Halo effective theory for α-α, 3-α and N-α interactions

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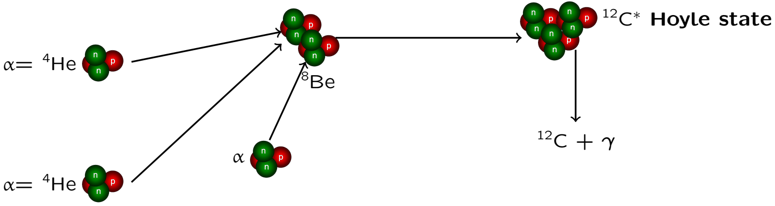
Introduction

Motivation: relevance of α particles



Issues in α interactions

- Rigorously QCD needed
- System consisting of 12 nucleons

Properties of α particles

Technique used: low-energy theory

Purpose of this work

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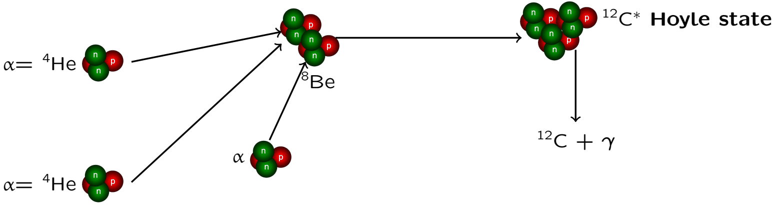

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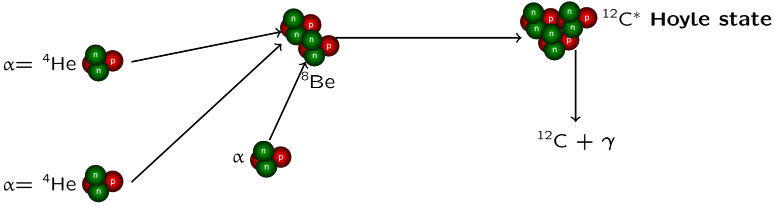

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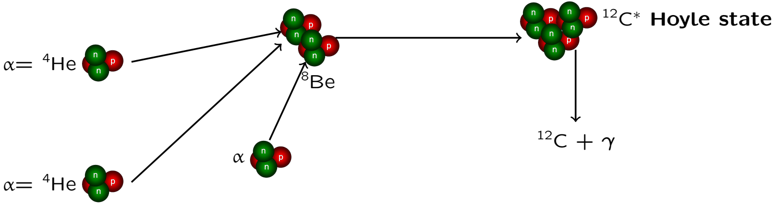

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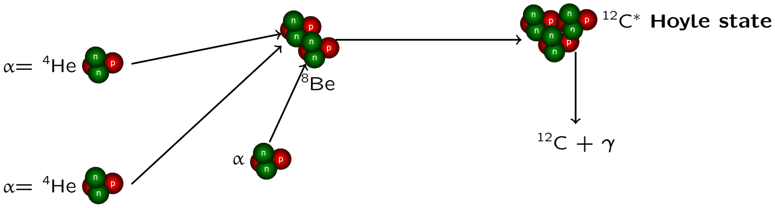

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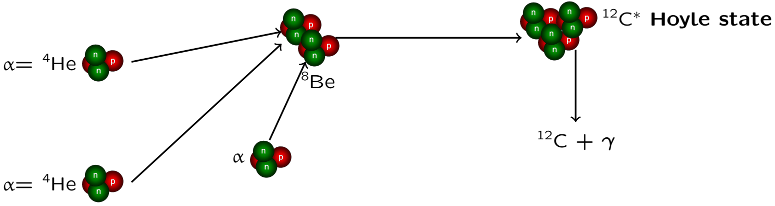

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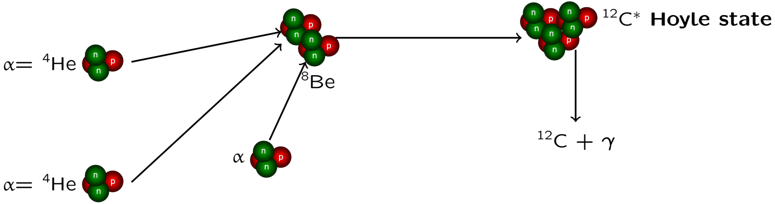

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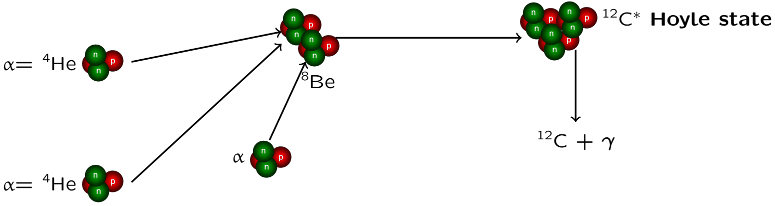

