

TOWARD SUPER-B:  
QED LOGS IN  $B \rightarrow X_{sll}$  & USES OF  $B \rightarrow \tau\nu$

ENRICO LUNGH  
INDIANA UNIVERSITY

SUPER-B, FRASCATI

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# OUTLINE

## (1) Precision calculations of inclusive $b \rightarrow sll$ decays:

- Present status of SM predictions and NP implications
- Electromagnetic corrections: issues and resolution (preliminary)

## (2) UT fit without semileptonic decays:

- Present status of the UT fit and hints for NP
- Issues with  $V_{cb}$  and  $V_{ub}$  from semileptonic decays
- Impact of a determination of  $B \rightarrow \tau \nu$  with super-B precision

# PART I: INCLUSIVE $B \rightarrow SLL$



T. Huber, E.L., M. Misiak and D. Wyler, hep-h/0512066

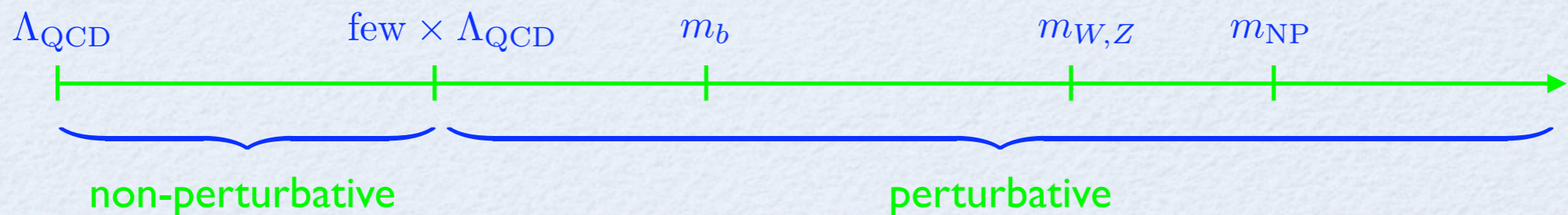
T. Huber, T. Hurth and E.L., arXiv:0712.3009

T. Huber, T. Hurth and E.L., in preparation

[Misiak; Buras, Munz;  
Bobeth, Misiak, Urban;  
Asatryan, Asatrian, Greub, Walker;  
Ghinculov, Hurth, Isidori, Yao;  
Bobeth, Gambino, Gorbahn, Haisch;  
Huber, Lunghi, Misiak, Wyler]

# EFFECTIVE LAGRANGIAN

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \left[ \sum_{i=1}^{10} C_i Q_i + \frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*} \sum_{i=1}^2 C_i (Q_i - Q_i^u) + \underbrace{\sum_{i=3}^6 C_{iQ} Q_{iQ} + C_b Q_b}_{\text{for QED corrections}} \right]$$



- Magnetic & chromo-magnetic

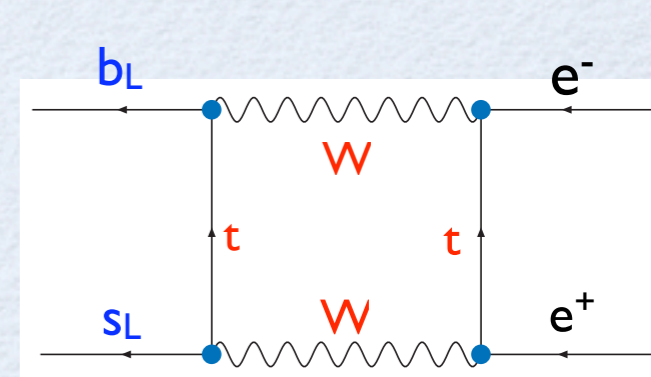
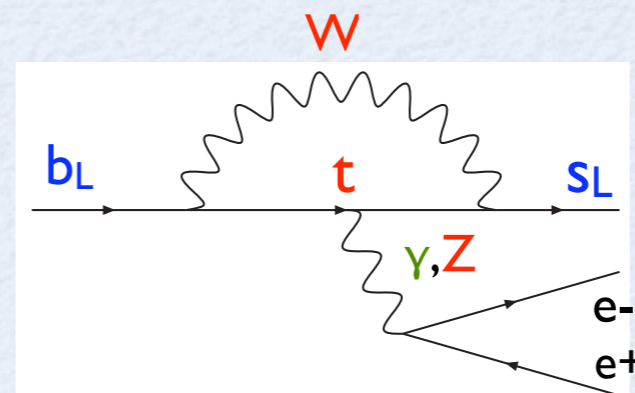
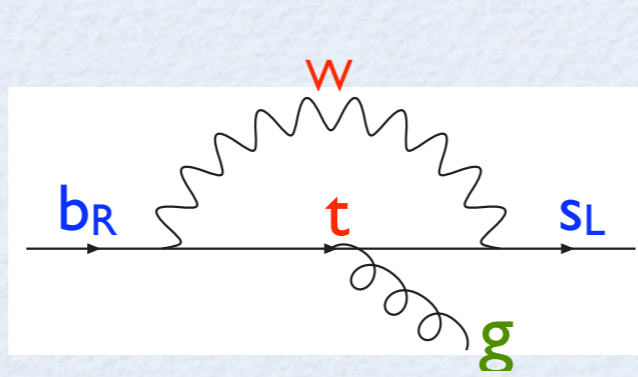
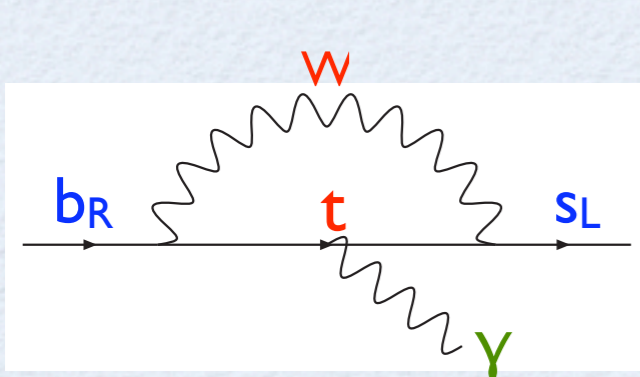
$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

$$Q_8 = \frac{g}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a$$

- Semileptonic

$$Q_9 = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \ell)$$

$$Q_{10} = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$



①

# GENERAL CONSIDERATIONS

$$\Gamma [\bar{B} \rightarrow X_s \ell^+ \ell^-] = \Gamma [\bar{b} \rightarrow X_s \ell^+ \ell^-] + O \left( \frac{\Lambda_{QCD}^2}{m_b^2}, \frac{\Lambda_{QCD}^3}{m_b^3}, \frac{\Lambda_{QCD}^2}{m_c^2}, \dots \right)$$

