

MICROSCOPIC OPTICAL POTENTIAL FOR ELASTIC PROTON- AND ANTI-PROTON-NUCLEUS SCATTERING FROM CHIRAL FORCES

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OPTICAL POTENTIAL

The OP provides a suitable framework to describe elastic nucleon-nucleus scattering

Its use can be extended to inelastic scattering and to calculate the cross section of a wide variety of nuclear reactions

In our models for QE electron and neutrino-nucleus scattering the OP describes FSI between the emitted nucleon and the residual nucleus

OPTICAL POTENTIAL

PHENOMENOLOGICAL: assume a form and a dependence on a number of adjustable parameters for the real and imaginary parts that characterize the shape of the nuclear density distribution and that vary with the nuclear energy and the nucleus mass number.

Parameters obtained through a fit to pA elastic scattering data

OPTICAL POTENTIAL

PHENOMENOLOGICAL: assume a form and a dependence on a number of adjustable parameters for the real and imaginary parts that characterize the shape of the nuclear density distribution and that vary with the nuclear energy and the nucleus mass number.

Parameters obtained through a fit to pA elastic scattering data

THEORETICAL: microscopic calculations require the solution of the full many-body nuclear problem. Some approximations are needed.

We do not expect better description of experimental data (at least for data in the database used to generate phen. OP) but greater predictive power when applied to situations where exp. data not available

M. Vorabbi, P. Finelli, C. Giusti, PRC 93 034619 (2016)

Theoretical optical potential derived from nucleon-nucleon chiral potentials

M. Vorabbi, P. Finelli, C. Giusti, PRC 93 034619 (2016)

Theoretical optical potential derived from nucleon-nucleon chiral potentials

PHYSICAL REVIEW

VOLUME 89, NUMBER 3

FEBRUARY 1, 1953

Multiple Scattering and the Many-Body Problem—Applications to Photomeson Production in Complex Nuclei*

KENNETH M. WATSON

Physics Department, Indiana University, Bloomington, Indiana

(Received October 1, 1952)

The Scattering of Fast Nucleons from Nuclei

A. K. Kerman

Massachusetts Institute of Technology, Cambridge, Massachusetts

H. McManus

Chalk River Laboratory, Chalk River, Ontario, Canada

and

R. M. Thaler

Los Alamos Scientific Laboratory, Los Alamos, New Mexico

Received May 27, 1959

PHYSICAL REVIEW

VOLUME 92, NUMBER 2

OCTOBER 15, 1953

The Elastic Scattering of Particles by Atomic Nuclei*

N. C. FRANCIS AND K. M. WATSON†

Department of Physics, Indiana University, Bloomington, Indiana

(Received June 1, 1953)

M. Vorabbi, P. Finelli, C. Giusti, PRC 93 034619 (2016)

Theoretical optical potential derived from nucleon-nucleon chiral potentials

Purpose: study the domain of applicability of microscopic chiral potentials to the construction of an OP

Theoretical framework for pA elastic scattering

We start from the full (A+1) body LS equation

$$T = T + VG_0(E)VT$$

Separation into two coupled integral equations

$$T = U + G_0(E)PT$$

$$U = V + VG_0(E)QU$$

T transition op. for
elastic scattering,

U OP

Free propagator

$$G_0(E) = (E - H_0 + i\epsilon)^{-1}$$

Projection operators

$$P + Q = 1$$

Free Hamiltonian

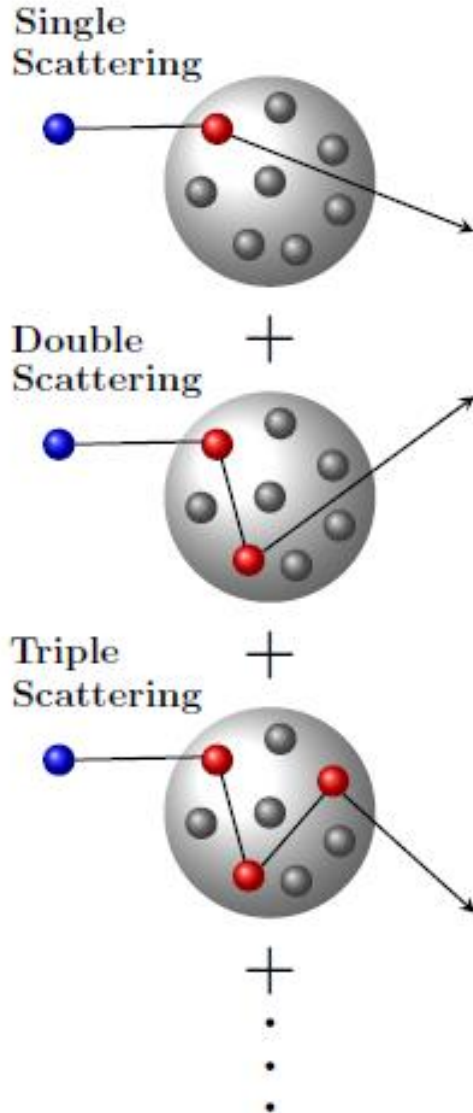
$$H_0 = h_0 + H_A$$

External interaction

$$V = \sum_{i=1}^A v_{0i}$$

The spectator expansion

Consistent framework to calculate U and T

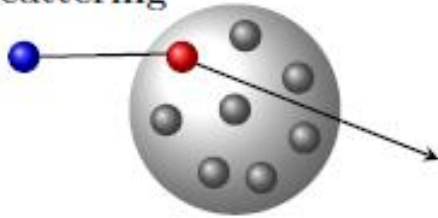


$$U = \sum_{i=1}^A \tau_i + \sum_{i,j \neq i}^A \tau_{ij} + \sum_{i,j \neq i, k \neq i,j}^A \tau_{ijk} + \dots$$

The spectator expansion

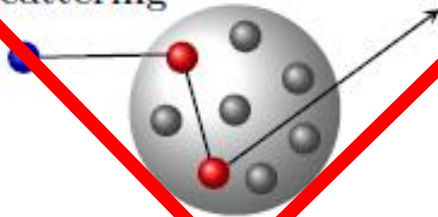
Consistent framework to calculate U and T

Single Scattering



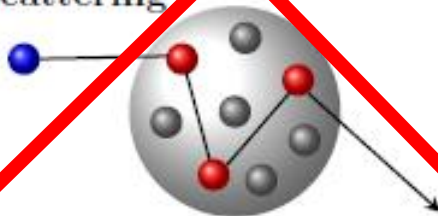
$$U = \sum_{i=1}^A \tau_i$$

Double Scattering



$$+ \sum_{i,j \neq i}^A \tau_{ij}$$

Triple Scattering

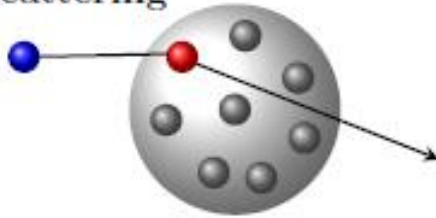


$$+ \sum_{i,j \neq i, k \neq i,j}^A \tau_{ijk} + \dots$$

The spectator expansion

Consistent framework to calculate U and T

Single
Scattering



$$U = \sum_{i=1}^A \tau_i$$

$$\tau_i = v_{0i} + v_{0i} G_0(E) Q \tau_i$$

Impulse Approximation

$$\tau_i \approx t_{0i}$$

The free NN t matrix

$$t_{0i} = v_{0i} + v_{0i}g_i t_{0i}$$

The free two-body propagator

$$g_i = \frac{1}{E - h_0 - h_i + i\epsilon}$$

$$U = \sum_{i=1}^A t_{0i}$$

Impulse Approximation

$$\tau_i \approx t_{0i}$$

The free NN t matrix

$$t_{0i} = v_{0i} + v_{0i}g_i t_{0i}$$

The free two-body propagator

$$g_i = \frac{1}{E - h_0 - h_i + i\epsilon}$$

$$U = \sum_{i=1}^A t_{0i}$$

We have to solve only 2-body equations

Optimum Factorization Approximation

$$U(\mathbf{q}, \mathbf{K}; \omega) = \frac{A-1}{A} \eta(\mathbf{q}, \mathbf{K}) \sum_{N=n,p} t_{pN} \left[\mathbf{q}, \frac{A+1}{A} \mathbf{K}; \omega \right] \rho_N(\mathbf{q})$$


NN t-matrix
NN interaction

n, p densities

$$\mathbf{q} = \mathbf{k}' - \mathbf{k}$$

$$\mathbf{K} = \frac{1}{2} (\mathbf{k}' + \mathbf{k})$$

Optimum Factorization Approximation

$$U(\mathbf{q}, \mathbf{K}; \omega) = \frac{A-1}{A} \eta(\mathbf{q}, \mathbf{K}) \sum_{N=n,p} t_{pN} \left[\mathbf{q}, \frac{A+1}{A} \mathbf{K}; \omega \right] \rho_N(\mathbf{q})$$


Moeller factor
imposes the Lorentz invariance of
the flux when passing from the
NA to the NN frame in which the
t matrices are evaluated

$$U(q, K; \omega) = U^c(q, K; \omega) + \frac{i}{2} \sigma \cdot q \times K U^{ls}(q, K; \omega)$$

central
spin-orbit

$$U^c(q, K; \omega) = \frac{A-1}{A} \eta(q, K) \sum_{N=n,p} t_{pN}^c \left[q, \frac{A+1}{A} K; \omega \right] \rho_N(q)$$

$$U^{ls}(q, K; \omega) = \frac{A-1}{A} \eta(q, K) \frac{A+1}{2A} \sum_{N=n,p} t_{pN}^{ls} \left[q, \frac{A+1}{A} K; \omega \right] \rho_N(q)$$

$$U(q, K; \omega) = \underbrace{U^c(q, K; \omega)}_{\text{central}} + \frac{i}{2} \sigma \cdot q \times K \underbrace{U^{ls}(q, K; \omega)}_{\text{spin-orbit}}$$

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NN t-matrix
NN interaction

n, p densities

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↑
 NN t-matrix
 NN interaction

↑
 n, p densities

■ **n, p densities** calculated within the RMF description of spherical nuclei using a DDME model

■ **NN interaction** chiral potentials...

