Spectra and electroweak response of light nuclei: a status report

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Outline

- The basic model of nuclear theory
- Chiral $2N$ and $3N$ interactions, nuclear spectra, and neutron matter EOS
- Nuclear electroweak currents and response at low and intermediate energies (QE CC response in ^{12}C)
- Outlook:
  - *Modeling $3N$ interactions*
  - *Validating weak current model*

The basic model

- Effective interactions:

$$H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2\, m_i} + \sum_{i<j=1}^{A} \underbrace{v_{ij}}_{\text{th+exp}} + \sum_{i<j<k=1}^{A} \overbrace{V_{ijk}}^{\text{th+exp}} + \cdots$$

- Assumptions:
  - *Quarks in nuclei are in color singlet states close to those of N's (and low-lying excitations: Δ's, ...)*
  - *Series of interactions converges rapidly*
  - *Dominant terms in v_{ij} and V_{ijk} are due to π exchange*

$$\text{leading } \pi N \text{ coupling} = \frac{g_A}{2\, f_\pi} \tau_a\, \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \phi_a(\mathbf{r})$$

- Effective electroweak currents:

$$j^{EW} = \sum_{i=1}^{A} j_i + \sum_{i<j=1}^{A} j_{ij} + \sum_{i<j<k=1}^{A} j_{ijk} + \cdots$$

χEFT formulation of the basic model

- χEFT is a low-energy approximation of QCD
- Lagrangians describing the interactions of π, N, ... are expanded in powers of Q/Λ_χ ($\Lambda_\chi \sim 1$ GeV)
- Their construction has been codified in a number of papers [1]

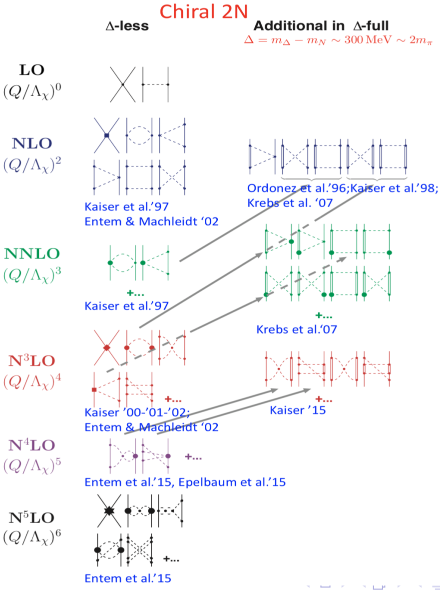
$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \dots \\ &\quad + \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \dots\end{aligned}$$

- $\mathcal{L}^{(n)}$ also include contact $(\overline{N}N)(\overline{N}N)$-type interactions parametrized by low-energy constants (LECs)
- Initial impetus to the development of χEFT for nuclei in the early nineties [2,3]

[1] Gasser and Leutwyler (1984); Gasser, Sainio, and Švarc (1988); Bernard *et al.* (1992); Fettes *et al.* (2000)

[2] Weinberg (1990–1992); [3] Park, Min, and Rho (1993; 1996)

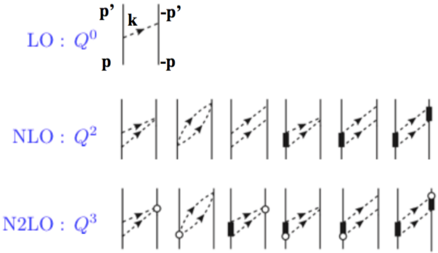
Chiral $2N$ interactions: current status



Local chiral $2N$ interactions with Δ's

Piarulli *et al.* (2015); Piarulli *et al.* (2016)

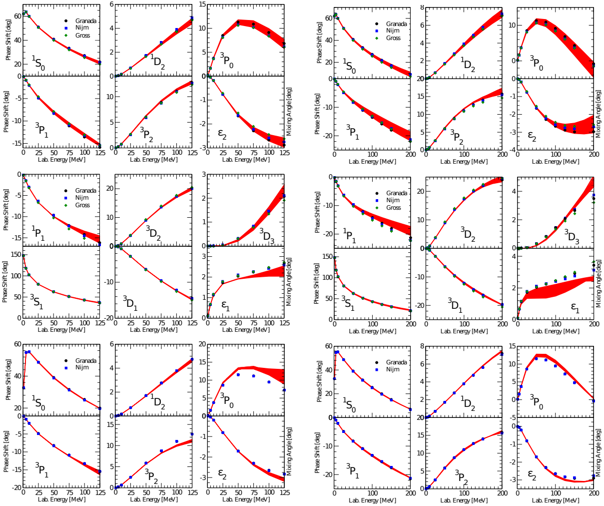
- Two-nucleon interaction: $v = v^{\rm EM} + v^{\rm LR} + v^{\rm SR}$
- EM component $v^{\rm EM}$ including corrections up to α^2
- Chiral OPE and TPE component $v^{\rm LR}$ with Δ's



- Short-range contact component $v^{\rm SR}$ up to order Q^4 parametrized by (2+7+11) IC and (2+4) IB LECs
- $v^{\rm SR}$ functional form taken as $C_{R_S}(r) \propto {\rm e}^{-(r/R_S)^2}$ with R_S=0.8 (0.7) fm for a (b) models

np ($T = 0$ and 1) and pp phase shifts

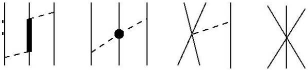
Ia-Ib: $E_{\mathrm{lab}} = 125$ MeV   IIa-IIb: $E_{\mathrm{lab}} = 200$ MeV



Local chiral $3N$ interactions with Δ's

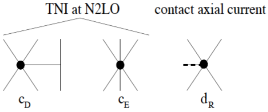
Piarulli *et al.* (2018)

- $3N$ interaction up to N2LO[1]:



- Type (1): fix c_D and c_E by fitting $E_0^{\rm exp}(^3\text{H})$ and nd doublet scattering length $a_{nd}^{\rm exp} = (0.645 \pm 0.010)$ fm

