

Continuum states of exotic nuclei studied by ab-initio methods

$^{18}\text{O}(^9\text{Be}, ^{11}\text{Be})^{16}\text{O}$ at 84 MeV at LNS-Catania

PHYSICAL REVIEW C **100**, 024617 (2019)

**Application of an *ab initio* S matrix to data analysis of transfer
reactions to the continuum populating ^{11}Be**

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Cortona, ottobre 2019

Background: weakly bound nuclei difficult to study because continuum states must be included **Nicole Vinh Mau, NPA592 33 (1995)**

- in ^{11}Be :

$$|1/2^+\rangle = 0.964|2s_{1/2} \otimes 0^+\rangle + 0.267|d_{5/2} \otimes 2^+\rangle, \quad (18)$$

$$|1/2^-\rangle = 0.746|p_{1/2} \otimes 0^+\rangle + 0.667|p_{3/2} \otimes 2^+\rangle, \quad (19)$$

$$|5/2^+\rangle = 0.896|d_{5/2} \otimes 0^+\rangle + 0.353|d_{5/2} \otimes 2^+\rangle + 0.269|2s_{1/2} \otimes 2^+\rangle; \quad (20)$$

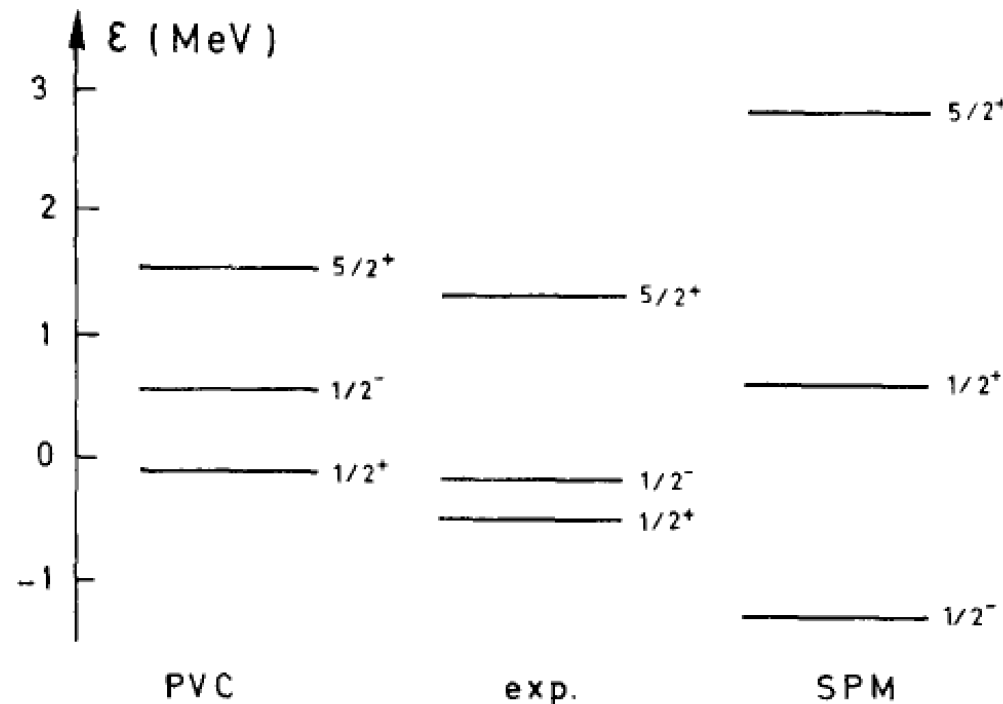


Fig. 2. The energy spectrum of ^{11}Be calculated with particle-vibration coupling (PVC) compared to the experimental (exp.) and single particle model (SPM) spectra.

At the beginning treated as bound states, now many different “fundamental” methods:

NCSM, Navratil, Barrett can treat the continuum, also Nazarewicz et al. (Gamow states and Berggren ensemble).

See Sofia Quaglioni review EPJP (2018) 133:385.

While the HO potential provides a means for eliminating the spurious motion of the c.m. even when working with single-particle coordinates and Slater-determinant basis states, its confining nature leads to a discrete spectrum of energies, and the wave functions of the computed eigenstates are all localized in space. Because of this, expansions based on the many-body HO states are well suited for the calculations of bound-state properties and narrow resonances, but are not adapted for the description of broad resonances or scattering states. Even the description of weakly-bound nuclei presenting extended configurations (such as halo nuclei) represents a challenge when working with HO functions, which present a rapid Gaussian falloff ($\propto \exp -ar^2$) at large distance r [40]. Alternative single particle basis states have been proposed in recent years in an attempt to alleviate this problem and enable the description of weakly-bound exotic nuclei and resonances within the single particle framework presented in this section. A few examples are the Berggren ensemble of single-particle states [41, 42], which includes bound, resonant and non-resonant scattering states, and the Coulomb-Sturmian functions [39], which are the solutions to a Sturm-Liouville problem associated with the Coulomb potential. As anticipated, such alternative basis states do not allow an exact removal of the spurious c.m. contributions to the dynamics.

The method presented in this section is an example of expansion technique. Expansion techniques make use of a large but finite number of many-body basis states. Convergence to the exact result is approached (variationally) by increasing the basis size. One of the main examples is the NCSM approach [43, 44], which will be discussed in some more detail in sect. 4.4.1, its important-truncation [45] and symmetry-adapted [46] versions, and the no-core Monte-Carlo shell model [47]. Other powerful and complementary *ab initio* methods that can tackle the A -nucleon problem

for bound states (with different regions of applicability) are, *e.g.*, the effective interaction hyperspherical harmonics (EIHH) expansion [29, 30], Green’s function Monte Carlo methods (GFMC) [48], the auxiliary field diffusion Monte Carlo method [49], coupled-cluster methods (CCM) [50, 51], the fermionic molecular dynamics approach [52], in medium similarity renormalization group [53, 54], self-consistent Green’s function theory [55] and its Gorkov generalization [56], and nuclear lattice EFT [57].

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- NCSMC calculations including chiral 3N (N³LO NN+N²LO 3NF400, NNLOsat)
 - n-¹⁰Be + ¹¹Be
 - ¹⁰Be: 0⁺, 2⁺, 2⁺ NCSM eigenstates
 - ¹¹Be: ≥6 π = -1 and ≥3 π = +1 NCSM eigenstates

$$|\Psi_A^{J^\pi T}\rangle = \sum_{\alpha} c_{\alpha}^{J^\pi T} |A\alpha J^\pi T\rangle + \sum_{\nu} \int dr r^2 \frac{\gamma_{\nu}^{J^\pi T}(r)}{r} \hat{A}_{\nu} |\Phi_{\nu r}^{J^\pi T}\rangle.$$

Can *Ab Initio* Theory Explain the Phenomenon of Parity Inversion in ^{11}Be ?

Angelo Calci,^{1,*} Petr Navrátil,^{1,†} Robert Roth,² Jérémy Dohet-Eraly,^{1,‡} Sofia Quaglioni,³ and Guillaume Hupin^{4,5}

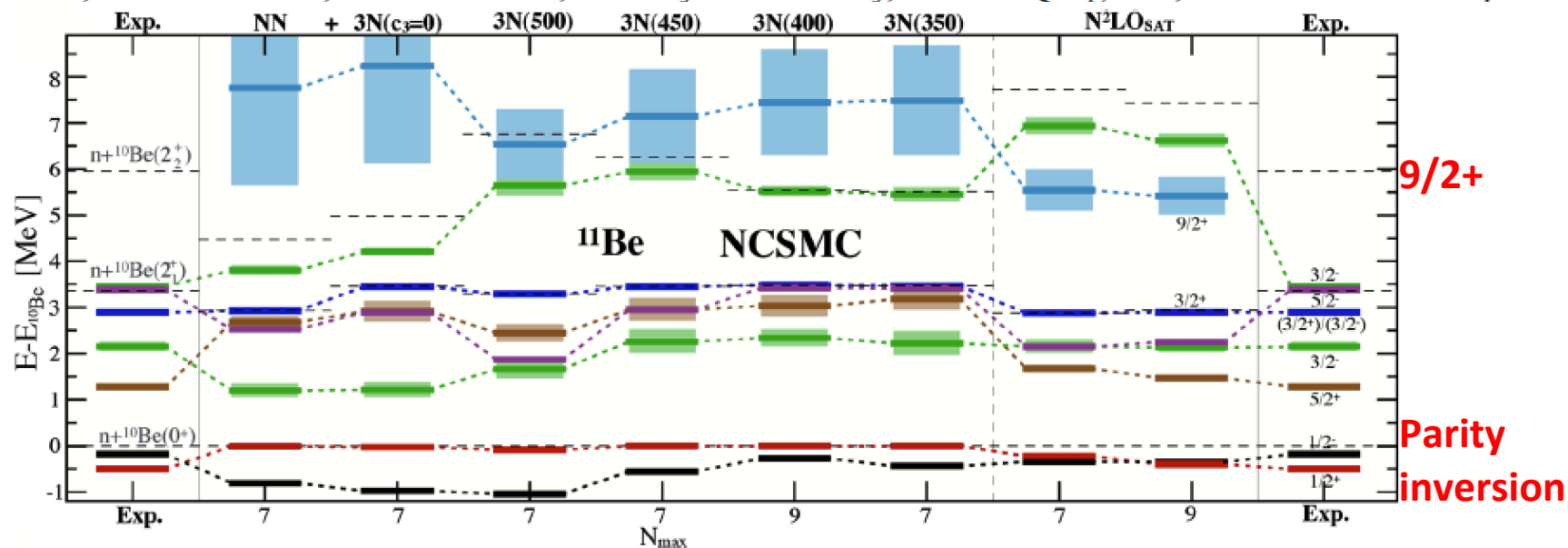


FIG. 2. NCSMC spectrum of ^{11}Be with respect to the $n + ^{10}\text{Be}$ threshold. Dashed black lines indicate the energies of the ^{10}Be states. Light boxes indicate resonance widths. Experimental energies are taken from Refs. [1,51].