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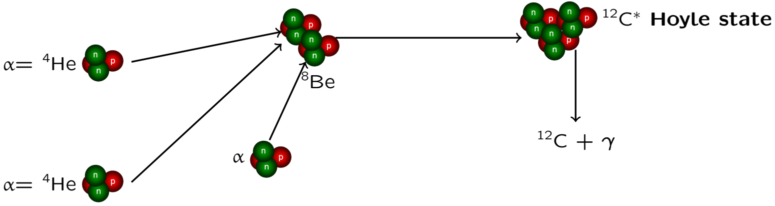

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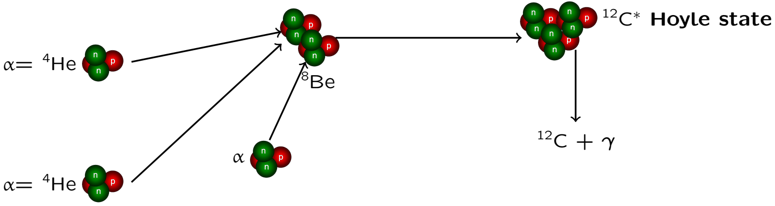

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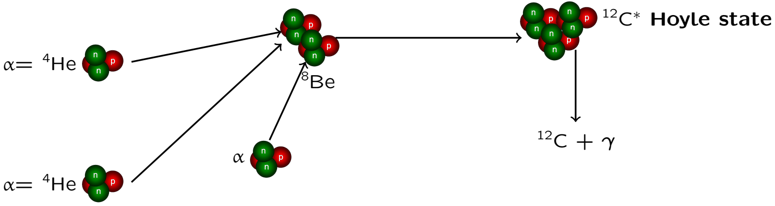

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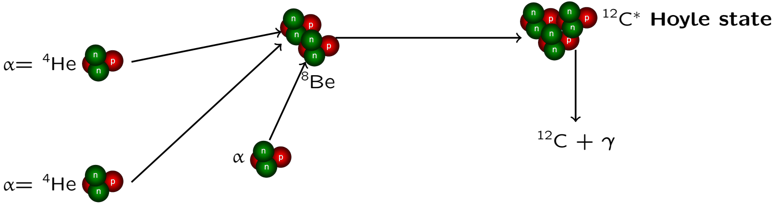

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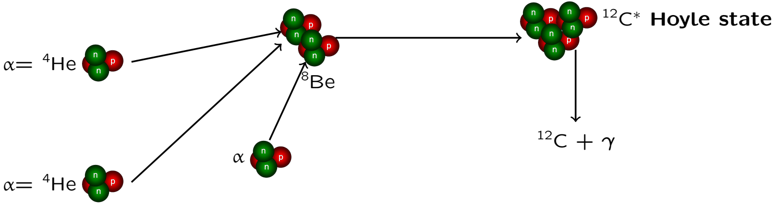

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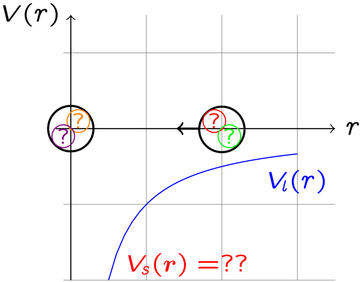
Effective theories

Why effective theories?



Issue

Short-distance (high-energy) theory is unknown or complicated

Purpose

Predicting with an arbitrary accuracy the low-energy observables regardless of the short-range behavior of the "fundamental" theory

Technique used

- Establishing the relevant degrees of freedom of the system
- Establishing the relevant scale ratios of the system
- Building a series expansion in powers of the scale ratios

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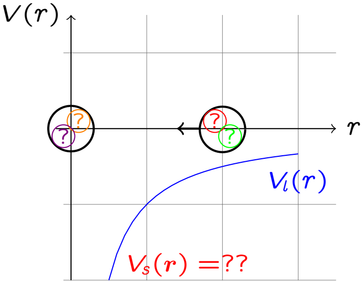

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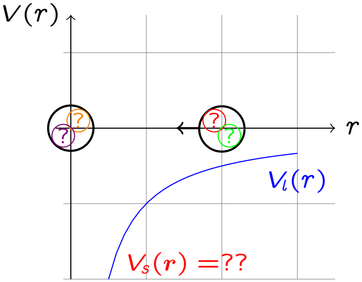

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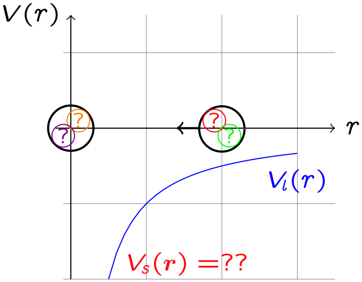

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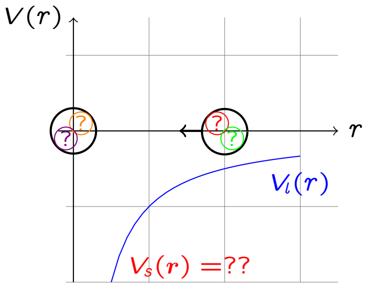

