From sum rules to integral transforms:

a personal tribute in memory of

Renzo Leonardi

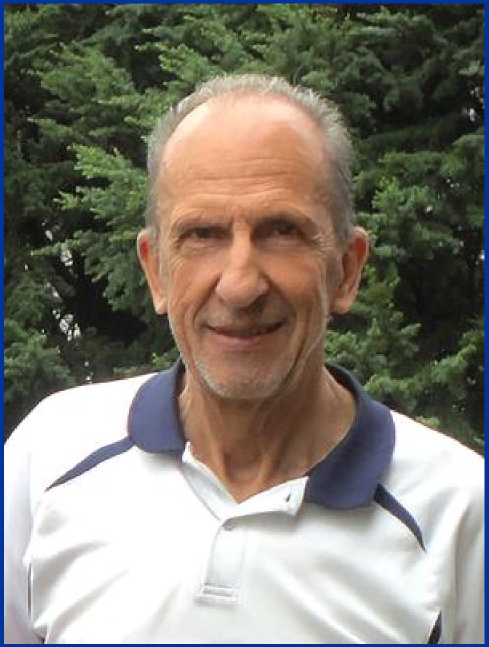
Giuseppina Orlandini

UNIVERSITÀ DI TRENTO and INFN

Outline

- a personal memory of **Renzo Leonardi**, a key person in our both national and international nuclear physics community, an eclectic personality as well as my "scientific father".
- I will try to trace a common thread between his first works on **"sum rules"** and the recent actvity on **“integral transforms”** of the Trento group.

Renzo Leonardi passed away on Saturday, July 6 2019



Renzo Leonardi was Born in Tuenno (Val di Non) on 24.06.1940

Tuenno

Lago di Tovel

Renzo Leonardi (n. 24.6.1940)

COPY from ORIGINAL

Curriculum Vitae

Carriera accademica

1959-63 Studi universitari a Pisa (Scuola Normale Superiore) e Laurea in Fisica all'Università di Bologna

1966 Nomina ad assistente ordinario alla cattedra di "Fisica Nucleare" dell'Università di Bologna

1968-73 Professore Incaricato, Università di Bologna

1971 Libera docenza in "Istituzioni di Fisica Teorica"

1972 Maturità didattica e scientifica nel concorso a cattedra di "Fisica Nucleare", Università di Genova

1973-76 Assistente Ordinario stabilizzato e Professore Incaricato stabilizzato dell'Università di Bologna

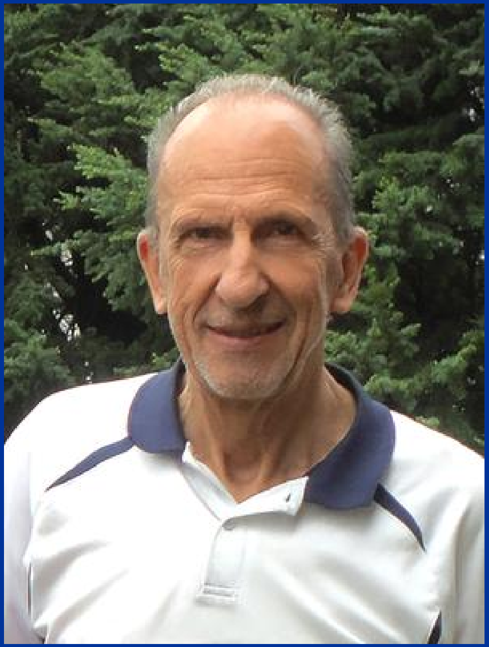
1976-77 Professore Straordinario di "Teoria delle reazioni nucleari", Università di Catania

1977-79 Professore Straordinario di "Fisica Nucleare" , Università di Trento

dal 1979 al 2008 Ordinario di Fisica Generale all'Università di Trento

dal 2009 Professore Emerito della Facoltà di Scienze dell'Università di Trento

HIS MAIN HERITAGE: "...non chi comincia ma chi persevera"



ECT*

1990: First idea and proposal

Oct 1992: recommendations of the "community meeting" held at Orsay (France) to place it in Trento

1993: operative

1997: ESF - NUPECC recognition

HIS MAIN HERITAGE: "...non chi comincia ma chi persevera"

Protontherapy Center - Trento

2002: **first idea and proposal**

2009: **the contract is undersigned for its construction**

2014: **consigned to Azienda Provinciale Servizi Sanitari (now about 300 patients/y)**

Cyclotron beam: 70-228 MeV
1-320 nA
50% duty-cycle
2 lines to *gantries*
1 line tp exp.hall

AN ECLECTIC PEARSONALITY...

R. Leonardi
“La Crucifixion de van Eyck et **l'étonnante lune** du retable de Gand”
Revue de l'art Paris 55(1):51-57 · March 2007

...WITH A PECULIAR APPROACH TO ART

R. Leonardi
"La Crucifixion de van Eyck et **l'étonnante lune** du retable de Gand"
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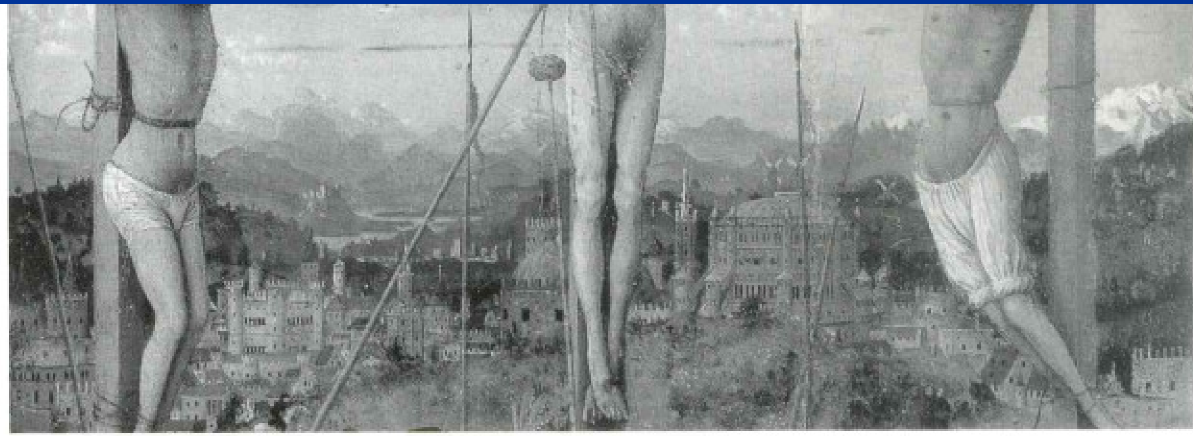
Abstract

When **Van Eyck** painted the mountainous landscape in the background of Met's panel of the Crucifixion, he most probably was **inspired by the view of the Alps** offered to the voyager coming from the north by the road crossing the Col de la Faucille between Chalonsur-Saône and Geneva. **This confirms Van Eyck's trip in the Savoie**. In this painting Van Eyck has also painted the moon at the very beginning of its declining phase. It is the first time that the moon is represented in Western painting as an astronomical rather than symbolic object. Its declining phase is undoubtedly linked to the fact that the Crucifixion took place right after the full moon of the Jewish Passover. Other examples are cited which suggest that Van Eyck's lenticular faithfulness went so far as to treat the moon like an astronomical object in coherence with the painted subject.



1. Partie supérieure de la *Crucifixion* par Jan Van Eyck. Metropolitan Museum of Art, New York. Détail du paysage de haute montagne.