NUCLEAR DENSITY DISTRIBUTION

- RMF using a DDME where the couplings between meson and baryon fields are functions of the density

DDME1 T. Nikšić, D. Vretenar, P. Finelli, P. Ring, PRC 66 (2002) 024306

- RMF: nucleus system of Dirac nucleons coupled to the exchange mesons (σ , ω , ρ) and elm fields through an effective Lagrangian, whose parameters adjusted to reproduce NM EOS and global properties of spherical closed-shell nuclei

$$U(q, K; \omega) = \underbrace{U^c(q, K; \omega)}_{\text{central}} + \frac{i}{2} \sigma \cdot q \times K \underbrace{U^{ls}(q, K; \omega)}_{\text{spin-orbit}}$$

$$U^c(q, K; \omega) = \frac{A-1}{A} \eta(q, K) \sum_{N=n,p} t_{pN}^c \left[q, \frac{A+1}{A} K; \omega \right] \rho_N(q)$$

$$U^{ls}(q, K; \omega) = \frac{A-1}{A} \eta(q, K) \frac{A+1}{2A} \sum_{N=n,p} t_{pN}^{ls} \left[q, \frac{A+1}{A} K; \omega \right] \rho_N(q)$$

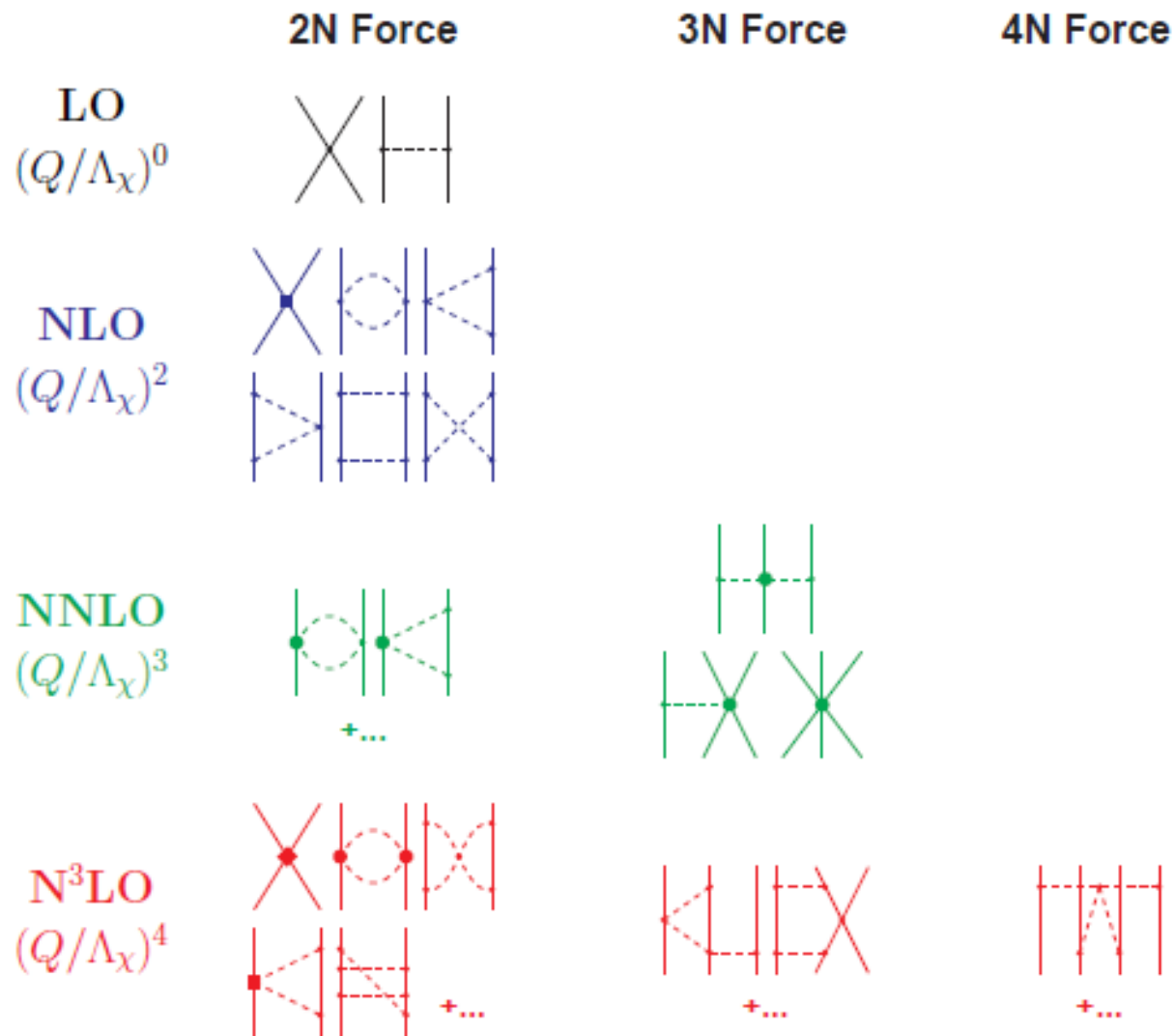
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■ **NN interaction** chiral potentials...

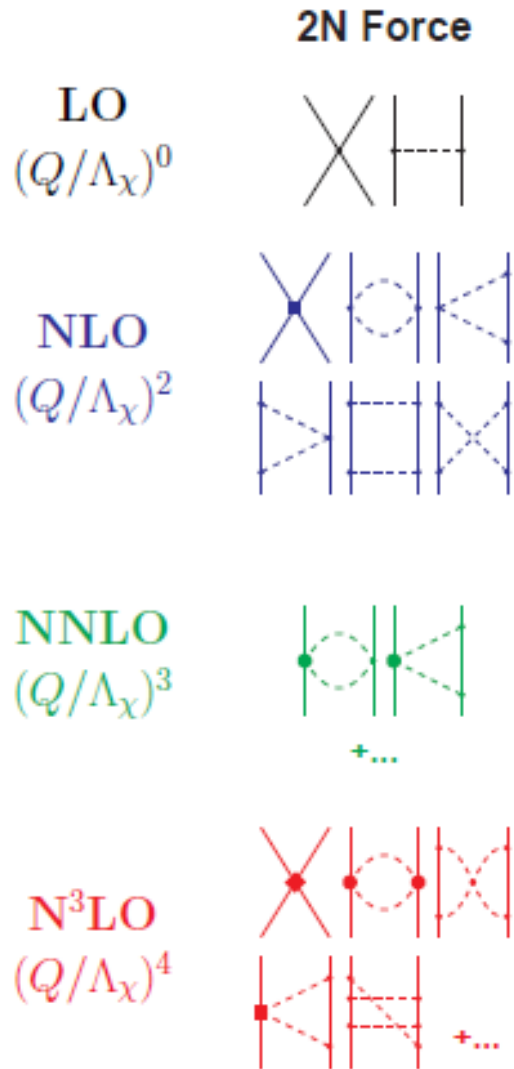
CHIRAL POTENTIAL

- When the concept of EFT was applied to low-energy QCD, ChPT was developed
- Within ChPT it became possible to implement chiral symmetry consistently in a theory of pionic and nuclear interactions
- The theory is based on a perturbative expansion in powers of $(Q/\Lambda_\chi)^n$ where Q is the magnitude of the three-momentum of the external particles or the pion mass and Λ_χ is the chiral symmetry breaking scale of the chiral EFT
- From the perturbative expansion only a finite number of terms contribute at a given order

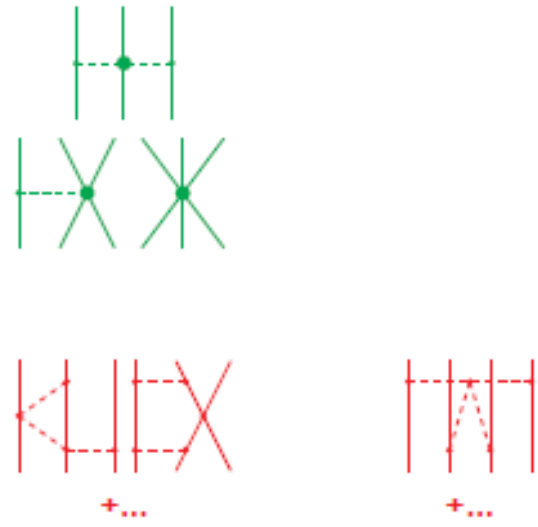
CHIRAL POTENTIAL



CHIRAL POTENTIAL



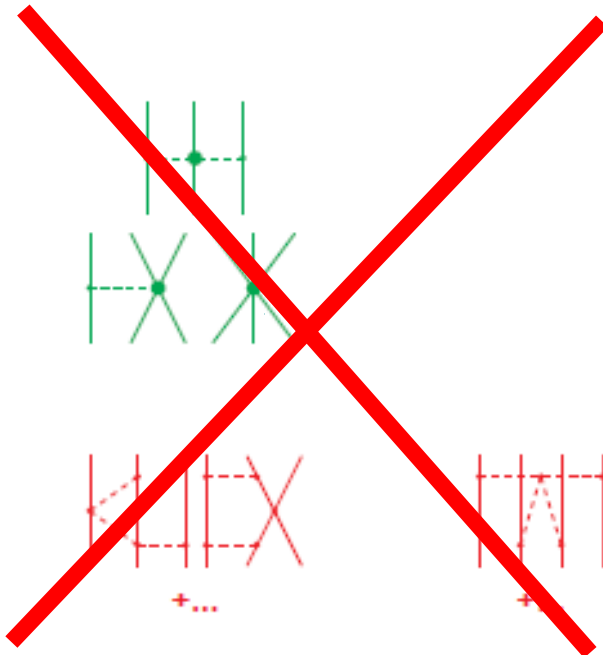
3N Force	4N Force
<p>3N forces start at 3rd order, 4N forces start at 4th order 2 and many-body forces are created on an equal footing and emerge in increasing order going to higher order</p>	



CHIRAL POTENTIAL

	2N Force	3N Force	4N Force
LO $(Q/\Lambda_\chi)^0$			
NLO $(Q/\Lambda_\chi)^2$			
NNLO $(Q/\Lambda_\chi)^3$			
N ³ LO $(Q/\Lambda_\chi)^4$			

CHIRAL POTENTIAL AT N³LO
ONLY 2N



CHIRAL POTENTIAL

Two different versions of chiral potentials at N³LO
Entem and Machleidt (EM) , Epelbaum et al. (EGM)

In general the integral in the LS eq . is divergent and needs to be regularized

Usual procedure: $V(k', k) \longmapsto V(k', k) e^{-(k'/\Lambda)^{2n}} e^{-(k/\Lambda)^{2n}}$

EM present results with $\Lambda = 450, 500, 600$ MeV

EGM present results with $\Lambda = 450, 550, 600$ MeV

and treat differently the short-range part of the 2PE contribution, that has an unphysically strong attraction.