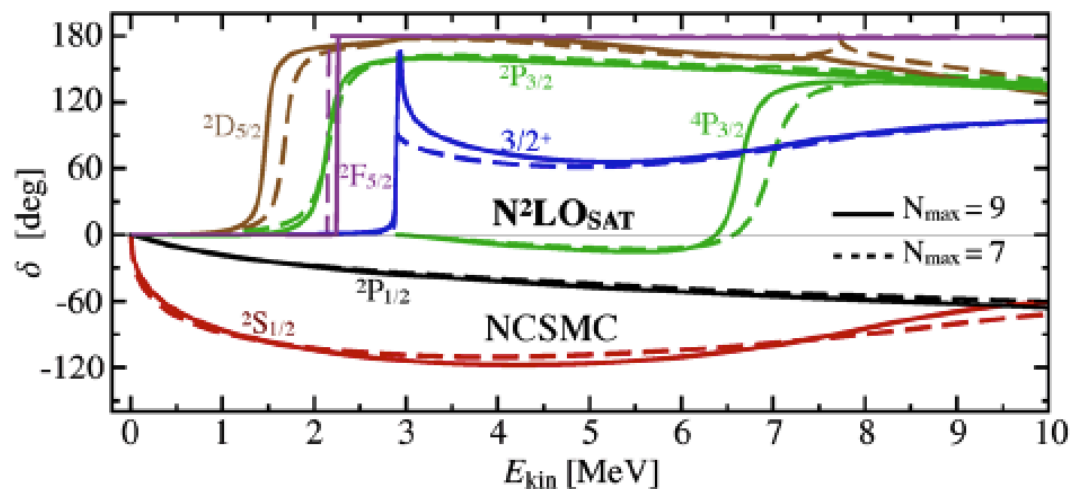


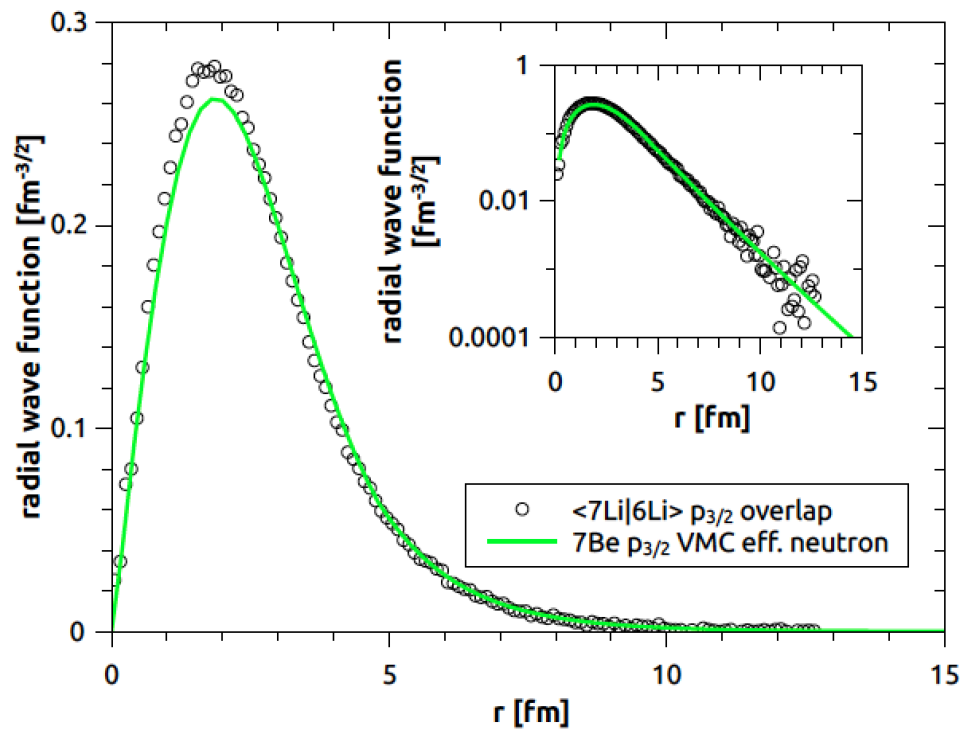
TABLE I. Excitation spectrum of ^{11}Be with respect to the $n + ^{10}\text{Be}$ threshold. Energies and widths are in MeV. The calculations are carried out at $N_{\text{max}} = 9$.

J^π	NCSMC				NCSMC-pheno		Experiment	
	$NN + 3N(400)$		$\text{N}^2\text{LO}_{\text{SAT}}$		$\text{N}^2\text{LO}_{\text{SAT}}$			
	E	Γ	E	Γ	E	Γ	E	Γ
$1/2^+$	-0.001	...	-0.40	...	-0.50	...	-0.50	...
$1/2^-$	-0.27	...	-0.35	...	-0.18	...	-0.18	...
$5/2^+$	3.03	0.44	1.47	0.12	1.31	0.10	1.28	0.1
$3/2^-_1$	2.34	0.35	2.14	0.21	2.15	0.19	2.15	0.21
$3/2^+$	3.48	...	2.90	0.014	2.92	0.06	2.898	0.122
$5/2^-$	3.43	0.001	2.25	0.0001	3.30	0.0002	3.3874	<0.008
$3/2^-_2$	5.52	0.20	6.62	0.29	5.72	0.19	3.45	0.01
$9/2^+$	7.44	2.30	5.42	0.80	5.59	0.62

Le funzioni d'onda ab-initio hanno una struttura complicata e spesso sono difficili da inserire anche solo numericamente in un codice standard che calcola reazioni di breakup o trasferimento.

Spesso sono fittate con delle funzione calcolate in una WF e poi se ne usa solo la parte asintotica.

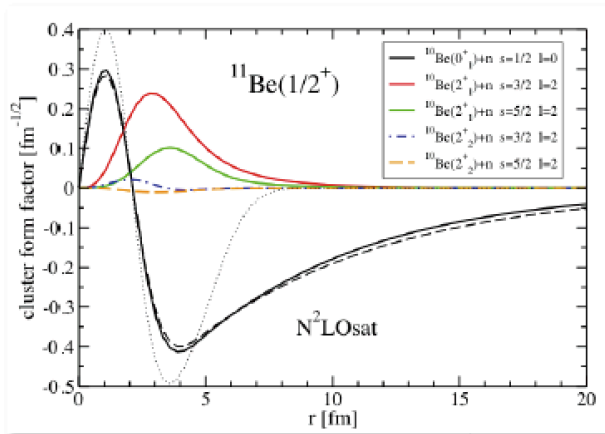
Es: G. Salvioni, Tesi Magistrale, 2014:



Oppure: Halo-EFT following [P. Capel, D. Phillips, and H.-W. Hammer, Phys. Rev. C 98 , 034610 (2018)]

Extract ANC from transfer data $^{14}\text{C}(d,p)^{15}\text{C}$, impose the halo-EFT wave function has the same ANC and then use it in breakup reaction calculations.

NCSMC wave functions of ^{11}Be used as input for other studies



PHYSICAL REVIEW C 98, 054602 (2018)

Systematic analysis of the peripherality of the $^{10}\text{Be}(d,p)^{11}\text{Be}$ transfer reaction and extraction of the asymptotic normalization coefficient of ^{11}Be bound states

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PHYSICAL REVIEW C 98, 034610 (2018)

Dissecting reaction calculations using halo effective field theory and *ab initio* input

P. Capel,^{1,2,3,4,*} D. R. Phillips,^{5,3,4,†} and H.-W. Hammer^{3,4,‡}

IOP Publishing
Journal of Physics G: Nuclear and Particle Physics
J. Phys. G: Nucl. Part. Phys. 46 (2019) 025104 (15pp)
<https://doi.org/10.1088/1361-6471/ab85b5>

Neutron disappearance inside the nucleus

H Ejiri¹ and J D Vergados²

Physics Letters B 879 (2019) 367–371
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Physics Letters B
www.elsevier.com/locate/physletb

Reliable extraction of the $dB(E1)/dE$ for ^{11}Be from its breakup at 520 MeV/nucleon

L. Moschini^{1,*}, P. Capel^{1,2}

PRL 117, 242501 (2016) PHYSICAL REVIEW LETTERS
week ending 9 DECEMBER 2016

Can *Ab Initio* Theory Explain the Phenomenon of Parity Inversion in ^{11}Be ?