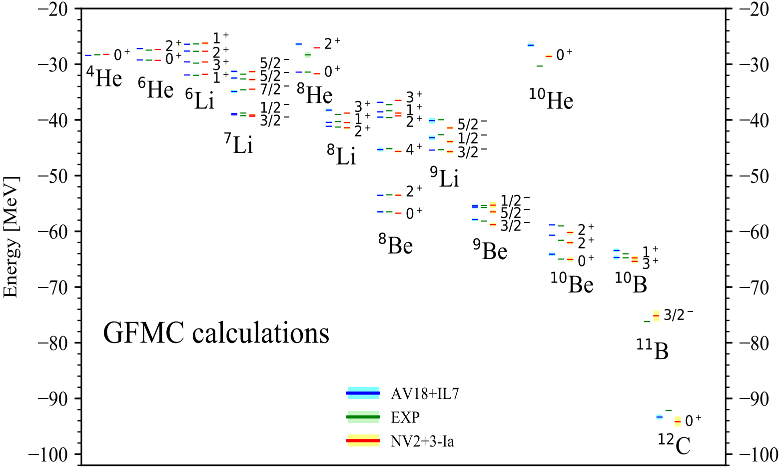
- Type (2): fix c_D and c_E by fitting $E_0^{\rm exp}(^3\text{H})$ and the GT$^{\rm exp}$ matrix element in ^{3}H β-decay



[1] Epelbaum *et al.* (2002)

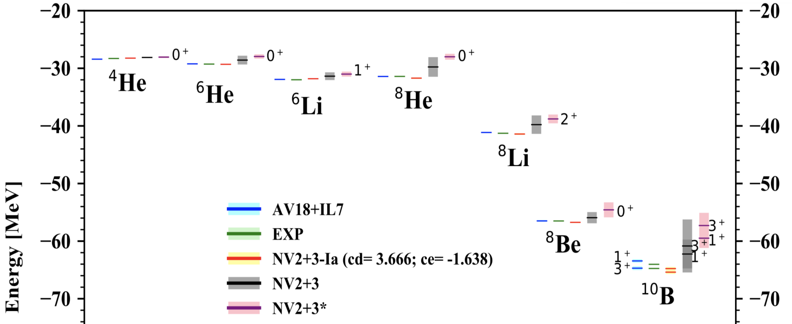
Spectra $A = 4$–12 nuclei with model Ia type (1)

Piarulli *et al.* (2018)



Model dependence: spectra

Piarulli *et al.* (unpublished)



Fit type (1)

Model	c_D	c_E
Ia	3.666	−1.638
Ib	−2.061	−0.982
IIa	1.278	−1.029
IIb	−4.480	−0.412

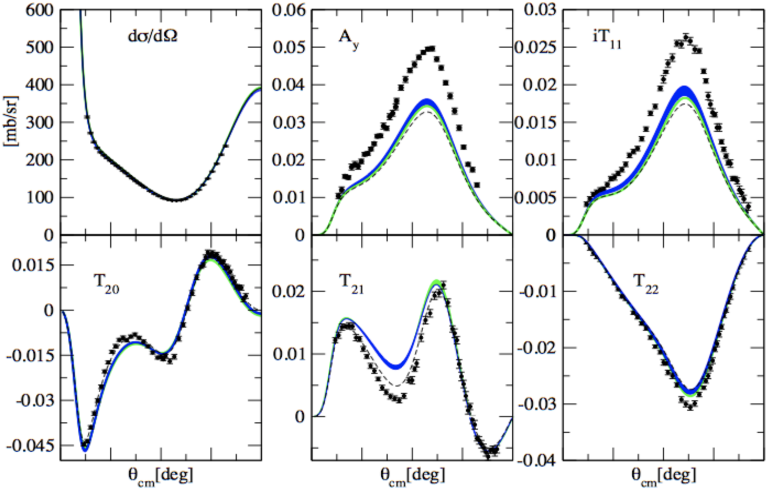
Fit type (2)

Model	c_D	c_E
Ia*	−0.635(255)	−0.09(8)
Ib*	−4.705(285)	0.550(150)
IIa*	−0.610(280)	−0.350(100)
IIb*	−5.250(310)	0.05(180)

Polarization observables in pd scattering

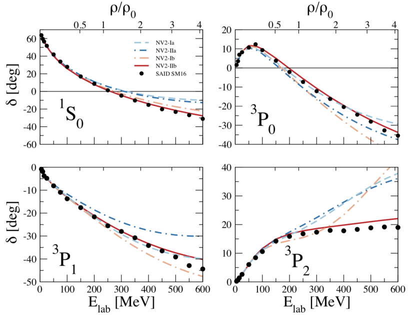
Girlanda, Kievsky, Marcucci, and Viviani (2018)

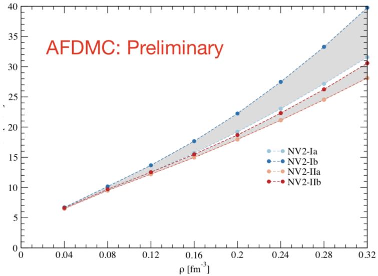
- HH calculation with type (1) Ia/b (IIa/b) green (blue)

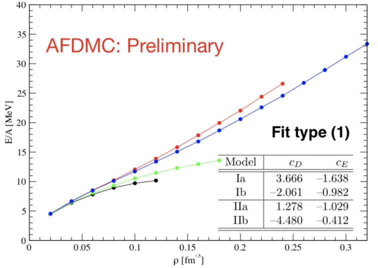


Neutron matter equation of state

Logoteta, Piarulli, Bombaci, Lovato, and Wiringa (unpublished)







