Magnetic Susceptibility of neutron stars from QMC

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In collaboration with:

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XVII Conference on Theoretical Nuclear Physics in Italy (TNPI)
Cortona - October 9th, 2019

Outline

(i) Overview.

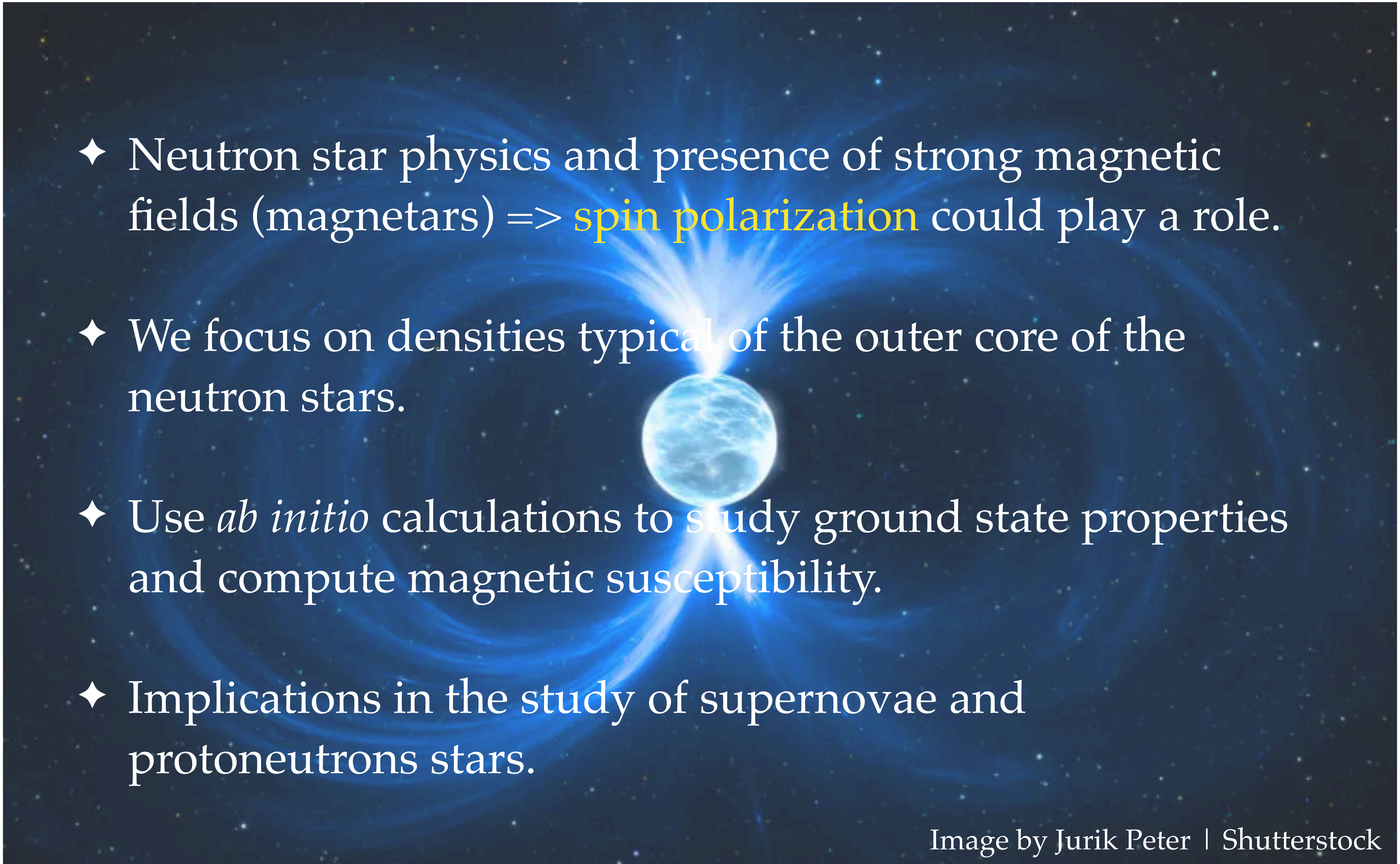
(ii) Introduction.

(iii) Previous results on the magnetic susceptibility.

(iv) Magnetic susceptibility.

(v) Conclusions.

Overview

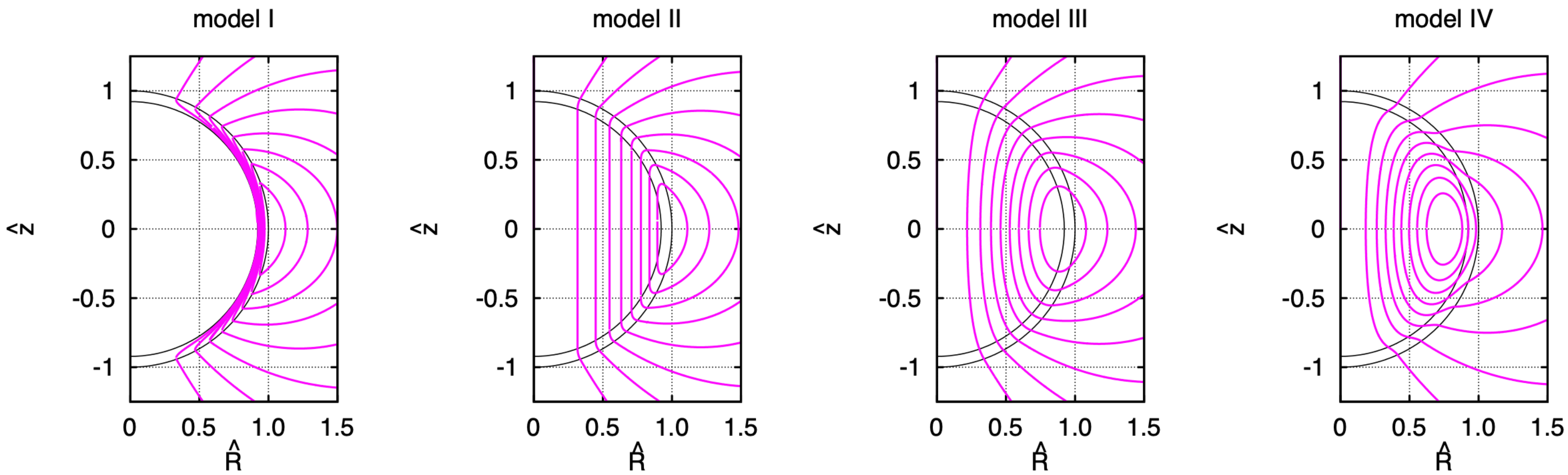


- Neutron star physics and presence of strong magnetic fields (magnetars) => spin polarization could play a role.
- We focus on densities typical of the outer core of the neutron stars.
- Use *ab initio* calculations to study ground state properties and compute magnetic susceptibility.
- Implications in the study of supernovae and protoneutrons stars.

Image by Jurik Peter | Shutterstock

Why are we doing this?

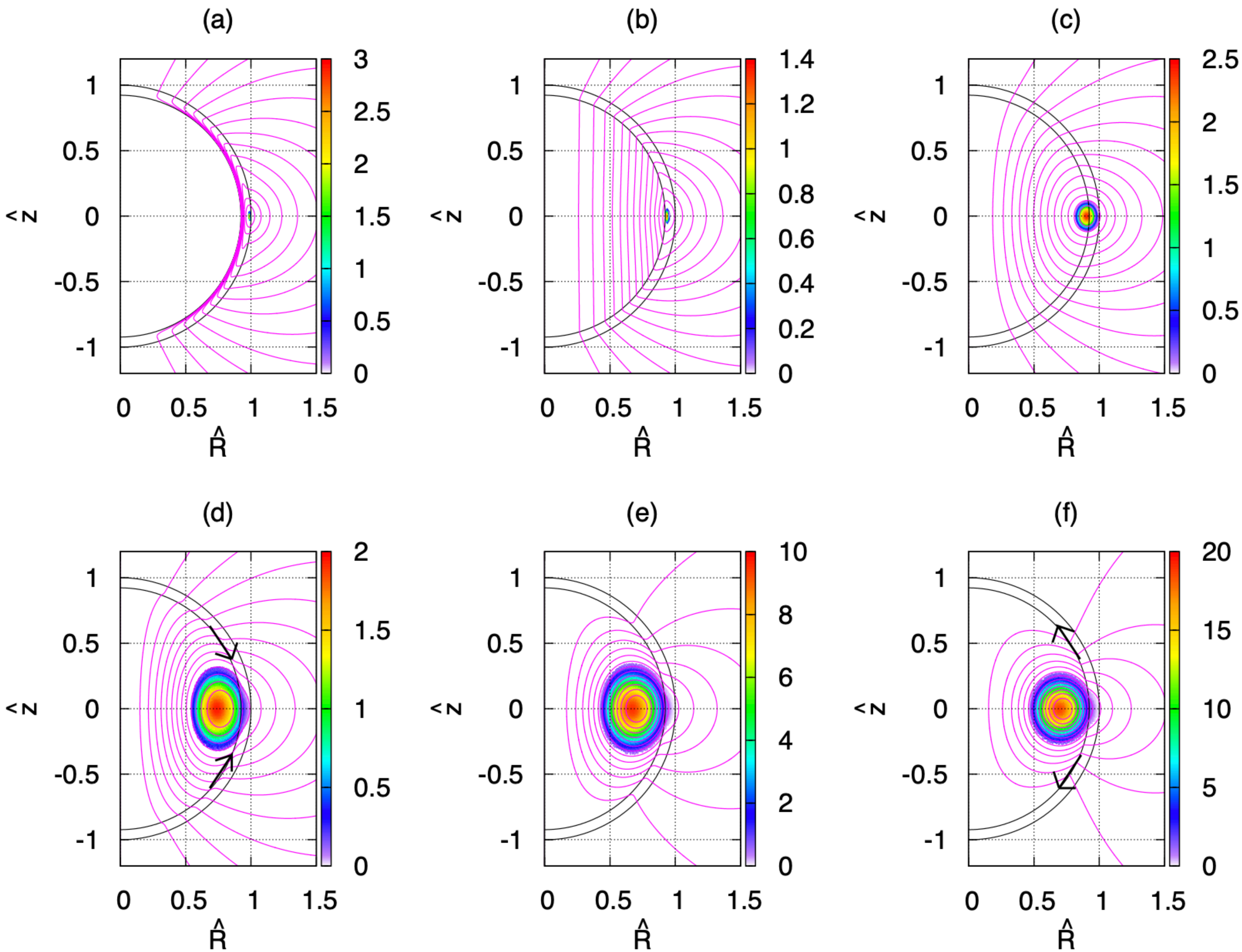
- Magnetars ~ 10% of young NS, with high magnetic field observed on the surface (~10^{14}/10^{15} G) => huge inferred internal magnetic field.



K. Fujisawa *et al.*, Mon. Not. **445**, 2777–2793 (2014)

- Calculate magnetic susceptibility to include in simulation of NS mergers and supernovae explosions.

Why are we doing this?



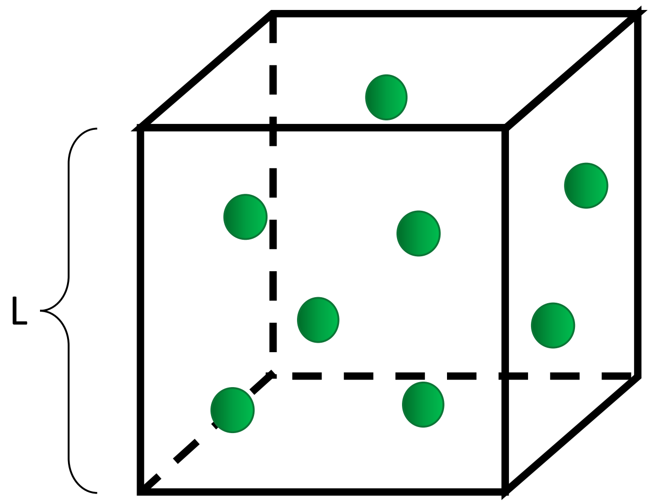
Nuclear Hamiltonian

$$H = T + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} + \cdots$$

non-relativistic system with effective nucleon-nucleon (NN) force and three nucleon interaction (NNN).

