Infrared Universality of ζ in asymptotically FLRW spacetime

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In progress

in collaboration with Takahiro Tanaka (Kyoto U.)

Infrared Universality in cosmology

History of Universe

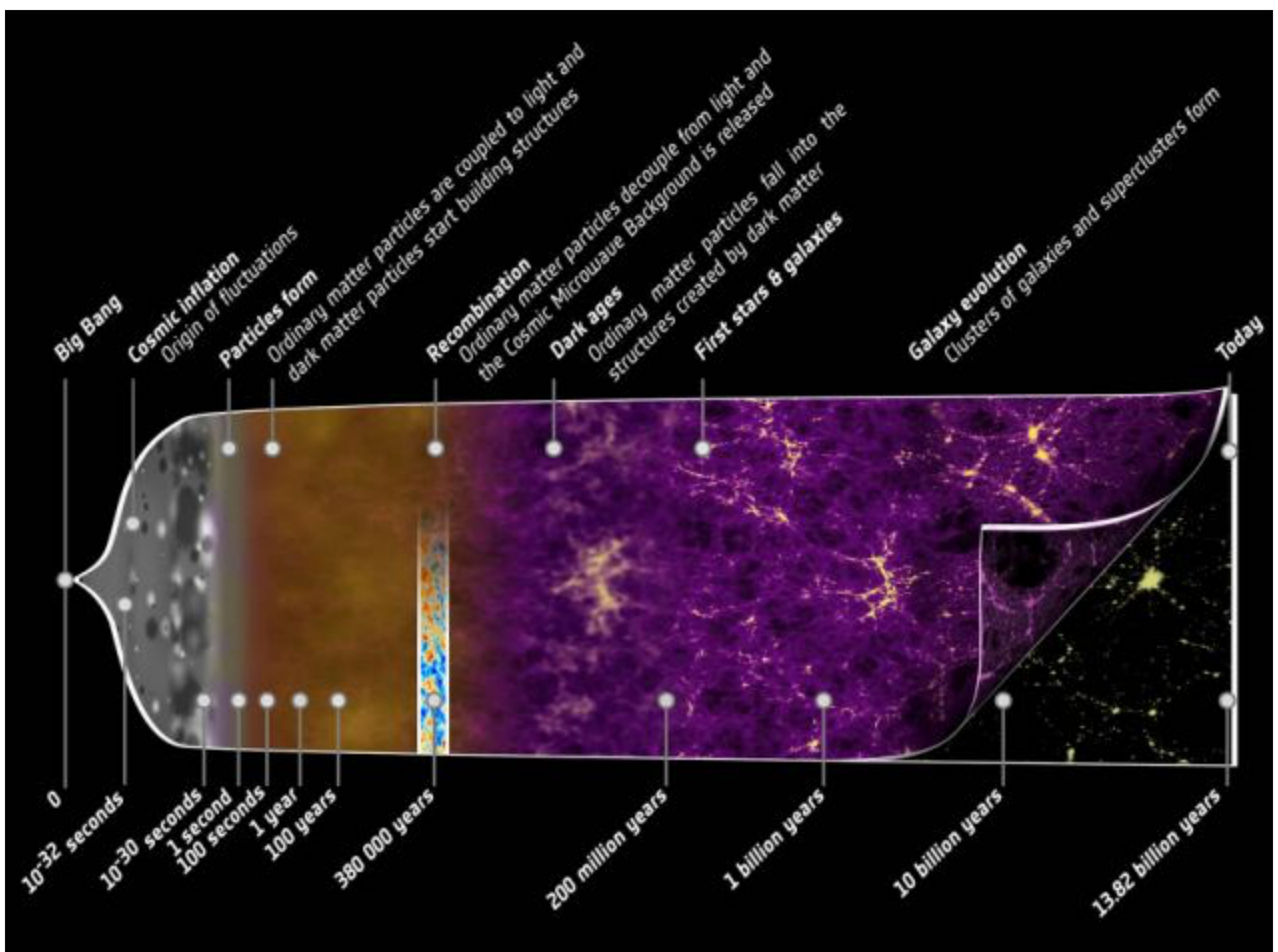
In cosmology, it is crucially important to under the evolution at large scales,

$$\frac{k}{aH} = \frac{1/H}{a/k} = \frac{\text{Hubble scale (curvature radius)}}{\text{physical length}} \ll 1$$

$k = |\boldsymbol{k}|$

$a(t)$:scale factor

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$



Basic idea of inflation

$$\ddot{a} > 0 \longrightarrow \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{aH}\right) = -\frac{\ddot{a}}{\dot{a}^2} < 0$$

$\lambda_{\mathrm{Hubble}} / \lambda_{\mathrm{phys}}$ decreases in time

Physical scale $\lambda_{\mathrm{phys}} \propto a$

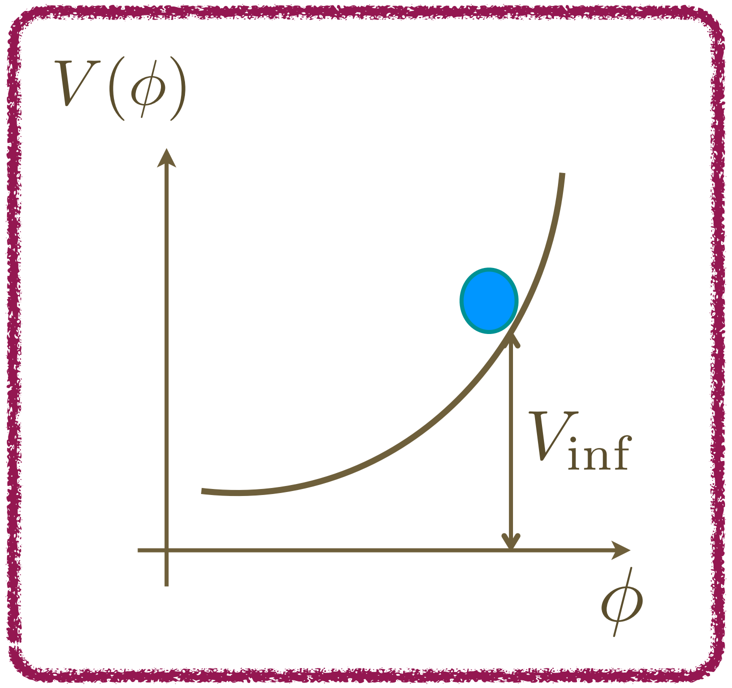
Hubble scale $\lambda_{\mathrm{Hubble}} \propto 1/H$

Simple example

slowly rolling scalar field

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \simeq V(\phi)$$

= constant



Primordial perturbations

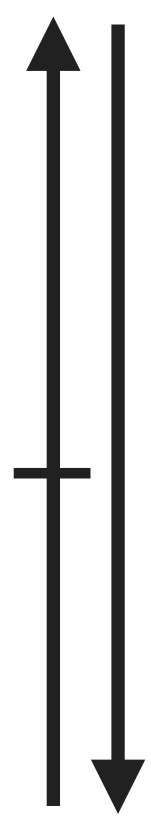
Unique window to explore models of inflation