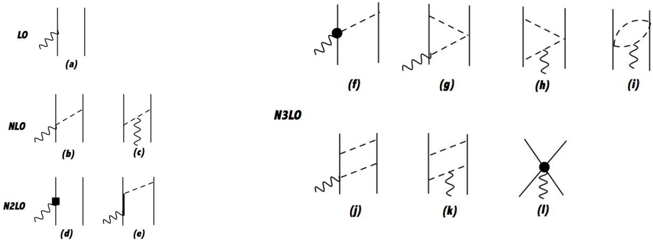
Model	c_D	c_E
Ia	3.666	−1.638
Ib	−2.061	−0.982
IIa	1.278	−1.029
IIb	−4.480	−0.412

- Sensitivity to $3N$ contact term:
  - *$c_E < 0$ repulsive in $A \leq 4$*
  - *but attractive in PNM*
- Cutoff sensitivity:
  - *modest in NV2 models*
  - *large in NV2+3 models*

EM operators up to one loop

Pastore *et al.* (2009; 2011); Kölling *et al.* (2009; 2011); Piarulli *et al.* (2013); Schiavilla *et al.* (2019)

- Contributions to $\mathbf{j}_\gamma$ up to order $e\,Q$:

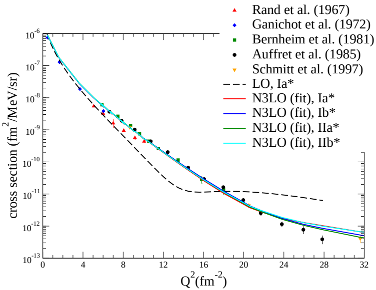


- Five unknown LEC's in $\mathbf{j}_\gamma$ at $e\,Q$ (solid dots) fixed by:
  - *$A = 2$–3 magnetic moments*
  - *fit to $d(e,e')pn$ at threshold*
- LO for ρ_γ at $e\,Q^{-3}$ and no OPE corrections at $e\,Q^{-1}$
- No unknown LEC's in ρ_γ up to $e\,Q$

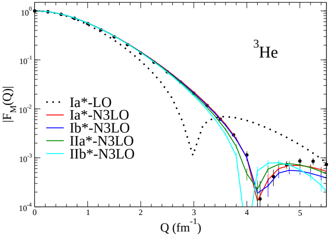
An example: magnetic structure of $A = 3$ nuclei

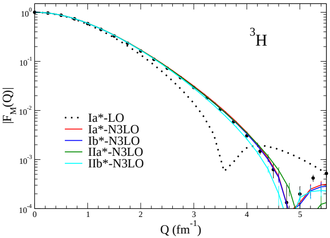
Schiavilla *et al.* (2019)

fit to $d(e, e')pn$ at threshold



Magnetic form factors

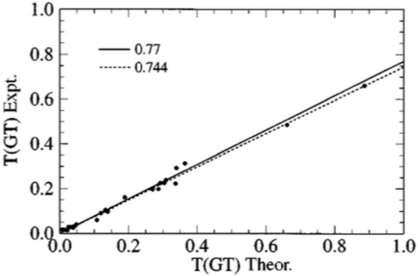




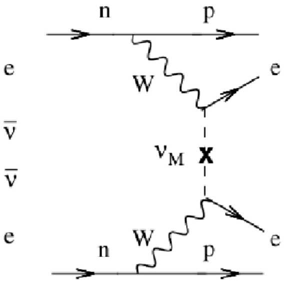
- Predicted f.f.'s are in agreement with exp data
- Modest cutoff dependence

Nuclear weak interactions at low energies

- Shell model in agreement with exp if $g_A^{\rm eff} \simeq 0.7\, g_A$
- Understanding “quenching” of g_A in nuclear β decays
- Relevant for neutrinoless 2β-decay since rate $\propto\ g_A^4$



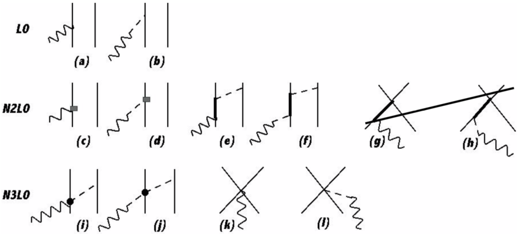
Martinez-Pinedo *et al.* (1996)

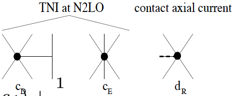


0ν–2β amplitude

Nuclear axial currents

Park *et al.* (1993,2003); Gazit *et al.* (2009); Baroni *et al.* (2016); Krebs *et al.* (2017); Schiavilla (unpublished)

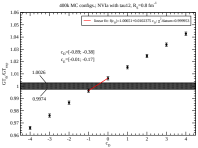


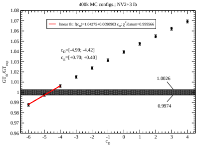


- Relation between c_D and d_R

$$d_R = -\frac{m}{4\, g_A\, \Lambda_\chi}\, c_D + \frac{m}{3}\,(c_3 + 2\, c_4) + \frac{1}{6}$$

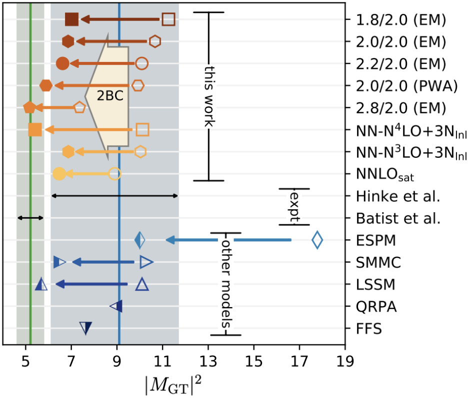
- Fix c_D via ^{3}H GT m.e. with axial current at N3LO





Gamow-Teller matrix element in ^{100}Sn β decay

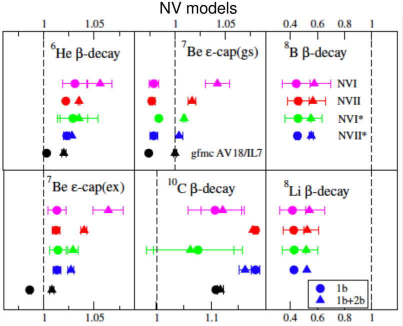
Gysbers *et al.* Nature Physics (2019)

