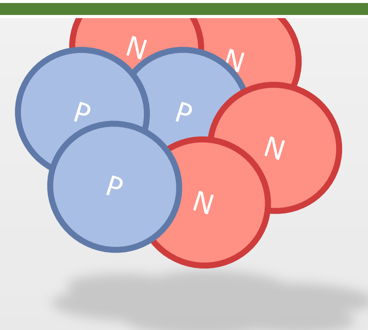
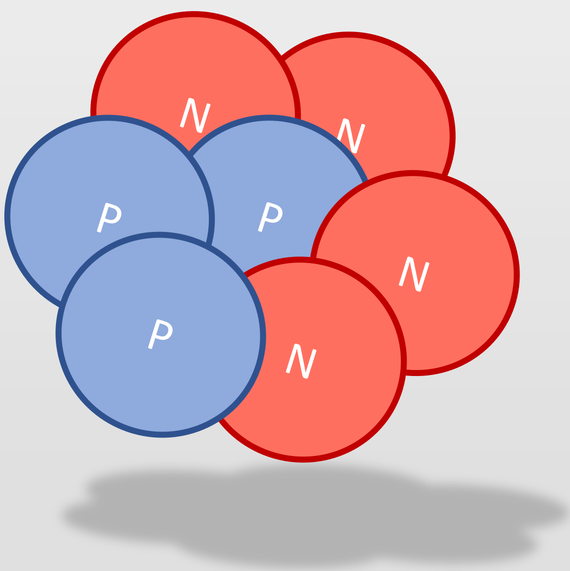
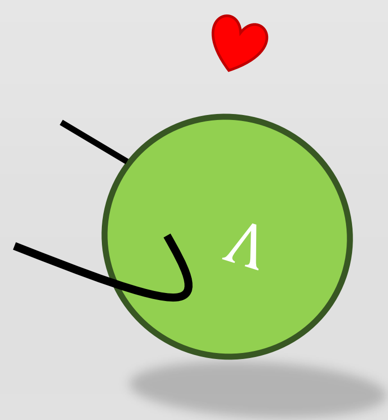
Single- and double-Λ hypernuclei in EFT($\not{\pi}$)







Lorenzo Contessi

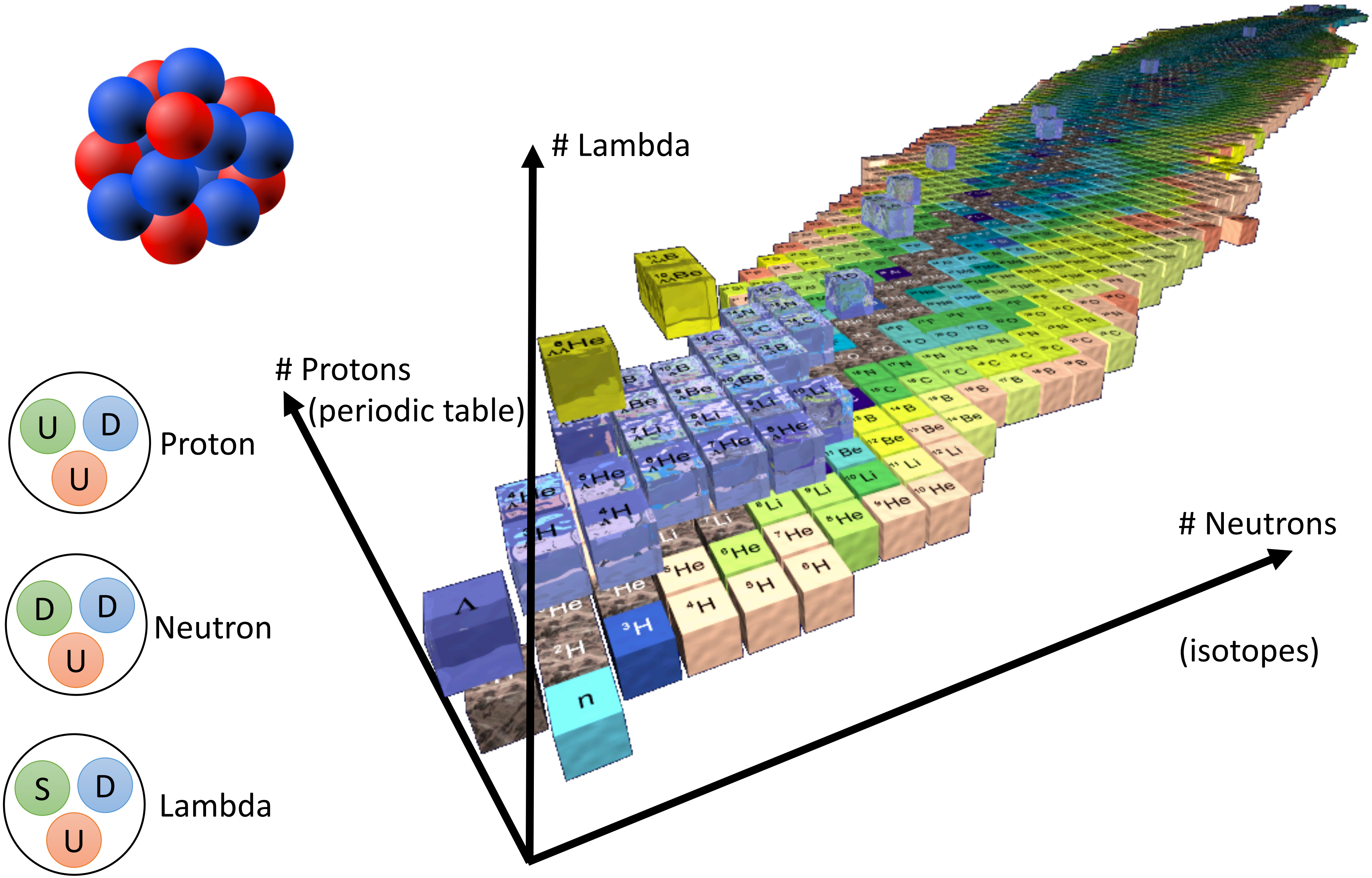
Martin Shäfer

Nir Barnea

Avraham Gal

Jiří Mareš

THE HEBREW UNIVERSITY OF JERUSALEM



Lambda
Protons
(periodic table)
Neutrons
(isotopes)
U
D
U
Proton
D
D
U
Neutron
S
D
U
Lambda

Phaseshift shortage

$N-\Lambda$

$\Lambda-\Lambda$

Experimentally **not known**
Theoretically **debated**

Unknown FEW-BODY

$nn\Lambda$ $np\Lambda\Lambda$ $n\Lambda\Lambda$ $nn\Lambda\Lambda$

${}^{5}_{\Lambda\Lambda}\mathrm{H}$

J-PARC **P75** proposal

Theoretical challenging

--- ---

Few input data to tune theories

↓

Minimal number of free parameters

Known FEW-BODY

${}^{4}_{\Lambda}\mathrm{H}\,(S=0)$ ${}^{3}_{\Lambda}\mathrm{H}$

${}^{4}_{\Lambda}\mathrm{H}\,(S=1)$ ${}^{5}_{\Lambda}\mathrm{He}$

Experimentally **known**
Theoretically hard to be described all together

Known Double-Λ

${}^{6}_{\Lambda\Lambda}\mathrm{He}$ ${}^{10}_{\Lambda\Lambda}\mathrm{Be}$

${}^{11}_{\Lambda\Lambda}\mathrm{Be}$

New experiments are planned

STAR collaboration

BES III

PANDA

J-parc

J-Lab

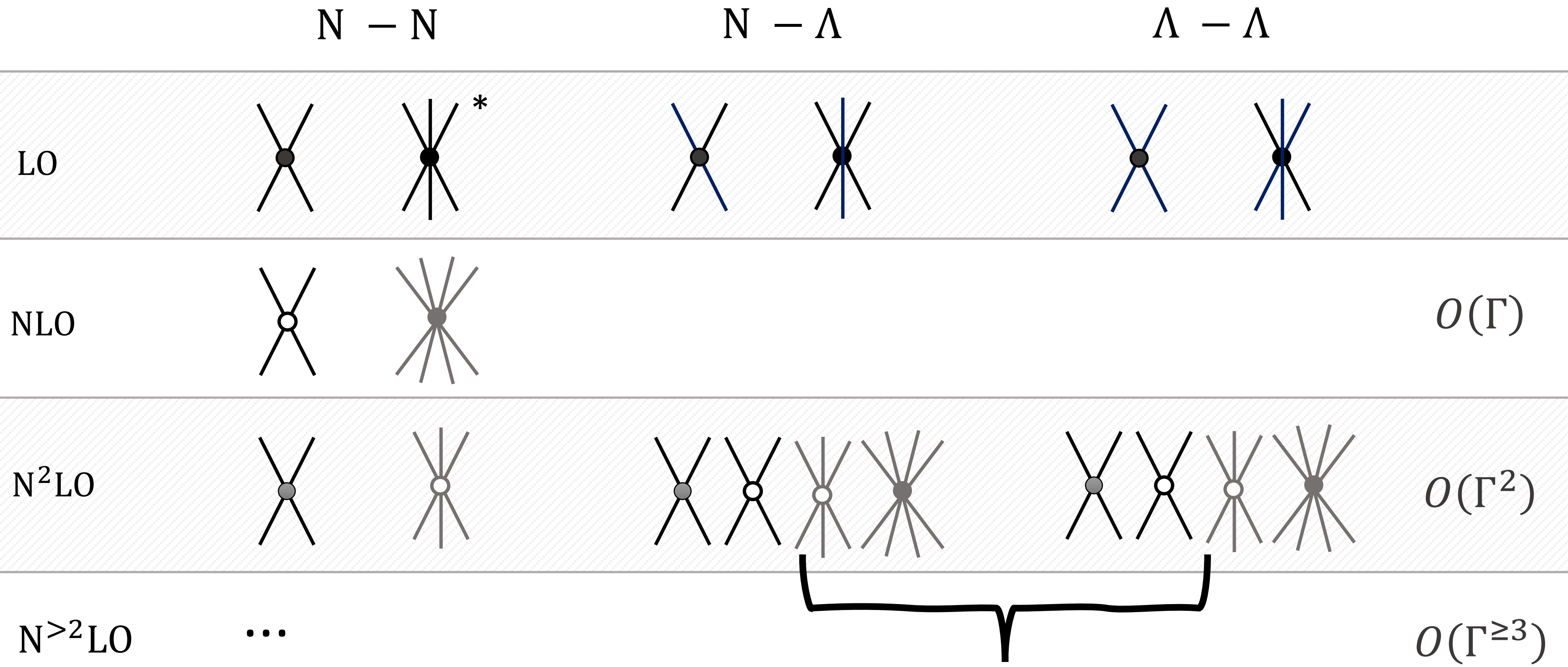
LHC

HALQCD

Abundant open queries

- **Description** of few-body hypernuclei
- **Double** Λ hypernuclei description
- **Life time** of $^{3}_{\Lambda}\mathrm{H}$ and $^{3}_{\Lambda}\mathrm{n}$
- **Charge symmetry breaking** ($^{4}_{\Lambda}\mathrm{H}$ - $^{4}_{\Lambda}$He)
- Λ^{*}(1405) **matter**
- **Neutron star** equation of state
- **...**

