AVENUES OF QUANTUM FIELD THEORY IN CURVED SPACETIME
MODENA, SEPTEMBER 9 2019

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THERMAL FIELD THEORY WITH ACCELERATION, ENTROPY CURRENT AND UNRUH EFFECT

MOTIVATIONS

- Relativistic hydrodynamics
  - Astrophysics and cosmology: expectation value of energy-momentum tensor at thermodynamic equilibrium with quantum corrections
  - Quark-Gluon Plasma as relativistic quantum fluid at local thermodynamic equilibrium with acceleration and vorticity
- Quantum Field Theory
  - Relativistic quantum effects at low temperature due to acceleration (Unruh effect)
- Rigorous derivation of the entropy current

OUTLINE

1. Equilibrium relativistic quantum statistical mechanics
2. Global thermodynamic equilibrium with acceleration
3. Thermal expectation values and Unruh effect

[F. Becattini Phys.Rev. D97 (2018) no.8, 085013]

4. Entropy current and extensivity
5. Entropy current at global equilibrium with acceleration
6. Entanglement entropy and Unruh effect

[F. Becattini and D.R. Phys.Rev. D99 (2019) no.12, 125011]

7. Summary

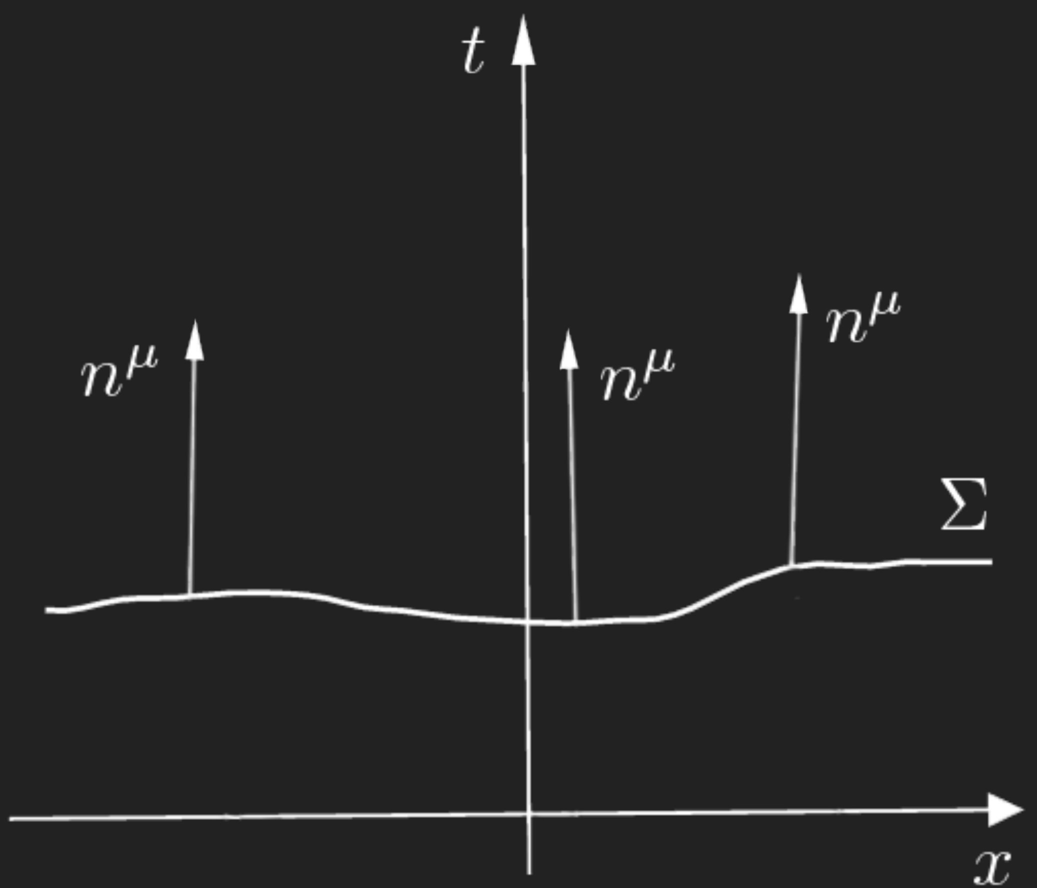
EQUILIBRIUM RELATIVISTIC QUANTUM STATISTICAL MECHANICS

- Thermal QFT: calculate expectation values of operators at thermodynamic equilibrium.
- Equilibrium density operator $\hat{\rho}$: maximize $S = -\,\mathrm{tr}(\hat{\rho}\log\hat{\rho})$ with constraints of fixed $\{\langle\hat{\mathcal{O}}_i\rangle\} = \{\mathrm{tr}(\hat{\rho}\hat{\mathcal{O}}_i)\}$ for a given set of operators $\{\hat{\mathcal{O}}_i\}$ introducing a set of Lagrange multipliers $\{\lambda_i\}$. Result:

$$\hat{\rho} = \frac{1}{Z}\exp\left[-\sum_i \lambda_i \hat{\mathcal{O}}_i\right].$$

- In relativity $\hat{\mathcal{O}}_i = \int_\Sigma \mathrm{d}\Sigma_\mu \hat{j}^\mu_i$ with $\mathrm{d}\Sigma_\mu = \mathrm{d}\Sigma\, n_\mu$ and $\nabla_\mu \hat{j}^\mu_i = 0$, hence

$$\hat{\rho} = \frac{1}{Z}\exp\left[-\int_\Sigma \mathrm{d}\Sigma_\mu \sum_i \lambda_i \hat{j}^\mu_i\right].$$



EQUILIBRIUM RELATIVISTIC QUANTUM STATISTICAL MECHANICS

Local Thermodynamic Equilibrium (LTE): maximize S with given densities $\{n_\mu \langle \hat{j}^\mu_i \rangle\}$ on Σ.