Infinite matter <=> N particles in a box with PBC

Approximation: only neutrons.



Magnetic Susceptibility

$$H = H_0 - \sum_i \vec{\sigma}_i \cdot \vec{b}$$

where we applied an external magnetic field $\vec{b} = \mu \vec{B}$ to the system.

$$\chi = -\rho\mu^2 \left. \frac{\partial^2 E_0(b)}{\partial b^2} \right|_{b=0}$$

Following approximation on S. Fantoni *et al.*, PRL **87**, 181101 (2001).

Magnetic Susceptibility

Using Pauli expansion in spin polarization: $\xi = - \left. \frac{\partial E_0(b)}{\partial b} \right|_{b=0}$

$$E(\xi) = E(0) - b\xi + \frac{1}{2}\xi^2 E''(0)$$

Minimizing the energy with respect to the spin polarization we obtain:

$$\chi = \mu^2 \rho \frac{1}{E''(0)}$$

Magnetic Susceptibility

How to compute $E''(0)$?

Using chain rule (J_z is the spin asymmetry, i.e. $N_\uparrow - N_\downarrow$):

$$E''(0) = \left[\frac{\partial \xi}{\partial J_z}\right]^{-2} \left\{ \frac{\partial^2 E_0}{\partial J_z^2} - \frac{\partial E_0}{\partial J_z} \left[\frac{\partial \xi}{\partial J_z}\right]^{-1} \frac{\partial^2 \xi}{\partial J_z^2} \right\}$$

Magnetic Susceptibility

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Magnetic Susceptibility

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$$E''(0) = \left[\frac{\partial \xi}{\partial J_z}\right]^{-2} \frac{\partial^2 E_0}{\partial J_z^2}$$

Magnetic Susceptibility

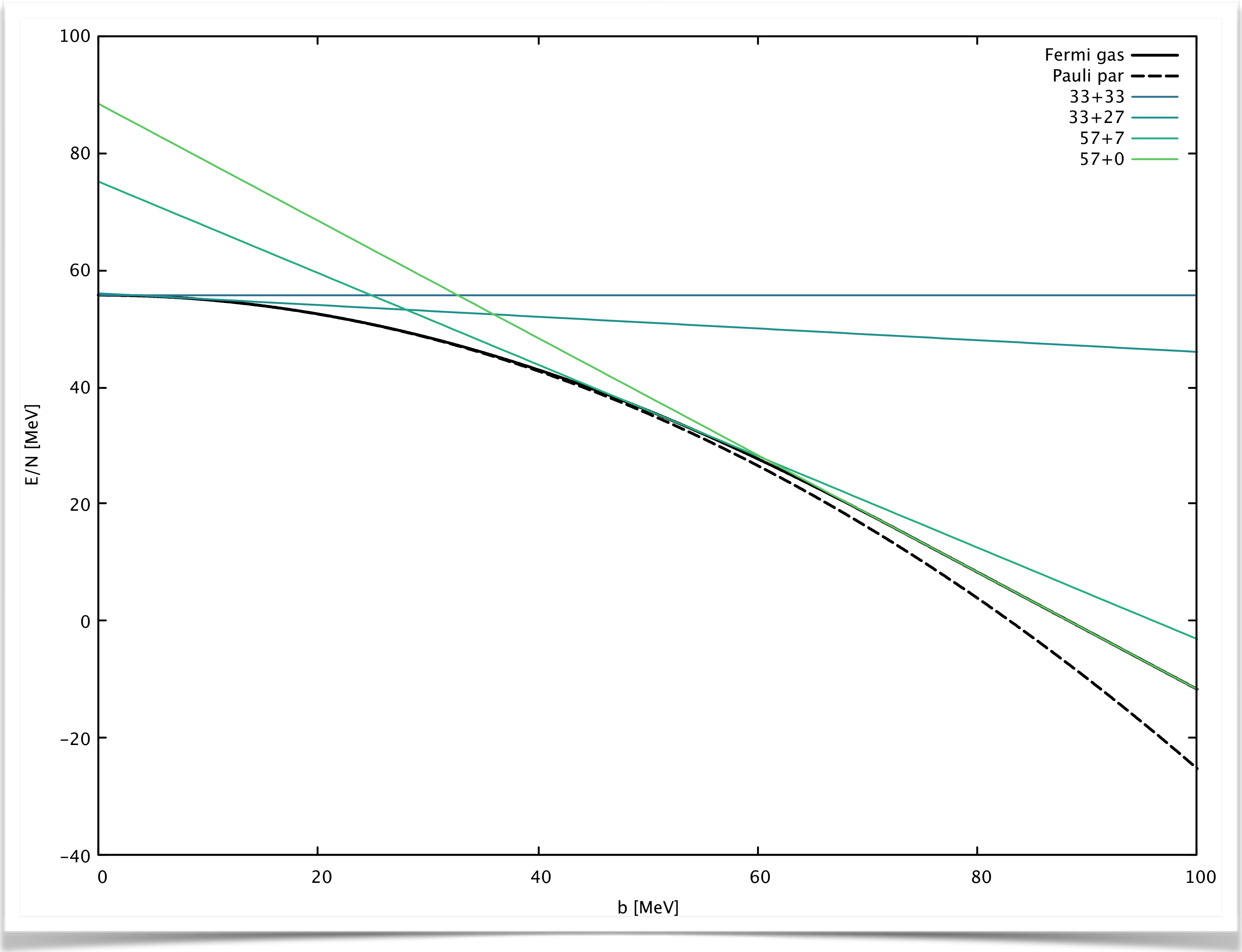
Approximate expressions (exact for infinite system with low b and J_z):

$$\frac{\partial \xi}{\partial J_z} \approx \frac{E_0\left(J_z = J_{z0}, b = 0\right) - E_0\left(J_z = J_{z0}, b = b_0\right)}{J_{z0} b_0}$$

$$\frac{\partial^2 E_0}{\partial J_z^2} \approx 2\frac{E_0\left(J_z = J_{z0}, b = 0\right) - E_0\left(J_z = 0, b = 0\right)}{J_{z0}^2}$$

Compute energies with AFDMC.

S. Fantoni *et al.*, PRL **87**, 181101 (2001) (PBC)



Magnetic Susceptibility

This approximation is reasonable under these assumptions:

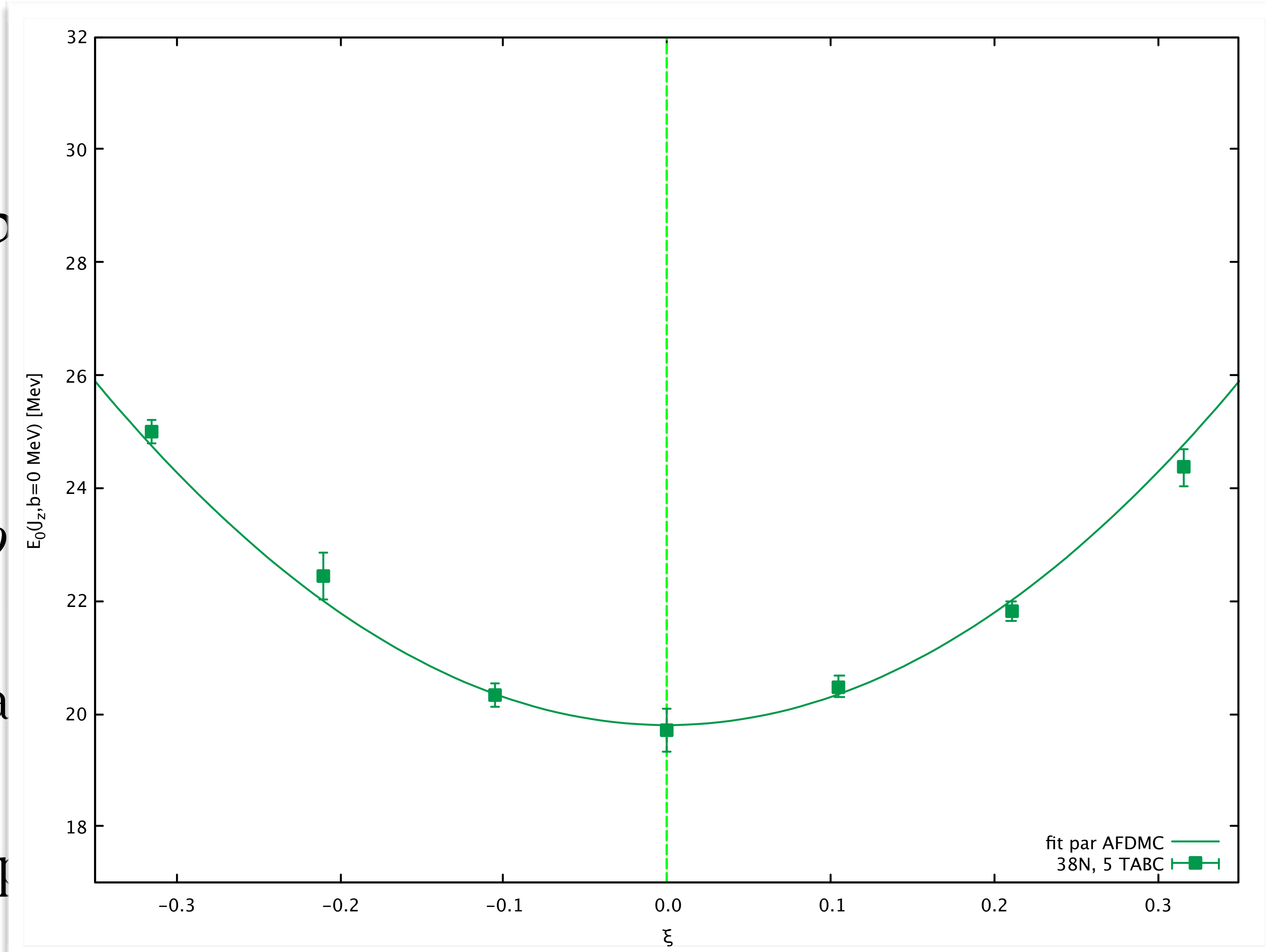
(i) for $b = 0$, $E_0(J_z, b)$ is quadratic in J_z

(ii) for a fixed J_z, $E_0(J_z, b)$ is linear in b

(iii) the polarization is linear in J_z

Magnetic Susceptibility

This app ions:



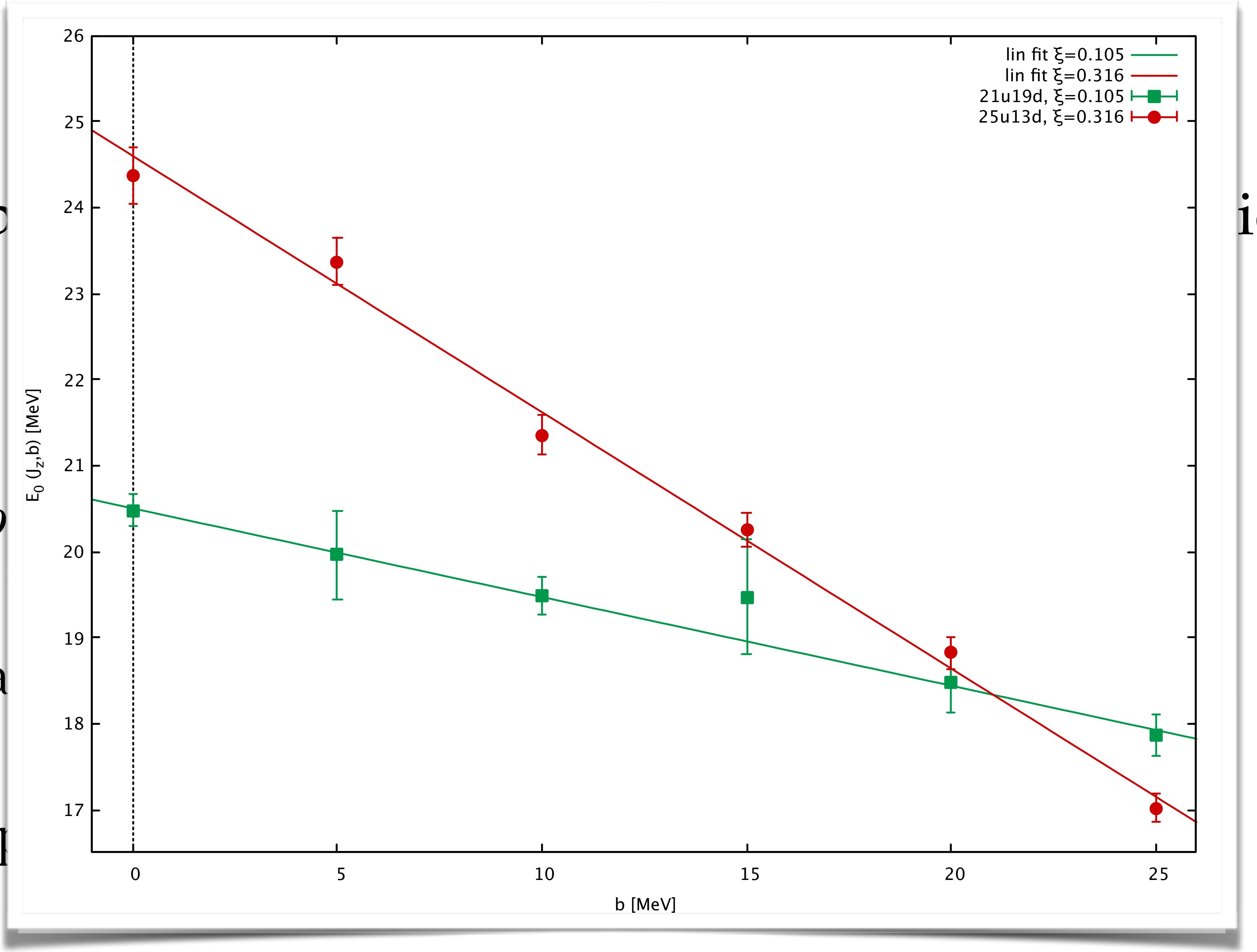
(i) for b

(ii) for a

(iii) the p

Magnetic Susceptibility

This app… …ions:



(i) for b…

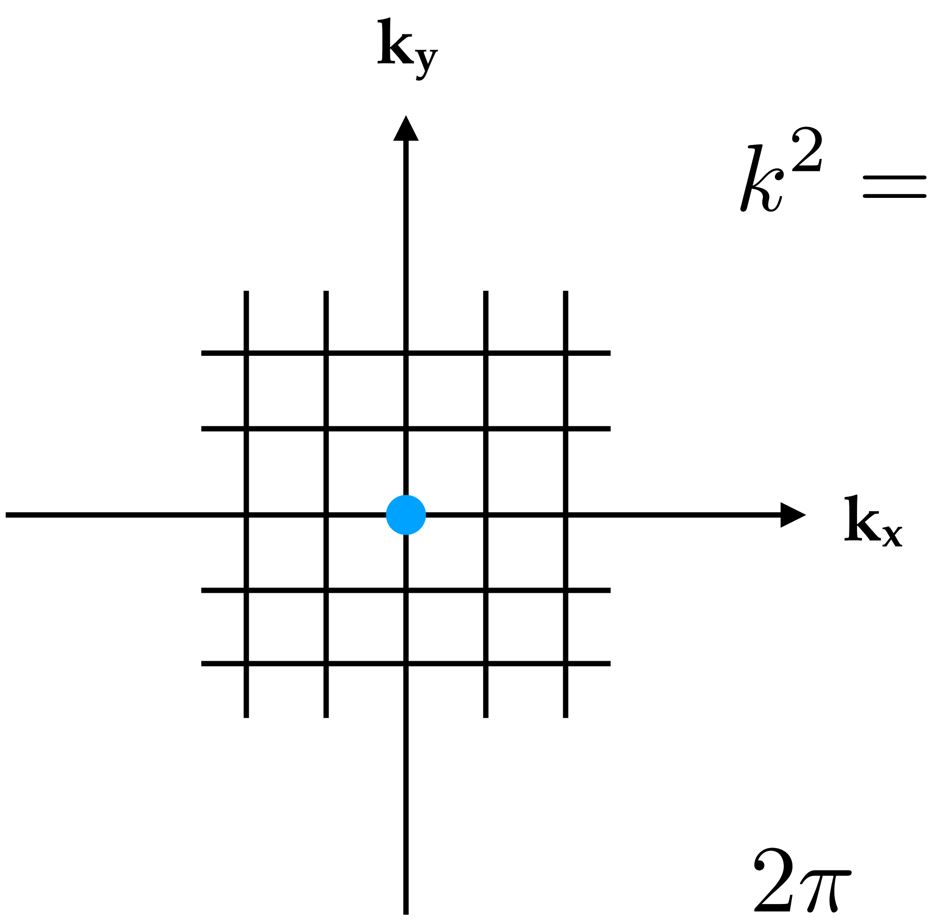
(ii) for a…

(iii) the …

Beyond PBC: TABC

In collaboration with:
✦ Diego Lonardoni @MSU/LANL

2D



$$k^2 = 0$$

$$a = \frac{2\pi}{L}$$

Beyond PBC: TABC

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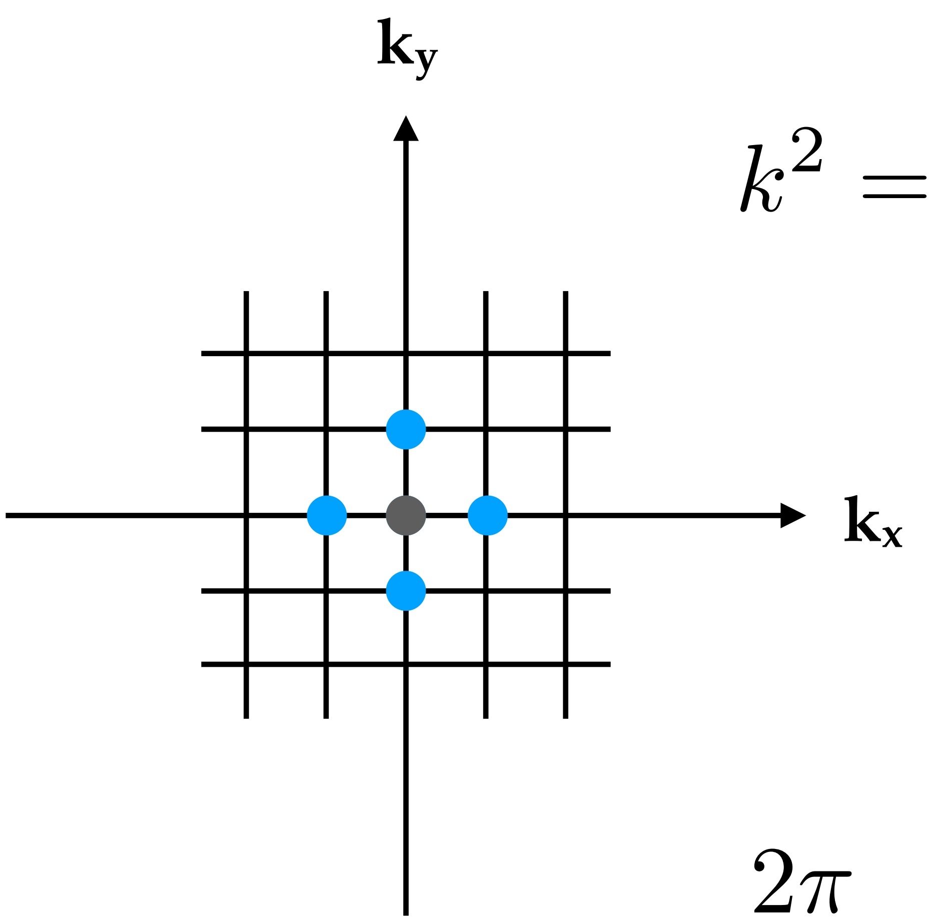
2D



$$k^2 = 1$$

$$a = \frac{2\pi}{L}$$

Beyond PBC: TABC

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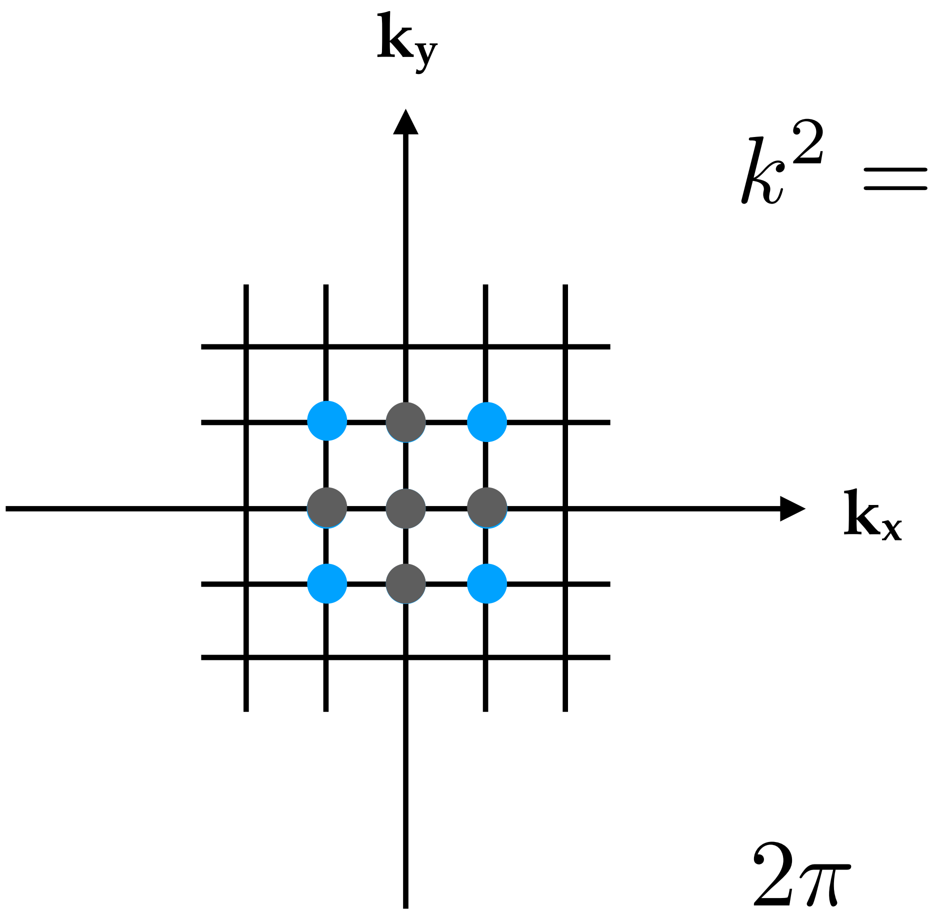
2D



$$k^2 = 2$$

$$a = \frac{2\pi}{L}$$

Beyond PBC: TABC

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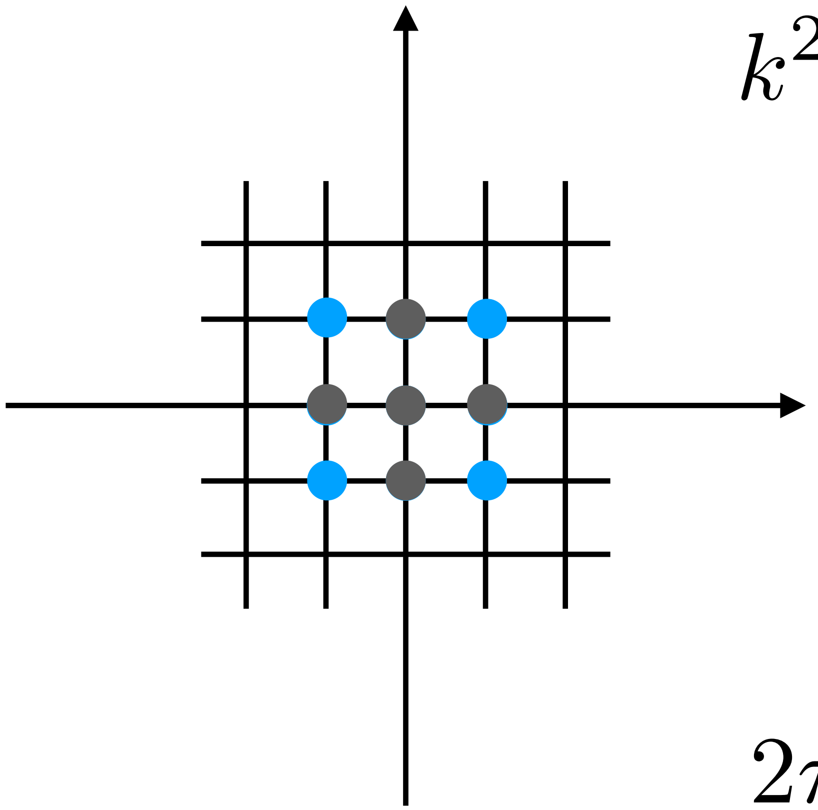
2D