Pionless powercounting

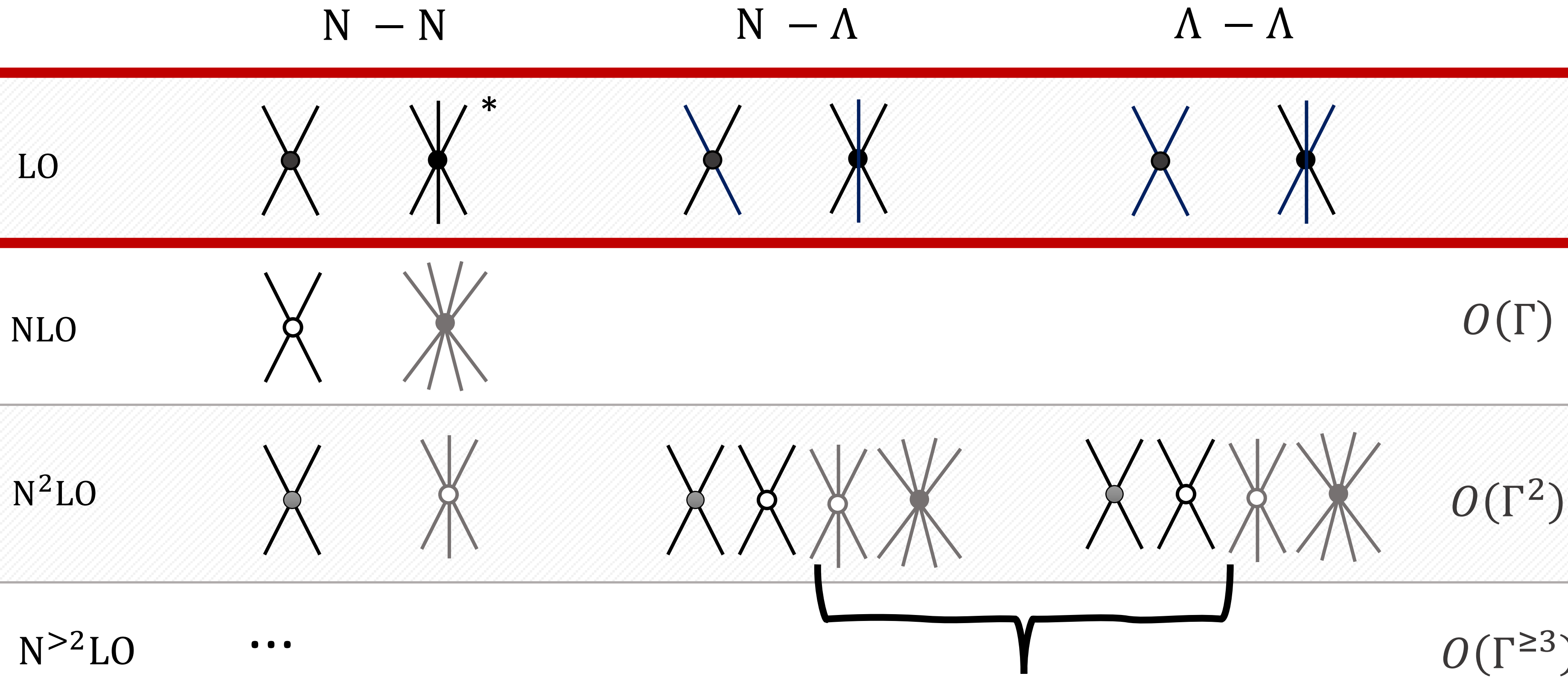


$$\Gamma_{NN} = \frac{Q}{m_\pi} = 0.5 \sim 0.8 \qquad \Gamma_{N\Lambda} = \frac{Q}{2\, m_\pi^{**}} \sim 0.2$$

B. Bazak, Four-Body Scale in Universal Few-Boson Systems, PRL 122.143001 (2019)
G.P. Lepage, How to renormalize the Schrodinger equation (1997)
van Kolck, U. Nucl.Phys. A645 (1999) 273-302
Chen, Jiunn-Wei et al. Nucl.Phys. A653 (1999)
S. K˙onig, H. W. Grießhammer, H. W. Hammer, and U. van Kolck J. Phys. G43, 055106 (2016)

* Three-body force is necessary to avoid Thomas collapse
** OPE not allowed

Pionless powercounting



$$\Gamma_{NN} = \frac{Q}{m_\pi} = 0.5 \sim 0.8$$

$$\Gamma_{N\Lambda} = \frac{Q}{2\,m_\pi^{**}} \sim 0.3$$

* Three body force is necessary to avoid Thomas collapse

** OPE not allowed

B. Bazak, Four-Body Scale in Universal Few-Boson Systems, PRL 122.143001 (2019)
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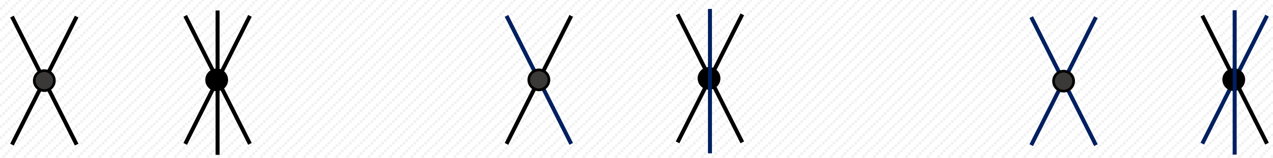
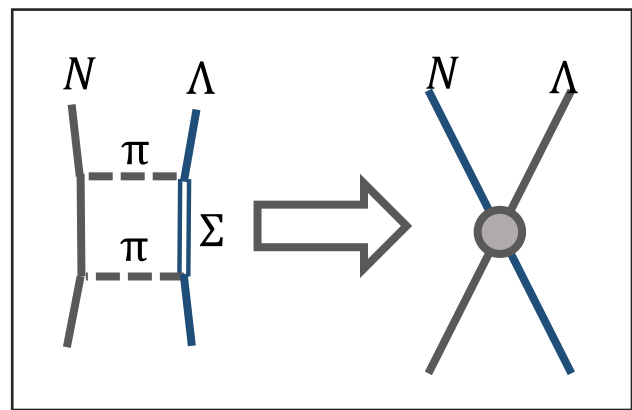
Pionless powercounting

	$N - N$	$N - \Lambda$	$\Lambda - \Lambda$

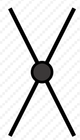
LO

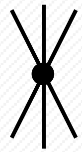


$N - N$	$N - \Lambda$	$\Lambda - \Lambda$
$S = 1, I = 0$	$S = 1, I = \frac{1}{2}$	$S = 0, I = 0$
$S = 0, I = 1$	$S = 0, I = \frac{1}{2}$	$S = \frac{1}{2}, I = \frac{1}{2}$
$S = \frac{1}{2}, I = \frac{1}{2}$	$S = \frac{1}{2}, I = 0$	
	$S = \frac{3}{2}, I = 0$	
	$S = \frac{1}{2}, I = 1$	



LO





Pionless at Leading Order

❖ **Interaction**:

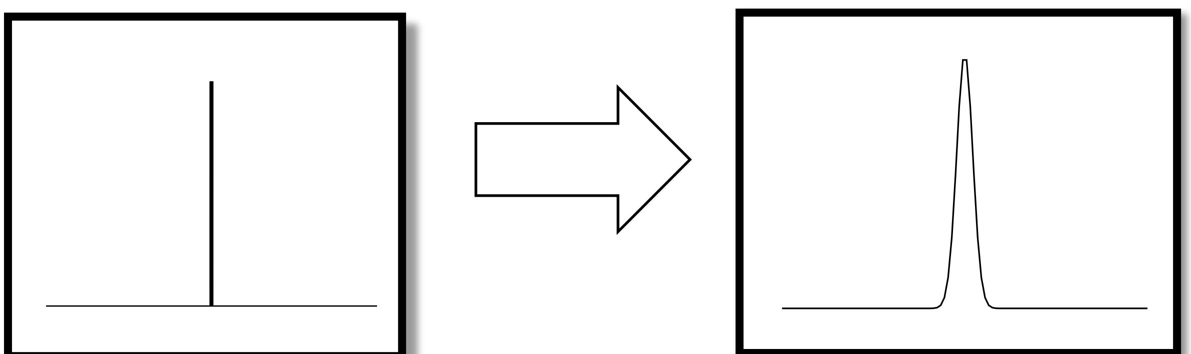
$$V = \sum_{ij} C_p\, \delta^p\left(\vec{r_i}, \vec{r_j}\right) + \sum_{ijk} D_p\, \delta^p\left(\vec{r_i}, \vec{r_j}, \vec{r_k}\right)$$

❖ **Interaction:**

$$V = \sum_{ij} C_p \, \delta^p(\vec{r_i}, \vec{r_j}) + \sum_{ijk} D_p \, \delta^p(\vec{r}_i, \vec{r}_j, \vec{r}_k)$$

p represent projections In the 2- and 3-body S-wave channels.

Regularization / Renormalization required



*Observables are **cut-off dependent:***

$$O_\lambda = O_\infty + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \frac{\gamma}{\lambda^3} + \cdots$$

$\boldsymbol{\lambda \to \infty}$ ***:**

- Regularization/model independent
- Observables are λ dependent

* $\boldsymbol{\lambda \gg M}$

LO 

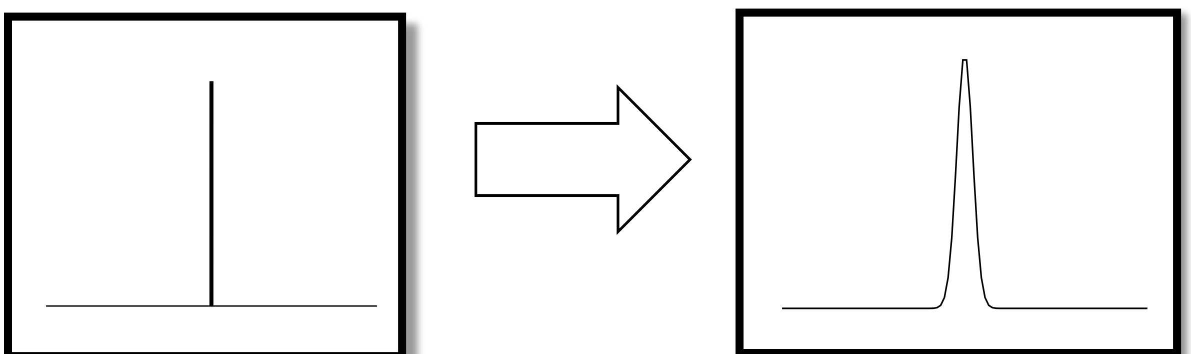
Pionless at Leading Order

❖ **Interaction:**

Projections in channels

$$V = \sum_{ij} C_p^{\lambda} P^p e^{-\frac{\lambda^2 r_{ij}^2}{4}} + \sum_{ijk} D_p^{\lambda} P^p \sum_{cyc} \left[e^{-\frac{\lambda^2 (r_{ij}^2 + r_{ik}^2)}{4}} \right]$$

Regularization / Renormalization required



*Observables are **cut-off dependent:***

$$O_\lambda = O_\infty + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \frac{\gamma}{\lambda^3} + \cdots$$

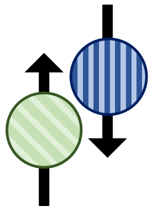
$\boldsymbol{\lambda \to \infty}$ *:

- Regularization/model independent
- Observables are λ dependent

* $\lambda \gg M$

Fitting input

Two body



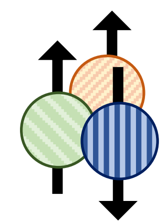
$2\times NN$ — S = 0, S = 1 → a_{NN}

$2\times N\Lambda$ — S = 0, S = 1 → $a_{N\Lambda}$

$\Lambda\Lambda$ → $a_{\Lambda\Lambda}$

No precise Experimental data ($a_{N\Lambda}$, $a_{\Lambda\Lambda}$)