- Given $n_\mu \langle \hat{T}^{\mu\nu} \rangle$ and $n_\mu \langle \hat{j}^\mu \rangle$ on Σ, result:

$$\hat{\rho}_{\rm LE} = \frac{1}{Z_{\rm LE}} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\hat{T}^{\mu\nu}\beta_\nu - \zeta \hat{j}^\mu \right) \right].$$

- β^μ *four-temperature* (timelike) such that:
  - $u^\mu = \beta^\mu / \sqrt{\beta^2}$ four-velocity
  - $T = 1/\sqrt{\beta^2}$ *proper temperature* i.e. measured by comoving thermometer, different from temperature measured by fixed thermometer $T_{\rm fix} = 1/\beta^0$.
- $\zeta = \mu / T$ with μ proper chemical potential

[Zubarev et al. 1979, Van Weert 1982]
[Becattini et al. 2015, Hayata et al. 2015]

EQUILIBRIUM RELATIVISTIC QUANTUM STATISTICAL MECHANICS

Require $\hat{\rho}$ to be time-independent:
Global Thermodynamic Equilibrium (GTE).

$$\hat{\rho} = \frac{1}{Z} \exp\left[- \int_\Sigma \mathrm{d}\Sigma_\mu \left(\hat{T}^{\mu\nu}\beta_\nu - \zeta \hat{j}^\mu \right) \right]$$

Time-independence

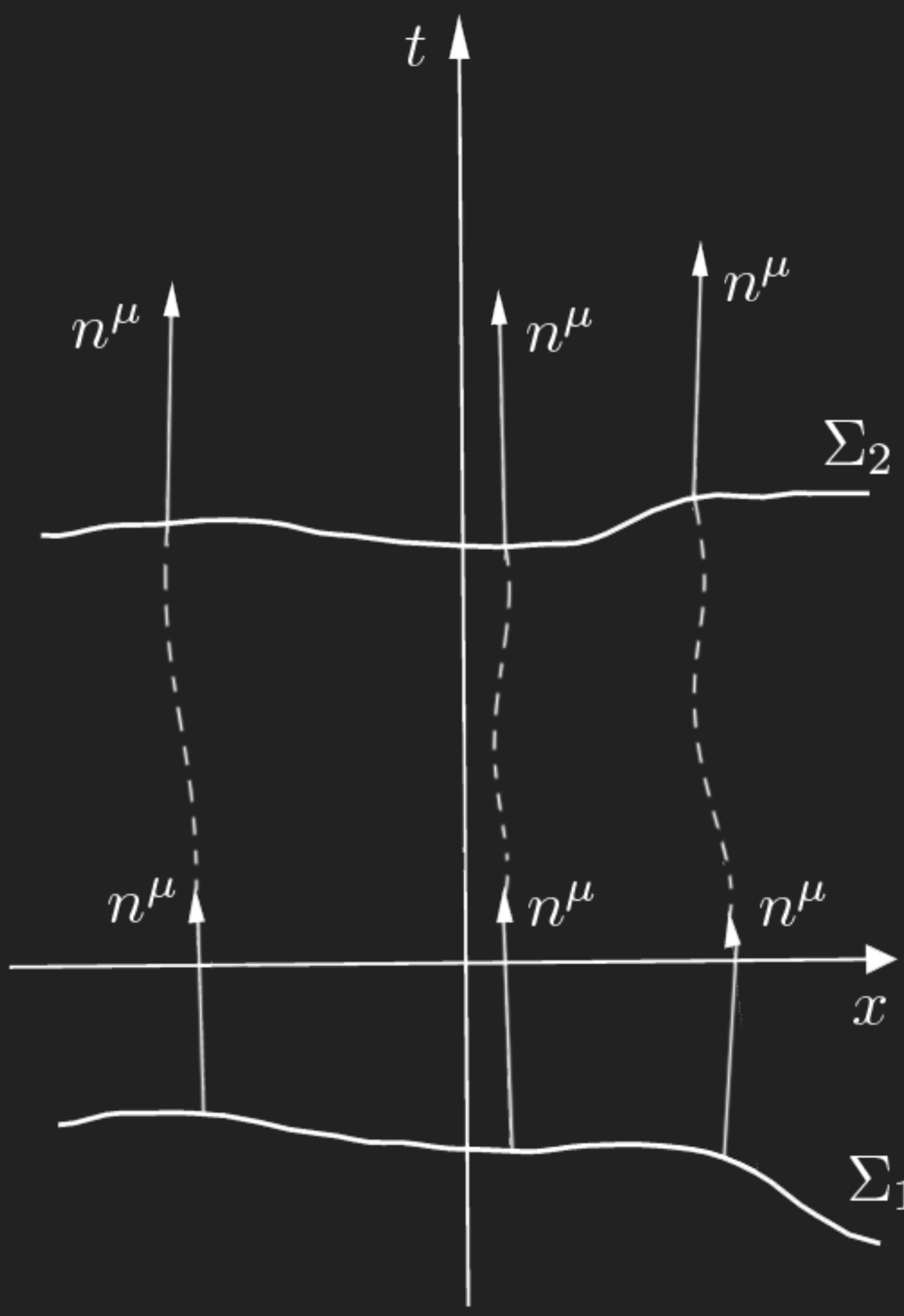
$\Updownarrow$

Σ-independence

$\Updownarrow$

$$\nabla_\mu \zeta = 0, \qquad \nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0$$

β^μ timelike Killing vector



GTE IN MINKOWSKI SPACETIME

Solution of Killing equation in Minkowski spacetime:

$$\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu$$

with b_μ constant and $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$ constant *thermal vorticity*, hence

$$\hat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \hat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\hat{J}^{\mu\nu} + \zeta \hat{Q}\right]$$

where $\hat{J}^{\mu\nu}$ are generators of Lorentz group.

Different choices of $(b_\mu, \varpi_{\mu\nu})$ correspond to different GTEs.

- *Homogeneous GTE:*

$$b_\mu = \frac{1}{T_0}(1,0,0,0), \qquad \varpi_{\mu\nu} = 0$$

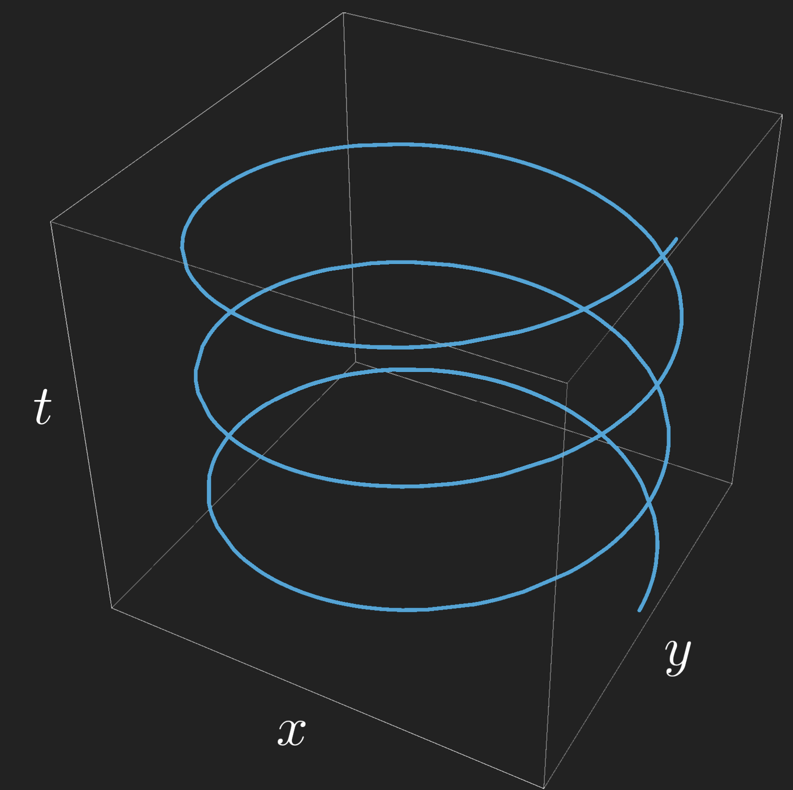
$$\beta_\mu = \frac{1}{T_0}(1,0,0,0), \qquad \hat{\rho} = \frac{1}{Z}\exp\left[-\frac{\hat{H}}{T_0}\right].$$