spin 0 (inflaton) ζ, spin 2 GWs γ_{ij}

<u>Def.</u> $$h_{ij} = a^2 e^{2\zeta} \left[e^{\gamma}\right]_{ij}$$

time slicing $\delta\phi = 0$ spatial coordinates $\partial^i \gamma_{ij} = 0$

ω_{phys}=k/a



time

Adiabatic

$|\dot{\omega}/\omega^2| \ll 1$

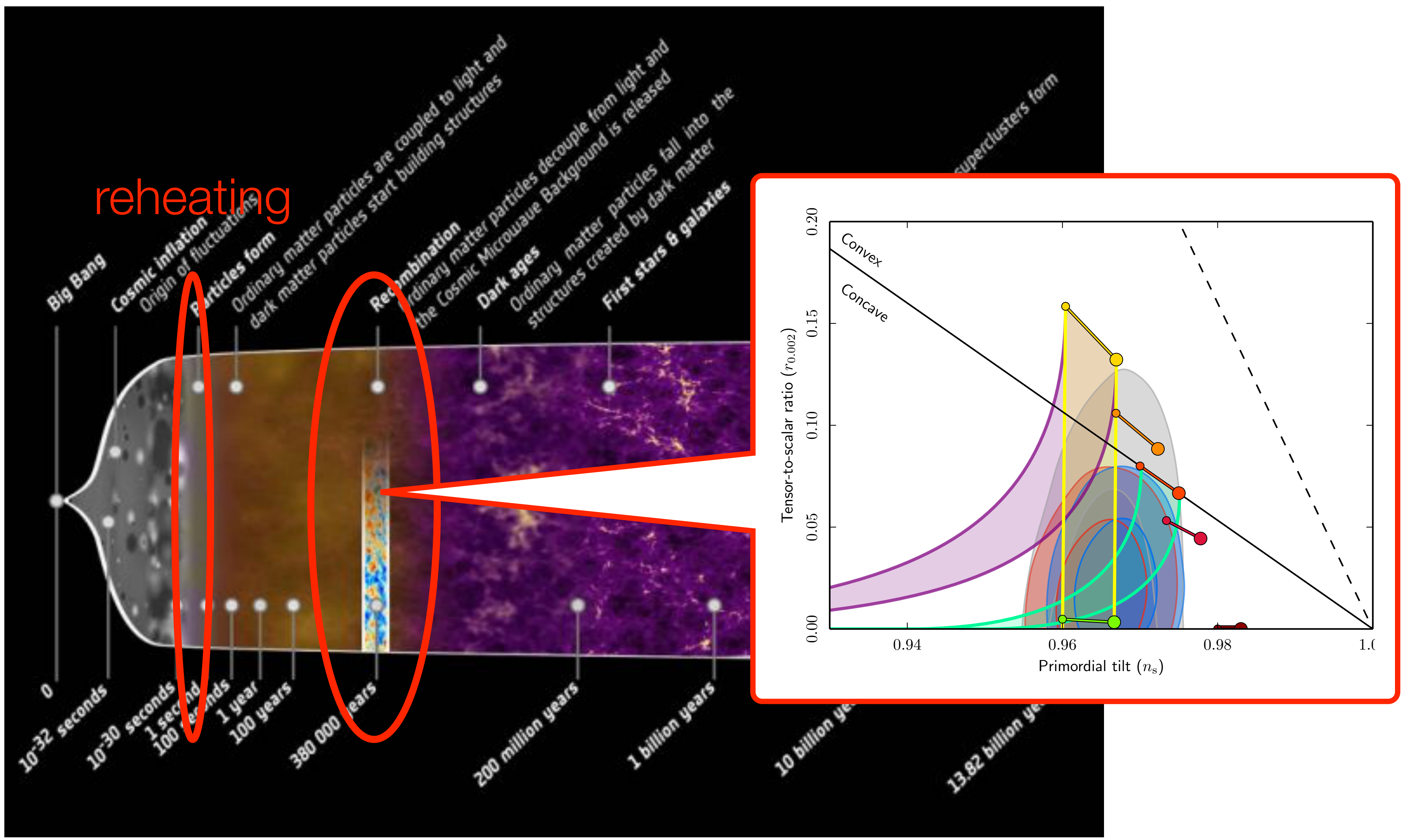
H

Non-adiabatic

$|\dot{\omega}/\omega^2| \gg 1$

Vacuum polarization ≠ 0

Solving large scale evolution



k/aH

Large scale evolution *k/aH* << 1

n_s-r plot

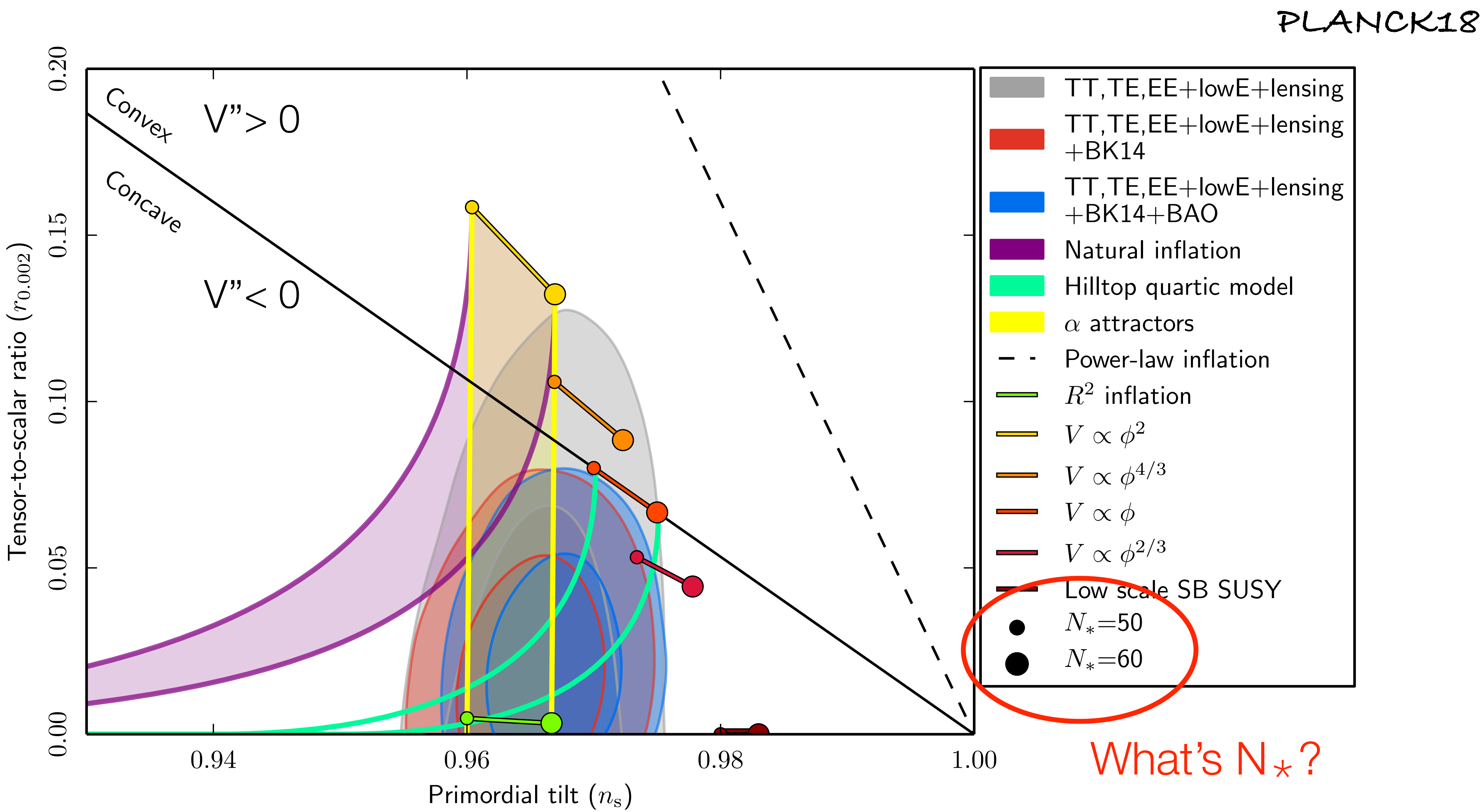
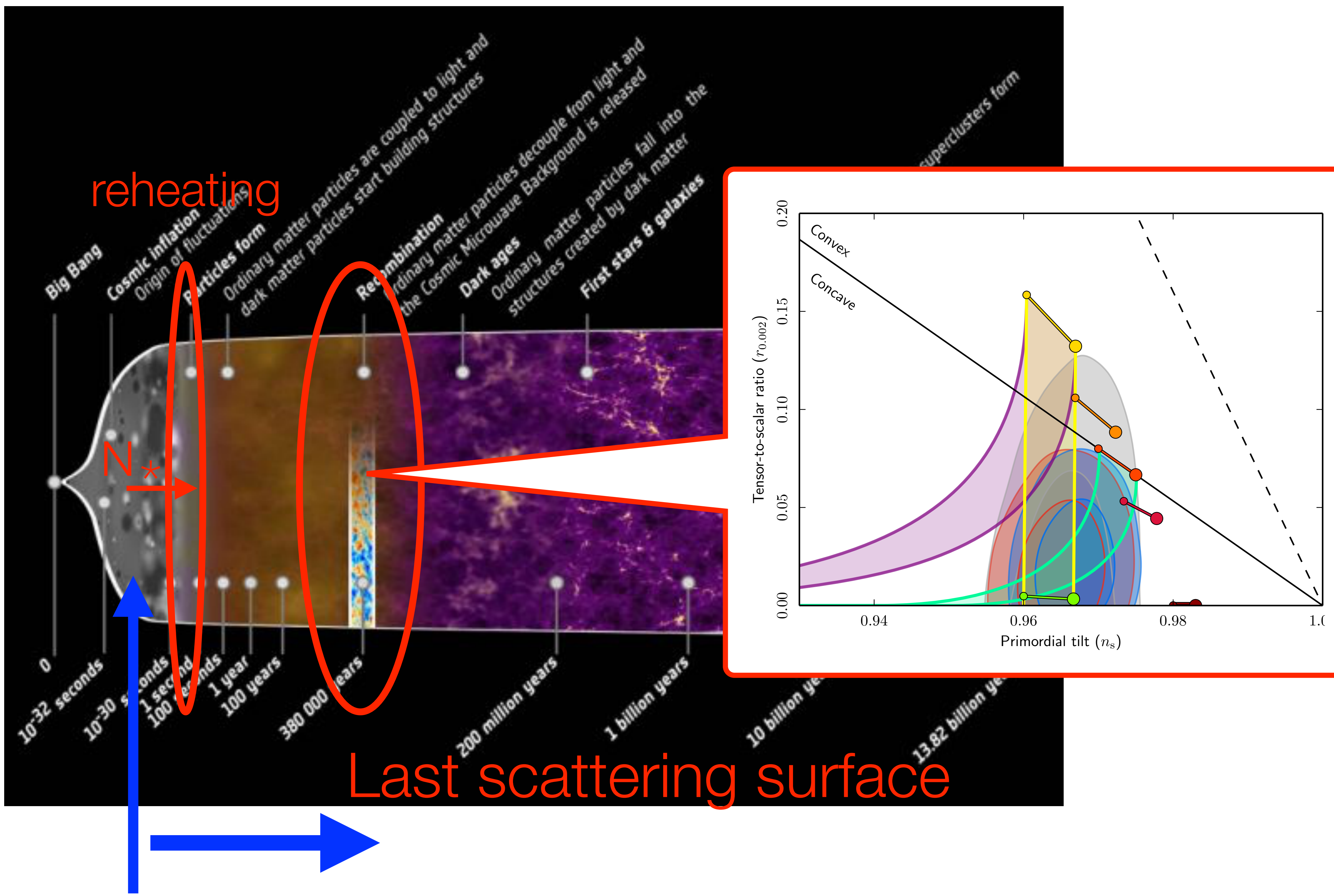


Fig. 8. Marginalized joint 68 % and 95 % CL regions for n_s and r at $k = 0.002\,\mathrm{Mpc}^{-1}$ from *Planck* alone and in combination with BK14 or BK14 plus BAO data, compared to the theoretical predictions of selected inflationary models. Note that the marginalized joint 68 % and 95 % CL regions assume $dn_s/d\ln k = 0$.