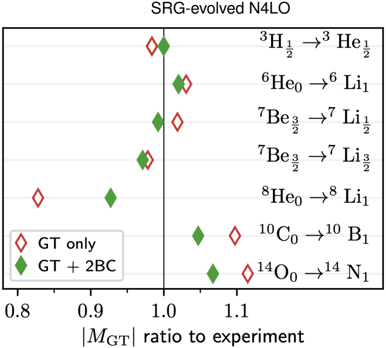


- Claim: g_A-quenching problem mostly resolved by corrections beyond LO

β decays in light nuclei

Pastore *et al.* PRC (2018); Pastore *et al.* (unpublished); Gysbers *et al.* Nature (2019)





- Both calculations constrained to reproduce ^{3}H GT m.e.
- Overall sign of corrections beyond LO is different in the two calculations (but for ^{8}He)
- Origin of this difference is unclear at this point in time (interactions? methods?)

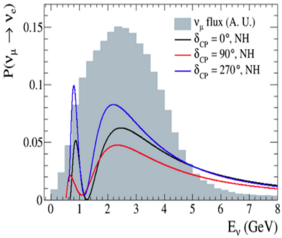
CC and NC ν-A scattering

- Large program in accelerator ν physics (MicroBooNE, NOνA, T2K, Minerνa, DUNE, . . .)

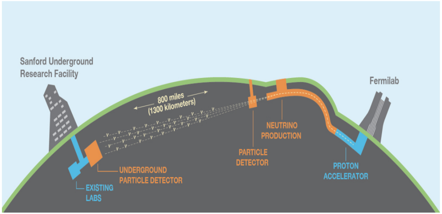
$$R^{\alpha}_{\beta}(E_{\text{obs}}) \propto \int dE\, \Phi_{\alpha}(E)\, P(\nu_{\alpha} \to \nu_{\beta}; E)\, \sigma_{\beta}(E, E_{\text{obs}})$$

- Determination of oscillation parameters depends crucially on our understanding of
  - ν *flux* $\Phi_{\alpha}(E)$
  - ν-A *cross section* $\sigma_{\beta}(E, E')$

DUNE ν_{μ} flux



DUNE



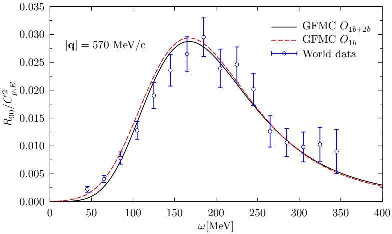
GFMC calculation of EM response in ^{12}C

Carlson and Schiavilla (1992); Lovato *et al.* (2013–2019)

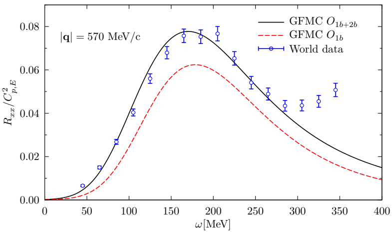
$$E_{\alpha\beta}(q,\tau) = \int_0^\infty \mathrm{d}\omega\, \mathrm{e}^{-\tau\omega}\, R_{\alpha\beta}(q,\omega) = \langle i|\, j^\dagger_\alpha(\mathbf{q})\, \mathrm{e}^{-\tau(H-E_i)}\, j_\beta(\mathbf{q})\, |i\rangle$$

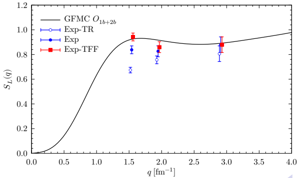
- Back to $R_{\alpha\beta}(q,\omega)$ by maximum entropy methods

longitudinal



transverse



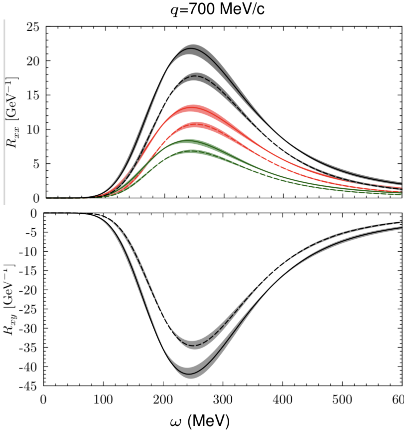


← Coulomb sum

Lovato *et al.* (unpublished)

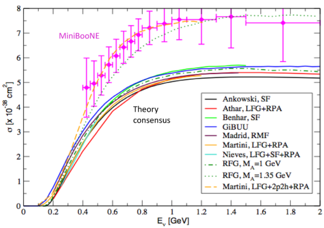
- Inclusive $\nu/\overline{\nu}$ $(-/+)$ cross section

$$\frac{d\sigma}{d\epsilon_l' d\Omega_l} \propto \left[v_{00}\, R_{00} + v_{zz}\, R_{zz} - v_{0z}\, R_{0z} + \overbrace{v_{xx}\, R_{xx} \mp v_{xy}\, R_{xy}}^{\text{dominant}} \right]$$



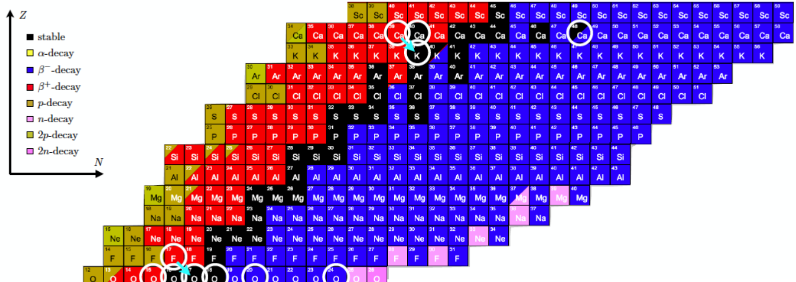
- Significant enhancement from EW 2b currents
- Towards an understanding of observed x-section?

