Relativistic hydrodynamics with spin tensor

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F. Becattini, W. Florkowski, E. S., Phys. Lett. B **789**, 419 (2019)
W. Florkowski, B. Friman, A. Jaiswal, E. S., Phys. Rev. C **97**, no. 4, 041901 (2018)

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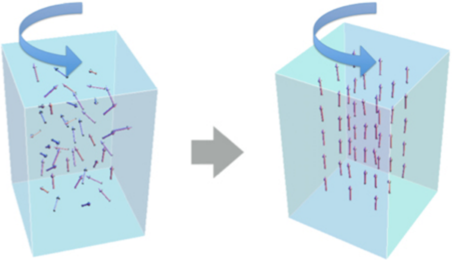
Outline

- Heavy-ion collisions and spin polarization
- What is the spin tensor?
- Why and when do we need the spin tensor in hydrodynamics?
- Is hydrodynamics invariant under different choices of energy-momentum and spin tensors?
- Can we find observables which are sensitive to different choices of energy-momentum and spin tensors?

Rotation and polarization

- Condensed matter: **Barnett effect**

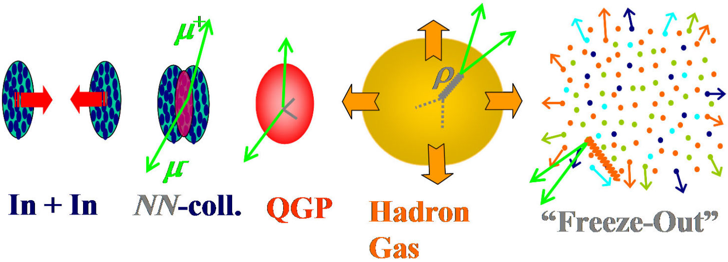


Picture by Mamoru Matsuo

Ferromagnet gets magnetized when it rotates

Can something like the Barnett effect happen in heavy-ion collisions? Yes!

Heavy-ion collisions

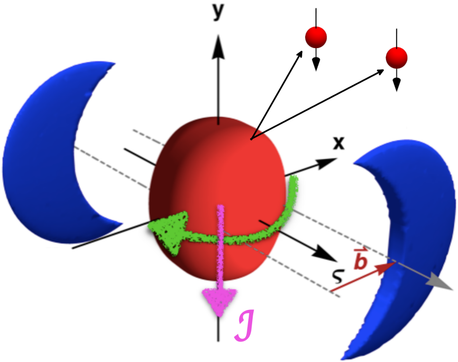


- QGP = Quark-gluon plasma - Quarks and gluons are deconfined

Relativistic hydrodynamics is a good effective theory

$$\partial_\mu T^{\mu\nu} = 0$$

Noncentral heavy-ion collisions

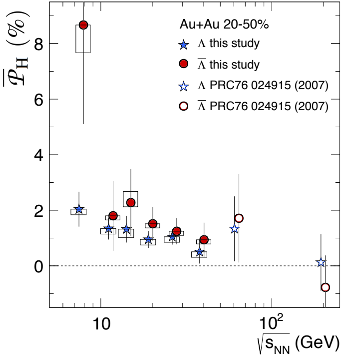


Picture from W. Florkowski, R. Ryblewski and A. Kumar, Prog. Part. Nucl. Phys. **108**, 103709 (2019)

Noncentral nuclear collisions $\Rightarrow$ Large global angular momentum
$\Rightarrow$ Vorticity of hot and dense matter $\Rightarrow$ particle polarization along vorticity

Experimental observation - Global Λ polarization

- Polarization along global angular momentum



L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

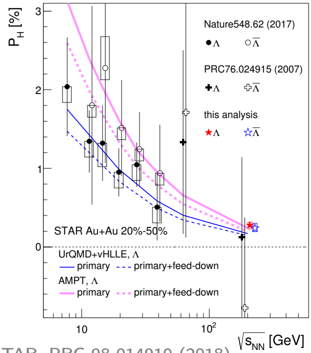
- Weak decay: $\Lambda \to p + \pi^-$ angular distr.: $dN/d\cos\theta = \frac{1}{2}(1 + \alpha|\vec{\mathcal{P}}_H|\cos\theta)$
- Quark-gluon plasma is the "most vortical fluid ever observed"

$$\omega = (P_\Lambda + P_{\bar{\Lambda}})k_B T/\hbar \approx (9+1) \times 10^{21} \text{s}^{-1}$$