↓  
*local OPE, optical theorem  
 quark-hadron duality*

↓  
 HQET

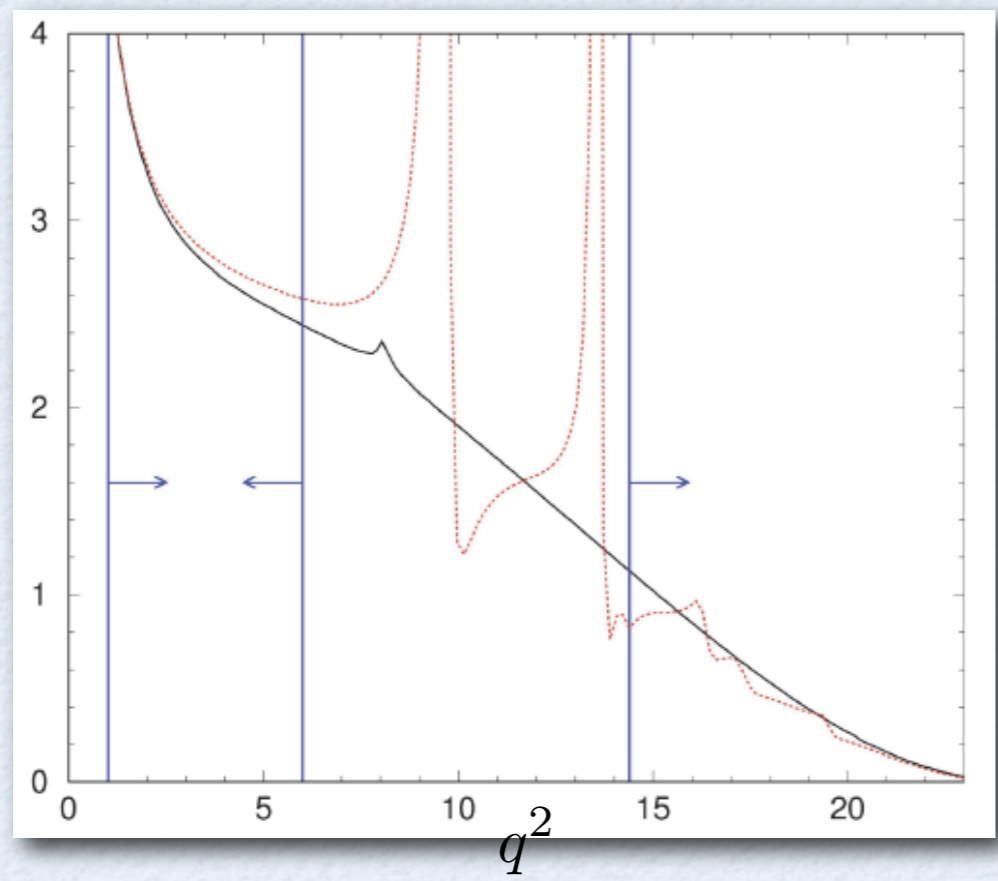
Phase space cuts introduce sensitivity to new scales, the rate becomes less inclusive and new non-perturbative effects appear

(I)  $q^2 = (p_{\ell^+} + p_{\ell^-})^2$  cut to remove  $B \rightarrow X_s (J/\psi, \psi') \rightarrow X_s \ell^+ \ell^-$  background

(II)  $M_{X_s} < [1.8, 2]$  GeV cut to remove the double semileptonic  $b \rightarrow c \ell^- \bar{\nu} \rightarrow s \ell^- \ell^+ \bar{\nu} \nu$  background

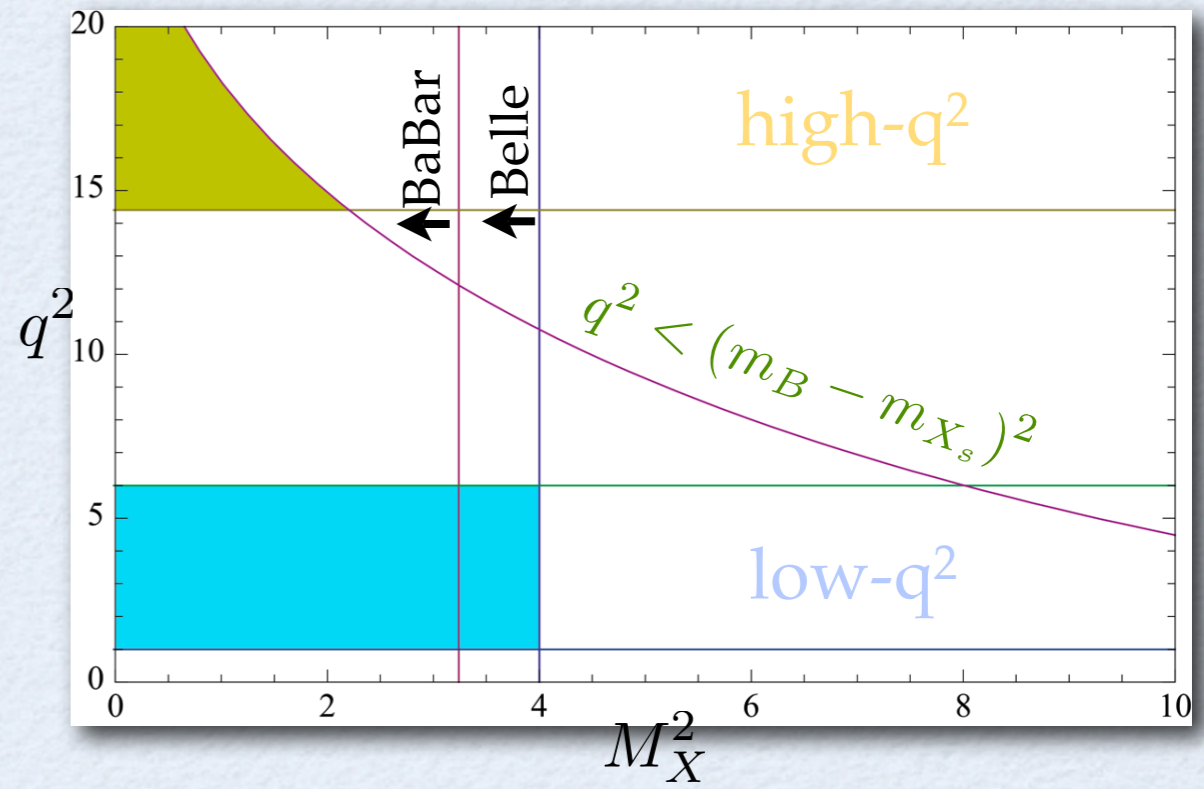
# CUTS

- Di-lepton invariant mass:



- low:  $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$
- high:  $q^2 > 14.4 \text{ GeV}^2$
- Model resonances with data
- Away from resonances expansion in  $1/m_c^2$

- Hadronic invariant mass:

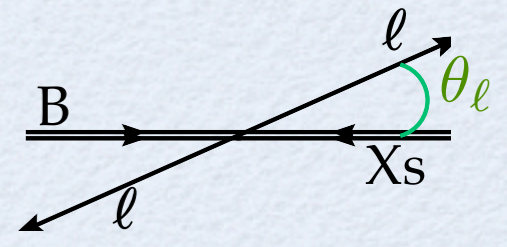


- high- $q^2$  region unaffected
- Experiments correct using Fermi motion model
- Leading power SCET suggests cuts are universal (same for  $b \rightarrow sll$  and  $b \rightarrow ulv$ )

# STATUS IN THE SM

- Known at NNLO in QCD and NLO in QED
- Double differential rate:

$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} \left[ (1+z^2)H_T(q^2) + 2zH_A(q^2) + 2(1-z^2)H_L(q^2) \right]$$



$$\frac{d\Gamma}{dq^2} = H_T + H_L \quad \frac{d\mathcal{A}_{FB}}{dq^2} = \frac{3}{4}H_A \quad z = \cos \theta_\ell$$

$$H_T \propto (1-\hat{s})^2 \hat{s} \left[ \left( C_9 + \frac{2}{\hat{s}} C_7 \right)^2 + C_{10}^2 \right]$$

$$H_L \propto (1-\hat{s})^2 \left[ (C_9 + 2C_7)^2 + C_{10}^2 \right]$$

➔

Independent combinations of WC's

- Standard approach: normalization to the full  $B \rightarrow X_u \ell \nu$  rate
- At high- $q^2$  it is convenient to normalize to the  $B \rightarrow X_u \ell \nu$  rate with

the same  $q^2$  cut:  $\mathcal{R}(s_0) = \int_{s_0}^1 d\hat{s} \frac{d\Gamma(\bar{B} \rightarrow X_s \ell^+ \ell^-)}{d\hat{s}} / \int_{s_0}^1 d\hat{s} \frac{d\Gamma(\bar{B}^0 \rightarrow X_u \ell \nu)}{d\hat{s}}$

