# Realistic shell-model calculations for neutrinoless double-beta decay: Where do we stand ?

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XVII Conference on Theoretical Nuclear Physics in Italy

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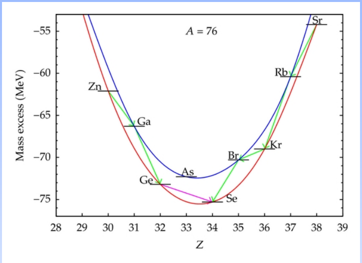
# Double $\beta$-decay

Double $\beta$-decay ($2\nu$ECEC) is the rarest process yet observed in nature.

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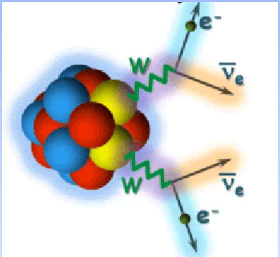


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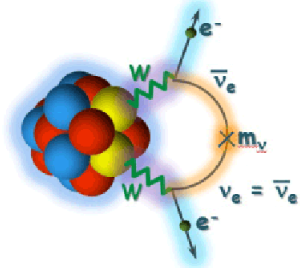


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# Neutrinoless double $\beta$-decay

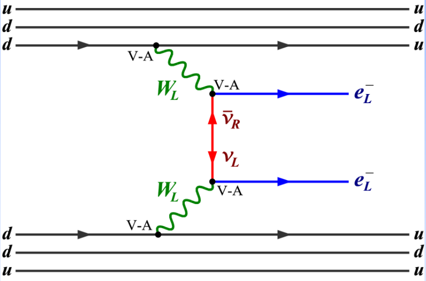
The detection of the $0\nu\beta\beta$ decay is nowadays one of the main targets in many laboratories all around the world, triggered by the search of "new physics" beyond the Standard Model.

Its detection $\Longrightarrow$

- would correspond to a violation of the conservation of the leptonic number
- may provide more informations on the nature of neutrinos (neutrino as a Majorana particle, determination of its effective mass, ..).

# Neutrinoless double $\beta$-decay

The inverse of the $0\nu\beta\beta$-decay half-life is proportional to the squared nuclear matrix element (NME).
This evidences the relevance to calculate the NME




$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2 \langle m_\nu \rangle^2$$

- $G^{0\nu}$ $\rightarrow$ phase-space factor
- $\langle m_\nu \rangle = \mid \sum_k m_k U_{ek}^2 \mid$ effective mass of the Majorana neutrino, $U_{ek}$ being the lepton mixing matrix

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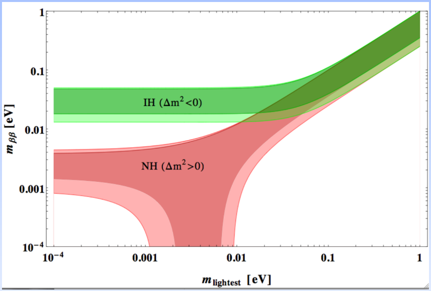


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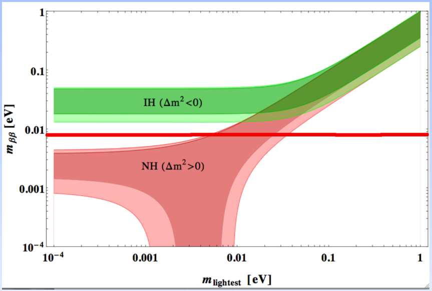


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- to exclude IH $\Rightarrow$ $m_\nu = 8\text{meV}$

# The calculation of the NME

The nuclear matrix element (NME) is expressed as

$$M^{0\nu} = M^{0\nu}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0\nu}_{F} + M^{0\nu}_{T} ~,$$

where

$$M^{0\nu}_{GT} =< 0^+_f \mid \sum_{m,n} \tau^-_m \tau^-_n H_{GT}(r_{mn}) \vec{\sigma}_m \cdot \vec{\sigma}_n \mid 0^+_i >$$

$$M^{0\nu}_{F} =< 0^+_f \mid \sum_{m,n} \tau^-_m \tau^-_n H_{F}(r_{mn}) \mid 0^+_i >$$

$$M^{0\nu}_{T} =< 0^+_f \mid \sum_{m,n} \tau^-_m \tau^-_n H_{T}(r_{mn}) \left[3 \left(\vec{\sigma}_m \cdot \hat{r}_{mn}\right) \left(\vec{\sigma}_n \cdot \hat{r}_{mn}\right) - \vec{\sigma}_m \cdot \vec{\sigma}_n\right] \mid 0^+_i >$$

# The calculation of the NME

$\Rightarrow$ make use of nuclear structure models

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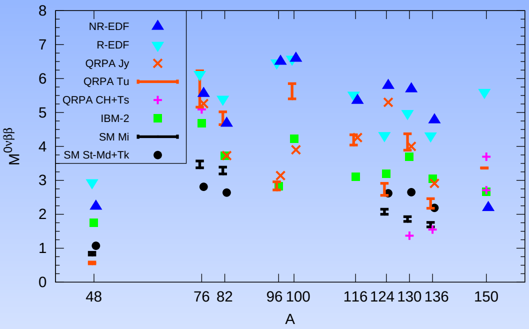


*Engel & Menendez Reports on Progress in Physics 80, 046301 (2017)*

- The spread of nuclear structure calculations evidences inconsistencies among results obtained with different models

# Realistic Shell-Model Calculations

Shell model $\Rightarrow$ well-established approach to obtain a microscopic description of both collective and single-particle properties of nuclei

## STRENGTH project: Napoli-Caserta unit

- L. Coraggio (INFN-NA)
- G. De Gregorio (Università "Vanvitelli" and INFN-NA)
- T. Fukui (INFN-NA)
- A. Gargano (INFN-NA)
- N. I. (Università "Vanvitelli" and INFN-NA)
- R. Mancino (Università "Vanvitelli" and INFN-NA)
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Focus on $^{76}$Ge, $^{82}$Se, $^{130}$Te, and $^{136}$Xe.

