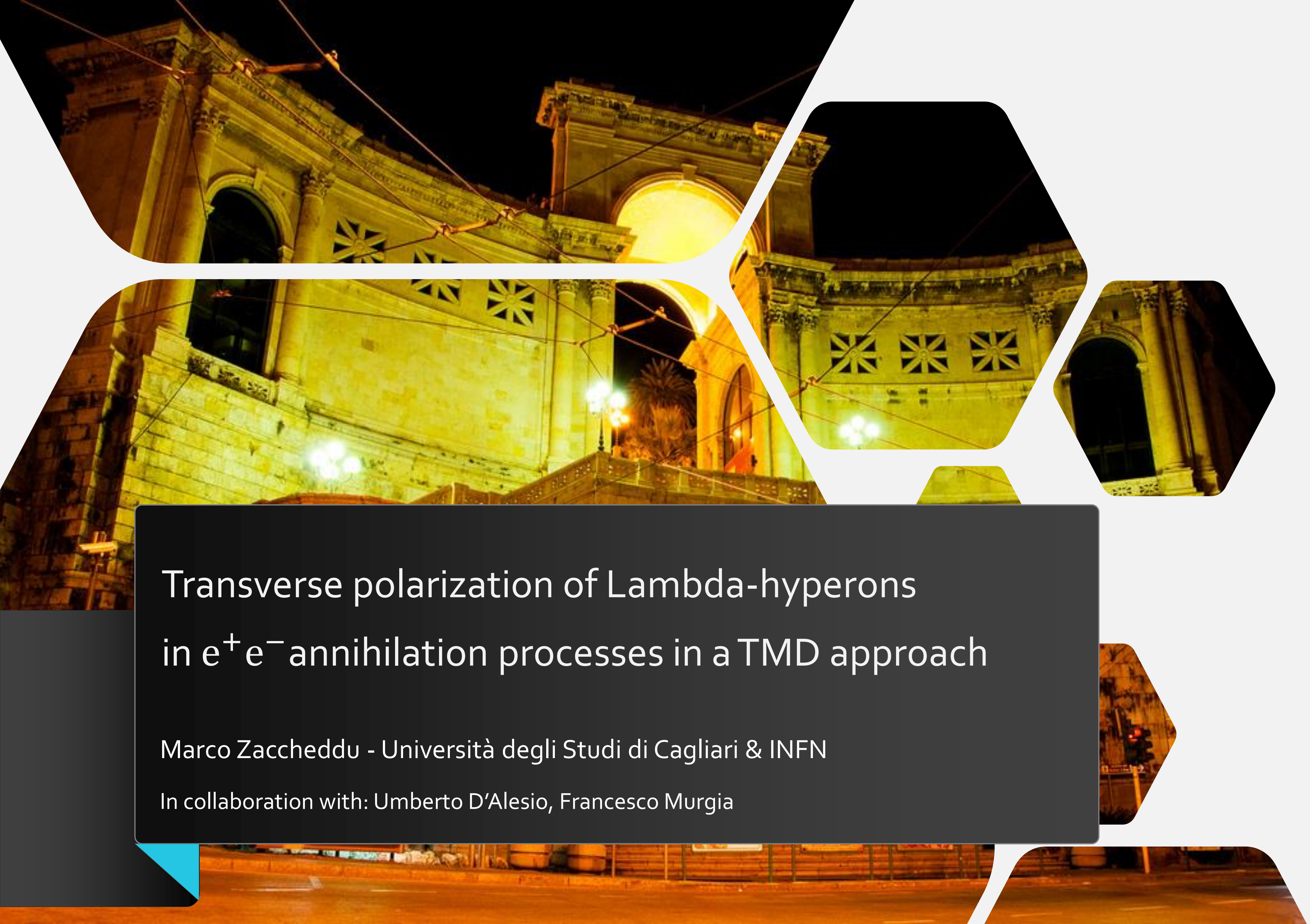


Transverse polarization of Lambda-hyperons in e^+e^- annihilation processes in a TMD approach

Marco Zaccheddu - Università degli Studi di Cagliari & INFN

In collaboration with: Umberto D’Alesio, Francesco Murgia

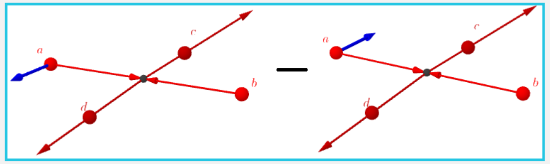
Introduction

Spin and transverse momentum effects expected negligible at high energies.

Single Spin Asymmetry (SSA) at partonic level: $a^{\uparrow}b \to cd$

$$a_N = \frac{d\sigma^{a^{\uparrow}b\to cd} - d\sigma^{a^{\downarrow}b\to cd}}{d\sigma^{a^{\uparrow}b\to cd} + d\sigma^{a^{\downarrow}b\to cd}}$$

$$a_N \propto \alpha_s \frac{m}{\sqrt{s}} \simeq \alpha_s \frac{m}{p_\perp}$$ [Kane, Pumplin, Repko 1978]



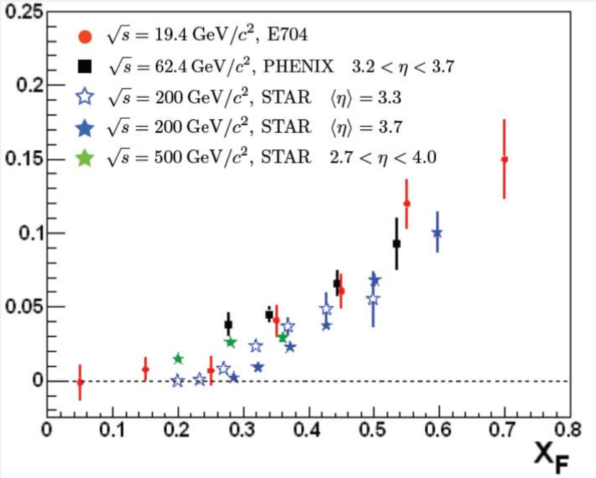
Collinear pQCD

SSAs at hadronic level: $A_N : \quad p^{\uparrow}p \longrightarrow hX$ $\quad P_T : \quad pp \longrightarrow h^{\uparrow}X$

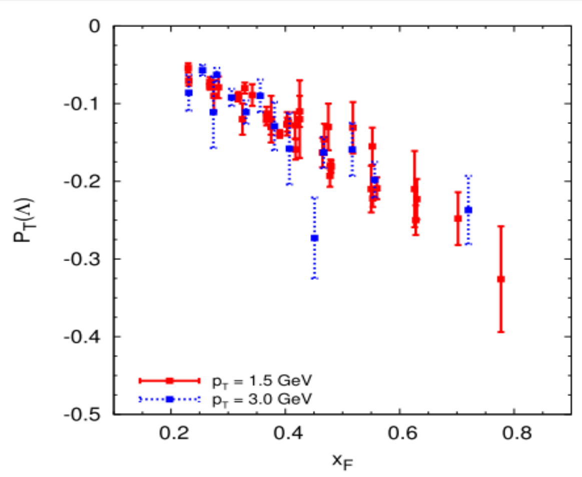
$$A_N/P_T = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \longrightarrow A_N \leq 1-2\%$$

Experimental Data: $A_N \simeq 20\%$ $\quad P_T \simeq 30\%$

$p^{\uparrow}p \to \pi X$



$pp \longrightarrow \Lambda^{\uparrow}X$



TMD factorisation

$$\underbrace{d\sigma^{AB\to CX}}_{\text{Hadronic cross sec.}} \propto \underbrace{d\sigma^{ab\to cd}}_{\text{Partonic cross sec.}} \otimes \begin{cases} f(x, k_\perp) & \text{TMD PDF} \\ D(z, k_\perp) & \text{TMD FF} \end{cases}$$

SSAs closely related to our understanding of:

- Strong interaction/confinement
- Inner structure of hadron in terms of quarks and gluons
- Hadronisation processes

Correlation between intrinsic transverse momenta and spin, leading to a generalisation of PDFs and FFs, with asymmetric azimuthal distributions.