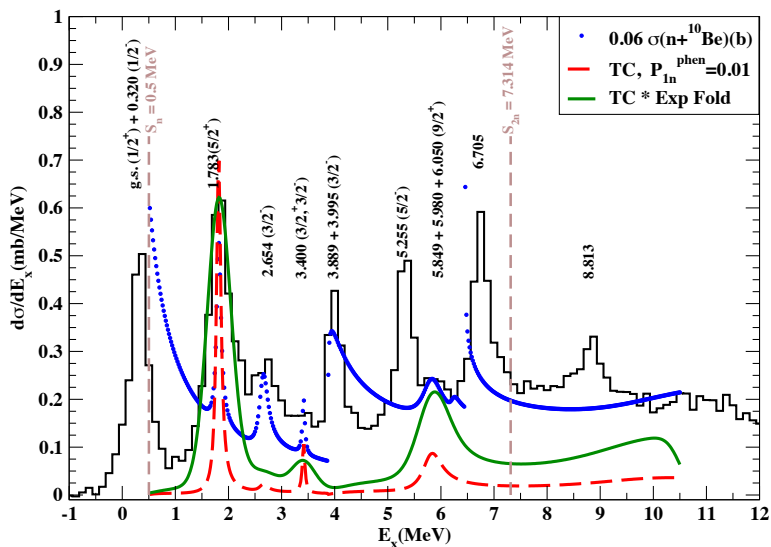
Angelo Calci,^{1,†} Petr Navrátil,^{1,†} Robert Roth,² Jérémy Dohet-Ebly,^{1,†} Sofia Quaglioni,³ and Guillaume Hupin^{4,2}

PHYSICAL REVIEW C 99, 054611 (2019)

Neutron transfer reactions in halo effective field theory

M. Schmidt,^{1,2} L. Platter,^{2,3} and H.-W. Hammer^{1,4}

18O(9Be,11Be)16O



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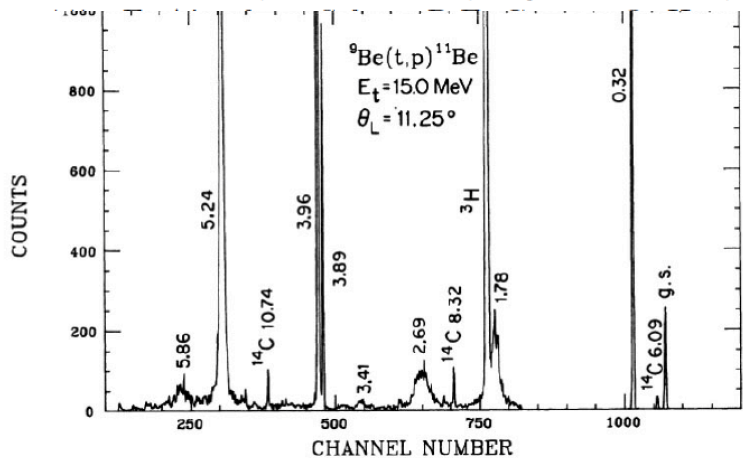


Fig. 2. Spectrum of the reaction ${}^9\text{Be}(t,p){}^{11}\text{Be}$ [116]. States in ${}^{11}\text{Be}$ are labeled by their excitation energies. Peaks from impurities in the target are labeled by final nucleus and their excitation energy.

K. T. SCHMITT *et al.*

PHYSICAL REVIEW C **88**, 064612 (2013)