GTE IN MINKOWSKI SPACETIME

- *GTE with rotation:*

$$b_\mu = \frac{1}{T_0}(1,0,0,0), \qquad \varpi_{\mu\nu} = \frac{\omega}{T_0}\left(g_{1\mu}g_{2\nu} - g_{1\nu}g_{2\mu}\right)$$

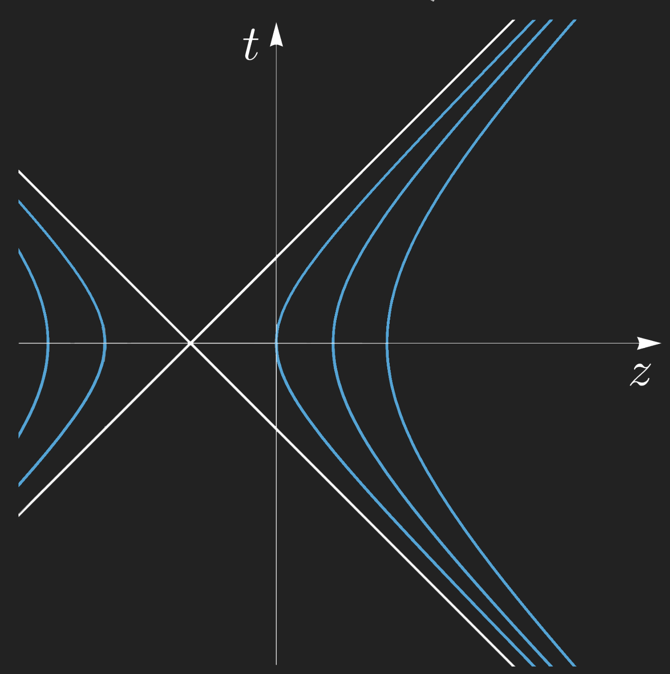
$$\beta_\mu = \frac{1}{T_0}(1,\mathbf{x}\times\boldsymbol{\omega}), \qquad \hat{\rho} = \frac{1}{Z}\exp\left[-\frac{\hat{H}}{T_0} + \frac{\omega}{T_0}\hat{J}_z\right]$$



- *GTE with acceleration:*

$$b_\mu = \frac{1}{T_0}(1,0,0,0), \qquad \varpi_{\mu\nu} = \frac{a}{T_0}\left(g_{0\nu}g_{3\mu} - g_{3\nu}g_{0\mu}\right)$$

$$\beta^\mu = \frac{1}{T_0}(1+az,0,0,at), \qquad \hat{\rho} = \frac{1}{Z}\exp\left[-\frac{\hat{H}}{T_0} + \frac{a}{T_0}\hat{K}_z\right]$$



GTE WITH ACCELERATION IN MINKOWSKI SPACETIME

- Shift $z' = z + 1/a$:

$$\beta^\mu = \frac{a}{T_0}(z', 0, 0, t).$$

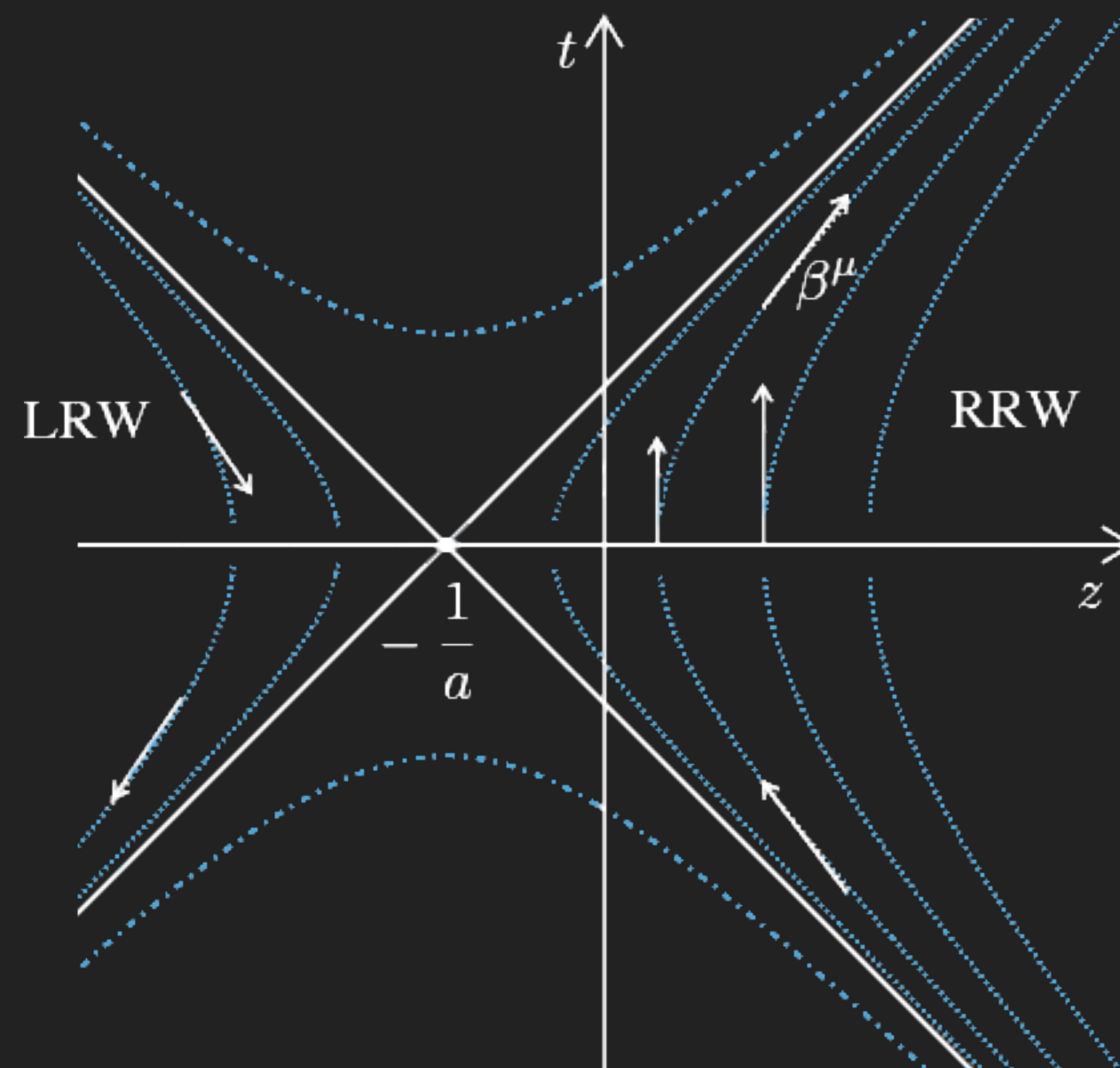
- Flow lines are hyperbolae with constant $z'^2 - t^2$:

$$T = \frac{T_0}{a\sqrt{z'^2 - t^2}} \quad \text{(\textit{Tolman's law})}$$

$$u^\mu = \frac{1}{\sqrt{z'^2 - t^2}}(z', 0, 0, t), \qquad A^\mu = \frac{1}{z'^2 - t^2}(t, 0, 0, z').$$

- Proper acceleration A^2 constant along flow lines, hence the name "*GTE with acceleration*".

- Define $\alpha^\mu = \frac{A^\mu}{T}$, thus $\alpha^2 = \frac{A^2}{T^2} = -\frac{a^2}{T_0^2}$.



- $|z'| = t$ is bifurcated *Killing horizon*: β^μ timelike and future-oriented only in *Right Rindler Wedge (RRW)*.

GTE WITH ACCELERATION IN MINKOWSKI SPACETIME

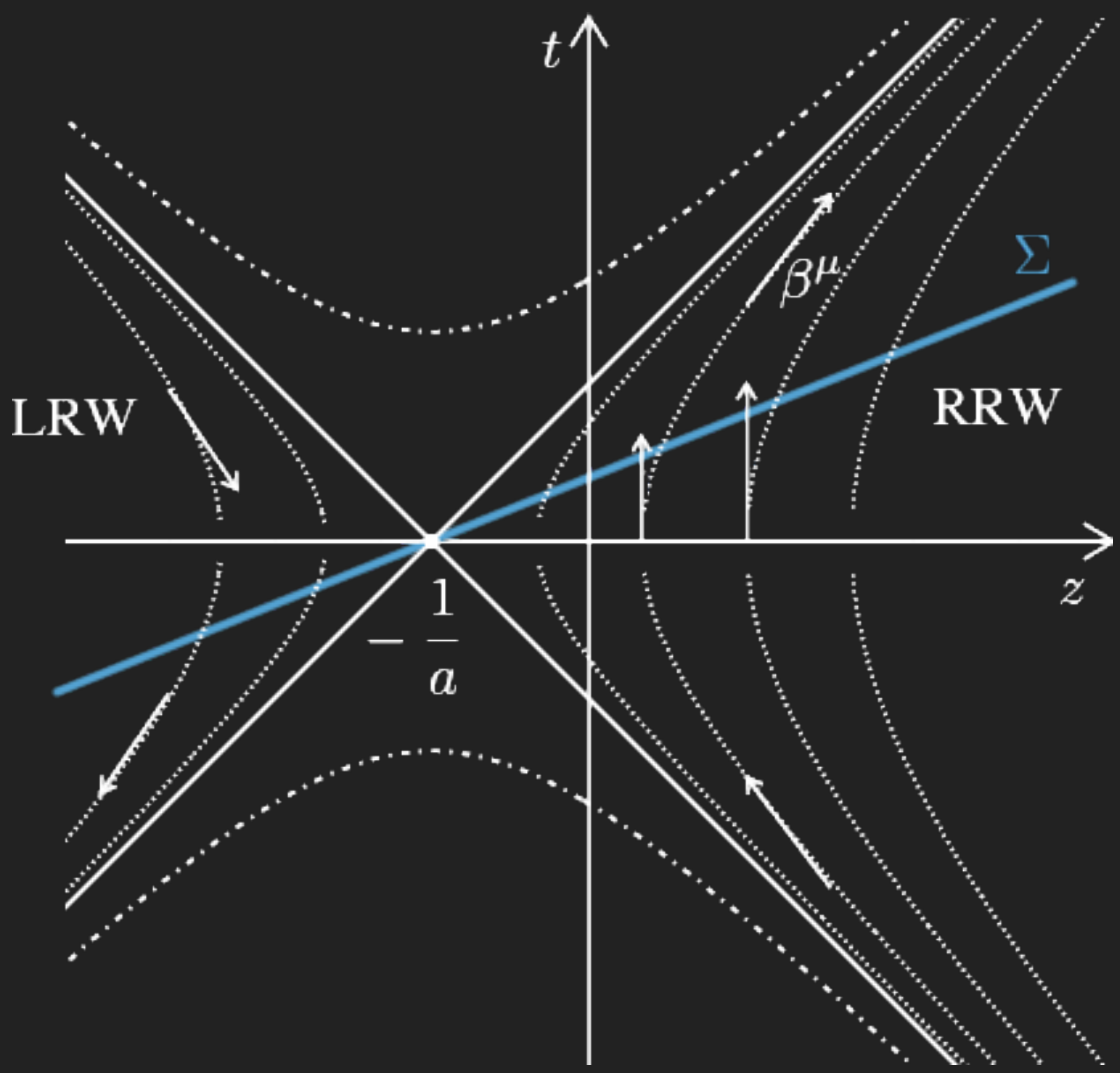
- Recall: $\hat{\rho} = \frac{1}{Z} \exp\left[-\int_\Sigma \mathrm{d}\Sigma_\mu \hat{T}^{\mu\nu} \beta_\nu \right]$ is Σ-independent.
- Notice: $\beta^\mu = 0$ at $z' = 0$.
- Consequence: For any Σ through $z' = 0$

$$\hat{\rho} = \hat{\rho}_\mathrm{R} \otimes \hat{\rho}_\mathrm{L}, \qquad [\hat{\rho}_\mathrm{R}, \hat{\rho}_\mathrm{L}] = 0$$

with $\hat{\rho}_{\mathrm{R/L}}$ involving DOFs only in RRW/LRW:

$$\hat{\rho}_\mathrm{R} = \frac{1}{Z_\mathrm{R}} \exp\left[-\int_{z'>0} \mathrm{d}\Sigma_\mu \hat{T}^{\mu\nu} \beta_\nu \right],$$

$$\hat{\rho}_\mathrm{L} = \frac{1}{Z_\mathrm{L}} \exp\left[-\int_{z'<0} \mathrm{d}\Sigma_\mu \hat{T}^{\mu\nu} \beta_\nu \right].$$