# STATUS IN THE SM

- SM predictions for the branching ratios:

$$\mathcal{B}_{\mu\mu}^{\text{low}} = (1.59 \pm 0.14) \times 10^{-6}$$

$$\mathcal{B}_{\mu\mu}^{\text{high}} = (2.4 \pm 0.7) \times 10^{-7}$$

$$\mathcal{B}_{ee}^{\text{low}} = (1.64 \pm 0.14) \times 10^{-6}$$

$$\mathcal{B}_{ee}^{\text{high}} = (2.1 \pm 0.6) \times 10^{-7}$$

scale,  $\alpha_s/m_b, m_t$

$m_b^{-3}$  parameters, scale

- New normalization:  $\mathcal{R}(14.4\text{GeV}^2)_{\mu\mu} = 2.29 \times 10^{-3} (1 \pm 0.13)$   
 $\mathcal{R}(14.4\text{GeV}^2)_{ee} = 1.94 \times 10^{-3} (1 \pm 0.16)$   $\rightarrow V_{ub}$

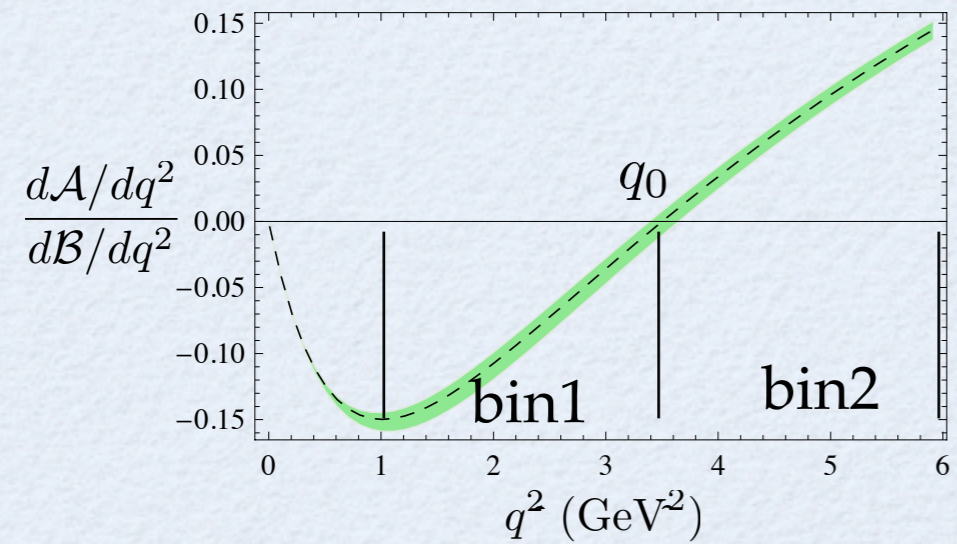
- Experimental world averages:

$$(\mathcal{B}_{ll}^{\text{low}})_{\text{exp}} = (1.60 \pm 0.51) \times 10^{-6}$$

$$(\mathcal{B}_{ll}^{\text{high}})_{\text{exp}} = (4.4 \pm 1.2) \times 10^{-7}$$

- Forward-Backward asymmetry:

NNLO + QED



$$(q_0^2)_{\mu\mu} = (3.50 \pm 0.12) \text{ GeV}^2$$

$$(q_0^2)_{ee} = (3.38 \pm 0.11) \text{ GeV}^2$$

$$(\bar{\mathcal{A}}_{\mu\mu})_{\text{bin1}} = [-9.1 \pm 0.9]\%$$

$$(\bar{\mathcal{A}}_{ee})_{\text{bin1}} = [-8.1 \pm 0.9]\%$$

$$(\bar{\mathcal{A}}_{\mu\mu})_{\text{bin2}} = [7.8 \pm 0.8]\%$$

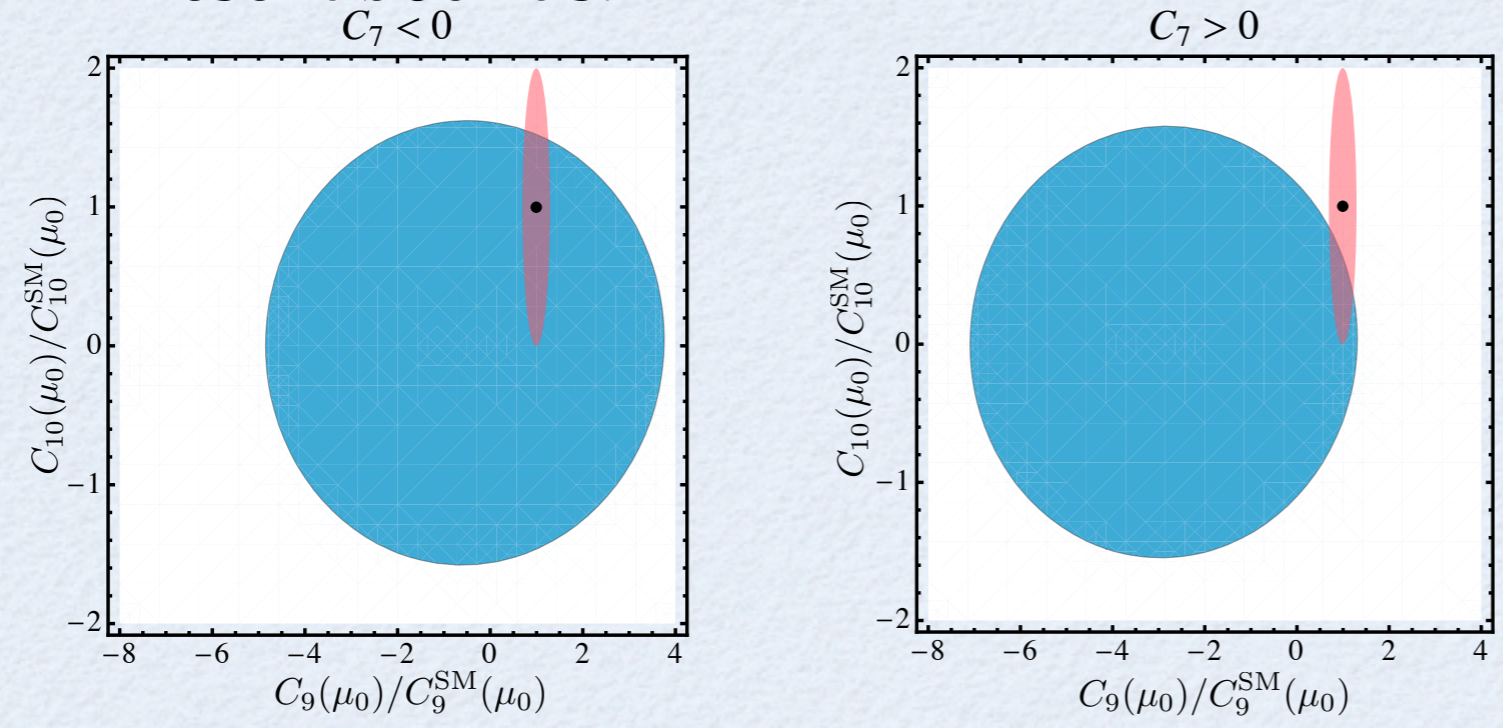
$$(\bar{\mathcal{A}}_{ee})_{\text{bin2}} = [8.3 \pm 0.6]\%$$