2. Ligne d'horizon des Alpes vue du Col de la Faucille (en septembre). Le Mont Blanc est sur la droite.

...WITH A PECULIAR APPROACH TO ART

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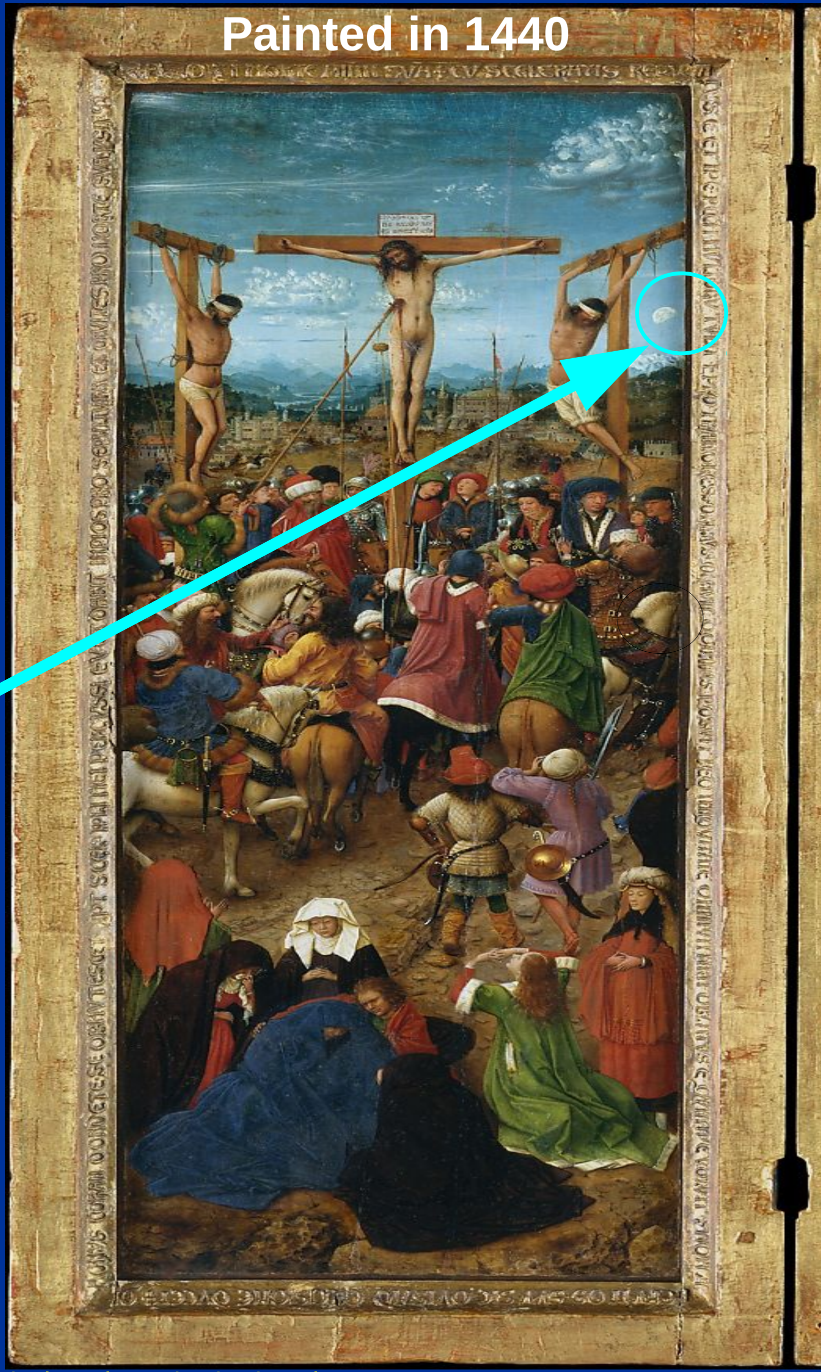
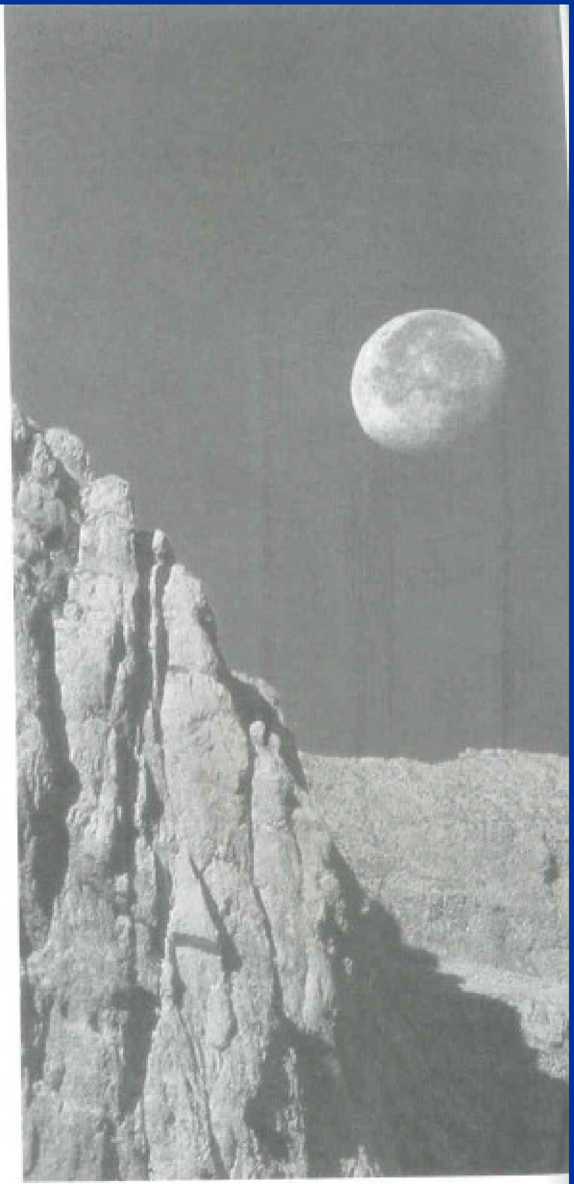


3. Détail du coucher de la lune dans la *Crucifixion* de Jan Van Eyck.
Les ombres sur la montagne sont correctement positionnées.



4. Le coucher de la lune au-dessus des Dolomites de Brenta
(région du Trentin, Italie) dans la matinée du 9 septembre 1998,
deux jours après la pleine lune.

In 2016 he broke his neck and got almost blind, but never lost his good mood


December 2018

Physics heritage:

Schema dell'attività scientifica

L'attività scientifica del candidato ha come filo conduttore la fisica nucleare e si è sviluppata essenzialmente su nove fronti, in parte, strettamente interdipendenti:

a) Studio di livelli eccitati T=2 in nuclei auto coniugati e ricerca delle relative funzioni d'onda

b) Studio dell'operazione di Time-reversal, invarianza e decadimento dei livelli T=2; Time-reversal e interazione in stato finale nel processo Gamma + Nucleo = N* + Nucleo residuo, Esperimenti di T-invarianza e interazioni in stato finale

c) Costruzione di nuove regole di somma e applicazione sistematica delle regole di somma a processi di eccitazione nucleare

    a. Teoria generale

    b. Applicazioni ai seguenti processi:

        i. Fotone-nucleo

        ii. Elettrone-nucleo

        iii. Muone-nucleo

        iv. Neutrino-nucleo

        v. Processi di eccitazione nucleare con doppio scambio di carica

        (π^+, π^-)

d) Studio dei momenti magnetici e delle transizioni super-permesse nei nuclei speculari e determinazione del rapporto g_a / g_v .

e) Studio sistematico tramite regole di somma delle proprietà collettive dei nuclei, in particolare, polarizzabilità, risonanze giganti di dipolo elettrico, struttura isotopica delle risonanze giganti di dipolo, risonanze giganti di monopolo e di quadrupolo isoscalari e isovettoriali, risonanze giganti di dipolo magnetico (isoscalari e isovettoriali).

f) Applicazioni di tecniche e regole di somma sviluppate per la fisica nucleare ad altri sistemi fisici; cluster metallici, sistemi atomici condensati.

g) Applicazioni di tecniche di regole di somma ai costituenti del nucleone.

h) Applicazioni della fisica nucleare nel settore dei beni culturali.

i) Applicazioni della fisica nucleare nel settore della oncologica.

authors (J.N.C.) (unpublished). Details are also given there of the $l_p=1$ transfer present in the 2^+ final state with a spectroscopic factor 0.008 ± 0.002.

[5]E. Newman *et al.*, Nucl. Phys. A100, 225 (1967).
[6]E. F. Gibson *et al.*, Phys. Rev. 155, 1194 (1967).
[7]F. Hinterberger *et al.*, Nucl. Phys. A111, 225 (1968).
[8]G. Harrison, Ph.D. thesis, University of Maryland, 1972 (unpublished).
[9]J. R. Shepard, P. D. Kunz, and J. J. Kraushaar, Phys. Lett. 56B, 135 (1975).
[10]DWUCK4 is an unpublished program kindly put at our disposal by P. D. Kunz of the University of Colorado.
[11]B. Zeidman, private communication.
[12]E. Newman and J. C. Hiebert, Nucl. Phys. A110, 366 (1968).
[13]J. C. van der Merwe, Ph.D. thesis, University of Stellenbosch, 1974 (unpublished).
[14]See N. S. Chant and J. N. Craig, University of Maryland Technical Report No. 76-059 (to be published), for a discussion of this program as well as a discussion of the normalization problems.
[15]W. T. Pinkston and G. R. Satchler, Nucl. Phys. 72, 641 (1965).
[16]R. H. Bassel, Phys. Rev. 149, 791 (1966).
[17]Other indications of the failure of DWBA at relatively high energies are discussed by C. C. Chang *et al.*, to be published, in their analysis of the reaction $^{12}C(d, {}^3He)$.

Microscopic Approach to the Monopole and Quadrupole Giant Resonances

E. Lipparini, G. Orlandini, and R. Leonardi
Facoltà di Scienze, Libera Università, Trento, Italy, and Istituto di Fisica dell'Università, Bologna, Italy
(Received 2 December 1975)

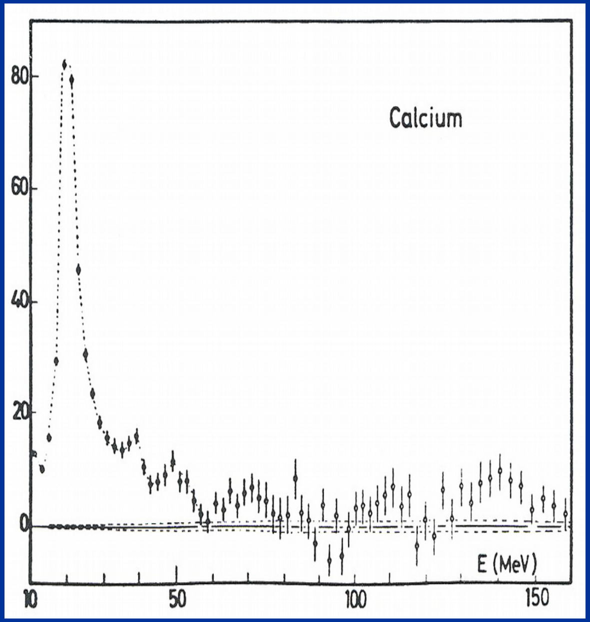
We present a microscopic approach to the monopole and quadrupole isoscalar giant resonances based on appropriate sum rules. The energies of both modes contain the same model-independent contribution $(\sqrt{2}\hbar\omega)$ and a contribution involving antisymmetrization effects and the nuclear potential. Several potentials are considered and the results of many other approaches are derived in a simple and compact way.