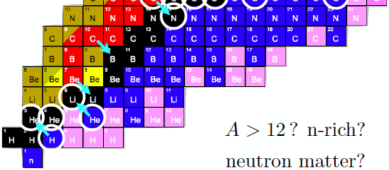
MiniBooNE CCQE



Outlook

Basic model
Nuclear χEFT
Chiral $2N$ interactions
Chiral $3N$ interactions
EW interactions
EW QE response
Outlook



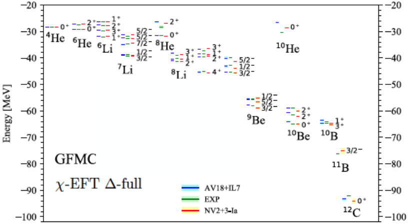


$A > 12$? n-rich?

neutron matter?

nuclear matter?

(error estimate?)



M. Piarulli *et al.*, arXiv:1707.02883

Slide by Lonardoni

Subleading contact $3N$ interaction

Girlanda, Kievsky, Marcucci, and Viviani (2011,2019)

- Lots of operators with two derivatives ...
- But Fierz identities and relativistic covariance lead to 10 independent operator structures; a possible choice:

$$\sum_{n=1}^{4} V_{ijk}^{(n)} = (E_1 + E_2\, \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + E_3\, \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + E_4\, \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\, \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j) \times \left[C''_{R_S}(r_{ij}) + 2\,\frac{C'_{R_S}(r_{ij})}{r_{ij}}\right] C_{R_S}(r_{jk}) + (j \rightleftharpoons k)$$

$$\sum_{n=5}^{6} V_{ijk}^{(n)} = (E_5 + E_6\, \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\, S_{ij} \left[C''_{R_S}(r_{ij}) - \frac{C'_{R_S}(r_{ij})}{r_{ij}}\right] C_{R_S}(r_{jk}) + (j \rightleftharpoons k)$$

$$\sum_{n=7}^{8} V_{ijk}^{(n)} = -2\,(E_7 + E_8\, \boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k)\, \frac{C'_{R_S}(r_{ij})}{r_{ij}} \left\{(\mathbf{L}\cdot\mathbf{S})_{ij}\,,\, C_{R_S}(r_{jk})\right\} + (j \rightleftharpoons k)$$

$$\sum_{n=9}^{10} V_{ijk}^{(n)} = (E_9 + E_{10}\, \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\, \boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}_{ik}\, \boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{jk}\, C'_{R_S}(r_{ik})\, C'_{R_S}(r_{jk}) + (j \rightleftharpoons k)$$

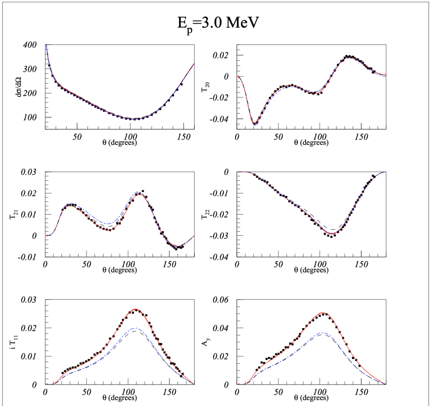
- The 10 subleading contact terms can be expressed as

$$V^{\rm CT2} = \widetilde{E}_1\, O_1^{(3/2)} + \sum_{i=2}^{10} \widetilde{E}_i\, O_i^{(1/2)}$$

Strategies to constrain the LECs

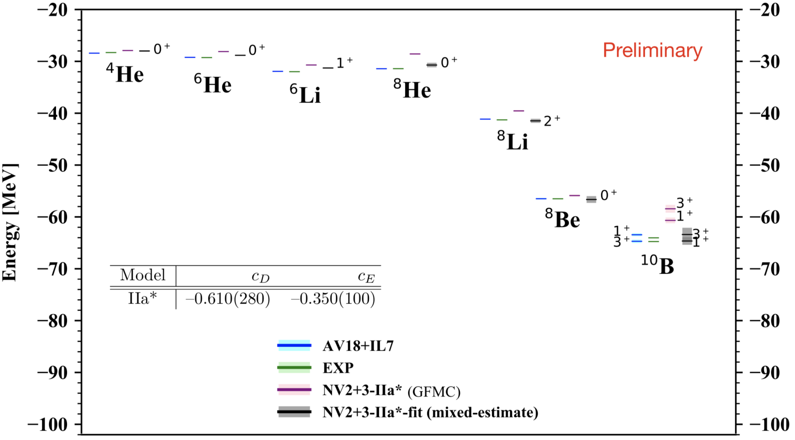
- Fix c_D by fitting exp GT m.e.'s in light nuclei
- Possible approaches to fix c_E and subleading LECs:
  - Nd *scattering observables at low energies*
  - *Spectra of light- and medium-weight nuclei and properties of nuclear/neutron matter*

Girlanda *et al.* (2019)



Subleading LECs from light-nuclei spectra

Piarulli *et al.* (unpublished)



- Tensor (E_5 and E_6 LECs) and spin-orbit (E_7 and E_8 LECs) terms are dominant

μ-capture: validating model of EW currents

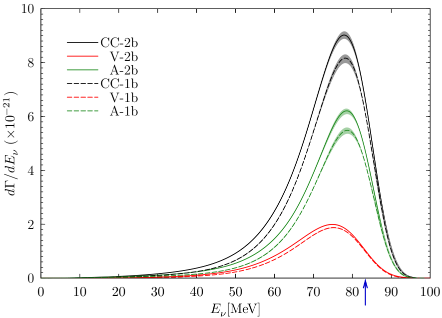
Lovato, Rocco, and Schiavilla (2019)