dominant uncertainty: scale



# MODEL INDEPENDENT NP REACH

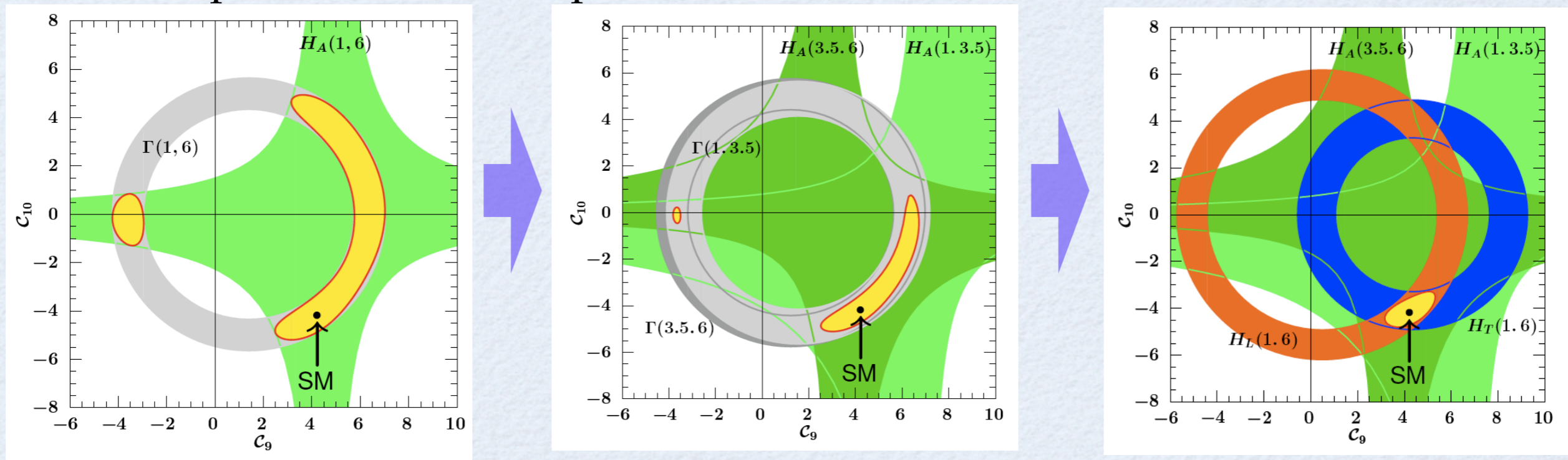
● Present bounds:



- $C_7$  constrained by  $b \rightarrow s\gamma$
- $C_7 > 0$  excluded in MFV
- pink ellipses are allowed ranges in the most general MSSM ( $(\delta_{23}^{d,u})_{LL,RR} \neq 0$ )

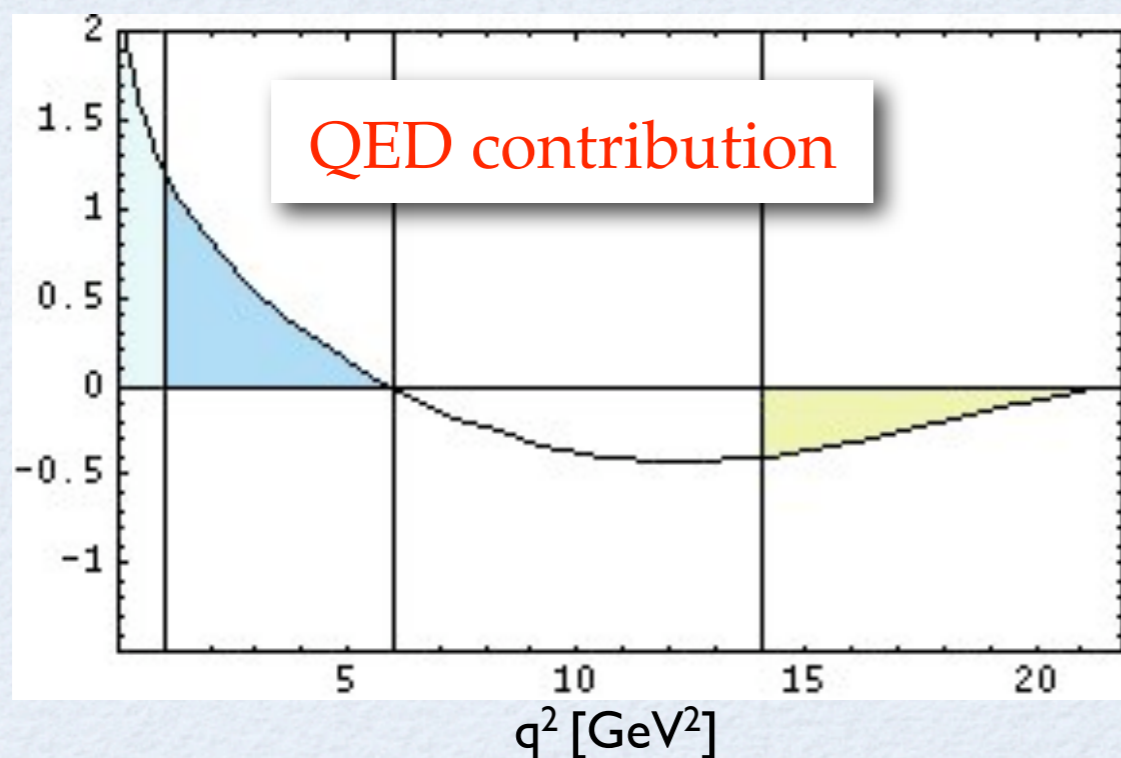
● Future possibilities (extrapolated to  $1 \text{ ab}^{-1}$ ):

[Lee, Ligeti, Stewart, Tackmann]



# QED LOGS: OVERVIEW

- The *rate is proportional to*  $\alpha_{em}^2(\mu^2)$ . Without QED corrections the scale  $\mu$  is undetermined  $\rightarrow \pm 4\%$  uncertainty
- Focus on corrections enhanced by large logarithms:
  - $\alpha_{em} \log(m_W/m_b) \sim \alpha_{em}/\alpha_s$  [WC, RG running] [Bobeth, Gambino, Gorbahn, Haisch]
  - $\alpha_{em} \log(m_\ell/m_b)$  [Matrix Elements]
- The differential rate is not IR safe with respect to photon emission the results in the presence of a physical collinear logarithm,  $\log(m_\ell/m_b)$



$$\text{virtual} = \frac{A_{\text{soft+collinear}}}{\epsilon^2} + \frac{B_{\text{collinear}} + B_{\text{soft}}}{\epsilon} + C$$

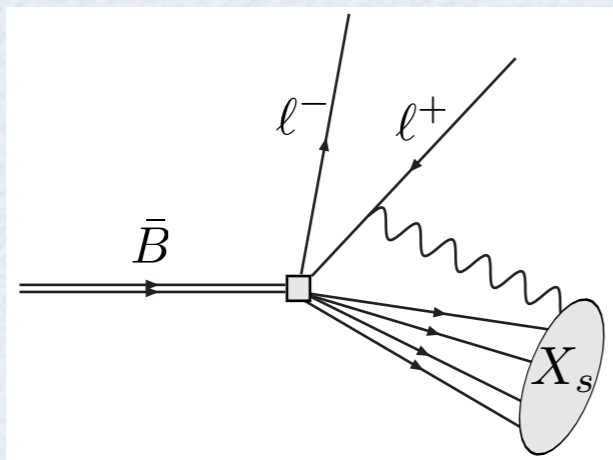
$$\text{real} = -\frac{A_{\text{soft+collinear}}}{\epsilon^2} - \frac{B'_{\text{collinear}} + B_{\text{soft}}}{\epsilon} + C'$$

$$\int dq^2 (B_{\text{collinear}} - B'_{\text{collinear}}) = 0$$

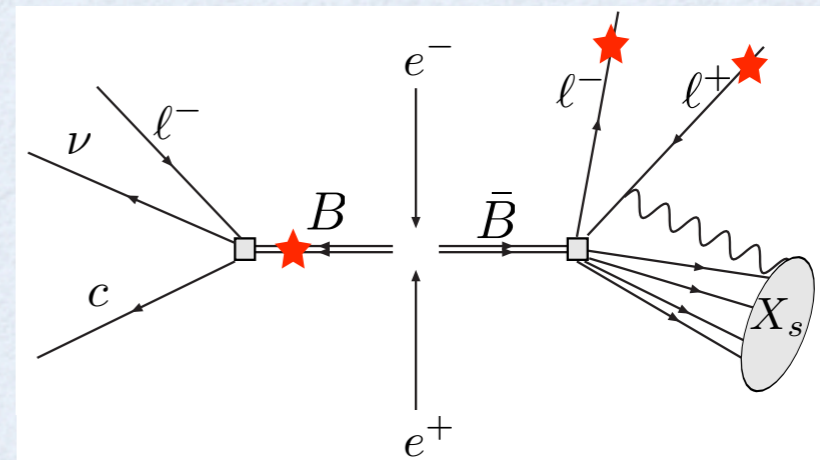
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# COMPARISON TO EXPERIMENTS