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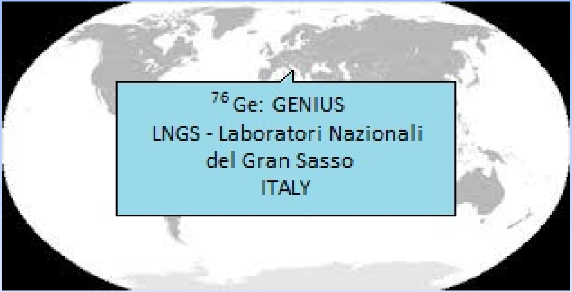

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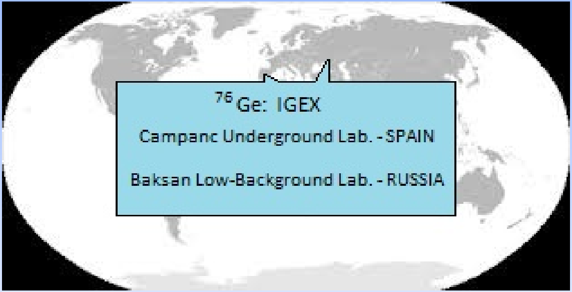


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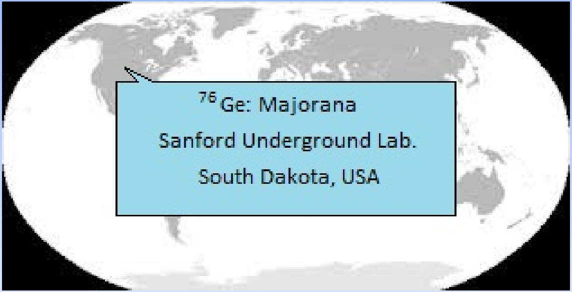

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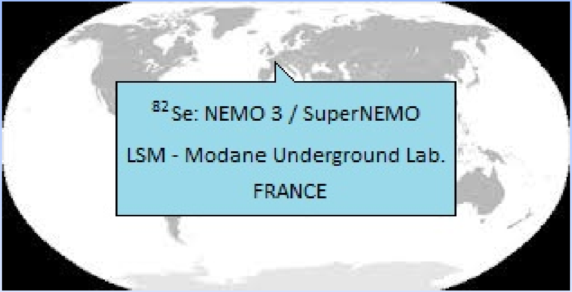


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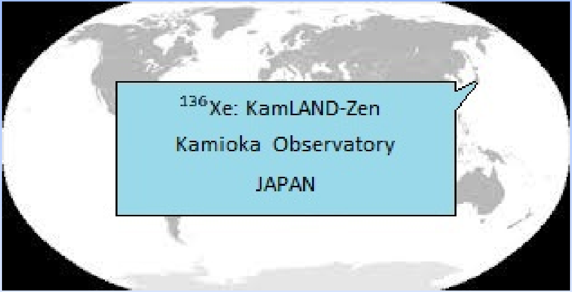


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# Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

**Two alternative approaches**

- phenomenological
- microscopic

$$V_{NN} \ (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

**Definition**

The eigenvalues of $H_{\text{eff}}$ belong to the set of eigenvalues of the full nuclear hamiltonian

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# Workflow for a realistic shell-model calculation

1. Choose a realistic *NN* potential (*NNN*)
2. Determine the model space better tailored to study the system under investigation
3. Derive the effective shell-model hamiltonian and operators by way of a many-body theory
4. Calculate the physical observables (energies, e.m. transition probabilities, ...)

# The shell-model effective hamiltonian

We start from the many-body hamiltonian $H$ defined in the full Hilbert space:

$$H = H_0 + H_1 = \sum_{i=1}^{A} (T_i + U_i) + \sum_{i<j} (V_{ij}^{NN} - U_i)$$

$$\left( \begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array} \right) \quad \begin{array}{c} \mathcal{H} = X^{-1} H X \\ \Longrightarrow \\ Q\mathcal{H}P = 0 \end{array} \quad \left( \begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array} \right)$$

$$H_{\mathrm{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow X = e^{\omega}$ with $\omega = \left( \begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array} \right)$

$$H_1^{\mathrm{eff}}(\omega) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P - $$

$$- PH_1Q \frac{1}{\epsilon - QHQ} \omega H_1^{\mathrm{eff}}(\omega)$$

# The perturbative approach to the shell-model $H^{\rm eff}$

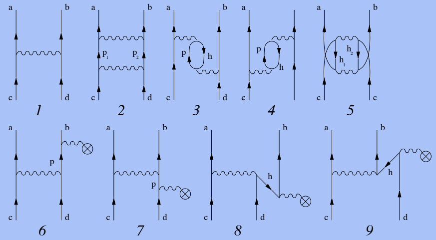
## The $\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

Exact calculation of the $\hat{Q}$-box is computationally prohibitive for many-body system $\Rightarrow$ we perform a perturbative expansion

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

## The diagrammatic expansion of the $\hat{Q}$-box

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# Effective operators

$\Phi_\alpha =$ eigenvectors obtained diagonalizing $H_{\mathrm{eff}}$ in the reduced model space $\Rightarrow |\Phi_\alpha\rangle = P|\Psi_\alpha\rangle$

$$\langle\Phi_\alpha|\hat{\Theta}|\Phi_\beta\rangle \neq \langle\Psi_\alpha|\hat{\Theta}|\Psi_\beta\rangle$$

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## Effective operator $\hat{\Theta}_{\rm eff}$: definition

$$\hat{\Theta}_{\rm eff} = \sum_{\alpha\beta} |\Phi_\alpha\rangle\langle\Psi_\alpha|\hat{\Theta}|\Psi_\beta\rangle\langle\Phi_\beta|$$

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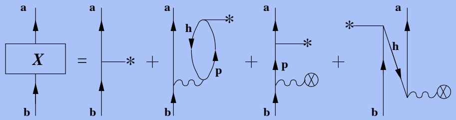
$\hat{\Theta}_{\rm eff}$ can be derived consistently in the MBPT framework