Conservation of ζ



$\mathcal{P}_\zeta(k)|_{k/aH\sim 1}$

Time conservation $\partial_t \zeta = \mathrm{O}((k/aH)^2)$

More on Conservation of ζ

Matter action $S_m = S_m[\, g_{\mu\nu}, \Phi \,]$

(d+1)-dim Diff invariance

$$0 = \delta_\xi S_m = \frac{\delta S_m}{\delta g_{\mu\nu}} \delta_\xi g_{\mu\nu} + \underline{\frac{\delta S_m}{\delta \Phi}} \delta_\xi \Phi = -2 \int d^{d+1}x \sqrt{-g}\, \xi_\nu \nabla_\mu T^{\mu\nu} \longrightarrow \nabla_\mu T^{\mu\nu} = 0$$

=0 (eom)

Lyth, Malik, & Sasaki (2004)

- Energy conservation $n_\nu \nabla_\mu T^{\mu\nu} = 0$
- Barotropic $p = p(\rho)$
- Asymp FLRW

$$\partial_t \rho + 3(\rho + P)(H + \dot{\zeta}) + \mathcal{O}(\epsilon^2) = 0 \longrightarrow \dot{\zeta} = \mathcal{O}(\epsilon^2)$$

slicing $\rho(t, x) = \rho(t)$

Weinberg's adiabatic mode

Adiabatic Modes in Cosmology

Steven Weinberg[1]
Theory Group, Department of Physics, University of Texas
Austin, TX, 78712

We show that the field equations for cosmological perturbations in Newtonian gauge always have an adiabatic solution, for which a quantity $\mathcal{R}$ is non-zero and constant in all eras in the limit of large wavelength, so that it can be used to connect observed cosmological fluctuations in this mode with those at very early times. There is also a second adiabatic mode, for which $\mathcal{R}$ vanishes for large wavelength, and in general there may be non-adiabatic modes as well. These conclusions apply in all eras and whatever the constituents of the universe, under only a mild technical assumption about the wavelength dependence of the field equations for large wave length. In the absence of
anisotropic ine
large waveleng
scale factor. W
what appears t
in synchronous
and Newtonian gauges suggest inequivalent assumptions about the behavior of the perturbations for large wavelength.

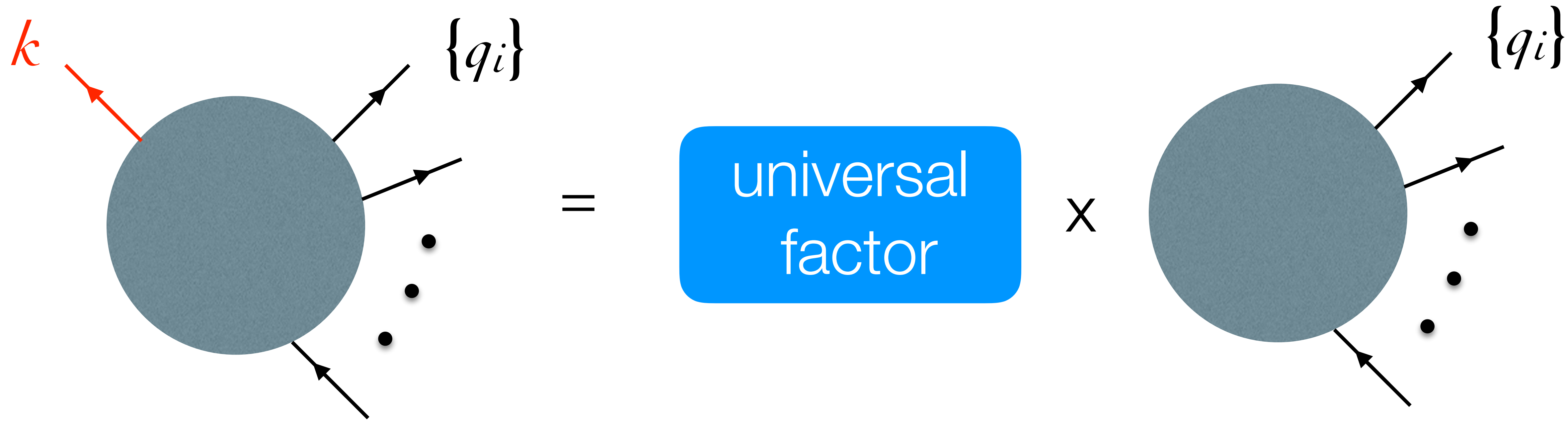
While it may not be the dominant solution in IR, the constant solution exists rather generically.

$$\dot{\zeta} \neq 0, \quad \zeta = \zeta_1 + \zeta_2 \qquad \dot{\zeta}_1 \sim 0$$

Consistency relation

correlation fun. with 1 soft ζ / γ_{ij} at time const. slicing

ζ / γ_{ij} n-point functions



Maldacena (2002), Creminelli&Zaldarriaga (2004), ...

e.g. $\langle \zeta(t_*, \mathbf{k}_1) \cdots \zeta(t_*, \mathbf{k}_n) \rangle = \delta(\mathbf{k}_1 + \cdots + \mathbf{k}_n)\, \mathcal{C}^{(n)}(\{\mathbf{k}_i\}_n)$

$$\lim_{k_n \to 0} \frac{\mathcal{C}^{(n)}(\{\boldsymbol{k}_i\}_n)}{P(k_n)} = -\left(\sum_{i=2}^{n-1} \boldsymbol{k}_i \cdot \partial_{\boldsymbol{k}_i} + 3(n-2) \right) \mathcal{C}^{(n-1)}(\{\boldsymbol{k}_i\}_{n-1})$$

Question

<u>IR "Universality"</u>

- Existence of const. solution in ζ (a.k.a. WAM)
- Consistency relation (~ soft theorem)
- Cancellation of IR divergence

Tanaka & Y.U. (09, 10,....), ...

holds rather generically, but not always.

e.g. WAM does not exist

- Solid inflation, 3 scalar fields w/ large scale anisotropic pressure

Enlich, Nicholas and Wang (11, 12)

Then, what is the condition that ensures IR "Universality"?

Outline

1) Introduction

2) IR physics in asymptotically flat spacetimes

($k^\mu \to 0$)

~ Learn from predecessors~

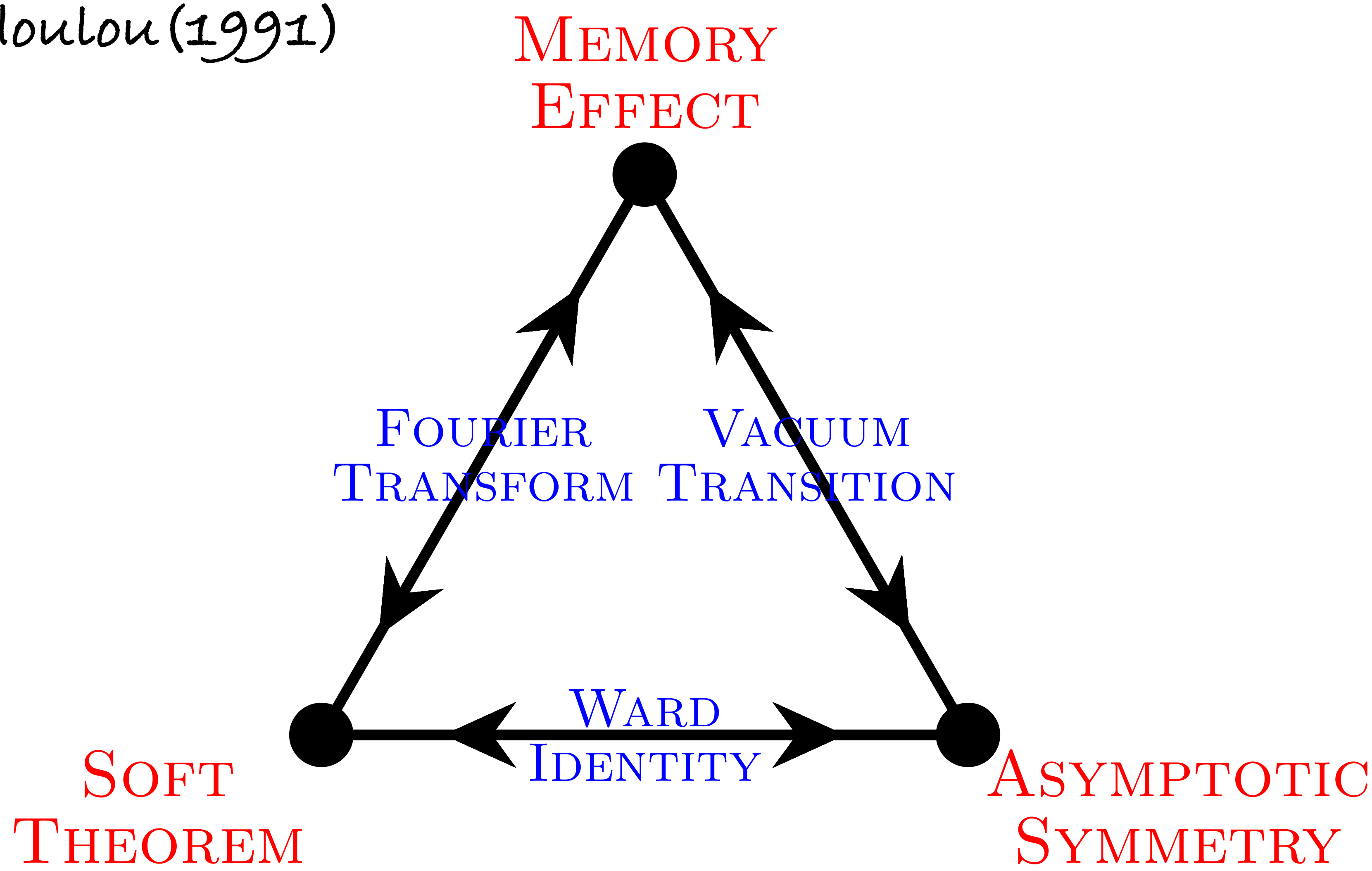
3) Conditions for IR "Universality"

