagger q_R$$

In the hadron Lagrangian we construct the terms invariant under G_χ:

- Lowest order $O(Q^2)$

$$\bar{N}\sigma_{\mu\nu}T_\pm^{\mu\nu}N \qquad T_\pm^{\mu\nu} = ut^{\mu\nu\,\dagger}u \pm u^\dagger t^{\mu\nu}u^\dagger$$

Only term C, P invariant $\to T_+^{\mu\nu}$

$$\mathcal{L}_{\pi N}^{(2)} = \tilde{c}_1\bar{N}\sigma_{\mu\nu}\langle T_+^{\mu\nu}\rangle N + \tilde{c}_2\bar{N}\sigma_{\mu\nu}\hat{T}_+^{\mu\nu}N$$

where $\tilde{c}_1$ and $\tilde{c}_2$ are new LECs

- Other terms $O(Q^3)$

$$\bar{N}\gamma^\mu\gamma_5[u^\nu, T_{+\mu\nu}]N + \bar{N}\gamma^\mu\{u^\nu, T_{-\mu\nu}\}N$$

  not considered for simplicity

Defining

$$J_1^{\mu} = \text{One body nuclear current}$$
$$J_2^{\mu} = \text{Two body nuclear current}$$
$$L_{\mu} = \text{WIMP current}$$

For the interaction of type X ($X=$ scalar, pseudoscalar, vector, axial and tensor), the transition amplitude between two nucleons and WIMP can be cast in the form

$$\begin{aligned}\langle \mathbf{p}_1'\mathbf{p}_2'\mathbf{k}' \,| T |\, \mathbf{p}_1\mathbf{p}_2\mathbf{k}\rangle &= \frac{C_{X+}}{\Lambda_S^2}\frac{1}{\Omega}\Big(\delta_{\mathbf{p}_1'+\mathbf{k}',\mathbf{p}_1+\mathbf{k}}\, J_1^{\mu}(\mathbf{p}_1\mathbf{p}_1')\delta_{\mathbf{p}_2,\mathbf{p}_2'} \\ &+ \delta_{\mathbf{p}_2'+\mathbf{k}',\mathbf{p}_2+\mathbf{k}}\, J_1^{\mu}(\mathbf{p}_2\mathbf{p}_2')\delta_{\mathbf{p}_1,\mathbf{p}_1'} \\ &+ \delta_{\mathbf{p}_1'+\mathbf{p}_2'+\mathbf{k}',\mathbf{p}_1+\mathbf{p}_2+\mathbf{k}}\, J_2^{\mu}(\mathbf{p}_1\mathbf{p}_2\mathbf{p}_1'\mathbf{p}_2')\Big) L_{\mu}(\mathbf{k}'\mathbf{k})\end{aligned}$$

Multipoles expansion of $J_\mu(x)$

We consider $\left\langle J' J'_z \right| \int e^{i\mathbf{q}\cdot\mathbf{x}} J_\mu(x) \cdot L^\mu \, d\mathbf{x} \left| J J_z \right\rangle$

$$J^\mu(x) = \delta(\mathbf{x} - \frac{\mathbf{r}}{2}) J_1^\mu(-i\nabla_1, -i\nabla_1 + \mathbf{q}) + \delta(\mathbf{x} + \frac{\mathbf{r}}{2}) J_1^\mu(-i\nabla_2, -i\nabla_2 + \mathbf{q})$$
$$+ \delta(\mathbf{x} - \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}) \left[N^\mu(-i\nabla_1 - \frac{\mathbf{q}}{2}, -i\nabla_2 - \frac{\mathbf{q}}{2}) f(\mathbf{r}) \right]$$

L can be expanded in spherical components as

$$\mathbf{L} = L_\lambda = \mathbf{L} \cdot \hat{\epsilon}_{\mathbf{q},\lambda} \qquad \hat{\epsilon}_{\mathbf{q},\lambda} = \text{set of 3 versor forming a "destrorse tern"}$$

$$\int e^{i\mathbf{q}\cdot\mathbf{x}} J_\mu(x) \cdot L^\mu \, d\mathbf{x} = \int d\mathbf{x} e^{i\mathbf{q}\cdot\mathbf{x}} \left(L_0 J_0 - L_z \hat{\epsilon}_{\mathbf{q},0} \cdot \mathbf{J} + \sum_{\lambda=\pm1} L_{-\lambda} \hat{\epsilon}_{\mathbf{q},\lambda} \cdot \mathbf{J} \right)$$