“Sum Rules provide useful yardsticks for measuring quantitatively the degree of collectivity of a given excited state”

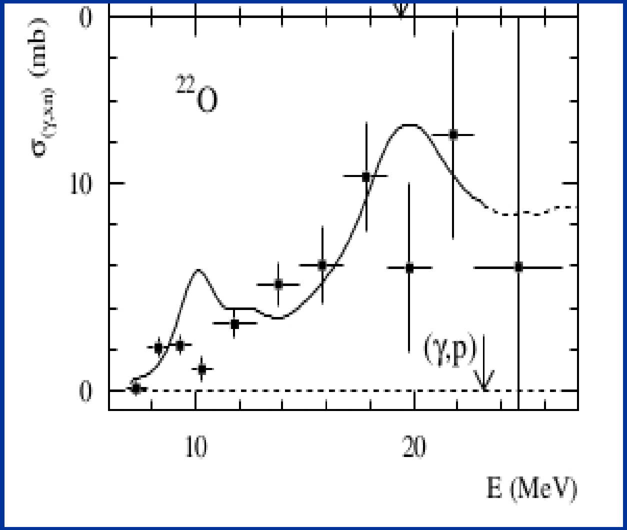
D.Rowe in “Nuclear Collective motion” 1970

Typical example: Photoabsorption cross section

$$\sigma(\omega) \sim \Sigma_n |\langle n|\Theta|0\rangle|^2\, \delta\,(\omega - E_n + E_0)$$



From J. Ahrens et al. 1975

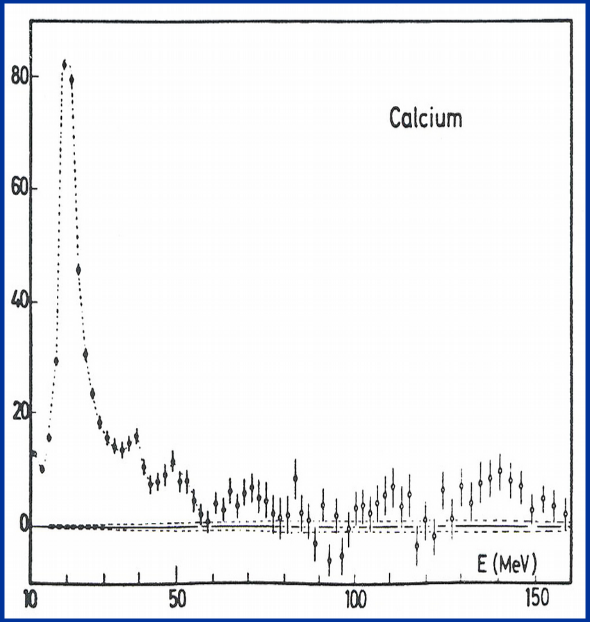


From A. Leistenschneider et al. 2001

Difficulties for many-body *ab initio* approaches

$$\sigma(\omega) \sim \Sigma_n |\langle n|\Theta|0\rangle|^2 \, \delta\,(\omega - E_n + E_0)$$

Many body states in the continuum



From J. Ahrens et al. 1975

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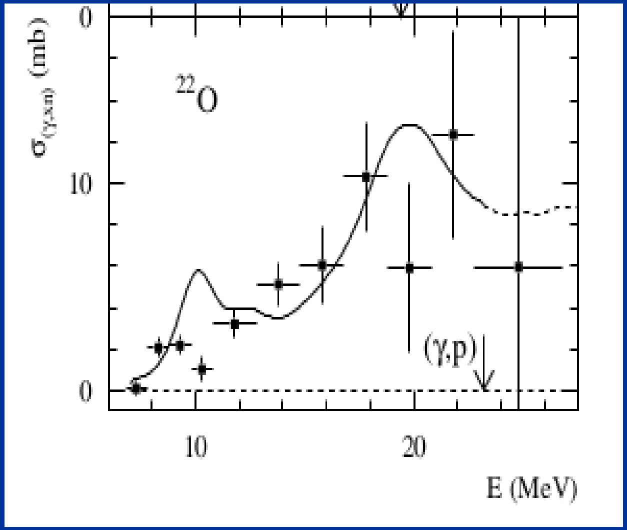

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Sum rule / moments approaches

Sum rule / moments approaches

$$m_k = \int d\omega\ \omega^k \sigma(\omega) = \int d\omega\ \omega^k \Sigma_n |<n|\Theta|0>|^2\ \delta\ (\omega\text{-}E_n+E_0)$$

Sum rule / moments approaches

$$m_k = \int d\omega \ \omega^k \sigma(\omega) = \int d\omega \ \omega^k \sum_n |<n|\Theta|0>|^2 \ \delta(\omega - E_n + E_0)$$

1) integrate in **dω** using delta function

$$\sum_n (E_n - E_0)^k < 0 \mid \Theta^+ \mid n > < n \mid \Theta \mid 0 >$$

Sum rule / moments approaches

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1) integrate in **dω** using delta function

$$\Sigma_n (E_n - E_0)^k < 0 \mid \Theta^+ \mid n > < n \mid \Theta \mid 0 >$$

2) use completeness $\Sigma_n |n><n| = I$

$$m_k = <0|\Theta^+ (H - E_0)^k \Theta |0>$$

Sum rule / moments approaches

$$m_k = \int d\omega\ \omega^k \sigma(\omega) = \int d\omega\ \omega^k \sum_n |\langle n|\Theta|0\rangle|^2\ \delta\,(\omega - E_n + E_0)$$

1) integrate in **dω** using delta function

$$\sum_n (E_n - E_0)^k \langle 0 | \Theta^+ | n \rangle \langle n | \Theta | 0 \rangle$$

2) use completeness $\sum_n |n\rangle\langle n| = I$

$$\boxed{m_k = \langle 0|\Theta^+ (H - E_0)^k \Theta |0\rangle}$$

Sum rule / moments approaches

Bound state

$$m_k = \langle 0|\Theta^+ (H-E_0)^k \Theta |0\rangle$$

Avoid calculating excited states, avoid the many-body continuum problem!

Bound state

$$m_0 = \int \sigma(\omega)\, d\omega = \langle 0|\ \Theta^+\ \Theta|0\rangle$$

$$m_1 = \int \sigma(\omega)\ \omega\ d\omega = \langle 0|\Theta^+\ (H-E_0)\ \Theta\ |0\rangle$$

$$m_2 = \int \sigma(\omega)\ \omega^2\ d\omega = \langle 0|\Theta^+\ (H-E_0)^2\ \Theta\ |0\rangle$$

.....

$$m_k = \int \sigma(\omega)\ \omega^k\ d\omega = \langle 0|\Theta^+\ (H-E_0)^k\ \Theta\ |0\rangle$$

Avoid calculating excited states, avoid the many-body continuum problem!

Sum Rules allow to know gross featrures

$$E_{peak} = \frac{m_1}{m_0} \qquad \frac{\int \sigma(\omega)\, \omega\, d\omega}{\int \sigma(\omega)\, d\omega} \qquad \frac{= \frac{1}{2} <0|\, [\Theta^+, [H, \Theta]]\, |0>}{= \frac{1}{2} \quad <0|\, \{\Theta^+, \Theta\} |0>}$$

$$\Gamma_{peak} = \sqrt{\frac{m_2}{m_0} - \frac{m_1^2}{m_0^2}}$$

Skewness involves m_3 etc...

The knowledge of all "sum rules" (moments) corresponds to the knowledge of the whole detailed cross section (distribution)

"Theory of moments": how to reconstruct the distribution from a *(limited!)* number of moments (m_0 m_1 m_2 ??)

limited because of problem of the *existence* of the moments

$$m_k = \int \sigma(\omega)\, \omega^k\, d\omega$$

high omega behavior of $\sigma(\omega)$??

Moments as integral transform

$$m_k = \int \sigma(\omega)\, \omega^k \, d\omega$$

Moments as integral transform:

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General integral transform:

$$\Phi(\tau) = \int d\omega \; K(\omega,\tau)\, \sigma(\omega)$$

Moments as integral transform

$$m_k = \int \sigma(\omega)\, \omega^k \, d\omega$$

General integral transform:

$$\Phi(\tau) = \int d\omega \; K(\omega,\tau)\, \sigma(\omega)$$

For the moments the kernel is
$K(\omega,\tau) = \omega^{\tau}$ with $\tau = k$ integer

Moments as integral transform

$$m_k = \int \sigma(\omega)\, \omega^k\, d\omega$$

Theory of moments: attempt to *invert* the integral transform

(with very limited information!)