Contents

- TMDs FF with Helicity Formalism
- $e^+e^- \to \mathrm{h}_1(\mathrm{jet})X$
- $e^+e^- \to h^{\uparrow}{}_1 h_2 X$
- Phenomenology
- Preliminary Fits

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with asymmetric azimuthal distributions.

New Data in e^+e^- from Belle: 150 data points

Observation of Transverse $\Lambda/\bar{\Lambda}$ Hyperon Polarization in e^+e^- Annihilation at Belle

We report the first observation of the spontaneous polarization of Λ and $\bar{\Lambda}$ hyperons transverse to the production plane in e^+e^- annihilation, which is attributed to the effect arising from a polarizing fragmentation function. For inclusive $\Lambda/\bar{\Lambda}$ production, we also report results with subtracted feed-down contributions from Σ^0 and charm. This measurement uses a dataset of 800.4 fb^{-1} collected by the Belle experiment at or near a center-of-mass energy of 10.58 GeV. We observe a significant polarization that rises with the fractional energy carried by the $\Lambda/\bar{\Lambda}$ hyperon.

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Benefits:

- No PDFs - Cleaner process
- Lambda self-analysing
- TMD Factorisation proven

Lambda Polarising FF:
first extraction !

Helicity Formalism: TMD Fragmentation Functions for quarks

From Quark Polarisation to Hadron Polarisation

$$\underbrace{\rho^{h,S_h}_{\lambda_h,\lambda'_h}}_{\text{Hadron h.m.}} \hat{D}_{h/q,s_q}(z,\mathbf{k}_{\perp h}) = \sum_{\lambda_q,\lambda'_q} \underbrace{\rho^{q,s_q}_{\lambda_q,\lambda'_q}}_{\text{Parton h.m.}} \hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{k}_{\perp h})$$

$$P^h_j \hat{D}_{h/q,s_q} = \hat{D}^h_{S_j/q,s_q} - \hat{D}^h_{-S_j/q,s_q} = \Delta\hat{D}^h_{S_j/s_q}.$$

Helicity density matrix

$$\rho^{i,s_i}_{\lambda_i,\lambda'_i} = \frac{1}{2}\begin{pmatrix} 1+P^i_z & P^i_x - iP^i_y \\ P^i_x + iP^i_y & 1-P^i_z \end{pmatrix}$$

8 independent TMD Fragmentation Functions

		Hadron		
	Pol. States	U	L	T
Quark	U	$\hat{D}_{h/q}$		$\Delta\hat{D}^h_{S_Y/q}$
	L		$\Delta\hat{D}^{h/q}_{S_Z/s_L}$	$\Delta\hat{D}^{h/q}_{S_X/s_L}$
	T	$\Delta^N D_{h/q^\uparrow}$	$\Delta\hat{D}^{h/q}_{S_Z/s_T}$	$\Delta\hat{D}^{h/q}_{S_X/s_T}$ / $\Delta^-\hat{D}^{h/q}_{S_Y/s_T}$

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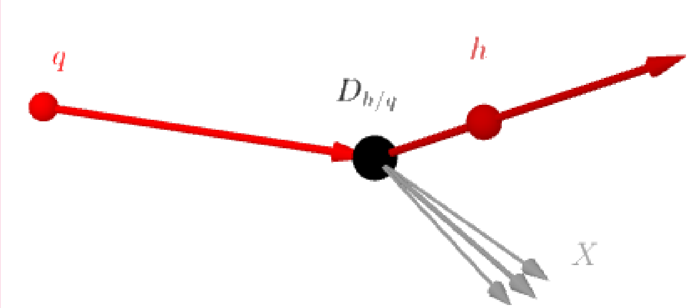
Unpolarized FF



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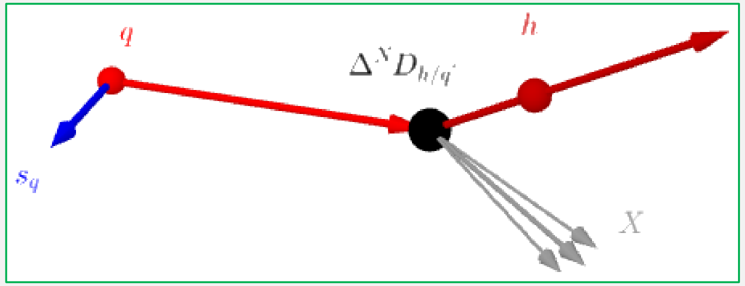
Collins FF



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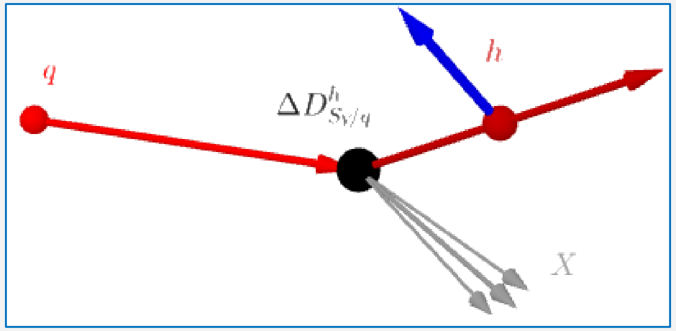
Polarising FF



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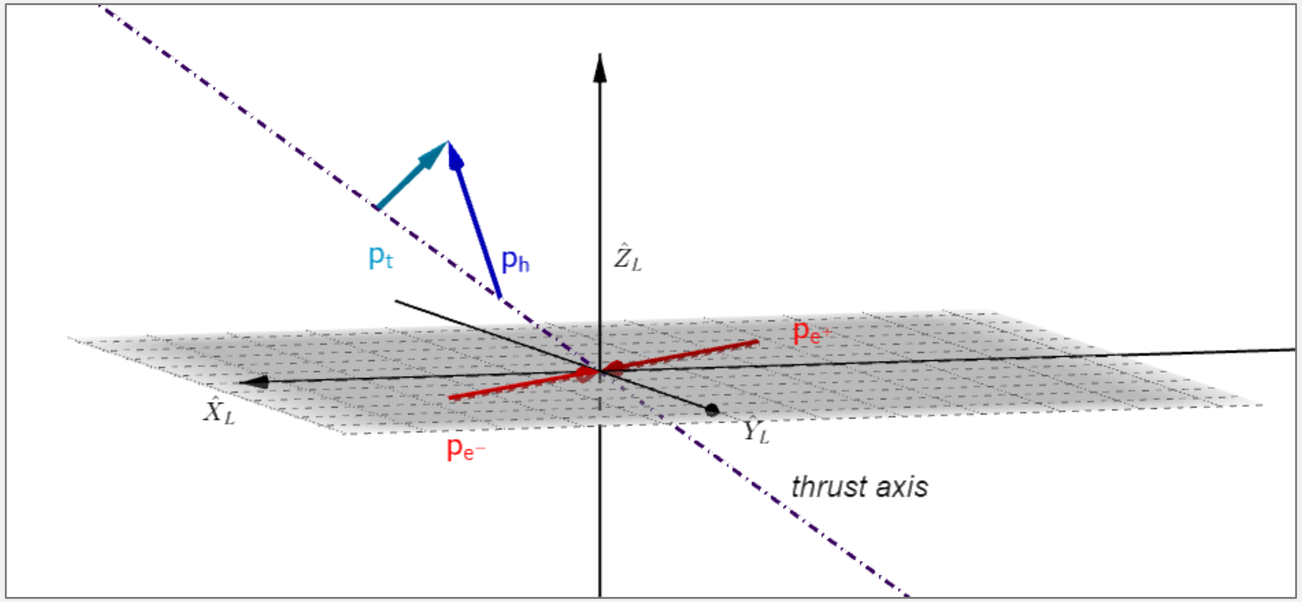
$e^+e^- \to \mathrm{h}_1(\mathrm{jet})X$

