Halo effective theory for α - α , 3- α and N- α interactions

Paolo Recchia



October 9, 2019

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 $3-\alpha$ and N- α interactions

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Purpose of this work

Building an "halo" effective theory for the α - α interaction and test it in 3- α and N- α interactions

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6 Co	nclusions			• • • •		≣ ୬୯୯

Introduction Summary Image: Effective theories Why effective theories? Cutoff regularization $\alpha - \alpha$ effective potential • Scales • LO • NLO and N ² LO • Regularization of the effective potential Data analysis $\alpha - \alpha$ • LO • NLO • L=2 state Image: N- α effective potential • Symmetries and potential • LO: S _{1/2} phase shifts • NLO: S _{1/2} phase shifts • NLO: P _{1/2} , P _{3/2} phase shifts • NLO: P _{1/2} , P _{3/2} phase shifts	Intro ○●	duction	Effective theories	α - α effective potential 0000	Data analysis α-α 0000	Data analysis 3-α 00	N- α effective potential 0000	Conclusions 00
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Effective theories Why effective theories?



Issue

Short-distance (high-energy) theory is unknown or complicated

Purpose

Predicting with an arbitrary accuracy the low-energy observables regardless of the short-range behavior of the "fundamental" theory

Technique used

- Establishing the relevant degrees of freedom of the system
- Establishing the relevant scale ratios of the system
- Building a series expansion in powers of the scale ratio







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The error due to the cutoff is corrected by introducing contact terms multiplied by some coupling constants

- Setting the accuracy (the power in p/A we stop)
 Establishing only the relevant operators for the accuracy required
 Considering the operators satisfying the underlying symmetries
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 The high control provide factor and a boothest (providence) researchest (interview).



Cutoff regularization

- Regularization of the theory by a cutoff Λ
- The effective theory is valid only for p. The effective theory is more simple $\sigma < \Lambda$ than the fundamental one
 - The error due to the cutoff is corrected by introducing contact terms multiplied by some coupling constants

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- Regularization of the theory by a cutoff A
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 - The error due to the cutoff is corrected by introducing contact terms multiplied by some coupling constants

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Setting the accuracy (the power in p/A we stop)
 Establishing only the relevant operators for the accuracy required
 Considering the operators satisfying the underlying symmetries

Only a finite numbers of coupling constants needed
































α - α effective potential

Formalism

The $\boldsymbol{\alpha}$ particles created and annihilated by the fields

$$arphi(x) = \int rac{\mathrm{d}^3 q}{(2\pi)^3} a_q \mathrm{e}^{-\mathrm{i} q \cdot x}
onumber \ arphi^\dagger(x) = \int rac{\mathrm{d}^3 q}{(2\pi)^3} a_q^\dagger \mathrm{e}^{\mathrm{i} q \cdot x}$$

Symmetries

Fields and gradients get combined in order to have operators

- normal ordered
- parity invariant
- O time-reversal invariant
- O invariant under Galilean transformation

Hamiltonian density:

 $\mathcal{H}^{(LO)} = a \varphi^{\dagger} \varphi \varphi^{\dagger} \varphi$

(except muthemore) O.L. of quilibring potential

α - α effective potential

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α - α effective potential

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 $\mathcal{H}^{(LO)} = \alpha \varphi^{\dagger} \varphi \varphi^{\dagger} \varphi$

(exage mutation) O.I. of quilibrium space)

 $V_{eff}^{\text{strong}(LO)} = a$

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α - α effective potential

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 $\mathcal{H}^{(0,0)}=aarphi^{\dagger}arphiarphi^{\dagger}arphi$

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α - α effective potential

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العمالية: tonian density: ${\cal R}^{(LO)}=a \varphi^{\dagger} \varphi \varphi^{\dagger} \varphi$

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α - α effective potential

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 $\mathcal{H}^{(ext{LO})} = a arphi^{\dagger} arphi arphi^{\dagger} arphi$

Effective potential up to LO (momentum space)