Other kernels??

a ***"good"*** **Kernel must allow the calculation of the transform** $\Phi(\tau)$ **and allow the "reconstruction" of** $\sigma(\omega)$

Sum rule / integ. Trans. approaches

$$m_k = \int d\omega \;\; \omega^k \sigma(\omega) = \int d\omega \;\; \omega^k \Sigma_n |<n|\Theta|0>|^2 \; \delta\, (\omega - E_n + E_0)$$

$\Phi\ (\tau)$

$K\ (\omega - E_0, \tau)$

Sum rule / Integ. Transf. approaches

$$m_k = \int d\omega \ \omega^k \sigma(\omega) = \int d\omega \ \omega^k \sum_n |<n|\Theta|0>|^2 \ \delta\,(\omega - E_n + E_0)$$

1) integrate in **dω** using delta function

$$\sum_n (E_n - E_0)^k < 0 \mid \Theta^+ \mid n > < n \mid \Theta \mid 0 >$$

$$\updownarrow$$

$$K\,(E_n - E_0, \tau)$$

Sum rule / Integ. Trans. approaches

$$m_k = \int d\omega \;\; \omega^k \sigma(\omega) = \int d\omega \;\; \omega^k \Sigma_n |<n|\Theta|0>|^2 \; \delta\,(\omega - E_n + E_0)$$

1) integrate in **dω** using delta function

$$\Sigma_n (E_n - E_0)^k < 0 \mid \Theta^+ \mid n > < n \mid \Theta \mid 0 >$$

2) use completeness $\Sigma_n |n><n| = I$

$$\boxed{\begin{array}{l} m_k = \; <0|\Theta^+ (H-E_0)^k \Theta \,|0> \\ K\,(H-E_0,\tau) \end{array}}$$

…namely, repeating the procedure (*use of completeness*)

$$m_k = \langle 0|\Theta^+ (H-E_0)^k \, \Theta \, |0\rangle$$

$$\Phi(\tau) = \langle 0| \, \Theta^+ \, K(H-E_0,\tau) \, \Theta|0\rangle$$

Ab initio methods have made great progress in calculating |0>

in essentially two ways:

1) GFMC *(and "variations")*

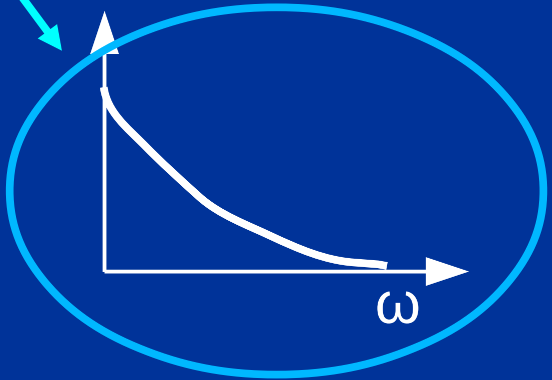
2) diagonalization on HO or HH basis *(and "variations")*

The same two methods can be used to calculate accurately

$$\Phi\,(\tau) = \langle 0|\ \Theta^{+}\ K\,(H-E_0,\tau)\ \Theta|0\rangle$$

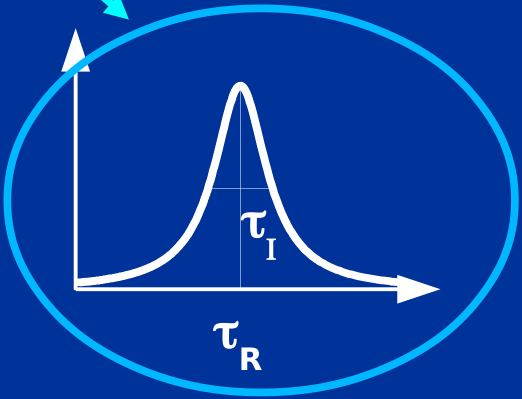
for two different Kernels

1: **Exponential Kernel:** $K(\omega,\tau) = e^{-\omega\tau}$ **τ real**



- $\Phi(\tau)$ calculated via **GFMC**
- requires a big effort to be inverted

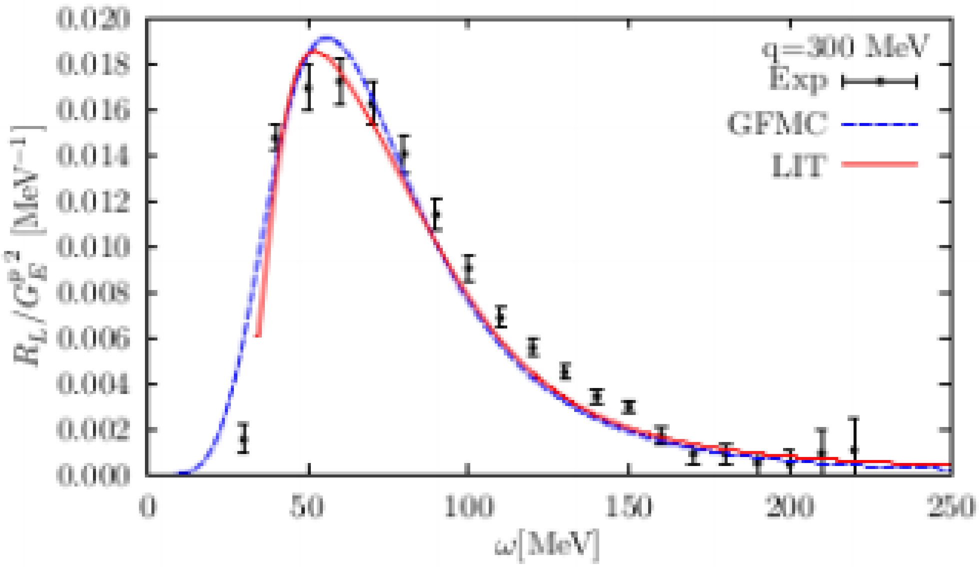
2: **Lorentzian Kernel:** $K(\omega,\tau) = [(\omega - \tau)(\omega - \tau)^*]^{-1}$



- **τ complex = $\tau_R + \iota\,\tau_I$**
- easy to invert
- $\Phi(\tau)$ calculated via **matrix diagonalization on bound** basis functions

estimation of accuracy

Longitudinal (e,e') response function of ^{4}He



Blue line: Exponential Kernel

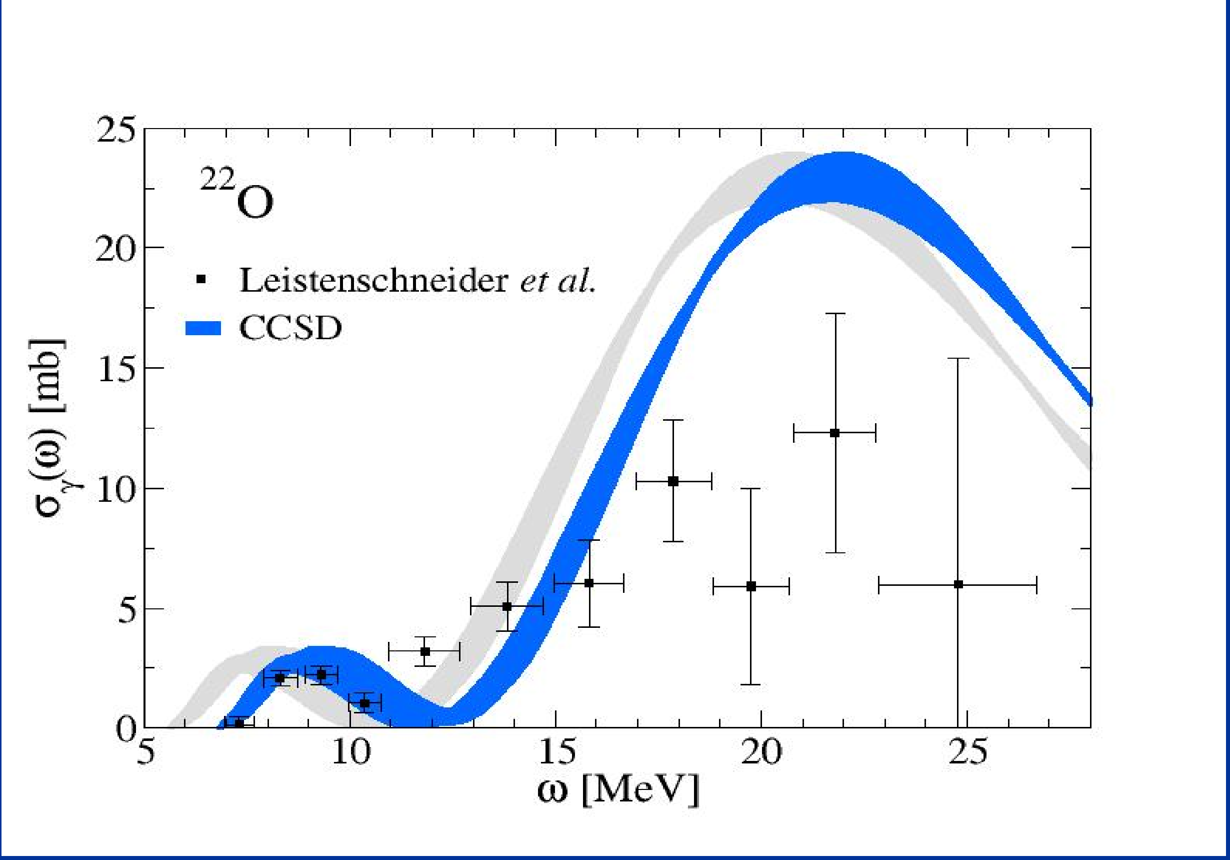
N. Rocco, W. Leidemann, A. Lovato, G. O.