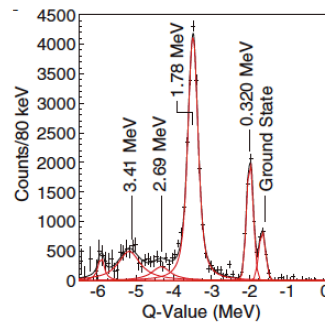
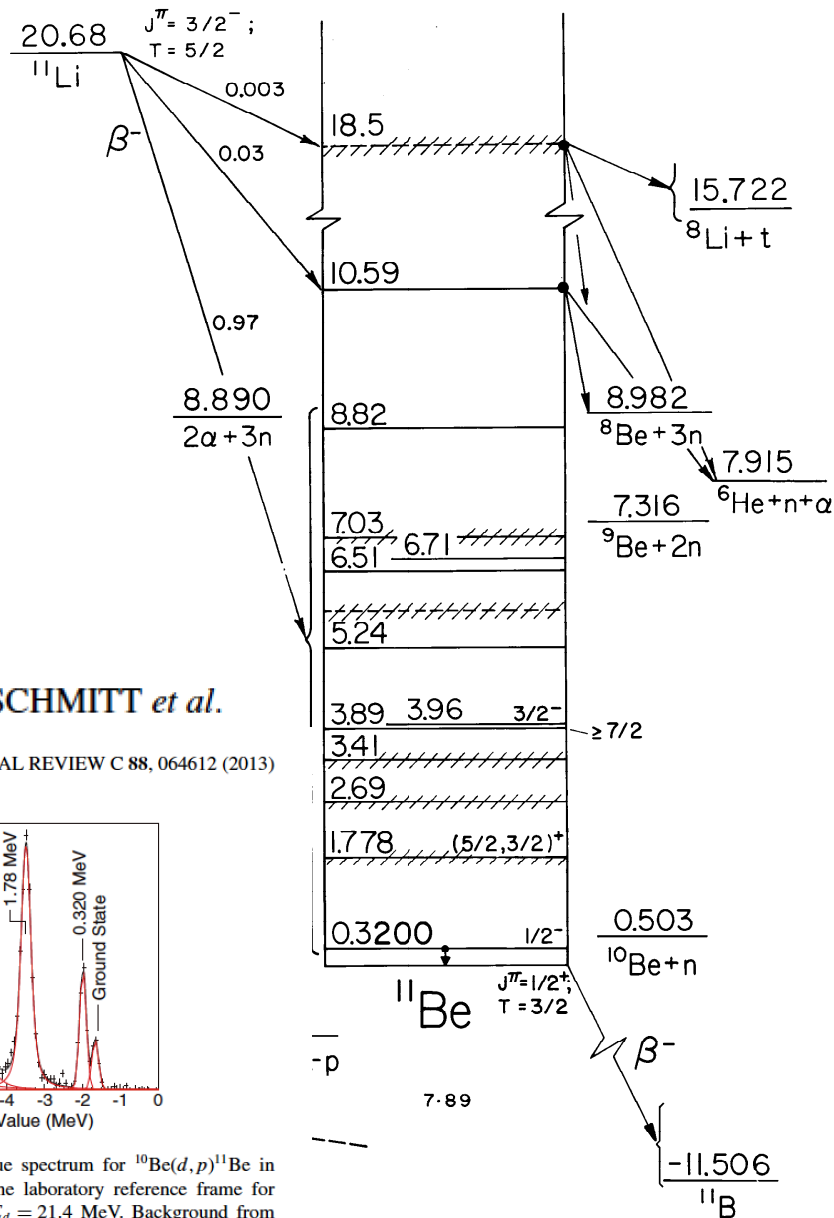
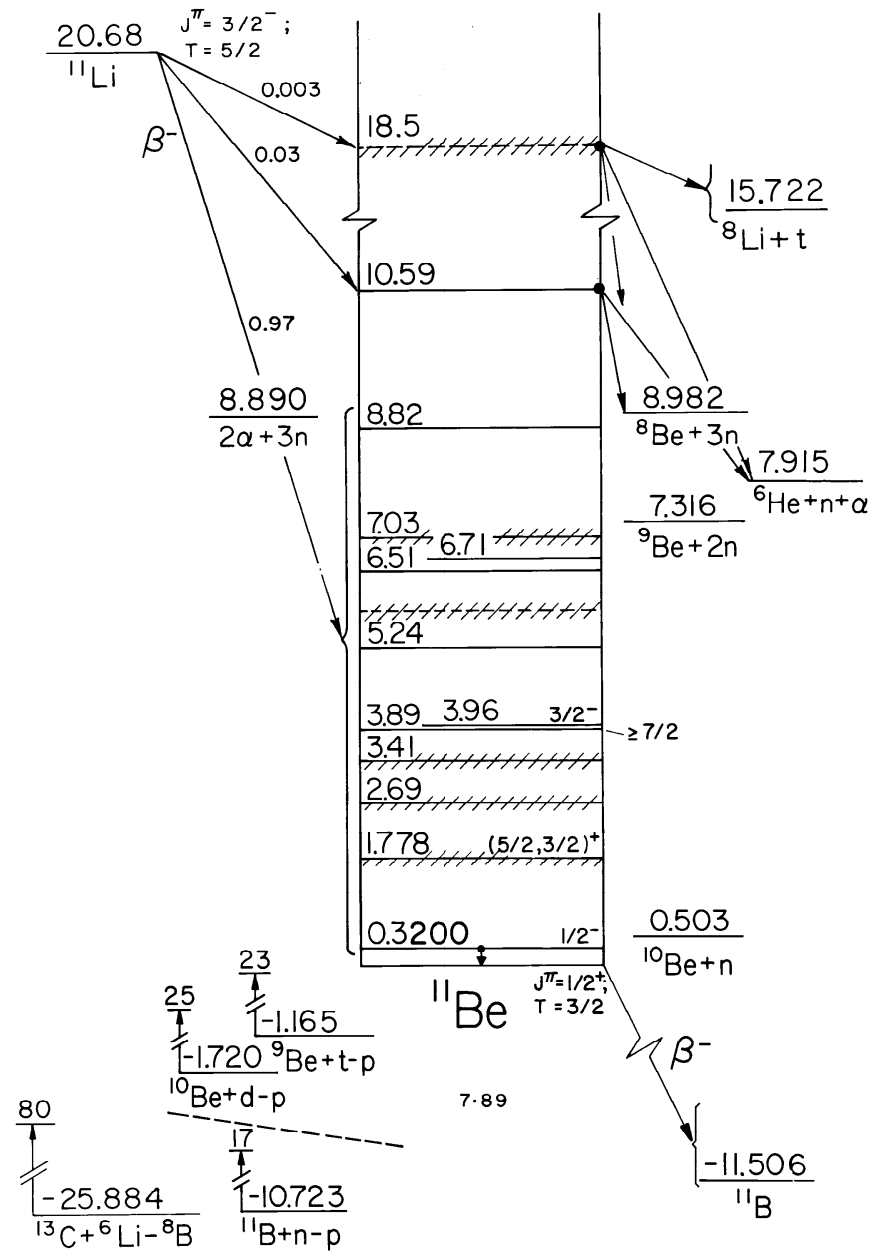
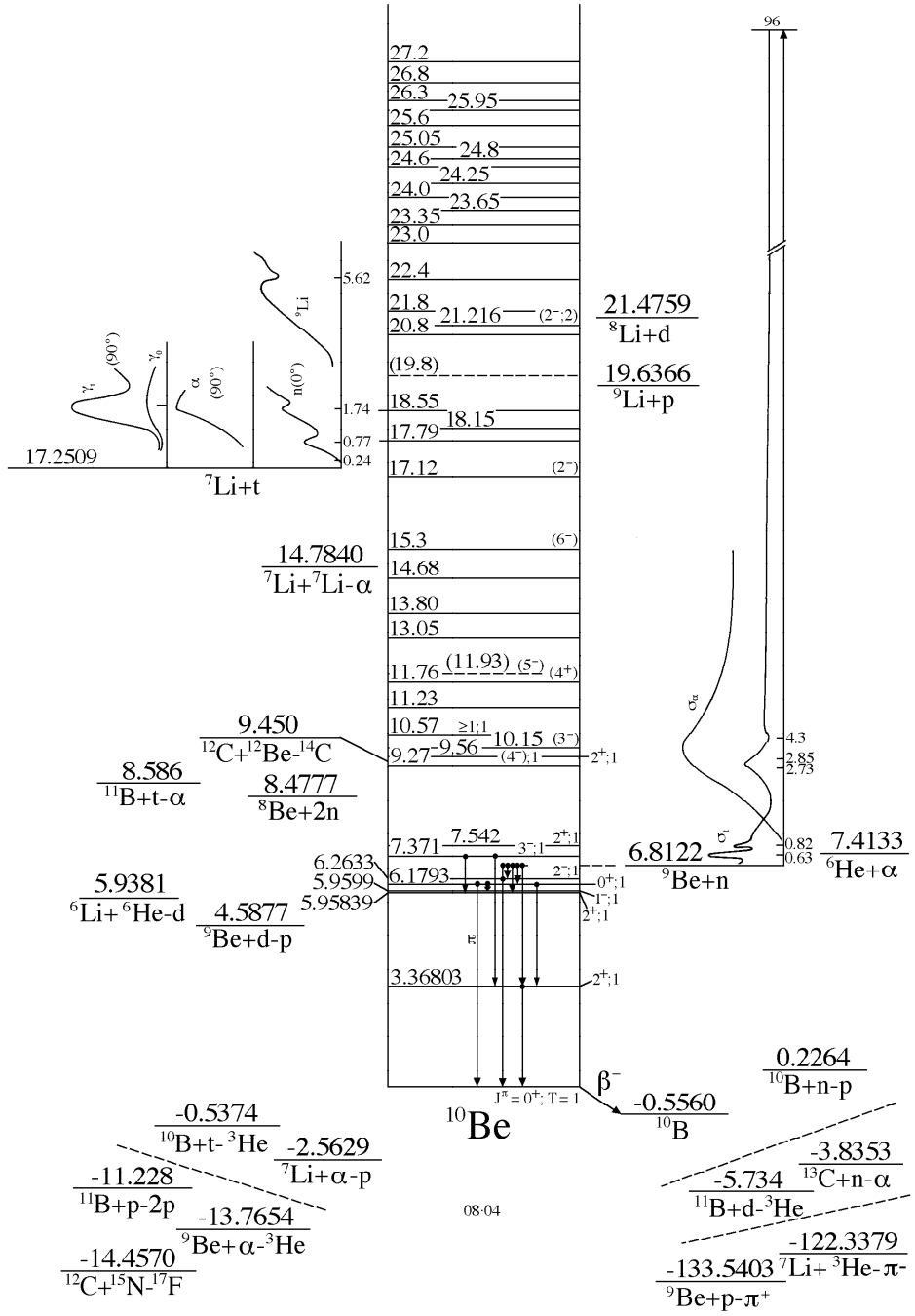
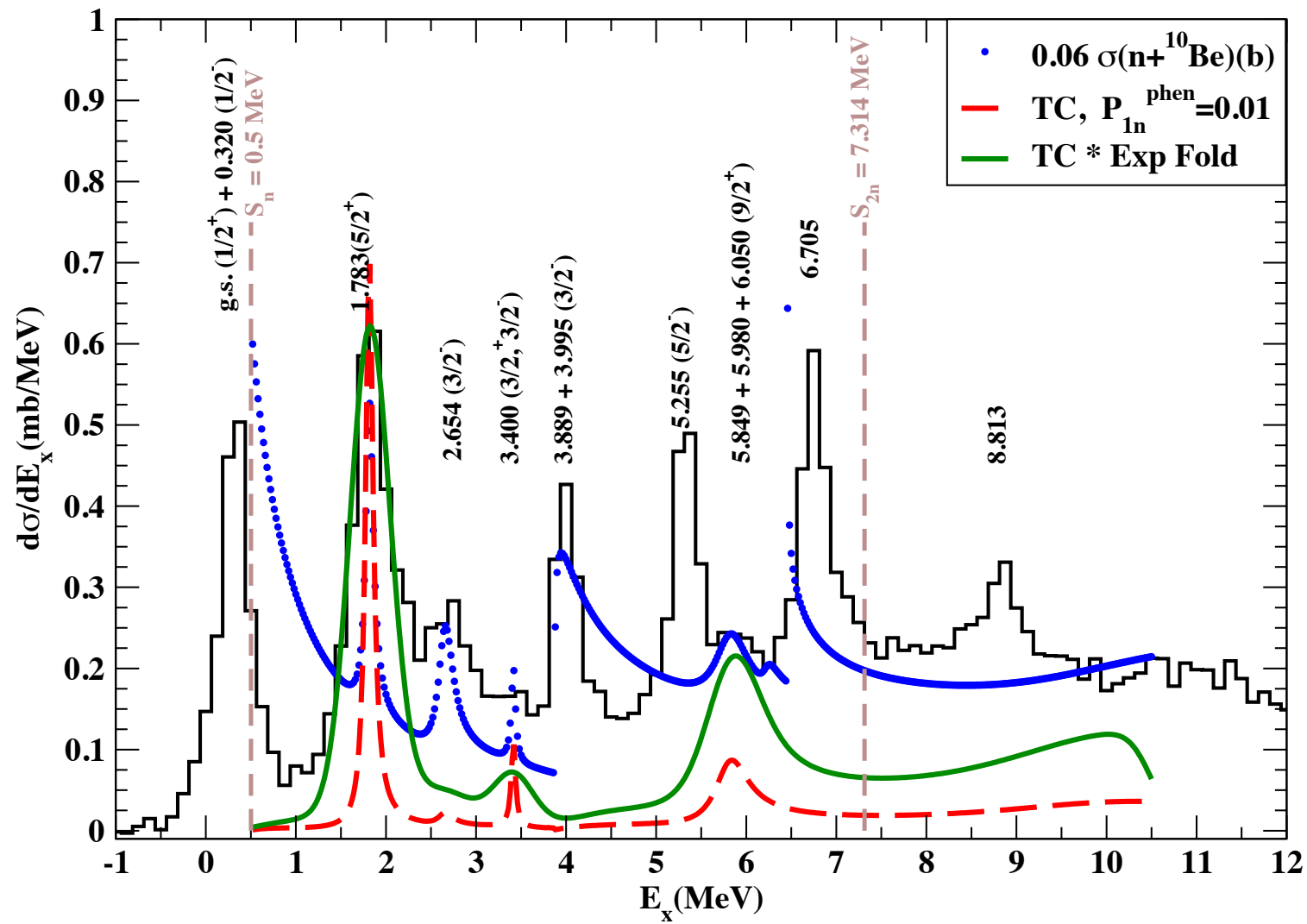