EM dimensional regularization

EGM spectral function regularization introduces an additional cutoff $\tilde{\Lambda}$ and give cut-off combinations: $(\Lambda, \tilde{\Lambda}) = (450, 500), (450, 700), (550, 600), (600, 600), (600, 700)$

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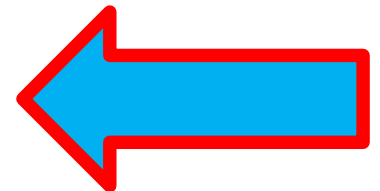
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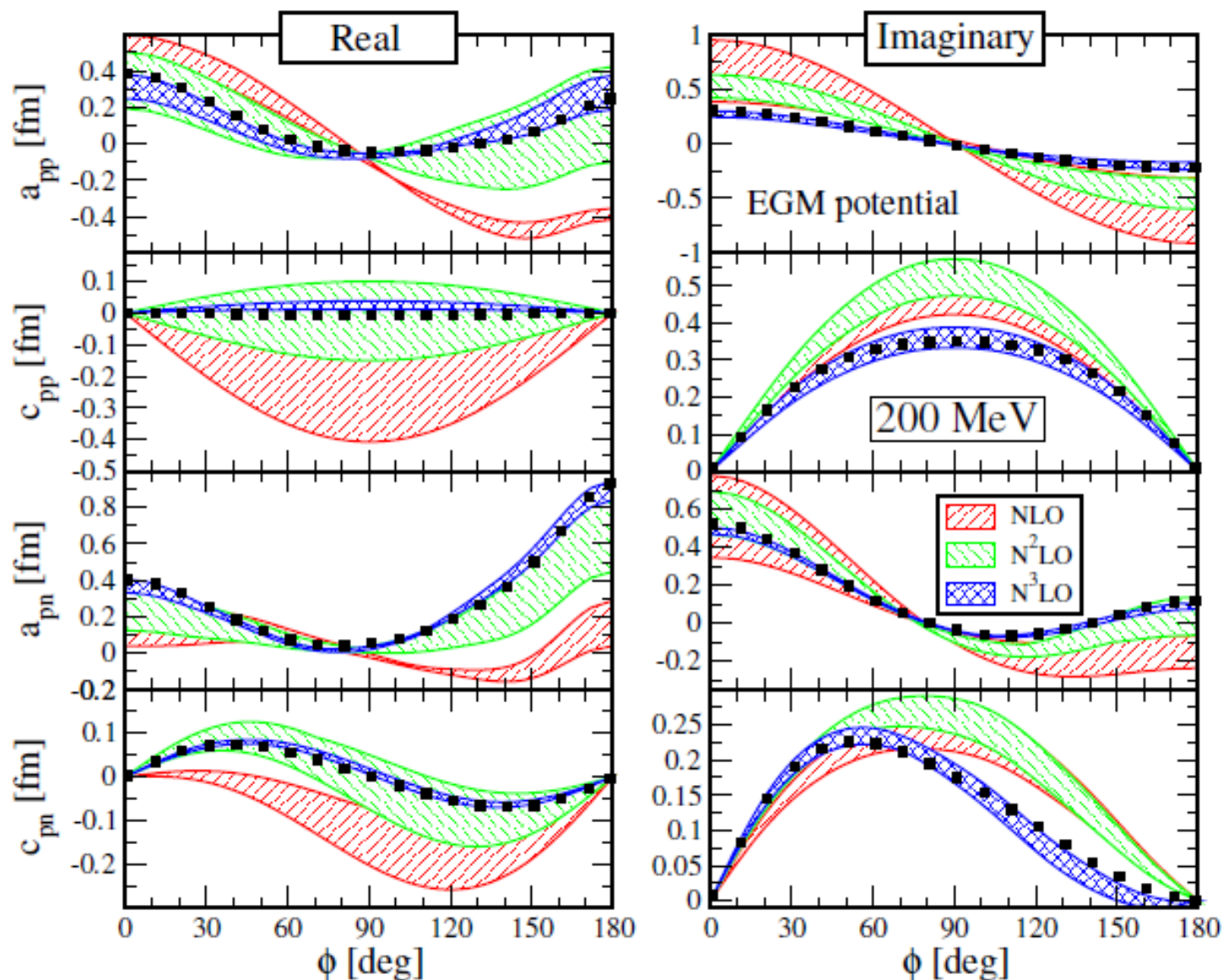
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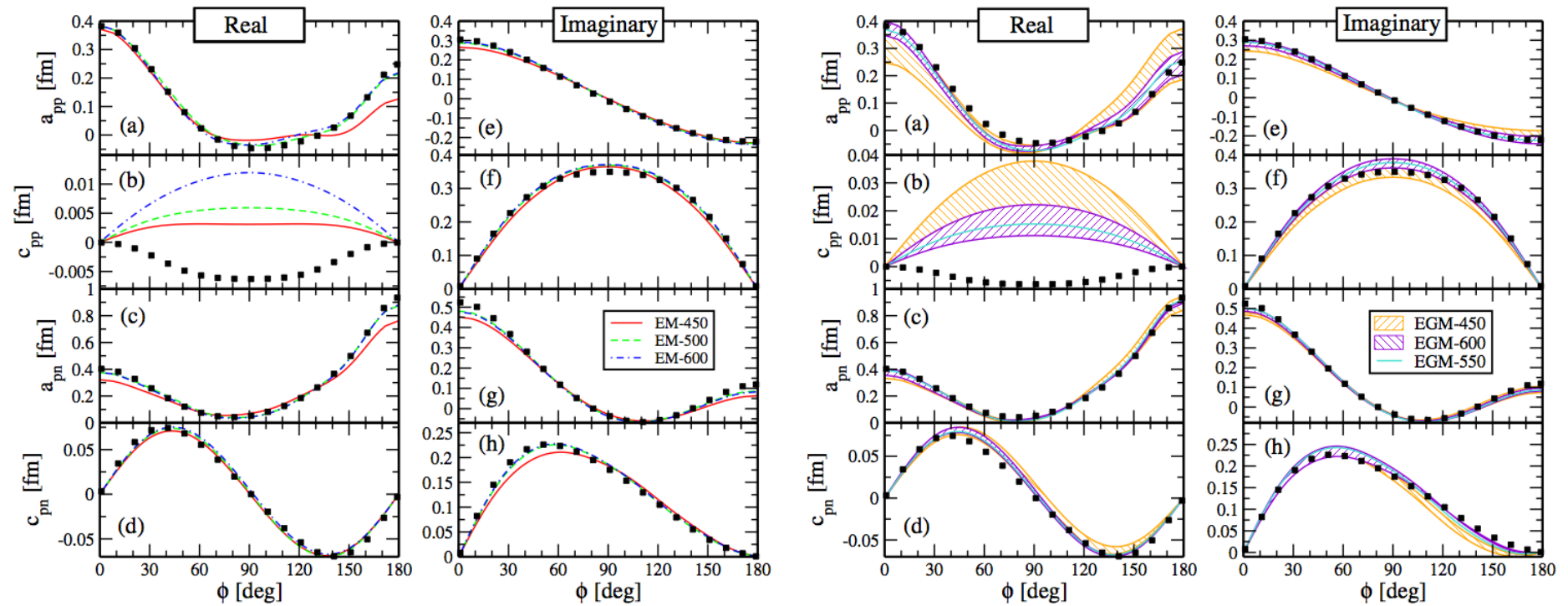
**sensitivity to the cutoff parameters
order by order convergence**



THE NUCLEON-NUCLEON AMPLITUDES



NN AMPLITUDES 200 MeV



M. Vorabbi, P. Finelli, C. Giusti, PRC 93 034619 (2016)

OP and scattering observables

The most general form of the amplitude for elastic p scattering from a spin 0 nucleus

$$M(k_0, \theta) = A(k_0, \theta) + \boldsymbol{\sigma} \cdot \hat{\mathbf{N}} C(k_0, \theta)$$