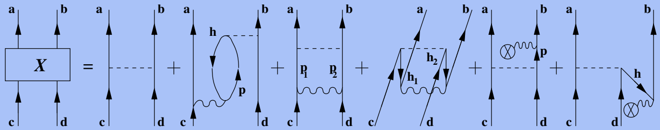
*K. Suzuki and R. Okamoto, Prog. Theor. Phys.* **93** *, 905 (1995)*

# The shell-model effective operators

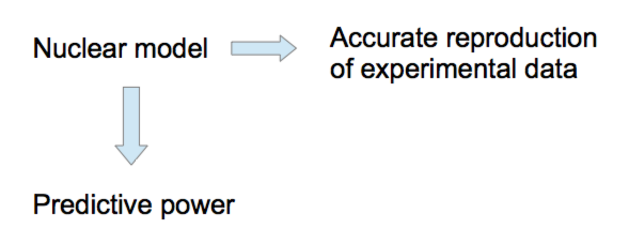
## One-body operator




## Two-body operator



# Nuclear models and predictive power




Realistic shell-model calculations for
$^{130}$Te, $^{136}$Xe, $^{76}$Ge and $^{82}$Se

$\Downarrow$

Test our approach calculating observables related to the GT strengths and $2\nu\beta\beta$ decay and comparing the results with data.

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# Realistic Shell-Model Calculations

- $^{76}$Ge, $^{82}$Se: four proton and neutron orbitals outside double-closed $^{56}$Ni $\longrightarrow 0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$
- $^{130}$Te, $^{136}$Xe: five proton and neutron orbitals outside double-closed $^{100}$Sn $\longrightarrow 0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2}$

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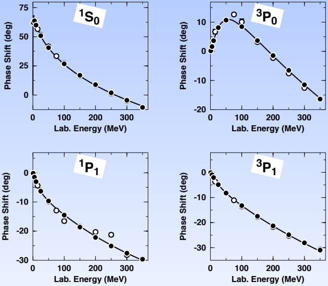
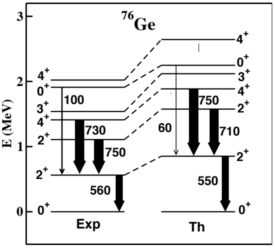
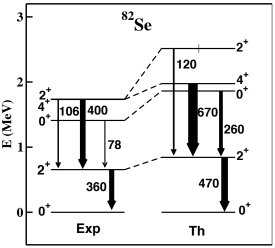
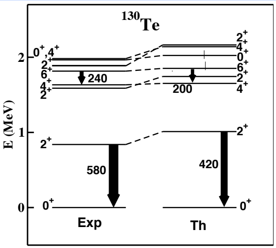
- Input $V_{NN}$: $V_{\mathrm{low-k}}$ derived from the high-precision *NN* CD-Bonn potential with a cutoff: $\Lambda = 2.6 \text{ fm}^{-1}$.
- $H_{\mathrm{eff}}$ obtained calculating the $Q$-box up to the 3rd order in $V_{\mathrm{low-k}}$
- Effective operators are consistently derived by way of the MBPT

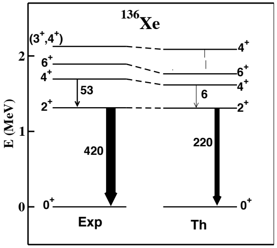


# Spectroscopic properties (B(E2)s in $e^2fm^4$)

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# GT$^-$ strength distribution

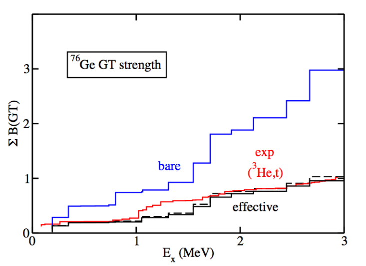
## Charge-exchange experiments

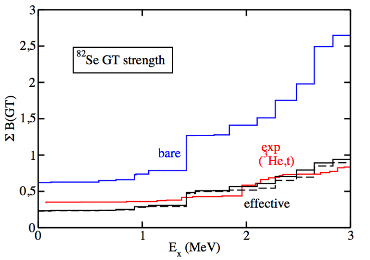
$$\left[\frac{d\sigma}{d\Omega}(q=0)\right] = \hat{\sigma} B_{exp}(GT)$$

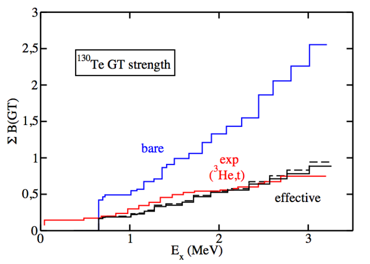
## Theory

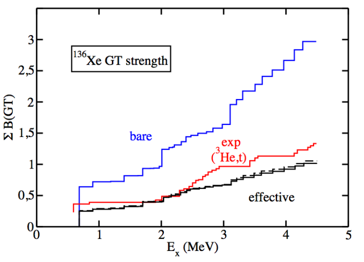
$$B_{th}(\mathrm{GT}) = \frac{\left| \langle \Phi_f | \sum_j \vec{\sigma}_j \vec{\tau}_j | \Phi_i \rangle \right|^2}{2J_i + 1}$$

# GT⁻ strength distribution

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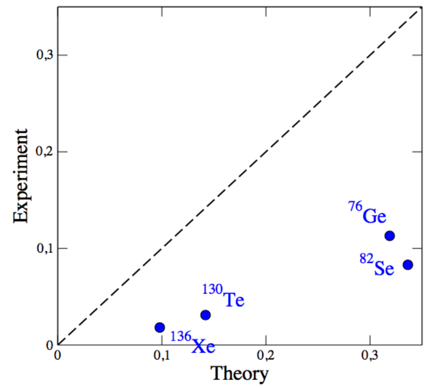
# $2\nu\beta\beta$ nuclear matrix elements

$$M_{2\nu}^{\mathrm{GT}} = \sum_n \frac{\langle 0_f^+ || \vec{\sigma}\tau^- || 1_n^+ \rangle \langle 1_n^+ || \vec{\sigma}\tau^- || 0_i^+ \rangle}{E_n + E_0}$$