Scattering Amplitudes

TMD Fragmentation Function

$$\rho^{h_1}_{\lambda_{h_1},\lambda'_{h_1}} \frac{d\sigma^{e^+e^-\to h_1(jet)X}}{d\cos\theta dz_1 d^2p_{\perp 1}} = \sum_{q_c}\sum_{\{\lambda\}} \frac{1}{32\pi s}\frac{1}{4}\hat{M}_{\lambda_c\lambda_d,\lambda_a\lambda_b}\hat{M}^*_{\lambda'_c\lambda_d,\lambda_a\lambda_b}\hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,p_{\perp 1})$$

Helicity matrix



Polarisation given along the hadron helicity axes:

$$\hat{X}_{h_1} = \hat{Y}_{h_1} \times \hat{Z}_{h_1}$$

$$\hat{Y}_{h_1} = \frac{\hat{q}_1 \times P_{h1}}{|\hat{q}_1 \times P_{h1}|}$$

$$\hat{Z}_{h_1} = \frac{P_{h1}}{|P_{h1}|}$$

$e^+e^- \to \mathrm{h}_1(\mathrm{jet})X$

For Spin-½ hadron production, two possibile cross sections :

Unpolarised hadron

$$\frac{d\sigma^{e^+e^-\to h_1(jet)X}}{d\cos\theta dz_1 d^2p_{\perp 1}} = \sum_{q_c} \frac{3e^4}{32\pi s} e_q^2 (1+\cos^2\theta)\hat{D}_{h/q}(z_1, p_{1\perp h_1})$$

Transversely polarised hadron

$$P_Y^{h_1}\frac{d\sigma^{e^+e^-\to h_1(jet)X}}{d\cos\theta dz_1 d^2p_{\perp 1}} = \sum_{q_c} \frac{3e^4}{32\pi s} e_q^2 (1+\cos^2\theta)\Delta D^{h_1}_{S_Y/q}(z_1, p_{1\perp h_1})$$

ratio →

Hadron polarisation

$$P_Y^{h_1} = \frac{\sum_q e_q^2 \Delta D^{h_1}_{S_Y/q}(z_1, p_{\perp h_1})}{\sum_q e_q^2 D_{h/q}(z_1, p_{\perp h_1})}$$

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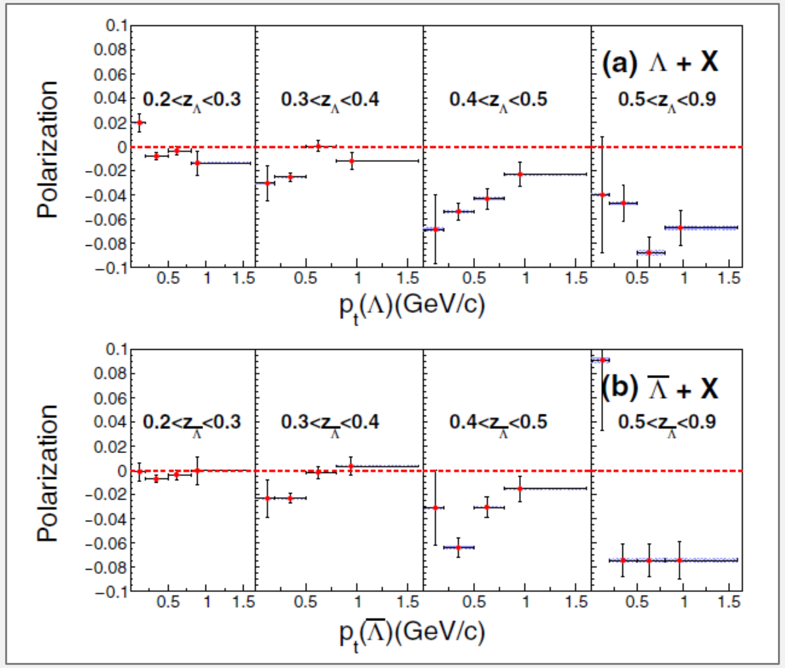


From experimental data it is possible to extract directly →

Lambda Polarising FF

$$\Delta\hat{D}^h_{S_Y/q}$$

$e^+e^- \to h^{\uparrow}{}_1 h_2 X$

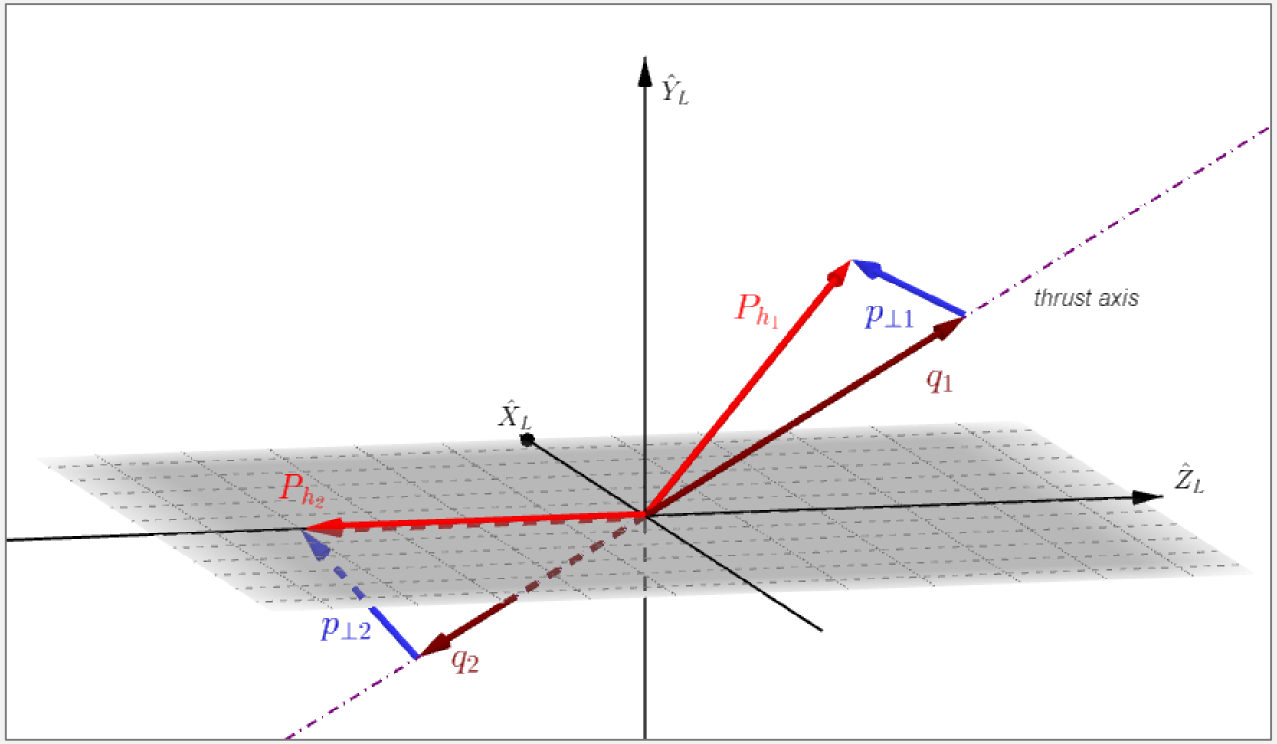
Helicity matrices

$$\rho^{h_1,S_{h_1}}_{\lambda_{h_1},\lambda'_{h_1}}\,\rho^{h_2,S_{h_2}}_{\lambda_{h_2},\lambda'_{h_2}}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta\, dz_1 d^2\mathbf{p}_{\perp h_1} dz_2 d^2\mathbf{p}_{\perp h_2}}$$

$$=\sum_{q_c}\sum_{\{\lambda\}}\frac{3}{32\pi s}\frac{1}{4}M_{\lambda_c\lambda_d,\lambda_a\lambda_b}M^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b}\hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp h_1})\hat{O}^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp h_2})$$