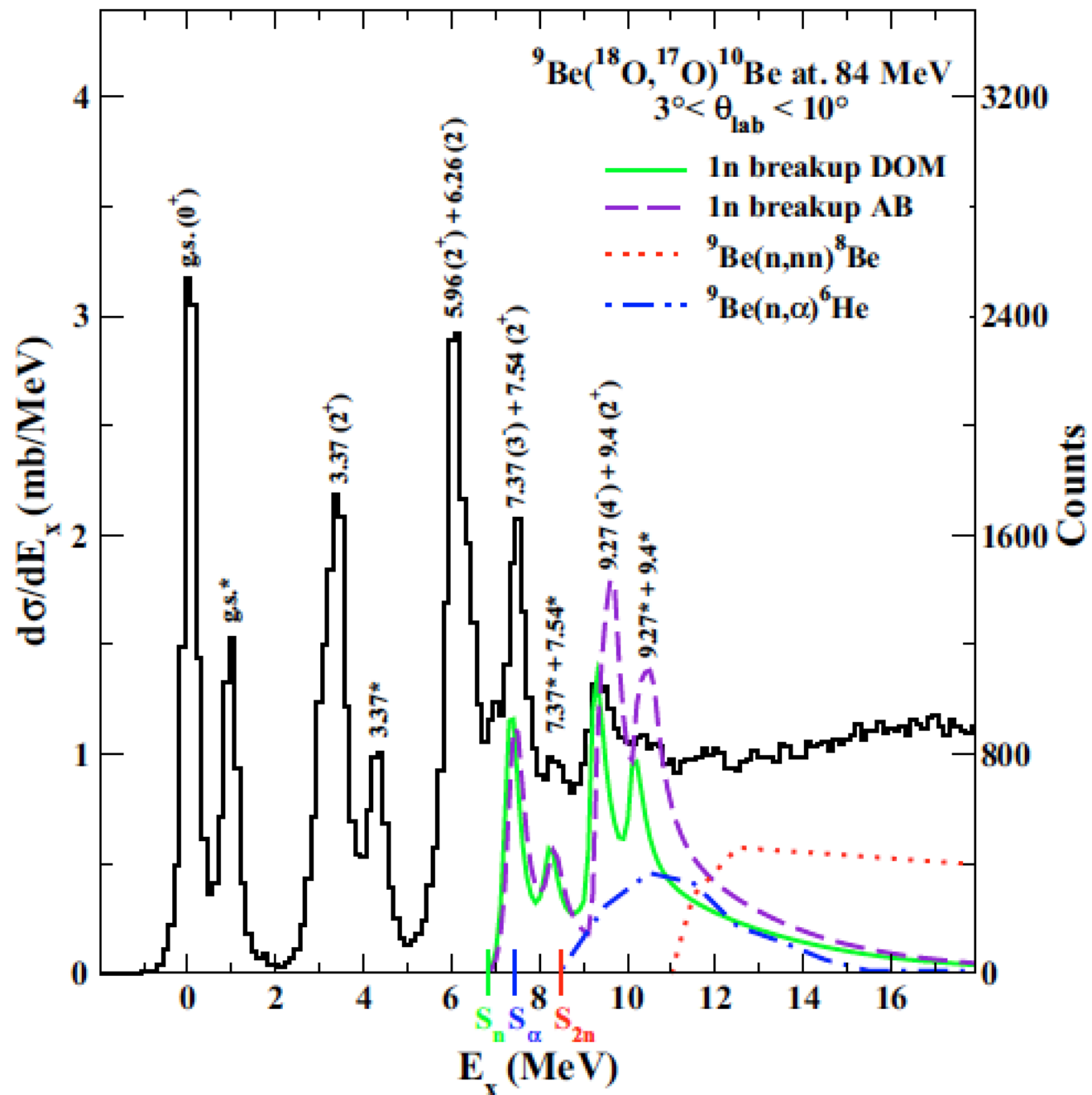
FIG. 6. (Color online) Q -value spectrum for ${}^{10}\text{Be}(d,p){}^{11}\text{Be}$ in inverse kinematics at 140° in the laboratory reference frame for an equivalent beam energy of $E_d = 21.4$ MeV. Background from fusion evaporation on ${}^{12}\text{C}$ was accounted for by subtracting data from reactions on a carbon target.