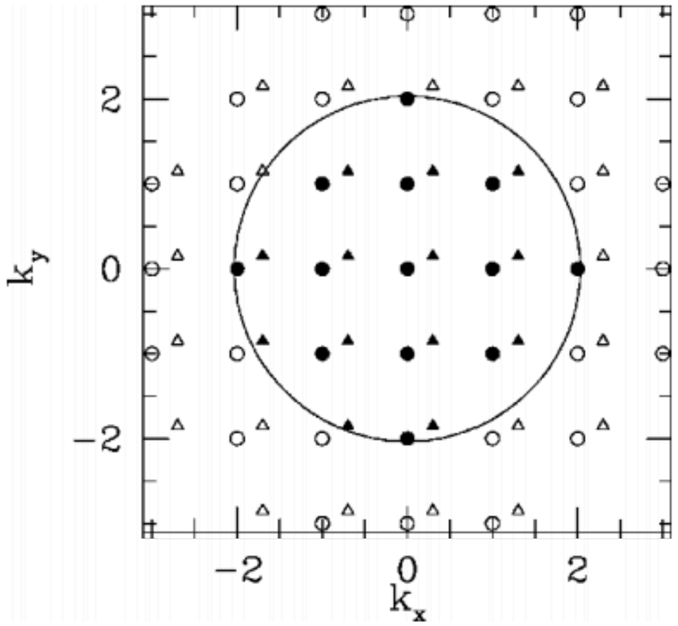


$$k^2 = 2$$

$$a = \frac{2\pi}{L}$$

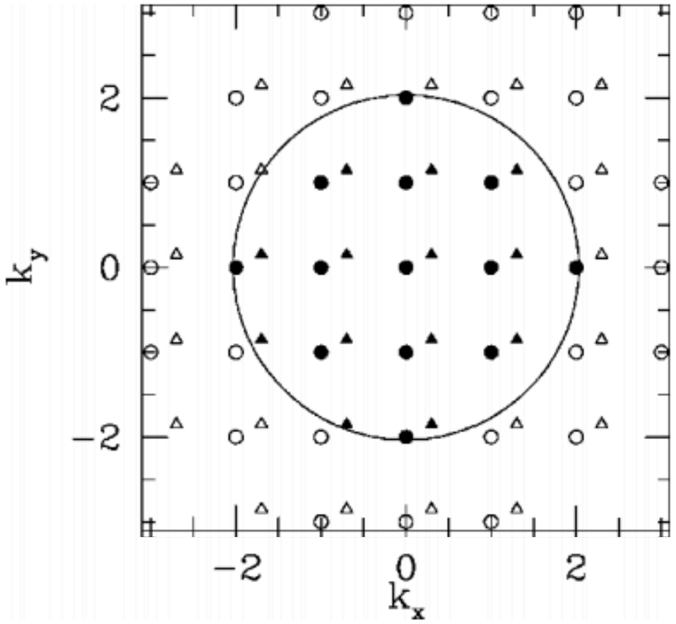
$$\theta_{TW} = \eta$$

$$\eta \in (-0.5, 0.5)$$



C. Lin *et al.*, Phys. Rev. E **64**, 016702 (2001)

Beyond PBC: TABC



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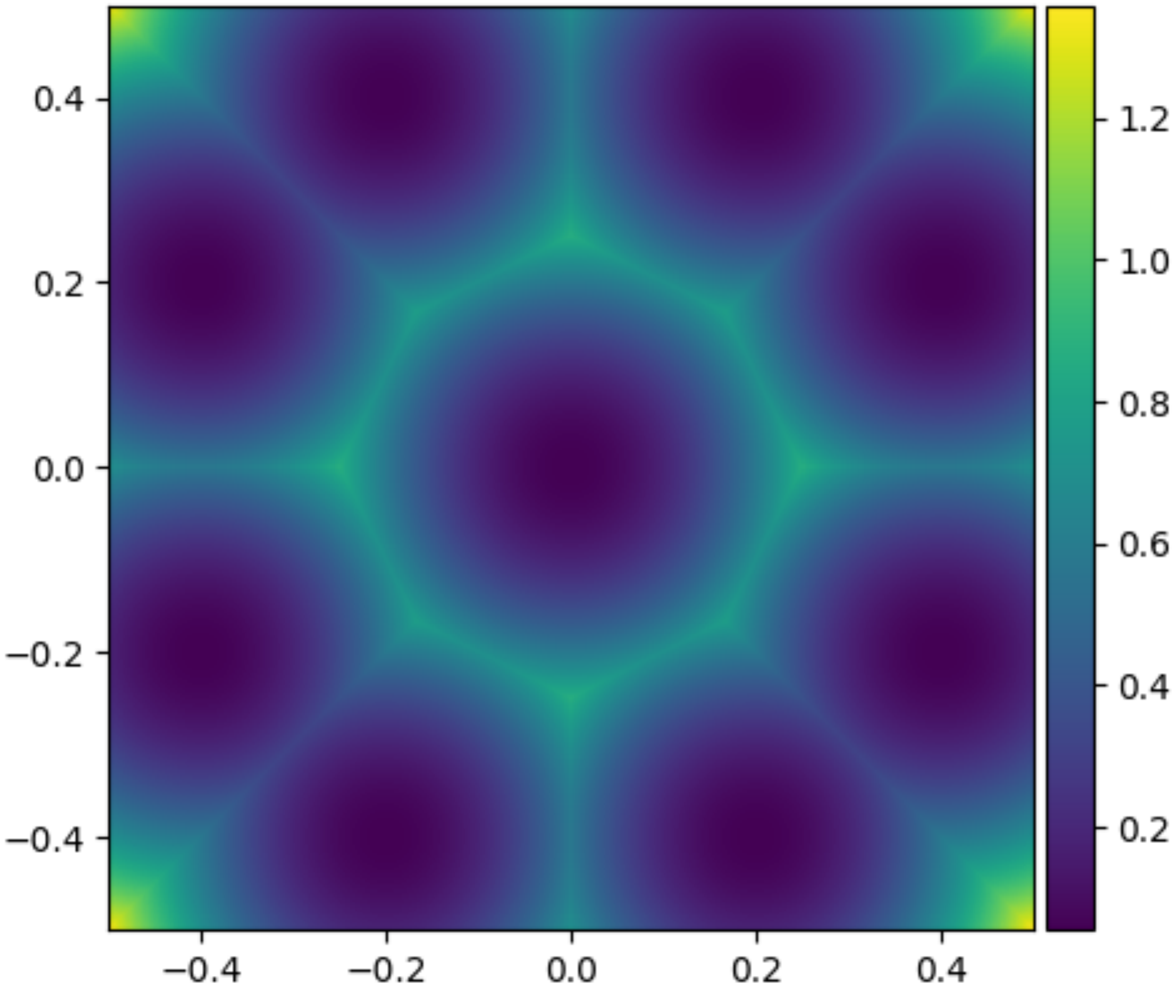
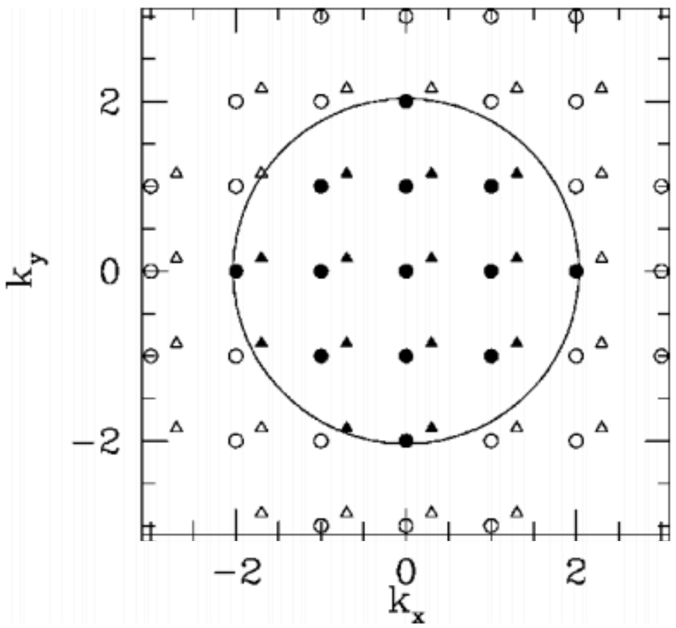
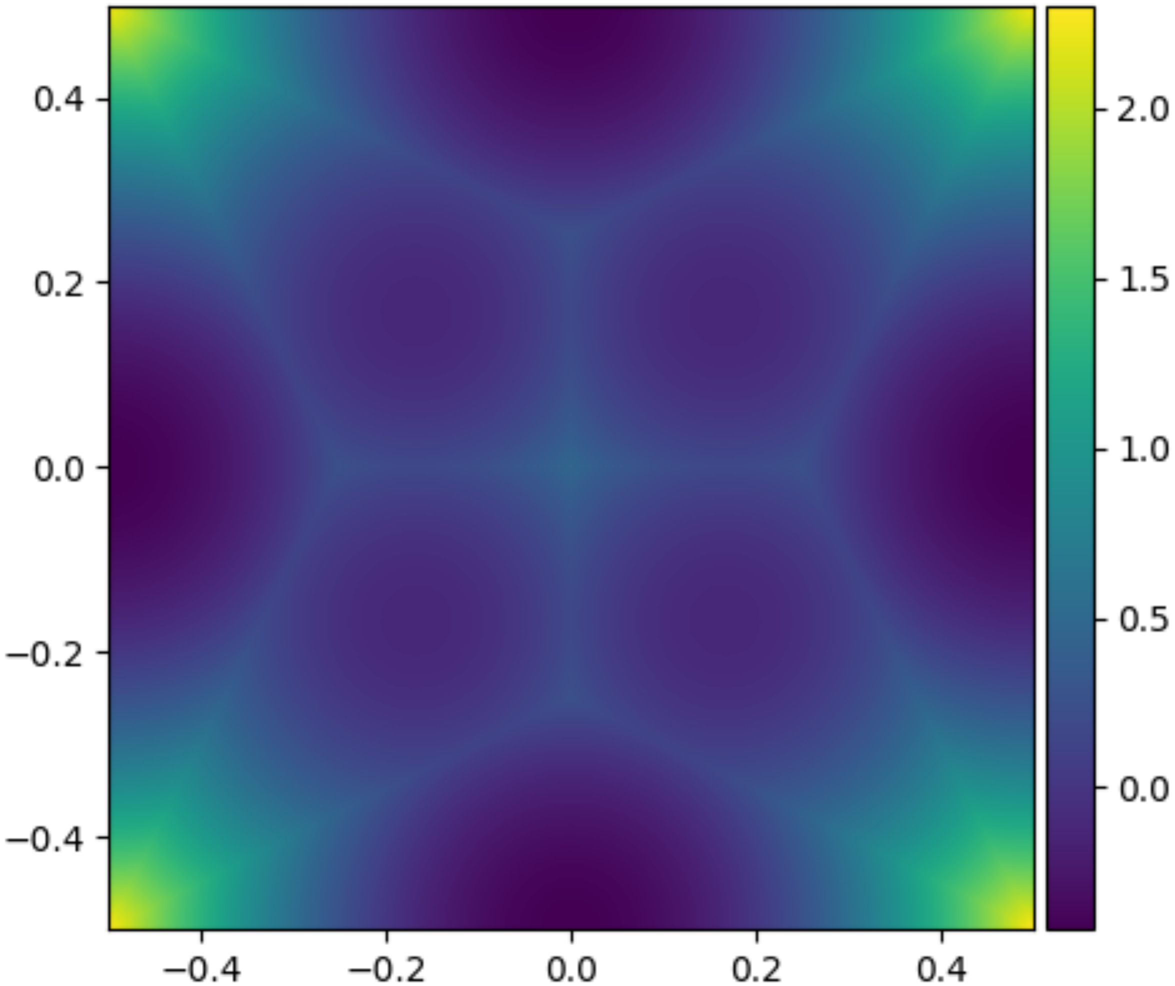