Blue dots: bare GT operator

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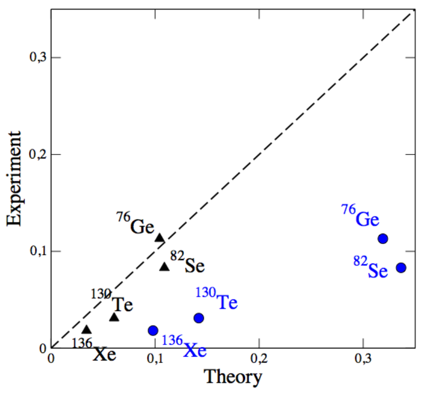


Blue dots: bare GT operator
Black triangles: effective GT operator


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# $2\nu\beta\beta$ nuclear matrix elements perturbative properties

- RSM calculations provide a satisfactory description of observed GT-strength distributions and $2\nu 2\beta$ NME
- what about perturbative properties ?

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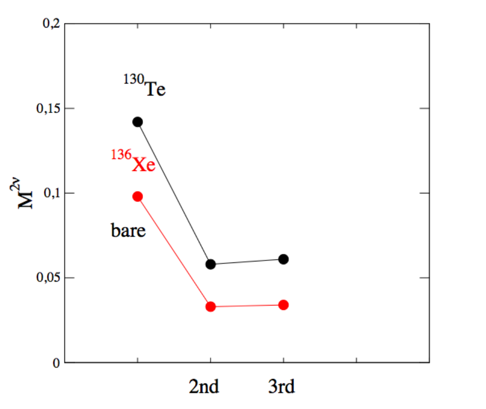


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# The calculation of the $0\nu\beta\beta$ NME

The NME is given by

$$M^{0\nu} = M^{0\nu}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0\nu}_{F} + M^{0\nu}_{T} \quad ,$$

The matrix elements $M^{0\nu}_{\alpha}$ are defined, for a SM calculation, as follows:

$$M^{0\nu}_{\alpha} = \sum_{j_p j_{p'} j_n j_{n'} J_\pi} TBTD\,(j_p j_{p'}, j_n j_{n'}; J_\pi)\, \langle j_p j_{p'}; J^\pi T \mid \tau^-_1 \tau^-_2 O^{\alpha}_{12} \mid j_n j_{n'}; J^\pi T \rangle$$

$$\text{with } \alpha = (GT,\ F,\ T)$$

The *TBTD* are the two-body transition-density matrix elements, and the Gamow-Teller $(GT)$, Fermi $(F)$, and tensor $(T)$ operators:

$$O^{GT}_{12} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r)$$
$$O^{F}_{12} = H_F(r)$$
$$O^{T}_{12} = [3\,(\vec{\sigma}_1 \cdot \hat{r})\,(\vec{\sigma}_1 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2]\, H_T(r)$$

# Light Majorana neutrino exchange

The neutrino potentials $H_\alpha$ are defined using the closure approximation

$$H_\alpha(r) = \frac{2R}{\pi} \int_0^\infty f_\alpha(qr) \frac{h_\alpha(q^2)}{q + \langle E \rangle} qdq$$

where $f_{F,GT}(qr) = j_0(qr)$ and $f_T(qr) = j_2(qr)$, $\langle E \rangle$ is the average energy used in the closure approximation.

- closure approximation
- higher order corrections (HOC)
- finite nucleon size corrections (FNS)

# Short-range correlations

## Empirical approach

$$\psi_{nl} \rightarrow [1 + f(r)]\, \psi_{nl} \qquad f(r) = -ce^{-ar^2}(1 - br^2)$$

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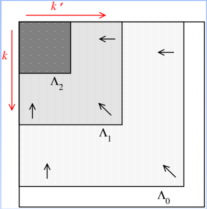
$$V_{NN}(k,k') \rightarrow V_{\rm low-k}(k,k') = \Omega^{-1} V_{NN}(k,k')\Omega$$

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# Short-range correlations

## Empirical approach

$$\psi_{nl} \to [1 + f(r)]\,\psi_{nl} \qquad f(r) = -ce^{-ar^2}(1 - br^2)$$

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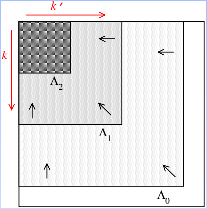


$$V_{NN}(k,k') \to V_{\mathrm{low-k}}(k,k') = \Omega^{-1} V_{NN}(k,k')\Omega$$

Consistently, we transform the $0\nu\beta\beta$ operator $\Theta$ by way of the same similarity transformation $\Omega$

$$\Theta(k,k') \to \Theta_{\mathrm{low-k}}(k,k') = \Omega^{-1}\Theta(k,k')\Omega$$

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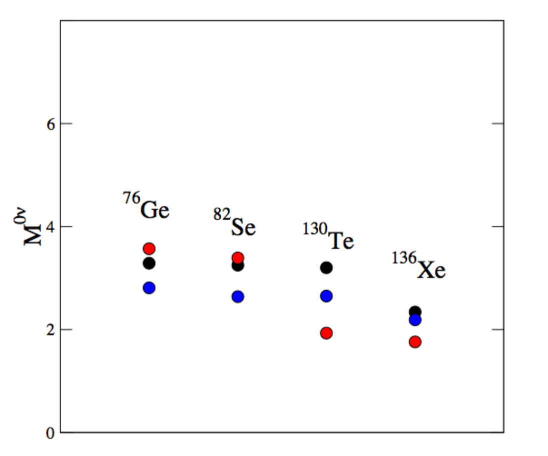
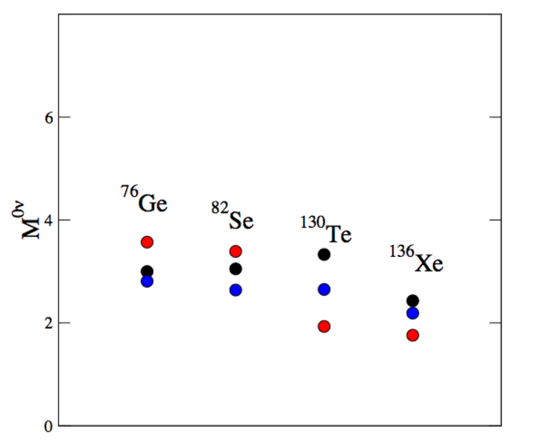
# Shell model calculations of $M^{0\nu}$