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Cutoff regularization

1. Regularization of the theory by a cutoff Λ
2. The error due to the cutoff is corrected by introducing contact terms multiplied by some coupling constants
3. Setting the accuracy (the power in p/Λ we stop)
4. Establishing only the relevant operators for the accuracy required
5. Considering the operators satisfying the underlying symmetries

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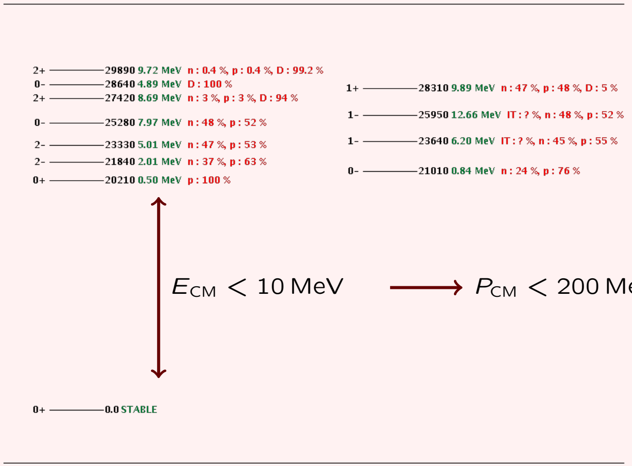
α-α effective potential
Scales

Excitation energy of α particle



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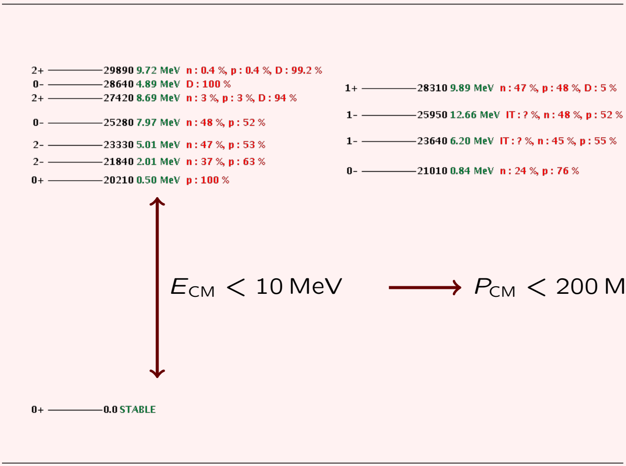

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α-α effective potential

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Formalism

The α particles created and annihilated by the fields

$$\varphi(\boldsymbol{x}) = \int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3} a_{\boldsymbol{q}} \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{x}}$$

$$\varphi^\dagger(\boldsymbol{x}) = \int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3} a^\dagger_{\boldsymbol{q}} \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{x}}$$

Symmetries

Fields and gradients get combined in order to have operators

1. normal ordered
2. parity invariant
3. time-reversal invariant
4. invariant under Galilean transformation

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The α particles created and annihilated by the fields

$$\varphi(\boldsymbol{x}) = \int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3} a_{\boldsymbol{q}} \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{x}}$$

$$\varphi^\dagger(\boldsymbol{x}) = \int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3} a^\dagger_{\boldsymbol{q}} \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{x}}$$

Symmetries

Fields and gradients get combined in order to have operators

1. normal ordered
2. parity invariant
3. time-reversal invariant
4. invariant under Galilean transformation

α-α effective potential
LO

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Hamiltonian density:

$$\mathcal{H}^{(\mathrm{LO})} = a\varphi^\dagger\varphi\varphi^\dagger\varphi$$

Effective potential up to LO (momentum space):

$$V_{\mathrm{eff}}^{\mathrm{strong\ (LO)}} = a$$

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Effective potential for α-α interaction
NLO and N^{2}LO

NLO operators

$$\nabla_{1,2} = \nabla(\varphi^\dagger\varphi)_{1,2}$$
$$O_1 = \nabla_1 \cdot \nabla_2$$
$$O_2 = \overleftrightarrow{\nabla}_1 \cdot \overleftrightarrow{\nabla}_2$$
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N^{2}LO operators

$$\nabla_{12} = \nabla_2 - \nabla_1 \quad , \quad \overleftrightarrow{\nabla} = \varphi^\dagger \overleftrightarrow{\nabla} \varphi$$
$$U_1 = \nabla_1^4 \quad , \quad U_2 = \nabla_1^2 \overleftrightarrow{\nabla}_{12}^2 \quad , \quad U_3 = \overleftrightarrow{\nabla}_{12}^4$$
$$U_4 = i\nabla_1^3 \overleftrightarrow{\nabla}_{12} \quad , \quad U_5 = i\nabla_1 \overleftrightarrow{\nabla}_{12}^3$$
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SYMMETRIES