- Consequence: If $x \in$ RRW, then

$$\langle \hat{\mathcal{O}}(x) \rangle = \mathrm{tr}(\hat{\rho} \hat{\mathcal{O}}(x)) = \mathrm{tr}_\mathrm{R}(\hat{\rho}_\mathrm{R} \hat{\mathcal{O}}(x)).$$

THERMAL EXPECTATION VALUES IN THE RRW AND UNRUH EFFECT

- Free scalar field theory in the RRW: Klein-Gordon equation

$$(\square + m^2)\hat{\psi} = 0.$$

- Introduce (hyperbolic) *Rindler coordinates*:

$$\tau = \frac{1}{2a}\log\left(\frac{z'+t}{z'-t}\right), \qquad \xi = \frac{1}{2a}\log\left[a^2\left(z'^2 - t^2\right)\right], \qquad \mathbf{x}_T = (x, y).$$

- Solution:

$$\hat{\psi} = \int_0^{+\infty} \mathrm{d}\omega \int_{\mathbb{R}^2} \mathrm{d}^2 k_T \left(u_{\omega,\mathbf{k}_T} \hat{a}^{\mathrm{R}}_{\omega,\mathbf{k}_T} + u^*_{\omega,\mathbf{k}_T} \hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_T} \right)$$

$$u_{\omega,\mathbf{k}_T} = \sqrt{\frac{1}{4\pi^4 a}\sinh\left(\frac{\pi\omega}{a}\right)}\,\mathrm{K}_{i\frac{\omega}{a}}\left(\frac{\mathrm{e}^{a\xi}}{a}\sqrt{\omega^2 - \mathbf{k}_T^2}\right)\mathrm{e}^{-i(\omega\tau - \mathbf{k}_T\cdot\mathbf{x}_T)}$$

orthonormalized with respect to Klein-Gordon inner product.

- $\hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_T}, \hat{a}^{\mathrm{R}}_{\omega,\mathbf{k}_T}$ are creation and annihilation operators.

[Crispino et al. 2008]

THERMAL EXPECTATION VALUES IN THE RRW AND UNRUH EFFECT

- Thermal expectation values (TEVs) of physical interest can be calculated once the following are known

$$\langle \hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_T} \hat{a}^{\mathrm{R}}_{\omega',\mathbf{k}'_T} \rangle = \frac{1}{\mathrm{e}^{\omega/T_0} - 1} \delta(\omega - \omega')\, \delta^2(\mathbf{k}_T - \mathbf{k}'_T)$$

$$\langle \hat{a}^{\mathrm{R}}_{\omega',\mathbf{k}'_T} \hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_T} \rangle = \left(\frac{1}{\mathrm{e}^{\omega/T_0} - 1} + 1 \right) \delta(\omega - \omega')\, \delta^2(\mathbf{k}_T - \mathbf{k}'_T)$$

$$\langle \hat{a}^{\mathrm{R}}_{\omega,\mathbf{k}_T} \hat{a}^{\mathrm{R}}_{\omega',\mathbf{k}'_T} \rangle = \langle \hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_T} \hat{a}^{\mathrm{R}\dagger}_{\omega',\mathbf{k}'_T} \rangle = 0.$$

- The $+1$ gives rise to divergences $\Rightarrow$ need renormalization.
- TEVs in Minkowski vacuum $|0_{\mathrm{M}}\rangle$: same TEVs as above with $T_0 = a/2\pi$. In particular

$$\langle 0_{\mathrm{M}} | \hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_T} \hat{a}^{\mathrm{R}}_{\omega',\mathbf{k}'_T} | 0_{\mathrm{M}} \rangle = \frac{1}{\mathrm{e}^{2\pi\omega/a} - 1} \delta(\omega - \omega')\, \delta^2(\mathbf{k}_T - \mathbf{k}'_T).$$

This is the content of the *Unruh effect*, and $a/2\pi$ is the *Unruh temperature*.

THERMAL EXPECTATION VALUES IN THE RRW AND UNRUH EFFECT

- TEVs of operators quadratic in the field, once the $|0_{\rm M}\rangle$ contribution is subtracted, vanish at $T_0 = a/2\pi$ and become negative for $T_0 < a/2\pi$. For instance the energy density

$$\rho_{\rm Minkowski} = \left(\langle \hat{T}^{\mu\nu} \rangle - \langle 0_{\rm M} | \hat{T}^{\mu\nu} | 0_{\rm M} \rangle \right) u_\mu u_\nu$$