 $V_{\rm eff}^{\rm strong (LO)} = 0$

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Effective potential up to LO (momentum space):

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Effective potential for α - α interaction NLO and N²LO



Effective potential for α - α interaction NLO and N²LO

NLO operators	N ² LO operators
$\nabla_{1,2} = \nabla(\varphi^{\dagger}\varphi)_{1,2}$ $O_1 = \nabla_1 \cdot \nabla_2$ $O_2 = \overleftarrow{\nabla}_1 \cdot \overleftarrow{\nabla}_2$ $O_3 = \overleftarrow{\nabla}_1 \cdot \overleftarrow{\nabla}_1$ $O_4 = i\nabla_1 \cdot \overleftarrow{\nabla}_2$	$\nabla_{12} = \nabla_2 - \nabla_1 , \overleftarrow{\nabla} = \varphi^{\dagger} \overleftarrow{\nabla} \varphi$ $U_1 = \nabla_1^4 , U_2 = \nabla_1^2 \overleftarrow{\nabla}_{12}^2 , U_3 = \overleftarrow{\nabla}_{12}^4$ $U_4 = i \nabla_1^3 \overleftarrow{\nabla}_{12} , U_5 = i \nabla_1 \overleftarrow{\nabla}_{12}^3$ $U_6 = (\nabla_1 \cdot \overleftarrow{\nabla}_{12}) (\nabla_1 \cdot \overleftarrow{\nabla}_{12})$
	SYMMETRIES ↓

Effective potential for α - α interaction NLO and N²LO

NLO operators	N ² LO operators					
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SYMMETRIES						
	*					
Effective potential up to N	N ² LO (momentum space)					
_						

$$V_{\rm eff}^{\rm strong (N^2LO)}(K,Q) = A + BK^2 + C_1K^4 + C_2K^2Q^2 + C_3(K \times Q)^2$$

with

 $Q = rac{p'+p}{2}$ $p = p_2 - p_1$ K = p' - p. $p' = p_2 - p'_1$

The coupling constants are dimensionful

Effective potential for α - α interaction NLO and N²LO

NLO operators	N ² LO operators			
$\nabla_{1,2} = \nabla(\varphi^{\dagger}\varphi)_{1,2}$ $O_1 = \nabla_1 \cdot \nabla_2$ $O_2 = \overleftarrow{\nabla}_1 \cdot \overleftarrow{\nabla}_2$ $O_3 = \overleftarrow{\nabla}_1 \cdot \overleftarrow{\nabla}_1$ $O_4 = i\nabla_1 \cdot \overleftarrow{\nabla}_2$	$\nabla_{12} = \nabla_2 - \nabla_1 , \overleftarrow{\nabla} = \varphi^{\dagger} \overleftarrow{\nabla} \varphi$ $U_1 = \nabla_1^4 , U_2 = \nabla_1^2 \overleftarrow{\nabla}_{12}^2 , U_3 = \overleftarrow{\nabla}_{12}^4$ $U_4 = i \nabla_1^3 \overleftarrow{\nabla}_{12} , U_5 = i \nabla_1 \overleftarrow{\nabla}_{12}^3$ $U_6 = (\nabla_1 \cdot \overleftarrow{\nabla}_{12}) (\nabla_1 \cdot \overleftarrow{\nabla}_{12})$			
SYMMETRIES				
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Effective potential up to N^2LO (momentum space)

$$V_{\rm eff}^{\rm strong~(N^2LO)}(K,Q) = A + BK^2 + C_1K^4 + C_2K^2Q^2 + C_3(K\times Q)^2$$
 with

 $Q = rac{p'+p}{2}$ $p = p_2 - p_1$ The coupling constants are dimensionful

Effective potential for α - α interaction Regularization of the effective potential

