The ^{6}Li ground state within the Hyperpsherical Harmonics basis

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Outline

- The Hyperspherical Harmonic(HH) method
- The ^{6}Li ground state
  - Convergence properties
  - Electromagnetic structure
  - $\alpha + d$ structure
- Conclusions and future prospective

The HH method (1)

$$H = \sum_i \frac{p_i^2}{2M} + \sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k) + \dots$$

Search for accurate solution of $H\Psi = E\Psi$

- Variational approach
- Expansion of Ψ on the basis of Hyperspherical Harmonics functions
- [A. Kievsky, S. Rosati, M. Viviani, L.E. Marcucci, and L. Girlanda J. Phys. G, **35**, 063101 (2008)]
- Applied for $A = 3, 4$ and now also for $A = 6$

The HH method general charcteristics

- Jacobi vectors $\Rightarrow$ CoM decoupling
- HH functions

$$L^2(\Omega)\mathcal{Y}_{[K]}(\Omega) = K(K + D - 2)\mathcal{Y}_{[K]}(\Omega)$$

- Transformation Coefficients
  - $\mathcal{Y}_{[K]}(\Omega') = \sum_{[K']}^{K=K'} a_{[K],[K']}\mathcal{Y}_{[K']}(\Omega)$
  - antisymmetrization
  - overcomplete basis $\Rightarrow$ orthogonalization

A new computational approach

$$\langle \Phi_\alpha | V(1,2) | \Phi_\beta \rangle = \sum_{[\alpha'],[\beta']} a_{[\alpha],[\alpha']} a_{[\beta],[\beta']} V_{[\alpha'],[\beta']}(1,2)$$

All correlations generated by the potential are constructed through the TC

- the coupling of quantum number are the same for any potential
- No need to save the matrix elements
  only the combination of the transformation coefficients
- $\sim$ 3 hours for constructing and diagonalize the Hamiltonian

Warning!

- We will use SRG evolved N^3LO500 *NN* interaction [1-2]
  - The Coulomb interaction is included as "bare" (not SRG evolved)
  - SRG evolution parameter $\Lambda = 1.2, 1.5, 1.8$ fm^{-1}
- Explorative study with $NNLO_{sat}$ [3]
- No 3-body forces (for now)
- We compute the mean values of "bare" operators

[1] S.K. Bogner, R.J. Furnstahl, and R.J. Perry, PRC **75**, 061001(R) (2007)
[2] D.R. Entem and R. Machleidt, PRC **68**, 041001(R) (2003)
[3] A. Ekström, *et al.*, PRC **91**,051301 (2015)

Selection of the states

- Include all the states up to a $K_{max} \Rightarrow$ FAIL!!
- NOT all the states gives the same contribution $\Rightarrow$ division by class [1]
  - Centrifugal barrier $\Rightarrow \ell_1 + \cdots + \ell_5 \leq 2$
  - two-body correlations are more important
- The ^{6}Li ground state is a $J^\pi = 1^+$ state

wave	class	int.	K_{max}
S	C1	two-body	14
	C3	many-body	10
D	C2	two-body	12
	C4	many-body	10
P	C5	all	8

[1] M. Viviani, *et al.*, PRC **71**,024006 (2005)

Extrapolation to $K = \infty$

- For each channel we define

$$\Delta E_{ch}(K) = E_{ch}(K) - E_{ch}(K+2)$$

- Fit the quantity ΔE_{ch} with [1]