Scattering observables

$$\frac{d\sigma}{d\Omega}(\theta) = |A(\theta)|^2 + |C(\theta)|^2$$

unpolarized differential cross section

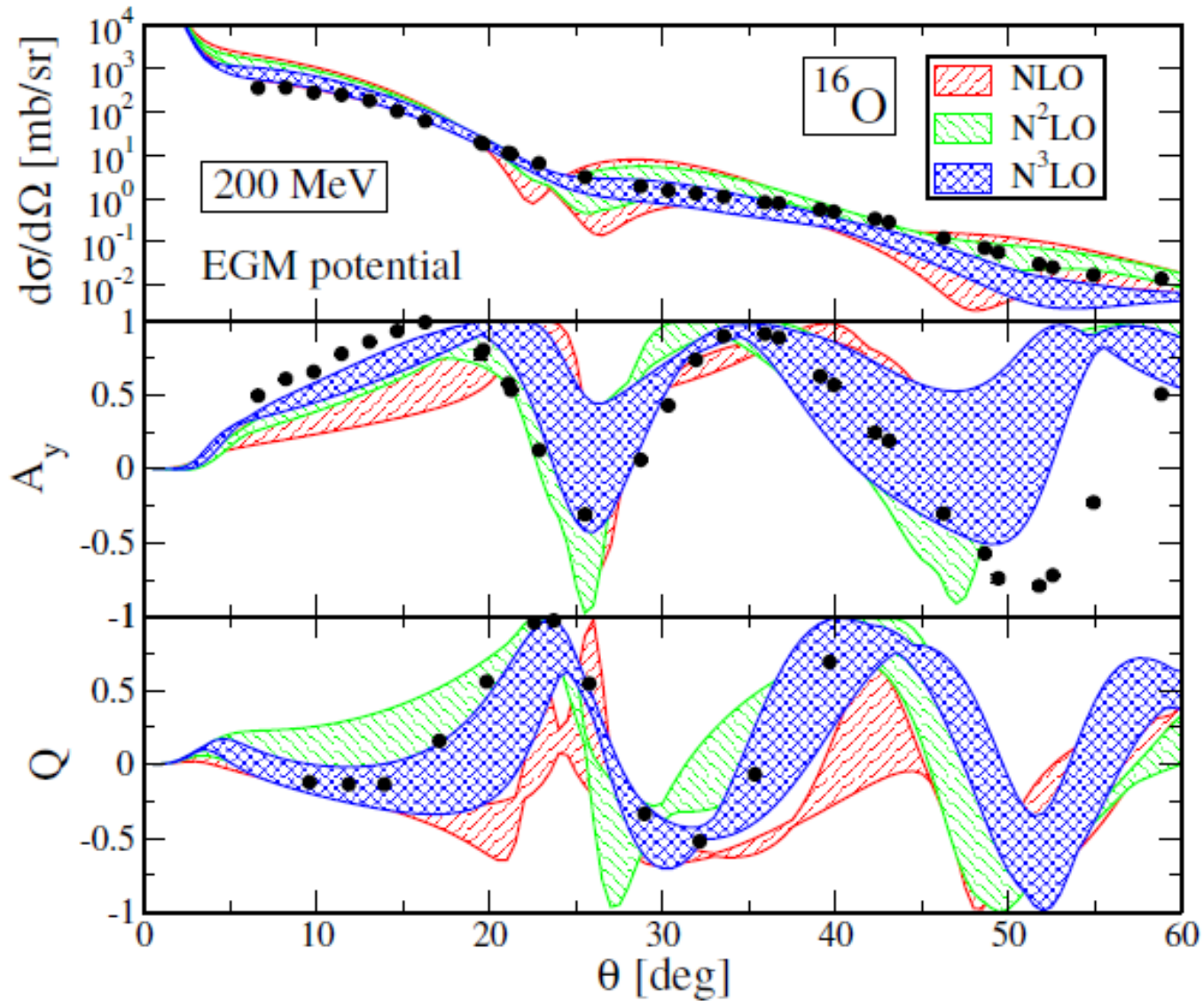
$$A_y(\theta) = \frac{2\text{Re}[A^*(\theta) C(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$

analyzing power

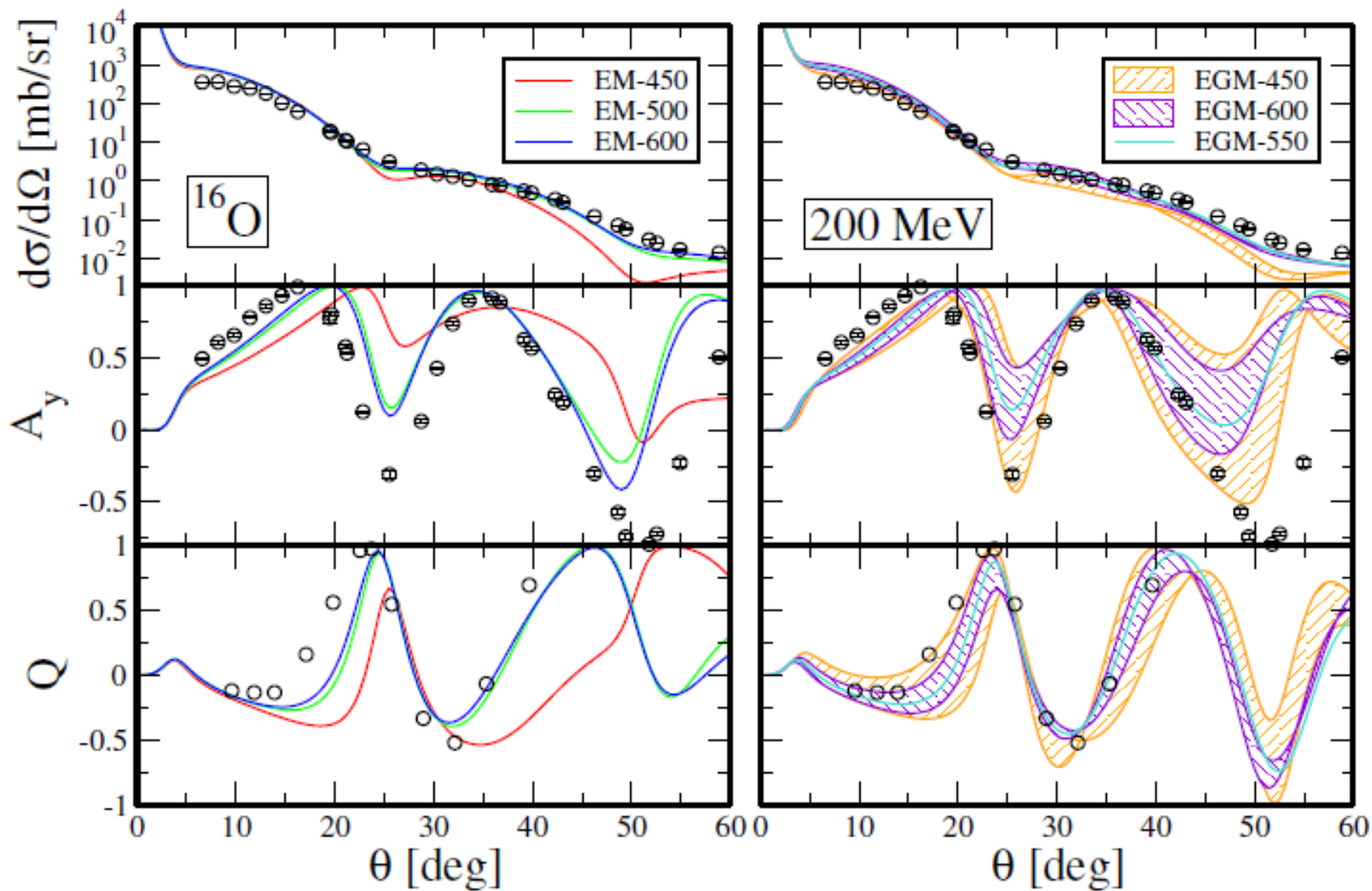
$$Q(\theta) = \frac{2\text{Im}[A(\theta) C^*(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$

spin rotation

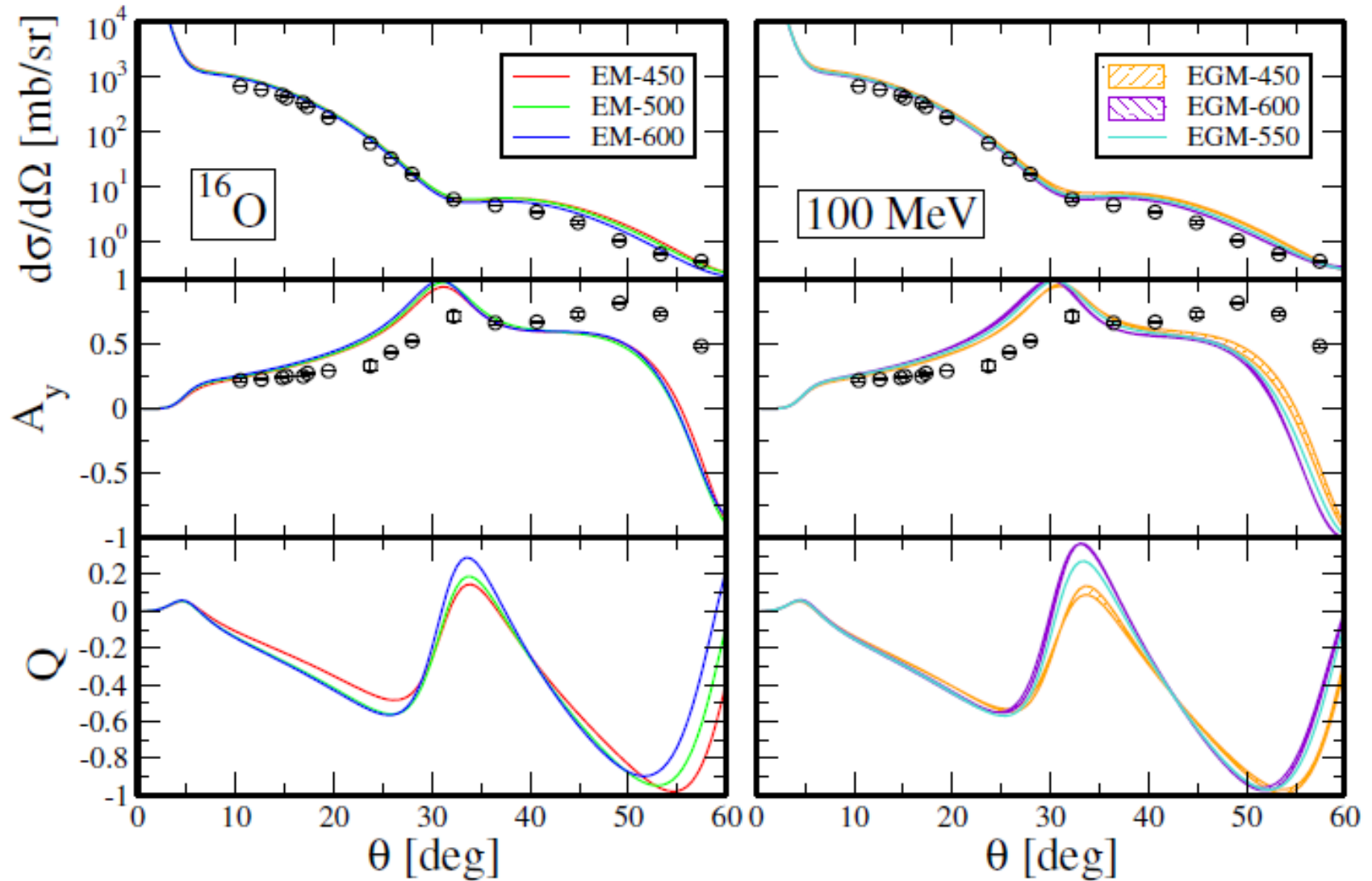
ELASTIC PROTON SCATTERING



ELASTIC P-A SCATTERING



ELASTIC P-A SCATTERING



M. Vorabbi, P. Finelli, and C. Giusti, Phys. Rev. C 93, 034619 (2016)

M. Vorabbi, P. Finelli, C. Giusti, PRC 96 044001 (2017)

Optical potential derived from nucleon-nucleon chiral potentials at N⁴LO

CHIRAL POTENTIAL AT N⁴LO

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q ⁰)		—	—
NLO (Q ²)		—	—
N ² LO (Q ³)			—
N ³ LO (Q ⁴)			
N ⁴ LO (Q ⁵)			