Dimensionless coupling constants

$$\begin{split} &\mathcal{V}_{\text{eff}}^{\text{strong}}(K,Q) = \frac{\tilde{A}}{\Lambda^2} + \frac{\tilde{B}}{\Lambda^4}K^2 + \\ &+ \frac{1}{\Lambda^6} [\tilde{C}_1 K^4 + \tilde{C}_2 K^2 Q^2 + \tilde{C}_3 (K \times Q)^2] \end{split}$$

$$V_{
m eff,\ strong}^{
m reg} = f_{\Lambda}(K^2) V_{
m eff}^{
m strong}(K,Q)$$

Gaussian regulator
$$f_{\wedge}(K^2)={
m e}^{-rac{K}{2 \wedge}}$$

$$V_{
m cout}(r) = rac{4lpha}{r} \stackrel{F.T.}{ o} rac{16\pilpha}{K^2} \stackrel{
m cutoff}{ o} rac{16\pilpha}{K^2} f_{\Lambda}(K)$$

$$\begin{split} V_{\rm eff}(r) &= \frac{4\alpha}{r} {\rm erf}\left(\frac{r}{a\sqrt{2}}\right) + a^2 \bar{A} \delta_a^{(3)}(r) + a^4 \bar{B} \nabla^2 \delta_a^{(3)}(r) + a^6 \Big[\bar{\mathcal{C}}_1 \nabla^4 \delta_a^{(3)}(r) \\ &- \bar{\mathcal{C}}_2 \nabla^2 \delta_a^{(3)}(r) \left(\frac{1}{2} \stackrel{\leftarrow}{\nabla}\right)^2 + \bar{\mathcal{C}}_3 \left(\frac{l(l+1)}{a^4} + \frac{2}{a^2} \left(\frac{1}{2} \stackrel{\leftarrow}{\nabla}\right)^2\right) \delta_a^{(3)}(r) \Big] \end{split}$$

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Effective potential for α - α interaction Regularization of the effective potential

Dimensionless coupling constants

$$egin{aligned} & \mathcal{M}^{ ext{strong}}(K,Q) = rac{ ilde{A}}{\Lambda^2} + rac{ ilde{B}}{\Lambda^4}K^2 + \ & + rac{1}{\Lambda^6}[ilde{C}_1K^4 + ilde{C}_2K^2Q^2 + ilde{C}_3(K imes Q)^2] \end{aligned}$$

$$V_{\mathrm{eff,\ strong}}^{\mathrm{reg}} = f_{\Lambda}(K^2) V_{\mathrm{eff}}^{\mathrm{strong}}(K,Q)$$

Gaussian regulator
$$f_{\wedge}(K^2) = \mathrm{e}^{-rac{K^2}{2\Lambda^2}}$$

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Effective potential for α - α interaction Regularization of the effective potential

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Regularization of the Coulomb potential

$$V_{\text{coul}}(r) = rac{4lpha}{r} \stackrel{F.T.}{\longrightarrow} rac{16\pilpha}{K^2} \stackrel{ ext{cutoff}}{\longrightarrow} rac{16\pilpha}{K^2} f_{\wedge}(K)$$

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Effective potential for α - α interaction Regularization of the effective potential

Dimensionless coupling constants

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$$V_{\text{coul}}(r) = rac{4lpha}{r} \stackrel{\text{F.T.}}{\longrightarrow} rac{16\pilpha}{K^2} \stackrel{ ext{cutoff}}{\longrightarrow} rac{16\pilpha}{K^2} f_{\wedge}(K)$$

Full effective potential (coordinate space)

$$\begin{split} \mathcal{V}_{\text{eff}}(r) &= \frac{4\alpha}{r} \text{erf}\Big(\frac{r}{a\sqrt{2}}\Big) + a^2 \tilde{A} \delta_a^{(3)}(r) + a^4 \tilde{B} \nabla^2 \delta_a^{(3)}(r) + a^6 \Big[\tilde{C}_1 \nabla^4 \delta_a^{(3)}(r) \\ &- \tilde{C}_2 \nabla^2 \delta_a^{(3)}(r) \Big(\frac{1}{2} \overleftrightarrow{\nabla}\Big)^2 + \tilde{C}_3 \Big(\frac{l(l+1)}{a^4} + \frac{2}{a^2} \Big(\frac{1}{2} \overleftrightarrow{\nabla}\Big)^2 \Big) \delta_a^{(3)}(r) \Big] \end{split}$$