Phys. Rev. C 97, 055501 (2018)

Red line: Lorentzian Kernel:

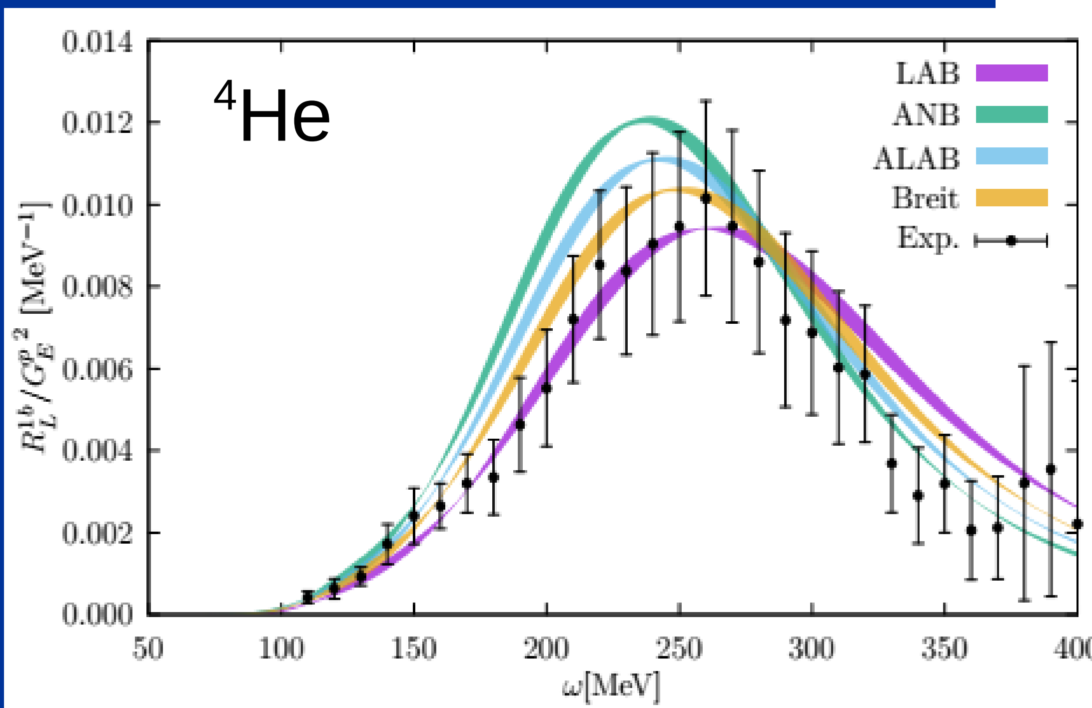
S. Bacca, N. Barnea, W. Leidemann, and G. O.
Phys. Rev. C 80, 064001 (2009)

Ex. of result with Lorentzian Kernel: photoabsorption in halo nucleus



S. Bacca, N. Barnea, G. Hagen, M. Miorelli, G. O. and T. Papenbrock
Phys. Rev. C 90, 064619 (2014)

Ex. of result with Exponential Kernel: a study of the frame dependence of electronscattering in view of ν-exp



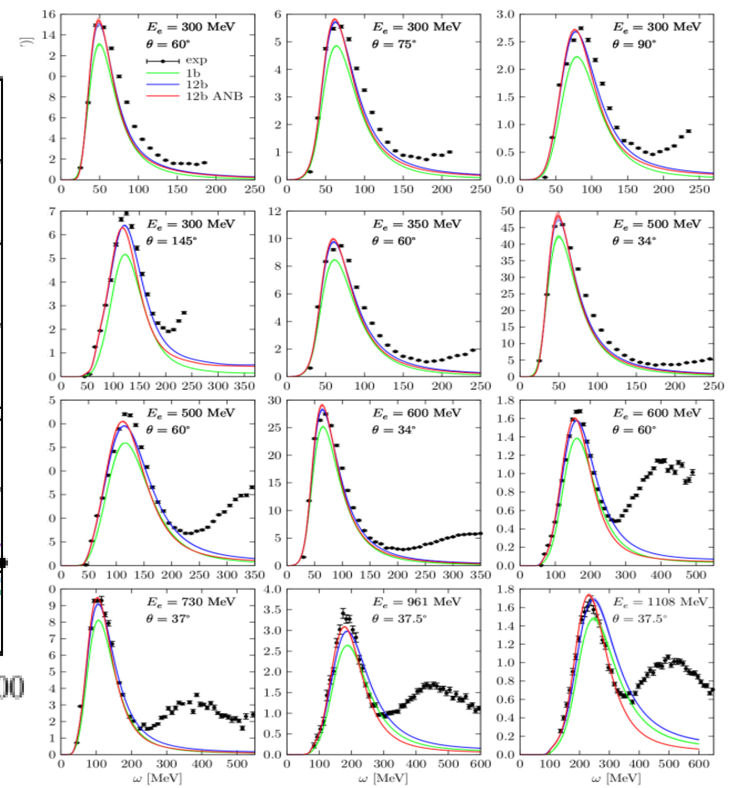


FIG. 7. Double-differential electron-^{4}He cross sections for different values of incident electron energy and scattering angle. The green and blue lines correspond to GFMC calculation were only one- body and one- plus two-body contributions in the electromagnetic currents are accounted for. The red line indicates one plus two-body current results obtained in the ANB frame, employing the two-body fragment model to account for relativistic kinematics. The experimental data are taken from Ref. [?].

N.Rocco, W.Leidemann, A. Lovato, G.O.
Phys. Rev. C 97, 055501 (2018)

Are there other kernels suitable for diagonalization methods?

$$\boxed{\Phi(\tau)} = \int \sigma(\omega)\, K(\omega,\tau)\, d\omega =$$

$$\boxed{\langle 0 |\ \Theta^{+}\ K(H-E_0,\tau)\ \Theta | 0 \rangle}$$

Yes, conditions required:

1) $$m_0 = \int \sigma(\omega)\, d\omega \quad < \infty$$

2) $$\Phi(\tau) \quad = \quad \int \sigma(\omega)\, K(\omega,\tau)\, d\omega \quad < \infty$$

3) $K(\omega,\tau)$ is a real positive definite function of ω
(or linear combinations)
e.g. Gaussians, wavelets etc.

More recent activity in Trento:

The LIT as ab initio method *with controlled resolution* and the conditions for reliable inversions

V. D. Efros, W. Leidemann , and V. Yu. Shalamova
Few-Body Syst. 60:35 (2019)

[W.Leidemann Phys. Rev. C 91, 054001 (2015)]

Conditions for reliable inversion:

$$\Phi = \boxed{\sum_\lambda L(\varepsilon_\lambda, \Gamma) \, |\langle\lambda| \, \Theta |0\rangle|^2}$$

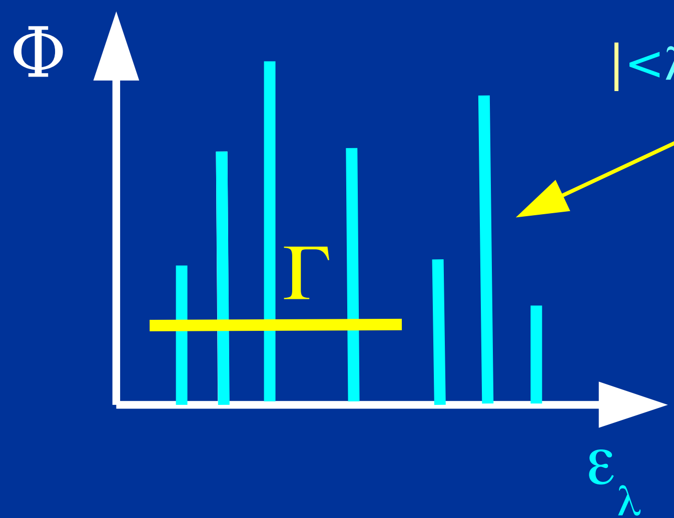
After diagonalization

$$H \, |\lambda\rangle = \varepsilon_\lambda \, |\lambda\rangle$$

Conditions for reliable inversion:

$$\Phi \;=\; \boxed{\;\Sigma_\lambda \;\; L(\varepsilon_\lambda,\Gamma)\; |\langle\lambda|\;\Theta|0\rangle|^2\;}$$

Get enough ε_λ, $|\lambda\rangle$ and $|\langle\lambda|\;\Theta|0\rangle|^2$ from the diagonalization of $H_{\mu\nu}$ in the energy region of interest within the "*resolution of the kernel*"



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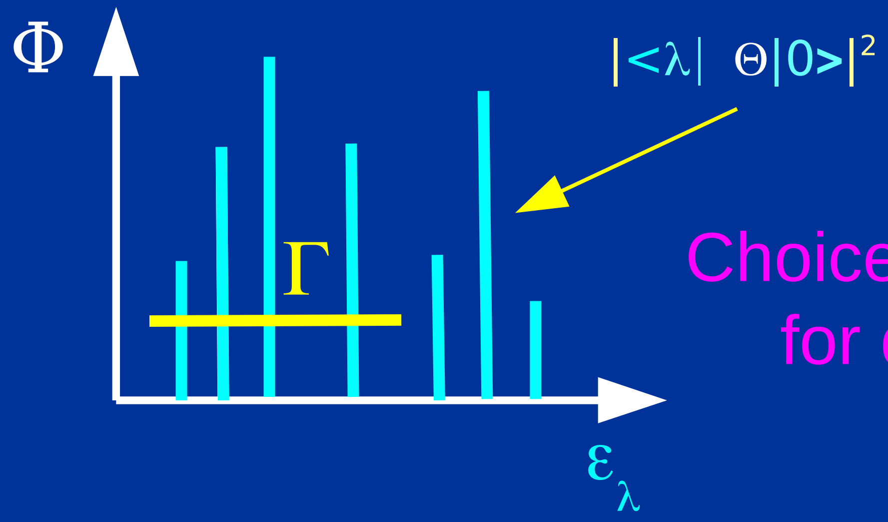


Choice of a suitable basis for diagonalization!!

More recent activity in Trento
in view of applications of the Lorentz Integral Transform to

- Hypernuclei, also with meson exc. based potentials with Λ-Σ couplings (*Sergio Deflorian, Fabrizio Ferrari Ruffino)*
- Reactions of astrophysical interest within cluster models and EFT potentials e.g.