- *Theory*  
include all bremsstrahlung photons into the  $X_s$  system:



- *Experiment (fully inclusive, Super-B only)*  
One B is identified; on the other side only the two leptons are reconstructed:



- *Experiment ( $X_s$  system reconstructed as a sum over exclusive states):*  
At BaBar (Belle) photons with energies smaller than 30 (20) MeV are not resolved. Photons emitted inside a small cone (35x50 mrad) around the electrons are identified and included in the event reconstruction. Events with any other photon ( $E > 30$  (20) MeV and outside of the cone) are vetoed.

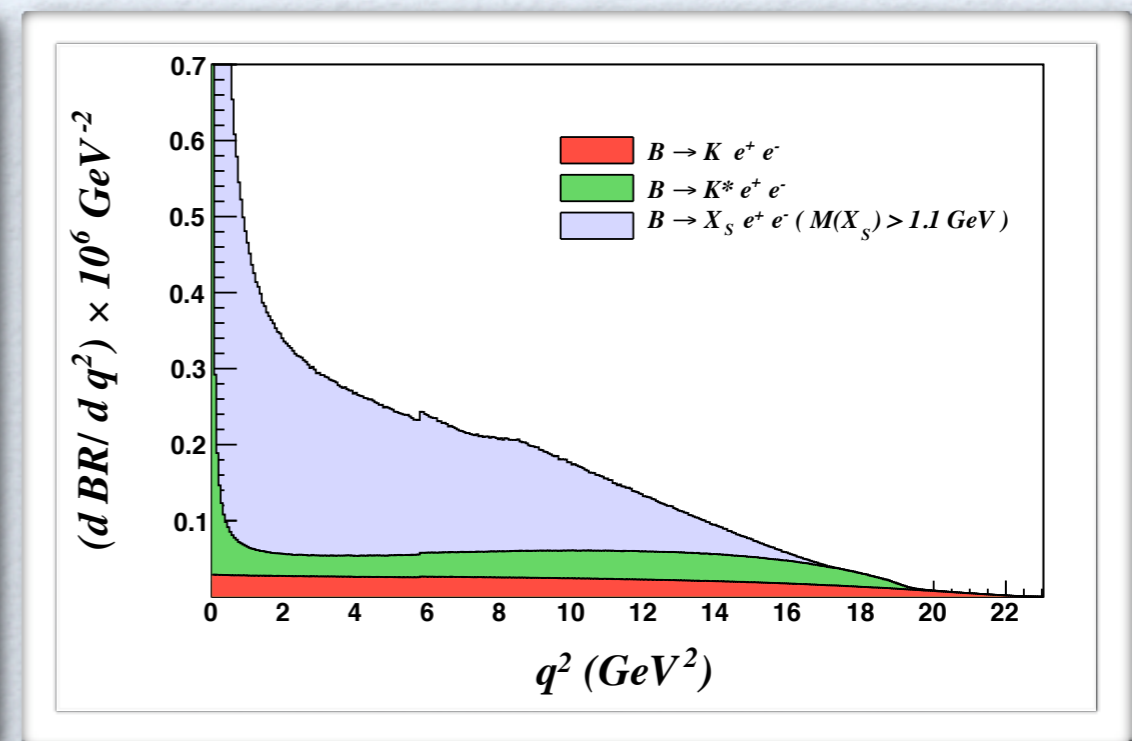
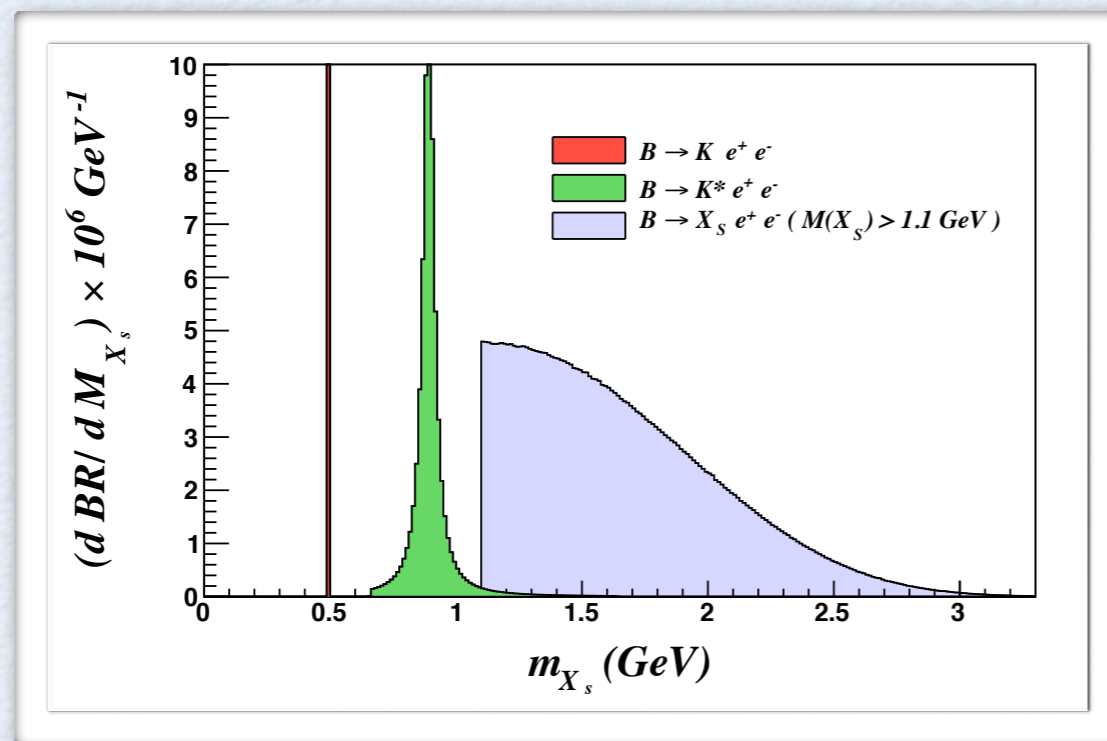
**Note:** at BaBar (Belle) photons inside the cone are (are not) included in the definition of the  $q^2$

- Measured rates are sensitive to the *soft photon cutoff* and to the *size of the cone*