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Effective potential for α - α interaction Regularization of the effective potential

Dimensionless coupling constants

Regularizatio

$$\begin{split} & \mathcal{A}^{\text{strong}}_{\text{eff}}(K,Q) = \frac{\tilde{A}}{\Lambda^2} + \frac{\tilde{B}}{\Lambda^4}K^2 + \\ & + \frac{1}{\Lambda^6}[\tilde{C}_1K^4 + \tilde{C}_2K^2Q^2 + \tilde{C}_3(K \times Q)^2] \end{split}$$

$$V_{\mathrm{eff, strong}}^{\mathrm{reg}} = f_{\wedge}(K^2) V_{\mathrm{eff}}^{\mathrm{strong}}(K, Q)$$

Gaussian regulator
$$f_{\Lambda}(K^2) = e^{-rac{K^2}{2\Lambda^2}}$$

Regularization of the Coulomb potential

$$V_{ ext{coul}}(r) = rac{4lpha}{r} \stackrel{\text{F.T.}}{\longrightarrow} rac{16\pilpha}{K^2} \stackrel{ ext{cutoff}}{\longrightarrow} rac{16\pilpha}{K^2} f_{\wedge}(K)$$

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The Schrödinger equation with potential V_{eff} has been solved

- Numerov's method for seeking the resonance condition of ⁸Be
- Kohn's variational method for fitting data with L = 0, 2 and energies lower than 6 MeV
- Study cutoff in the range [130 MeV, 150 MeV]



$\Delta B \approx 0.12$

$\Delta B \approx 0.74$

Up to LO the effective theory predict S-wave phase shifts better than D-wave ones



S-wave phase shifts

LO



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 $\Delta B \approx 0.74$

D-wave phase shifts

Up to LO the effective theory predict S-wave phase shifts better than D-wave ones

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S-wave phase shifts

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Paolo Recchia	(INPHYNI)
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Data	analysis ($\gamma_{-}\alpha$				
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Data analysis α - α

Preliminary observations

The NLO's coupling constant has been fitted to low-energy data

The NLO predicts the D-wave phase shifts better than the LO
 Theory with cutoff 130 MeV provides best predictions

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$\Delta B \approx 0.03$

4 times less than LO

9 times less than LO

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			Data analysis α - α	Data analysis 3- α		Conclusions

Data analysis α - α

Preliminary observations

- Only central term acting on states with $l \neq 0$
- Th NLO and N²LO coupling constants has been sought by fitting low-energy data

a NELCO consult forgether the accuracy for Sevene phase shifts a NELCO improve the accuracy for Device phase shifts

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Data	analysis (x-α				

N²LO

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N²LO doesn't improve the accuracy for S-wave phase shifts N²LO improve the accuracy for D-wave phase shifts



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S-wave phase shifts



D-wave phase shifts



 $\Delta B \approx 0.04$

comparable with NLO



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S-wave phase shifts



D-wave phase shifts



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Halo α interactions

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Data G-wave	analysis					

The theoretical predictions follow the experimental trend
 The theoretical error is greater than the cases of S-wave and D-wave

The α - α effective potential which we have used carries only central terms



G-wave

Theory up to NLO



Theory up to N^2LO



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 $\Delta B \approx 0.24$

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Theory up to NLO



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The α - α effective potential which we have used carries only central terms

- \bullet Only two-body $\alpha\text{-}\alpha$ potential up to NLO by fitting data with energy less than 1 MeV
- The Ritz variational method for bound state searching



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$\Lambda(MeV)$	$E_0(MeV)$
100	-1.17820
120	-1.06630
140	-1.16920
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Introduction Effective theories $\alpha - \alpha$ effective potential $\alpha = \alpha$ offective potenti