↓

Effective potential up to N^{2}LO (momentum space)

$$V_{\text{eff}}^{\text{strong (N}^2\text{LO)}}(\boldsymbol{K},\boldsymbol{Q}) = A + B\boldsymbol{K}^2 + C_1\boldsymbol{K}^4 + C_2\boldsymbol{K}^2\boldsymbol{Q}^2 + C_3(\boldsymbol{K}\times\boldsymbol{Q})^2$$

with

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The coupling constants are dimensionful

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Effective potential for α-α interaction

Regularization of the effective potential

Dimensionless coupling constants

$$V_{\text{eff}}^{\text{strong}}(\boldsymbol{K},\boldsymbol{Q}) = \frac{\tilde{A}}{\Lambda^2} + \frac{\tilde{B}}{\Lambda^4}\boldsymbol{K}^2 + $$
$$+ \frac{1}{\Lambda^6}[\tilde{C}_1\boldsymbol{K}^4 + \tilde{C}_2\boldsymbol{K}^2\boldsymbol{Q}^2 + \tilde{C}_3(\boldsymbol{K}\times\boldsymbol{Q})^2]$$

Regularization

$$V_{\text{eff, strong}}^{\text{reg}} = f_\Lambda(\boldsymbol{K}^2)V_{\text{eff}}^{\text{strong}}(\boldsymbol{K},\boldsymbol{Q})$$

Gaussian regulator $f_\Lambda(\boldsymbol{K}^2) = e^{-\frac{\boldsymbol{K}^2}{2\Lambda^2}}$

Regularization of the Coulomb potential

$$V_{\text{coul}}(r) = \frac{4\alpha}{r} \xrightarrow{F.T.} \frac{16\pi\alpha}{\boldsymbol{K}^2} \xrightarrow{\text{cutoff}} \frac{16\pi\alpha}{\boldsymbol{K}^2} f_\Lambda(\boldsymbol{K})$$

Full effective potential (coordinate space)

$$V_{\text{eff}}(r) = \frac{4\alpha}{r}\text{erf}\left(\frac{r}{a\sqrt{2}}\right) + a^2\tilde{A}\delta_a^{(3)}(r) + a^4\tilde{B}\nabla^2\delta_a^{(3)}(r) + a^6\left[\tilde{C}_1\nabla^4\delta_a^{(3)}(r) - \tilde{C}_2\nabla^2\delta_a^{(3)}(r)\left(\frac{1}{2}\overleftrightarrow{\nabla}\right)^2 + \tilde{C}_3\left(\frac{l(l+1)}{a^4} + \frac{2}{a^2}\left(\frac{1}{2}\overleftrightarrow{\nabla}\right)^2\right)\delta_a^{(3)}(r)\right]$$

The Schrödinger equation with potential V_{eff} has been solved

- Numerov's method for seeking the resonance condition of ^{8}Be
- Kohn's variational method for fitting data with $L = 0, 2$ and energies lower than 6 MeV
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Data analysis α-α

LO

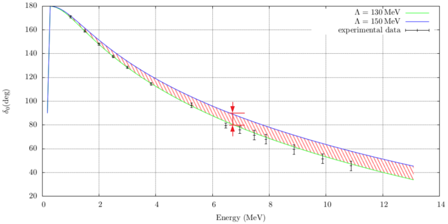
$\Delta B \approx 0.12$

$\Delta B \approx 0.74$

Up to LO the effective theory predict S-wave phase shifts better than D-wave ones

Data analysis α-α
LO

S-wave phase shifts



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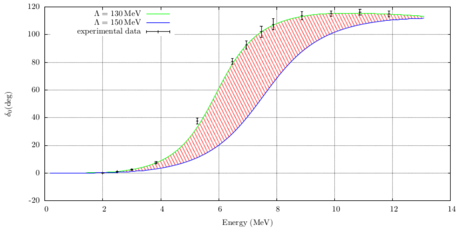
D-wave phase shifts



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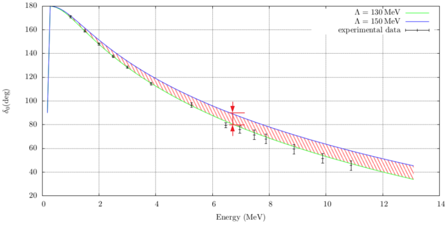

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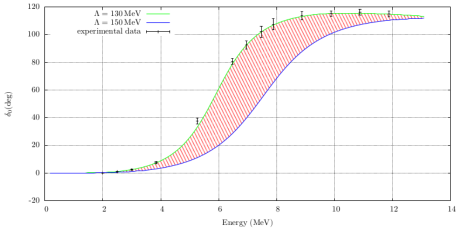

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Preliminary observations

The NLO's coupling constant has been fitted to low-energy data

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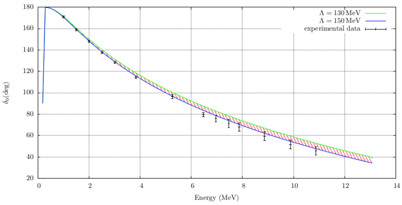