$$\Delta E_{ch}(K) = A_{ch} e^{-b_{ch} K}(1 - e^{-2b_{ch}})$$

- Missing energy for channel

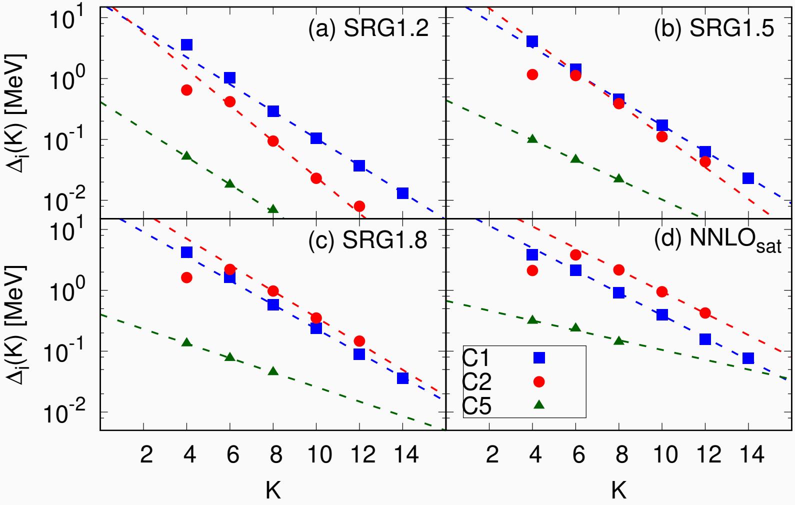
$$\Delta E_{ch}(\infty) = \sum_{K=\overline{K}}^{\infty} \Delta E(K) = A_{ch} e^{-b_{ch}\overline{K}}$$

- $\overline{K}$ maximum K used for the channel
- Extrapolated energy

$$E(\infty) = E(\overline{K}) - \sum_{ch} \Delta E_{ch}(\infty)$$

[1] S.K. Bogner *et al.*, NPA **801**, 21 (2008)

Convergence of the classes



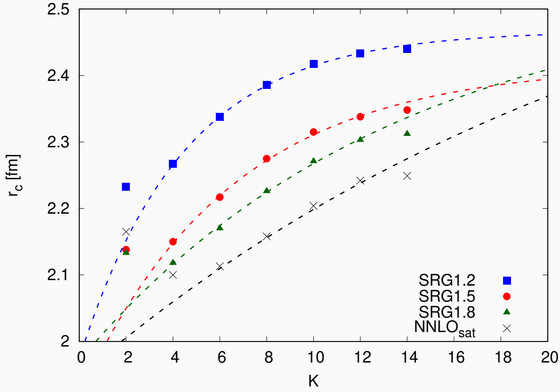
Final extrapolation

Λ [fm^{-1}]	$E(\overline{K})$ [MeV]	$E(\infty)$ [MeV]	Ref. [1]	Exp.
SRG1.2	-31.735	-31.767(7)	-31.85(5)	-31.99
SRG1.5	-32.699	-32.789(15)	-33.00(5)	-31.99
SRG1.8	-32.093	-32.305(25)	-32.8(1)	-31.99
NNLO$_{sat}$	-29.77	-30.71(15)	–	-31.99

- The errors come form the fit
- Results of Ref. [1] extrapolated from $N_{max} = 10$

[1] E.D Jungerson, P. Navrátil and R.J. Furnstahl, PRC **83**, 034301 (2011)

Charge radius



	SRG1.2	SRG1.5	SRG1.8	NNLO$_{sat}$	Ref. [1]	Exp.
r_c [fm]	2.47(1)	2.42(3)	2.54(11)	2.7(3)	2.40(6)	2.540(28)

[1] CDB2k-SRG1.5 C. Forssén, E. Caurier, P. Navrátil, PRC **71**, 021303 (2009)

Magnetic dipole moment

$$^6\text{Li} \simeq \alpha + d \Rightarrow \mu_z(^6\text{Li}) \simeq \mu_z(d)$$

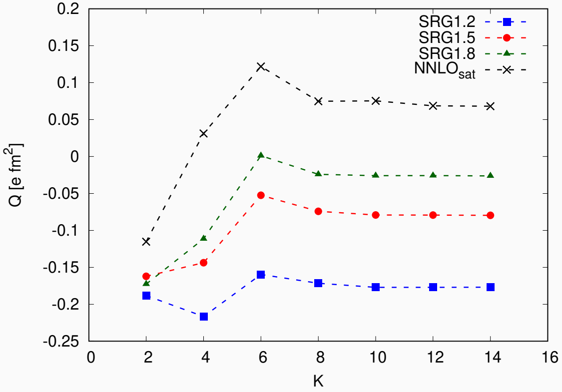
Experiment tell us $\mu_z(^6\text{Li}) < \mu_z(d)$