Three body



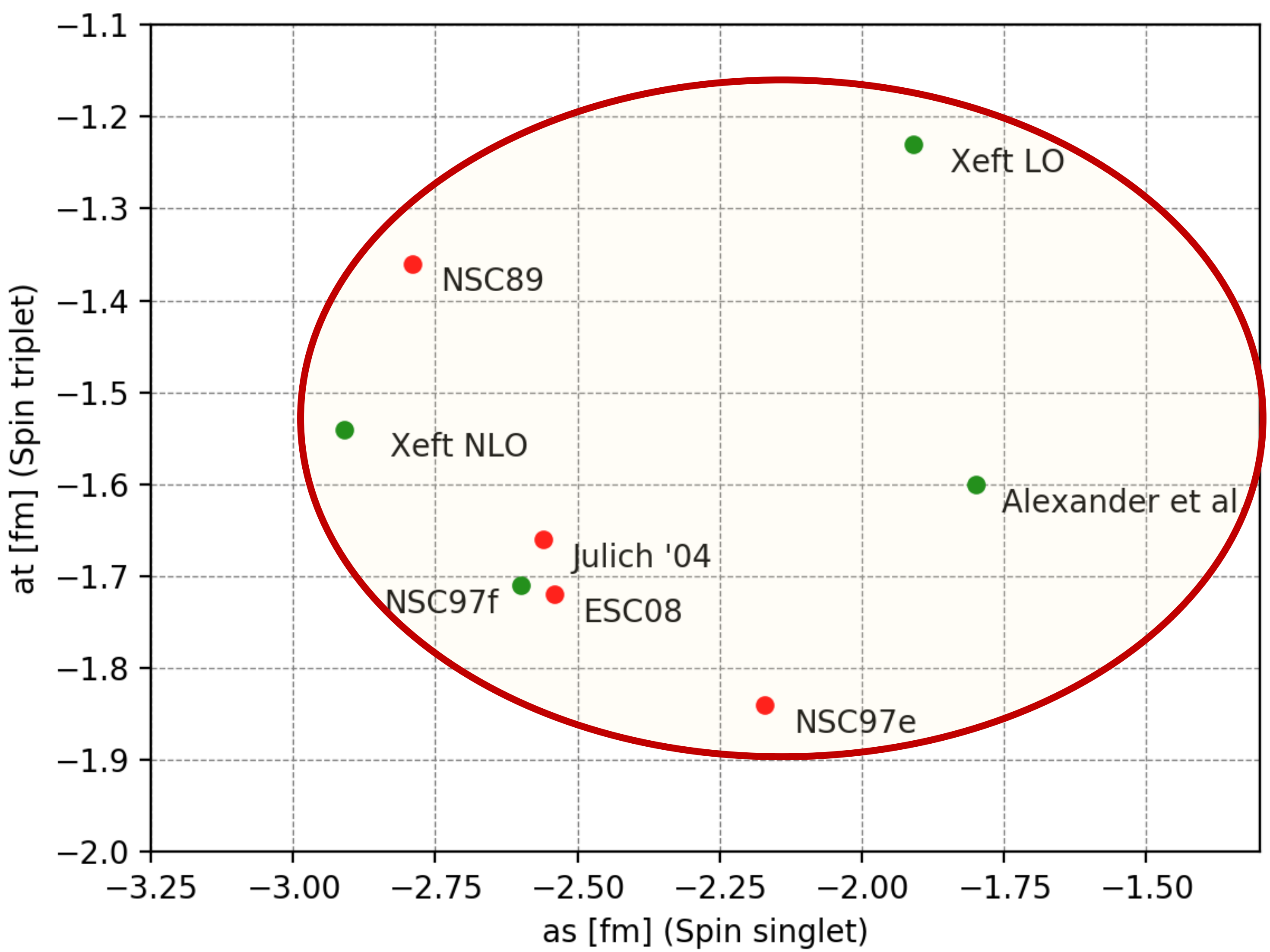
NNN $^{3}\mathrm{H}$

$3\times NN\Lambda$:
- $^{3}_{\Lambda}\mathrm{H}$
- ~~$^{3}_{\Lambda}\mathrm{H}(S=2/3)$~~ $\rightarrow$ $^{4}_{\Lambda}\mathrm{H}(S=0, I=1/2)$
- ~~$^{3}_{\Lambda}\mathrm{He}$~~ $\rightarrow$ $^{4}_{\Lambda}\mathrm{H}(S=1, I=1/2)$

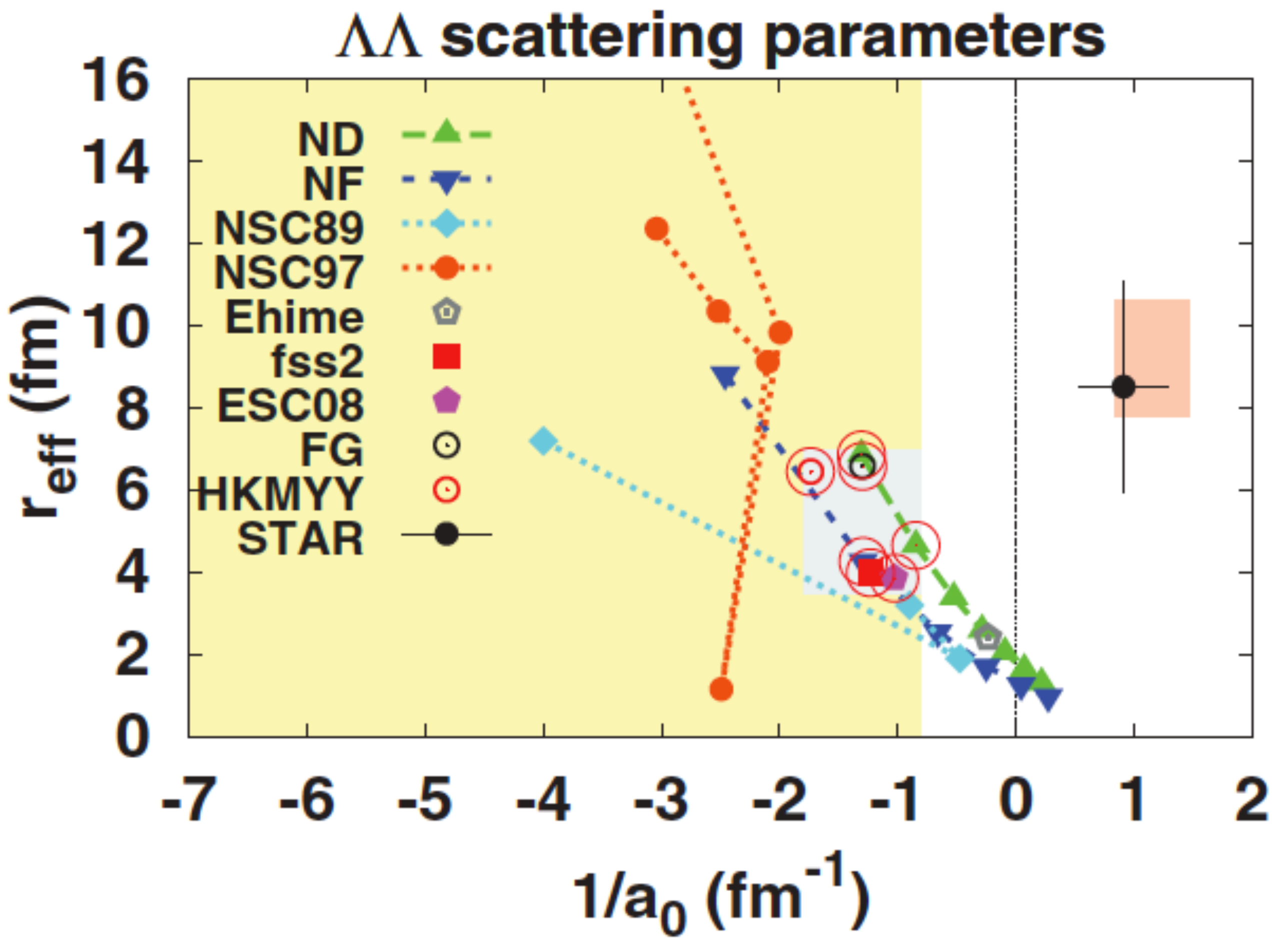
$N\Lambda\Lambda$ ~~$^{3}_{\Lambda\Lambda}\mathrm{H}$~~ $\rightarrow$ $^{6}_{\Lambda\Lambda}\mathrm{He}$

N-Λ scattering length

A. Gal et al. - Strangeness in nuclear physics - Rev.Mod.Phys. 88 (2016) no.3, 035004



ΛΛ Scattering data



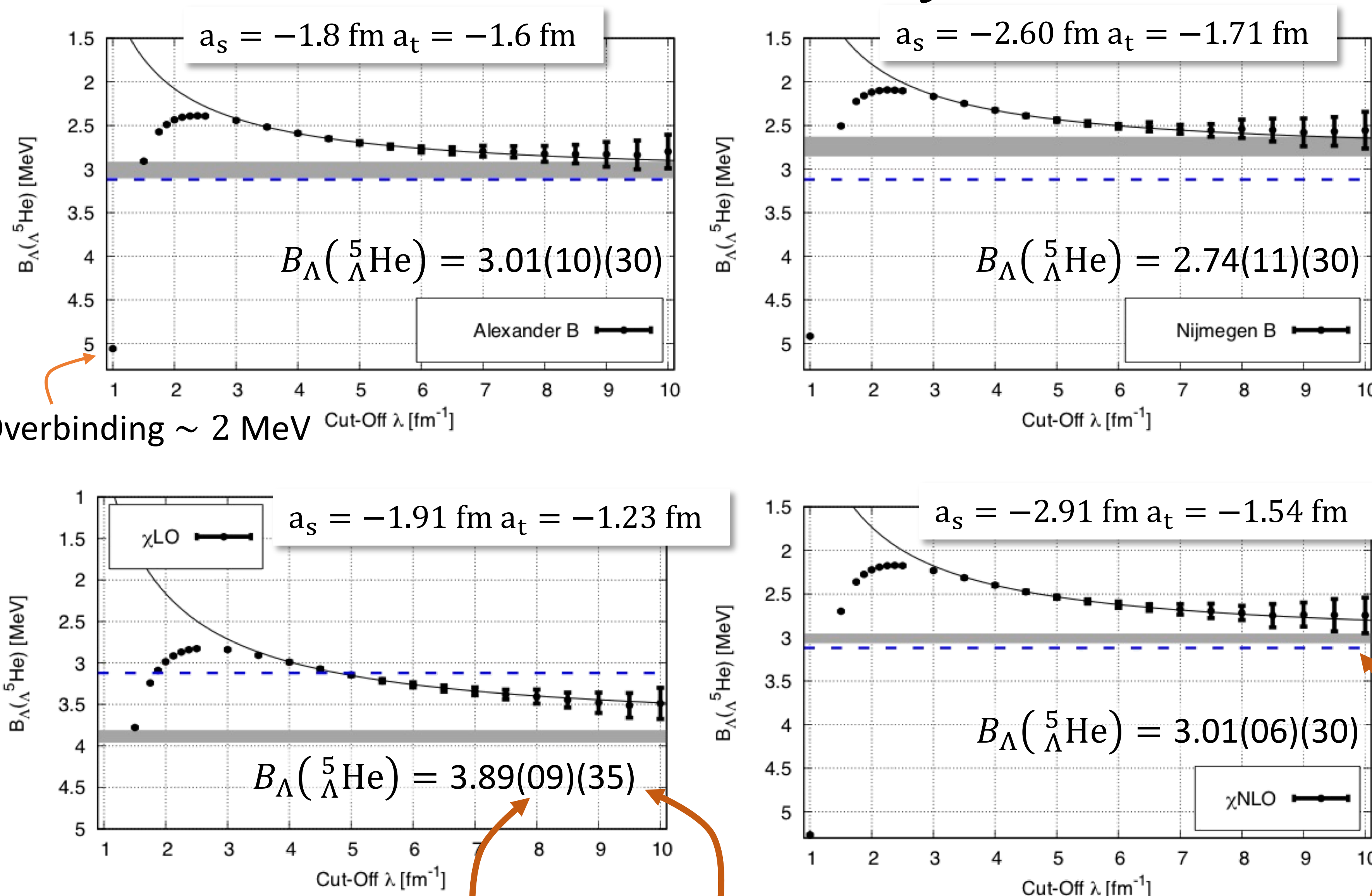
PHYSICAL REVIEW C 91, 024916 (2015)

$$a_{\Lambda\Lambda} \sim \{-0.5 \,..\, -1.9\}\ \text{fm}$$

Results

- ❖ $^{5}_{\Lambda}\mathbf{He}$
- ❖ $^{5}_{\Lambda\Lambda}\mathbf{H}$
- ❖ $\boldsymbol{np\Lambda\Lambda}$