15.0-MeV triton beam from the University of Pennsylvania tandem

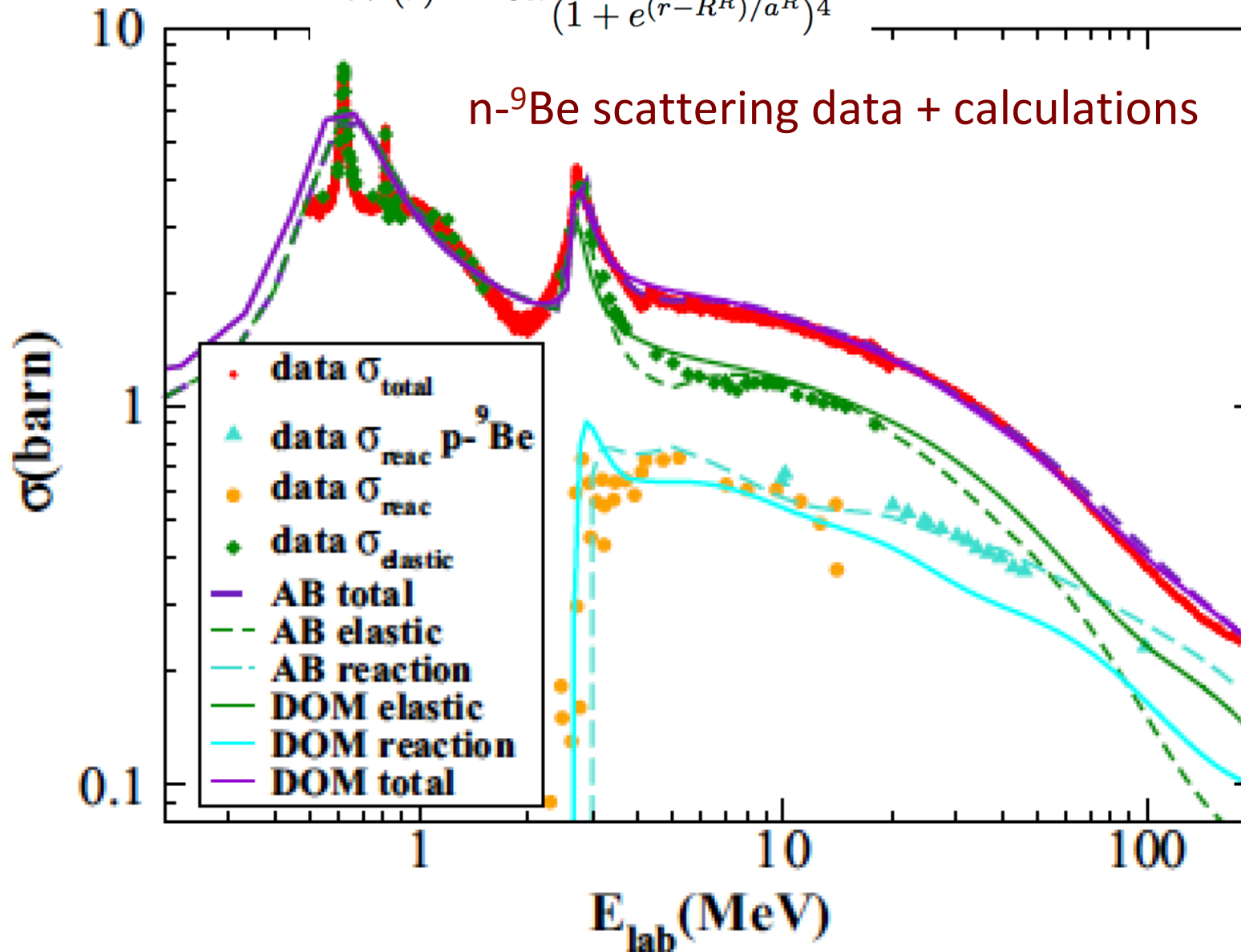




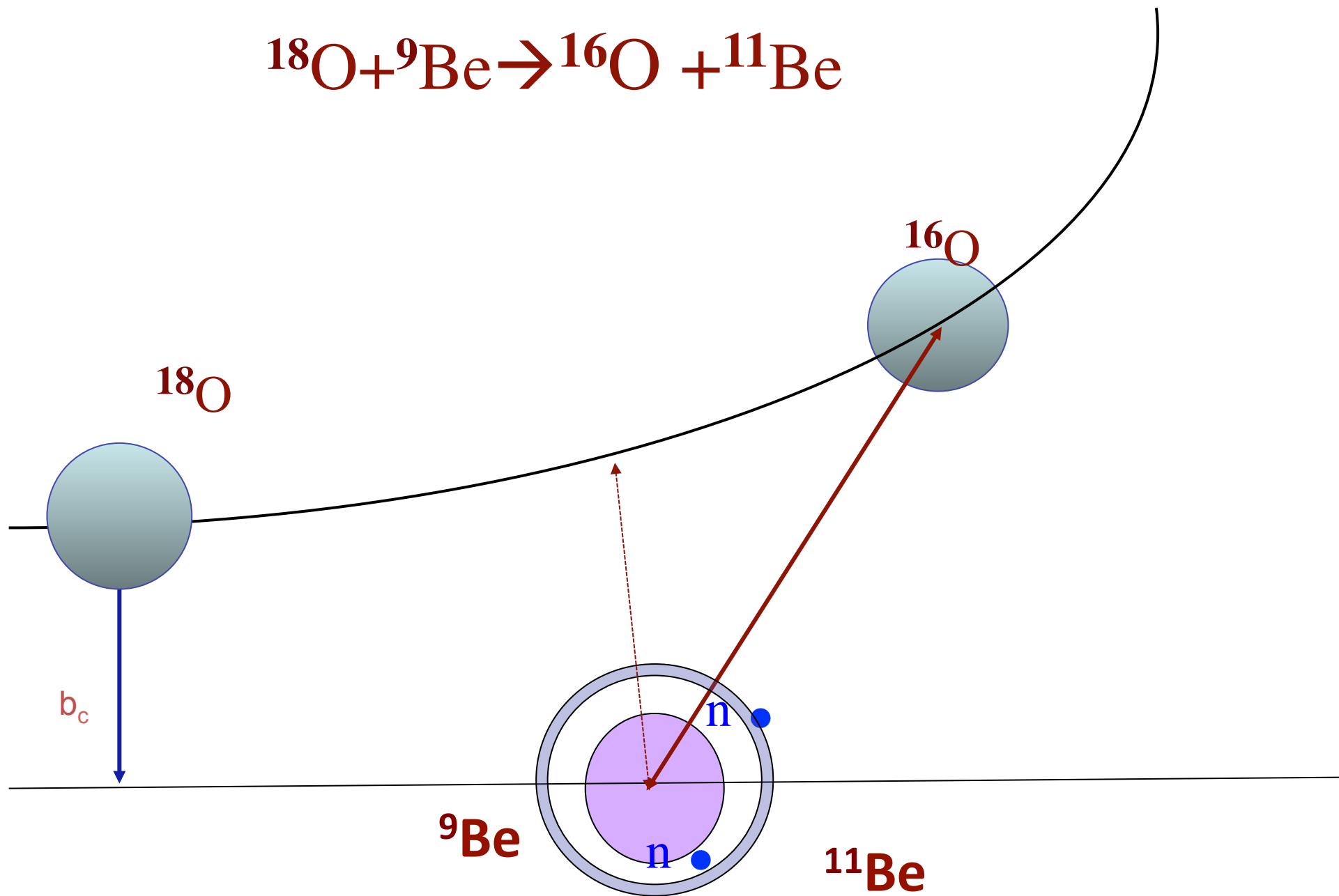


A.B & R. J. Charity, PRC89, 024619 (2014)

Resonances described by $\delta V(r) = 16\alpha \frac{e^{2(r-R^R)/a^R}}{(1 + e^{(r-R^R)/a^R})^4}$ consistent with dispersive contribution



Two-step reaction model
equivalent to applying second
order perturbation theory justified
by the small probabilities



transfer and inelastic excitations

A consistent formalism for all breakup reaction mechanisms

The core-target movement is treated in a semiclassical way, but neutron-target and/or neutron-core with a full QM method.

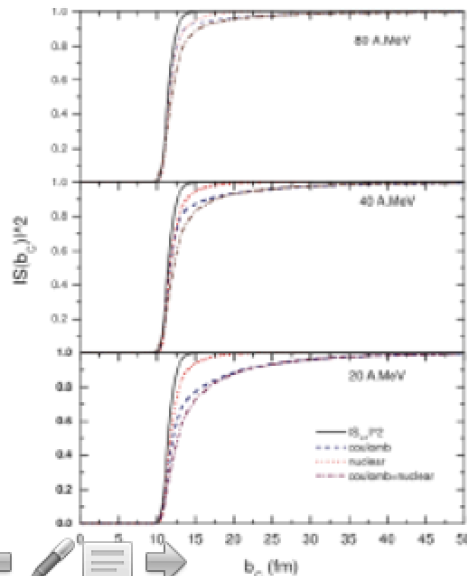
AB and DM Brink, PRC38, 1776 (1988), PRC43, 299 (1991), PRC44, 1559 (1991).

Early eikonal model: I. Tanihata, Prog. Part. Nucl. Phys. 35, 505 (1995), halo-core decoupling.

$$\frac{d\sigma}{d\xi} = C^2 S \int_0^\infty db_c \frac{dP_{-n}(b_c)}{d\xi} P_{ct}(b_c) P_{n1}^{phen}(R_s)$$

⊗

$$\xi \rightarrow \varepsilon_f, k_z, P_{||} \quad \text{also} \quad ANC = \sqrt{C^2 S C_i^2}$$



Use of the simple parametrization

$$P_{ct}(b_c) = |S_{ct}|^2 = e^{(-\ln 2 \exp[(R_s - b_c)/a])},$$

$$R_s \approx r_s (A_p^{1/3} + A_t^{1/3}) \quad r_s \approx 1.4 \text{ fm}$$

'strong absorption radius'

AB&F.Carstoiu, NPA706 (2002) 322

AB&A.Ibraheem, NPA748 (2005) 4

Transfer bewtween bound states

$$\begin{aligned}\sigma_{1n} &= \int_0^\infty d\mathbf{b}_c |S_{ct}(b_c)|^2 P_{n1}(b_c) \\ &= \int_{R_s}^\infty dd_c (d_c - a_c) P_{n1}(d_c) \\ &= \pi \frac{(R_s - a_c)}{\eta} P_{n1}(R_s),\end{aligned}\tag{2}$$

and

$$b_c^2 = d_c^2 - 2d_c a_c$$

$$P_{n1}(b_c) = P_{n1}(R_s) e^{-2\eta(d_c - R_s)}$$

$$P_{n1}^{phen}(R_s) = \frac{\sigma_{1n}^{exp}}{\pi \frac{(R_s - a_c)}{\eta}}$$

which is obtained when the core-target S-matrix is calculated in the sharp cut off approximation:

$$|S_{ct}(b_c)|^2 = 1 \quad \text{if} \quad b_c > R_s$$

$$|S_{ct}(b_c)|^2 = 0 \quad \text{if} \quad b_c < R_s.$$

η is a kinematical parameter depending on the initial and final neutron separation energies and the energy of relative motion [53],

$$\eta = \sqrt{\gamma_i^2 + k_1^2}$$

where $\gamma_i^2 = 2mS_{i(1,2)}\hbar^2$ and $S_{i(1,2)}$ is the bound state initial separation energy of the first and second neutron respectively. $k_1^2 = (Q + 1/2mv^2)^2/(\hbar v)^2$ where $Q = \varepsilon_{i(1,2)} - \varepsilon_{f(1,2)}$ is the Q-value. The $\varepsilon_{i(1,2)}$ are the negative initial binding energies of neutron n1 and n2 in their bound states, while $\varepsilon_{f(1,2)}$ is the negative final energy of the first step neutron in ^{10}Be and is the positive

How accurate is this method?

Reaction	Energy	$R_s(r_{0s})$	j_i	l_i	j_f	l_f	S_i	S_f	C_i	C_f	C^2S_i	C^2S_f	σ_{th}	σ_{ex}	
	A.MeV	fm					MeV	MeV	$\text{fm}^{-\frac{1}{2}}$	$\text{fm}^{-\frac{1}{2}}$			mb	mb	
$^{14}\text{O}(d, t)^{13}\text{O}_{gs}$	18	5.53(1.46)	3/2	1	1/2	0	23.2	6.26	17.32	2.14	1.4	1.3	1.6	1.6	[7]
$^{14}\text{O}(d, ^3\text{He})^{13}\text{N}_{gs}$	18	6.01(1.58)	1/2	1	1/2	0	4.63(6.05)	5.49	2.12	2.14	1.15	1.3	8.7	8.7	[7]
$^{12}\text{C}(^{13}\text{C}, ^{12}\text{C})^{13}\text{C}$	50	6.18(1.37)	3/2	1	1/2	1	18.7	4.95	8.2	1.89	2.6	...	0.92	0.92	[19]
$^{12}\text{C}(^{12}\text{C}, ^{13}\text{C})^{11}\text{C}_{gs}$	50	6.18(1.37)	3/2	1	1/2	1	18.7	4.95	8.2	1.89	2.6	...	0.92	0.92	[19]
	35	6.05(1.34), 6.35(1.39)									1.9, 2.8	...	2.5	2.5	[19]
	25	6.36(1.41)									2.2	...	3.5	3.5	[19]
	20	6.31(1.40)									1.8	...	3.8	3.8	[20]
	10	6.38(1.41)									2.1	...	4.9	4.9	[21]
$^{18}\text{O}(^9\text{Be}, ^{10}\text{Be})^{17}\text{O}_{gs}$	4.6	6.18(1.37)	5/2	2	3/2	1	8.04	6.81	1.73	2.3	1.7	2.73	2.1	2.1	[22]

[7] F. Flavigny, N. Keeley, A. Gillibert, and A. Obertelli
Phys. Rev. C 97, 034601 (2018), and references therein.