E. Epelbaum et al. . PRL 115 122391 (2015), EPJA 51 53 (2015) **EKM**

D.R. Entem et al. PRC 91 014002 (2015), PRC 96 024004 (2017) **EMN**

CHIRAL POTENTIAL AT N⁴LO

NN

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N ² LO (Q ³)			—
N ³ LO (Q ⁴)			
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D.R. Entem et al. PRC 91 014002 (2015), PRC 96 024004 (2017) **EMN**

M. Vorabbi, P. Finelli, C. Giusti, PRC 93 034619 (2017)

Theoretical optical potential derived from nucleon-nucleon chiral potentials at N⁴LO

Purpose: check the convergence and assess the theoretical errors associated with the truncation of the chiral expansion in the construction of an OP

M. Vorabbi, P. Finelli, C. Giusti, PRC 93 034619 (2017)

Theoretical optical potential derived from nucleon-nucleon chiral potentials at N⁴LO

Purpose: check the convergence and assess the theoretical errors associated with the truncation of the chiral expansion in the construction of an OP

Results: robust convergence has been reached at N⁴LO, agreement with data neither better nor worse than with chiral NN potentials at N³LO

Comparison with phenomenological OP

- investigate and compare predictive power of our microscopic OP and of phenomenological OP in comparison with exp. data in a wider range of nuclei, including isotopic chains

M. Vorabbi, P. Finelli, C. Giusti, PRC 98 064602 (2019)

Proton-Nucleus Elastic Scattering: Comparison
between Phenomenological and Microscopic Optical
Potentials

Comparison with phenomenological OP

- investigate and compare predictive power of our microscopic OP and of phenomenological OP in comparison with exp. data in a wider range of nuclei, including isotopic chains

PHENOMENOLOGICAL OP

parameters fitted to data,
data very well described in
particular situations.

Investigate capability to
describe data in different
situations

MICROSCOPIC OP

obtained from a model and
approximations
may be less able to describe
specific data
should have a greater
predictive power for
situations for which data not
yet available

Comparison phenomenological and microscopic NROP

PHENOMENOLOGICAL OP

GLOBAL given in a wide range of nuclei and energies

NROP up to ~ 200 MeV, for higher energies it is generally believed that the Schroedinger picture should be taken over by a Dirac approach. Global ROP available up to ~ 1 GeV

NROP Koning et al. NPA 713 231 (2003) (KD) for nuclei $24 \leq A \leq 209$ and energies from 1 keV to 200 MeV, recently extended to 1 GeV, to test at which energy the predictions of a phen. NROP fail

Calculations with TALYS (ECIS-06)

MICROSCOPIC OP

chiral potentials at N^4 LO describe NN scattering data up to 300 MeV and our OP can be used up to ~ 300 MeV

Comparison phenomenological and microscopic NROP

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NROP Koning et al. NPA 713 231 (2003) (KD) for $24 \leq A \leq 200$ and energies from 0 to 200 MeV recently extended to 300 MeV at which energy the phen. NROP fail

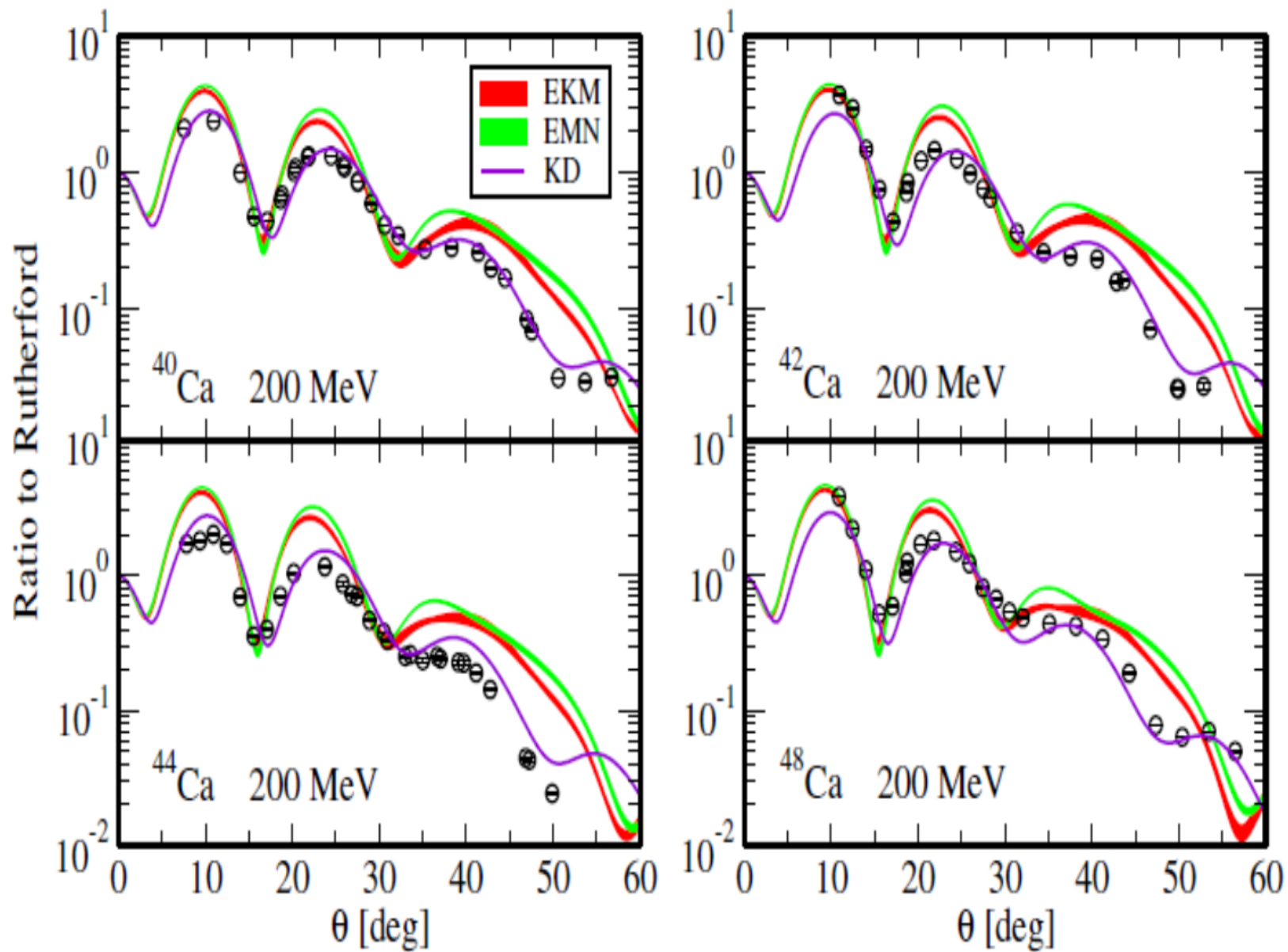
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MICROSCOPIC OP