Great Red Spot of Jupiter $10^{-4}\,\text{s}^{-1}$,
Turbulent flow superfluid He-II $150\,\text{s}^{-1}$, Superfluid nanodroplets $10^7\,\text{s}^{-1}$

Theory - Global vs longitudinal Λ polarization

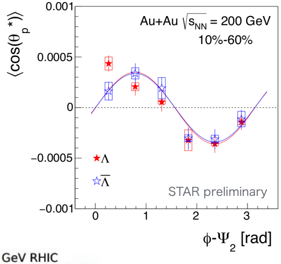
Global - along J

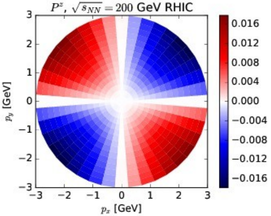


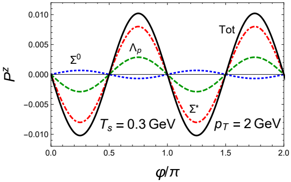
STAR, PRC 98 014910 (2018)

$$\Pi^{\mu}(x,p) \propto (1-n_F)\epsilon^{\mu\nu\alpha\beta} p_{\nu} \varpi_{\alpha\beta}$$

Longitudinal - along beam axis







F. Becattini, I Karpenko, PRL 120 012302;
F. Becattini, G. Cao, E. S. EPJC 79 741

Problem: Opposite sign in longitudinal polarization!

Does spin play a dynamical role?

Goal: Relativistic hydrodynamics (classical) with spin (quantum)

Starting point: Quantum field theory

Canonical energy-momentum and spin tensors

Lagrangian $\Rightarrow$ Poincaré symmetry $\Rightarrow$ Noether's th. $\Rightarrow$ Conservation laws

- **Conservation of energy and momentum:**
  Canonical energy-momentum tensor $\widehat{T}_C^{\mu\nu}(x)$

$$\partial_\mu \widehat{T}_C^{\mu\nu}(x) = 0$$

- **Conservation of total angular momentum:**
  Canonical total angular momentum tensor ("orbital"+"spin")

$$\widehat{J}_C^{\lambda,\mu\nu}(x) = x^\mu \widehat{T}_C^{\lambda\nu}(x) - x^\nu \widehat{T}_C^{\lambda\mu}(x) + \widehat{S}_C^{\lambda,\mu\nu}(x)$$

$$\partial_\lambda \widehat{J}_C^{\lambda,\mu\nu}(x) = 0 \Longrightarrow \partial_\lambda \widehat{S}_C^{\lambda,\mu\nu}(x) = \widehat{T}_C^{\nu\mu}(x) - \widehat{T}_C^{\mu\nu}(x)$$

Pseudo-gauge transformations

- Total energy-momentum and angular momentum must be fixed

$$\widehat{P}^{\mu} = \int d^3\Sigma_\lambda \, \widehat{T}^{\lambda\mu}(x) \qquad \widehat{J}^{\mu\nu} = \int d^3\Sigma_\lambda \, \widehat{J}^{\lambda,\,\mu\nu}(x)$$

- Densities are not uniquely defined
  $\Longrightarrow$ **Pseudo-gauge transformations:**

F. W. Hehl, Rep. Mat. Phys. 9, 55 (1976)

$$\widehat{T}'^{\,\mu\nu}(x) = \widehat{T}^{\mu\nu}(x) + \frac{1}{2}\partial_\lambda \left[\widehat{\Phi}^{\lambda,\,\mu\nu}(x) + \widehat{\Phi}^{\mu,\,\nu\lambda}(x) + \widehat{\Phi}^{\nu,\,\mu\lambda}(x)\right]$$

$$\widehat{S}'^{\,\lambda,\,\mu\nu} = \widehat{S}^{\lambda,\,\mu\nu}(x) - \widehat{\Phi}^{\lambda,\,\mu\nu}(x)$$