Infrared triangle

Strominger+ (13,14, ...)

review 1703.05448

Christodoulou (1991)



Bloch & Nordsieck (1937)

Weinberg (1965)

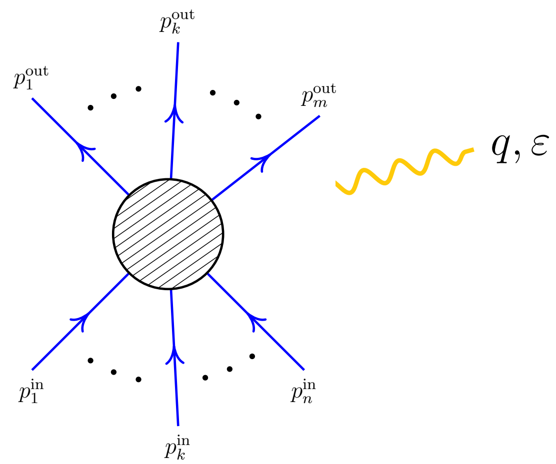
Bondi, Metzner, Sachs (1962)

Soft theorem

for QED Bloch & Nordsieck (1937)

for gravitons Weinberg (1965)

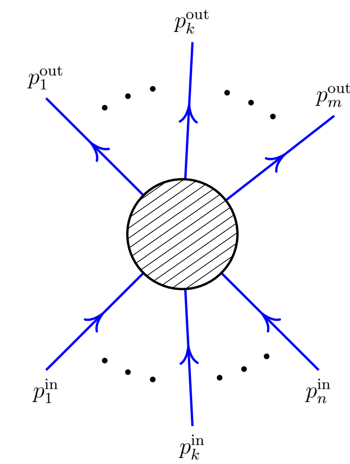
S matrix w/soft leg



=

universal factor

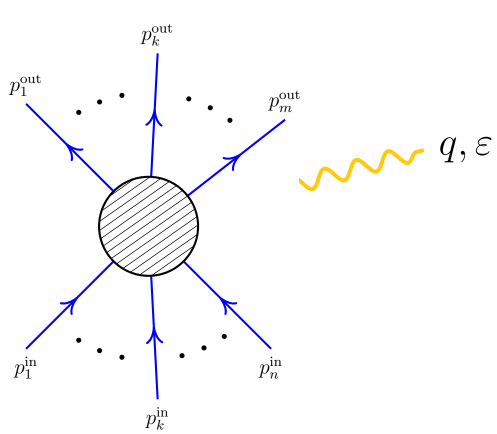
x



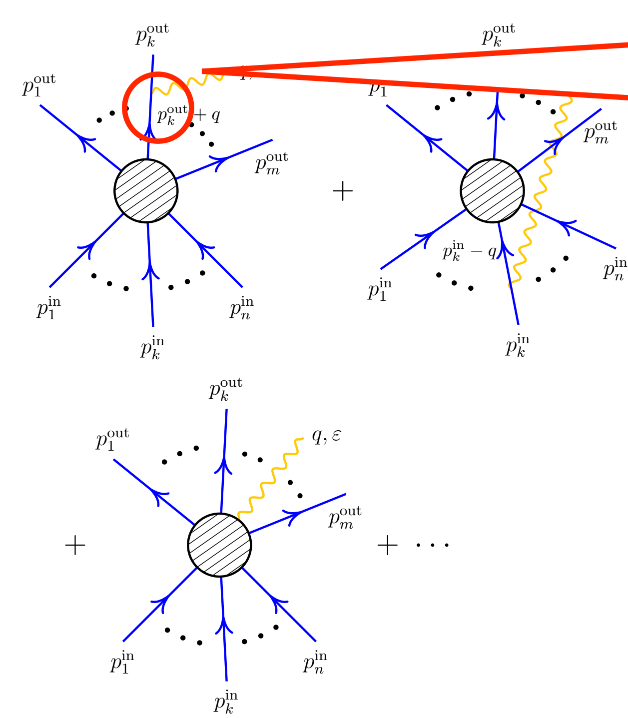
S matrix
(n→m scattering)

Soft photon theorem

$$\langle \mathrm{out}|a_+^{\mathrm{out}}(\vec{q})\mathcal{S}|\mathrm{in}\rangle = e\left[\sum_{k=1}^{m}\frac{Q_k^{\mathrm{out}}p_k^{\mathrm{out}}\cdot\varepsilon^+}{p_k^{\mathrm{out}}\cdot q} - \sum_{k=1}^{n}\frac{Q_k^{\mathrm{in}}p_k^{\mathrm{in}}\cdot\varepsilon^+}{p_k^{\mathrm{in}}\cdot q}\right]\langle \mathrm{out}|\mathcal{S}|\mathrm{in}\rangle + \mathcal{O}(q^0)$$



⟶



propagator

$$\frac{-i}{(p+q)^2+m^2} = \frac{-i}{p^2+2p\cdot q+q^2+m^2} = \frac{-i}{2p\cdot q}$$

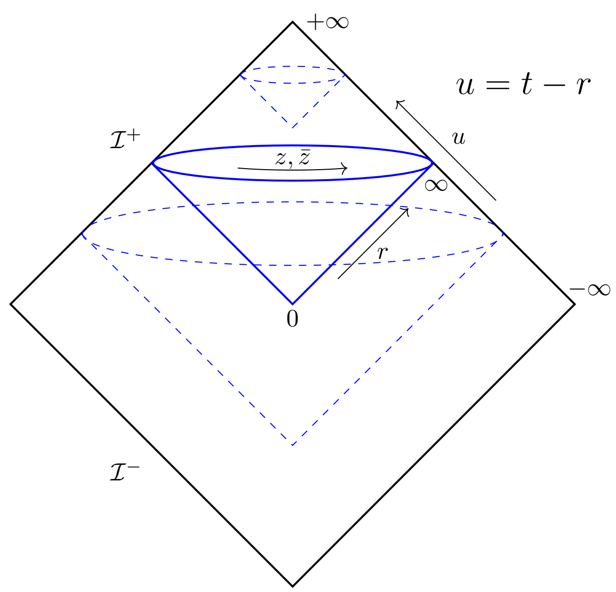
vertex factor

$$ie\varepsilon^\mu 2Qp_\mu$$

Asymptotic symmetry △

Bondi, Metzner, Sachs (1962)

BMS group: Symmetry in asymptotically flat sp.

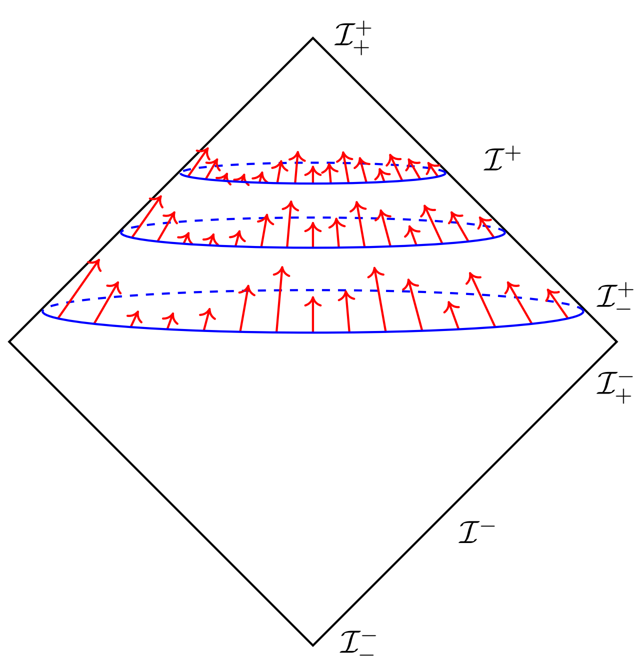


fixed (t, z, z̄), taking r → ∞ : approaching to $\mathcal{I}^+$

$$ds^2 = \underbrace{-du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}}_{\to M_4} + \frac{2m_B}{r}du^2 + \underbrace{rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2}_{\to GW} + \ \ldots$$

Supertranslations

$$\zeta = f\partial_u - \frac{1}{r}\left(D^z f\partial_z + D^{\bar{z}} f\partial_{\bar{z}}\right) + D^z D_z f\partial_r + \ldots, \qquad f(z, \bar{z})$$

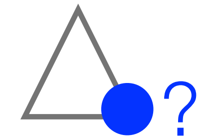


M4 ⟶ supertranslation $C_{zz} = -2D_z^2 C$

$C_{zz} = 0$ C → C + f

Strominger+ (13,14, ...)