- Inclusive muon capture rate can be expressed in terms of five response functions

$$\frac{d\Gamma}{dE_\nu} = \frac{G_V^2}{2\pi}\,|\psi(0)|^2\,E_\nu^2\left[R_{00}(E_\nu)+R_{zz}(E_\nu)+R_{0z}(E_\nu)+\,R_{xx}(E_\nu)-R_{xy}(E_\nu)\right]$$

- Euclidean $E_{\alpha\beta}(q,\tau)$ calculable by GFMC with inversion by maximum-entropy methods to obtain back $R_{\alpha\beta}(\omega)$

$$E_{\alpha\beta}(q,\tau) = \int_0^\infty d\,\overline{\omega}\,\mathrm{e}^{-\tau\,\overline{\omega}}\,R_{\alpha\beta}(q,\overline{\omega}) = \overline{\sum}_i\langle i|O^{\beta\dagger}(\mathbf{q})\mathrm{e}^{-\tau(H-E_i)}O^{\alpha}(\mathbf{q})|i\rangle$$



- Proof-of-principle: ^{4}He
- Theory: 265(9) s^{-1} (1b)
  306(9) s^{-1} (2b)
- Exp poorly known:
  358 $\pm$ 174 s^{-1}

The team

- The ANL/JLAB-ODU/LANL/Lecce/Pisa/WASHU collaboration members:

A. Baroni (LANL)
J. Carlson (LANL)
S. Gandolfi (LANL)
L. Girlanda (U-Salento)
A. Kievsky (INFN-Pisa)
D. Lonardoni (LANL)
A. Lovato (ANL/INFN-Trento)
L.E. Marcucci (U-Pisa)
S. Pastore (WASHU)
M. Piarulli (WASHU)
S.C. Pieper† (ANL)
N. Rocco (FNAL/ANL)
R. Schiavilla (JLab/ODU)
M. Viviani (INFN-Pisa)
R.B. Wiringa (ANL)

- Computational resources from *ANL LCRC*, *LANL Open Supercomputing*, and *NERSC*

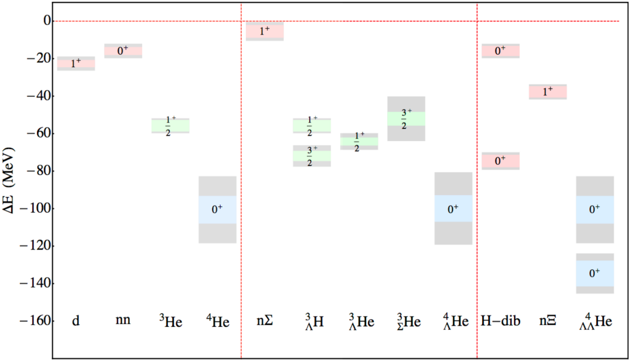
† Deceased

Backup slides

Few-nucleon systems from LQCD

NPLQCD collaboration: Beane *et al.* (2013); Chang *et al.* (2015); Savage *et al.* (2017)

- NPLQCD spectra calculations ($m_\pi \approx 800$ MeV)

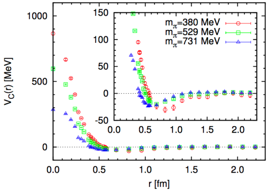


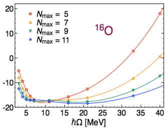
- NPLQCD calculations of magnetic moments and weak transitions in few-nucleon systems also available

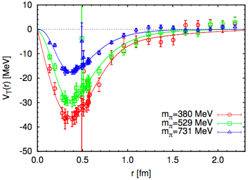
$2N$ interactions from LQCD and nuclear spectra

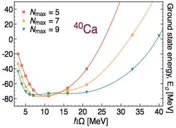
HAL collaboration: Aoki *et al.* (2012); McIlroy *et al.* (2017)

- Central and tensor interactions (left)
- Resulting ground-state energies of ^{16}O and ^{40}Ca (right)









General considerations

- Time-ordered perturbation theory (TOPT):

$$\langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\,\eta} H_1 \right)^{n-1} \mid i \rangle$$

- Momentum scaling of contribution

$$\underbrace{\left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1 \text{ scaling}} \times \underbrace{Q^{-(N-N_K-1)}\, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integrations}}$$

- Each of the N_K energy denominators involving only nucleons is of order Q^{-2}
- Each of the other $N - N_K - 1$ energy denominators involving also pion energies is expanded as

$$\frac{1}{E_i - E_I - \omega_\pi} = -\frac{1}{\omega_\pi} \left[1 + \frac{E_i - E_I}{\omega_\pi} + \frac{(E_i - E_I)^2}{\omega_\pi^2} + \dots \right]$$

- Power counting:

$$T = T^{LO} + T^{NLO} + T^{N^2LO} + \dots\,, \quad \text{and} \quad T^{N^n LO} \sim (Q/\Lambda_\chi)^n\, T^{LO}$$

From amplitudes to interactions

Pastore *et al.* (2009); Pastore *et al.* (2011)

- Construct v such that when inserted in LS equation

$$v + v\,G_0\,v + v\,G_0\,v\,G_0\,v + \dots \qquad G_0 = 1/(E_i - E_I + i\,\eta)$$

leads to T-matrix order by order in the power counting

- Assume

$$v = v^{(0)} + v^{(1)} + v^{(2)} + \dots \qquad v^{(n)} \sim (Q/\Lambda_\chi)^n\, v^{(0)}$$

- Determine $v^{(n)}$ from

$$\begin{aligned} v^{(0)} &= T^{(0)} \\ v^{(1)} &= T^{(1)} - \left[v^{(0)}\,G_0\,v^{(0)}\right] \\ v^{(2)} &= T^{(2)} - \left[v^{(0)}\,G_0\,v^{(0)}\,G_0\,v^{(0)}\right] - \left[v^{(1)}\,G_0\,v^{(0)} + v^{(0)}\,G_0\,v^{(1)}\right] \end{aligned}$$