Scattering Amplitudes

TMD Fragmentation Functions



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Polarisation vector

$$\mathcal{P}^{h_1} = P^{h_1}_x \hat{X}_1 + P^{h_1}_y \hat{Y}_1 + P^{h_1}_z \hat{Z}_1$$

Advantages:

- hadron 2 allows to weight the FF of hadron 1 from (anti)quark 1 ;

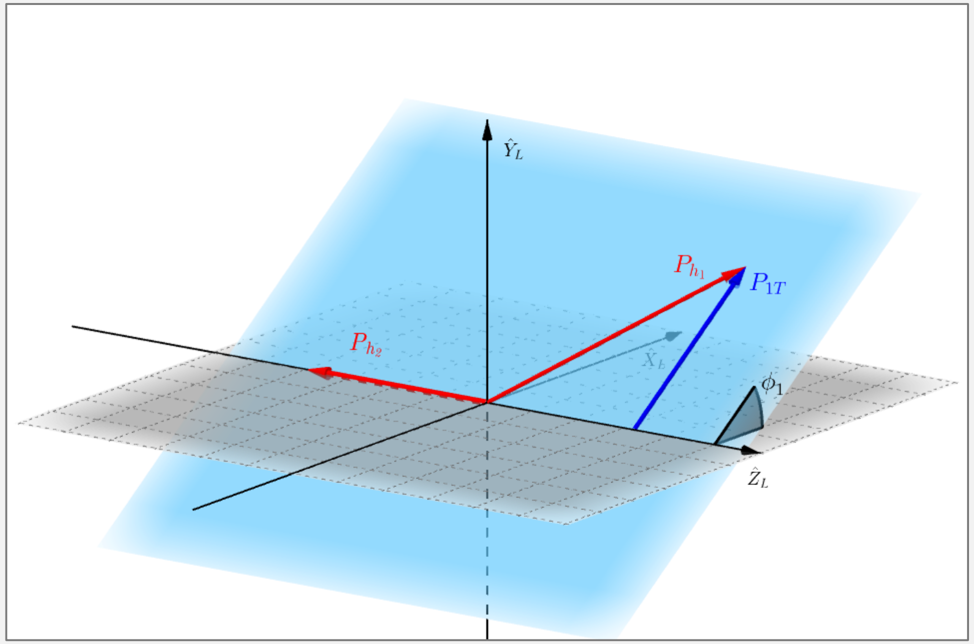
Example:

- h1 :Λ(u,d,s) – h2: π^+(u,dbar) ;

Enhancement of Λ from d quark (at moderate z)

Polarisation: $e^+e^- \to h^{\uparrow}{}_1 h_2 X$

Experimental configuration:



- Polarisation measured perpendicularly to the plane of the 2 hadrons
- Matching between theory and experiment

The polarisation is measured along:

$$\hat{n} = -\hat{P_{h_2}} \times \hat{P_{h_1}}$$

The polarisation projection along $\hat{n}$:

$$\mathcal{P}^{h_1} \cdot \hat{n} = P_x^{h_1} \cos\widetilde{\phi} + P_y^{h_1} \sin\widetilde{\phi}$$

This projection depends *deeply* on the kinematical variables

Polarisation: $e^+e^- \to h^{\uparrow}{}_1 h_2 X$

Experimental data depend only on energy fractions, z:

We introduce for the FF the following parametrisations:

$$\Delta D^h_{S_Y/q}(z,p_\perp) = \Delta D^h_{S_Y/q}(z)\sqrt{2e}\,\frac{p_\perp}{M_p}\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_p}}{\pi\langle p_\perp^2\rangle_h}$$

z dependence

Gaussian dependence on $p_\perp$

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_h}}{\pi\langle p_\perp^2\rangle_h}$$

$$\frac{1}{\langle p_\perp^2\rangle_p} = \frac{1}{M_p^2} + \frac{1}{\langle p_\perp^2\rangle_h}$$

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$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_h}}{\pi\langle p_\perp^2\rangle_h} \qquad \frac{1}{\langle p_\perp^2\rangle_p} = \frac{1}{M_p^2} + \frac{1}{\langle p_\perp^2\rangle_h}$$

Performing a change of variable and an integration over all transverse momenta

$$d^2p_{\perp 1} \longrightarrow dP_{1T}d\phi_1 \qquad \int dP_{1T}d\phi_1 dp_{\perp 2}d\varphi_2$$

The polarisation is measured along:

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The polarisation projection along $\hat{n}$:

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$$P^{h_1}\cdot\hat{n} :: \frac{\sum_q \Delta D^h_{S_Y/q}(z)D_{h_2/\bar{q}}(z_2)}{\sum_q D_{h_1/q}(z_1)D_{h_2/\bar{q}}(z_2)} \times \frac{\sqrt{2e\pi}}{2M_p}\frac{\langle p_\perp^2\rangle_p^2}{\langle p_\perp^2\rangle_1}\frac{z_2}{\sqrt{z_1^2\langle p_\perp^2\rangle_2 + z_2^2\langle p_\perp^2\rangle_p}}$$

Phenomenology and Fit

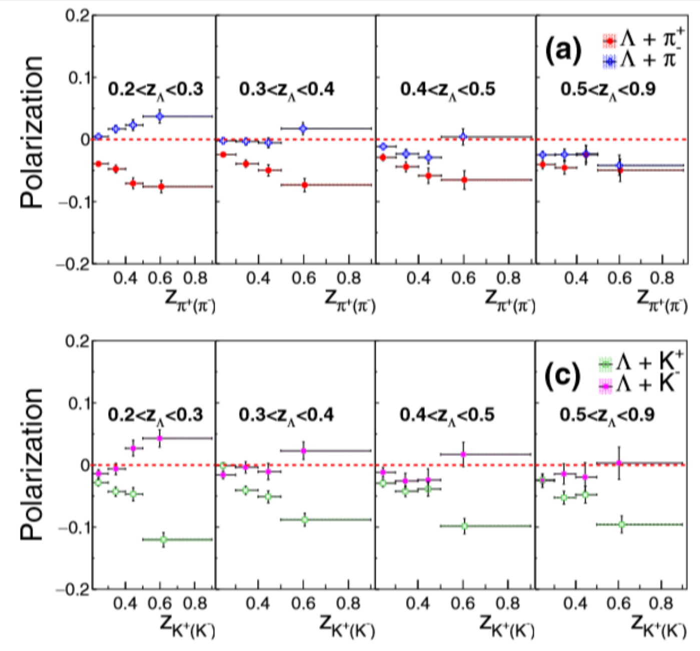
Polarising parametrization:

Unpolarized FF

$$\Delta D^h_{S_Y/q}(z) = \mathcal{N}^p_q(z) \underbrace{D_{h/q}(z)}$$

$$\mathcal{N}^p_q(z) = N^p_q z^{\alpha_q}(1-z)^{\beta_q} \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

- Normalisation factor: N^p_q , $|N^p_q| \leq 1$
- Shape for high and low z: $\alpha_q \quad \beta_q$



Data selection to Fit:

- Lambda-hadron
- 100 data points

- Λ(u,d,s) (+ h)

- h = π(u,d)
- h = K(u,s)