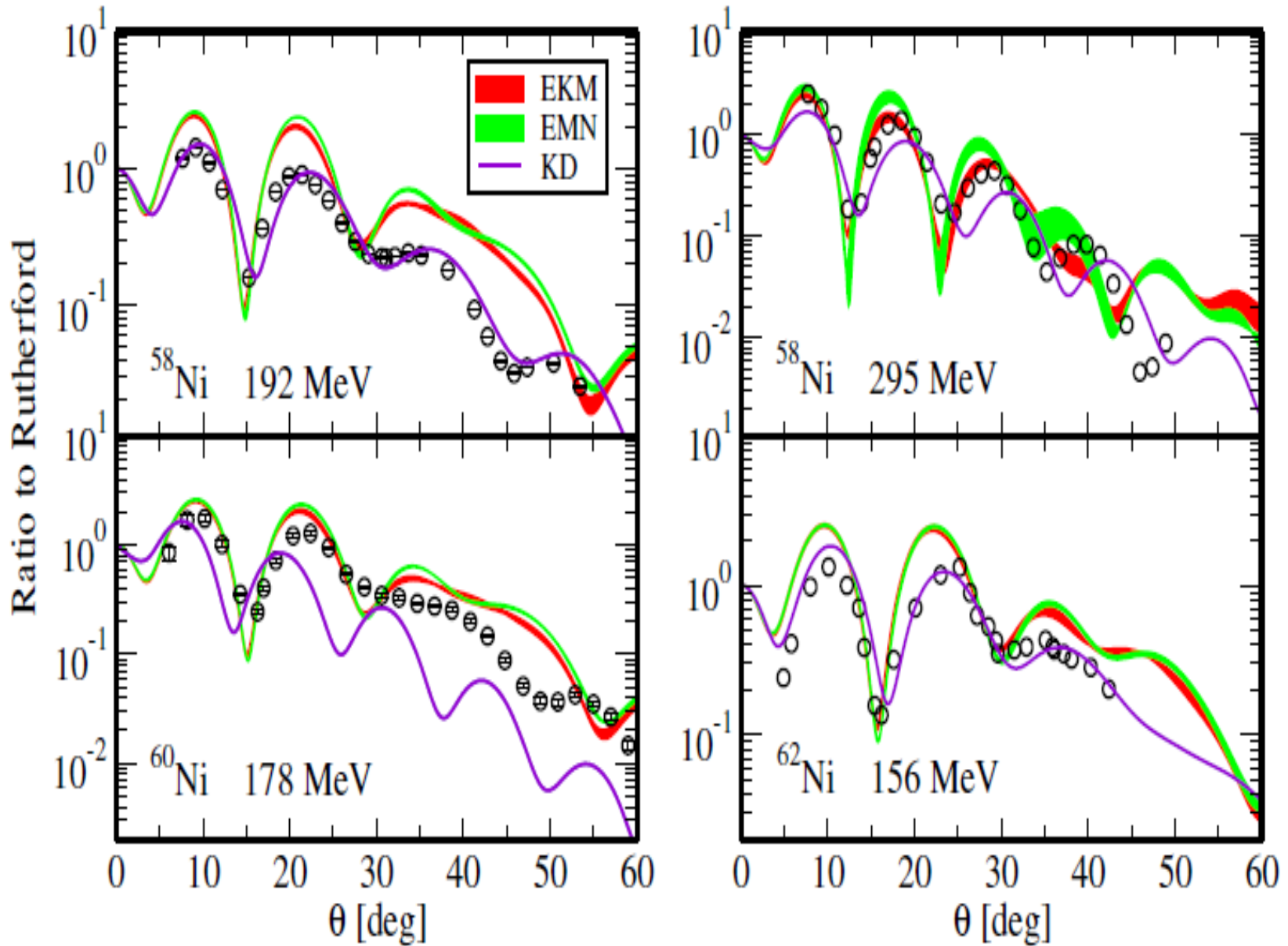
chiral potentials at N^4 LO describe NN scattering data up to 300 MeV and our OP can be used up to ~ 300 MeV