$n+\alpha+\alpha \longrightarrow {}^9\mathrm{Be} + \gamma$ (*Paolo Andreatta, Elena Filandri)*

Much of the work has been devoted to modify the HH basis used for potentials in configuration space to account for different masses and symmetries and EFT (p-space) potentials

Some ideas have been taken from our partners of the I.S. fbs in Pisa,
(NSHH, p-space HH),
however with *variations*

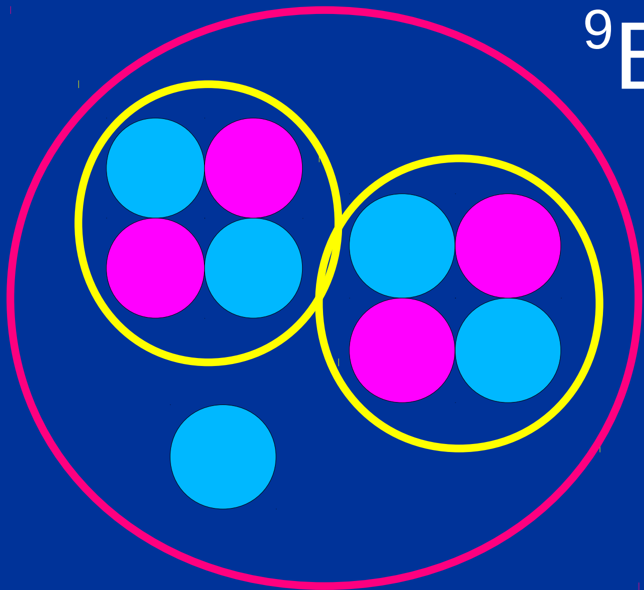
Code that deals with
both *mom.* and *conf.* space potentials
particles of both *equal* and *different masses*
particles of both *equal* and *different statistics*

Sergio Deflorian
Fabrizio Ferrari Ruffino
Paolo Andreatta

Λ-hypernuclei benchmark

Fabrizio Ferrari Ruffino

$V_{NN} + V_{YN}$	System	NSHH	FY	GEM
AV8'	^{2}H	[-2.226(1)]	-2.226(1)	-
AV8'+sNSC97f	$^{3}_{\Lambda}H$	-2.413(3)	-2.415(1)	-2.42(1)
	B_{Λ}	0.187(3)	0.189(1)	0.19(1)
AV8'	^{3}H	-7.76(0)	-7.76(0)	-7.77(1)
AV8'+sNSC97f	$^{4}_{\Lambda}H$	-10.08(2)	-	-10.10(1)
	B_{Λ}	2.32(2)	-	2.33(1)



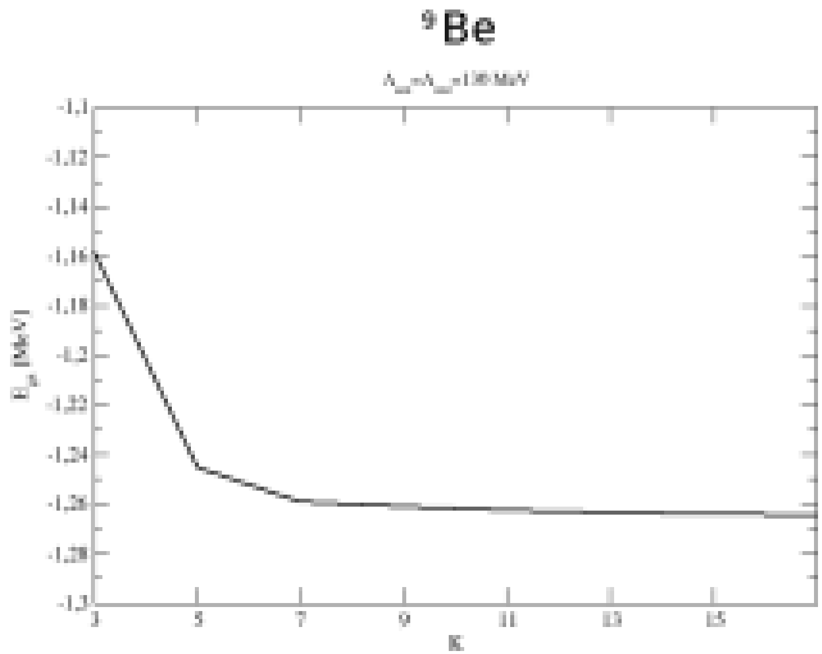
^{9}Be

EFT potential:

$$V(p, p') = \lambda_0 + \lambda_1 (p^2 + p'^2)$$

- Partial wave expansion ($\alpha\alpha$ **l=0** α**n l=1**)
- Gaussian regulator (Λ limited by Wigner bounds)
- λ_0 , λ_1 from on shell T-matrix with the effective range expansion up to k^2 order
- One gets **2** possible solutions ($+\lambda_0$, λ_1) ($-\tilde{\lambda}_0$, $\tilde{\lambda}_1$)

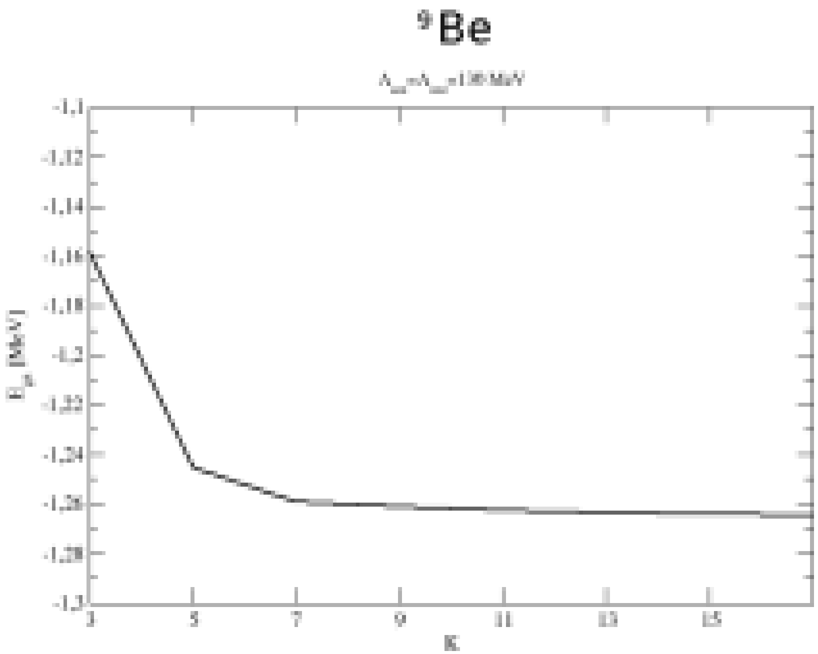
Convergence



Elena Filandri

K	E_{gr}[MeV]
3	-1.1582708
5	-1.2453006
7	-1.2593364
9	-1.2611099
11	-1.2624669
13	-1.2634732
15	-1.2641629
17	-1.2646203