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# COMPARISON TO EXPERIMENTS

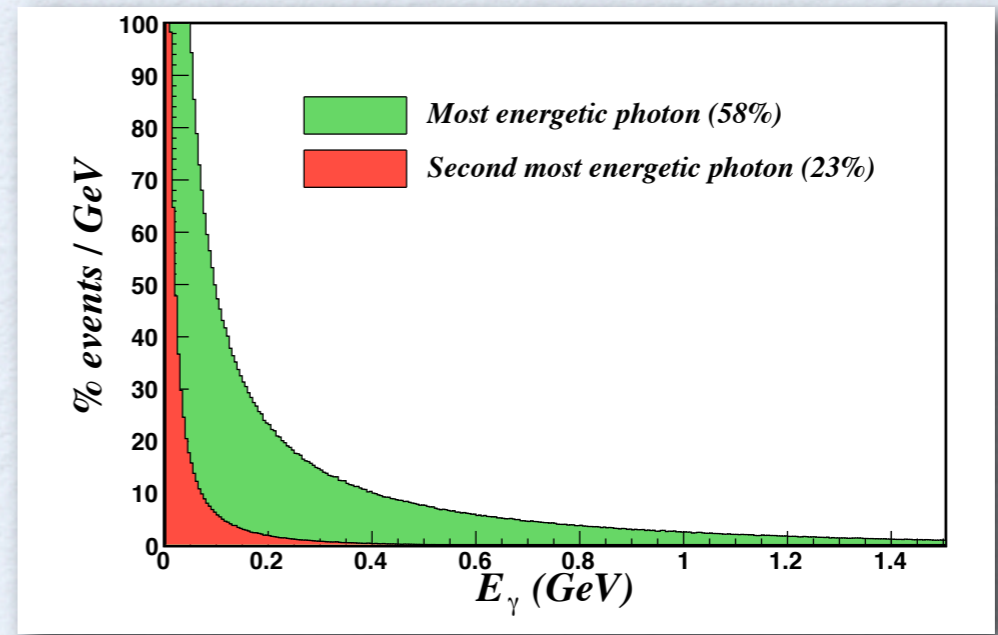
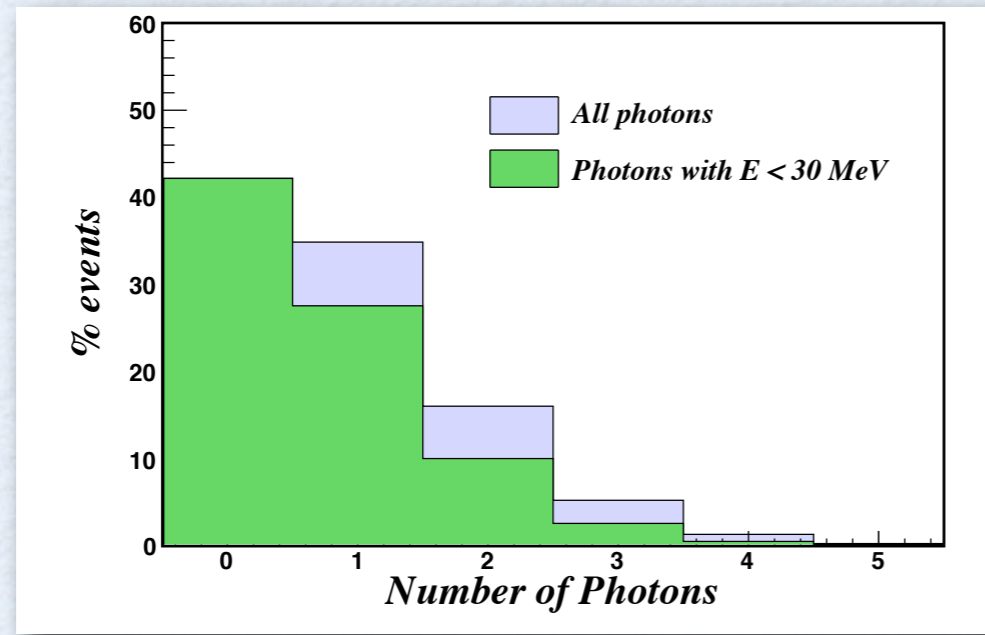
- Use BaBar  $b \rightarrow sll$  MonteCarlo to study the effects of the photon cuts [Many thanks to Kevin Flood, Owen Long and Chris Schilling]
- Inclusive  $b \rightarrow sll$  events are obtained combining fully inclusive events with  $M_{X_s} > 1.1$  GeV with  $B \rightarrow K^{(*)} ll$  exclusive samples to cover the low  $M_{X_s}$  (high  $q^2$  region):



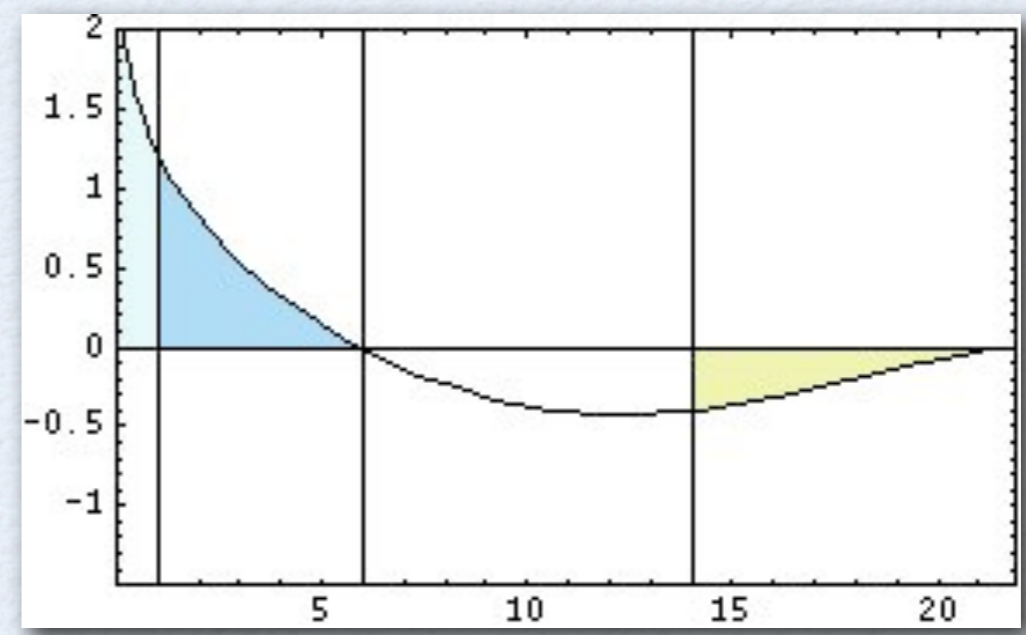
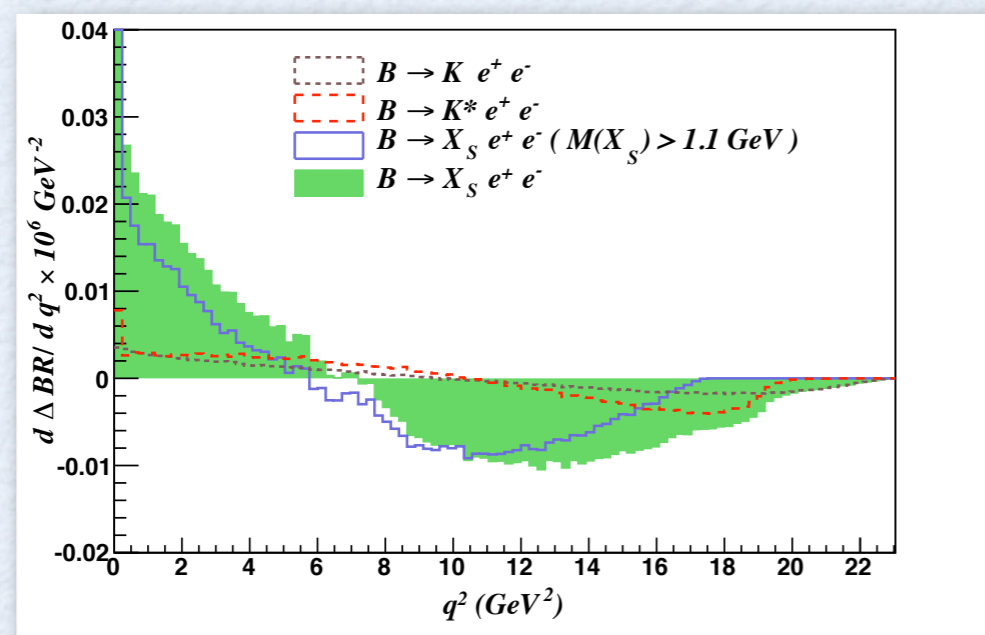
- Spectator effects are described using a Fermi motion model [Ali,Hiller]
- The  $X_s$  system is hadronized with JetSet
- Photons are modeled using PHOTOS (hard cut-off for  $E_\gamma = 150 \text{ eV}$ )