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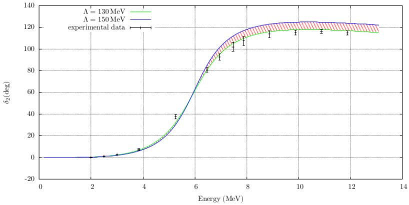

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4 times less than LO

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9 times less than LO

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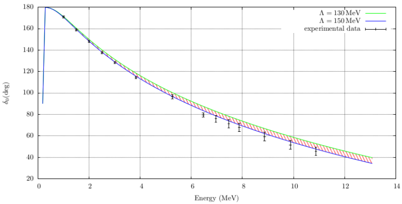

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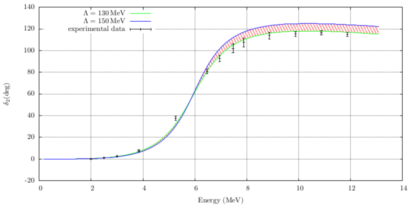

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N^2LO

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- Only central term acting on states with $l \neq 0$
- Th NLO and N^2LO coupling constants has been sought by fitting low-energy data

Data analysis α-α

N^2LO

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- N^2LO doesn't improve the accuracy for S-wave phase shifts
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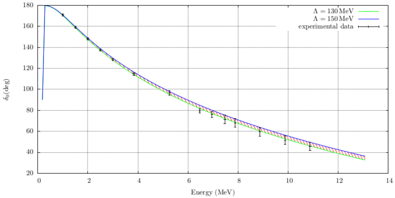

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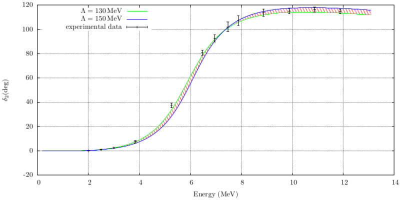

$\Delta B \approx 0.04$

comparable with NLO

$\Delta B \approx 0.01$

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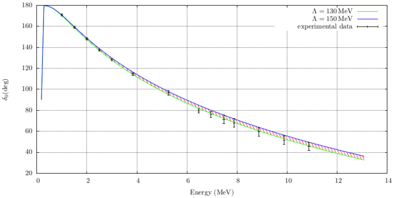

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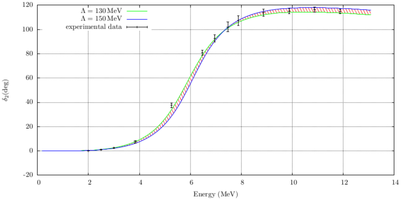

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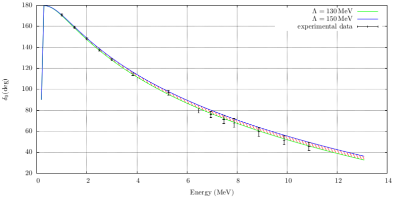
- N^{2}LO doesn't improve the accuracy for S-wave phase shifts
- N^{2}LO improve the accuracy for D-wave phase shifts

Data analysis α-α
N^2LO

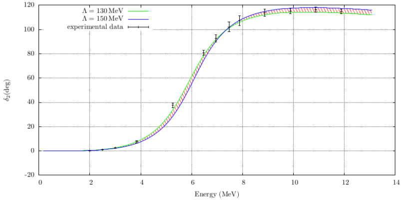
Preliminary observations

- Only central term acting on states with $l \neq 0$
- Th NLO and N^2LO coupling constants has been sought by fitting low-energy data

S-wave phase shifts



D-wave phase shifts



$\Delta B \approx 0.04$

comparable with NLO

$\Delta B \approx 0.01$

4 times less than NLO

- N^2LO doesn't improve the accuracy for S-wave phase shifts
- N^2LO improve the accuracy for D-wave phase shifts

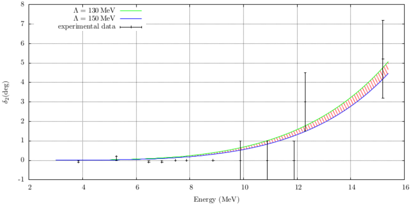
Data analysis
G-wave

- The theoretical predictions follow the experimental trend
- The theoretical error is greater than the cases of S-wave and D-wave

The α-α effective potential which we have used carries only central terms

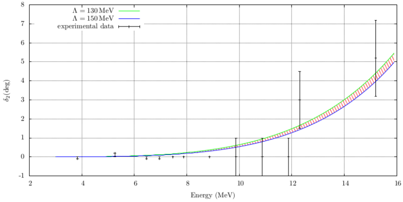
Data analysis
G-wave

Theory up to NLO



$\Delta B \approx 0.24$

Theory up to N^{2}LO



$\Delta B \approx 0.20$

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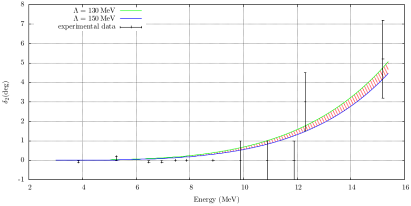
G-wave