[8] F. Flavigny, private communication.

[19] J.S. Winfield et al., Phys Lett. B203 345 (1988).

[20] H.G. Bohlen et al., Z. Phys. A 322 241 (1985).

[21] N. Anyas-Weiss et al., Phys. Rep. 12 203 (1974).

[22] D. Carbone et al., private communication.

Transfer to the continuum

First order time dependent perturbation theory amplitude:

$$A_{fi} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} dt \langle \phi_f(\mathbf{r}) | V(\mathbf{r}) | \phi_i(\mathbf{r} - \mathbf{R}(t)) \rangle e^{-i(\omega t - mvz/\hbar)} \quad (1)$$

$$\omega = \varepsilon_i - \varepsilon_f + \frac{1}{2}mv^2$$

$$\begin{aligned} \frac{dP_{-n}(b_c)}{d\varepsilon_f} &= \frac{1}{8\pi^3} \frac{m}{\hbar^2 k_f} \frac{1}{2l_i + 1} \sum_{m_i} |A_{fi}|^2 \\ &\approx \frac{4\pi}{2k_f^2} \sum_{j_f, \nu} (|1 - \bar{S}_{j_f, \nu}|^2 + 1 - |\bar{S}_{j_f, \nu}|^2) (2j_f + 1) \mathcal{F}, \end{aligned}$$

$$\mathcal{F} = (1 + F_{l_f, l_i, j_f, j_i}) B_{l_f, l_i} \quad B_{l_f, l_i} = \frac{1}{4\pi} \left[\frac{k_f}{mv^2} \right] |C_i|^2 \frac{e^{-2\eta b_c}}{2\eta b_c} M_{l_f}$$

Wave functions

Final continuum state:

$$\phi_f(\mathbf{r}) = C_f k \frac{i}{2} (h_{l_f}^{(+)}(kr) - \bar{S}_{l_f} h_{l_f}^{(-)}(kr)) Y_{l_f, m_f}(\Omega_f),$$

$\bar{S}_{l_f}(\epsilon_f)$ is an optical model n-t (n-core in fragmentation reactions) S-matrix.

or using the potential $V = V_{nt} + V_{eff}$ sum of the neutron-target optical and Coulomb potentials, a distorted wave of the eikonal-type

$$\phi_f(\mathbf{r}, \mathbf{k}) = \exp \{ i\mathbf{k} \cdot \mathbf{r} + i\chi_{eik}(\mathbf{r}, t) \} \quad (3)$$

the eikonal phase shift is simply

$$\chi_{eik}(\mathbf{r}, t) = \frac{1}{\hbar} \int_t^\infty V(\mathbf{r}, \mathbf{R}(t')) dt'. \quad (4)$$

Initial state:

$$\phi_i(\mathbf{r}) = -C_i i^l \gamma h_{l_i}^{(1)}(i\gamma r) Y_{l_i, m_i}(\Omega_i).$$

L. Lo Monaco and DM Brink JPG11, 935, 1985; A. Mukhamedzhanov PRC 84, 044616, 2011; I. Thomposon talk at DREB2012

(Pisa).

CONCLUSIONS

- Ab-initio structure calculations including the continuum of a system $n+N$ contain all scattering information usually obtained with an optical potential, at least as far as low relative energies of the $n+N$ system are concerned.
- In the structure community phase shifts are usually discussed. The calculations provide in fact an S-matrix.
- The ^{11}Be nucleus has recently been discussed as $n+^{10}\text{Be}$. This is very useful because free scattering data do not exist due to the difficulties in obtaining ^{10}Be targets.
- We have used the $n-^{10}\text{Be}$ ab-initio S-matrix to describe an experimental spectrum from $2n$ transfer $^{18}\text{O}(^9\text{Be},^{11}\text{Be})^{16}\text{O}$ at 84 MeV at LNS-Catania.
- The two-step model used seems to work ok also as far as the absolute cross sections are concerned.
- Unfortunately $2n$ simultaneous transfer and $2n+^9\text{Be}$ structures cannot be described.

$$|S_{NN}(\mathbf{b})|^2 = e^{2\chi_I(\mathbf{b})} \quad (2)$$

is the probability that the nucleus-nucleus (NN) scattering is elastic for a given impact parameter \mathbf{b} .

The imaginary part of the eikonal phase shift is given by

$$\begin{aligned} \chi_I(\mathbf{b}) &= \frac{1}{\hbar v} \int dz W^{NN}(\mathbf{b}, z) \\ &= \frac{1}{\hbar v} \int dz \int d\mathbf{r}_1 W^{nN}(\mathbf{r}_1 - \mathbf{r}) \rho(\mathbf{r}_1), \end{aligned} \quad (3)$$

where W^{NN} is negative defined as

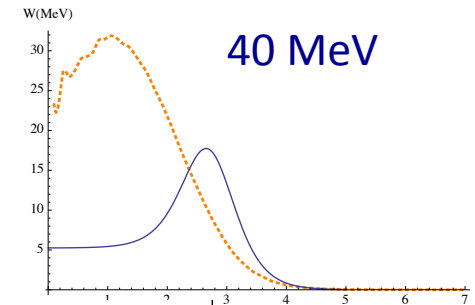
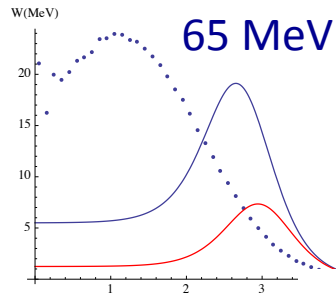
$$W^{NN}(\mathbf{r}) = \int d\mathbf{b}_1 W^{nN}(\mathbf{b}_1 - \mathbf{b}, z) \int dz_1 \rho(\mathbf{b}_1, z_1). \quad (4)$$

This quantity is the imaginary part of the single-folded optical potential given in terms of a nucleon-nucleus (nN) optical potential $W^{nN}(\mathbf{r})$ and the matter density $\rho(\mathbf{b}_1, z_1)$ of the other nucleus. In the single-folding method, $W^{nN}(\mathbf{r})$ can be the imaginary part of a phenomenological nucleon-target potential such as the (DOM) or the (AB) potentials of Ref. [3]. In the double-folding method, W^{NN} is obtained from the microscopic densities $\rho_{p,t}(\mathbf{r})$ for the projectile and target, respectively, and an energy-dependent nucleon-nucleon (nn) cross section σ_{nn} , i.e.,

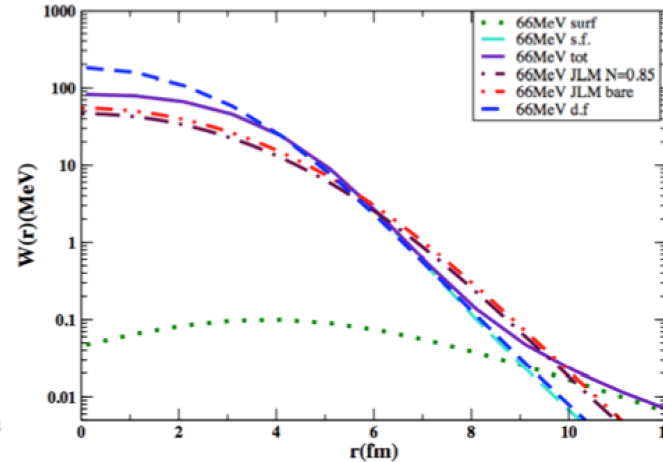
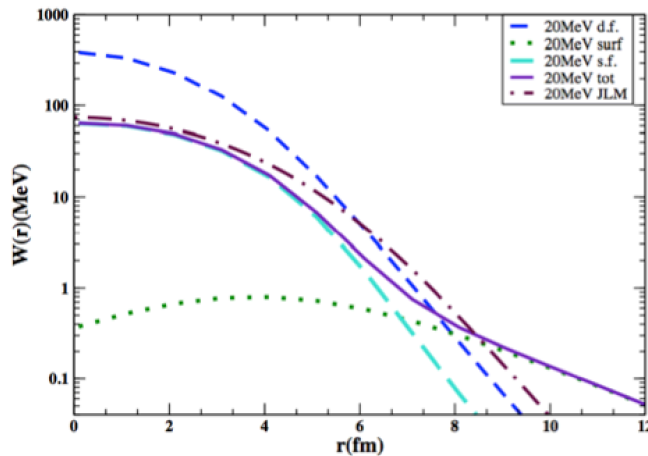
$$W^{NN}(\mathbf{r}) = -\frac{1}{2} \hbar v \sigma_{nn} \int d\mathbf{b}_1 \rho_p(\mathbf{b}_1 - \mathbf{b}, z) \int dz_1 \rho_t(\mathbf{b}_1, z_1). \quad (5)$$