$^{5}_{\Lambda}\mathrm{He}$: Λ separation energy



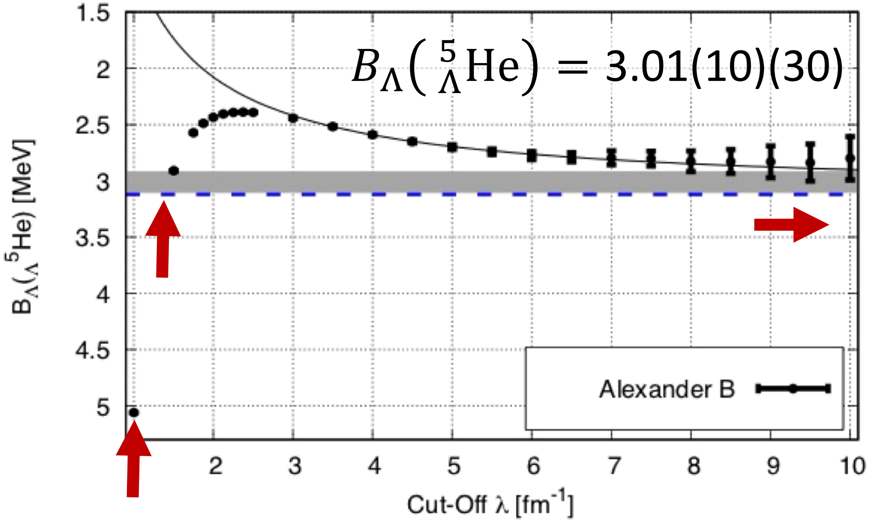
Overbinding ~ 2 MeV

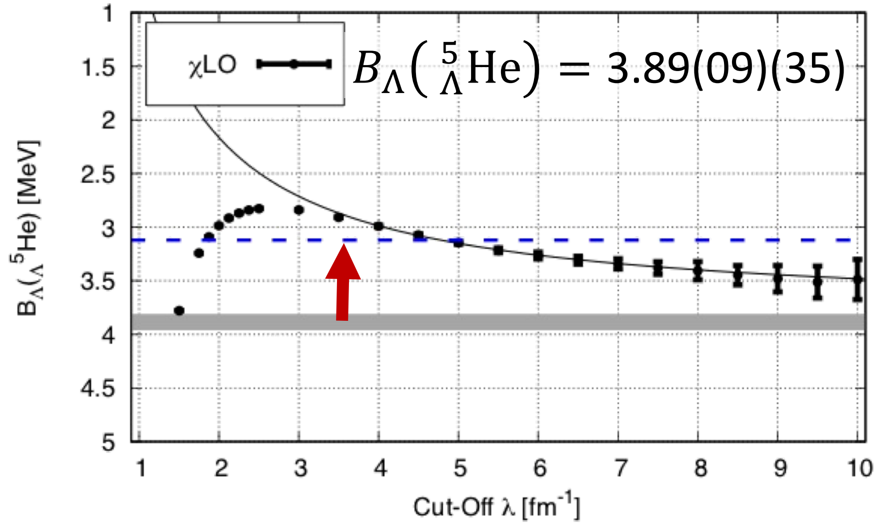
Numerical uncertainty

Theory uncertainty

Experimental: 3.12(2) MeV

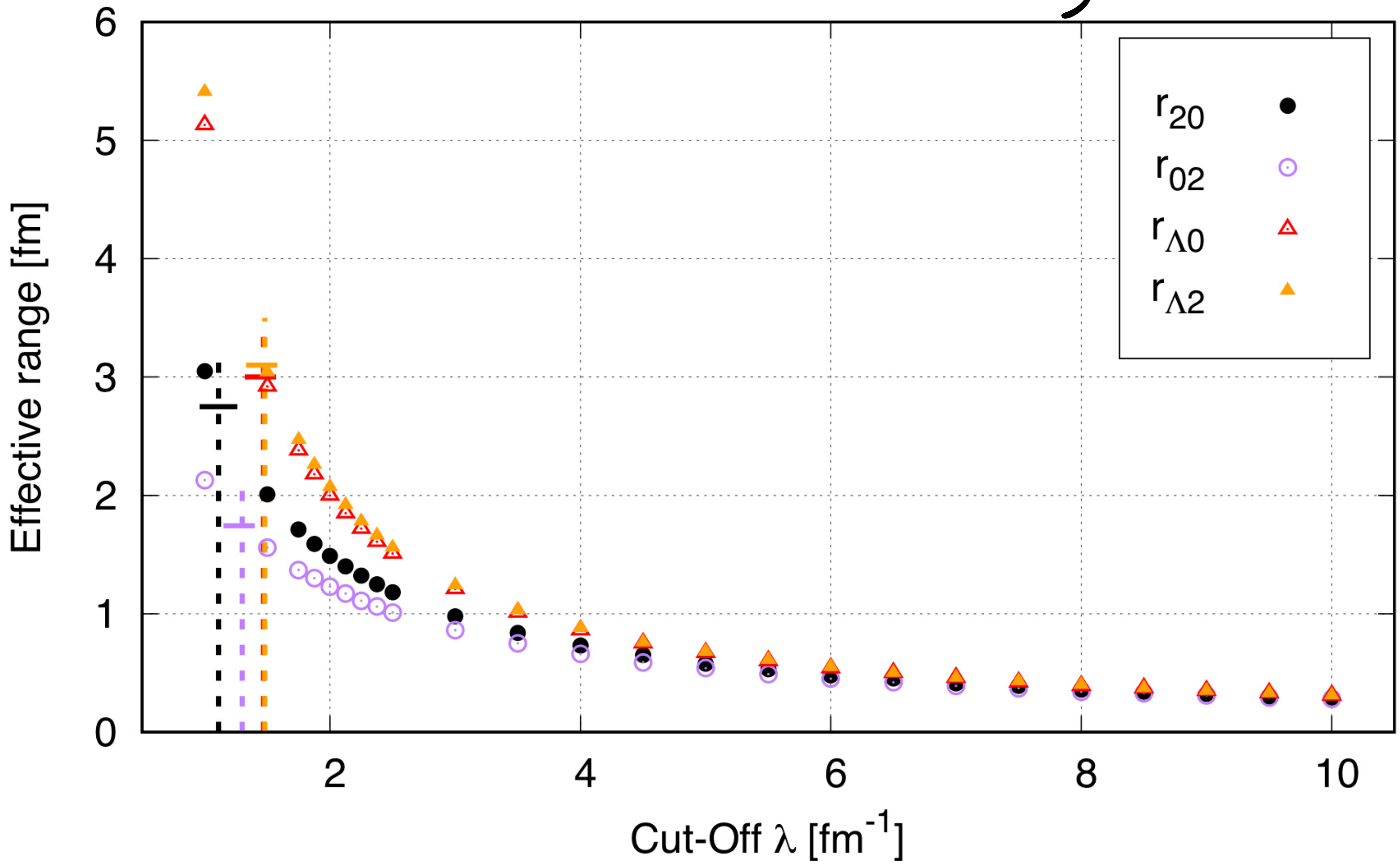
Relevant cut-offs





- $\lambda \gg M \sim 2\, m_\pi$ Is the **standard choice** for a renormalizable EFT.
- $2 < \lambda < 4\ \mathrm{fm}^{-1}$ is between $\mathbf{M \sim 2\, m_\pi}$ and the closest **not-included vector meson**.
- $\lambda \sim 1\ \mathrm{fm}^{-1} \rightarrow$ **Many similarities** with models that **overbind** ${}^5_\Lambda\mathrm{He}$**.**
- $\lambda \sim 1.5\ \mathrm{fm}^{-1}$ **describes r_0** and effectively takes into account sub-leading orders.