Theory up to NLO



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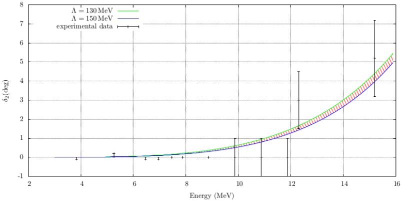

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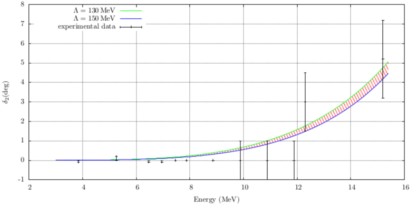

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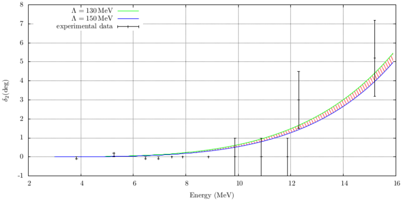

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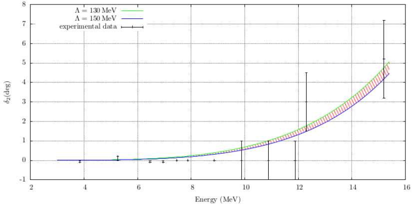

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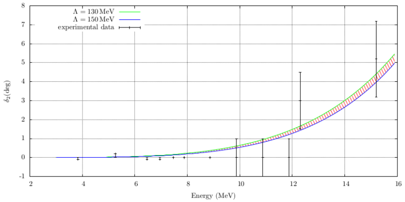

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Data analysis 3-α [Francesco Alemanno, Laurea Triennale in Fisica 2016, Università del Salento]

Ground state L=0

Preliminarily observations

- Only two-body α-α potential up to NLO by fitting data with energy less than 1 MeV
- The Ritz variational method for bound state searching

Λ(MeV)	E_0(MeV)
100	-1.17820
120	-1.06630
140	-1.16920
160	-1.20128
180	-1.12625
200	-1.30226

Drawback

Wrong binding energy (-7.26 MeV)

Borromean system

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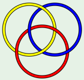

Data analysis 3-α [Francesco Alemanno, Laurea Triennale in Fisica 2016, Università del Salento]

L=2 state

Preliminarily observations

- A LO hyper-central three body term has been added

$$V_{3\text{body}}(\rho) = C\Lambda e^{\frac{\rho^2}{2a^2}}$$

- Its coupling constant has been fitted to get the binding energy of the ^{12}C ground state

Drawback

The 2^+ state hasn't never been found for each cutoff

Observations

- α-α potential has been built at very low energy (≤ 1 MeV)
- Only hyper-central terms have been considered

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Observations

- α-α potential has been built at very low energy ($\lesssim 1$ MeV)
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N-α effective potential [Sara Murciano, Laurea Triennale in Fisica 2016, Università del Salento]

Symmetries and potential

Symmetries

Fields and gradients get combined in order to have operators

1. normal ordered
2. parity invariant
3. time-reversal invariant
4. invariant under Galilean transformation
5. **internal rotation (isospin)**

↓

Effective potential up to NLO

$$V_{\text{eff}}^{\text{strong (NLO)}}(\boldsymbol{K}, \boldsymbol{Q}, \boldsymbol{\sigma}) = A + B_1 \boldsymbol{K}^2 + B_2 \boldsymbol{Q}^2 + B_3 i \boldsymbol{\sigma} \cdot \boldsymbol{Q} \times \boldsymbol{K}$$

Phaseshifts analysis

- Only n-α phaseshifts data to test strong interaction
- Study cutoff in the range [130 MeV, 150 MeV]
- Fit of experimental data with $l = 0, 1$ and energies lower than 6 MeV

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$\Downarrow$

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Phaseshifts analysis

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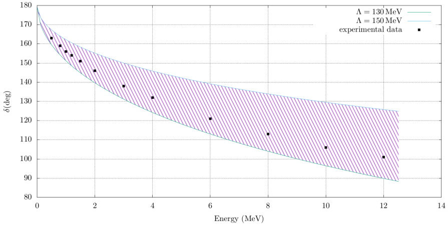
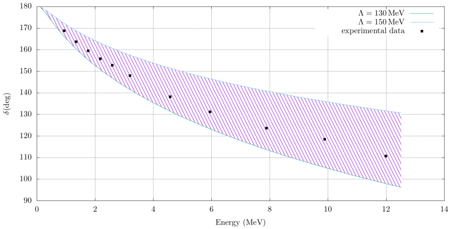
LO: $S_{1/2}$ phase shifts

n-α phase shifts



$\Delta B \approx 0.22$

p-α phase shifts



$\Delta B \approx 0.18$

The data trend is well reproduced

Relatively high cutoff dependent theory as α-α

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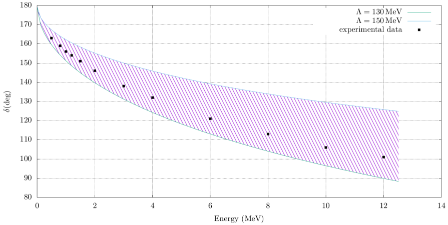