Each of the above term can be written as a sum of irreducible tensors $\mathbf{T}_l$: The multipoles [Walecka, 1995]

$$\hat{R}(\alpha, \beta, \gamma) T_{lm} \hat{R}^{-1}(\alpha, \beta, \gamma) = \sum_{m'=-l}^{l} T_{lm'} \mathcal{D}^l_{m',m}(\alpha, \beta, \gamma)$$

(α, β, γ) are the Eulero angles and

$$\hat{R}(\alpha, \beta, \gamma) = e^{-i\alpha \hat{J}_z} e^{-i\beta \hat{J}_y} e^{-i\gamma \hat{J}_z} \qquad \mathcal{D}^l_{m,m'}(\alpha, \beta, \gamma) = \langle lm | \, \hat{R}(\alpha, \beta, \gamma) \, | lm' \rangle$$

The multipoles that enter the expansion for the current are of the charge (C), longitudinal (L), electric (E), or magnetic (M) type:

$$\mathcal{P} T^E_{lm} \mathcal{P}^{-1} = (-)^l T^E_{lm}, \qquad \mathcal{P} T^M_{lm} \mathcal{P}^{-1} = (-)^{l+1} T^M_{lm} \qquad \mathcal{P} = \text{parity}$$

$$\langle \psi_{L,n} | -\frac{\nabla^2}{2\mu} | \psi_{L',n'} \rangle \equiv T^{n,n'}_{L,L'}$$

$$T^{n,n'}_{L,L'} = \delta_{L,L'} \frac{\gamma^2}{2\mu} \left[(L(L+1)+n') J^{(2)}_{n,n'} + (n'+1) J^{(1)}_{n,n'} - \frac{1}{4}\delta_{n,n'} - \sqrt{n'(n'+1)} J^{(2)}_{n,n'-1} \right]$$

$J^{(1)}_{n,n'} = N_n N_{n'} \int_0^\infty drr^2 \frac{L^{(2)}_n(\gamma r) L^{(2)}_{n'}(\gamma r)}{\gamma r} e^{-\gamma r}$ $\qquad$ $J^{(2)}_{n,n'} = N_n N_{n'} \int_0^\infty drr^2 \frac{L^{(2)}_n(\gamma r) L^{(2)}_{n'}(\gamma r)}{(\gamma r)^2} e^{-\gamma r}$

and are calculated numerically with Gaussian integration [Abramowitz & Stegun, 1970] with N_l integration points.

$$\langle \psi_{L,n} | V | \psi_{L',n'} \rangle \equiv V^{n,n'}_{L,L'}$$

Local potential: $V^{n,n'}_{L,L'} = N_n N_{n'} \int_0^\infty drr^2 e^{-\gamma r/2} L^{(2)}_n(\gamma r) v_{L,L'}(r) e^{-\gamma r/2} L^{(2)}_{n'}(\gamma r)$

$$v_{L,L'}(r) = \int d\hat{\mathbf{r}} \xi^\dagger_{T,0} \left[Y_L(\hat{\mathbf{r}}) \chi_S \right]^\dagger_{JJ_z} V(\mathbf{r}) \left[Y_{L'}(\hat{\mathbf{r}}) \chi_{S'} \right]_{JJ_z} \xi_{T',0} \qquad S = S' = 1,\ T = T' = 0,\ J = 1$$

Non-local potential: $V^{n,n'}_{L,L'} = N_n N_{n'} \int_0^\infty drr^2 \int_0^\infty dr' r'^2 e^{-\gamma r/2} L^{(2)}_n(\gamma r) v_{L,L'}(r,r') e^{-\gamma r'/2} L^{(2)}_{n'}(\gamma r')$

$$v_{L,L'}(r,r') = \int d\hat{\mathbf{r}} d\hat{\mathbf{r}}' \xi^\dagger_{T,0} \left[Y_L(\hat{r}) \chi_S \right]^\dagger_{JJ_z} V(\mathbf{r},\mathbf{r}') \left[Y_{L'}(\hat{\mathbf{r}}') \chi_{S'} \right]_{JJ_z} \xi_{T',0}$$

The angular matrix elements are calculated analytically, while integrals in r are calculated numerically using a Gaussian quadrature formula with N_l integration points.