10 particles

Beyond PBC: TABC

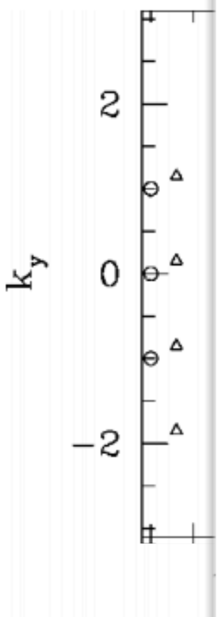


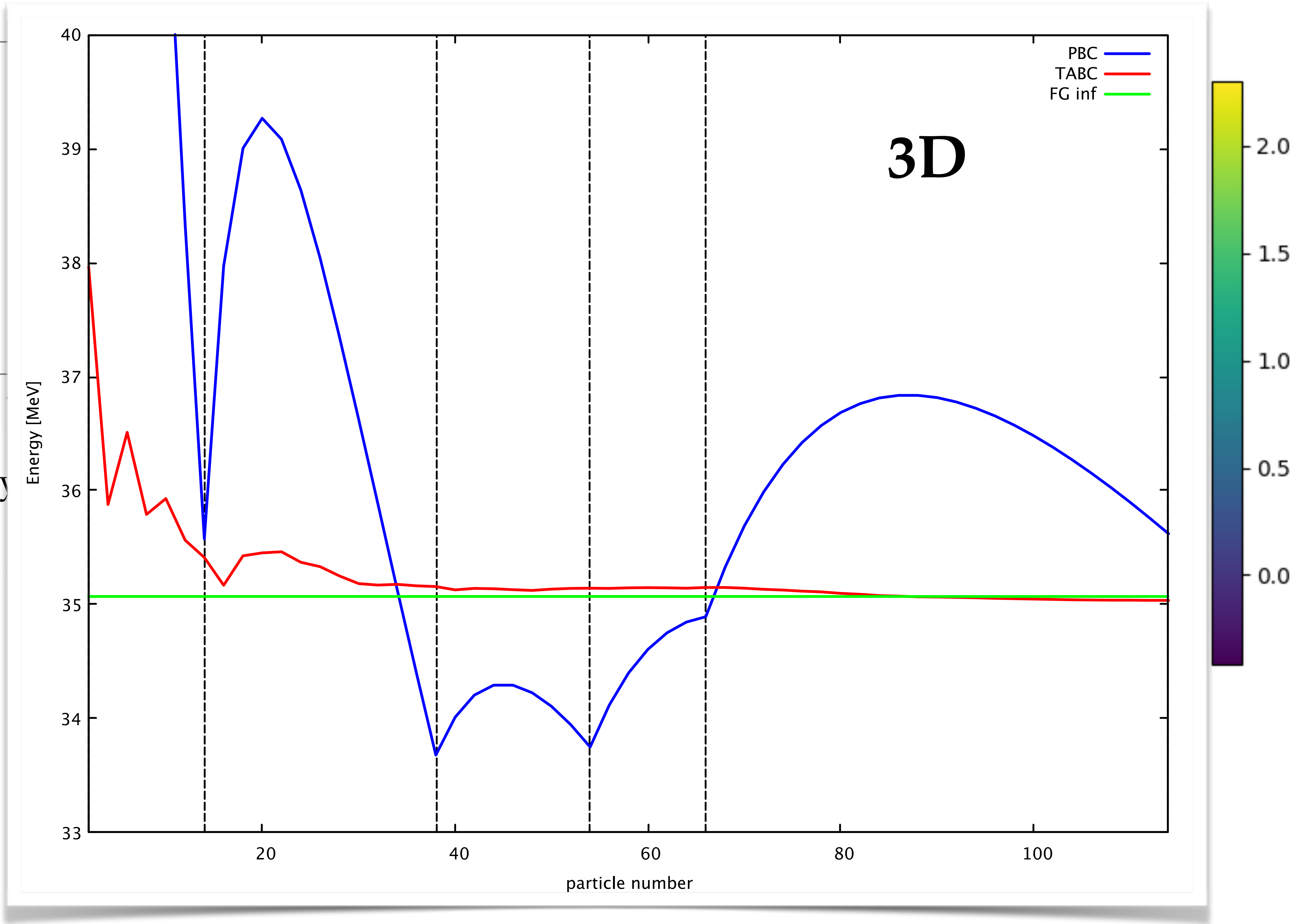
C. Lin *et al.*, Phys. Rev. E **64**, 016702 (2001)