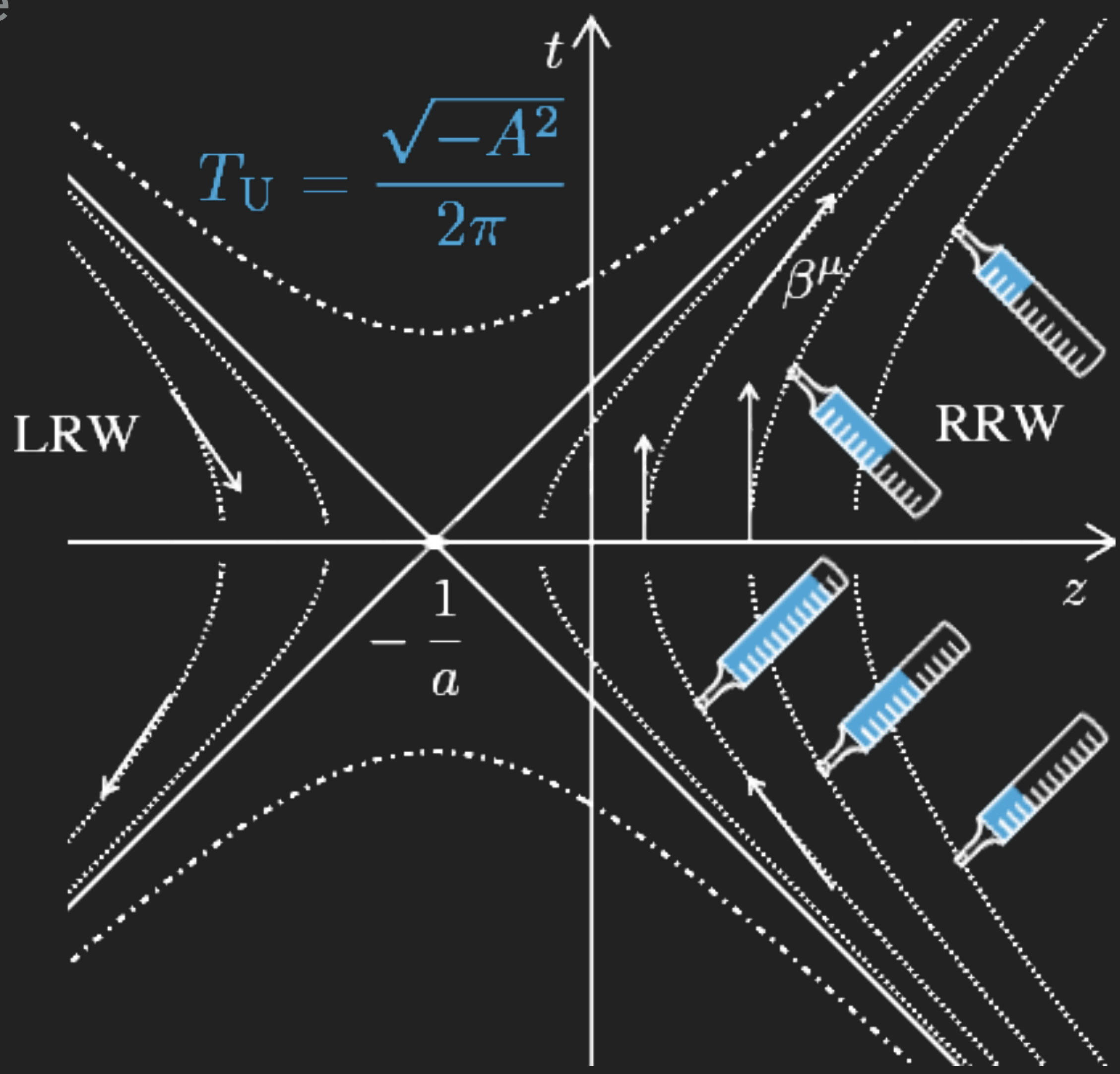
turns out to be

$$\rho_{\rm Minkowski} = \left(\frac{\pi^2}{30} - \frac{\alpha^2}{12} \right) T^4 \left[1 - \frac{\alpha^4}{(2\pi)^4} \right]$$

where at $T_0 = a/2\pi$ we have $\alpha^2 = -(2\pi)^2$.

- At $T_0 = a/2\pi$ the proper temperature is

$$\frac{A^2}{T^2} = -\frac{a^2}{T_0^2} \qquad \Rightarrow \qquad T = \frac{\sqrt{-A^2}}{2\pi} = T_{\rm U}.$$



ENTROPY CURRENT

- What is the entropy current?
  - Vector current s^μ that makes the entropy S extensive:

$$S = -\operatorname{tr}(\hat{\rho}\log\hat{\rho}) = \int_\Sigma \mathrm{d}\Sigma_\mu s^\mu.$$

- Why is it interesting?
  - Enters the local version of the second law of thermodynamics.
  - Postulated ingredient of Israel's relativistic hydrodynamics.
  - Responsible for the constitutive equations of the conserved currents.
- What is the problem with it?
  - Not TEV of a current dependent on quantum fields, unlike charge currents.
  - Therefore, in Israel's theory is postulated but not *derived*.
- We put forward a method to derive it including quantum corrections.

[Israel 1976]

ENTROPY CURRENT AND EXTENSIVITY

Recall: At LTE

$$\hat{\rho}_{\rm LE} = \frac{1}{Z_{\rm LE}} \exp\left[-\int_\Sigma {\rm d}\Sigma_\mu \left(\hat{T}^{\mu\nu}\beta_\nu - \zeta\hat{j}^\mu\right)\right],$$

hence

$$S = -\,{\rm tr}(\hat{\rho}_{\rm LE}\log\hat{\rho}_{\rm LE}) = \log Z_{\rm LE} + \int_\Sigma {\rm d}\Sigma_\mu \left(\langle\hat{T}^{\mu\nu}\rangle_{\rm LE}\beta_\nu - \zeta\langle\hat{j}^\mu\rangle_{\rm LE}\right).$$

where $\langle\hat{\mathcal{O}}\rangle_{\rm LE} = {\rm tr}(\hat{\rho}_{\rm LE}\hat{\mathcal{O}})$. If there is ϕ^μ such that (extensivity of $\log Z_{\rm LE}$)

$$\log Z_{\rm LE} = \int_\Sigma {\rm d}\Sigma_\mu \phi^\mu,$$

then

$$s^\mu = \phi^\mu + \langle\hat{T}^{\mu\nu}\rangle_{\rm LE}\beta_\nu - \zeta\langle\hat{j}^\mu\rangle_{\rm LE}.$$

ϕ^μ is *thermodynamic potential current*.

ENTROPY CURRENT AND EXTENSIVITY

- Define *local equilibrium operator*

$$\hat{\Upsilon} = \int_\Sigma \mathrm{d}\Sigma_\mu \left(\hat{T}^{\mu\nu}\beta_\nu - \zeta \hat{j}^\mu \right).$$

- Introduce λ as

$$\hat{\rho}_{\mathrm{LE}} = \frac{\mathrm{e}^{-\hat{\Upsilon}}}{Z_{\mathrm{LE}}} \mapsto \hat{\rho}_{\mathrm{LE}}(\lambda) = \frac{\mathrm{e}^{-\lambda\hat{\Upsilon}}}{Z_{\mathrm{LE}}(\lambda)}$$

such that $\hat{\rho}_{\mathrm{LE}}(\lambda = 1) = \hat{\rho}_{\mathrm{LE}}$.

- Derive $\log Z_{\mathrm{LE}}(\lambda)$ with respect to λ:

$$\frac{\partial \log Z_{\mathrm{LE}}(\lambda)}{\partial \lambda} = - \int_\Sigma \mathrm{d}\Sigma_\mu \left(\langle \hat{T}^{\mu\nu} \rangle_{\mathrm{LE}}(\lambda)\, \beta_\nu - \zeta \langle \hat{j}^\mu \rangle_{\mathrm{LE}}(\lambda) \right)$$

with $\langle \hat{\mathcal{O}} \rangle_{\mathrm{LE}}(\lambda) = \mathrm{tr}(\hat{\rho}_{\mathrm{LE}}(\lambda)\, \hat{\mathcal{O}})$.