Effective range

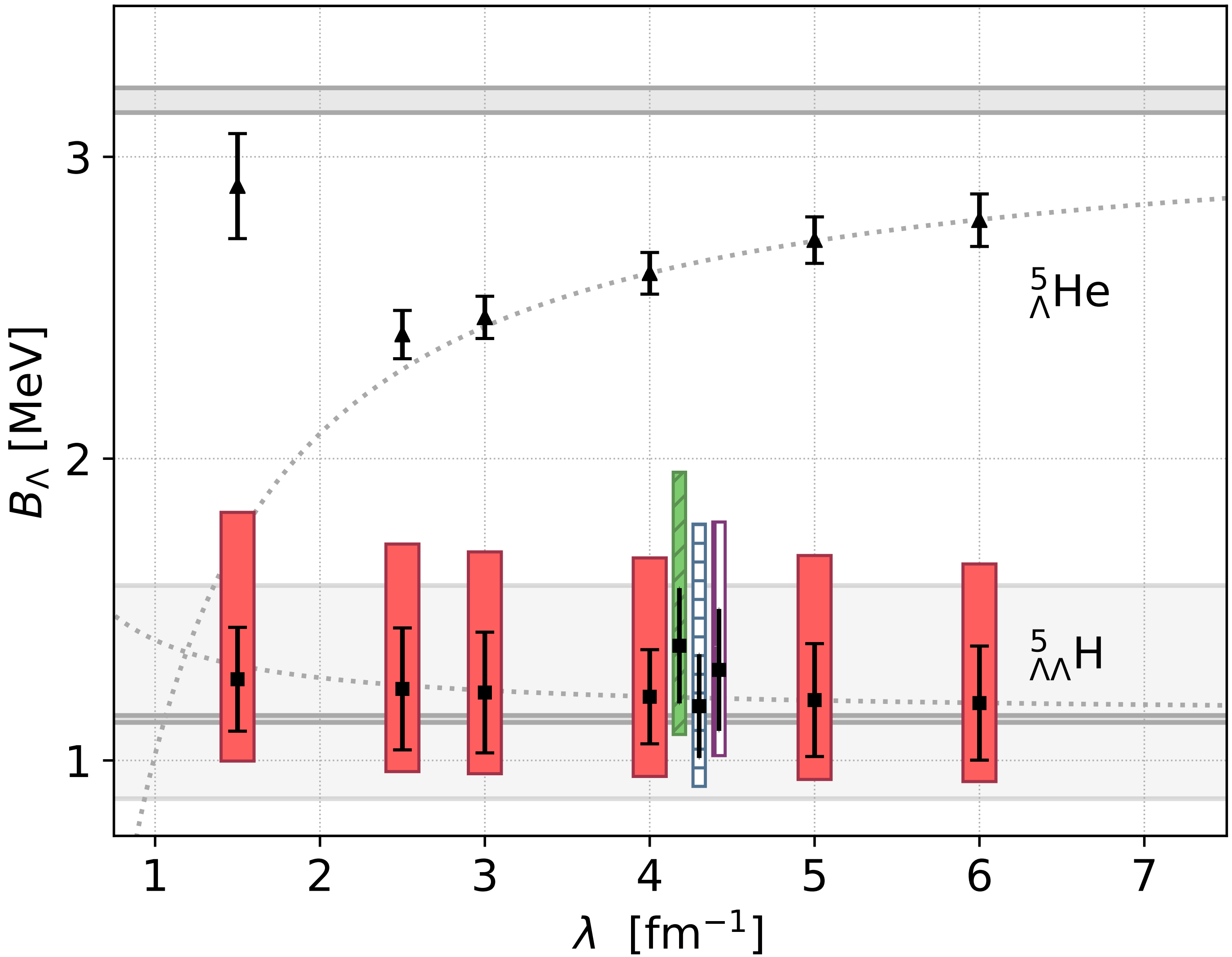


Dashed lines represent cut-off that fit the experimental values

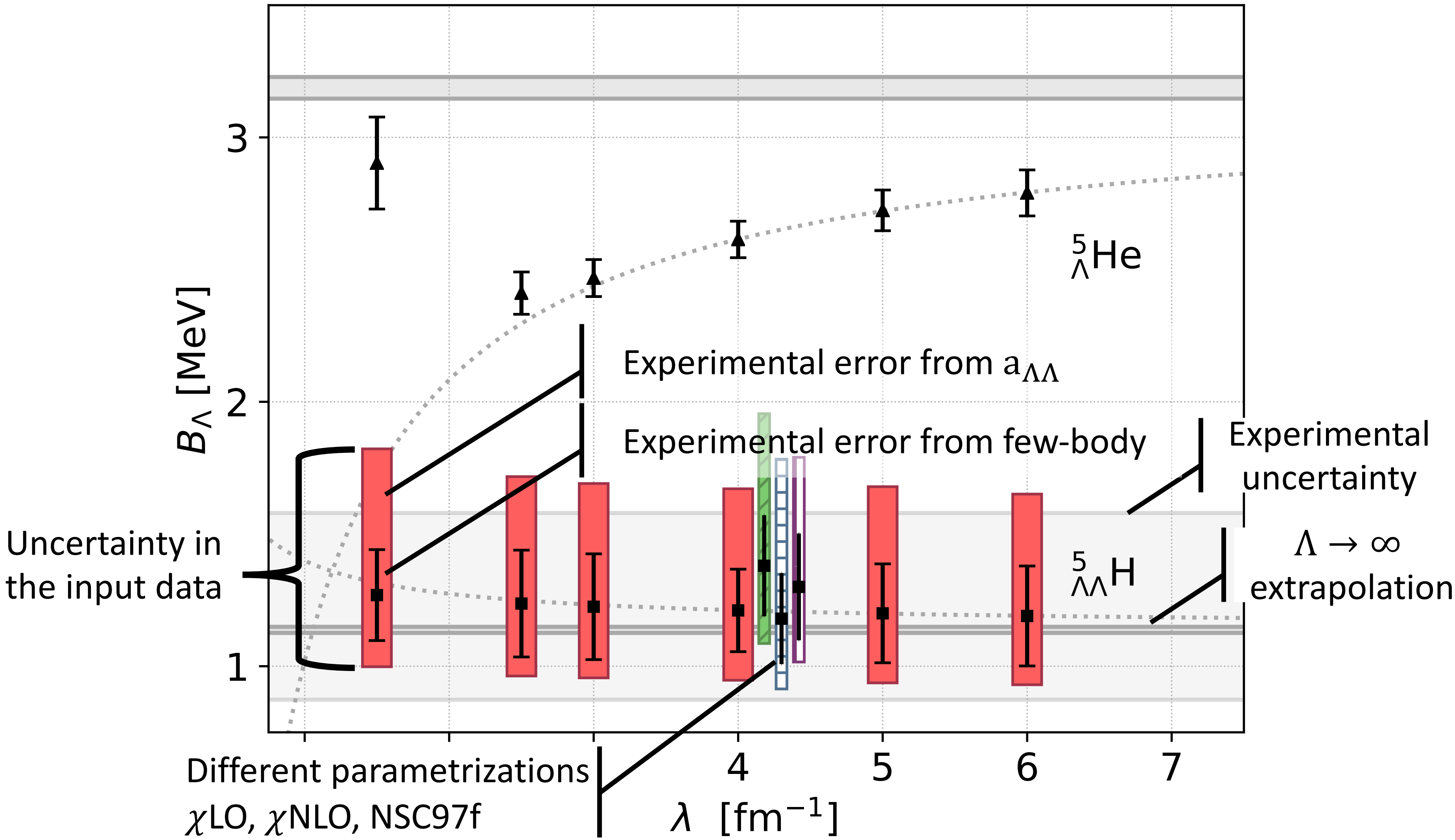
$$\lambda\left(r_0^{(20)}\right) = 1.11\ \mathrm{fm}^{-1}, \qquad \lambda\left(r_0^{(\Lambda 0)}\right) = 1.47\mathrm{fm}^{-1},$$

$$\lambda\left(r_0^{(02)}\right) = 1.30\mathrm{fm}^{-1}, \qquad \lambda\left(r_0^{(\Lambda 2)}\right) = 1.48\mathrm{fm}^{-1}.$$

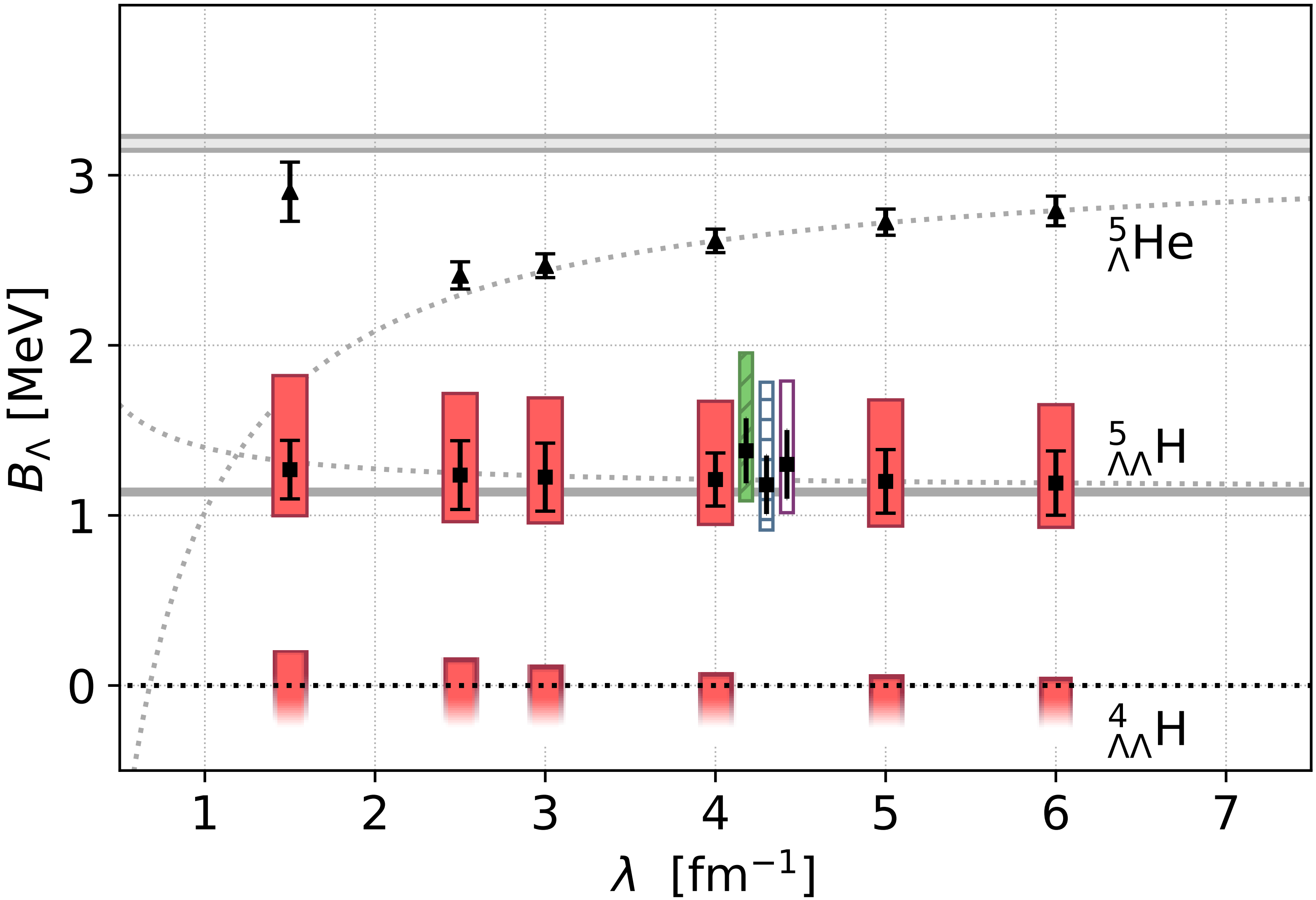
$^{5}_{\Lambda\Lambda}$H



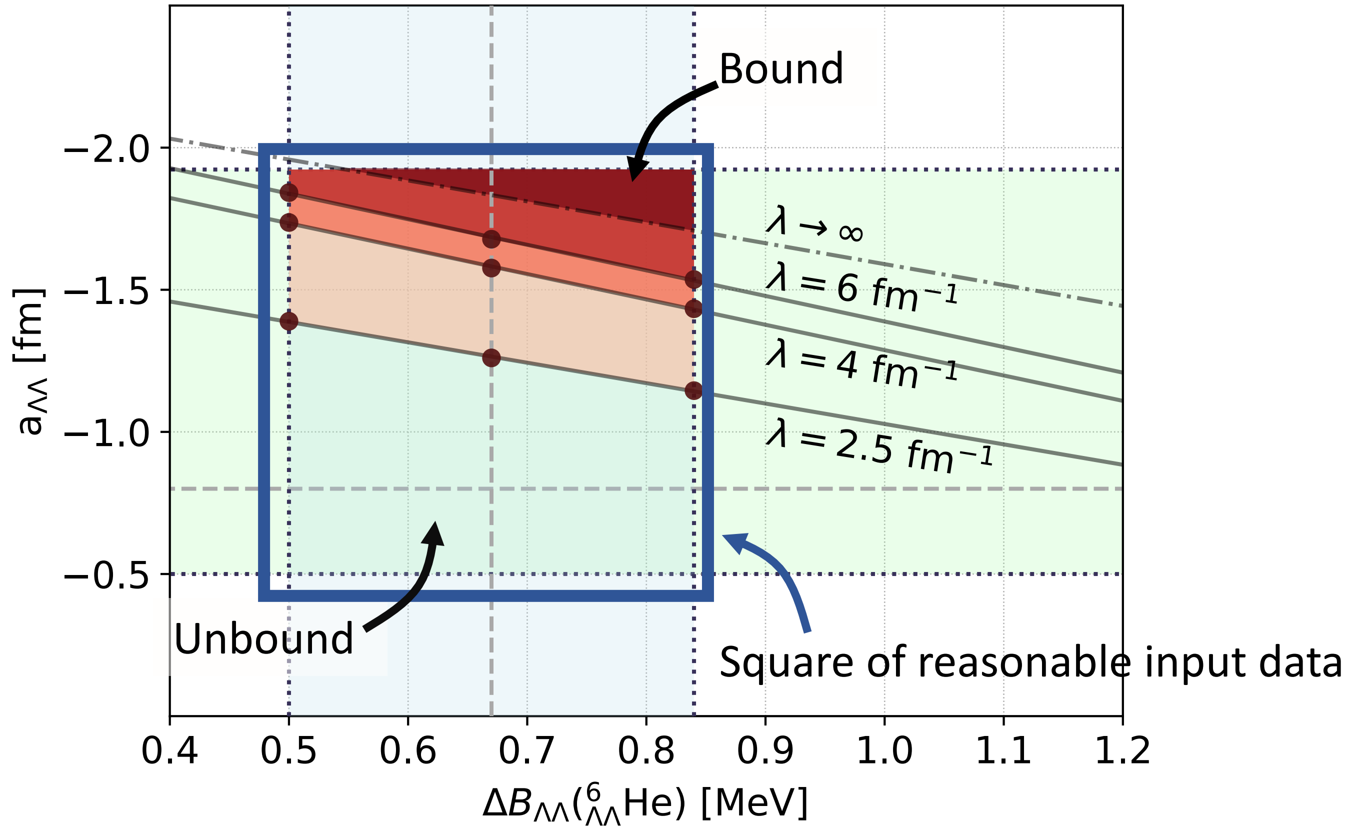
$$B_\Lambda\left({}^{\,5}_{\Lambda\Lambda}\mathrm{H}\right) = 1.14(1)^{+(44)}_{-(26)}\ \mathrm{MeV}$$



$^{4}_{\Lambda\Lambda}H$



${}^{4}_{\Lambda\Lambda}\mathrm{H}$



$$\Delta B_{\Lambda\Lambda}\left({}^{6}_{\Lambda\Lambda}\mathrm{He}\right) = B_{\Lambda\Lambda}\left({}^{6}_{\Lambda\Lambda}\mathrm{He}\right) - 2\,B_{\Lambda}\left({}^{5}_{\Lambda}\mathrm{He}\right)$$

${}^{4}_{\Lambda\Lambda}\mathrm{H}$ is bound/unbound depending to the theory input