# COMPARISON TO EXPERIMENTS

- PHOTOS generates photons with large multiplicity. Most of the energy is carried by a single photon:



- Comparison between event sets generated with and without PHOTOS yields:



# NUMERICAL RESULTS

- Impact of real photon emission on integrated observables (in bracket the analytical result - without charmonium effects):

	$\mu\mu$	$ee$
low $q^2$	+1.5% (+2.0%)	+3.6% (+5.2%)
high $q^2$	-4.4% (-6.8%)	-12.9% (-17.6%)

- Good agreement taking into account that the MC sample contains events with multiple photon emissions
- Inclusion of the leading photon into the  $q^2$  leads to integration over IR safe regions of the phase space and log-enhanced QED corrections vanish
- The weights of the K and K\* samples relatively to the Xs ( $M_{X_s} > 1.1$  GeV) one have to be supplemented externally (not built in the MC). We find:

$$X_K^{ee} = (8 \pm 2)\% \quad X_K^{\mu\mu} = (11 \pm 2)\%$$

$$X_{K^*}^{ee} = (15 \pm 3)\% \quad X_{K^*}^{\mu\mu} = (25 \pm 4)\%$$

$$X_{X_s(M_{X_s} > 1.1\text{GeV})}^{\ell\ell} = 100\% - X_K^{\ell\ell} - X_{K^*}^{\ell\ell}$$

# INTERPRETATION

- Fully inclusive (analytical) vs sum over exclusive (BaBar / Belle):

	low( $\mu\mu$ )	low( $ee$ )	high( $\mu\mu$ )	high( $ee$ )
Fully inclusive	$1.59 \times 10^{-6}$	$1.64 \times 10^{-6}$	$2.4 \times 10^{-7}$	$2.1 \times 10^{-7}$
BaBar	$-(7.5 \pm 0.2)\%$	$-(13.5 \pm 0.2)\%$	$-(5.2 \pm 0.1)\%$	$-(0.7 \pm 0.3)\%$
Belle	$-(8.4 \pm 0.2)\%$	$-(13.4 \pm 0.2)\%$	$-(6.2 \pm 0.1)\%$	$-(8.1 \pm 0.1)\%$

- We have checked that inclusion of the second most energetic photon in the cone does not affect the above results
- Effect induced by the soft photon cut (30 / 20 MeV for BaBar / Belle)
- Order of magnitude of the effect agrees with Sudakov double log:

$$\sigma_{\text{measured}} \simeq \sigma_0 \left( 1 - \frac{\alpha_{\text{em}}}{\pi} \log \frac{m_b^2}{m_\ell^2} \log \frac{m_b^2}{E_{\text{cut}}^2} \right) \Rightarrow \sigma_0 \left| \exp \left[ - \frac{\alpha_{\text{em}}}{2\pi} \log \frac{m_b^2}{m_\ell^2} \log \frac{m_b^2}{E_{\text{cut}}^2} \right] \right|^2$$

- The MC sample contains events with up to 12 photons: Sudakov resummation is effectively implemented

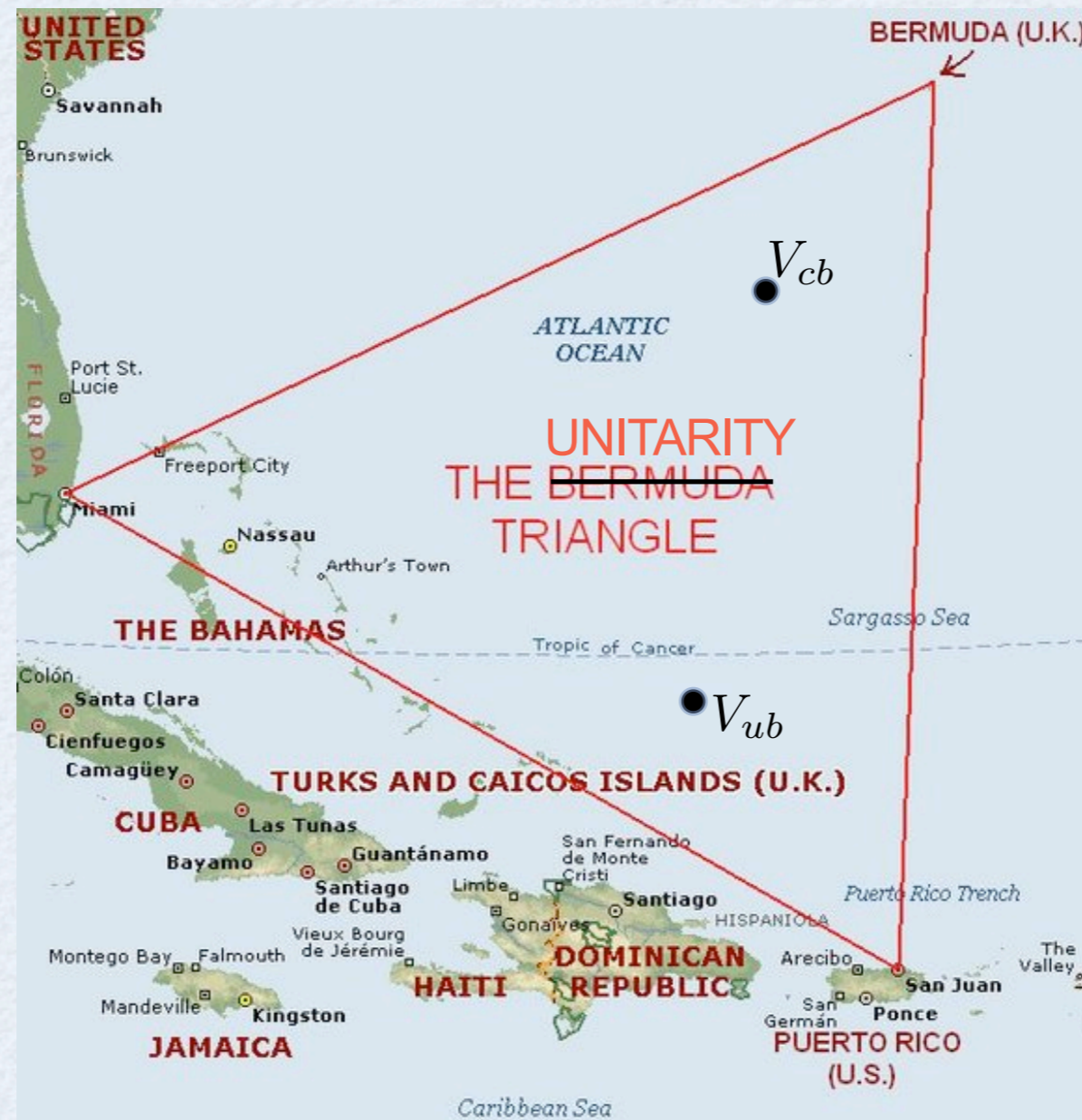
# ALTERNATIVE SELECTION

- Previous results obtained using the following cuts:
  - $\mu^+\mu^-$  final state  $\rightarrow E_\gamma < 30$  MeV
  - $e^+e^-$  final state  $\rightarrow E_\gamma < 30$  MeV or  $\gamma_1$  in a (35,50) mrad cone around  $e^\pm$
- Alternative possibility:
  - Construct  $m_{ES} = \sqrt{E_{beam}^2 - \vec{P}_B^2}$  and  $\Delta E = E_B - E_{beam}$  for each event
  - Impose  $m_{ES} > 5.2$  GeV and  $|\Delta E| < 100$  MeV
  - Note that  $p_B = (E_B, \vec{P}_B)$  is the reconstructed B meson momentum in the colliding electrons COM frame
    - $\mu^+\mu^-$  final state  $\rightarrow p_B = p_{X_s} + p_{\mu^+} + p_{\mu^-}$
    - $e^+e^-$  final state  $\rightarrow p_B = p_{X_s} + p_{e^+} + p_{e^-} + p_\gamma \rightarrow E_\gamma > 30$  MeV & in the cone