Results of the comparison in the energy range 150-330 MeV

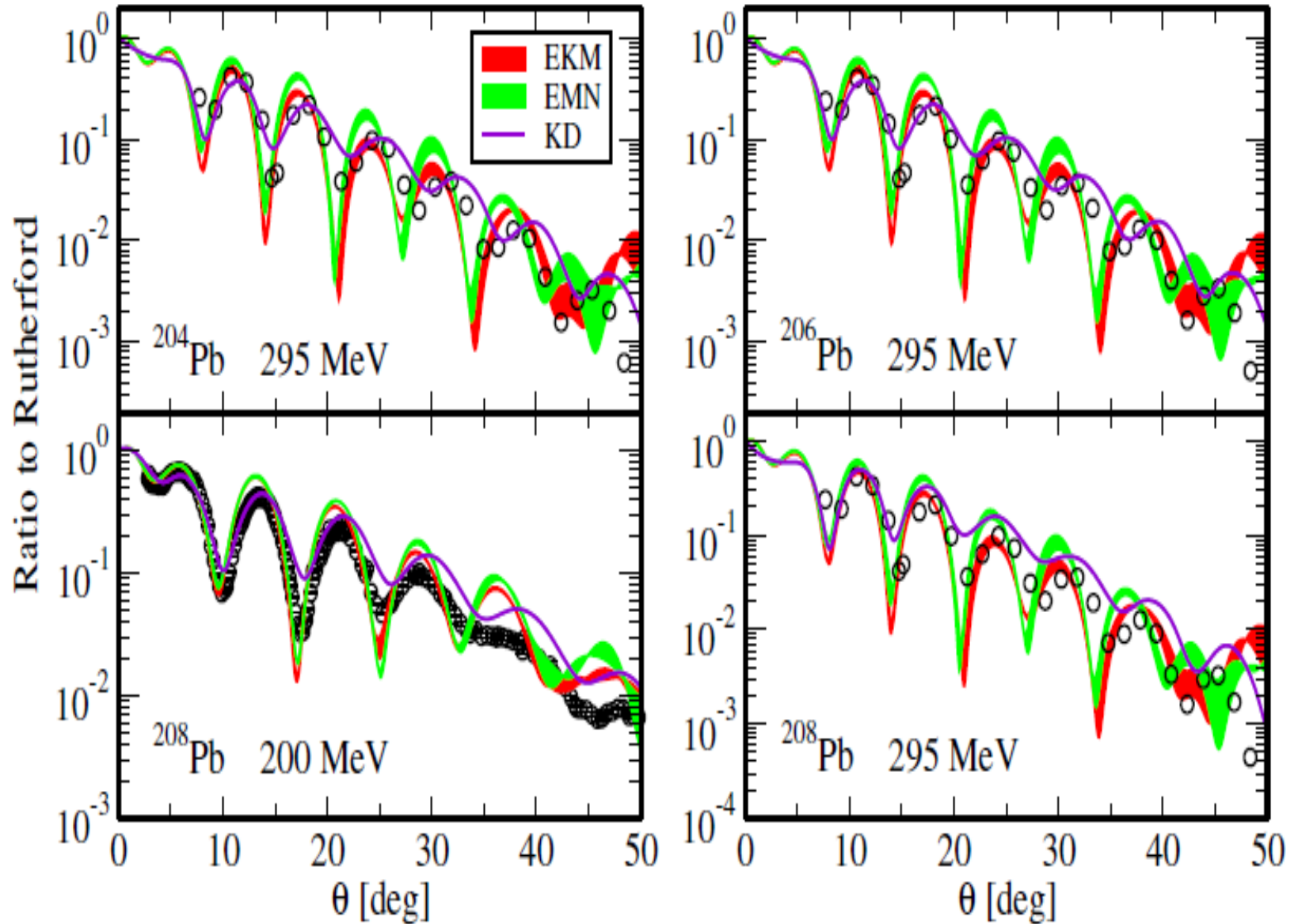
Comparison with phenomenological OP



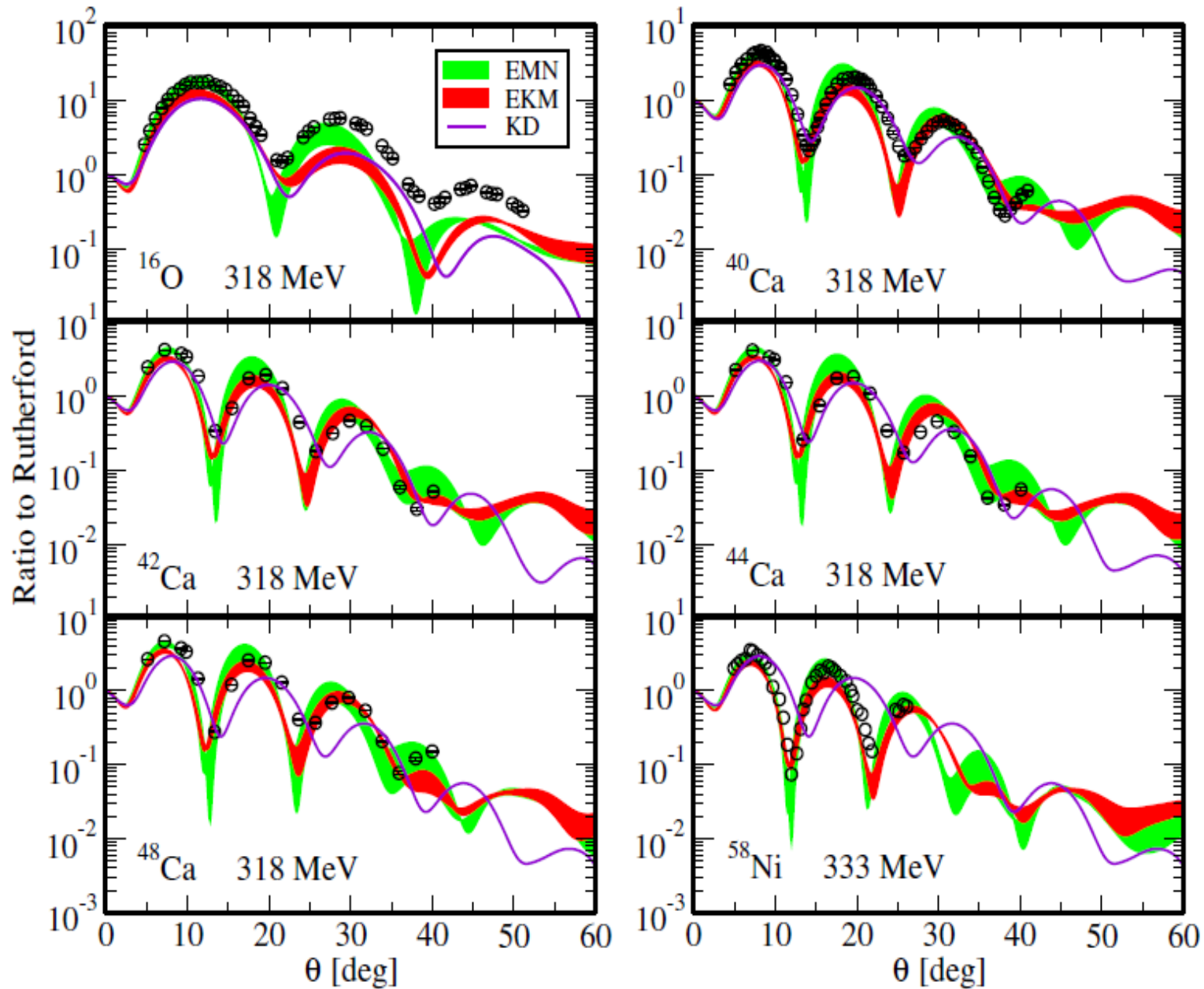
Comparison with phenomenological OP



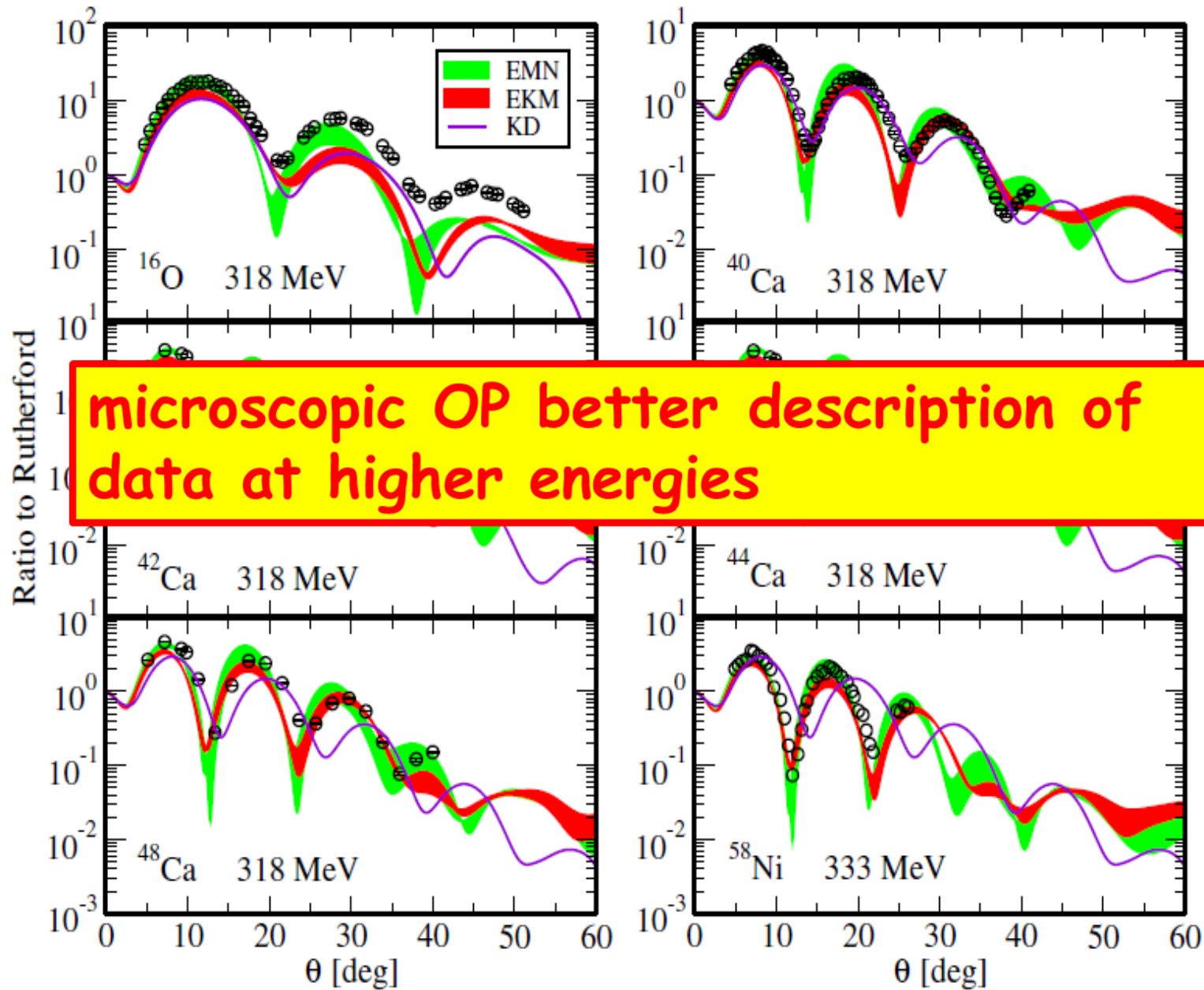
Comparison with phenomenological OP



Comparison with phenomenological OP



Comparison with phenomenological OP



microscopic OP better description of data at higher energies

PROSPECTS...

- model can be improved

Microscopic optical potentials derived from *ab initio* translationally invariant nonlocal one-body densities

Michael Gennari*

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TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

Matteo Vorabbi,[†] Angelo Calci, and Petr Navrátil[‡]

TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada
(Dated: December 11, 2017)

Background: The nuclear optical potential is a successful tool for the study of nucleon-nucleus elastic scattering and its use has been further extended to inelastic scattering and other nuclear reactions. The nuclear density of the target nucleus is a fundamental ingredient in the construction of the optical potential and thus plays an important role in the description of the scattering process.

Purpose: In this work we derive a microscopic optical potential for intermediate energies using *ab initio* translationally invariant nonlocal one-body nuclear densities computed within the no-core shell model (NCSM) approach utilizing two- and three-nucleon chiral interactions as the only input.

Methods: The optical potential is derived at first-order within the spectator expansion of the non-relativistic multiple scattering theory by adopting the impulse approximation. Nonlocal nuclear densities are derived from the NCSM one-body densities calculated in the second quantization. The translational invariance is generated by exactly removing the spurious center-of-mass (COM) component from the NCSM eigenstates.

Results: The ground state local and nonlocal densities of ${}^4,6,8\text{He}$, ${}^{12}\text{C}$, and ${}^{16}\text{O}$ are calculated and applied to optical potential construction. The differential cross sections and the analyzing powers for the elastic proton scattering off of these nuclei are then calculated for different values of the incident proton energy. The impact of nonlocality and the COM removal is discussed.

Conclusions: The use of nonlocal densities has a substantial impact on the differential cross sections and improves agreement with experiment in comparison to results generated with the local densities especially for light nuclei. For the halo nuclei ${}^6\text{He}$ and ${}^8\text{He}$, the results for the differential cross section are in a reasonable agreement with the data although a more sophisticated model for the optical potential is required to properly describe the analyzing powers.

PACS numbers: 24.10.-i; 24.10.Ht; 24.70.+s; 25.40.Cm; 21.60.De; 27.10.+h; 27.20.+n

M. Gennari, M. Vorabbi, A. Calci, P. Navratil, PRC 97 034619 (2018)

PROSPECTS...

- the model can be improved
- 3N forces, medium effects

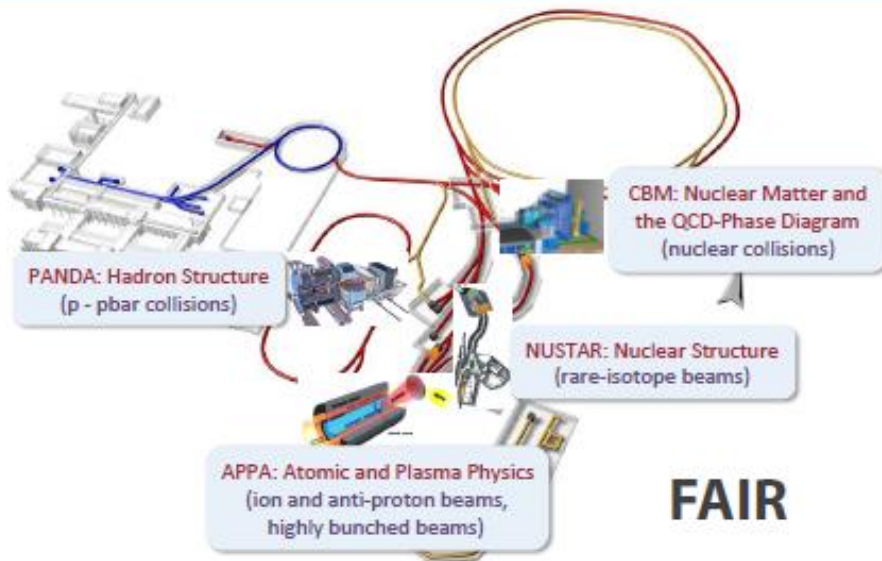
PROSPECTS...

- the model can be improved
- folding integral
- 3N forces, medium effects
- optical potential for elastic antiproton-nucleus scattering

M. Vorabbi, M. Gennari, P. Finelli, C. Giusti, P. Navratil, [arXiv:1906.11984](https://arxiv.org/abs/1906.11984)

OP for $\bar{p}A$ scattering

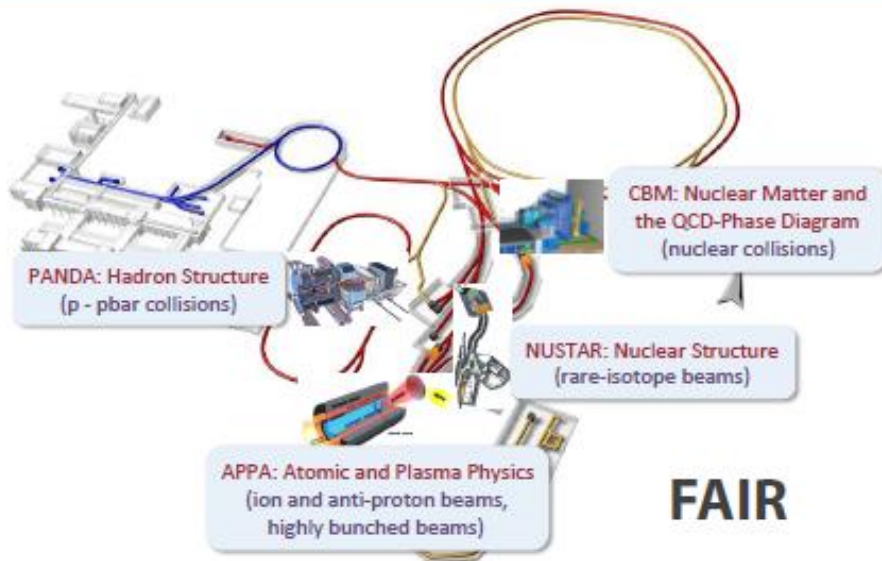
Antiproton physics



Facilities ELENA (CERN) and FAIR (Darmstadt) under construction, experiments on $\bar{p}A$ scattering will experience a new renaissance

OP for $\bar{p}A$ scattering

Antiproton physics



Facilities ELENA (CERN) and FAIR (Darmstadt) under construction, experiments on $\bar{p}A$ scattering will experience a new renaissance