$^{4}_{\Lambda\Lambda}\mathrm{H}$

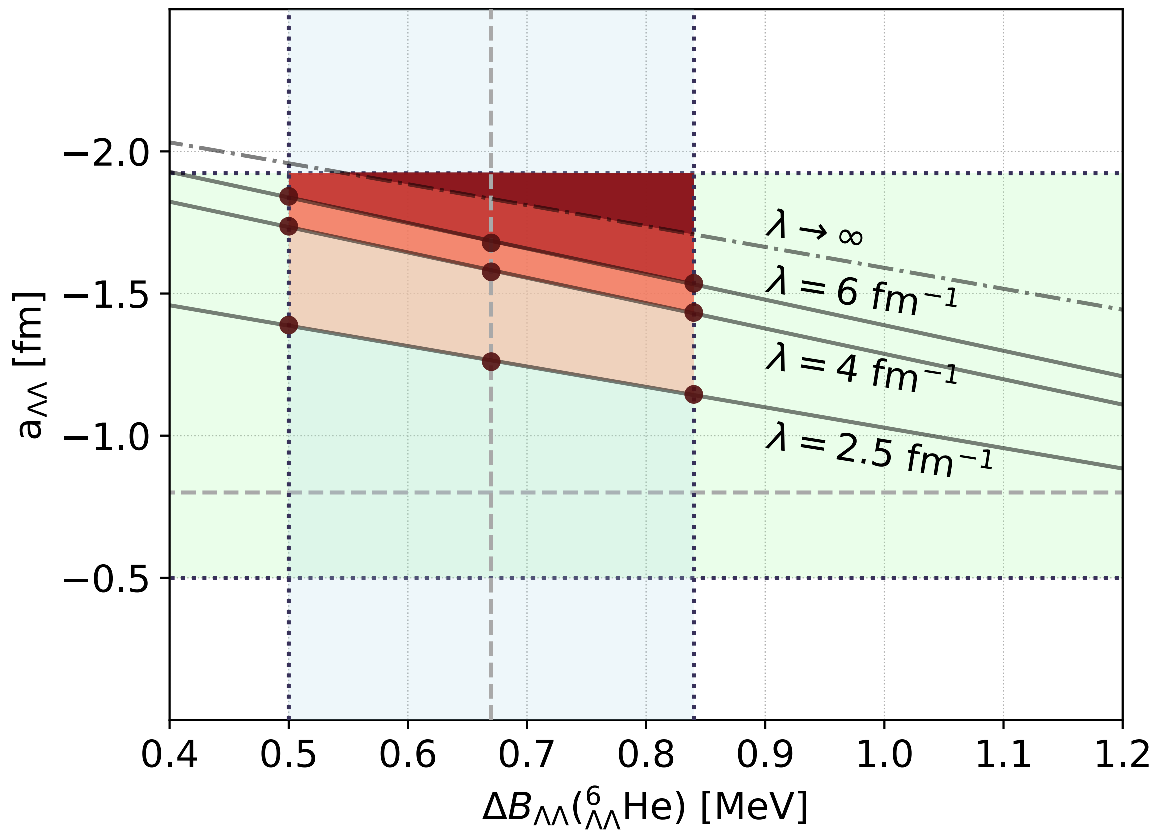


TABLE I: Λ separation energies $B_\Lambda({}^{A}_{\Lambda\Lambda}\mathrm{Z})$ for A=3–6, calculated using $a_{\Lambda\Lambda}=-0.8$ fm, cutoff $\lambda=4$ fm^{-1} and the Alexander[B] ΛN interaction model [18]. In each row a $\Lambda\Lambda N$ LEC was fitted to the underlined binding energy constraint.

Constraint (MeV)	${}^{3}_{\Lambda\Lambda}\mathrm{n}$	${}^{4}_{\Lambda\Lambda}\mathrm{n}$	${}^{4}_{\Lambda\Lambda}\mathrm{H}$	${}^{5}_{\Lambda\Lambda}\mathrm{H}$	${}^{6}_{\Lambda\Lambda}\mathrm{He}$
$\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})=\underline{0.67}$	–	–	–	1.21	3.28
$B_\Lambda({}^{4}_{\Lambda\Lambda}\mathrm{H})=\underline{0.05}$	–	–	0.05	2.28	4.76
$B({}^{4}_{\Lambda\Lambda}\mathrm{n})=\underline{0.10}$	–	0.10	0.86	4.89	7.89
$B({}^{3}_{\Lambda\Lambda}\mathrm{n})=\underline{0.10}$	0.10	15.15	18.40	22.13	25.66

Not bound

$nn\Lambda$
$n\Lambda\Lambda$
$nn\Lambda\Lambda$

In a nutshell

${}^{3}_{\Lambda}\mathrm{H}$

${}^{4}_{\Lambda}\mathrm{H}$ ${}^{4}_{\Lambda}\mathrm{H}^{*}$

${}^{5}_{\Lambda}\mathrm{He}$

It is possible to describe them all together.
(No overbinding problem!)

${}^{5}_{\Lambda\Lambda}\mathrm{He}$

Solidly bound!

$$B_{\Lambda}\left({}^{5}_{\Lambda\Lambda}\mathrm{He}\right) = 1.14(1)^{+(44)}_{-(26)}$$

$np\Lambda\Lambda$

Bound?
Need more precise
$\Lambda\Lambda$ scattering data!

$nn\Lambda$

$nn\Lambda\Lambda$

$n\Lambda\Lambda$

Unbound

Conclusions

General

- $\not{\pi}$-EFT can be applied **successfully to Λ hypernuclei**:
  (no catastrophic failure, truncation error of $\sim 10\%$ at LO).
- **7** new input data that **can** be fix on **experimental** data!
- **Overcomes overbinding** problem (comprehensive description of $A \leq 5$ Λ-hyperons)

Predictions

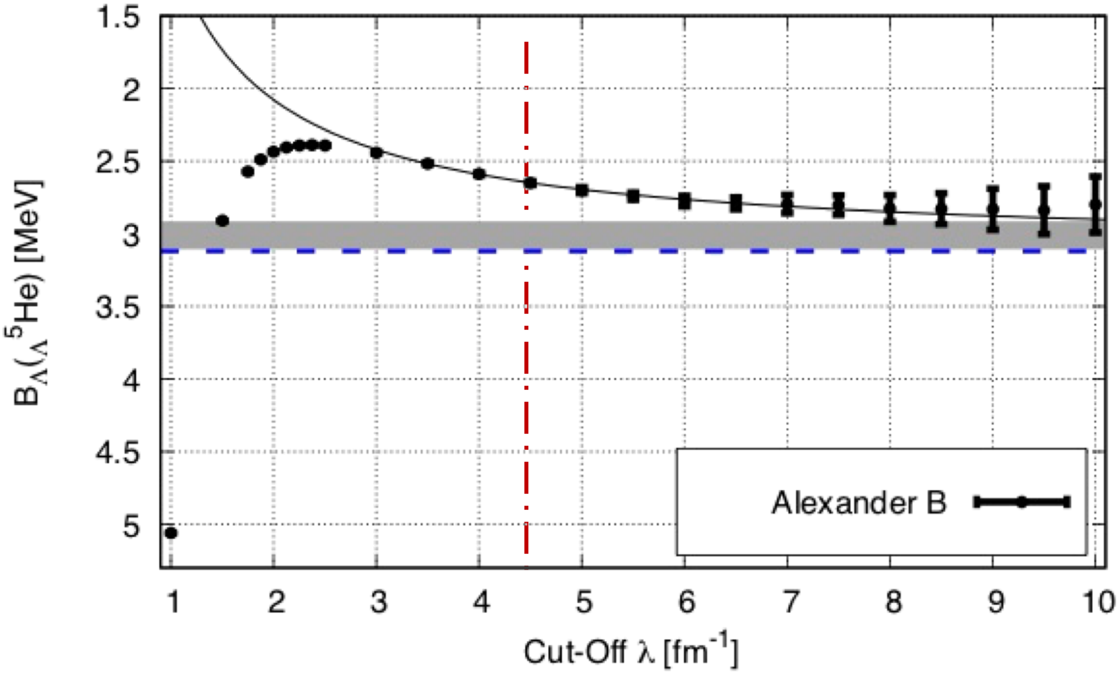
- **No** boundstate in $\boldsymbol{nn\Lambda}, \boldsymbol{np\Lambda}\left(S=\frac{3}{2}\right), \boldsymbol{n\Lambda\Lambda}$ or $\boldsymbol{nn\Lambda\Lambda}$
- $\boldsymbol{np\Lambda\Lambda}$ might be bound for large $a_{\Lambda\Lambda} < -1.5$ fm
- $^{5}_{\Lambda\Lambda}\mathbf{He}$ **bound** ($\boldsymbol{B}\left({}^{5}_{\Lambda\Lambda}\mathbf{He}\right) = \mathbf{1.14(1)}^{+(44)}_{-(26)}$ MeV)

Prospective

- **Extend** this approach to **A > 6** systems.
- Nuclear **NLO**.
- Include **subleading contributions**
  (explicit Ξ mixing, effective range, ..).

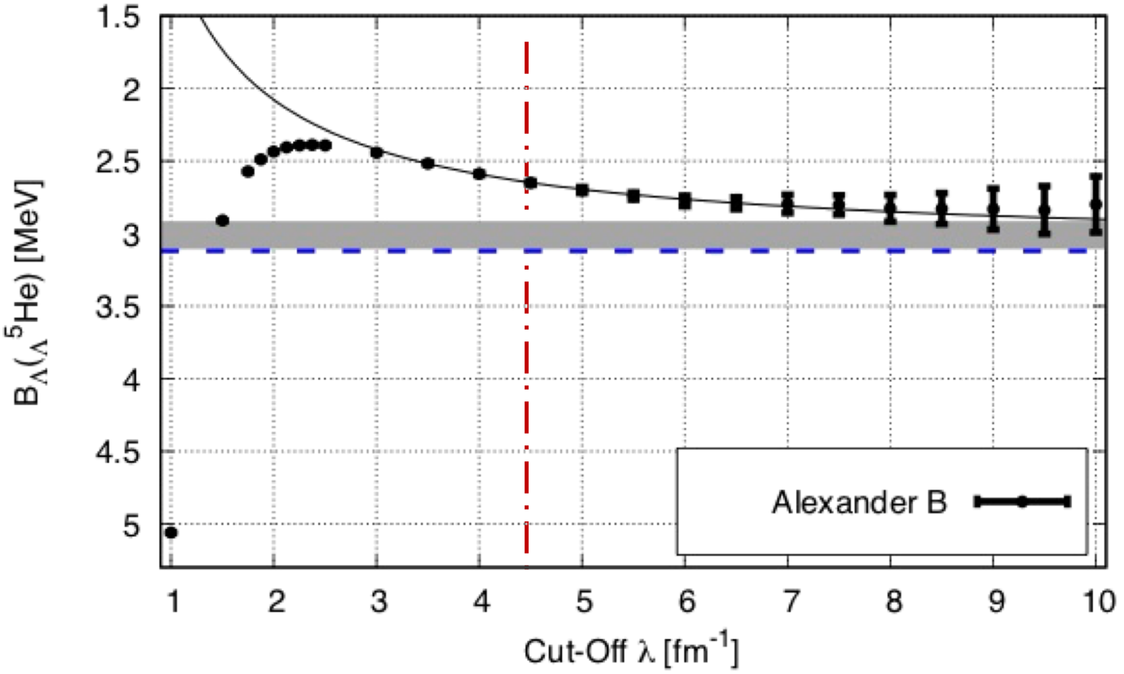
Thanks

Error sources



Numeric error	(From computational method)	~ 0.01 MeV
Extrapolation uncertainty	$(O_\lambda = O_\infty + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \frac{\gamma}{\lambda^3} + \cdots)$	From $\lambda = 4\text{fm}^{-1} \sim 0.1$ MeV
Fitting uncertainty	(Random error of the fit procedure)	~ 0.01 MeV
Experimental uncertainty	(Error propagated from experiments)	$\sim 6\ \%$
Truncation error	$(\sim \left(\frac{Q}{M}\right)^2)$	$\sim 9 - 50\ \%$ (Depends on the scales)

Error sources



Numeric error	(From computational method)	~ 0.01 MeV
Extrapolation uncertainty	$(O_\lambda = O_\infty + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \frac{\gamma}{\lambda^3} + \cdots)$	From $\lambda = 4\text{fm}^{-1} \sim 0.1$ MeV
Fitting uncertainty	(Random error of the fit procedure)	~ 0.01 MeV
Experimental uncertainty	(Error propagated from experiments)	$\sim 6\,\%$
Truncation error	$(\sim \left(\frac{Q}{M}\right)^2)$	$\sim$ 9% (Depends on the scales)

$^{5}_{\Lambda}$He: Overbinding problem

	$B_\Lambda(^{3}_{\Lambda}H)$	$B_\Lambda(^{4}_{\Lambda}H_{g.s.})$	$B_\Lambda(^{4}_{\Lambda}H_{exc.})$	$B_\Lambda(^{5}_{\Lambda}He)$
Exp.	0.13(5) [4]	2.16(8) [5]	1.09(2) [6]	3.12(2) [4]
DHT [7]	0.10	2.24	0.36	$\geq$ 5.16
AFDMCa	-	1.97(11) [8]	-	5.1(1) [9]
AFDMCb’	0.23(9) [13]	1.95(9) [13]	-	2.60(6) [13]
χEFTa	0.11 [10]	2.31 (3) [11]	0.95(15) [11]	5.82(2) [12]
χEFTb	-	2.13 (3) [11]	1.39(15) [11]	4.43(2) [12]

All the energies are in MeV.