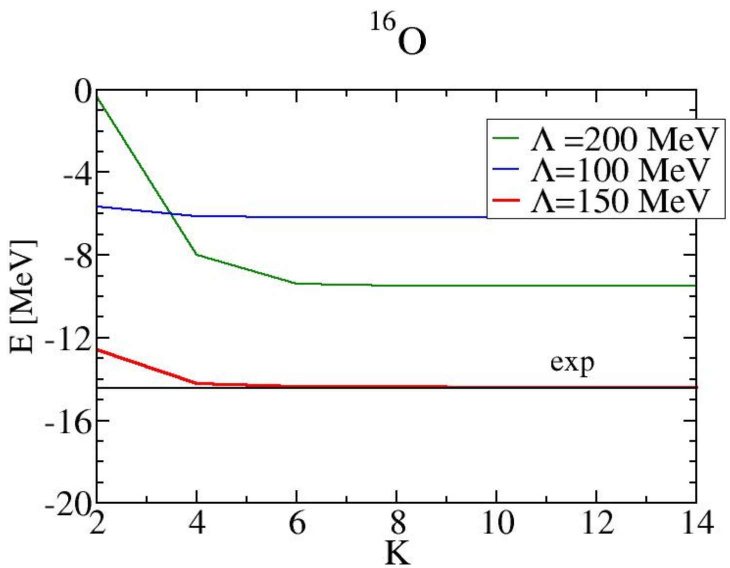
Convergence



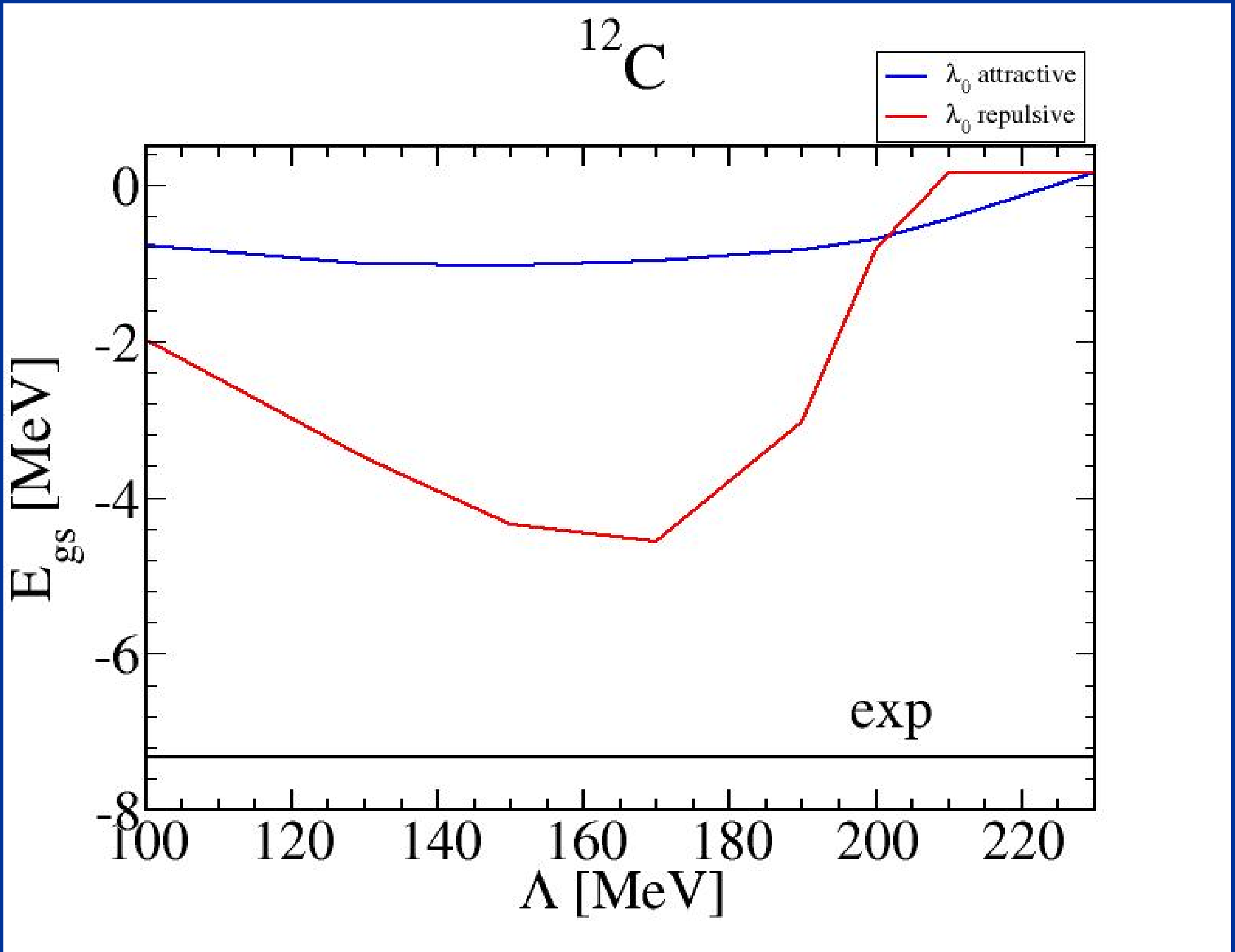
Elena Filandri

K	E_{gs}[MeV]
3	-1.1582708
5	-1.2453006
7	-1.2593364
9	-1.2611099
11	-1.2624669
13	-1.2634732
15	-1.2641629
17	-1.2646203

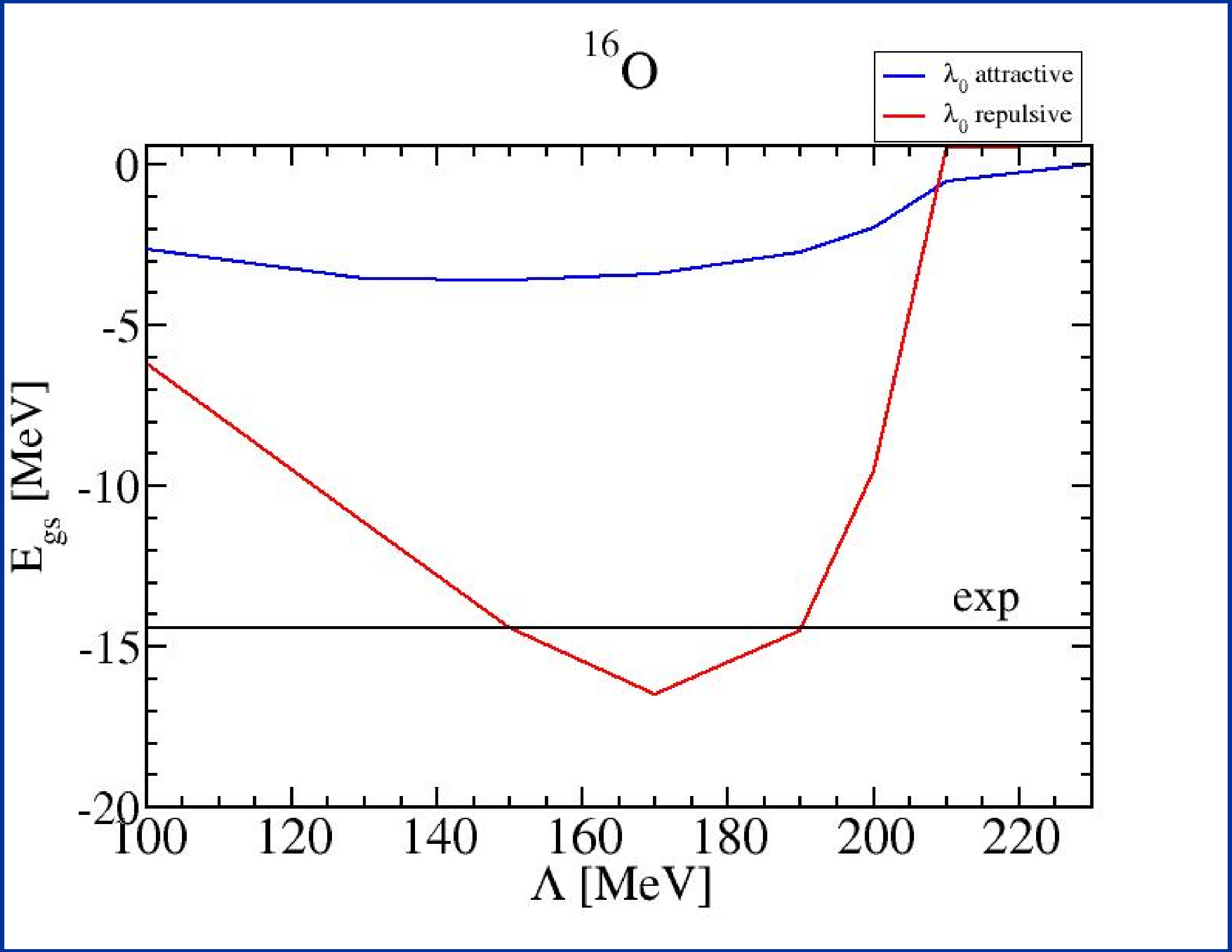
K	E_{gs}[MeV]
2	-12.569698
4	-14.215357
6	-14.345383
8	-14.364673
10	-14.370449
12	-14.375405
14	-14.377645