	$\mu_z(d)$	$\mu_z(^6\text{Li})$
SRG1.2	0.872	0.865
SRG1.5	0.868	0.860
SRG1.8	0.865	0.856
NNLO_{sat}	0.860	0.850
Exp.	0.857	0.822

- Negative contribution only from the $L = 2$ $S = 1$ component $\Rightarrow$ NOT SUFFICIENT
- We need two body currents contribution!! [1]

[1] R. Schiavilla, *et al.*, PRC **99**, 034005 (2019)

Electric quadrupole moment



	SRG1.2	SRG1.5	SRG1.8	NNLO_{sat}	Ref. [1]	Exp.
Q [e fm^2]	-0.177	-0.080	-0.026	+0.068	-0.066(40)	-0.0806(8)

- Large cancellations between different K

[1] CDB2k-SRG1.5 C. Forssén, E. Caurier, P. Navrátil, PRC **71**, 021303 (2009)

Electric quadrupole moment

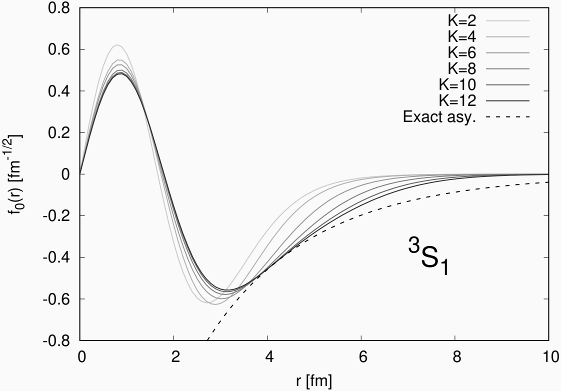
Contribution of the various waves

	$S-D$	$D-D$	$P-P$	$P-D$
SRG1.2	-0.173	-0.022	0.009	0.009
SRG1.5	-0.080	-0.021	0.012	0.010
SRG1.8	-0.028	-0.020	0.012	0.010
NNLO$_{sat}$	0.058	-0.016	0.015	0.011

- Direct connection with the strength of the tensor term in the potential
- Two-body currents contribution could be necessary!!

$\alpha + d$ form factor

$$\frac{f_L(r)}{r} = \langle \left[(\Psi_\alpha \otimes \Psi_d)_S \, Y_L(\hat{r}) \right]_J | \Psi_{^6\mathrm{Li}} \rangle$$



N^3LO500-SRG1.5

$\alpha + d$ form factor

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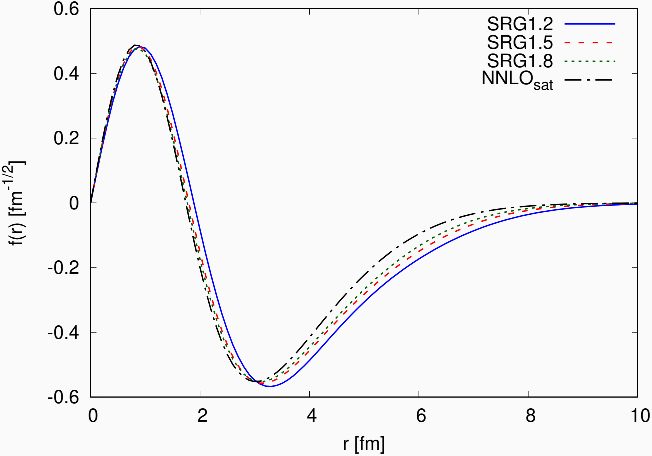