WT of supertranslation = Soft graviton theorem

Large gauge transformations 

see, e.g., Harvey (96)

GTs $\begin{cases} \text{Small GTs } \mathcal{G}_S & g \to \mathbf{1} \ (\text{ in } |x| \to \infty) \quad g \in \mathcal{G}_S \\ \text{Large GTs } \mathcal{G}_L & g \not\to \mathbf{1} \ (\text{ in } |x| \to \infty) \quad g \in \mathcal{G}_L \end{cases}$

e.g. Large GTs in U(1) gauge theory

$$A^\mu(x) \to A'^\mu(x) = A^\mu(x) + \partial^\mu \lambda(x)$$

Lorentz gauge $\quad \partial_\mu A'^\mu(x) = 0$

- Small GTs fixed by $\quad \partial_\mu \partial^\mu \lambda(x) = -\partial_\mu A^\mu(x)$

- Large GTs $\quad \partial_\mu \partial^\mu \lambda(x) = 0$

$$\lambda(x) = \sum_{\mu_1 \cdots \mu_n} \underline{C_{\mu_1 \cdots \mu_n}} x^{\mu_1} \cdots x^{\mu_n}$$

symmetric traceless tensor

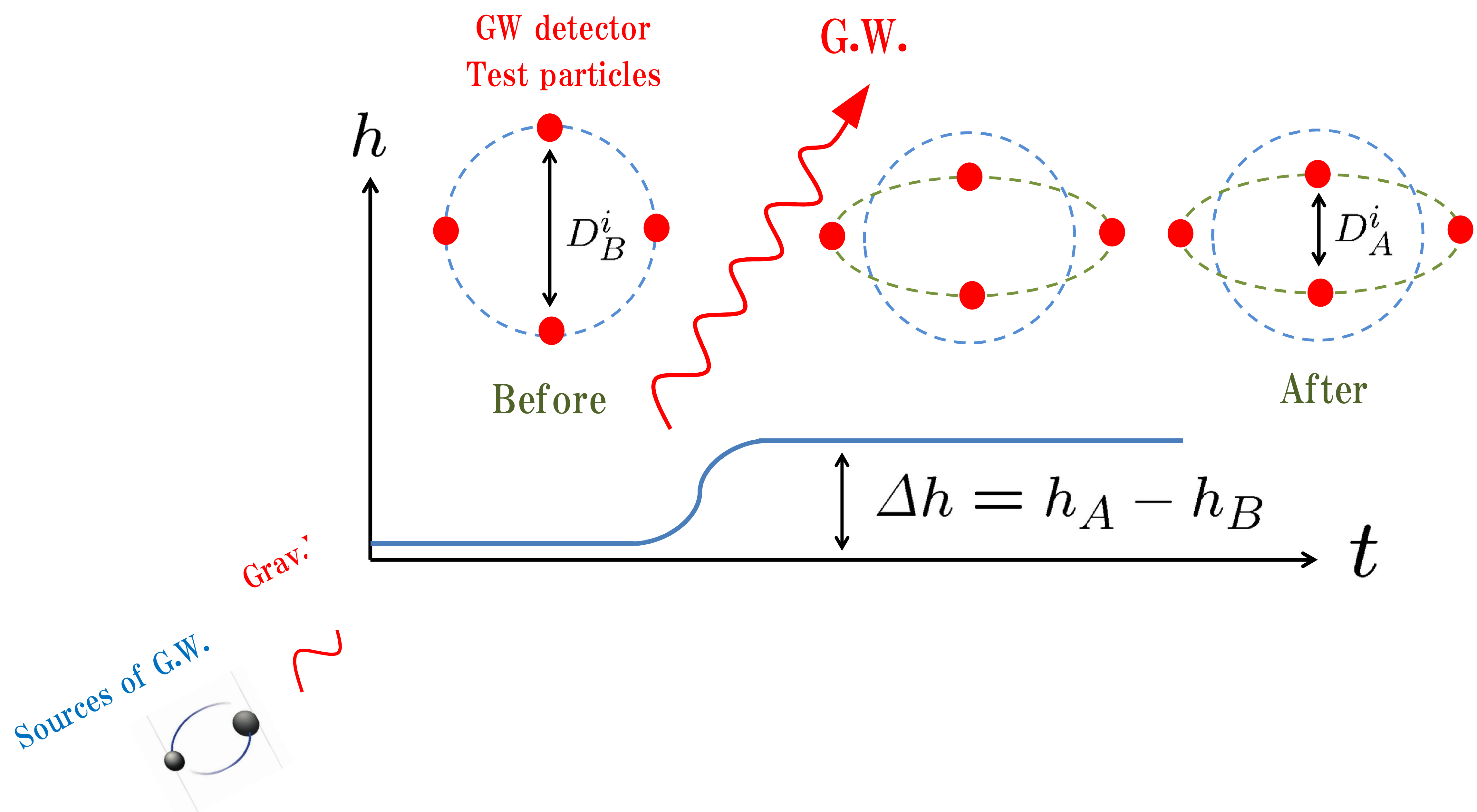
Memory effects

Zel'dovich & Polnarev (1974)

Christodoulou (1991)

Gravitational memory effect: Non-oscillatory contributions to GW amplitude

(Christodoulou effect)



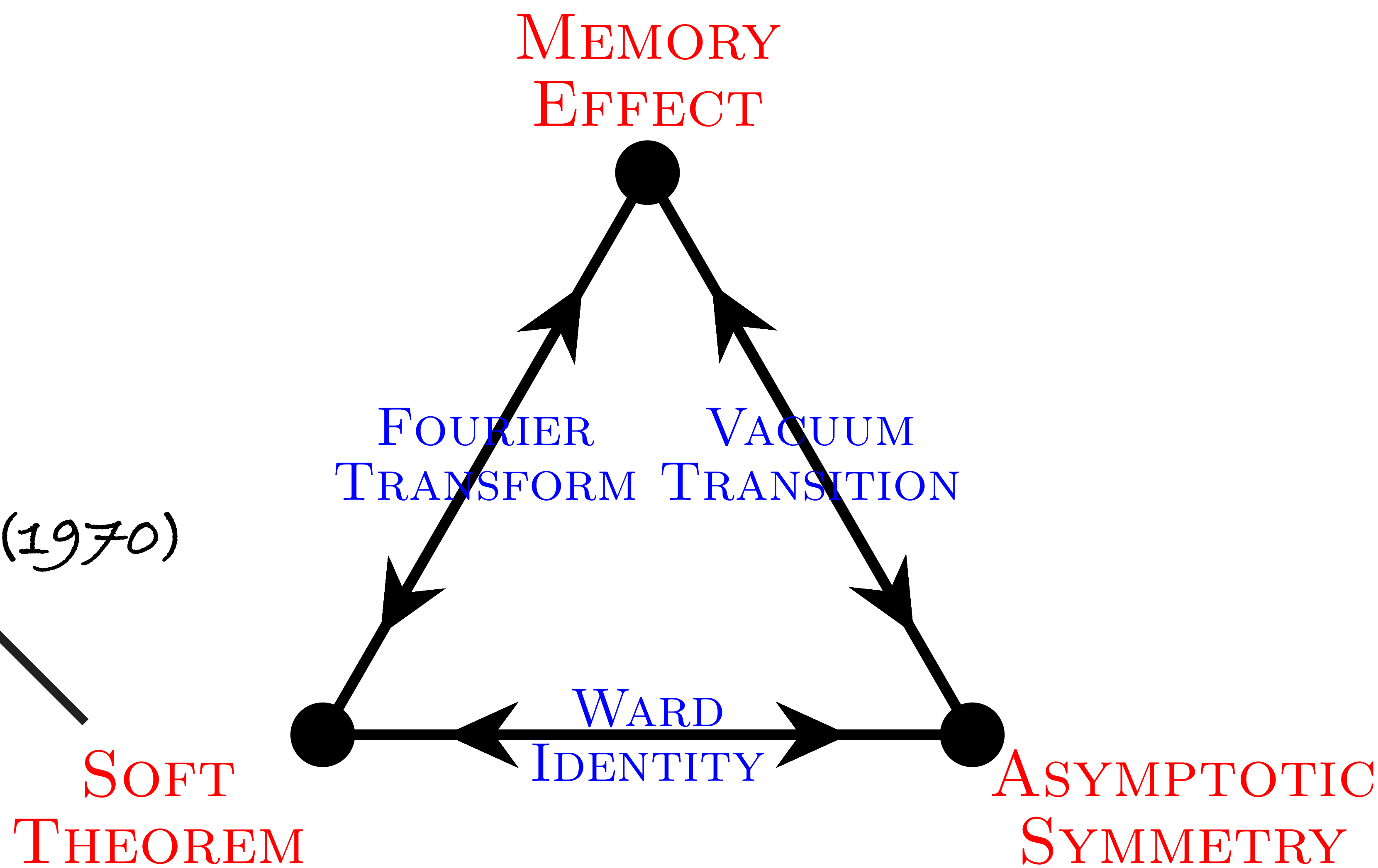
from talk slide of A.Ishibashi

Infrared triangle +

Strominger+ (13,14, ...)

review 1703.05448

Christodoulou (1991)



Cancellation of IR divergence

Faddeev-Kulish (1970)

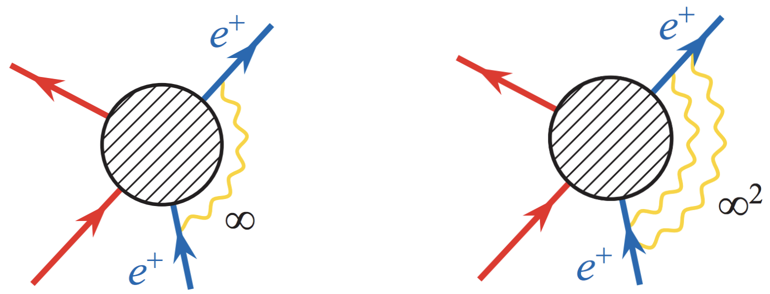
Bloch & Nordsieck (1937)

Weinberg (1965)

Bondi, Metzner, Sachs (1962)

Cancellation of IR div.