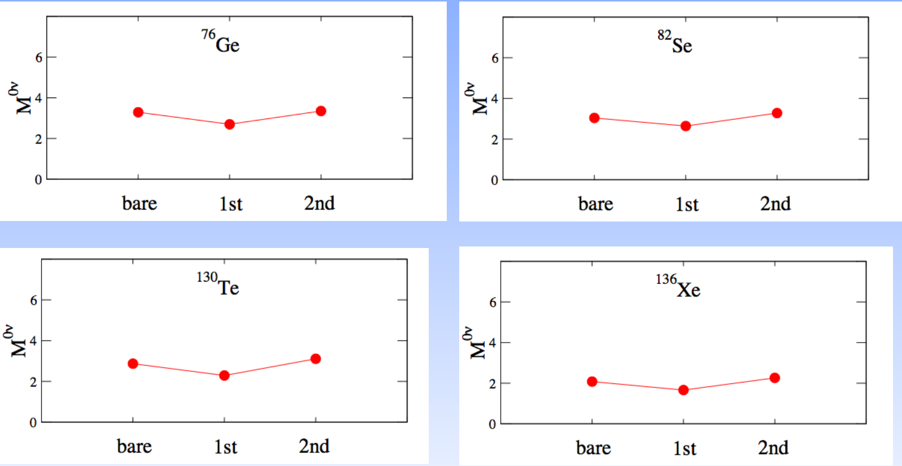

- Blue dots: Madrid-Strasbourg group, $0\nu\beta\beta$ operator
- Red dots: Horoi *et al.*, $0\nu\beta\beta$ operator
- Black dots: RSM, bare $0\nu\beta\beta$ operator

# Shell model calculations of $M^{0\nu}$




- Blue dots: Madrid-Strasbourg group, $0\nu\beta\beta$ operator
- Red dots: Horoi *et al.*, $0\nu\beta\beta$ operator
- Black dots: RSM, effective $0\nu\beta\beta$ operator

# Shell model calculations of $M^{0\nu}$

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# $J$-pair decomposition

It is possible to decouple $M^{0\nu}$ in terms of the angular momentum $J^{\pi}$ of the neutron and proton pairs

$$M^{0\nu}_{\alpha} = \sum_{j_p j_{p'} j_n j_{n'} J^{\pi}} TBTD\left(j_p j_{p'}, j_n j_{n'}; J^{\pi}\right) \left\langle j_p j_{p'}; J^{\pi} \mid \tau^{-}_1 \tau^{-}_2 O^{\alpha}_{12} \mid j_n j_{n'}; J^{\pi} \right\rangle$$

$$= \sum_{J^{\pi}} M^{0\nu}_{\alpha}(J^{\pi})$$

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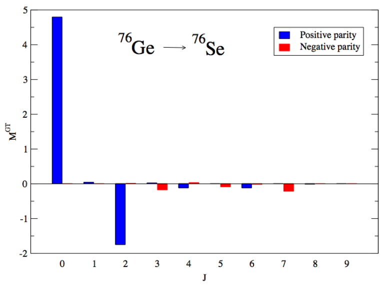
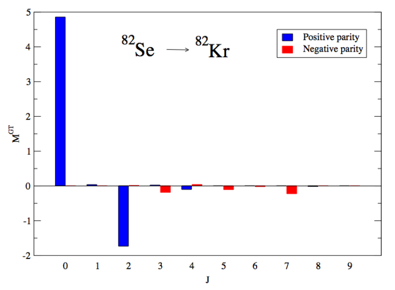
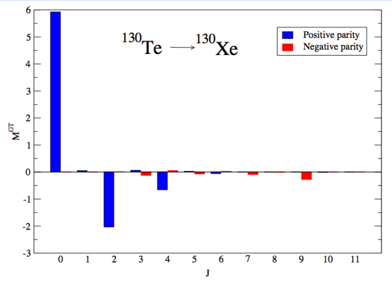
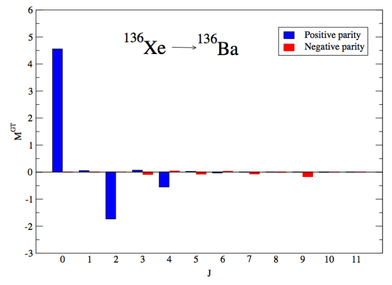
# *J*-pair decomposition

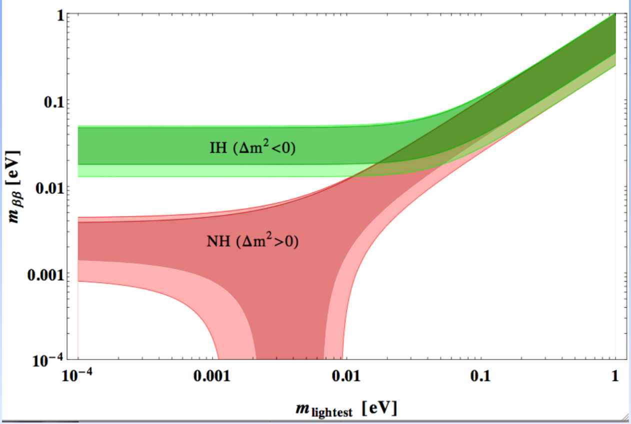


# Experimental upper bounds & Sensitivity

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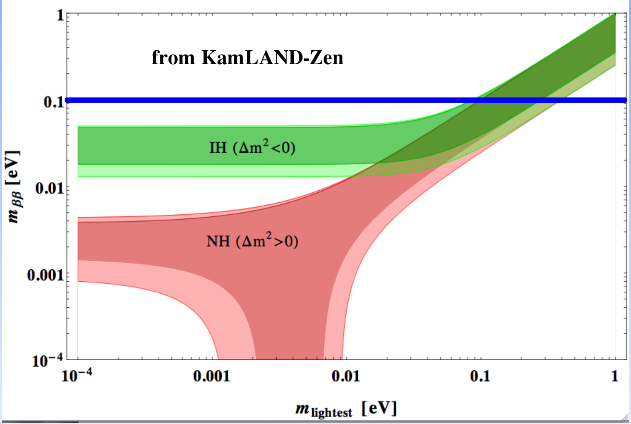