Phenomenology and Fit

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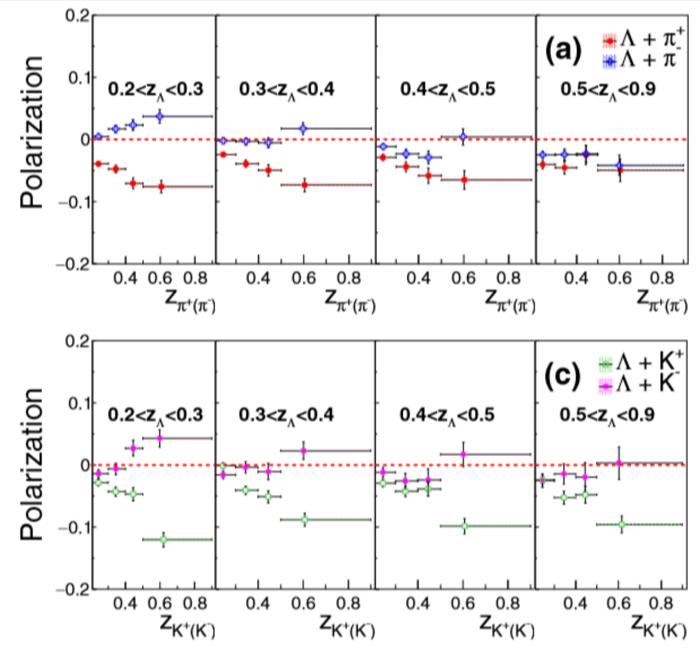

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Flav.	N^{p}_{q}	α_q	β_q	$\langle p_\perp^2\rangle_p$
u	N^{p}_{u}		β_u	$\langle p_\perp^2\rangle_p$
d	N^{p}_{d}			
s	N^{p}_{s}	α_s		
sea	N^{p}_{sea}		β_{sea}	

8 parameters

Phenomenology and Fit

Polarising parametrization:

Unpolarized FF

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$$\mathcal{N}^p_q(z) = N^p_q z^{\alpha_q}(1-z)^{\beta_q} \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

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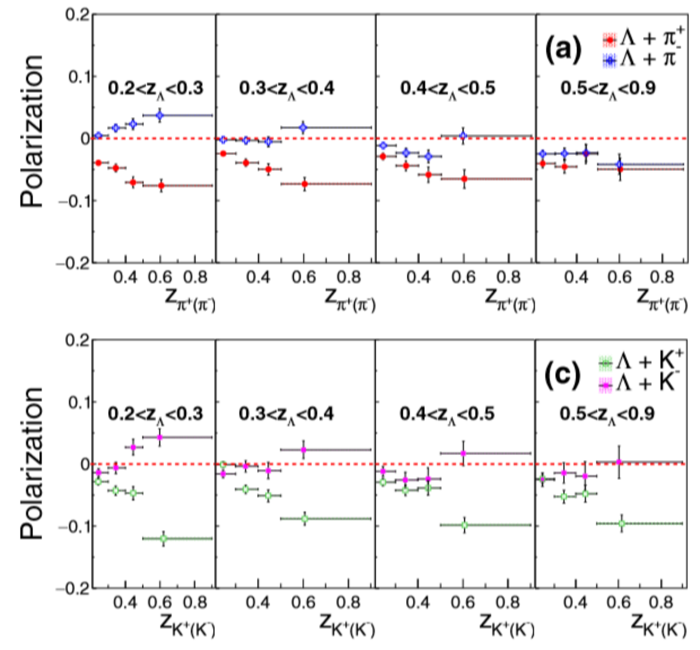

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Fitted Parameters Value	
Nu	0.51 $\pm$ 0.09
Nd	-0.45 $\pm$ 0.09
Ns	-0.30 $\pm$ 0.10
Nsea	-1.00 $\pm$ 0.25
α_s	1.6 $\pm$ 0.3
$\beta_{\{sea\}}$	6.1 $\pm$ 0.6
β_u	3.0 $\pm$ 0.3
$p_\perp$	0.11 $\pm$ 0.03

Minimisation method: MINUIT

8 parameters

$\chi^2/dof = 1.3$

Statistical Uncertainty Band

Multivariate Normal Distribution

MINUIT:
- Best fit parameters $\mu : \mathcal{N}_q^p \, \alpha_q \;\; \beta_q \; \langle p_\perp^2 \rangle_p$
- Covariance matrix Σ
- Minimum Chi-square χ^2

$\mu \; , \; \Sigma \longrightarrow$

$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

Generate a random set of parameters

x: ($\mathcal{N}_q^p \;\; \alpha_q \;\; \beta_q \;\; \langle p_\perp^2 \rangle_p$) $\longrightarrow$

Calculate the Chi-square

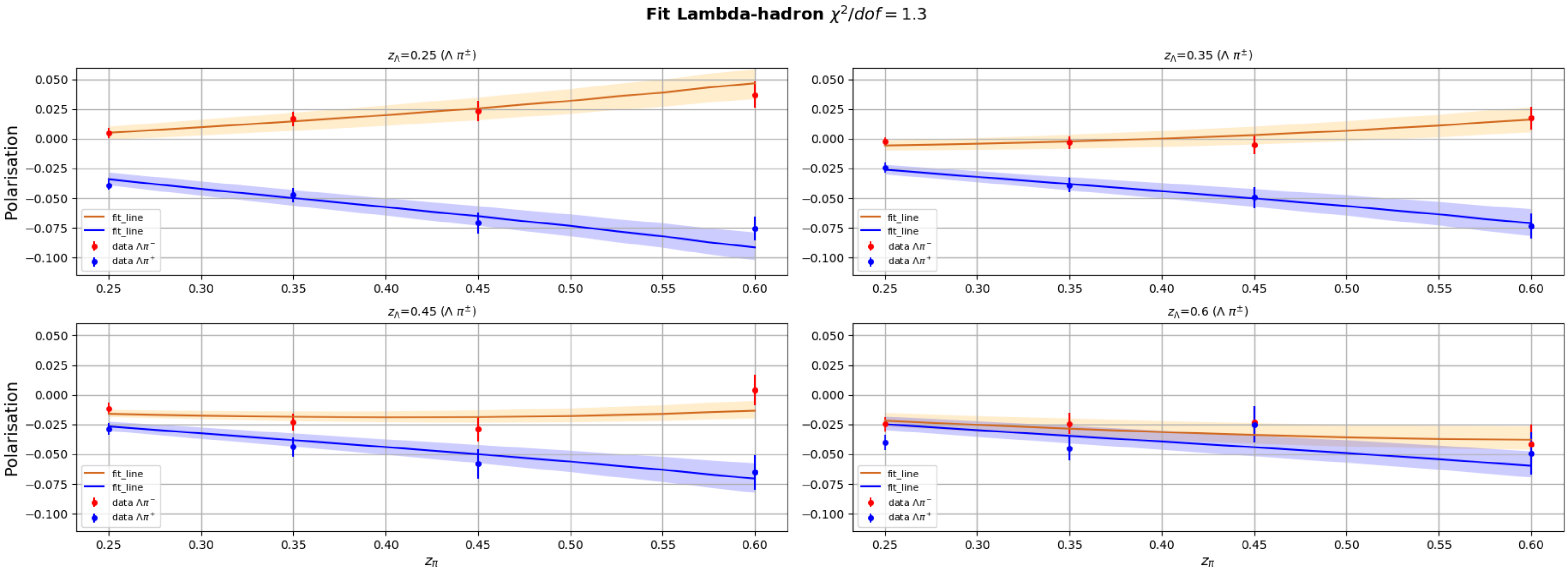
χ'^2 $\longrightarrow$

Keep set if

$$\chi^2 \leq \chi'^2 \leq \chi^2 + \Delta\chi^2$$

- Minimum Chi-square: χ^2 = 120,85
- Confidence interval $2\sigma \rightarrow$ 95,5% : $\Delta\chi^2$ = 15,79 for 8 parameters ($\boldsymbol{\chi}^2$ -distribution)