[7] R.H. Dalitz, R.C. Herndon, and Y.C. Tang, Nucl. Phys. B 47, 109 (1972).
[8] D. Lonardoni, F. Pederiva, and S. Gandolfi, Phys. Rev. C 89, 014314 (2014).
[9] D. Lonardoni, S. Gandolfi, and F. Pederiva, Phys. Rev. C 87, 041303(R) (2013).
[10] R. Wirth et al., Phys. Rev. Lett. 113, 192502 (2014).
[11] D. Gazda and A. Gal, Phys. Rev. Lett. 116, 122501 (2016); D. Gazda and A. Gal, Nucl. Phys. A 954, 161 (2016).
[12] R. Wirth and R. Roth, Phys. Lett. B 779, 336 (2018). We thank Roland Wirth for providing us with these values.
[13] D. Lonardoni arXiv:1711.07521v2 & Private comunication.
[15] H. Nemura, Y. Akaishi, and Y. Suzuki, Phys. Rev. Lett. 89, 142504 (2002); see also Y. Akaishi, T. Harada.

N-Λ scattering data

Alexander et al. : $a_s = -1.8$ fm
$a_t = -1.6$ fm

G. Alexander, U. Karshon, A. Shapira, et al. Phys. Rev. 173, 1452 (1968)

Sechi-Zorn et al. : $0 > a_s > -9$ fm
$-0.8 > a_t > -3.2$ fm

Sechi-Zorn, B., B. Kehoe, J. Twitty, and R. A. Burnstein, 1968, Phys. Rev. 175, 1735.

TABLE VII ΛN scattering lengths and effective ranges (in fm) for several YN interaction models. For the EFT models, these refer to Λp and to cutoff parameter of 600 MeV.

Model	Reference	a^s	r_0^s	a^t	r_0^t
NSC89	Maessen, Rijken, and de Swart (1989)	-2.79	2.89	-1.36	3.18
NSC97e	Rijken, Stoks, and Yamamoto (1999)	-2.17	3.22	-1.84	3.17
NSC97f	Rijken, Stoks, and Yamamoto (1999)	-2.60	3.05	-1.71	3.33
ESC08c	Nagels, Rijken, and Yamamoto (2015b)	-2.54	3.15	-1.72	3.52
Jülich '04	Haidenbauer and Meißner (2005)	-2.56	2.75	-1.66	2.93
EFT (LO)	Polinder, Haidenbauer, and Meißner (2006)	-1.91	1.40	-1.23	2.20
EFT (NLO)	Haidenbauer *et al.* (2013)	-2.91	2.78	-1.54	2.72

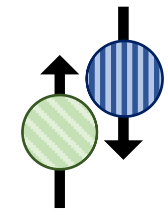
A. Gal et al. - Strangeness in nuclear physics - Rev.Mod.Phys. 88 (2016) no.3, 035004

Two body

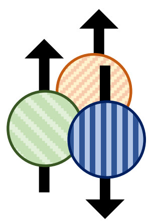
LEC	State	Fitting	
C_{02}	$S=1\,, I=0$	$^{2}\mathrm{H}$	✓
C_{20}	$S=0\,, I=1$	$\mathrm{N}-\mathrm{N}$	✓
C_{01}	$S=1\,, I=\frac{1}{2}$	$\Lambda-\mathrm{N}$	~
C_{21}	$S=0\,, I=\frac{1}{2}$	$\Lambda-\mathrm{N}$	~
C_{00}	$S=0\,, I=0$	$\Lambda-\Lambda$	~

Boundstate (C_{02})

Scattering (C_{20}, C_{01}, C_{21}, C_{00})



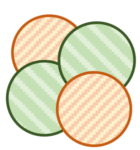
Three body

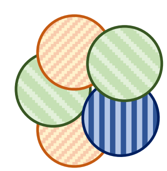


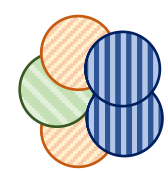
Boundstates

LEC	State		Fitting	
D_{11}	$S=\frac{1}{2}\,,$	$I=\frac{1}{2}$	$^{3}\mathrm{H}$	✓
D_{01}	$S=\frac{1}{2}\,,$	$I=0$	$^{3}_{\Lambda}\mathrm{H}$	✓
D_{03}	$S=\frac{1}{2}\,,$	$I=1$	$^{4}_{\Lambda}\mathrm{H}_{S=0\,,I=\frac{1}{2}}$	✓
D_{21}	$S=\frac{3}{2}\,,$	$I=0$	$^{4}_{\Lambda}\mathrm{H}_{S=1\,,I=\frac{1}{2}}$	✓
$D_{11}^{\Lambda\Lambda N}$	$S=\frac{1}{2}\,,$	$I=\frac{1}{2}$	$^{6}_{\Lambda\Lambda}\mathrm{He}$	✓

Predictions

$B(\,^{4}\mathrm{He})$ 

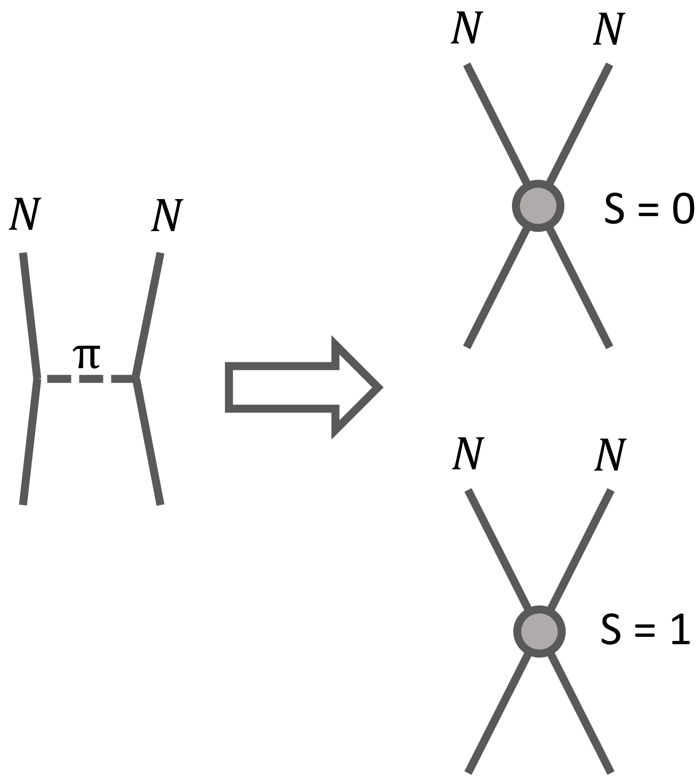
$B(\,^{5}_{\Lambda}\mathrm{He})$ 

$B(\,^{5}_{\Lambda\Lambda}\mathrm{He})$ 

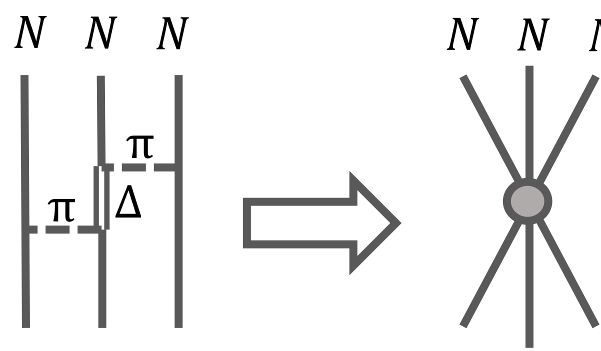
π-EFT (N)

$\boldsymbol{M}$ = Theory break-scale
$\boldsymbol{Q}$ = Typical exchanged momentum
B = Typical binding per particle

2-Body



3-Body



$\mathbf{B} \sim \boldsymbol{7\ MeV}$

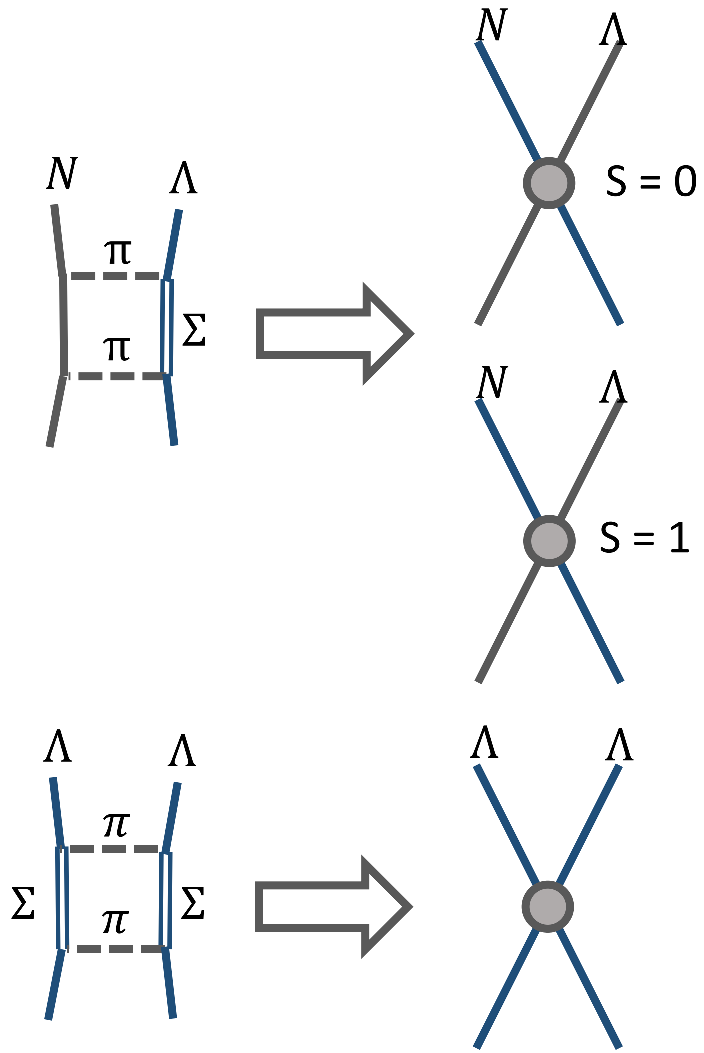
$\boldsymbol{M \sim m_\pi}$

$$\delta LO = \left(\frac{Q}{M}\right) \sim 50\%$$

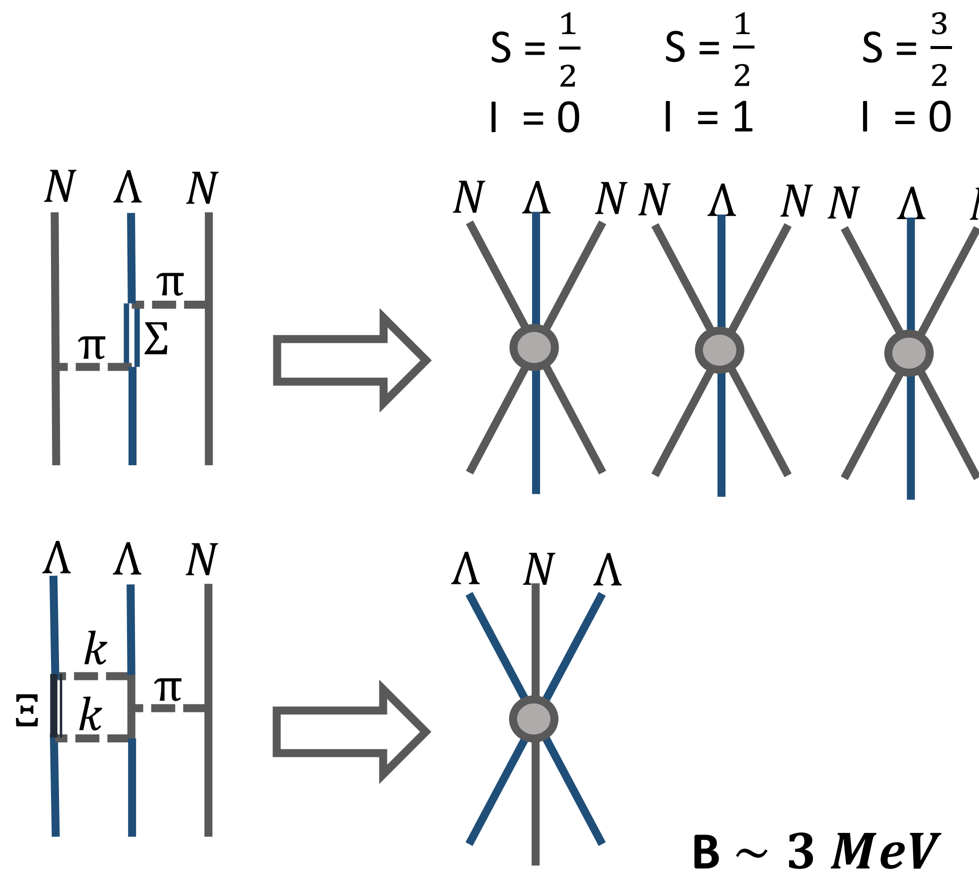
π-EFT (Λ)