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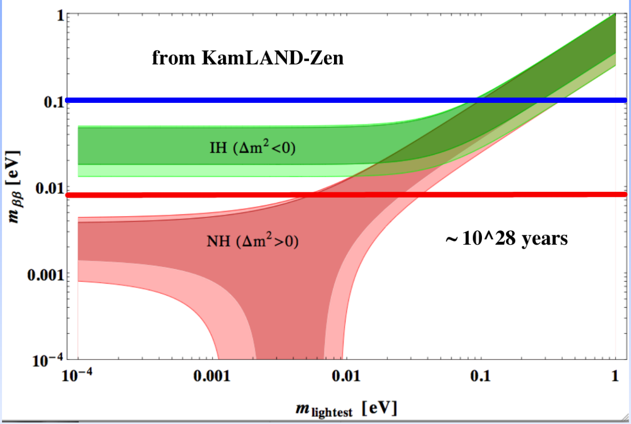
# Experimental upper bounds & Sensitivity

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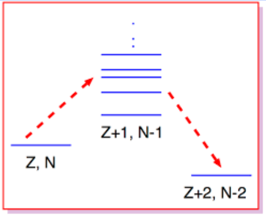
# Perspectives

- $H_{\mathrm{eff}}$ derived from chiral two- and three-body potentials: effects of chiral two-body currents (for both $2\nu\beta\beta$ and $0\nu\beta\beta$ decays)
- Beyond closure approximation




$$M_{\alpha}^{0\nu} = \sum_{j_p j_{p'} j_n j_{n'} J_k^{\pi}} \left\langle 0_f \middle| a^{\dagger}_{j_{p'}} a_{j_{n'}} \middle| J_k^{\pi} \right\rangle \left\langle J_k^{\pi} \middle| a^{\dagger}_{j_p} a_{j_n} \middle| 0_i \right\rangle \langle j_p j_{p'} \mid \tau_1^- \tau_2^- O^{\alpha}(E_k) \mid j_n j_{n'} \rangle \tag{1}$$

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# Realistic shell-model calculations for neutrinoless double-beta decay: Where do we stand ?

Nunzio Itaco

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Istituto Nazionale di Fisica Nucleare - Sezione di Napoli

XVII Conference on Theoretical Nuclear Physics in Italy

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# The shell-model effective operators

$$\Theta_{\rm eff} = (P + \hat{Q}_1 + \hat{Q}_1 \hat{Q}_1 + \hat{Q}_2 \hat{Q} + \hat{Q} \hat{Q}_2 + \cdots)(\chi_0 + \chi_1 + \chi_2 + \cdots) \ ,$$

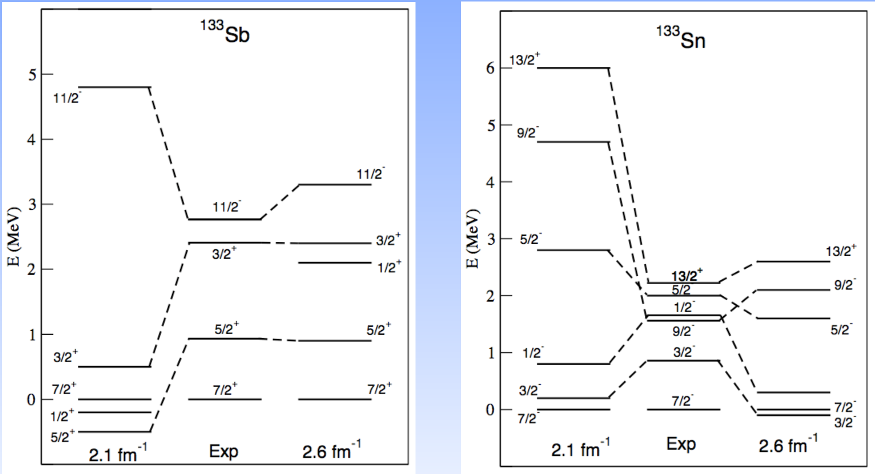
We arrest the $\chi$ series at $\chi_0$

$$\chi_0 = (\hat{\Theta}_0 + h.c.) + \Theta_{00}$$

$$\begin{aligned} \hat{\Theta}_0 &= P\Theta P + P\Theta Q \frac{1}{\epsilon_0 - QHQ} QH_1 P \ , \\ \hat{\Theta}_{00} &= PH_1 Q \frac{1}{\epsilon_0 - QHQ} Q\Theta Q \frac{1}{\epsilon_0 - QHQ} QH_1 P \end{aligned}$$

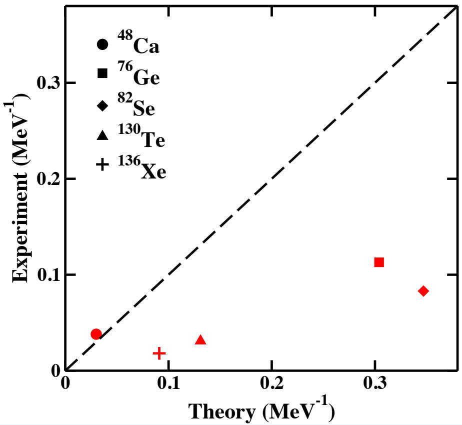
and expand it perturbatively.