Fit: Lambda-pion

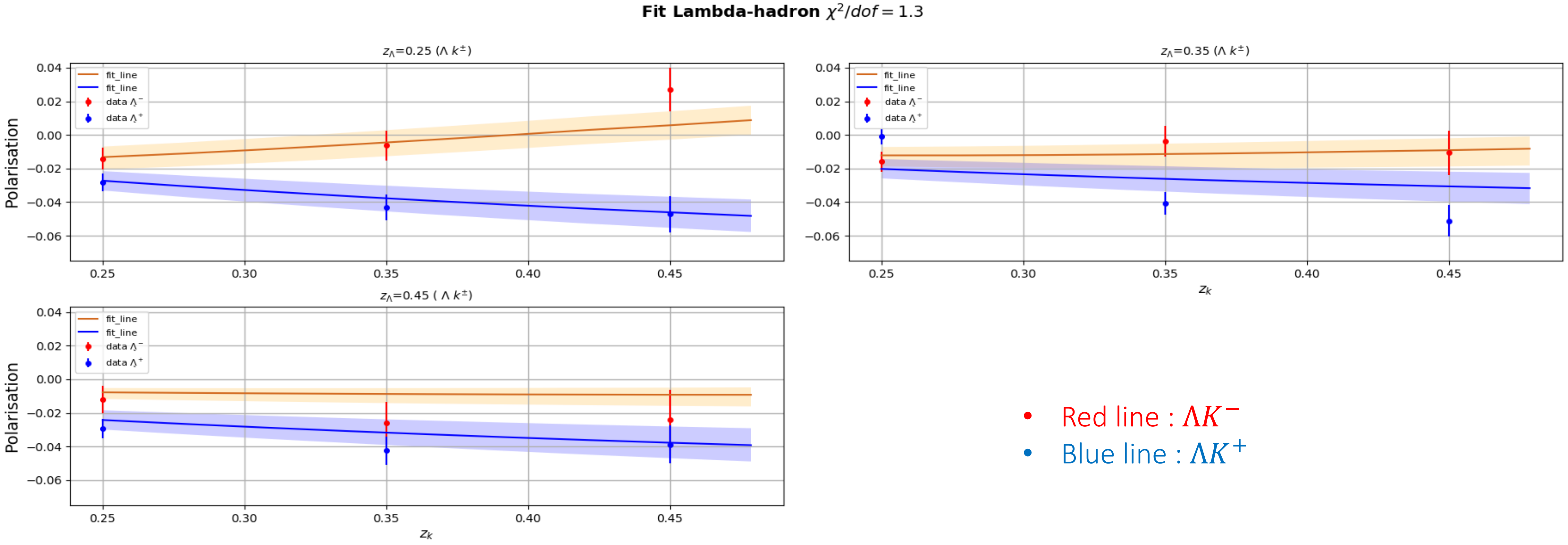


- Red line : $\Lambda\pi^-$
- Blue line : $\Lambda\pi^+$

z_Λ bins : (0.25,0.35,0.45,0.6)

Preliminary

Fit: Lambda-kaon

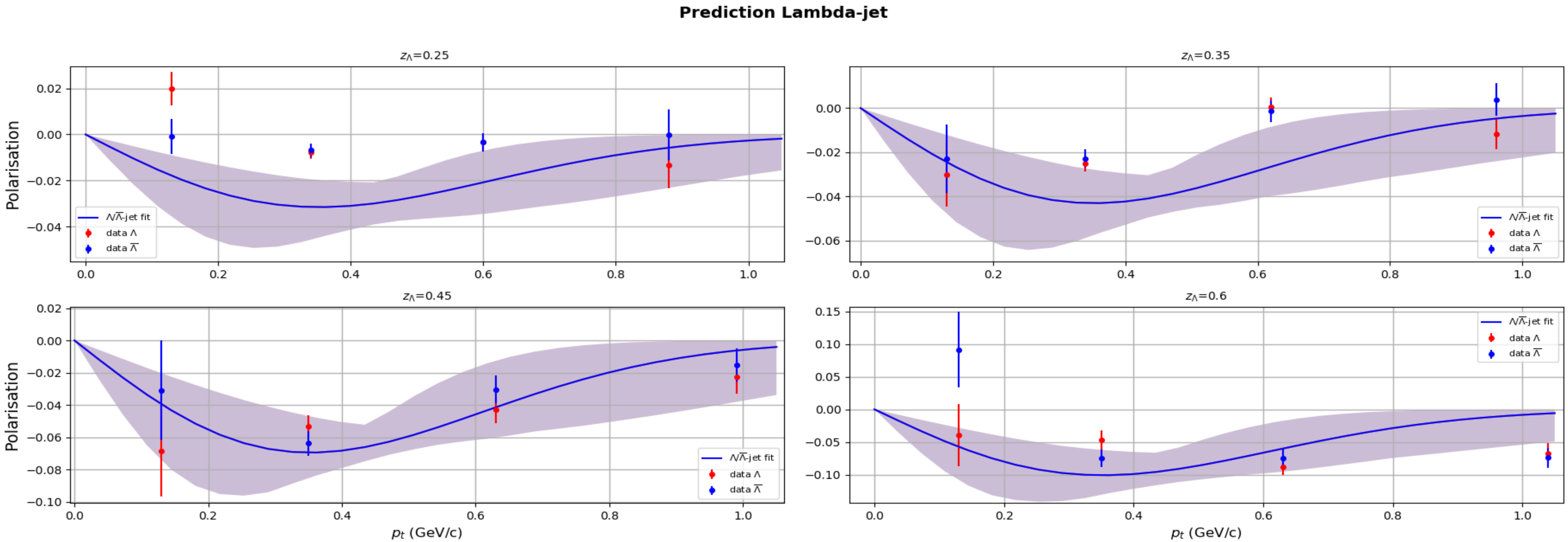


- Red line : ΛK^-
- Blue line : ΛK^+

z_Λ bins : (0.25,0.35,0.45)