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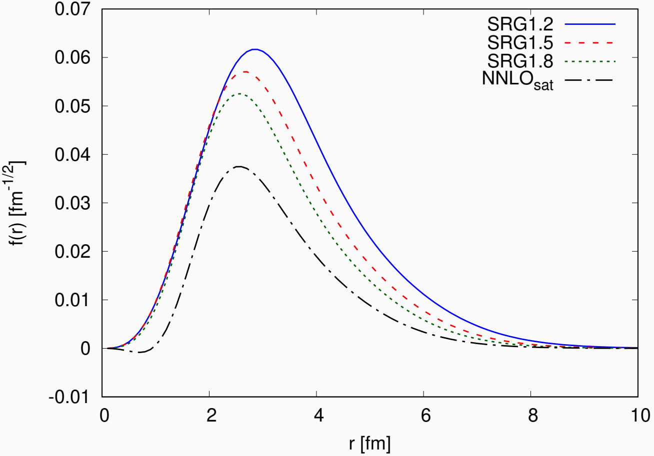

ANC and spectroscopic factor (preliminary)

$$C_L = \frac{f_L(r)}{W_{-\eta,L+1/2}(2kr)} \qquad \mathcal{S}_L = \int_0^\infty dr\,|f_L(r)|^2$$

	B_c [MeV]	C_0 [fm$^{-1/2}$]	C_2 [fm$^{-1/2}$]	$\mathcal{S}_0 + \mathcal{S}_2$
SRG1.2	2.985(7)	$-3.96(2)$	0.101(1)	0.914
SRG1.5	2.385(15)	$-3.05(3)$	0.056(1)	0.871
SRG1.8	1.684(7)	$-2.34(6)$	0.030(1)	0.841
Exp.	1.4743	$-2.91(9)$	0.077(18)	0.85(4)

- The ANCs depend on the binding energy B_c and only
- The spectroscopic factor represents the percentage of $\alpha + d$ clusterizzation of ^{6}Li

$B_c = B_{^6\text{Li}} - B_\alpha - B_d$, $k = \sqrt{2\mu B_c/\hbar^2}$ and $\eta = 2.88\mu/k$

Conclusions and future prospective

- Extension of the HH method to $A > 4$ is possible
    - Time consumption for coupling the TC $\sim 30.000h$

      $\Rightarrow$ technically possible to increase the basis
    - ~ 3 human hours for building and solving the ^{6}Li Hamiltonian

      $\Rightarrow$ easy to test various potentials
- Ground state of ^{6}Li within the HH approach
    - Nicely coverged for SRG potentials
    - Magnetic dipole moment and electric quadrupole moment
        - strongly depend on tensor forces
        - need two-body currents!!
    - Towards scattering: the $\alpha + d$ clusterizzation
        - ANCs and spectroscpic factors

Pisa group

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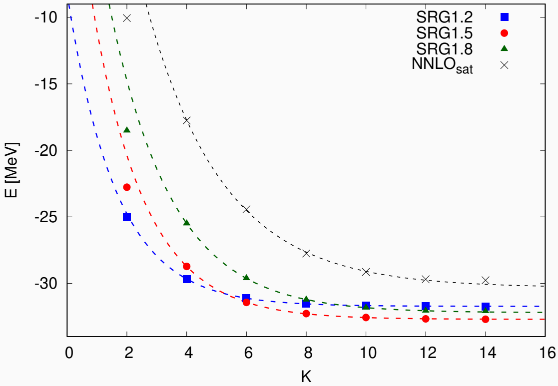
Calculations supported by

CINECA

Convergence



- Exponential behavior [1]

$$E(K) = E(\infty) + Ae^{-bK}$$

[1] S.K. Bogner *et al.*, NPA **801**, 21 (2008)