● new chiral $\bar{N}N$ interaction recently derived up to $N^3\text{LO}$
Dai, Haidenbauer, Meißner, JHEP 78 (2017)

OP for $\bar{p}A$ scattering

$$U(\mathbf{q}, \mathbf{K}; \omega) = \sum_{N=n,p} \int d^3P \eta(\mathbf{P}, \mathbf{q}, \mathbf{K}) \\ \times t_{\bar{p}N} \left[\mathbf{q}, \frac{1}{2} \left(\frac{A+1}{A} \mathbf{K} + \sqrt{\frac{A-1}{A}} \mathbf{P} \right); \omega \right] \\ \times \rho_N \left(\mathbf{P} + \frac{1}{2} \sqrt{\frac{A-1}{A}} \mathbf{q}, \mathbf{P} - \frac{1}{2} \sqrt{\frac{A-1}{A}} \mathbf{q} \right)$$

$$\mathbf{q} = \mathbf{k}' - \mathbf{k}$$

$$\mathbf{K} = \frac{1}{2} (\mathbf{k}' + \mathbf{k})$$

Moeller factor

$\bar{N}N$ t-matrix
 $\bar{N}N$ chiral interaction

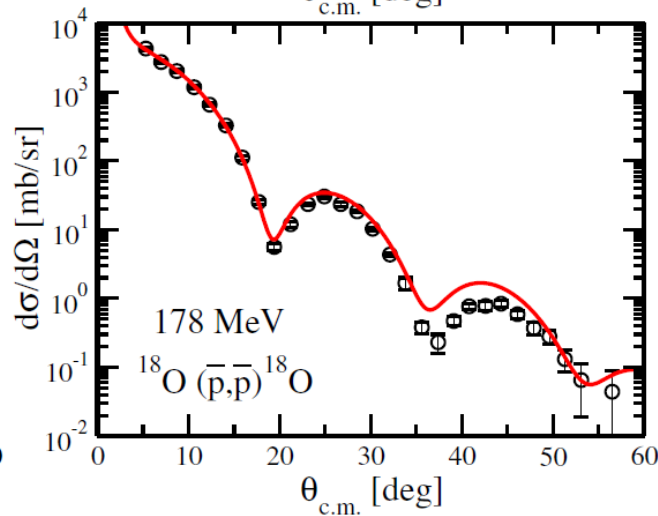
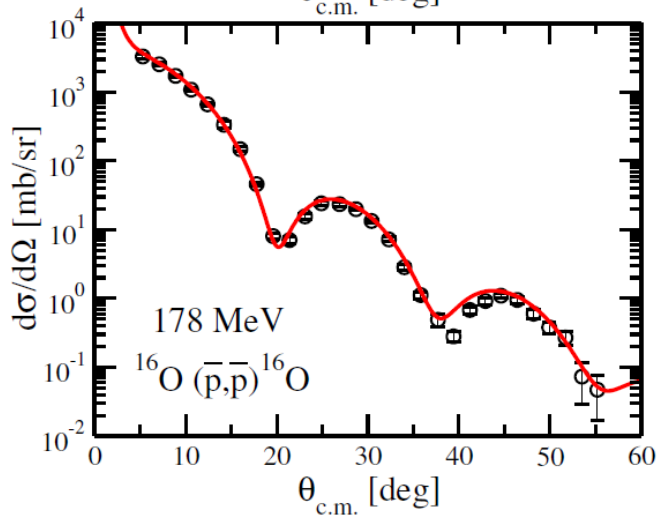
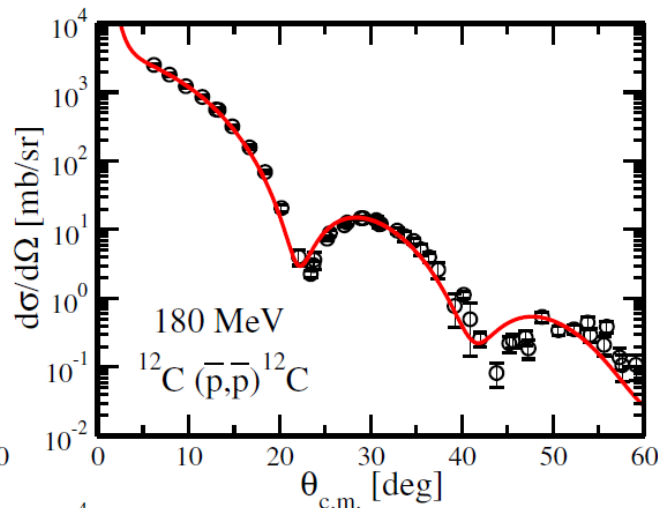
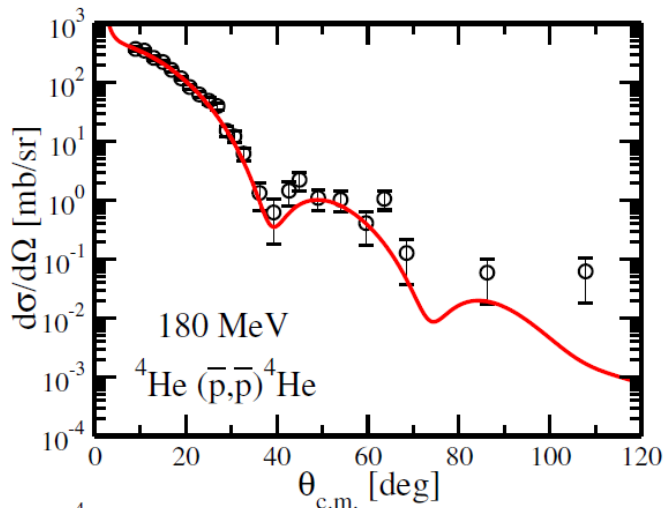
non local n, p densities

folding integral

CHIRAL FORCES

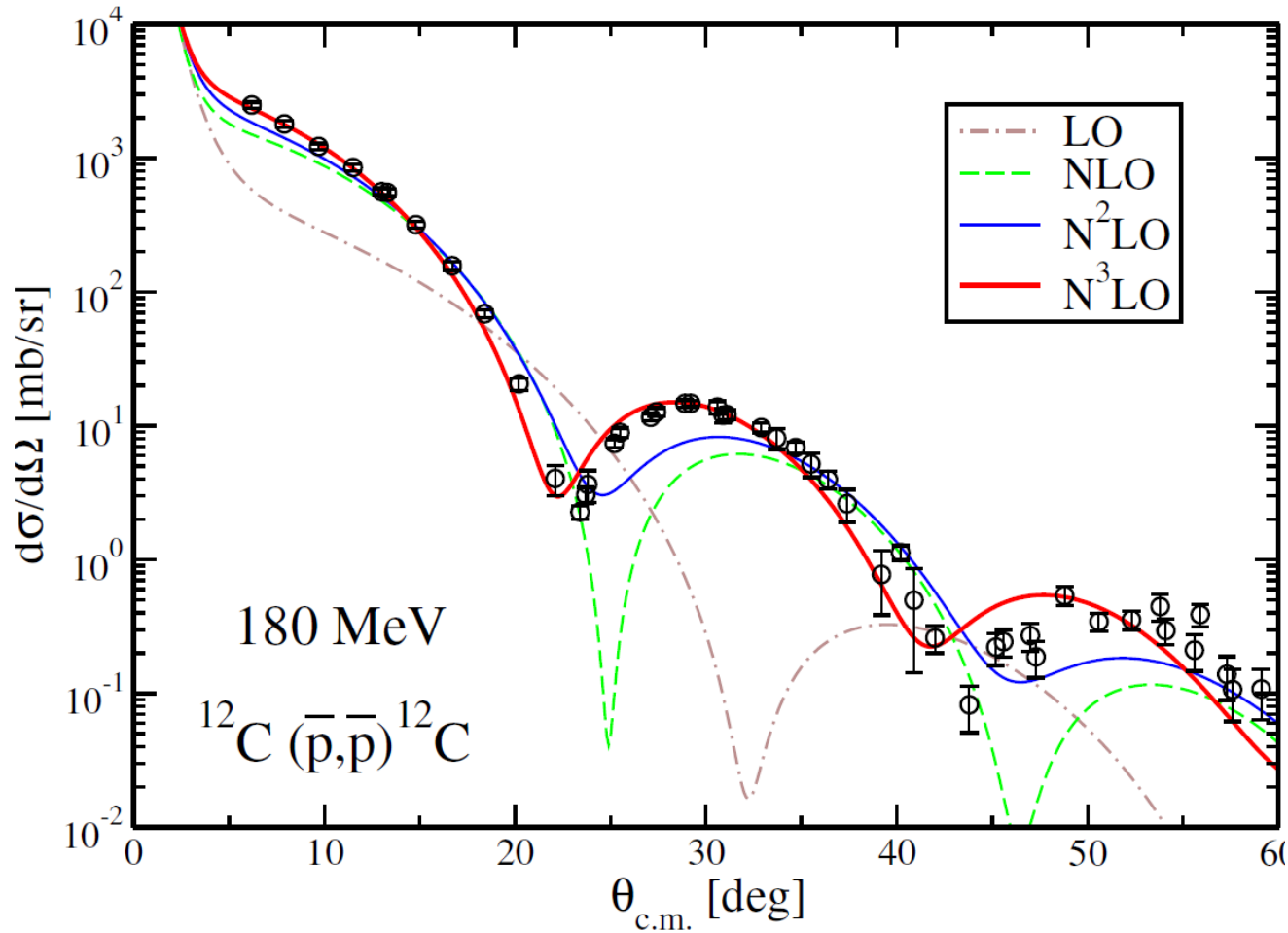
- Target density: one-body translationally invariant densities computed within the *ab initio* NCSM approach using NN - N^4 LO500, Entem et al. PRC 96 024004(2017) and 3N- N^2 LO, Navratil, Few-Body Syst. 41 117 (2007)
- Scattering matrix: $\bar{N}N$ - N^3 LO Dai, Haidenbauer, Meißner, JHEP 78 (2017)

\bar{p} A scattering



data from LEAR

$\bar{p} A$ scattering



data from LEAR