Leave $\widehat{P}^{\mu}$ and $\widehat{J}^{\mu\nu}$ invariant

- Belinfante's case ($\widehat{\Phi}^{\lambda,\,\mu\nu}(x) = \widehat{S}_C^{\lambda,\,\mu\nu}(x)$)

$$\widehat{T}_B^{\mu\nu}(x) = \widehat{T}_C^{\mu\nu}(x) + \frac{1}{2}\partial_\lambda \left[\widehat{S}_C^{\lambda,\,\mu\nu}(x) + \widehat{S}_C^{\mu,\,\nu\lambda}(x) + \widehat{S}_C^{\nu,\,\mu\lambda}(x)\right]$$

$$\widehat{S}_B^{\lambda,\,\mu\nu}(x) = 0$$

Example - Dirac theory

- Dirac Lagrangian

$$\mathcal{L}(x) = \frac{i}{2}\bar{\hat{\psi}}(x)\gamma^\mu \overleftrightarrow{\partial}_\mu \hat{\psi}(x) - m\bar{\hat{\psi}}(x)\hat{\psi}(x)$$

- Canonical case

$$\hat{T}_C^{\mu\nu}(x) = \frac{i}{2}\bar{\hat{\psi}}(x)\gamma^\mu \overleftrightarrow{\partial}^\nu \hat{\psi}(x) - g^{\mu\nu}\mathcal{L}(x)$$

$$\hat{S}_C^{\lambda,\mu\nu}(x) = \frac{1}{4}\bar{\hat{\psi}}(x)(\gamma^\lambda\sigma^{\mu\nu} + \sigma^{\mu\nu}\gamma^\lambda)\hat{\psi}(x)$$

with $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

- Belinfante case

$$\hat{T}_B^{\mu\nu}(x) = \frac{i}{4}\bar{\hat{\psi}}(x)(\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu)\hat{\psi}(x) - g^{\mu\nu}\mathcal{L}(x)$$

$$\hat{S}_B^{\lambda,\mu\nu}(x) = 0$$

Local equilibrium - Belinfante

- Maximization of entropy

$$S = -\mathrm{tr}(\widehat{\rho}_B \log \widehat{\rho}_B)$$

- Constraints on energy and momentum

$$n_\mu \mathrm{tr}\left[\widehat{\rho}_B\, \widehat{T}_B^{\mu\nu}(x)\right] = n_\mu T_B^{\mu\nu}(x)$$

n^μ - vector orthogonal to hypersurface Σ

- Density operator

$$\widehat{\rho}_B = \frac{1}{Z} \exp\left[-\int_\Sigma d\Sigma_\mu \widehat{T}_B^{\mu\nu}(x)\beta_\nu(x)\right],$$

β_μ - Lagrange multiplier for conservation of energy and momentum

Zubarev, 1979, Ch, Van Weert 1982
F. Becattini, L. Bucciantini, E. Grossi and L. Tinti, Eur. Phys. J. C **75**, no. 5, 191 (2015)

Do we need a constraint for the conservation of total angular momentum?

$$n_\mu \mathrm{tr}\left[\widehat{\rho}_B\, \widehat{J}_B^{\mu,\lambda\nu}(x)\right] = n_\mu \mathrm{tr}\left[\widehat{\rho}_B \left(x^\lambda \widehat{T}_B^{\mu\nu}(x) - x^\nu \widehat{T}_B^{\mu\lambda}(x)\right)\right] = n_\mu J_B^{\mu,\lambda\nu}(x)$$

No, it is redundant (it follows from constraint on energy and momentum)

Global equilibrium - Belinfante

- Density operator must be stationary

$$\partial_\mu \widehat{T}_B^{\mu\nu} \beta_\nu = \widehat{T}_B^{\mu\nu} (\partial_\mu \beta_\nu) = 0$$

- Global equilibrium conditions

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0$$

$$\beta_\nu = b_\nu + \Omega_{\nu\lambda} x^\lambda$$

$$\Omega_{\mu\nu} = \text{constant}$$

Local equilibrium - Canonical

- Maximization of entropy

$$S = -\mathrm{tr}(\widehat{\rho}_C \log \widehat{\rho}_C)$$

- Constraints on energy and momentum

$$n_\mu \mathrm{tr}\left[\widehat{\rho}_C \, \widehat{T}^{\mu\nu}_C(x)\right] = n_\mu T^{\mu\nu}_C(x)$$

Spin tensor $\Longrightarrow$ Constraint on total angular momentum

$$n_\mu \mathrm{tr}\left(\widehat{\rho}_C \, \widehat{J}^{\mu,\lambda\nu}_C\right) = n_\mu \mathrm{tr}\left[\widehat{\rho}_C \left(x^\lambda \widehat{T}^{\mu\nu}_C - x^\nu \widehat{T}^{\mu\lambda}_C + S^{\mu,\lambda\nu}_C\right)\right] = n_\mu J^{\mu,\lambda\nu}_C$$