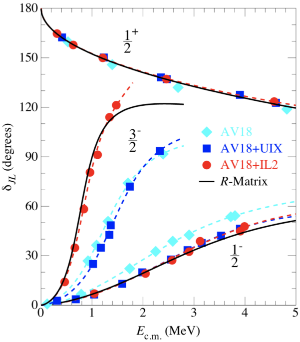
and so on, where

$$v^{(m)}\,G_0\,v^{(n)} \sim (Q/\Lambda_\chi)^{m+n+1}$$

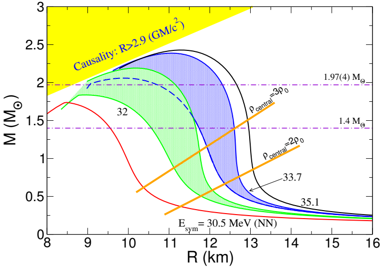
Failures of basic model with $2N$ interactions

- Spectra of light nuclei (underbinding problem)
- Low-energy scattering in few-nucleon systems
- Nuclear matter $E_0(\rho)$ and neutron star masses

n-α scattering



neutron matter EOS



Ab initio methods utilized by our group

- Hyperspherical harmonics (HH) expansions for $A=3$ and 4 bound and continuum states

$$|\psi_V\rangle = \sum_\mu c_\mu \underbrace{|\phi_\mu\rangle}_{\text{HH basis}} \quad \text{and} \quad c_\mu \ \text{from} \ E_V = \frac{\langle\psi_V|H|\psi_V\rangle}{\langle\psi_V|\psi_V\rangle}$$

- Quantum Monte Carlo for $A > 4$ bound states

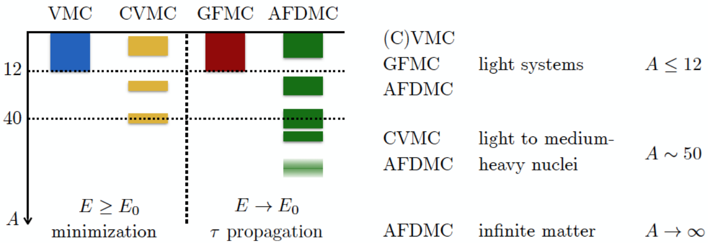


Figure by Lonardoni

GFMC for $A \leq 12$

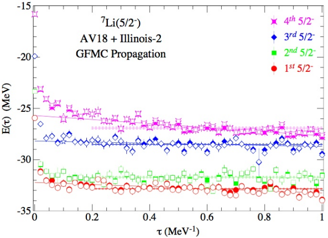
RMP review by Carlson *et al.* (2015)

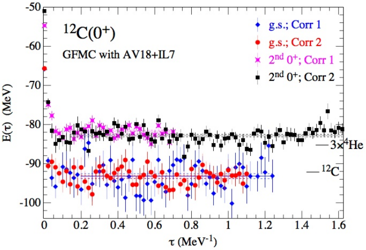
- Propagation in imaginary time

$$E_0 = \lim_{\tau \to \infty} \frac{\langle \psi_V | H\, \mathrm{e}^{-\tau H} | \psi_V \rangle}{\langle \psi_V | \mathrm{e}^{-\tau H} | \psi_V \rangle}$$

- Exponential growth with A (in ^{12}C st-states $\sim 4 \times 10^6$)

$$\psi_V = \sum_{s \leq 2^A} \sum_{t \leq 2^A} \phi_{st}(\mathbf{r}_1, \ldots, \mathbf{r}_A)\, \chi_{st}(1, \ldots, A)$$



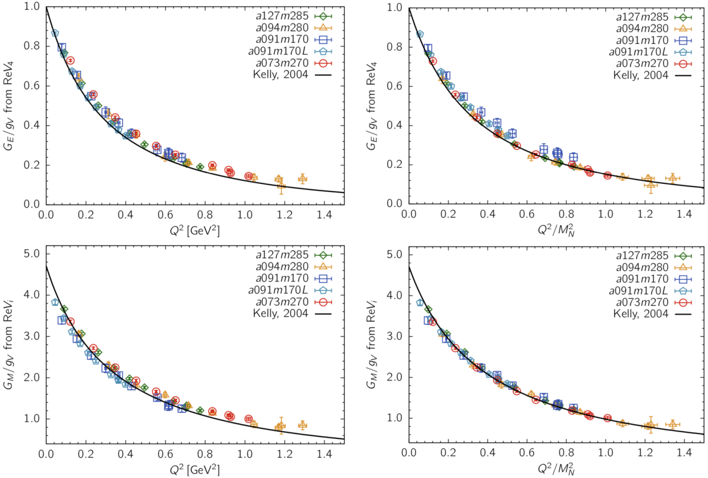


Isovector nucleon EM f.f.'s from LQCD

Gupta from ECT* talk (2019): $\overline{u}(p') \left(\gamma^\mu F_1 + \sigma^{\mu\nu} q_\nu \frac{F_2}{2m} \right) u(p)$

Clover-on-clover data

NME unpublished: 5 ensembles with ~2000 configs each

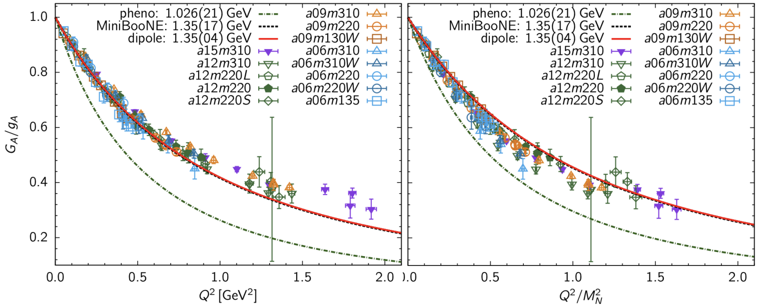


Isovector nucleon axial f.f. from LQCD

Gupta from ECT* talk (2019): $\overline{u}(p') \left(\gamma^\mu G_A + q^\mu \frac{\widetilde{G}_P}{2m} \right) \gamma_5 \, u(p)$