IR divergence in QED

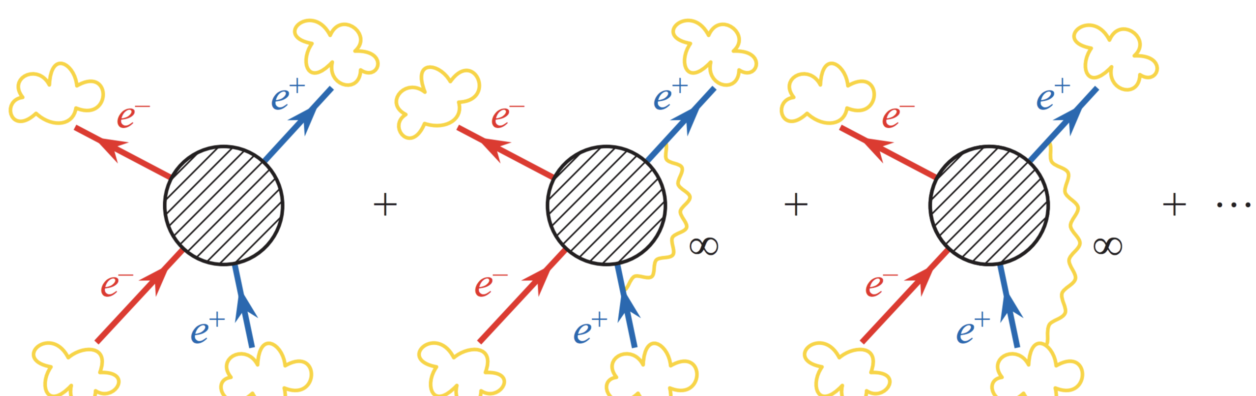


S-matrix element → 0

in IR cutoff → 0

Faddeev-Kulish (1970)

Dressing IR photons which are too IR to be detected = Degenerated state



IR divs. pair-wise cancellation

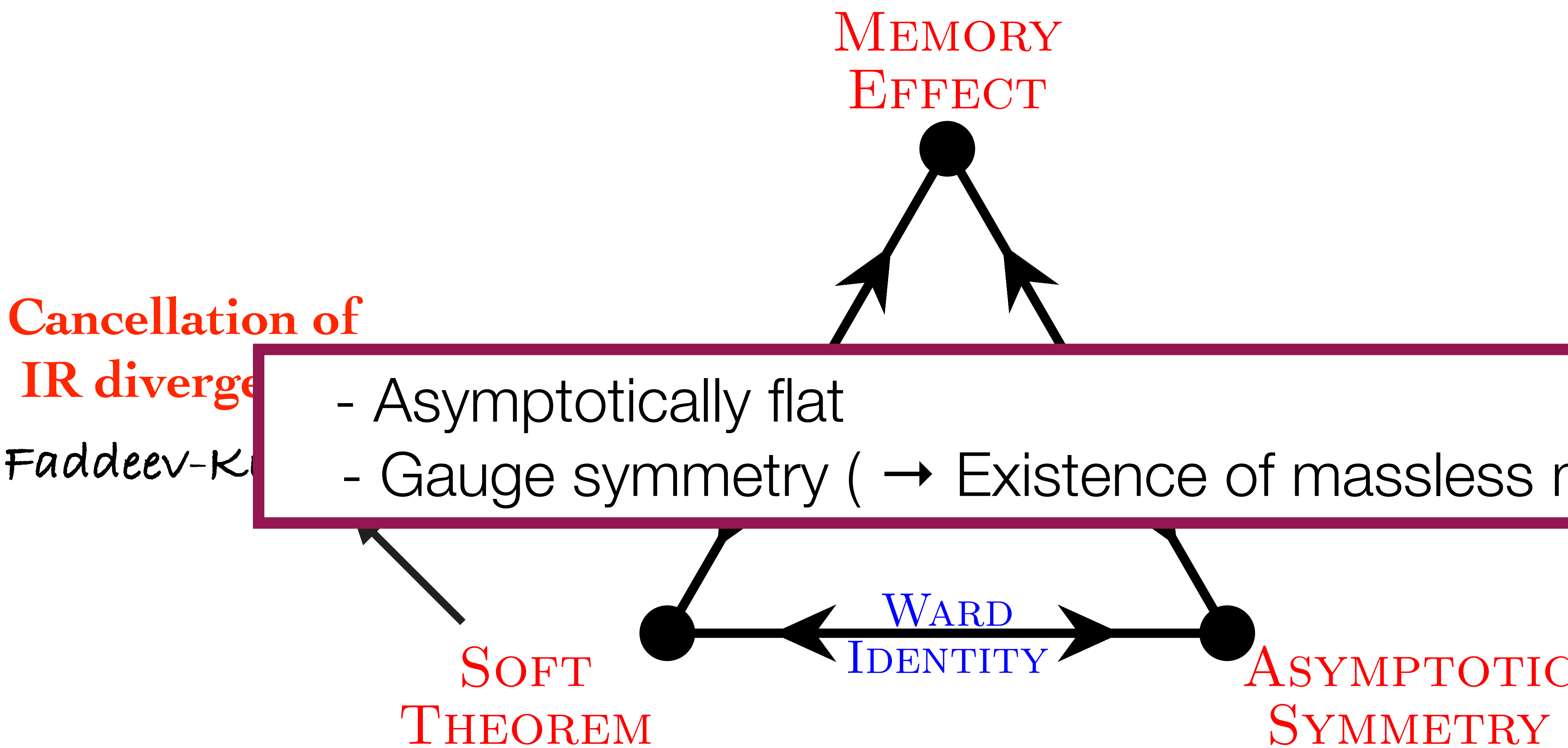
see also Kinoshita-Lee-Nauenberg for non-abelian

Infrared triangle +

Strominger+ (13,14, ...)

review 1703.05448

Christodoulou (1991)



- Asymptotically flat
- Gauge symmetry (→ Existence of massless mode)

Bloch & Nordsieck (1937)

Weinberg (1965)

Bondi, Metzner, Sachs (1962)

Outline

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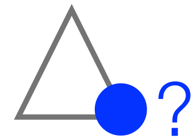
2) IR physics in asymptotically flat spacetimes

~ Learn from predecessors~

3) Conditions for IR “Universality”

Large GTs for *ζ/γ*$_{ij}$ 

Large: $|x^i| \to \infty$ at each time slicing

Invariance under Large GTs that preserves [asym FLRW]

(Counter part of BMS in cosmology)

- Dilatation transformation $\quad x^i \to e^s x^i$

$$\zeta(t,\,\boldsymbol{x}) \to \zeta_s(t,\,\boldsymbol{x}) = \zeta(t,\,e^{-s}\boldsymbol{x}) - s$$

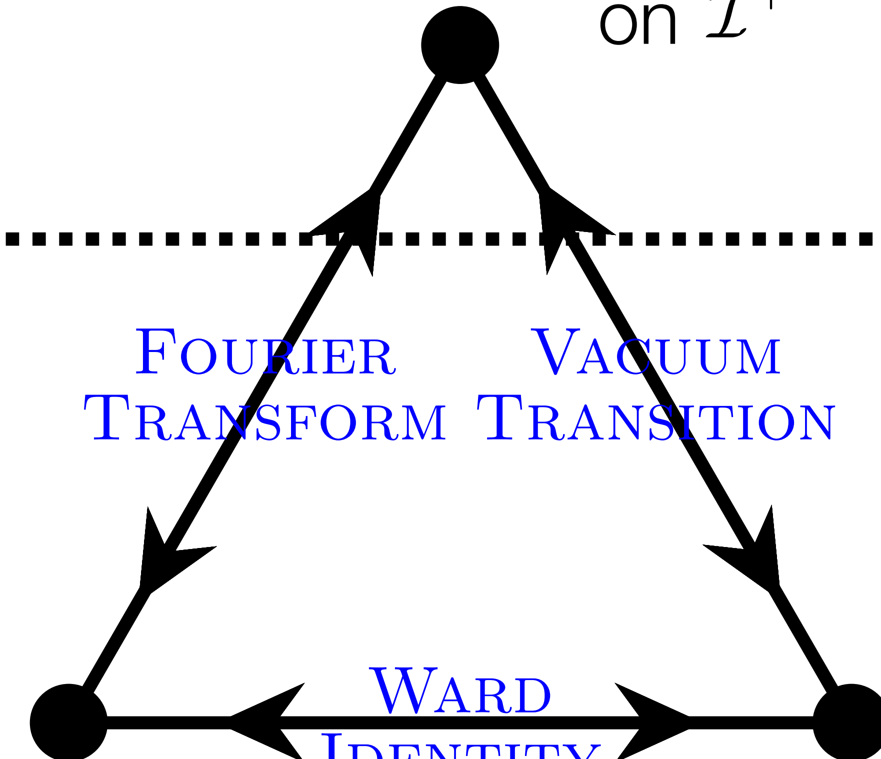
Very roughly speaking…..

dilatation inv. ensures “shift symmetry” of ζ

Infrared physics in cosmology

MEMORY EFFECT on $\mathcal{I}^+$

?
on spatial infinity



Cancellation of IR divergence

Tanaka & Y.U. (15, 17)

Consistency Relation ~ SOFT THEOREM

Maldacena(02)

Hinterbichler et al.(13)

ASYMPTOTIC SYMMETRY

Invariance under Large spatial GTs

e.g. $x^i \to e^s x^i$

Const. solution of ζ/γ_{ij}

Tanaka & Y.U. (15, 17), Garriga & Y.U.(16)

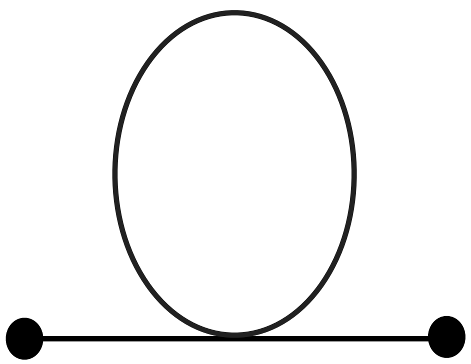
Cancellation of IR div.

For massless fields in (quasi) dS, accumulation of IR modes, *k/aH* <<1, leads to logarithmic divergence.

Ford&Parker(77), Allen&Folacci(87),...., Tsamis &Woodard(94, 97),....

e.g. $\mathcal{L}_\zeta^{(4)} \ni \zeta^2(\partial\zeta)^2$

in (*d*+1)-dim spacetimes

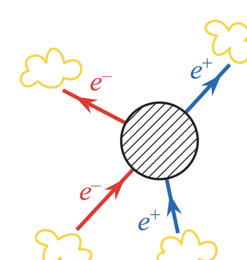


$$\text{(Loop integral)} \sim \int d^dk\,|\zeta_k|^2 \sim \int \frac{dk}{k}$$

scale inv. spectrum in IR

in QED

Limitation on detectable IR modes

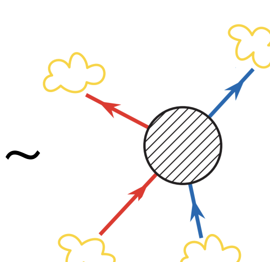


IR reg.

in cosmology

Limitation on detectable IR ***k*** modes

~ spatial distance

~ 

IR reg. w/CR

Tanaka & Y.U. (09, 10,....), ...