M = Theory break-scale
Q = Typical exchanged momentum
B = Typical binding per particle

2-Body



3-Body



$\mathbf{B} \sim 3\ \boldsymbol{MeV}$
$\boldsymbol{M} \sim 2\ \boldsymbol{m_\pi}$

$$\delta LO = \left(\frac{Q}{M}\right)^2 \sim 9\%$$

$$V_{2b}^{\lambda} = \sum_{ij} e^{-\left(\frac{r_{ij}\,\lambda}{2}\right)^2} \left[\, C_{10}^{\lambda}\, P_{[S=1,I=0]}^{NN} + C_{01}^{\lambda}\, P_{[S=0,I=1]}^{NN} \right]$$

Three body force

The energy of the $A \geq 3$ depends on λ

When $\lambda \rightarrow \infty \longrightarrow \mathrm{E}_{3b} \rightarrow -\infty$

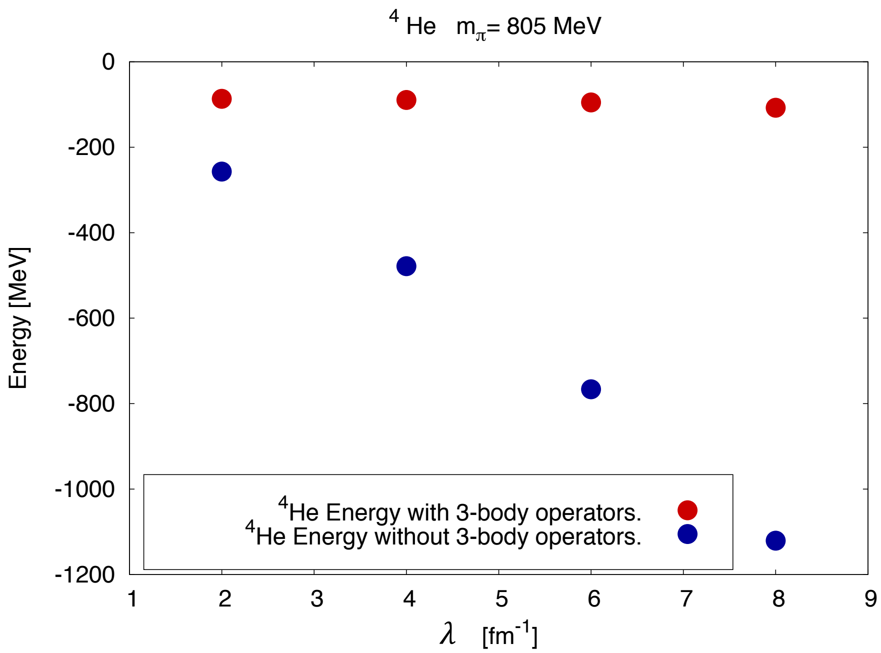
NO PREDICTIVE POWER!

Three-body force stabilize

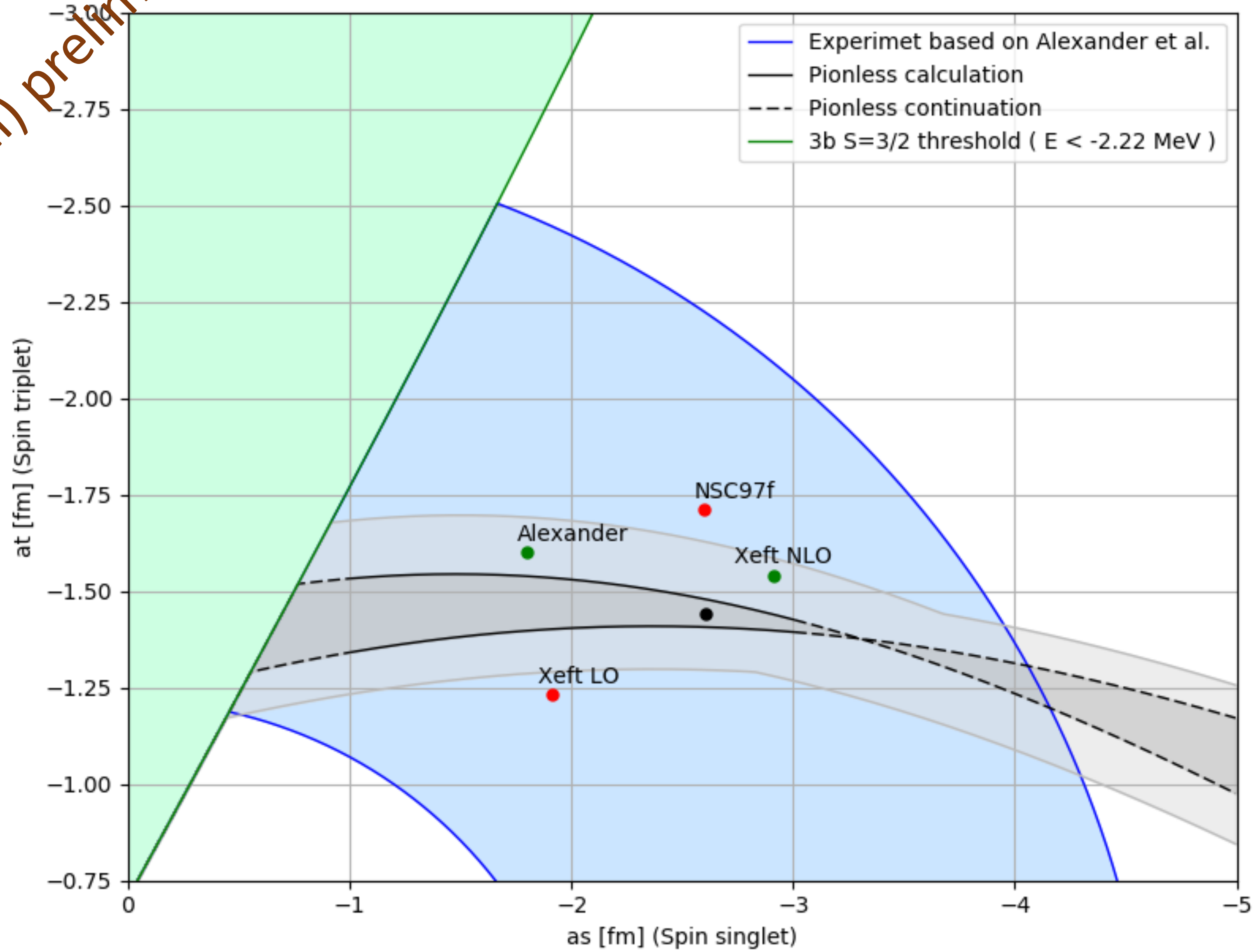
$A \geq 3$ systems

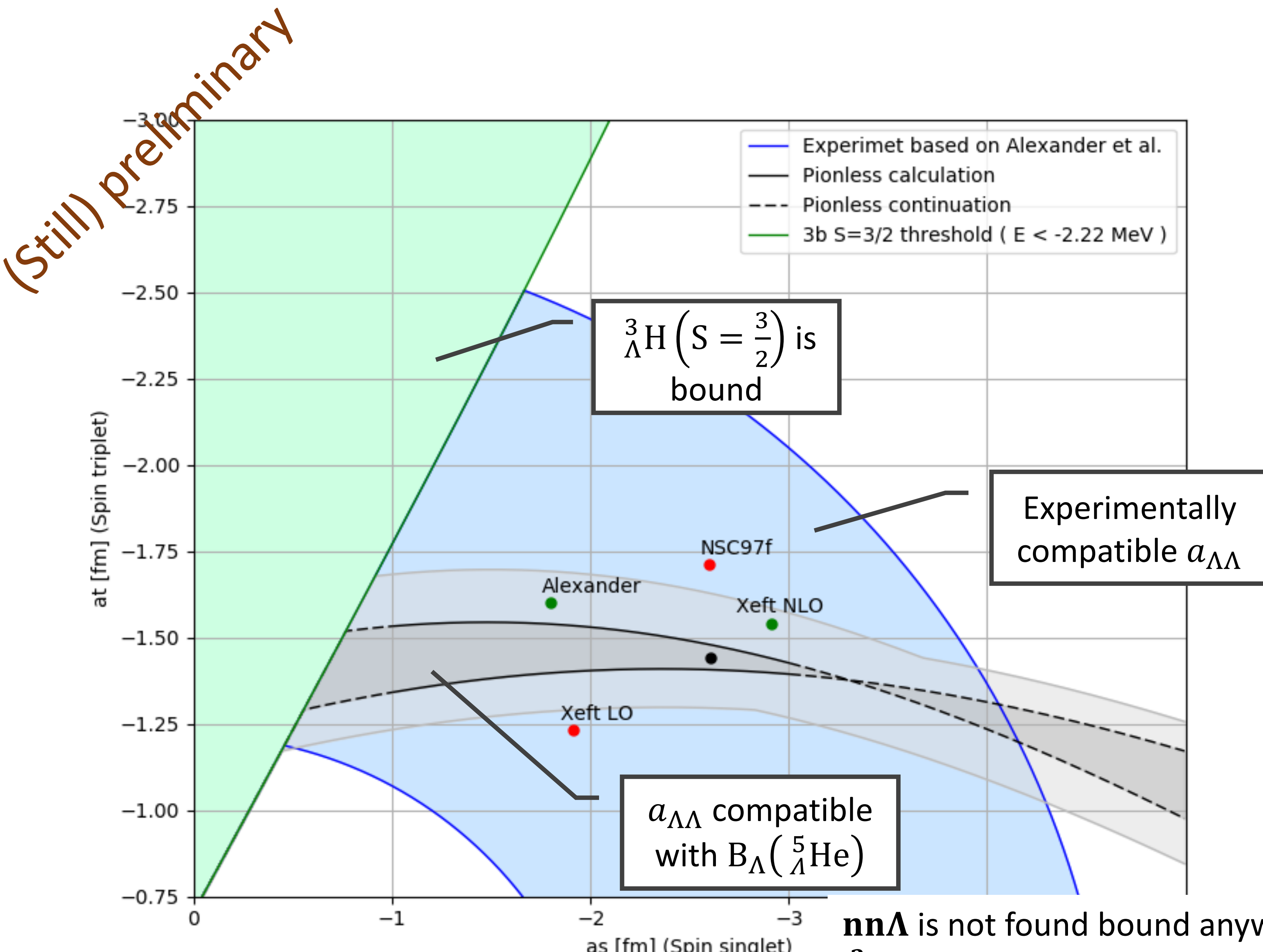
The three body force require

Regularization/Renormalization as well.



(Still) preliminary





$\mathbf{nn\Lambda}$ is not found bound anywhere

$^{3}_{\Lambda}\mathbf{H}$ is bound for "unusual" a_s and a_t