16 particles

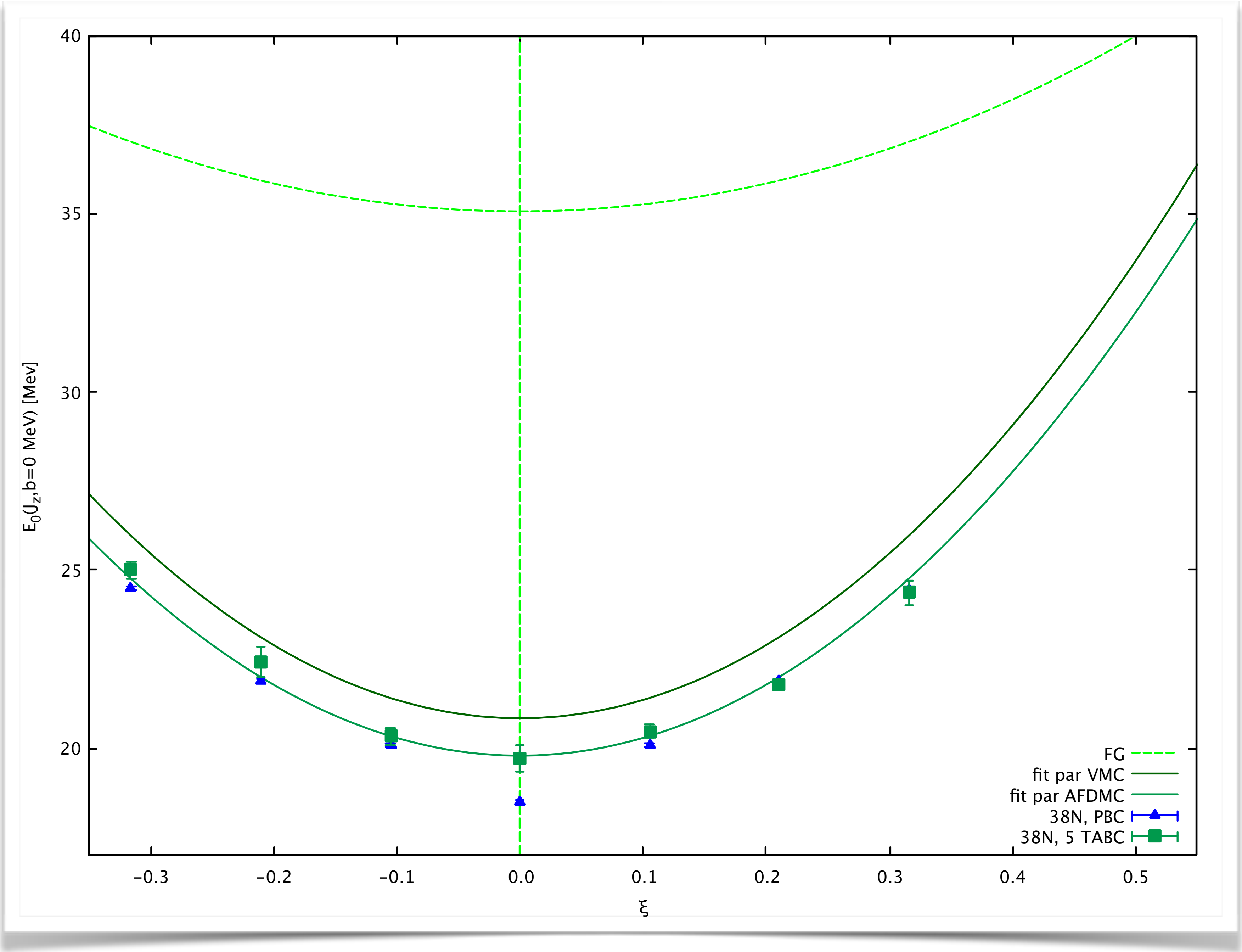
Beyond PBC: TABC



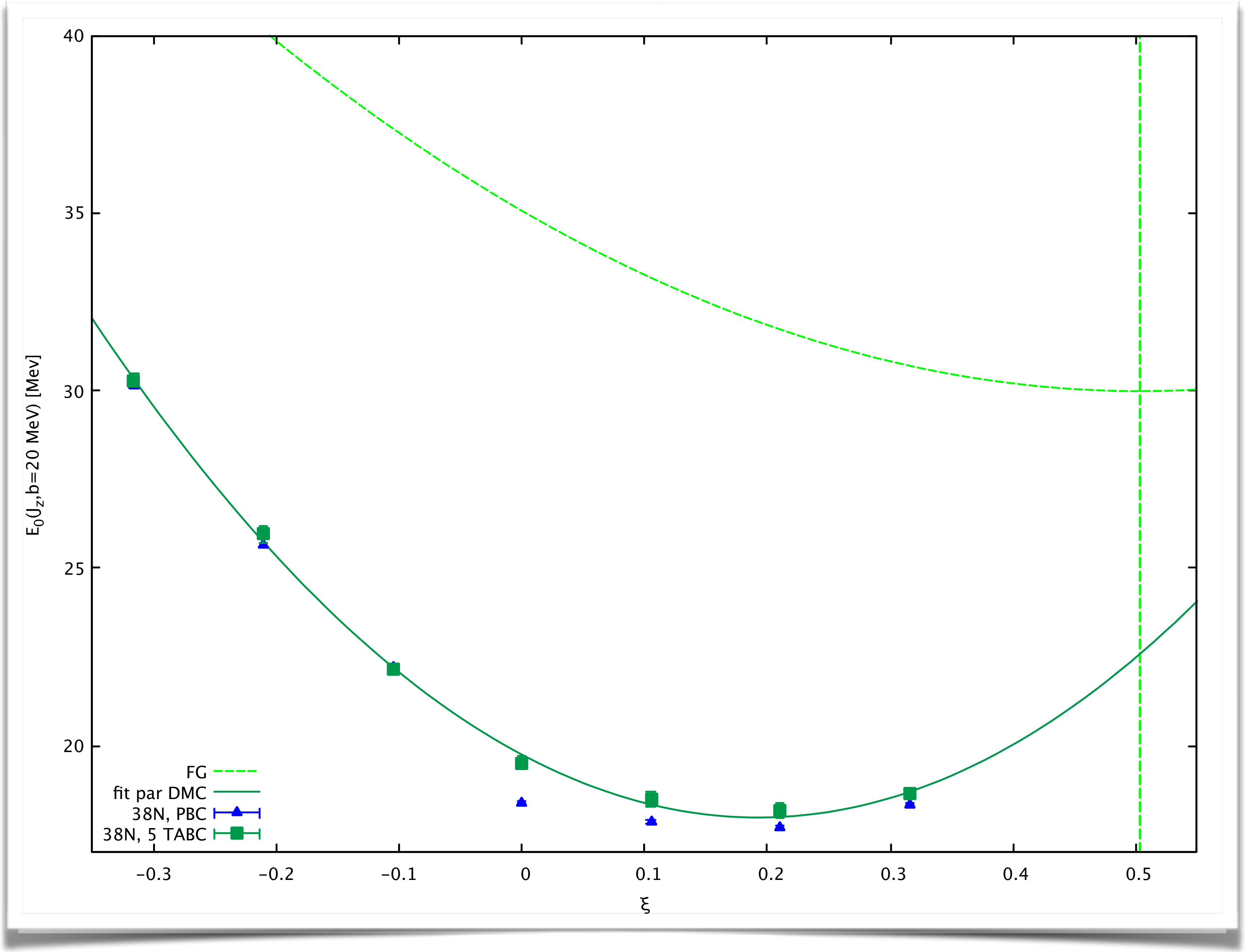


C. Lin *et al.*, Phy

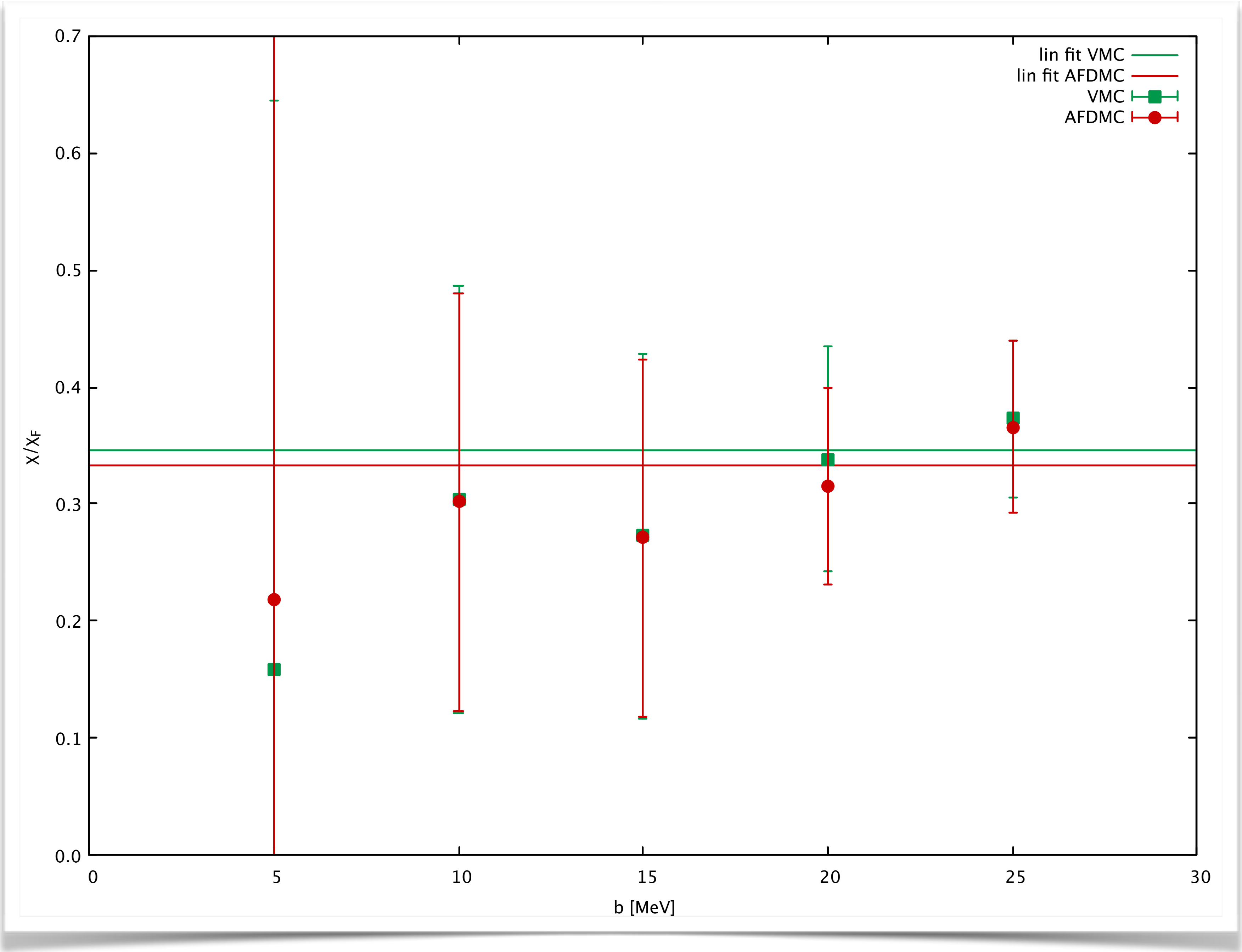
What can we do with TABC?



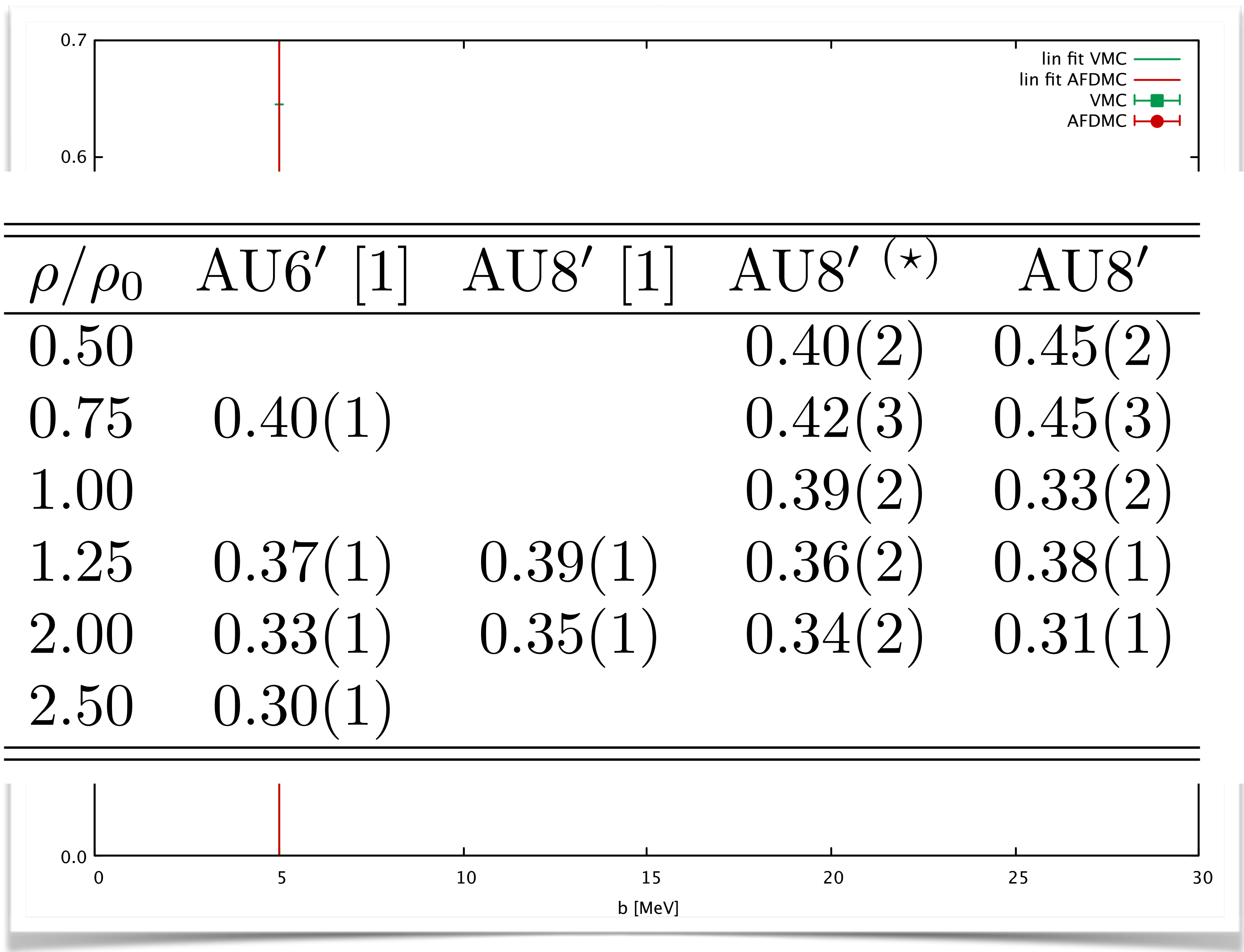
What can we do with TABC?