# The choice of the cutoff $\Lambda = 2.6\ \mathrm{fm}^{-1}$




L. C., A. Gargano, and N. Itaco, JPS Conf. Proc. **6**, 020046 (2015)

# $2\nu\beta\beta$ nuclear matrix elements




Red dots: bare GT operator

| Decay | Expt. | Bare |
|---|---|---|
| $^{48}Ca \rightarrow ^{48}Ti$ | $0.038 \pm 0.003$ | 0.030 |
| $^{76}Ge \rightarrow ^{76}Se$ | $0.113 \pm 0.006$ | 0.304 |
| $^{82}Se \rightarrow ^{82}Kr$ | $0.083 \pm 0.004$ | 0.347 |
| $^{130}Te \rightarrow ^{130}Xe$ | $0.031 \pm 0.004$ | 0.131 |
| $^{136}Xe \rightarrow ^{136}Ba$ | $0.0181 \pm 0.0007$ | 0.0910 |

Experimental data from *A. S. Barabash, Nucl. Phys. A* ***935***, *52 (2015)*

# Perturbative properties of the GT effective operator

## Convergence with respect the number of intermediate states

Selection rules of the GT operator make the convergence of the effective one with respect to $N_{\rm max}$ very fast.

The third decimal digit value of $M^{2\nu}_{\rm GT}$, calculated with effective operator at third order, does not change from $N_{\rm max} = 12$ on.

## Order-by-order convergence

| Decay | 1st ord $M^{2\nu}_{\rm GT}$ | 2nd ord $M^{2\nu}_{\rm GT}$ | 3rd ord $M^{2\nu}_{\rm GT}$ | Expt. |
|---|---|---|---|---|
| $^{130}\mathrm{Te} \rightarrow ^{130}\mathrm{Xe}$ | 0.142 | 0.058 | 0.061 | $0.031 \pm 0.004$ |
| $^{136}\mathrm{Xe} \rightarrow ^{136}\mathrm{Ba}$ | 0.0975 | 0.0325 | 0.0341 | $0.0181 \pm 0.0007$ |

More than 60% from 1st → 2nd order
Less than 5% from 2nd → 3rd order

*L.C., L. De Angelis, T. Fukui, A. Gargano, and N. Itaco, Phys. Rev. C* ***95****, 064324 (2017)*

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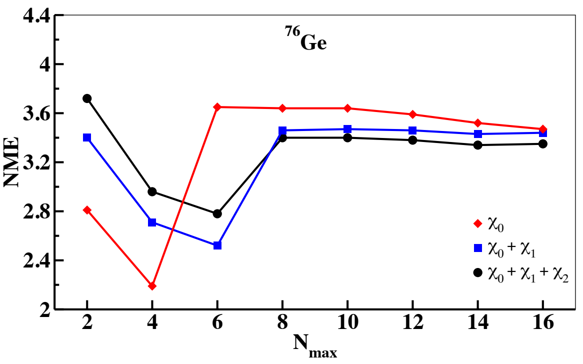
# Perturbative behavior with respect $N_{max}$




- $\chi_0$ contribution is not enough to obtain convergence with respect the number of intermediate states.
- $\chi_1$ and $\chi_2$ contributions are needed to obtain convergent results.

# Order-by-order perturbative behavior

| Decay | | $M^{0\nu}_{GT}$ | $M^{0\nu}_{F}$ | $M^{0\nu}$ |
|---|---|---|---|---|
| $^{48}Ca \rightarrow ^{48}Ti$ | | | | |
| | 1st order | 0.32 | -0.13 | 0.41 |
| | 2nd order | 0.53 | -0.16 | 0.63 |
| | 3rd order | 0.25 | -0.13 | 0.33 |
| $^{76}Ge \rightarrow ^{76}Se$ | | | | |
| | 1st order | 2.76 | -0.52 | 3.09 |
| | 2nd order | 2.30 | -0.62 | 2.70 |
| | 3rd order | 2.91 | -0.69 | 3.35 |
| $^{82}Se \rightarrow ^{82}Kr$ | | | | |
| | 1st order | 2.72 | -0.51 | 3.04 |
| | 2nd order | 2.25 | -0.61 | 2.64 |
| | 3rd order | 2.85 | -0.68 | 3.28 |
| $^{130}Te \rightarrow ^{130}Xe$ | | | | |
| | 1st order | 2.51 | -0.57 | 2.87 |
| | 2nd order | 1.83 | -0.72 | 2.29 |
| | 3rd order | 2.65 | -0.72 | 3.11 |
| $^{136}Xe \rightarrow ^{136}Ba$ | | | | |
| | 1st order | 1.82 | -0.41 | 2.08 |
| | 2nd order | 1.33 | -0.52 | 1.66 |
| | 3rd order | 1.93 | -0.53 | 2.26 |

The perturbative behavior is not satisfactory as for the single-$\beta$ decay operator: third-order contribution is rather large compared to the second order one

# The blocking effect

## Gamow-Teller two-body matrix elements

| Decay | $j_a j_b j_c j_d ; J = 0^+$ | ladder | 3b (a) | 3p-1h | 3b (b) |
|---|---|---|---|---|---|
| $^{48}\mathrm{Ca} \rightarrow {}^{48}\mathrm{Ti}$ | | | | | |
| | $0f_{7/2}0f_{7/2}0f_{7/2}0f_{7/2}$ | -0.334 | 0.004 | 0.260 | -0.017 |
| $^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}$ | | | | | |
| | $0g_{9/2}0g_{9/2}0f_{5/2}0f_{5/2}$ | 0.154 | -0.241 | -1.078 | 0.234 |
| | $0g_{9/2}0g_{9/2}1p_{3/2}1p_{3/2}$ | 0.185 | -0.246 | -0.214 | 0.048 |
| $^{82}\mathrm{Se} \rightarrow {}^{82}\mathrm{Kr}$ | | | | | |
| | $0g_{9/2}0g_{9/2}0f_{5/2}0f_{5/2}$ | 0.157 | -0.337 | -1.096 | 0.335 |
| | $0g_{9/2}0g_{9/2}1p_{3/2}1p_{3/2}$ | 0.189 | -0.263 | -0.219 | 0.058 |
| $^{130}\mathrm{Te} \rightarrow {}^{130}\mathrm{Xe}$ | | | | | |
| | $0h_{11/2}0h_{11/2}0g_{7/2}0g_{7/2}$ | 0.171 | -0.202 | -0.948 | 0.297 |
| $^{136}\mathrm{Xe} \rightarrow {}^{136}\mathrm{Ba}$ | | | | | |
| | $0h_{11/2}0h_{11/2}0g_{7/2}0g_{7/2}$ | 0.178 | -0.264 | -0.997 | 0.381 |