- Density operator

$$\begin{aligned}\widehat{\rho}_C &= \frac{1}{Z} \exp\left[-\int_\Sigma d\Sigma_\mu \left(\widehat{T}^{\mu\nu}_C(x) b_\nu(x) - \frac{1}{2} \widehat{J}^{\mu,\lambda\nu}_C(x) \Omega_{\lambda\nu}(x)\right)\right] \\ &= \frac{1}{Z} \exp\left[-\int_\Sigma d\Sigma_\mu \left(\widehat{T}_C(x)^{\mu\nu} \beta_\nu(x) - \frac{1}{2} \widehat{S}^{\mu,\lambda\nu}_C(x) \Omega_{\lambda\nu}(x)\right)\right]\end{aligned}$$

$\Omega_{\lambda\nu}$ - (Antisymmetric) Lagrange multiplier for conservation of total angular momentum

Global equilibrium - Canonical

- Asymmetric EM tensor $\widehat{T}_C^{\mu\nu} = \widehat{T}_S^{\mu\nu} + \widehat{T}_A^{\mu\nu}$ with $\widehat{T}_S^{\mu\nu} = \widehat{T}_S^{\nu\mu}$, $\widehat{T}_A^{\mu\nu} = -\widehat{T}_A^{\nu\mu}$
- Density operator must be stationary

$$\frac{1}{2}\widehat{T}_S^{\mu\nu}(\partial_\mu\beta_\nu+\partial_\nu\beta_\mu)+\frac{1}{2}\widehat{T}_A^{\mu\nu}(\partial_\mu\beta_\nu-\partial_\nu\beta_\mu)-\frac{1}{2}(\partial_\mu\widehat{S}_C^{\mu,\lambda\nu})\Omega_{\lambda\nu}-\frac{1}{2}\widehat{S}_C^{\mu,\lambda\nu}(\partial_\mu\Omega_{\lambda\nu}) = 0$$

- Global equilibrium conditions:

$$\Omega_{\mu\nu} = \text{constant}$$

$$\partial_\mu\beta_\nu+\partial_\nu\beta_\mu = 0$$

$$\Omega_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu) \quad \text{thermal vorticity}$$

$$\beta_\nu = b_\nu + \Omega_{\nu\lambda}x^\lambda$$

We used $\partial_\mu\widehat{S}_C^{\mu,\lambda\nu} = -2\widehat{T}_A^{\lambda\nu}$

F. Becattini, Phys. Rev. Lett. **108**, 244502 (2012)

Observables

Consequences on some measurable quantities?

$$\mathrm{tr}(\widehat{\rho}_B\,\widehat{O}) = \mathrm{tr}(\widehat{\rho}_C\,\widehat{O})?$$

$$\widehat{\rho}_B = \widehat{\rho}_C?$$

In global equilibrium yes!
In local equilibrium in general not!

Local equilibrium - Canonical vs Belinfante

- Start with Canonical

$$\widehat{\rho}_C = \frac{1}{Z}\exp\left[-\int_\Sigma d\Sigma_\mu \left(\widehat{T}_C^{\mu\nu}\beta_\nu - \frac{1}{2}\widehat{S}_C^{\mu,\lambda\nu}\Omega_{\lambda\nu}\right)\right]$$

- PS to Belinfante: $\widehat{T}_B^{\mu\nu} = \widehat{T}_C^{\mu\nu} + \frac{1}{2}\partial_\lambda(\widehat{S}_C^{\lambda,\,\mu\nu} + \widehat{S}_C^{\mu,\,\nu\lambda} + \widehat{S}_C^{\nu,\,\mu\lambda}), \quad \widehat{S}_B^{\lambda,\,\mu\nu} = 0$
- Canonical density operator becomes

$$\widehat{\rho}_C = \frac{1}{Z}\exp\left[-\int_\Sigma d\Sigma_\mu \left(\widehat{T}_B^{\mu\nu}\beta_\nu - \frac{1}{2}(\Omega_{\lambda\nu} - \varpi_{\lambda\nu})\widehat{S}_C^{\mu,\lambda\nu} + \frac{1}{2}\xi_{\lambda\nu}\left(\widehat{S}_C^{\lambda,\mu\nu} + \widehat{S}_C^{\nu,\mu\lambda}\right)\right)\right]$$

with $\varpi_{\lambda\nu} \equiv \frac{1}{2}(\partial_\nu\beta_\lambda - \partial_\lambda\beta_\nu), \qquad \xi_{\lambda\nu} \equiv \frac{1}{2}(\partial_\nu\beta_\lambda + \partial_\lambda\beta_\nu)$