Results using approximation



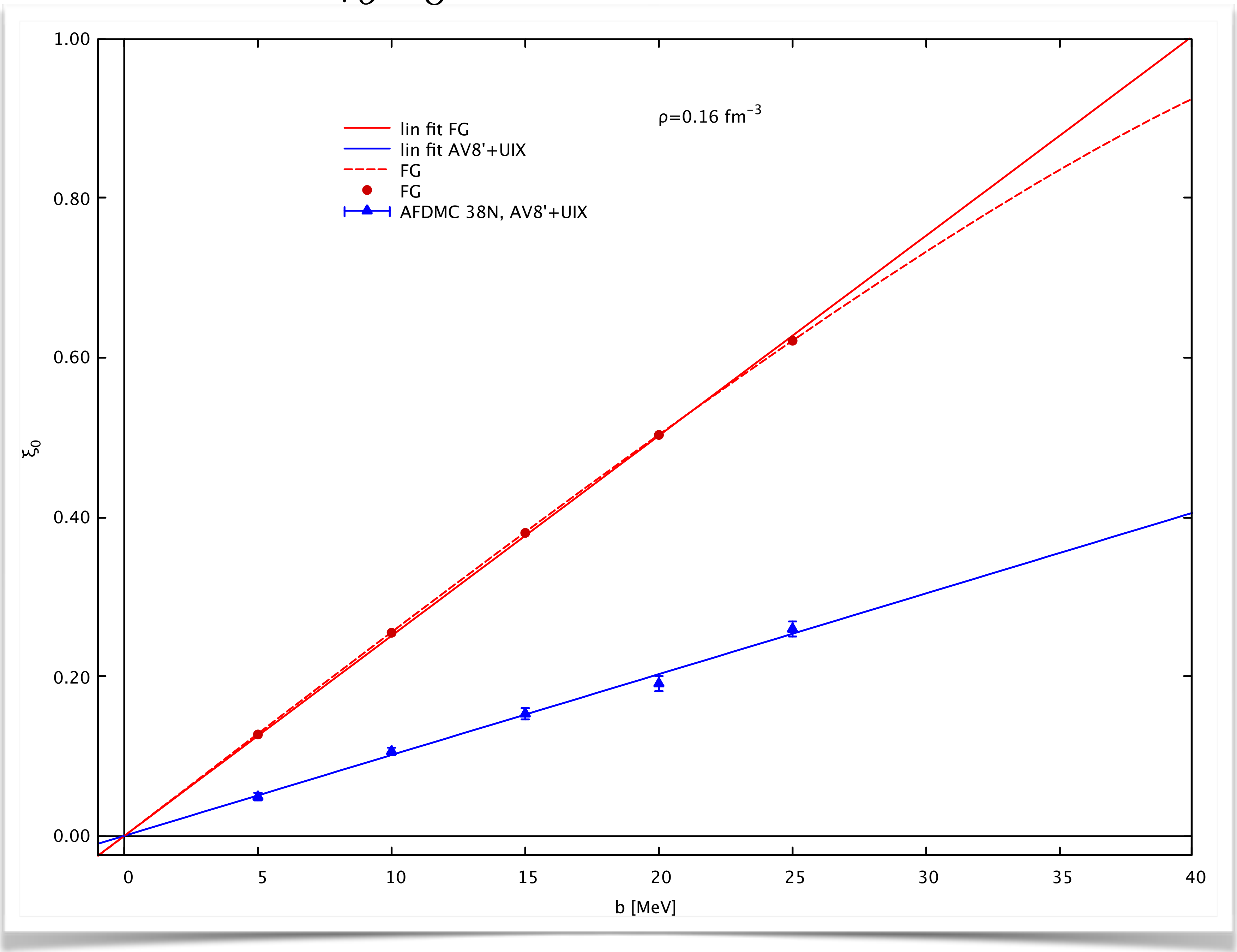
Results using approximation



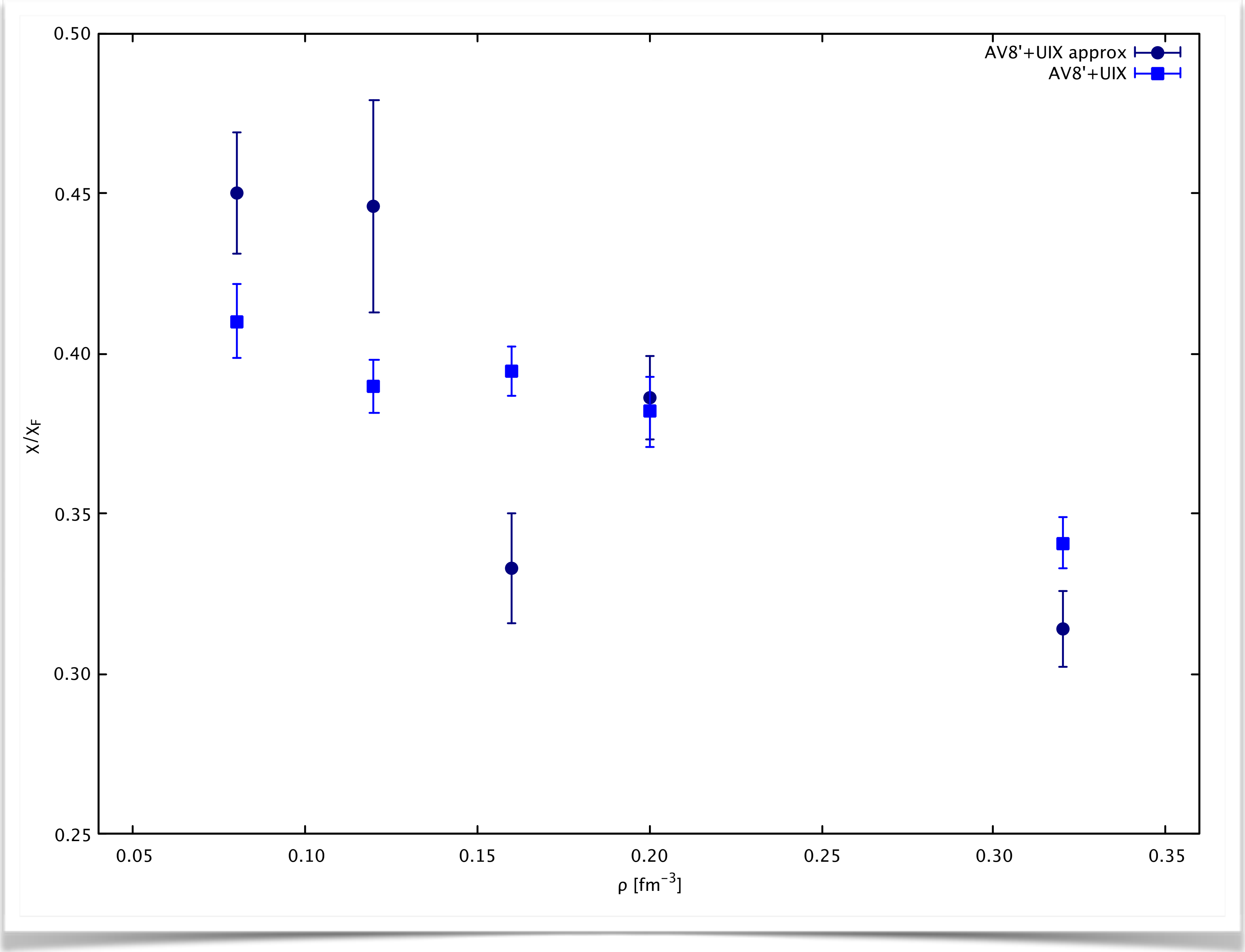
ρ/ρ_0	AU6′ [1]	AU8′ [1]	AU8′ $^{(\star)}$	AU8′
0.50			0.40(2)	0.45(2)
0.75	0.40(1)		0.42(3)	0.45(3)
1.00			0.39(2)	0.33(2)
1.25	0.37(1)	0.39(1)	0.36(2)	0.38(1)
2.00	0.33(1)	0.35(1)	0.34(2)	0.31(1)
2.50	0.30(1)			

We can do better

$$\chi = -\rho\mu^2 \frac{\partial^2 E_0(b)}{\partial b^2}\bigg|_{b=0} \xrightarrow{\xi = -\left.\frac{\partial E_0(b)}{\partial b}\right|_{b=0}} \chi = \rho\mu^2 \frac{\partial \xi}{\partial b}$$



Magnetic susceptibility ratio



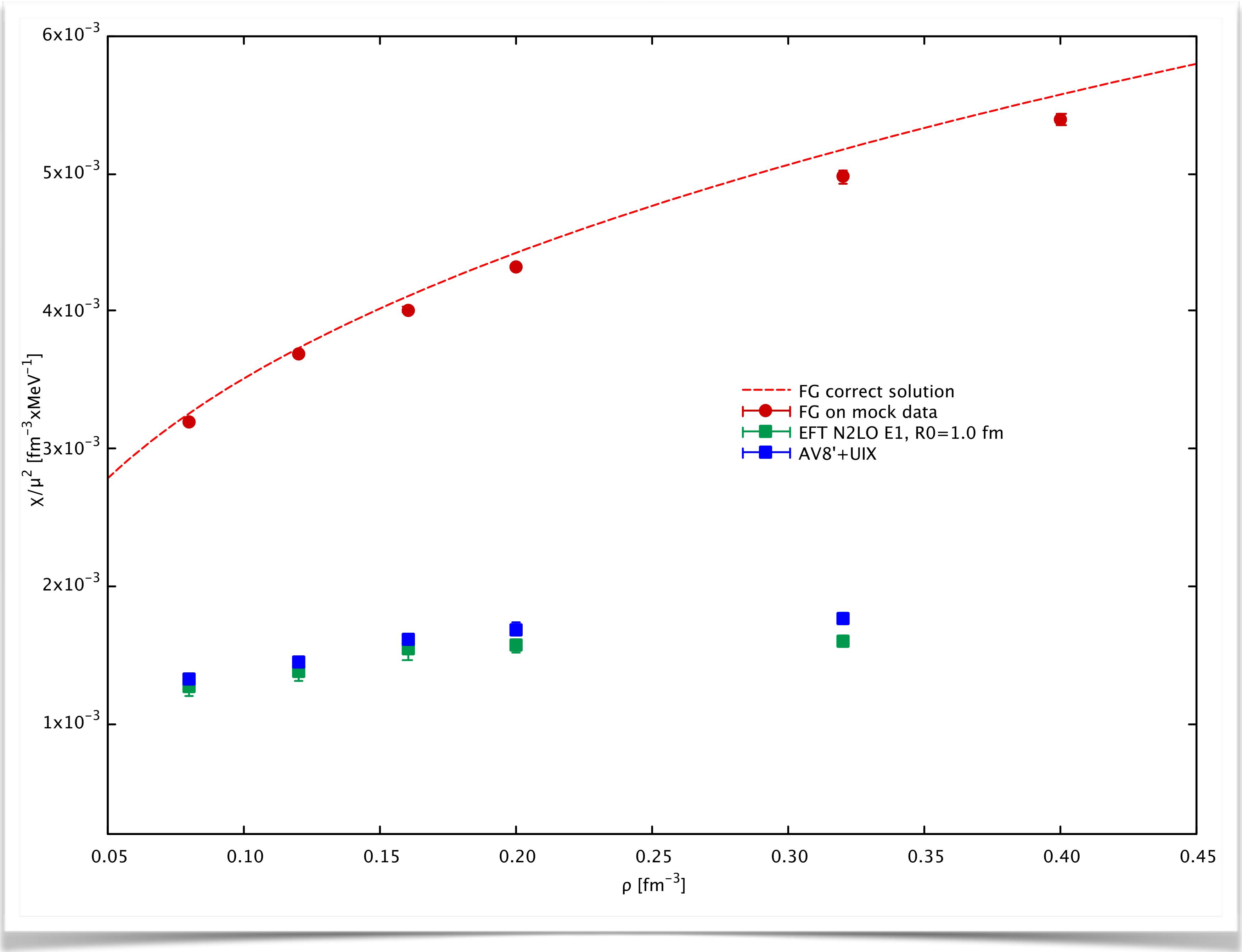
Nuclear Potentials

Two kinds of potential:

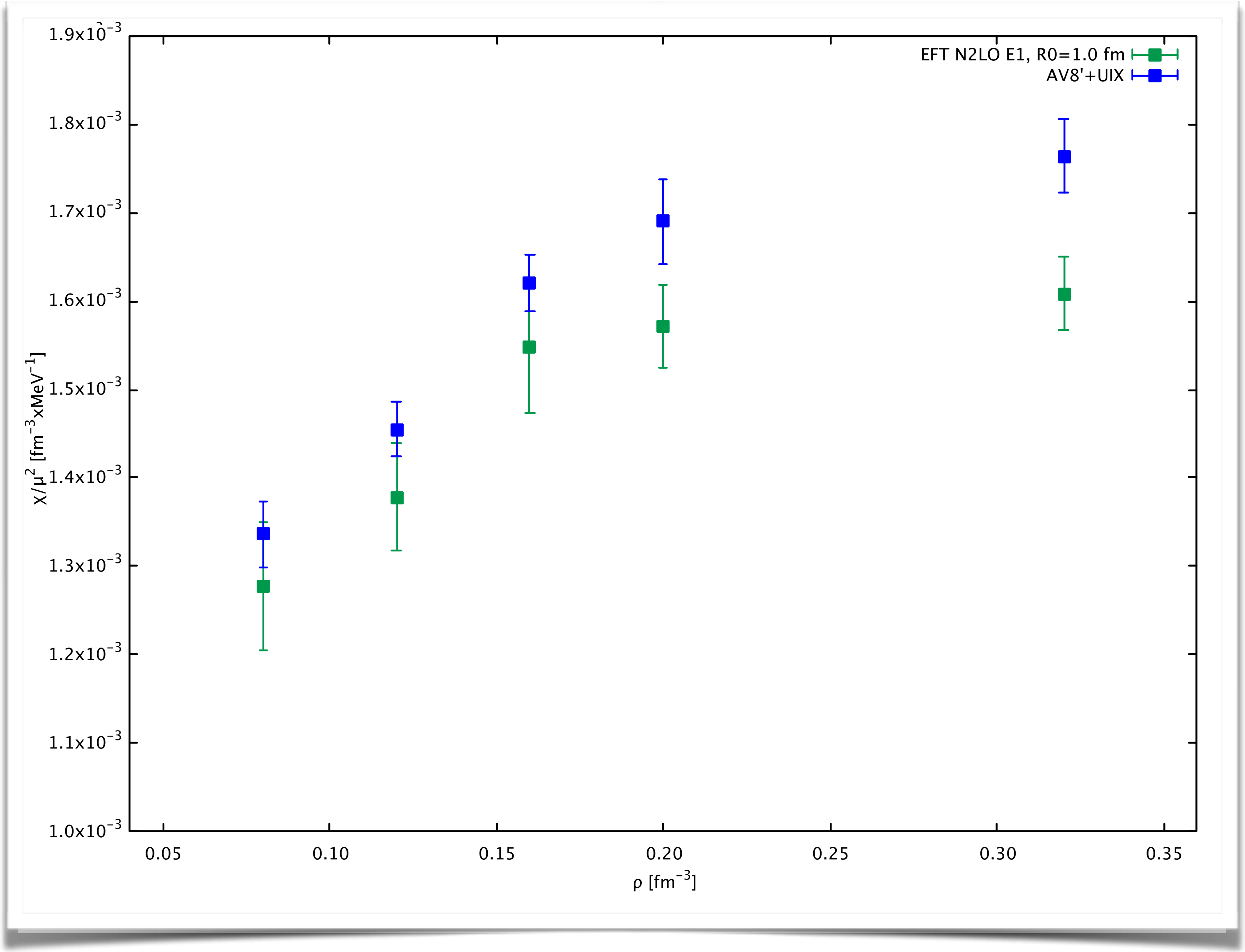
- Phenomenological AV8′+UIX.
- Local Chiral Effective Field Theory at N2LO [Lynn et al. PRL 116, 062501 (2016)].

Ground State energies computed by means of Auxiliary Field Diffusion Monte Carlo (AFDMC).