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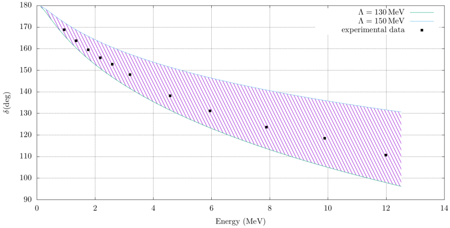

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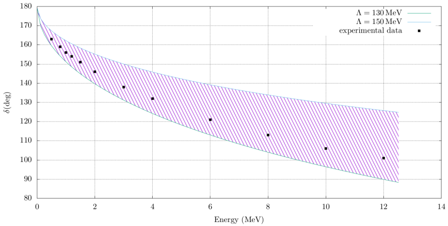

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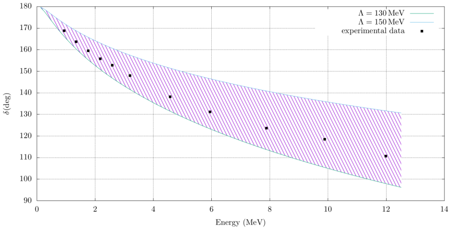

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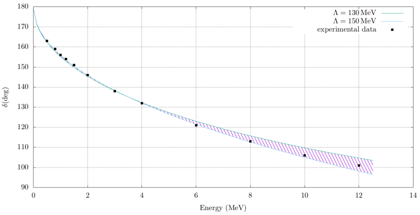

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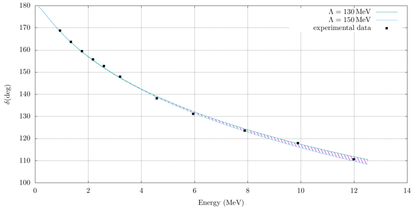

$\Delta B \approx 0.004$

- The NLO greatly improve the "goodness" of the theory
- Theory with cutoff around 130 MeV provides good predictions as α-α

- Theory up to NLO
- Only central potential terms

N-α effective potential [Sara Murciano, Laurea Triennale in Fisica 2016, Università del Salento]

NLO: $S_{1/2}$ phase shifts

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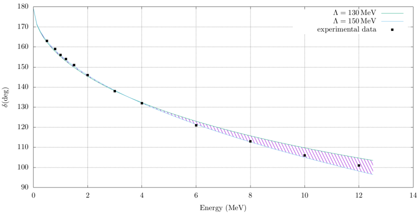

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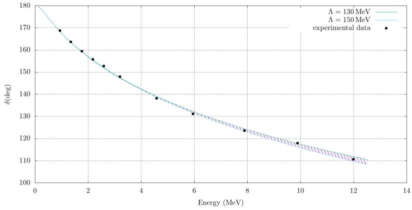

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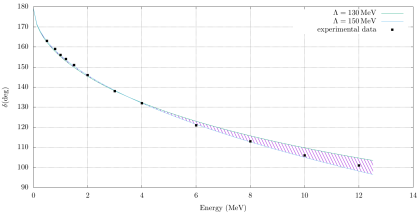

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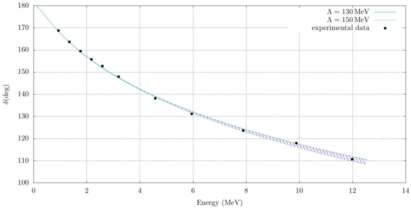

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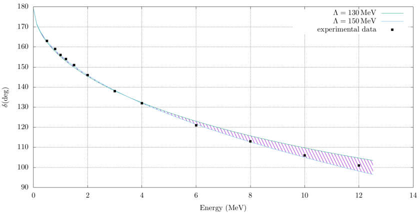
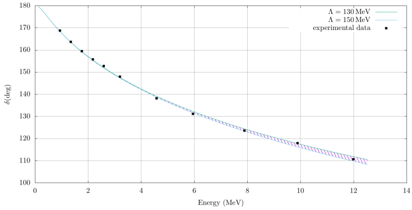
NLO: $S_{1/2}$ phase shifts

n-α phase shifts



$\Delta B \approx 0.01$

p-α phase shifts



$\Delta B \approx 0.004$

- The NLO greatly improve the "goodness" of the theory
- Theory with cutoff around 130 MeV provides good predictions as α-α

- Theory up to NLO
- Only central potential terms

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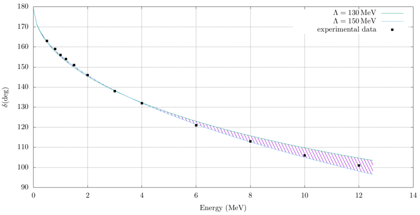