ENTROPY CURRENT AND EXTENSIVITY

- Integrate in λ from some λ_0 to $\lambda = 1$ recalling $\log Z_{\rm LE}(\lambda = 1) = \log Z_{\rm LE}$ and exchange λ-integration with Σ-integration

$$\log Z_{\rm LE} - \log Z_{\rm LE}(\lambda_0) = -\int_\Sigma {\rm d}\Sigma_\mu \int_{\lambda_0}^1 {\rm d}\lambda \left(\langle \hat{T}^{\mu\nu} \rangle_{\rm LE}(\lambda)\, \beta_\nu - \zeta \langle \hat{j}^\mu \rangle_{\rm LE}(\lambda) \right).$$

- If there exists λ_0 such that $\log Z_{\rm LE}(\lambda_0) = 0$, then

$$\phi^\mu = -\int_{\lambda_0}^1 {\rm d}\lambda \left(\langle \hat{T}^{\mu\nu} \rangle_{\rm LE}(\lambda)\, \beta_\nu - \zeta \langle \hat{j}^\mu \rangle_{\rm LE}(\lambda) \right).$$

- Assume: $\hat{\Upsilon}$ bounded from below with Υ_0 non-degenerate lowest eigenvalue and $|0\rangle$ corresponding eigenvector.
- Shift $\hat{\Upsilon} \mapsto \hat{\Upsilon}' = \hat{\Upsilon} - \Upsilon_0 = \hat{\Upsilon} - \langle 0 | \hat{\Upsilon} | 0 \rangle$ and see that $\hat{\rho}_{\rm LE}$ (hence S) is invariant.
- Consequence: $Z'_{\rm LE} = Z_{\rm LE}[\hat{\Upsilon}']$ is such that $\log Z'_{\rm LE}(\lambda_0 = +\infty) = 0$.

ENTROPY CURRENT AND EXTENSIVITY

- Conclusion: If $\hat{\Upsilon}$ is bounded from below and the lowest eigenvalue Υ_0 is non-degenerate, then $\log Z_{\mathrm{LE}}$ is extensive and ϕ^μ is given by

$$\phi^\mu = \int_1^{+\infty} \mathrm{d}\lambda \left[\left(\langle \hat{T}^{\mu\nu} \rangle_{\mathrm{LE}}(\lambda) - \langle 0 | \hat{T}^{\mu\nu} | 0 \rangle \right) \beta_\nu - \zeta \left(\langle \hat{j}^\mu \rangle_{\mathrm{LE}}(\lambda) - \langle 0 | \hat{j}^\mu | 0 \rangle \right) \right].$$

In this case, s^μ exists and reads

$$s^\mu = \phi^\mu + \left(\langle \hat{T}^{\mu\nu} \rangle_{\mathrm{LE}} - \langle 0 | \hat{T}^{\mu\nu} | 0 \rangle \right) \beta_\nu - \zeta \left(\langle \hat{j}^\mu \rangle_{\mathrm{LE}} - \langle 0 | \hat{j}^\mu | 0 \rangle \right).$$

- Result: We showed that $\log Z_{\mathrm{LE}}$ is extensive under general hypotheses and provided a method to calculate the entropy current at LTE.

ENTROPY CURRENT AT GTE WITH ACCELERATION

Ingredients at GTE with acceleration:

- $\langle \hat{T}^{\mu\nu} \rangle - \langle 0 | \hat{T}^{\mu\nu} | 0 \rangle = F_1 \beta^\mu \beta^\nu + F_2 g^{\mu\nu} + F_3 \alpha^\mu \alpha^\nu$ for symmetry reasons, with $F_i = F_i(\beta^2, \alpha^2)$.
- $|0\rangle = |0_{\mathrm{R}}\rangle$ is *Rindler vacuum* ($|0\rangle \neq |0_{\mathrm{M}}\rangle$), since

$$\hat{\rho}_{\mathrm{R}} = \frac{1}{Z_{\mathrm{R}}} \exp\left[-\frac{1}{T_0} \sum_i \omega_i \hat{a}_i^{\mathrm{R}\dagger} \hat{a}_i^{\mathrm{R}} \right].$$

<u>Recall</u>: $\phi^\mu = \int_1^{+\infty} \mathrm{d}\lambda \left(\langle \hat{T}^{\mu\nu} \rangle(\lambda) - \langle 0 | \hat{T}^{\mu\nu} | 0 \rangle \right) \beta_\nu$. Thus

$$\left(\langle \hat{T}^{\mu\nu} \rangle - \langle 0 | \hat{T}^{\mu\nu} | 0 \rangle \right) \beta_\nu = \left(F_1 \beta^2 + F_2 \right) \beta^\mu = \rho_{\mathrm{Rindler}}\, \beta^\mu$$

with ρ_{Rindler} the *energy density*. Hence

$$\phi^\mu = \int_1^{+\infty} \mathrm{d}\lambda\, \rho(\lambda)\, \beta^\mu.$$

ENTROPY CURRENT AT GTE WITH ACCELERATION

For free real massless scalar field:

$$\rho_{\text{Rindler}} = \left(\langle \hat{T}^{\mu\nu} \rangle - \langle 0 \,|\, \hat{T}^{\mu\nu} \,|\, 0 \rangle \right) u_\mu u_\nu = \frac{\pi^2}{30\beta^4} - \frac{\alpha^2}{12\beta^4}.$$

Result:

$$\phi^\mu = \left(\frac{\pi^2}{90\beta^4} - \frac{\alpha^2}{12\beta^4} \right) \beta^\mu, \qquad s^\mu = \left(\frac{2\pi^2}{45\beta^4} - \frac{\alpha^2}{6\beta^4} \right) \beta^\mu.$$

- Notice: $\nabla_\mu s^\mu = 0$, i.e. vanishing entropy production rate, as expected at GTE.
- The terms proportional to $\alpha^2 = \dfrac{A^2}{T^2} \left(\dfrac{\hbar}{c k_{\mathrm{B}}} \right)^2$ are quantum corrections.