Magnetic susceptibility



Magnetic susceptibility



Simple model

Assuming an energy-density function of the kind:

$$E(\rho, \xi) = E_0(\rho) + \xi^2 \left(E_1(\rho) - E_0(\rho)\right)$$

And recalling that the magnetic susceptibility can be computed as:

$$\chi = \mu^2 \rho \frac{1}{E''(0)}$$

Simple model

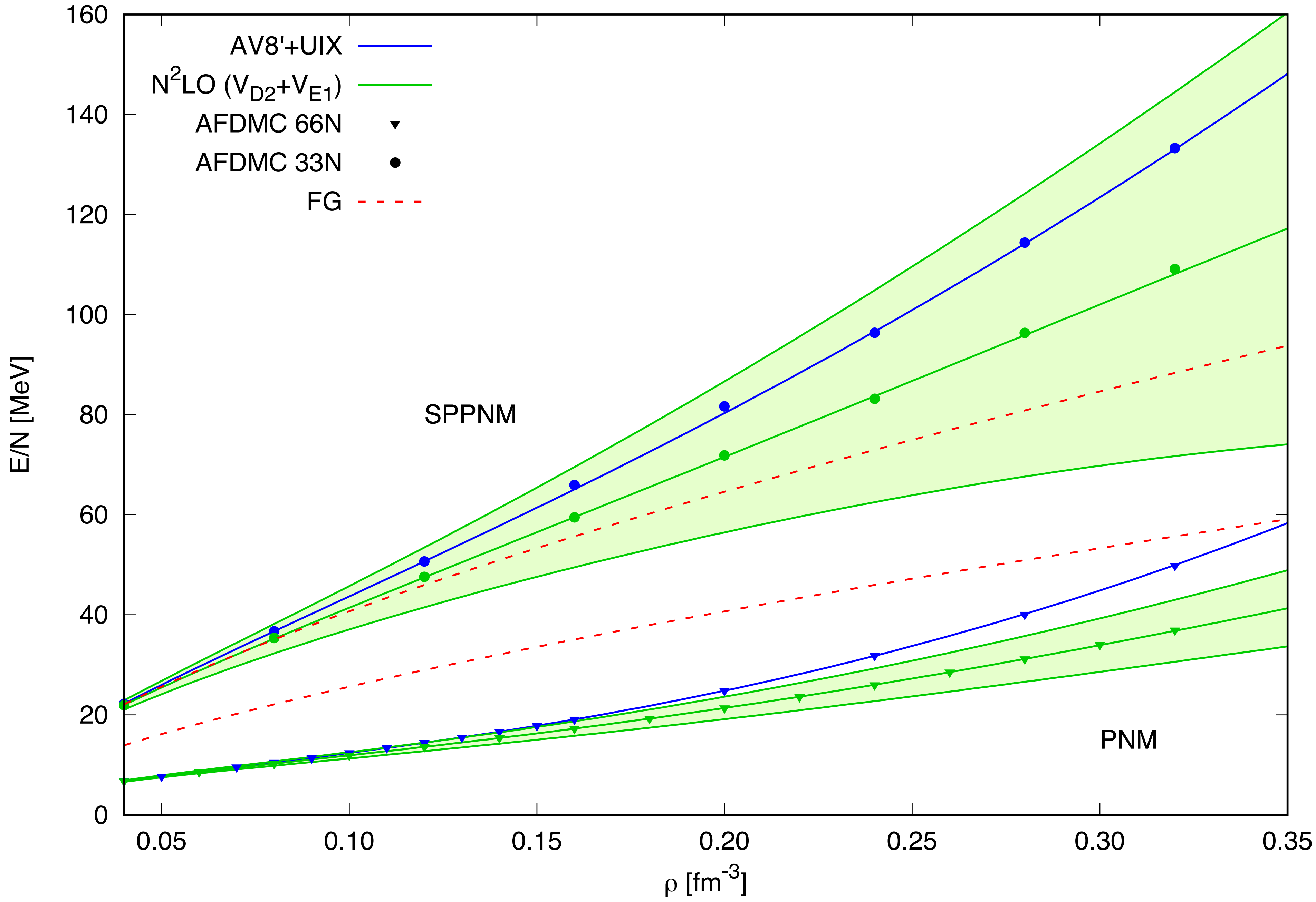
Assuming an energy-density function of the kind:

$$E(\rho, \xi) = E_0(\rho) + \xi^2 \underbrace{(E_1(\rho) - E_0(\rho))}_{SSE(\rho)}$$

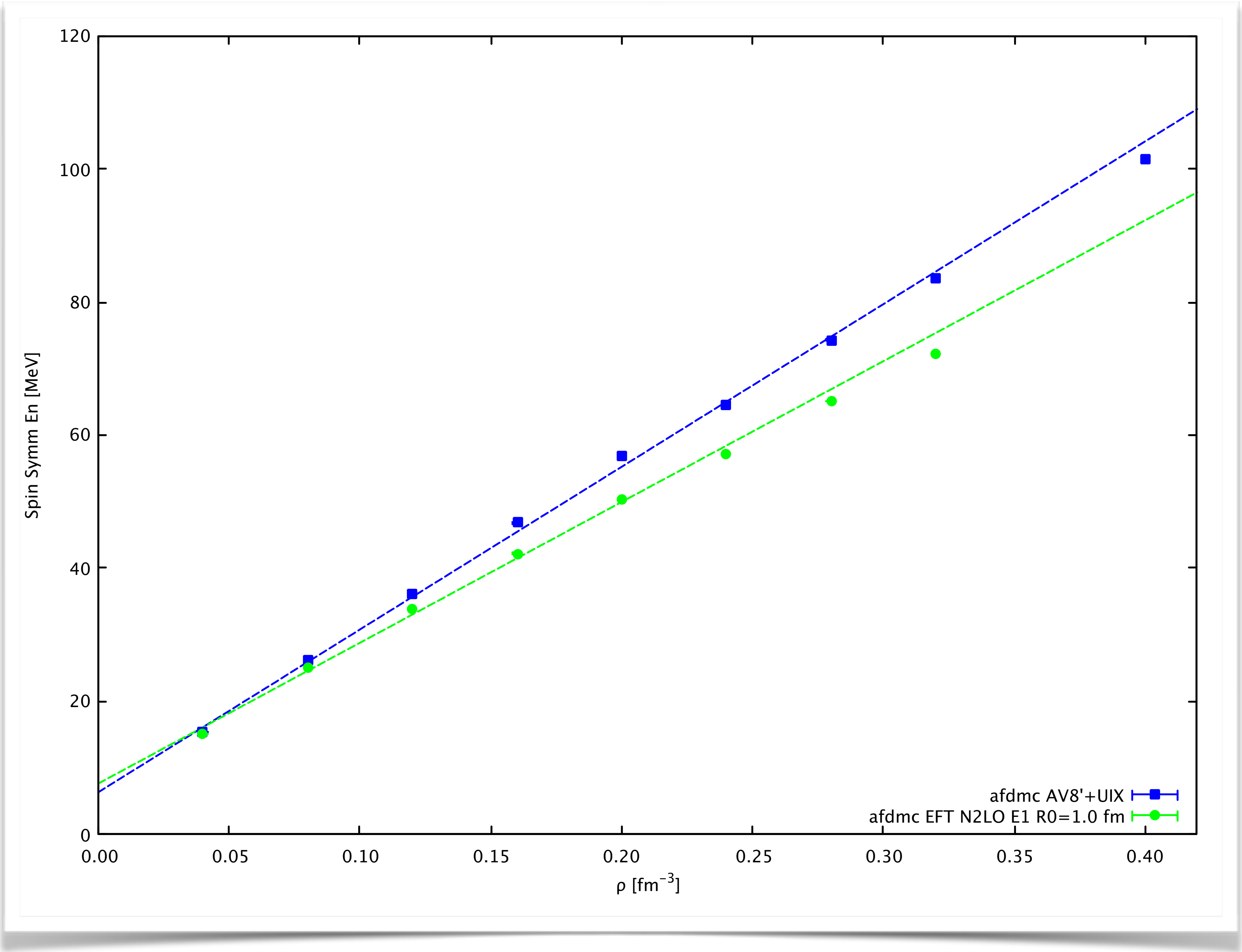
And recalling that the magnetic susceptibility can be computed as:

$$\chi = \mu^2 \rho \frac{1}{E''(0)}$$

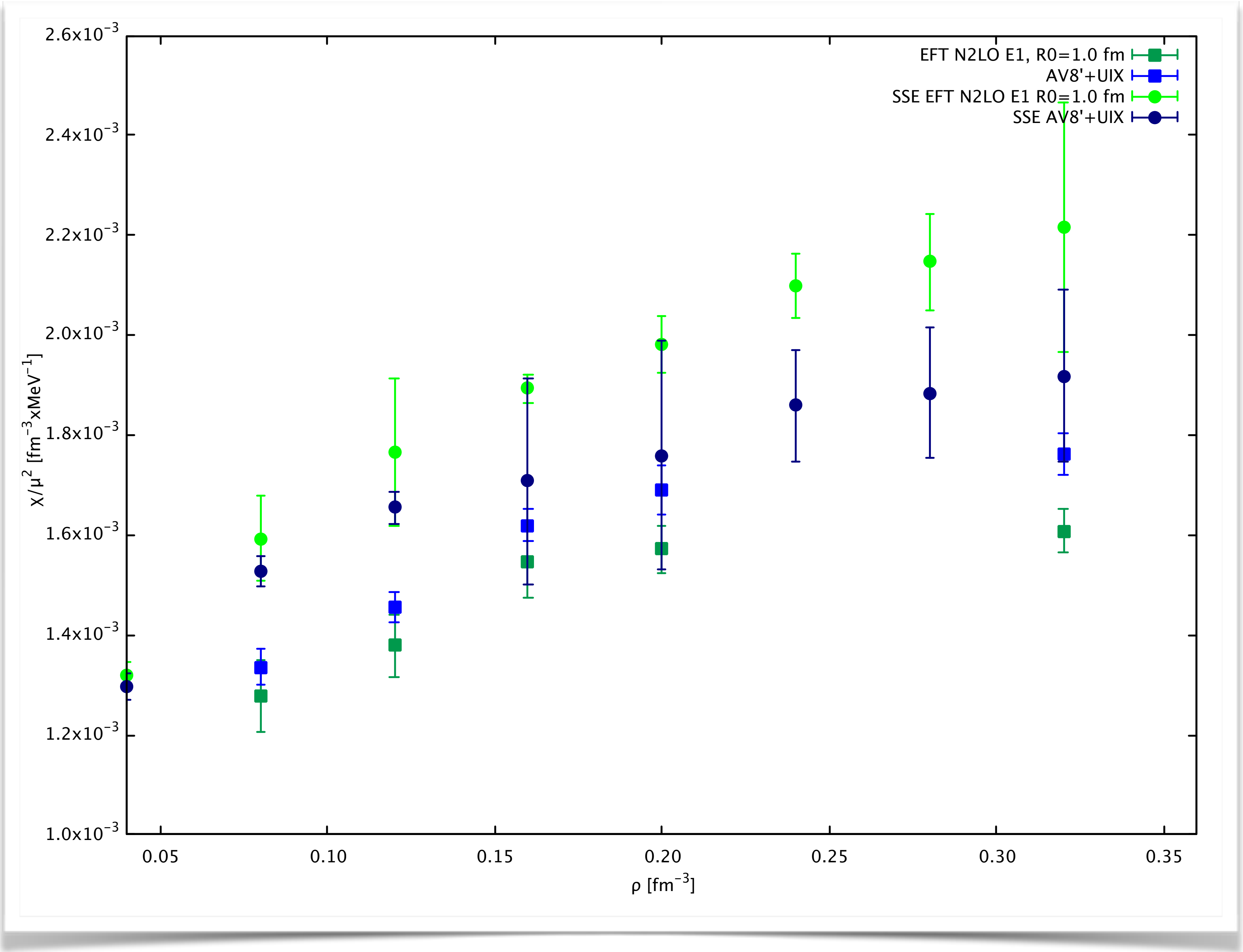
Simple model



Simple model



Simple model



Summary

- Using TABC to study partially polarized systems reducing finite-size effects and arbitrary particle numbers.
- Magnetic susceptibility computed 'correctly' with *ab initio* methods.
- Phenomenological and Chiral interactions give similar results, some differences at high densities.