$n+{}^9\text{Be}$

... SF VMC density & σ_{np} & σ_{pp} -
parametrized according to C. A. Bertulani, and C. De
Conti, Phys. Rev. C81 (2010) 064603.
AB phenomenological, DOM



${}^9\text{C}+{}^9\text{Be}$

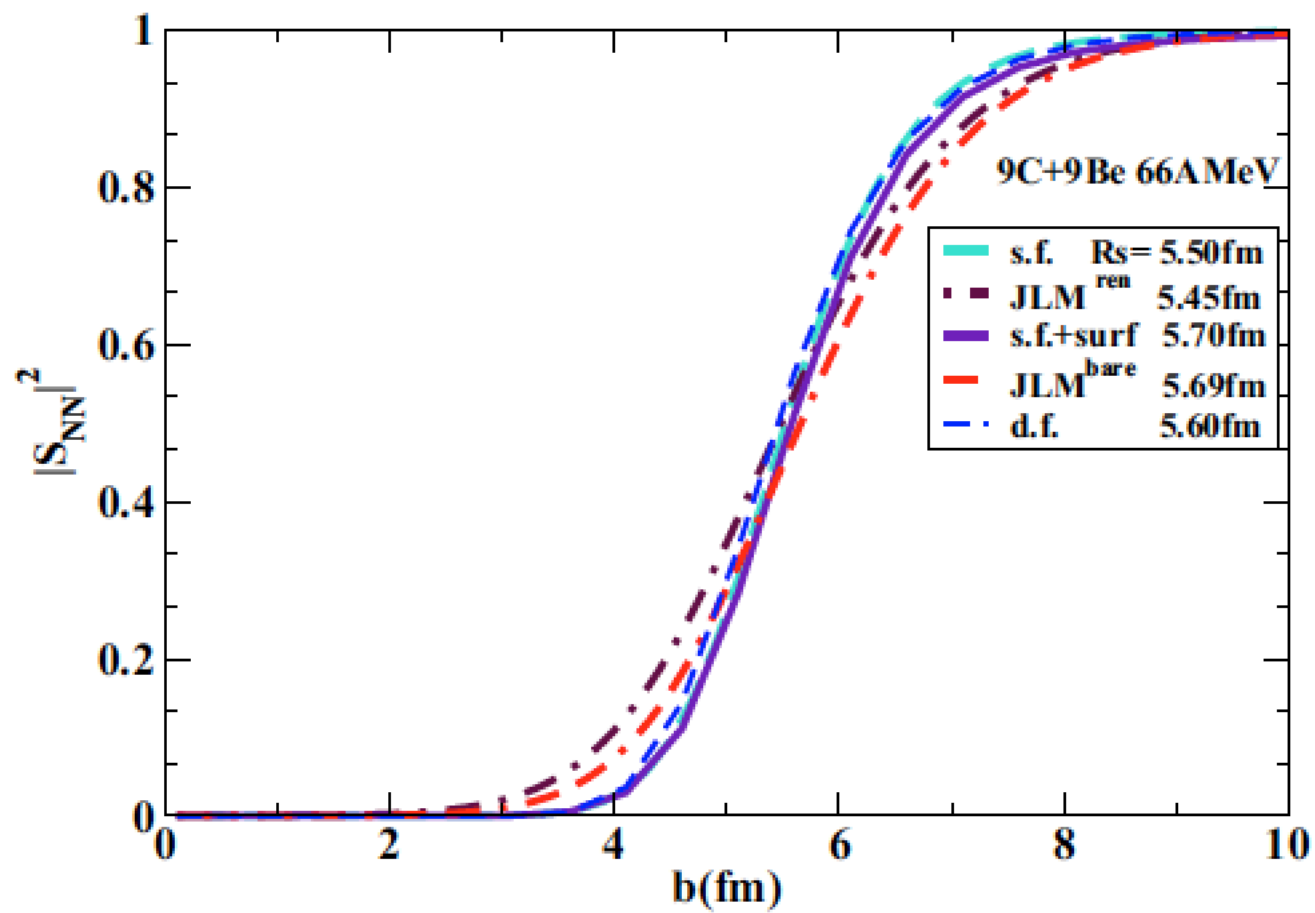


E_{lab} (A.MeV)	$J_{d.f.HF}$ (MeV fm ³)	rms (fm)	J_{JLM}^{ren} (MeV fm ³)	rms (fm)	$J_{s.fold}^{+surf}$ (MeV fm ³)	rms (fm)	$J_{s.fold}$ (MeV)	rms	W_{surf}
20	656	3.55	259	4.42	198	4.72	172	3.81	0.8
38	437	3.55	212	4.40	272	4.29	255	3.85	0.5
66	301	3.55	143	4.31	248	3.96	245	3.86	0.1
83	261	3.55	147	4.27	232.2	3.87	231.7	3.85	0.015

$$W(r) = -4a^i W_{surf} \frac{d}{dr} \frac{1}{1 + e^{(r-R^i)/a^i}}. \quad (9)$$

with very small strength ($W_{surf} = 0.8$ to 0.015 MeV), the radius has been taken as $R^i = 3.8$ fm, while the diffuseness has been taken large, according to [1; 4], and equal to $a^i = 1/(2\sqrt{2mS_p/\hbar}) = 2$ fm for ${}^9\text{C}$, since $S_p = 1.3$ MeV.

A. B. and F. Carstoiu,
Nucl. Phys. A **706**, 322
(2002).



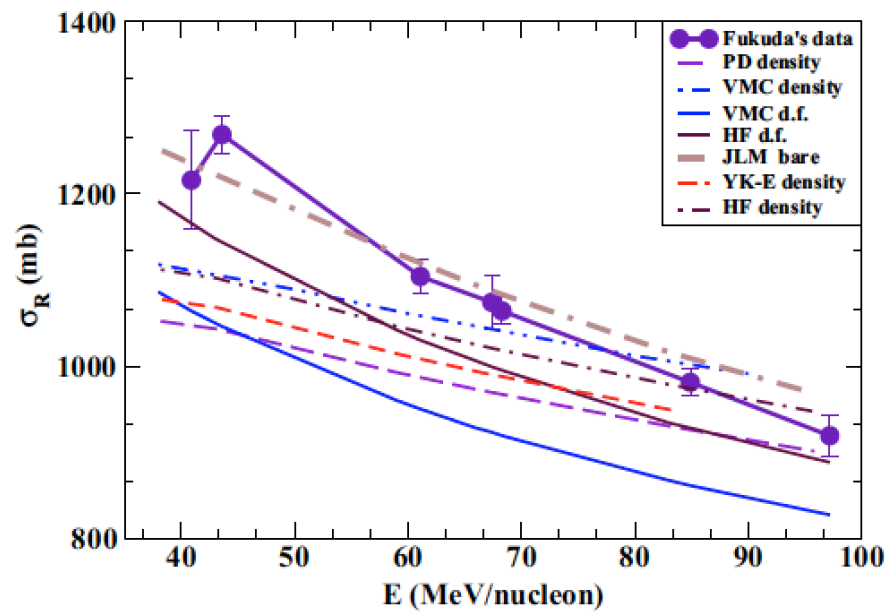


FIG. 2. (Color online) Reaction cross-section σ_R versus energy E for ^9C with various density functionals.

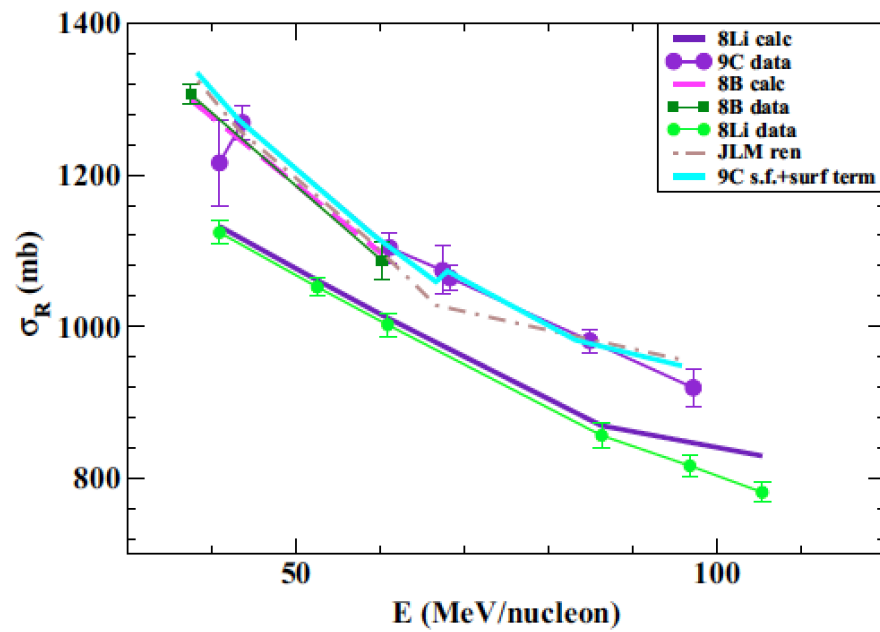


FIG. 3. (Color online) Reaction cross-section σ_R versus energy E for ^8Li and ^9C with various density functionals.