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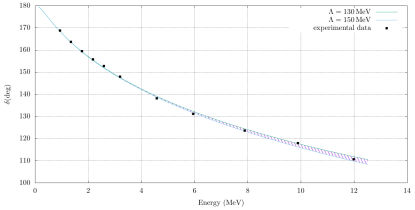

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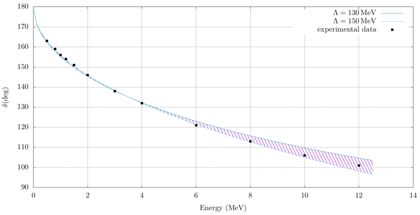

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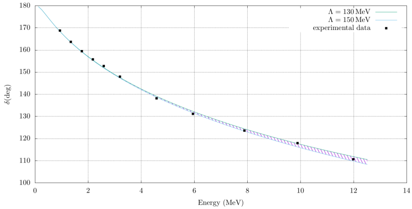

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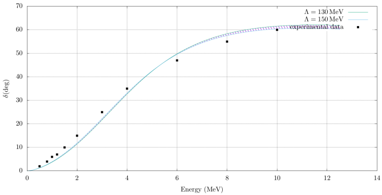
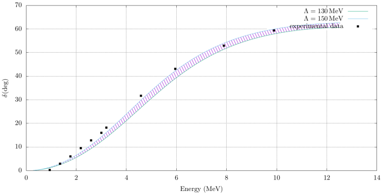
NLO: $P_{1/2}$, $P_{3/2}$ phase shifts

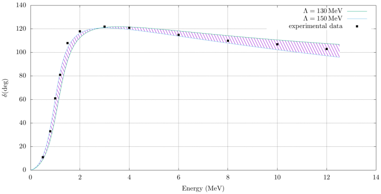
n-α phase shifts: $P_{1/2}$



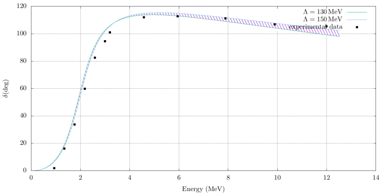
p-α phase shifts: $P_{1/2}$



n-α phase shifts: $P_{3/2}$



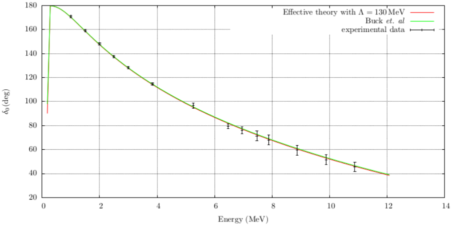
p-α phase shifts: $P_{3/2}$



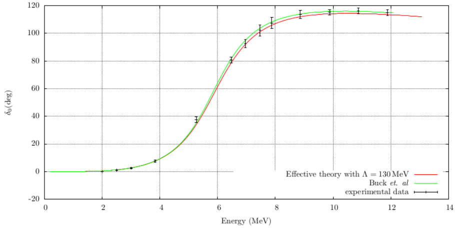
Conclusions

Comparison with a phenomenological potential

S-wave



D-wave



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- The phenomenological potentials are built by fitting a large range of data, while the effective theory uses only the low-energy ones
- The building of an effective theory isn't equal to fit a Fundamental theory

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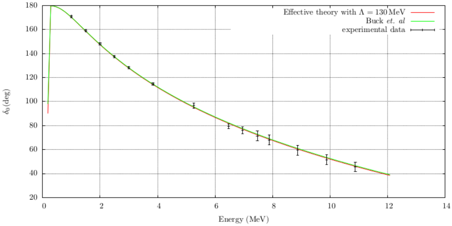

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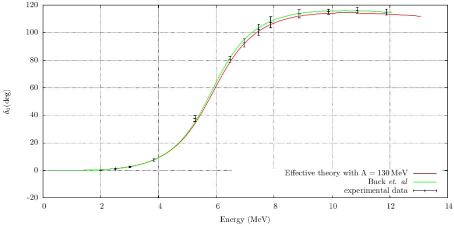

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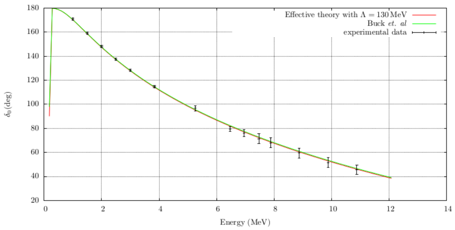

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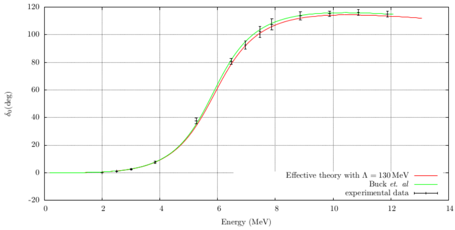

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1. Inserting non-central terms
    - improves the predictions in higher L channel
    - improves the accuracy at high energies
2. System consisting of "core" α-α and "halo" N: ^{9}Be
3. 3-α system
4. Studying the Hoyle state in a low-energy effective theory

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