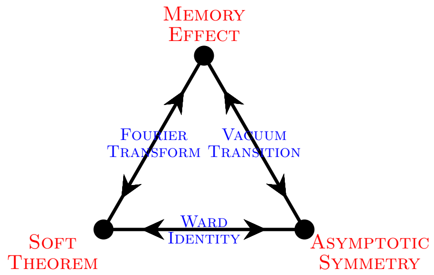
Conditions for IR "Universality"

QFT in asymptotically flat spacetimes

- Gauge invariance

- Asymptotically Flat spacetimes

→ IR triangle



QFT in cosmology

- Gauge invariance: spatial Diff. invariance

- Asymptotically FLRW spacetimes

→ IR "Universality" ?

Gradient expansion

Gradient expansion   *Salopek & Bond (1990), Shibata & Sasaki (1999)*

Physical scale of inhomogeneity of our focus L >> $1/H$

(or coarse graining scale)

$$\epsilon \equiv 1/(HL)$$

In local theory, we identity ϵ exp. as derivative exp.

(d+1)-dim line element & Gauge choice

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \qquad g_{ij} = a^2 e^{2\zeta}\gamma_{ij} \qquad \det[\gamma] = 1$$

- spatial coordinates: Transverse $\gamma_{ij}{}^{|j} = 0$

recall FLRW $ds^2 = -dt^2 + a^2(t)\bar{\gamma}_{ij}(\mathbf{x})dx^i dx^j$

Asymptotically FLRW spacetime

Asymp FLRW condition

$$\left| e^{-\bar{\zeta}} \frac{N_i}{aN} \right| = \mathcal{O}(\epsilon)$$

spatial average at reference time $t_\star$

$$\bar{\zeta} \equiv \frac{\int d^d\boldsymbol{x}\, \zeta(t_\star, \boldsymbol{x})}{\int d^d\boldsymbol{x}}$$

$$x^i \to e^{-s} x^i \qquad \bar{\zeta} \to \bar{\zeta} - s$$

* Do not impose condition on time derivative of γ_{ij}

Lyth, Malik & Sasaki (2004)

* At linear order, 【AsympFLRW】 is a condition on shear. $k\sigma_{\rm g} \sim \partial_i N_i / a$

* $\bar{\zeta}$ is introduced for the dilatation invariance.

(* For (d+1)-Diff, the above condition is imposed on K=dH slicing.)

Classical Lagrangian

【Spatial Diff. invariance】(& 【Locality】)

$$S = \int d^{d+1}x\sqrt{-g}\,[\mathcal{L}_{\rm g} + \mathcal{L}_{\rm matter}]$$

with

$$\mathcal{L}_{\rm g} = \frac{1}{16\pi G}\left[\left(K^i{}_j K^j{}_i - \beta_1 K^2\right) + \beta_2{}^s R + \mathcal{O}(\epsilon^2 \text{ w/o } N_i, \epsilon^3 \text{ w/} N_i)\right]$$

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - D_i N_j - D_j N_i) \qquad K \equiv \gamma^{ij} K_{ij}$$

* Matter sector ∋ Integer spin fields w/sDiff + locality

recall cosmological collier program   Arkani-Hamed & Maldacena (2015)

Bottom-line argument

Tanaka & Y.U. (in progress)

(d+1)-dim Diff H-const & M-const $\longrightarrow$ $\frac{\dot{\zeta}}{H} = \mathcal{J} + \mathcal{O}(\epsilon)$

$\mathcal{J}$ is described only by other independent dynamical fields.

All terms w/ ζ are O(ε)

- dilatation inv. ~ shift sym. of ζ
- asymp FLRW

(for quantum system)
e.g. during inflation

(ex) Single scalar field, classical linear perturbation

H-const $\frac{\dot{\zeta}}{H} = \frac{2(d-1)H^2}{16\pi G\dot{\bar{\phi}}^2}\frac{\partial_k N_k}{a^2 H} + \mathcal{O}(\epsilon^2) \underline{= \mathcal{O}(\epsilon^2)}$ 【AsympFLRW】

similarly for HL gravity w/d-dim Diff $\frac{\dot{\zeta}}{H} = \mathcal{J} + \mathcal{O}(\epsilon)$

Izumi&Mukohyama (11)
Gumrukcuoglu et al. (11)
Armendariz-Picon et al (10)
Arai, Sibiryakov, Y.U. (18)

spin-0 — Tanaka & Y.U. (2015)

arbitrary integer spin — Tanaka & Y.U. (in progress)

Soft theorem

Soft theorem: WT for dilatation (+ locality)

$$\left(-\sum_{i=1}^{n}\partial_{\boldsymbol{k}_i}\boldsymbol{k}_i+\sum_{\alpha}\mathcal{S}_\alpha\right)\langle\varphi_\pm^{(\mathrm{S})}(t_1,\,\boldsymbol{k}_1)\cdots\varphi_\pm^{(\mathrm{S})}(t_n,\,\boldsymbol{k}_n)\rangle_{\varphi^{(\mathrm{S})}}$$

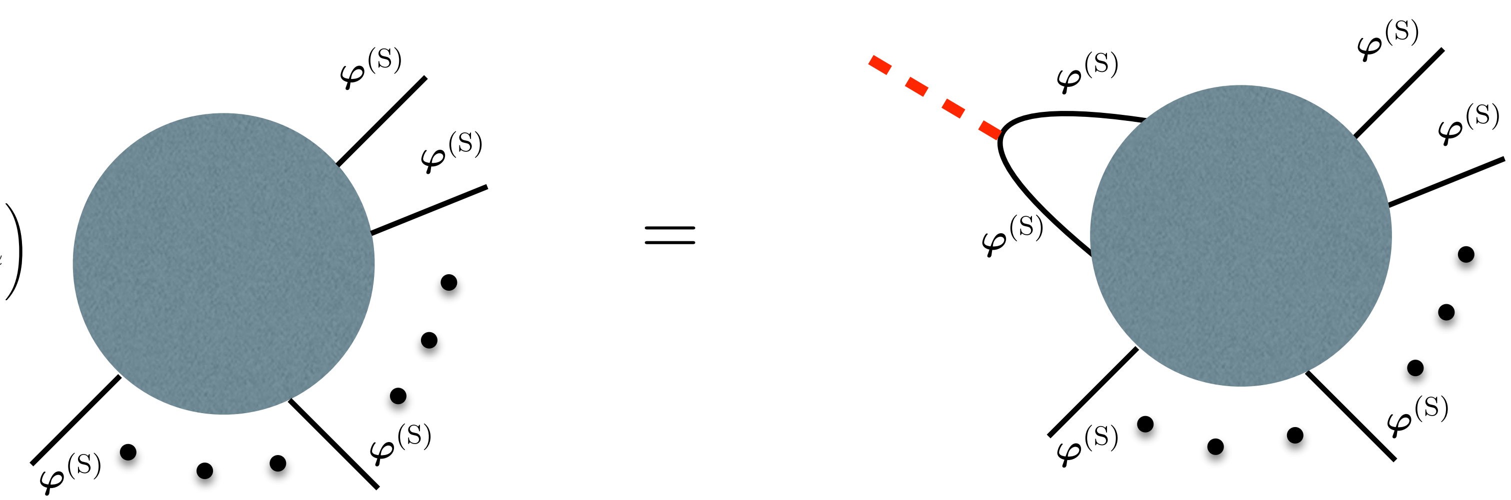
$$=\int dt\left\langle\varphi_\pm^{(\mathrm{S})}(t_1,\,\boldsymbol{k}_1)\cdots\varphi_\pm^{(\mathrm{S})}(t_n,\,\boldsymbol{k}_n)\frac{\delta iS_{\mathrm{int}}[\varphi_+^{(\mathrm{L})},\,\varphi_+^{(\mathrm{S})}]}{\delta\zeta_+^{(\mathrm{L})}(t,\,\boldsymbol{k}_\mathrm{L})}\bigg|_{\varphi_+^{(\mathrm{L})}=0}\right\rangle_{\varphi^{(\mathrm{S})}}-\int dt\left\langle\varphi_\pm^{(\mathrm{S})}(t_1,\,\boldsymbol{k}_1)\cdots\varphi_\pm^{(\mathrm{S})}(t_n,\,\boldsymbol{k}_n)\frac{\delta iS_{\mathrm{int}}[\varphi_-^{(\mathrm{L})},\,\varphi_-^{(\mathrm{S})}]}{\delta\zeta_-^{(\mathrm{L})}(t,\,\boldsymbol{k}_\mathrm{L})}\bigg|_{\varphi_-^{(\mathrm{L})}=0}\right\rangle_{\varphi^{(\mathrm{S})}}$$

$$\zeta(t,\mathbf{k}_\mathrm{L})=\zeta^{\mathrm{WAM}}(\mathbf{k}_\mathrm{L})+\int dt'\,\mathcal{J}(t',\mathbf{k}_\mathrm{L})+\mathcal{O}(\epsilon)$$

Consistency relation

$\varphi^{(\mathrm{S})}$: short (UV) modes

$$\left(\sum_{i=1}^{n}\boldsymbol{x}_i\cdot\partial_{\boldsymbol{x}_i}+\sum_{\alpha}\mathcal{S}_\alpha\right)$$



amputated long (IR) mode

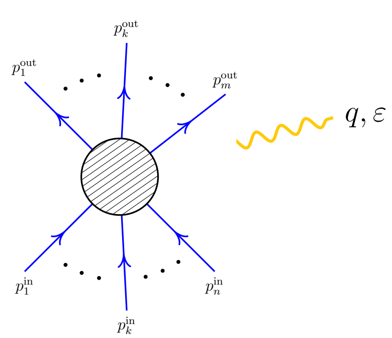
Why do we need additional condition?