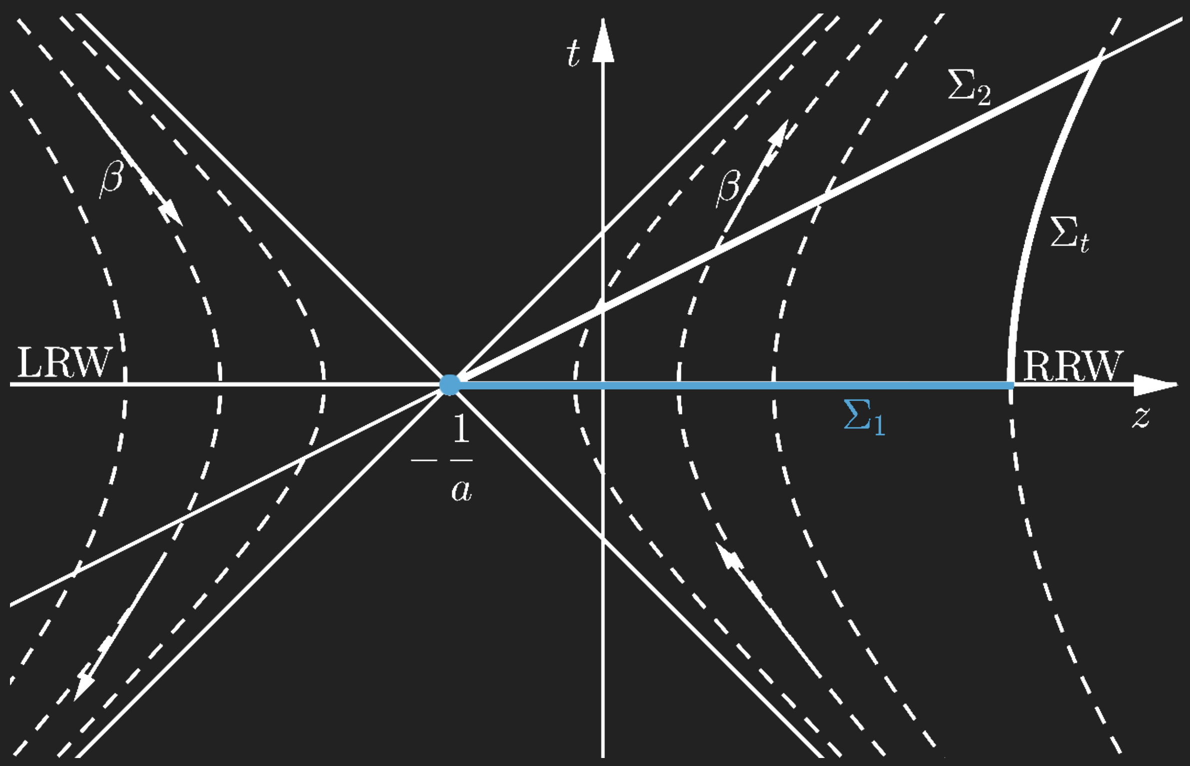
ENTANGLEMENT ENTROPY AND UNRUH EFFECT

At GTE, S is independent of choice of Σ.
Integrating s^μ in RRW on $\Sigma_1 = \{t = 0, z' \geq 0\}$:

$$S_{\rm R} = \int_{\mathbb{R}^2} {\rm d}x\,{\rm d}y \left(\frac{2\pi^2}{45} - \frac{\alpha^2}{12} \right) \frac{T_0^3}{a^3} \lim_{z'\to 0} \frac{1}{2z'^2}.$$

- *Area law,*
- Divergence as $z' \to 0$.

[Bombelli et al. 1986]



- $\nabla_\mu s^\mu = 0$ at GTE $\Rightarrow$ there is *potential* $\varsigma^{\mu\nu} = -\varsigma^{\nu\mu}$ such that $s^\mu = \nabla_\nu \varsigma^{\mu\nu}$, therefore $S_{\rm R}$ is

$$S_{\rm R} = -\frac{1}{4} \int_{\partial\Sigma} {\rm d}S^{\rho\sigma} \sqrt{|g|}\, \epsilon_{\mu\nu\rho\sigma}\, \varsigma^{\mu\nu}.$$

i.e. surface integral. Solution:

$$\varsigma^{\mu\nu} = \frac{s}{2\alpha^2}(\beta^\mu \alpha^\nu - \beta^\nu \alpha^\mu)$$

with $s = s^\mu u_\mu$ *entropy density*.

[Wald 1993]

ENTANGLEMENT ENTROPY AND UNRUH EFFECT

- Recall: At GTE with acceleration, $\hat{\rho} = \hat{\rho}_{\rm R} \otimes \hat{\rho}_{\rm L}$ with $\hat{\rho}_{\rm R} = {\rm tr}_{\rm L}(\hat{\rho})$ and $\hat{\rho}_{\rm L} = {\rm tr}_{\rm R}(\hat{\rho})$.
- Consequence: $S_{\rm R} = -\,{\rm tr}_{\rm R}(\hat{\rho}_{\rm R} \log \hat{\rho}_{\rm R})$ is *entanglement entropy* of RRW with LRW.

- Recall: At GTE with acceleration, $T_{\rm U}$ is absolute lower bound for T.
- Consequence: Non-vanishing entropy current in Minkowski vacuum

$$s^\mu(T_{\rm U}) = \frac{32\pi^2}{45} T_{\rm U}^3 u^\mu.$$

- Notice: s^μ depends on choice of $\hat{T}^{\mu\nu}$. Usually two choices

$$\hat{T}^{\mu\nu}_{\rm can} = \partial^\mu \hat{\psi} \partial^\nu \hat{\psi} - \mathscr{L} g^{\mu\nu}, \qquad \hat{T}^{\mu\nu}_{\rm imp} = \hat{T}^{\mu\nu}_{\rm can} - \frac{1}{6}\left(\partial^\mu \partial^\nu - g^{\mu\nu} \Box\right) \hat{\psi}^2$$

with $\mathscr{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \hat{\psi} \partial_\nu \hat{\psi} - \frac{1}{2} m^2 \hat{\psi}^2$, hence two different entropy currents (for $m = 0$)

$$s^\mu_{\rm can} = \left(\frac{2\pi^2}{45\beta^4} - \frac{\alpha^2}{6\beta^4} \right) \beta^\mu, \qquad s^\mu_{\rm imp} = \frac{2\pi^2}{45\beta^4} \beta^\mu.$$

SUMMARY

- Studied QFT at GTE with acceleration.
- Accelerated observers in Minkowski vacuum see thermal radiation (Unruh effect).
- Unruh temperature is an absolute lower bound for proper temperature.
- Method to derive the entropy current.
- Calculation of entropy current at GTE with acceleration.
- Relation with Unruh effect and entanglement entropy.

THANK YOU FOR YOUR ATTENTION