As we expect:

- 3-body (a) diagram reduces the contribution of the 2-body ladder diagram
- 3-body (b) diagram reduces the contribution of the 2-body 3p-1h (core polarization) diagram

# The blocking effect

| Decay | | $M^{0\nu}_{GT}$ | $M^{0\nu}_{F}$ | $M^{0\nu}$ | |
|---|---|---|---|---|---|
| $^{48}Ca \rightarrow ^{48}Ti$ | | | | | |
| | 3rd order | 0.25 | -0.13 | 0.33 | |
| | 3rd order + blocking | 0.27 | -0.12 | 0.35 | 0% |
| $^{76}Ge \rightarrow ^{76}Se$ | | | | | |
| | 3rd order | 2.91 | -0.69 | 3.35 | |
| | 3rd order + blocking | 2.55 | -0.70 | 3.00 | 10% |
| $^{82}Se \rightarrow ^{82}Kr$ | | | | | |
| | 3rd order | 2.85 | -0.68 | 3.28 | |
| | 3rd order + blocking | 2.60 | -0.71 | 3.05 | 7% |
| $^{130}Te \rightarrow ^{130}Xe$ | | | | | |
| | 3rd order | 2.65 | -0.72 | 3.11 | |
| | 3rd order + blocking | 2.83 | -0.79 | 3.33 | 7% |
| $^{136}Xe \rightarrow ^{136}Ba$ | | | | | |
| | 3rd order | 1.93 | -0.53 | 2.26 | |
| | 3rd order + blocking | 2.06 | -0.58 | 2.43 | 8% |

Obviously, the blocking effect is stronger for decays involving nuclei with a larger number of valence nucleons

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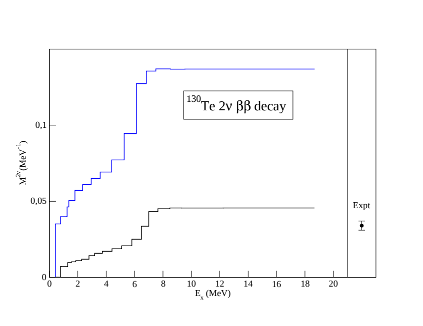
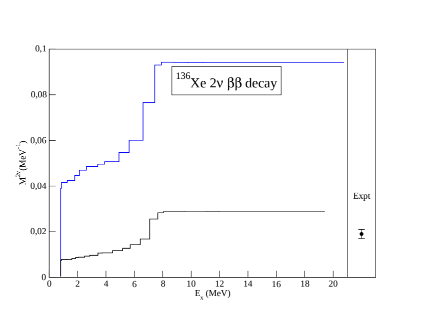
# Summing intermediate states

Lanczos strength function method

*E. Caurier, et al, Rev Mod. Phys.* ***77*** *, 427 (2005)*

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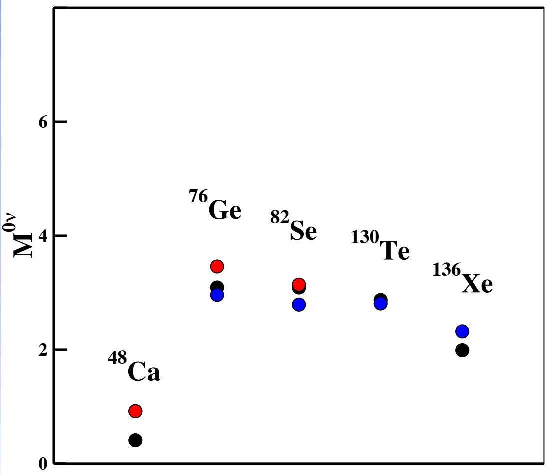
# Shell model calculations of $M^{0\nu}$: no SRC




- Blue dots: Madrid-Strasbourg group, bare $0\nu\beta\beta$ operator
- Red dots: Horoi *et al.*, bare $0\nu\beta\beta$ operator
- Black dots: RSM, bare $0\nu\beta\beta$ operator

# The Neutrinoless Double Beta Decay Matrix Element

The form factors $h_\alpha(q^2)$ are

$$
\begin{aligned}
h_F(q^2) &= g_V^2(q^2) \\
h_{GT}(q^2) &= \frac{g_A^2(q^2)}{g_A^2}\left[1-\frac{2}{3}\frac{q^2}{q^2+m_\pi^2}+\frac{1}{3}\left(\frac{q^2}{q^2+m_\pi^2}\right)^2\right] \\
&\quad +\frac{2}{3}\frac{g_M^2(q^2)}{g_A^2}\frac{q^2}{4m_p^2}\,, \\
h_T(q^2) &= \frac{g_A^2(q^2)}{g_A^2}\left[\frac{2}{3}\frac{q^2}{q^2+m_\pi^2}-\frac{1}{3}\left(\frac{q^2}{q^2+m_\pi^2}\right)^2\right] \\
&\quad +\frac{1}{3}\frac{g_M^2(q^2)}{g_A^2}\frac{q^2}{4m_p^2}\,. \qquad (2)
\end{aligned}
$$

# The Neutrinoless Double Beta Decay Matrix Element

The $g_{V,A,M}$ form factors can include nucleon finite size effects, which in the dipole approximation are given by

$$g_V(q^2) = \frac{g_V}{\left(1 + q^2/\Lambda_V^2\right)^2},$$
$$g_M(q^2) = (\mu_p - \mu_n) g_V(q^2),$$
$$g_A(q^2) = \frac{g_A}{\left(1 + q^2/\Lambda_A^2\right)^2}. \quad (3)$$

Here $g_V = 1$, $g_A = 1.25$, $(\mu_p - \mu_n) = 3.7$, $\Lambda_V = 850\ MeV$, and $\Lambda_A = 1086\ MeV$.