- When is $\rho_C = \rho_B$?

$$\widehat{\rho}_B = \frac{1}{Z}\exp\left[-\int_\Sigma d\Sigma n_\mu \widehat{T}_B^{\mu\nu}\beta_\nu\right]$$

1. β_μ is the same in both cases
2. $\Omega_{\lambda\nu}$ coincides with thermal vorticity, $\Omega_{\lambda\nu} = \varpi_{\lambda\nu} \equiv \frac{1}{2}(\partial_\nu\beta_\lambda - \partial_\lambda\beta_\nu)$
3. $\xi_{\lambda\nu} \equiv \frac{1}{2}(\partial_\nu\beta_\lambda + \partial_\lambda\beta_\nu) = 0$ or $\widehat{S}_C^{\lambda,\mu\nu} + \widehat{S}_C^{\nu,\mu\lambda} = 0$

Equivalence in global equilibrium!

Polarization in relativistic heavy-ion collisions

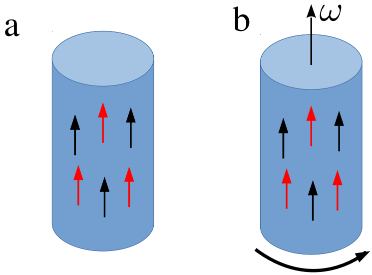
- Observable: single-particle polarization matrix

$$\Theta(p)_{\sigma,\sigma'} = \mathrm{tr}(\widehat{\rho}\, a^\dagger(p)_\sigma a(p)_{\sigma'}),$$

  Only $\widehat{\rho}$ can depend on pseudo-gauge!

- Local equilibrium: $\Theta(p)_C \neq \Theta(p)_B$
- Global equilibrium: $\Theta(p)_C = \Theta(p)_B$

Polarized neutral system - Physical meaning of $\Omega_{\mu\nu}$



- a) Fluid at rest with constant temperature with particles and antiparticles polarized in the same direction $\beta^\mu = (1/T)(1, \mathbf{0}) \Rightarrow \varpi = 0$

> Belinfante's "gauge" does not imply that polarization vanishes, but rather it is locked to thermal vorticity

> In the Canonical "gauge" one needs the spin potential to describe hydro evolution, but only if spin density relaxes "slowly" to equilibrium

- In general, in local equilibrium $\Omega_{\lambda\nu} \neq \varpi_{\lambda\nu} = \frac{1}{2}(\partial_\nu \beta_\lambda - \partial_\lambda \beta_\nu)$

Hydrodynamics with spin tensor

- 10 equations of motion 4 usual hydro + 6 due to total angular momentum conservation

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

- 10 unknowns: 4 + 6 additional independent fields (spin potential)

$$\beta^\mu \qquad \Omega^{\mu\nu}$$

$\Omega^{\mu\nu}$ and β^μ evolve separately

W. Florkowski, B. Friman, A. Jaiswal, and E. S., Phys. Rev. C **97**, no. 4, 041901 (2018)
W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, and E. S., Phys. Rev. D **97**, no. 11, 116017 (2018)
W. Florkowski, F. Becattini, and E. S., Acta Phys. Polon. B **49**, 1409 (2018)

Summary

- Local equilibrium or non-equilibrium thermodynamics is sensitive to the choice of different sets of energy-momentum and spin tensors
- Particle polarization may be sensitive to different choices of energy-momentum and spin tensors
- Hydrodynamics with spin tensor is needed if spin density relaxes "slowly" to equilibrium
- Hydrodynamics with spin tensor: 6 additional fields $\Omega_{\mu\nu}$ (spin potential) to be evolved
- **Outlook**: Derive hydrodynamics with spin tensor from quantum kinetic theory

  N. Weickgenannt, X. L. Sheng, E. S., Q. Wang and D. H. Rischke, "Kinetic theory for massive spin-1/2 particles from the Wigner-function formalism," Phys. Rev. D 100, no. 5, 056018 (2019)