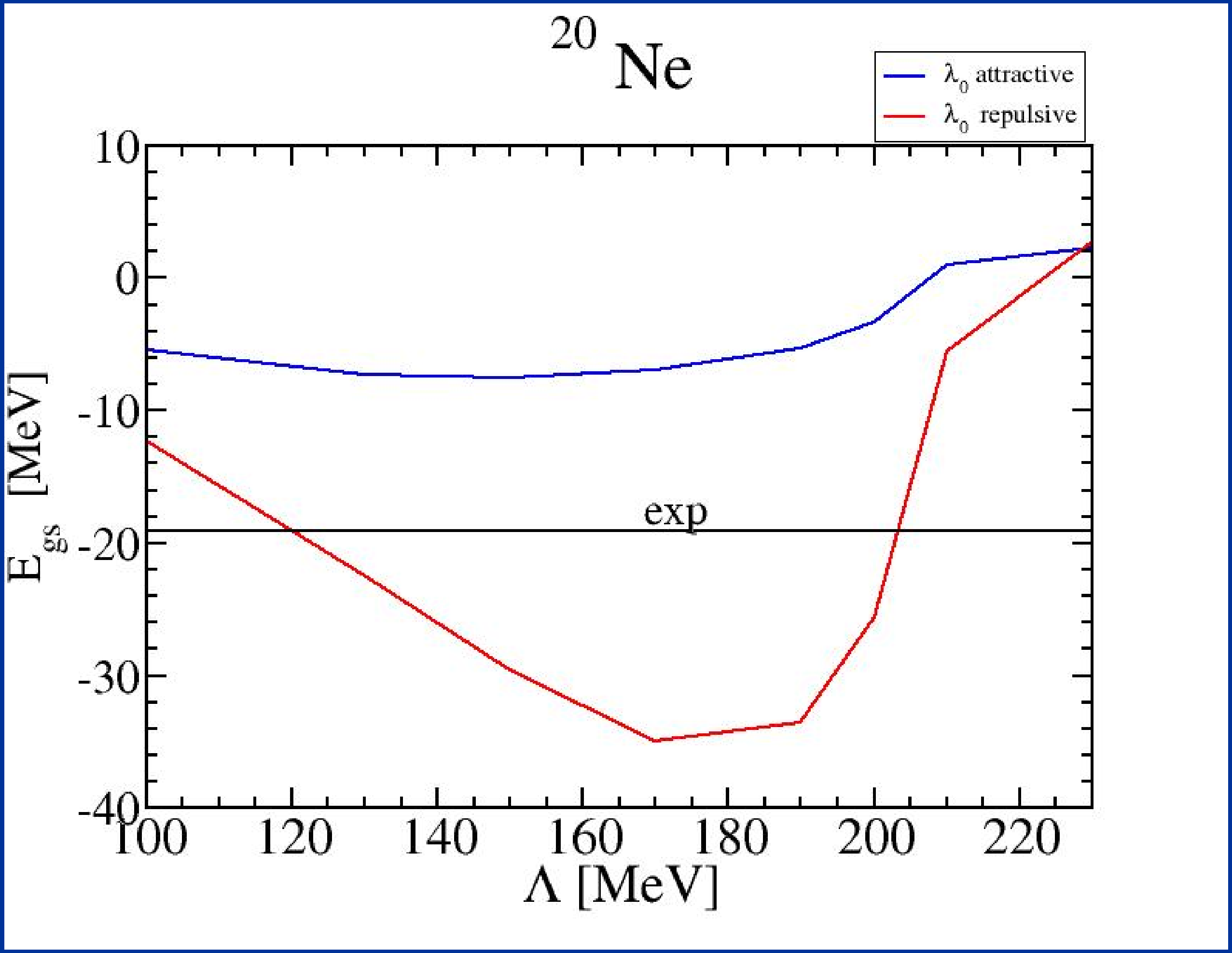
Cut off dependence



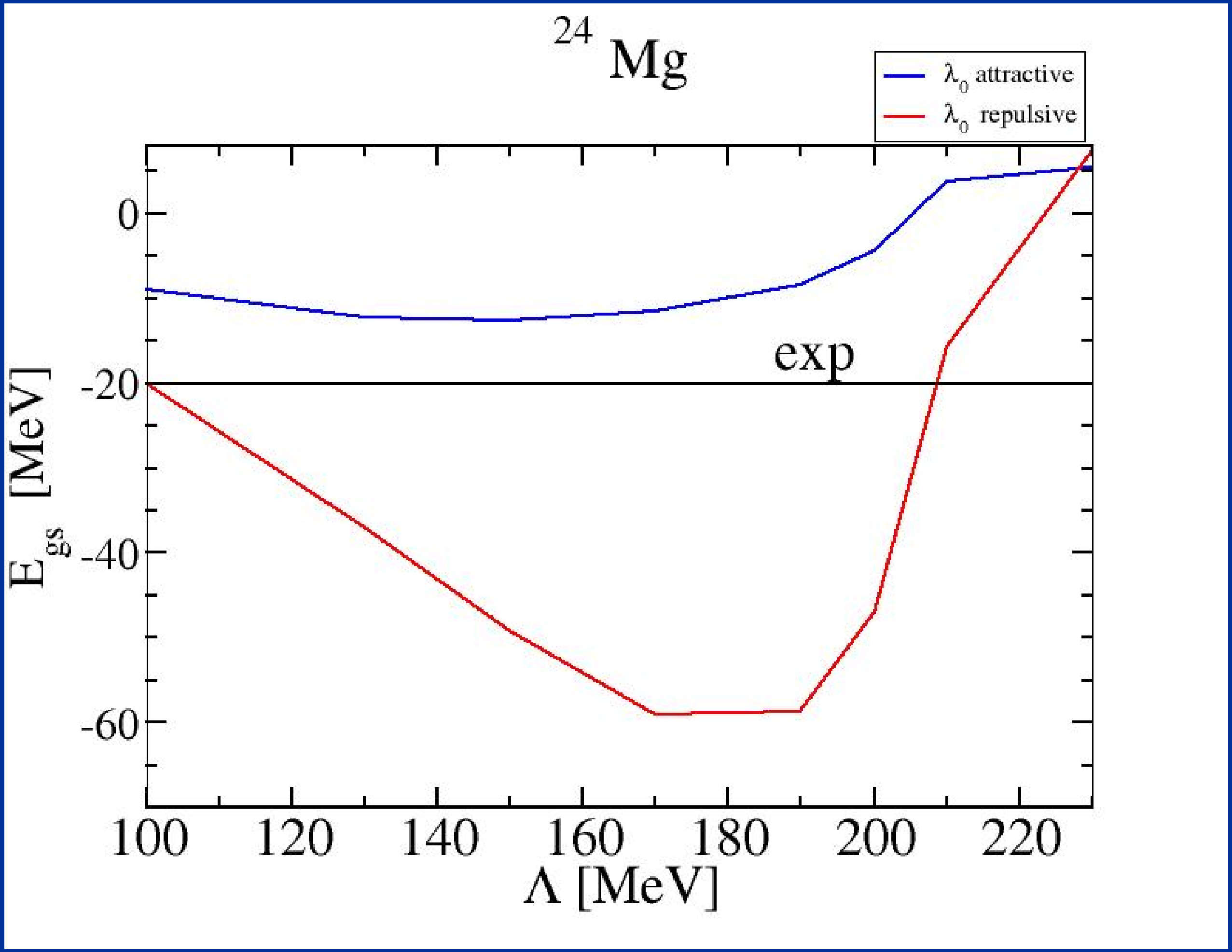
Cut off dependence



Cut off dependence



Cut off dependence



Importance of 3-body forces (in progress)

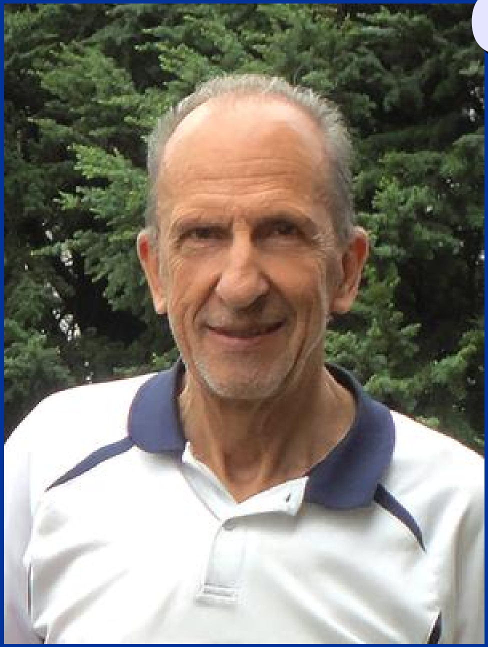
In most cases values of Λ exist which reproduce exp. B.E.

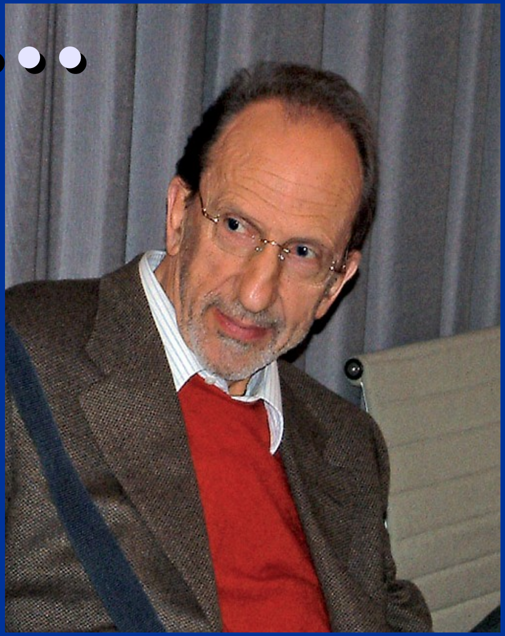
useful for preliminar **integral transform** results (in progress)

Conclusion

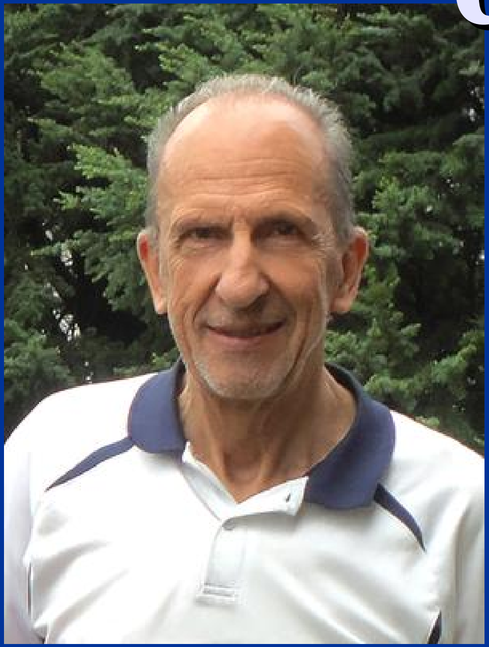
From sum rules to integral transforms…

Grazie Renzo, da me per questo...





… e da tutti noi per questo!!!



If we had to deal with a **“confined”** system one could represent H on **bound states eigenfunctions $|\nu>$**

$$\langle 0|\Theta^{+}\ K(H-E_0,\tau)\ \Theta|0\rangle = \Phi(\tau) =$$

$$\Sigma_{\mu\nu}\ \langle 0|\ \Theta^{+}\ |\mu\rangle\ \langle\mu|\ K(H_{\mu\nu}-E_0,\tau)|\nu\rangle\langle\nu|\ \Theta|0\rangle$$

If we had to deal with a **"confined"** system one could represent H on **bound states eigenfunctions |ν >**

$$\langle 0|\Theta^{+}\ K(H\text{-}E_0,\tau)\ \Theta|0\rangle = \Phi(\tau) =$$

$$\sum_{\mu\nu} \langle 0|\ \Theta^{+}\ |\mu\rangle\ \langle\mu|\ K(H_{\mu\nu}\text{-}E_0,\tau)|\nu\rangle\langle\nu|\ \Theta|0\rangle$$

After diagonalizing $H_{\mu\nu}$ the transform would be simply

$$\boxed{\sum_{\lambda}\ K(\varepsilon_{\lambda} - E_0,\tau)\ |\langle\lambda|\ \Theta|0\rangle|^2 = \Phi(\tau)}$$

If we had to deal with a **"confined"** system one could represent H on **bound states eigenfunctions $|\nu>$**

$$\langle 0|\Theta^+ \; K(H-E_0,\tau) \; \Theta|0\rangle = \Phi(\tau) =$$

$$\Sigma_{\mu\nu} \; \langle 0| \; \Theta^+ \; |\mu\rangle \; \langle\mu| \; K(H_{\mu\nu}-E_0,\tau)|\nu\rangle\langle\nu| \; \Theta|0\rangle$$

After diagonalizing $H_{\mu\nu}$ the transform would be simply

$$\boxed{\Sigma_\lambda \quad K(\varepsilon_\lambda - E_0,\tau) \; |\langle\lambda| \; \Theta|0\rangle|^2 = \Phi(\tau)}$$

(Up to convergence!)

THE GOOD NEWS:

The representation of H on **b.s. eigenfunctions $|\nu>$**
and therefore the calculation of the transform via

$$\Phi(\tau) = \boxed{\Sigma_\lambda \quad K(\varepsilon_\lambda - E_0, \tau)\ |\langle\lambda|\ \Theta|0\rangle|^2}$$

is **allowed** for **specific kernels $K(\omega,\tau)$!**

No approximation!