Soft theorem ⟶ Consistency relation

(relation for correlators)

★ $$\zeta(t, \mathbf{k}_{\mathrm{L}}) = \zeta^{\mathrm{WAM}}(\mathbf{k}_{\mathrm{L}}) + \int dt' \, \mathcal{J}(t', \mathbf{k}_{\mathrm{L}}) + \mathcal{O}(\epsilon)$$

<u>QFT in asymptotically flat spacetimes</u>

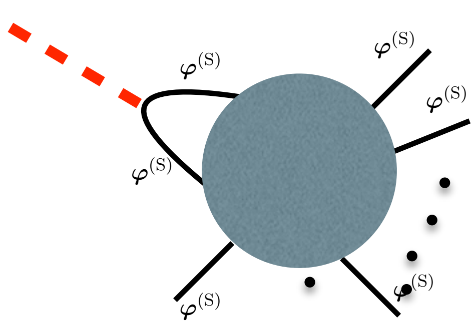


LSZ reduction formula (, Lorentz symmetry)

Inserting soft legs. $\frac{-i}{(p+q)^2+m^2} = \frac{-i}{p^2+2p\cdot q+q^2+m^2} = \frac{-i}{2p\cdot q}$ & $ie\varepsilon^{\mu} 2Q p_{\mu}$

~ BMS

<u>QFT in cosmology</u>



LSZ asymptotic states (, Lorentz symmetry)

only when ★ holds, dilatation describes soft mode insertion

Summary

【Spatial Diff.】

dilatation

【 Asymp FLRW】

- ✕ solid inflation
- ✕ anisotropic inflation

【 Locality】

- ✕ non-BD

Soft theorem

$\not\downarrow$ No LSZ

★

Consistency relation

Existence of const. sol (WAM)

$$\zeta(t,\mathbf{k}_{\mathrm{L}}) = \zeta^{\mathrm{WAM}}(\mathbf{k}_{\mathrm{L}}) + \underline{\int dt'\, \mathcal{J}(t',\mathbf{k}_{\mathrm{L}}) + \mathcal{O}(\epsilon)}$$

- multi-field inflation
- violation of SR

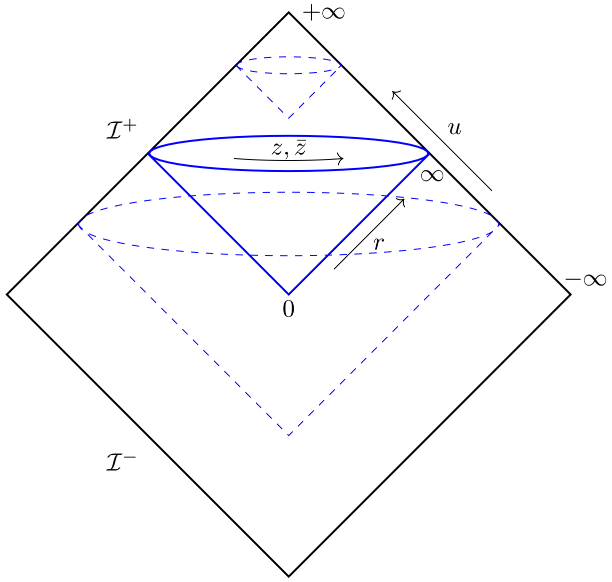
back-up slide

Coordinates at $\mathcal{I}^{\pm}$

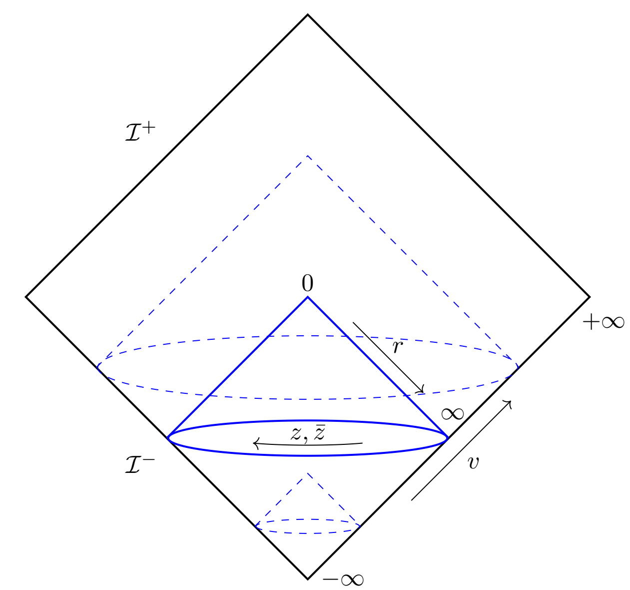
Minkowski space

$$ds^2 = -dt^2 + (d\vec{x})^2$$

$\mathcal{I}^+$



$\mathcal{I}^-$



$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \qquad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$

$$ds^2 = -dv^2 + 2dvdr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$

$$r^2 = (\vec{x})^2\,, \quad u = t - r \qquad x^1 + ix^2 = \frac{2rz}{1+z\bar{z}}\,, \quad x^3 = r\frac{1-z\bar{z}}{1+z\bar{z}}$$

$$v = t + r \qquad x^1 + ix^2 = -\frac{2rz}{1+z\bar{z}}\,, \quad x^3 = -r\frac{1-z\bar{z}}{1+z\bar{z}}$$

For a given z, anti-podal relation between $\mathcal{I}^+$ and $\mathcal{I}^-$.

Existence of const. solution w/ (d+1)-dim Diff

Choosing time slicing $K = dH$ $\longrightarrow$ $N = 1 + \frac{\dot{\zeta}}{H} + \mathcal{O}(\epsilon^2)$

recall δN formalism Starobinsky(82), Sasaki-Stewardt(95)

H-const $\frac{\delta\rho_{\rm tot}}{\bar{\rho}} = \mathcal{O}(\epsilon)$ $\delta\rho_{\rm total} \equiv \delta\rho + \delta\rho_{\rm TT}$ $\delta\rho_{\rm TT} \equiv \frac{A^i{}_j A^j{}_i}{16\pi G}$

M-const Remove one field e.g. scalar field system $\phi^1 = \phi^1(\phi^2, \phi^3, \cdots)$

Tanaka & Y.U. (in progress)

ζ w/o $\mathcal{O}(\epsilon^2)$ appears only through N (or δN) in $\frac{\delta\rho_{\rm tot}}{\bar{\rho}} = \mathcal{O}(\epsilon)$

【Spatial Diff.】 & 【Asymp FLRW】

$\longrightarrow$ $\frac{\dot{\zeta}}{H} = \mathcal{J} + \mathcal{O}(\epsilon)$ also for inhomogeneous ζ 【locality】

Extension to quantum theory

$\boldsymbol{\varphi} \equiv \{\delta g,\, \varphi^\alpha\}$ metric perturbations & matter fields

short modes $\boldsymbol{\varphi}^{(\mathrm{S})}(t,\,\boldsymbol{x}) \equiv \int \frac{d^d\boldsymbol{k}}{(2\pi)^{\frac{d}{2}}}\theta(k-k_c(a))e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,\boldsymbol{\varphi}(t,\,\boldsymbol{k})$

long modes $\boldsymbol{\varphi}^{(\mathrm{L})}(t,\,\boldsymbol{x}) \equiv \boldsymbol{\varphi}(t,\,\boldsymbol{x}) - \boldsymbol{\varphi}^{(\mathrm{S})}(t,\,\boldsymbol{x})$

Stochastic inflation
Starobinsky (86)

Smeared field in gradient exp. corresponds to $\boldsymbol{\varphi}^{(\mathrm{L})}$

Influence functional

Feynman & Vernon (63)
Feynman & Hibbs (65)

$$iS_{\mathrm{eff}}\left[\boldsymbol{\varphi}_+^{(\mathrm{L})},\,\boldsymbol{\varphi}_-^{(\mathrm{L})}\right] \equiv \ln\left[\int D^g\boldsymbol{\varphi}_+^{(\mathrm{S})}\int D^g\boldsymbol{\varphi}_-^{(\mathrm{S})}\; e^{iS_{\mathrm{tot}}[\boldsymbol{\varphi}_+^{(\mathrm{L})},\boldsymbol{\varphi}_+^{(\mathrm{S})}]-iS_{\mathrm{tot}}[\boldsymbol{\varphi}_-^{(\mathrm{L})},\boldsymbol{\varphi}_-^{(\mathrm{S})}]}\right]$$

effective action w/ influence of $\boldsymbol{\varphi}^{(\mathrm{S})}$

Property 2

Arkani-Hamed&Maldacena (15)

$$B_\phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3 = \mathbf{k}_L) = \sum_{\ell=0,2,\ldots} A_\ell \mathcal{P}_\ell(\hat{\mathbf{k}}_L \cdot \hat{\mathbf{k}}_S) \left(\frac{k_L}{k_S}\right)^\Delta P_\phi(k_L) P_\phi(k_S) \left[1 + \mathcal{O}\left(\frac{k_L^2}{k_S^2}\right)\right]$$

$$\Delta_\pm = \frac{3}{2} \pm \sqrt{\left(s - \frac{1}{2}\right)^2 - \frac{m^2}{H^2}} = \frac{3}{2} \pm i\mu$$

- Non-local contributions → Non-analyticity

- Dilution between the Hubble crossing times of k_L and k_S

  → $(k_L / k_S)^{3/2}$

- A_l contains two suppressions

  1) Boltzmann suppression

  2) Weak interaction, suppressed by SR parameters