Preliminarily observations

• A LO hyper-central three body term has been added

 $V_{\rm 3body}(\rho) = C \Lambda e^{rac{
ho^2}{2a^2}}$

• Its coupling constant has been fitted to get the binding energy of the ¹²C ground state

Drawback

The 2 ⁺ state hasn't never been found for each cutoff

Observations

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Symmetries

Fields and gradients get combined in order to have operators

- normal ordered
- 2 parity invariant
- time-reversal invariant
- invariant under Galilean transformation
- internal rotation (isospin)

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Effective potential up to NLO

 $V_{
m eff}^{
m strong (NLO)}(K,Q,\sigma) = A + B_1 K^2 + B_2 Q^2 + B_3 i \sigma \cdot Q imes K$

Phaseshifts analysis

Only n-a phaseshifts data to test strong interaction

hudy cutoff in the range 130 MeV 150 MeV

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 effective
 potential
 [Sara
 Murciano, Laurea
 Triennale in Fisica 2016,

 Università
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 Symmetries
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N- α effective potential [Sara Murciano, Laurea Triennale in Fisica 2016,

Università del Salento] Symmetries and potential

Symmetries

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N- α effective potential [Sara Murciano, Laurea Triennale in Fisica 2016,

Università del Salento] Symmetries and potential

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Effective potential up to NLO

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Phaseshifts analysis

Only n-a phaseshifts data to test strong interaction

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Phaseshifts analysis

- Only $n-\alpha$ phaseshifts data to test strong interaction
- Study cutoff in the range [130 MeV, 150 MeV]
- Fit of experimental data with L = 0, 1 and energies lower than 8 MeV

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N- α effective potential [Sara Murciano, Laurea Triennale in Fisica 2016,

Università del Salento] Symmetries and potential

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LO: $S_{1/2}$ phase shifts

$n-\alpha$ phase shifts



$p-\alpha$ phase shifts



 $\Delta B \approx 0.22$

 $\Delta B \approx 0.18$

The data trend is well reproduced
Relatively high cutoff dependent theory as α-α
Paolo Recchia (INPHYNI) Halo α interactions October 9, 2019 17/21

Introduction Effective theories $\alpha \sim \alpha$ effective potential $\Delta \alpha = 0$ analysis $\alpha \sim \alpha$ offective potential $\Delta \alpha = 0$ of $\alpha = 0$ of

$n-\alpha$ phase shifts



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Introduction Effective theories $\alpha \sim \alpha$ effective potential Data analysis $\alpha \sim \alpha$ Data analysis $3 \sim \alpha$ N- α effective potential Conclusions $0 \sim 0$ N- α effective potential Conclusions $0 \sim 0$ N- α effective potential [Sara Murciano, Laurea Triennale in Fisica 2016, Università del Salento] LO: $S_{1/2}$ phase shifts

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Introduction Effective theories $\alpha - \alpha$ effective potential 0 = 0 and 0 =

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The NLO greatly improve the "goodness" of the theory
 Theory with cutoff around 130 MeV provides good predictions as α-α

Introduction Effective theories $\alpha - \alpha$ effective potential 0 = 0 and 0 =

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n- α phase shifts: $P_{1/2}$



p- α phase shifts: $P_{1/2}$



n- α phase shifts: $P_{3/2}$



p- α phase shifts: $P_{3/2}$



Introduction Effective theories $\alpha - \alpha$ effective potential Data analysis $\alpha - \alpha$ Data analysis $3 - \alpha$ N- α effective potential Conclusions 00 00 000 000 000 000 000 000

Conclusions Comparison with a phenomenological potential

S-wave



D-wave



- An effective theory allows us to formally justify the Gaussian shape of a phenomenological potential
- The phenomenological potentials are built by fitting a large range of data, while the effective theory uses only the low-energy ones
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Concl	usions					

Inserting non-central terms

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- improves the accuracy at high energies
-) System consisting of "core" α - α and "halo" N: ⁹Be
- 3-α system
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