Clover-on-HISQ data

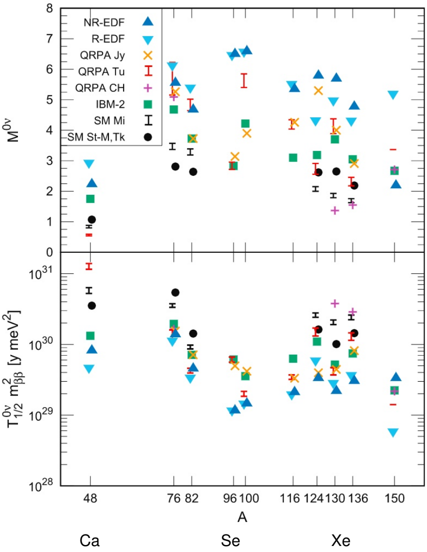
PNDME unpublished



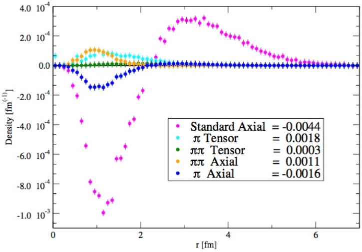
NOTE: The two dipole curves with $M_A = 1.35$ and $M_A = 1.026$ are drawn only as a reference to quantify spread and uncertainty in the lattice data

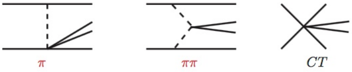
Nuclear matrix element in $0\nu\text{–}2\beta$ decay

Engel and Menendez (2017)



Test case[1]: ^{8}He$(0^+;2) \rightarrow {}^8$Be$(0^+;2)$





standard axial $\propto \tau_i^+ \tau_j^+ \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$

[1] Mereghetti *et al.* (2017)

Event generators I

- Event generators (EGs) are one of the most important components in the analysis of ν experiments
- EGs estimate $\overline{\sigma}_\beta(E, E_{\text{obs}})$ by modeling the nucleus
- Assuming the nuclear model is correct, EGs provide:
  - information on how signal and background events appear in detectors
  - means for estimating systematic errors on measurements
  - estimates of the final state composition (used to determine the incoming neutrino energy)
- GENIE is the most widely used EG (ArgoNeut, MINOS, MicroBooNE, MINERvA, NOvA, T2K, SBND, DUNE)

Event generators II

- Steps in oscillation analysis:
  - Measure $E_{\rm obs}$ and event topology in the near detector
  - Use nuclear model (via EG) to infer from these initial E and topology
  - Project initial E distribution perturbed via an oscillation hypothesis that changes $\Phi(E)$ at the far detector
  - Following ν initial interaction at the far detector, use nuclear model (via EG) to deduce $E_{\rm obs}$ and topology
  - Compare these with actual measurements in the far detector
- Critical dependence on nuclear model even with a near detector

Projecting in isospin T=1/2 and 3/2 channels

Girlanda, private communication

$$P_{1/2} = \frac{1}{2} - \frac{1}{6}\left(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k + \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k\right) \quad \text{and} \quad P_{3/2} = 1 - P_{1/2}$$

- The (single) LO contact term ($\propto c_E$ in standard notation) can be expressed as

$$V^{\rm CT0} = \widetilde{c}_E \, P_{1/2}$$

- The 10 subleading contact terms can be expressed as

$$V^{\rm CT2} = \widetilde{E}_1 \, O_1^{(3/2)} + \sum_{i=2}^{10} \widetilde{E}_i \, O_i^{(1/2)}$$

- There is not much flexibility to constrain the $T = 3/2$ component of the $3N$ contact interaction . . .
- But analysis relies on the use of Fierz identities and is valid up to cutoff effects

Reducing the number of subleading LECs

Girlanda, private communication

Implications from large-N_c limit:

- NN contact at LO: C_1, $C_4 \sim N_c$ and C_2, $C_3 \sim 1/N_c$

$$\begin{aligned}\mathcal{L} &= -C_1 N^\dagger N\, N^\dagger N - C_2 N^\dagger \sigma_i N\, N^\dagger \sigma_i N - C_3 N^\dagger \tau_a N\, N^\dagger \tau_a N - C_4 N^\dagger \sigma_i \tau_a N\, N^\dagger \sigma_i \tau_a N \\ &= -\underbrace{(C_1 - 2\,C_3 - 3\,C_4)}_{C_S} N^\dagger N\, N^\dagger N - \underbrace{(C_2 - C_3)}_{C_T} N^\dagger \sigma_i N\, N^\dagger \sigma_i N\end{aligned}$$

- $3N$ contact at LO[1]: D_1, D_4, $D_6 \sim N_c$, but only a single independent operator with associated LEC $\sim N_c$

$$\mathcal{L} = -\sum_{i=1}^{6} D_i\, O_i \qquad O_i = N^\dagger N\, N^\dagger N\, N^\dagger N \ \text{ and } 5 \text{ more}$$

- $3N$ subleading contact[2]: an analysis similar to above shows E_2, E_3, E_5 and E_9 vanish in the large-N_c limit

$$\mathcal{L} = -\sum_{i=1}^{10} E_i\, O_i \qquad O_i = \boldsymbol{\nabla}\left(N^\dagger N\right) \cdot \boldsymbol{\nabla}\left(N^\dagger N\right) N^\dagger N \ \text{ and } 9 \text{ more}$$

[1] Phillips and Schat (2013); [2] Girlanda, unpublished

NC responses and cross sections in ^{12}C

Lovato *et al.* (2018)

- Inclusive $\nu/\overline{\nu}$ $(-/+)$ cross section

$$\frac{d\sigma}{d\epsilon'_l d\Omega_l} \propto \left[v_{00}\, R_{00} + v_{zz}\, R_{zz} - v_{0z}\, R_{0z} + \overbrace{v_{xx}\, R_{xx} \mp v_{xy}\, R_{xy}}^{\text{dominant}} \right]$$