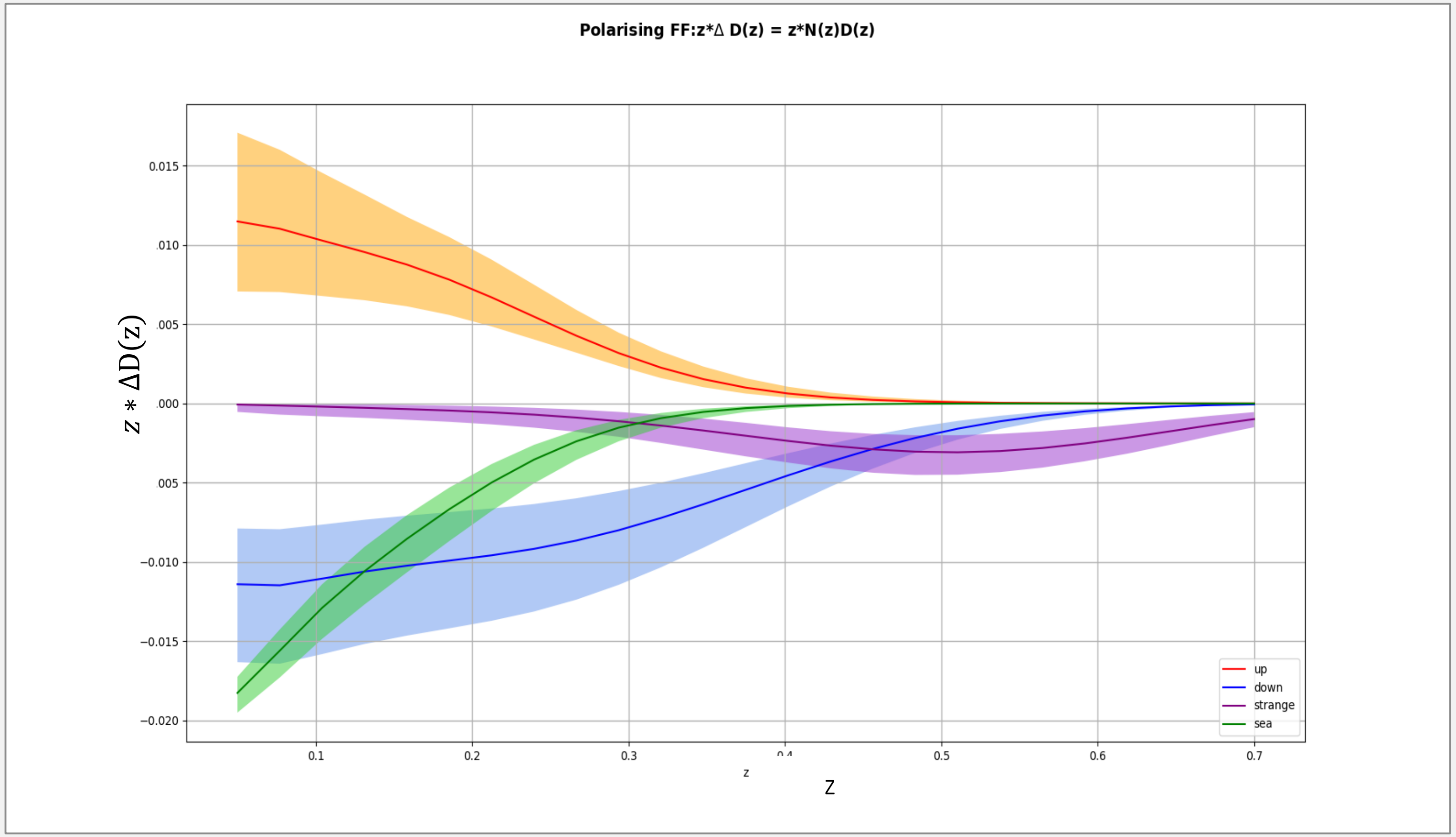
Preliminary

Prediction for Lambda-jet



Preliminary

Polarising FFs



- Up
- Down
- Strange
- Sea

Preliminary

up vs. down:

- different normalisation and shape (improve the fit)
- Relative opposite sign
- Comparable size at intermediate z
- Faster decrease of up w.r.t down at large z (cancellation vs. role of electric charges)

strange

- dominant at large z
- small at low z

sea

- Large at small z

Conclusions

- Complete helicity formalism within a TMD approach for $e^+e^- \to \mathrm{h}_1(\mathrm{jet})X$ and $e^+e^- \to h^{\uparrow}{}_1 h_2 X$
- First extraction of Lambda Polarising FF from Belle data : D'Alesio, Murgia, Zaccheddu

Outlook

- Different Gaussian width for the unpolarised and/or the polarising FF:
  z dependence, flavour dependence.
- Functional form of the polarising FF.
- Predictions for proton-proton collisions
- Comparison with existing data and previous extractions

Grazie

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$$\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 d^2p_{\perp 1}dz_2 d^2p_{\perp 2}}$$
$$=\frac{6e^4e_q^2}{64\pi\hat{s}}\Big\{D_{h_1/q}(z_1,p_{\perp 1})D_{h_2/\bar{q}}(z_2,p_{\perp 2})\Big(1+\cos^2\theta\Big)$$
$$+\frac{1}{4}\sin^2\theta\Delta^N D_{h_1/q^\uparrow}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\cos(2\varphi_2+\phi_1^{h_1})\Big\}$$

$$P_Y^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 d^2p_{\perp 1}dz_2 d^2p_{\perp 2}}$$
$$=\frac{6e^4e_q^2}{64\pi\hat{s}}\Big\{\Delta D_{S_Y/q}^{h_1}(z_1,p_{\perp 1})D_{h_2/\bar{q}}(z_2,p_{\perp 2})\Big(1+\cos^2\theta\Big)$$
$$+\frac{1}{2}\sin^2\theta\Delta^- D_{S_Y/s_T}^{h_1}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\cos(2\varphi_2+\phi_1^{h_1})\Big\}$$

$$P_X^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 d^2p_{\perp 1}dz_2 d^2p_{\perp 2}}$$
$$=\frac{3e^4e_q^2}{64\pi\hat{s}}\Delta D_{S_X/s_T}^{h_1}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\sin^2\theta\sin(2\varphi_2+\phi_1^{h_1})$$

$$P_Z^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 d^2p_{\perp 1}dz_2 d^2p_{\perp 2}}$$
$$=\frac{3e^4e_q^2}{64\pi\hat{s}}\Delta D_{S_Z/s_T}^{h_1}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\sin^2\theta\sin(2\varphi_2+\phi_1^{h_1})$$

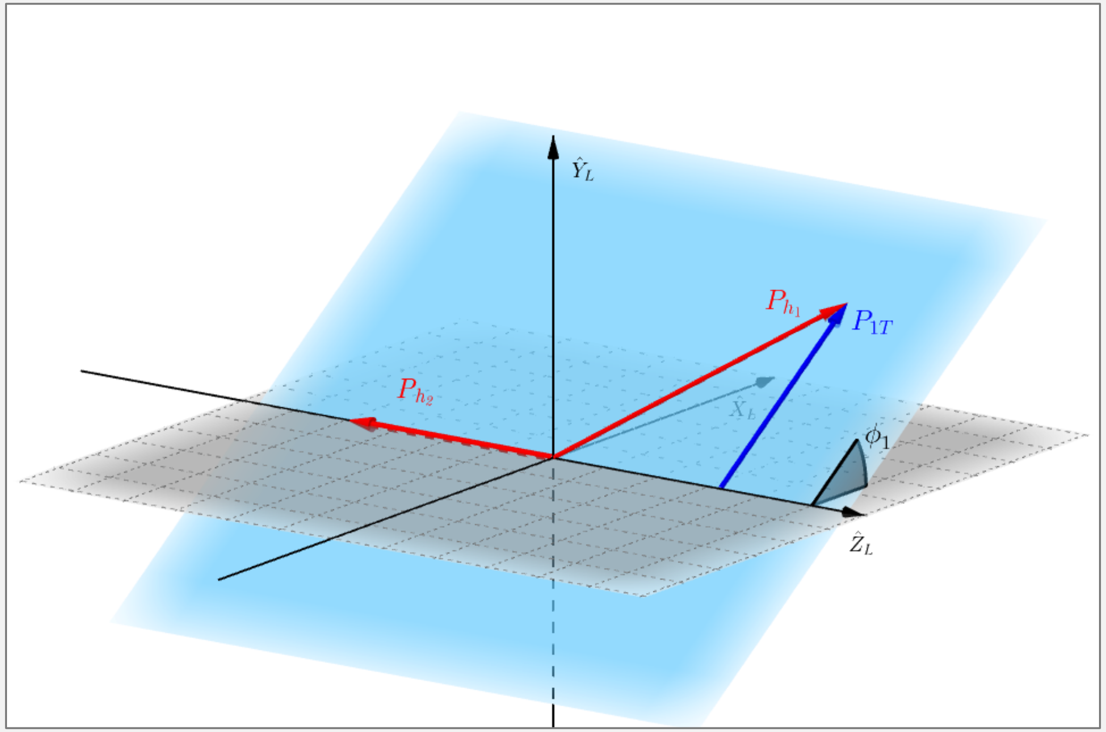
The polarisation is measured along:

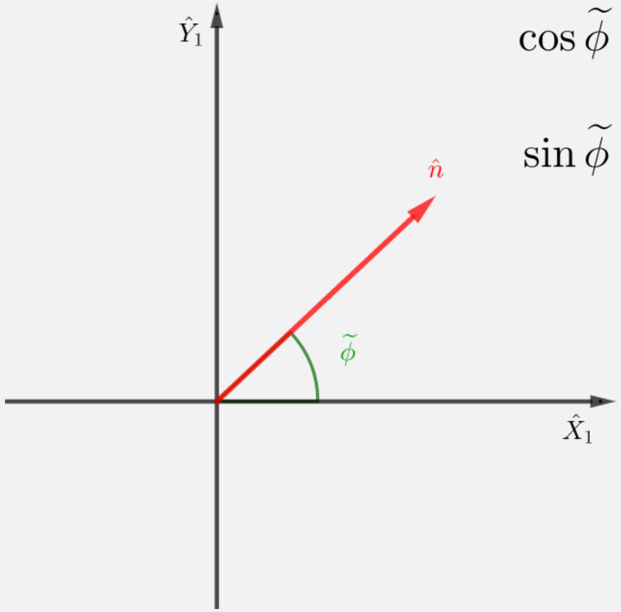
$$\hat{n} = -\hat{P_{h_2}} \times \hat{P_{h_1}}$$

Exp. Datas depend only on energy fraction z_1, z_2

The polarisation projection along $\hat{n}$:

$$\mathcal{P}^{h_1} \cdot \hat{n} = P_x^{h_1} \cos\widetilde{\phi} + P_y^{h_1} \sin\widetilde{\phi}$$





$$\cos\widetilde{\phi} \simeq \frac{z_{h_1}}{z_{h_2}} \frac{p_{\perp 2}}{p_{\perp 1}} \sin(\phi_1 - \varphi_2)$$

$$\sin\widetilde{\phi} \simeq \frac{P_{1T}}{p_{\perp 1}} - \frac{z_{h_1}}{z_{h_2}} \frac{p_{\perp 2}}{p_{\perp 1}} \cos(\phi_1 - \varphi_2)$$

$$\begin{cases} P_1 = \left(P_{1T}\cos\phi_1, P_{1T}\sin\phi_1, P_{1L}\right) \\ p_{\perp 1}^2 \simeq P_{1T}^2 + \left(\frac{z_1}{z_2}\right)^2 p_{\perp 2}^2 - 2\frac{z_1}{z_2} P_{1T} p_{\perp 2} \cos(\phi_1 - \varphi_2) \end{cases}$$

Phenomenology

From datas we can extract different information, particularly:

- $\Lambda(jet)X$: Lambda polarising width $\langle p_\perp^2 \rangle_p$
- $\Lambda\pi X$: Polarising FF (u,d)
- $\Lambda k X$: Polarising FF (u,s)

Fitted parameters:

Flav.	$\mathcal{N}_q^p$	α_q	β_q	$\langle p_\perp^2 \rangle_p$
u	$\mathcal{N}_u^p$			
d	$\mathcal{N}_d^p$			
s	$\mathcal{N}_s^p$	α_s		$\langle p_\perp^2 \rangle_p$
sea	$\mathcal{N}_{sea}^p$		β_{sea}	

Polarising parametrization:

$$\Delta D^h_{S_Y/q}(z) = \mathcal{N}_q^p(z) D_{h/q}(z)$$

$$\mathcal{N}_q^p(z) = N_q^p z^{\alpha_q}(1-z)^{\beta_q} \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

Since we did not have a set of FF that separates the Λ and $\bar{\Lambda}$:

$$D_q^\Lambda = D_q^{\Lambda^0} + D_q^{\bar{\Lambda}^0} \longrightarrow D_q^\Lambda = D_q^{\Lambda^0} + D_{\bar{q}}^{\Lambda^0}$$

$$D_{\bar{q}}^{\Lambda^0} = (1-z)^\alpha D_q^{\Lambda^0}$$

$$D_q^{\Lambda^0} = \frac{1}{1+(1-z)^\alpha} D_q^\Lambda \qquad \alpha = 1, 2$$

$$D_{\bar{q}}^{\Lambda^0} = \frac{(1-z)^\alpha}{1+(1-z)^\alpha} D_q^\Lambda$$

FF properties

$$\hat{D}_{h/q}(z,\mathbf{k}_{\perp,h}) = D_{h/q} = (D^{++}_{++} + D^{++}_{--})$$
$$\hat{D}_{h/q,s_T}(z,\mathbf{k}_{\perp,h}) = \hat{D}_{h/q} + \frac{1}{2}\Delta\hat{D}_{h/q,s_T}$$
$$\Delta\hat{D}_{h/q,s_T}(z,\mathbf{k}_{\perp,h}) = \Delta^N D_{h/q^\uparrow}\sin(\phi_{s_q}-\phi_h) = 4ImD^{++}_{+-}\sin(\phi_{s_q}-\phi_h) \quad [Collins]$$
$$\Delta\hat{D}^{h/q}_{S_Z/s_L}(z,\mathbf{k}_{\perp,h}) = \Delta D^{h/q}_{S_Z/s_L} = (D^{++}_{++} - D^{++}_{--})$$
$$\Delta\hat{D}^{h/q}_{S_Z/s_T}(z,\mathbf{k}_{\perp,h}) = \Delta D^{h/q}_{S_Z/s_T}\cos(\phi_{s_q}-\phi_h) = 2ReD^{++}_{+-}\cos(\phi_{s_q}-\phi_h)$$
$$\Delta\hat{D}^{h/q}_{S_X/s_L}(z,\mathbf{k}_{\perp,h}) = \Delta D^{h/q}_{S_X/s_L} = 2ReD^{+-}_{++}$$
$$\Delta\hat{D}^{h/q}_{S_X/s_T}(z,\mathbf{k}_{\perp,h}) = \Delta D^{h/q}_{S_X/s_T}\cos(\phi_{s_q}-\phi_h) = (D^{+-}_{+-} + D^{+-}_{-+})\cos(\phi_{s_q}-\phi_h)$$
$$\Delta\hat{D}^{h}_{S_Y/q}(z,\mathbf{k}_{\perp,h}) = \Delta D^{h}_{S_Y/q} = -2ImD^{+-}_{++} \quad [Polarizing]$$
$$\Delta\hat{D}^{h/q}_{S_Y/s_T}(z,\mathbf{k}_{\perp,h}) = \Delta\hat{D}^{h/q}_{S_Y/c} + \Delta^-\hat{D}^{h/q}_{S_Y/s_T}$$
$$\Delta^-\hat{D}^{h/q}_{S_Y/s_T}(z,\mathbf{k}_{\perp,h}) = \Delta^- D^{h/q}_{S_Y/s_T}\sin(\phi_{s_q}-\phi_h) = (D^{+-}_{+-} - D^{+-}_{-+})\sin(\phi_{s_q}-\phi_h)$$

$$D^{++}_{++} = D^{--}_{--}$$
$$D^{++}_{--} = D^{--}_{++}$$
$$D^{++}_{+-} = -D^{--}_{-+}$$
$$D^{++}_{-+} = -D^{--}_{+-}$$
$$D^{+-}_{++} = -D^{-+}_{--}$$
$$D^{-+}_{++} = -D^{+-}_{--}$$
$$D^{+-}_{+-} = D^{-+}_{-+}$$
$$D^{+-}_{-+} = D^{-+}_{+-}$$
$$D^{++}_{+-} = (D^{++}_{-+})^*$$
$$D^{+-}_{++} = (D^{-+}_{++})^*$$

$$P_T \cdot \hat{n} = P^{h_1}_X \cos\widetilde{\phi} + P^{h_1}_Y \sin\widetilde{\phi}$$