	low( $\mu\mu$ )	low( $ee$ )	high( $\mu\mu$ )	high( $ee$ )
Fully inclusive	$1.59 \times 10^{-6}$	$1.64 \times 10^{-6}$	$2.4 \times 10^{-7}$	$2.1 \times 10^{-7}$
BaBar (alternative)	$-(5.5 \pm 0.2)\%$	$-(10.4 \pm 0.2)\%$	$-(2.7 \pm 0.1)\%$	$-(2.8 \pm 0.3)\%$
Belle (alternative)	$-(5.5 \pm 0.2)\%$	$-(8.9 \pm 0.2)\%$	$-(2.7 \pm 0.1)\%$	$-(3.5 \pm 0.1)\%$



# PART 2: UT WITHOUT SEMILEPTONIC DECAYS



E.L. and A. Soni, arXiv:0903.5059

J. Laiho, E.L., R. van de Water, arXiv:0910.2928

E.L. and A. Soni, arXiv:0912.0002

Pay tribute to:

UTfit

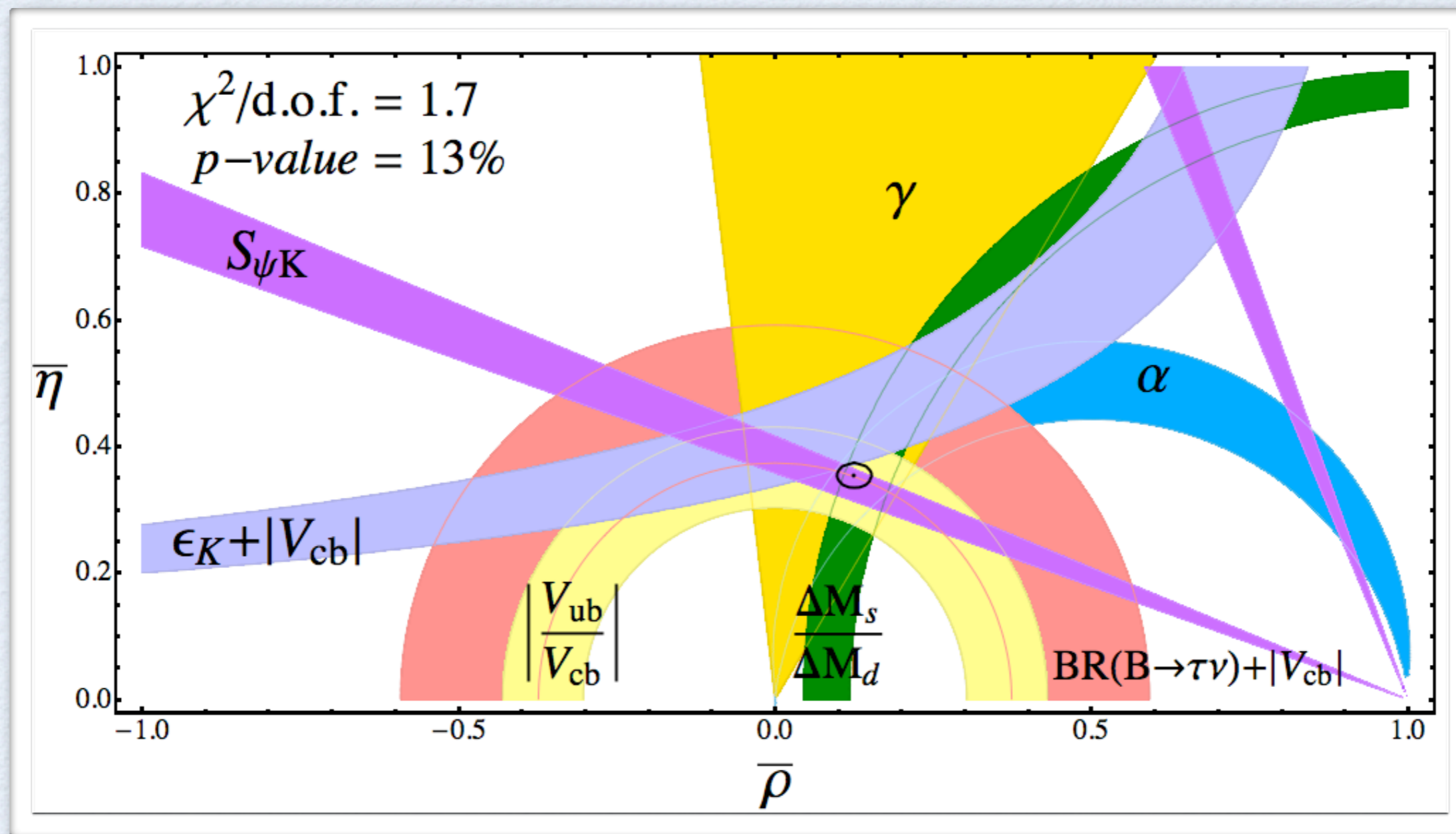
CKMfitter

Buras, Guadagnoli

...

# UNITARITY TRIANGLE

- Present status of the Unitarity Triangle fit:



- Note the ubiquitous use of  $|V_{cb}|$  for the individual constraints

# STATUS OF $V_{XB}$

- Presently we have discrepancies at the (1-2) $\sigma$  level between exclusive and inclusive determinations of  $|V_{cb}|$  and  $|V_{ub}|$

- $b \rightarrow c$ :

$$|V_{cb}|_{\text{excl}} = (38.6 \pm 1.2) \times 10^{-3}$$

$$|V_{cb}|_{\text{incl}} = (41.31 \pm 0.76) \times 10^{-3}$$

- Error on exclusive  $V_{cb}$  from  $D^*$  data rescaled to take into account bad experimental  $\chi^2$
- Inclusive calculation quite mature (NNLO and  $\alpha_s/m_b^2$  not in fit yet)
- Issues with violation of quark-hadron duality (D and  $D^*$  represent 70% of the spectrum)

- $b \rightarrow u$ :

$$|V_{ub}|_{\text{excl}} = (34.2 \pm 0.37) \times 10^{-4}$$

$$|V_{ub}|_{\text{incl}} = (40.3 \pm 1.5_{-2.5}^{+2.0}) \times 10^{-4}$$

- Inclusive calculation is sensitive to non-local effects (Shape function)
- New NNLO corrections seem to push the  $|V_{ub}|$  to higher central values (!)

# REMOVING $V_{XB}$

- $|V_{ub}|$  not essential to the fit (main effect is to favor NP in  $\varepsilon_K$  rather than in  $B_d$  mixing)
- $|V_{cb}|$  seems essential to  $\varepsilon_K$ ,  $B \rightarrow \tau\nu$  and  $\Delta M_{B_s}$ :

$$\Delta M_{B_s} = \chi_s f_{B_s}^2 \hat{B}_{B_s} A^2 \lambda^4$$

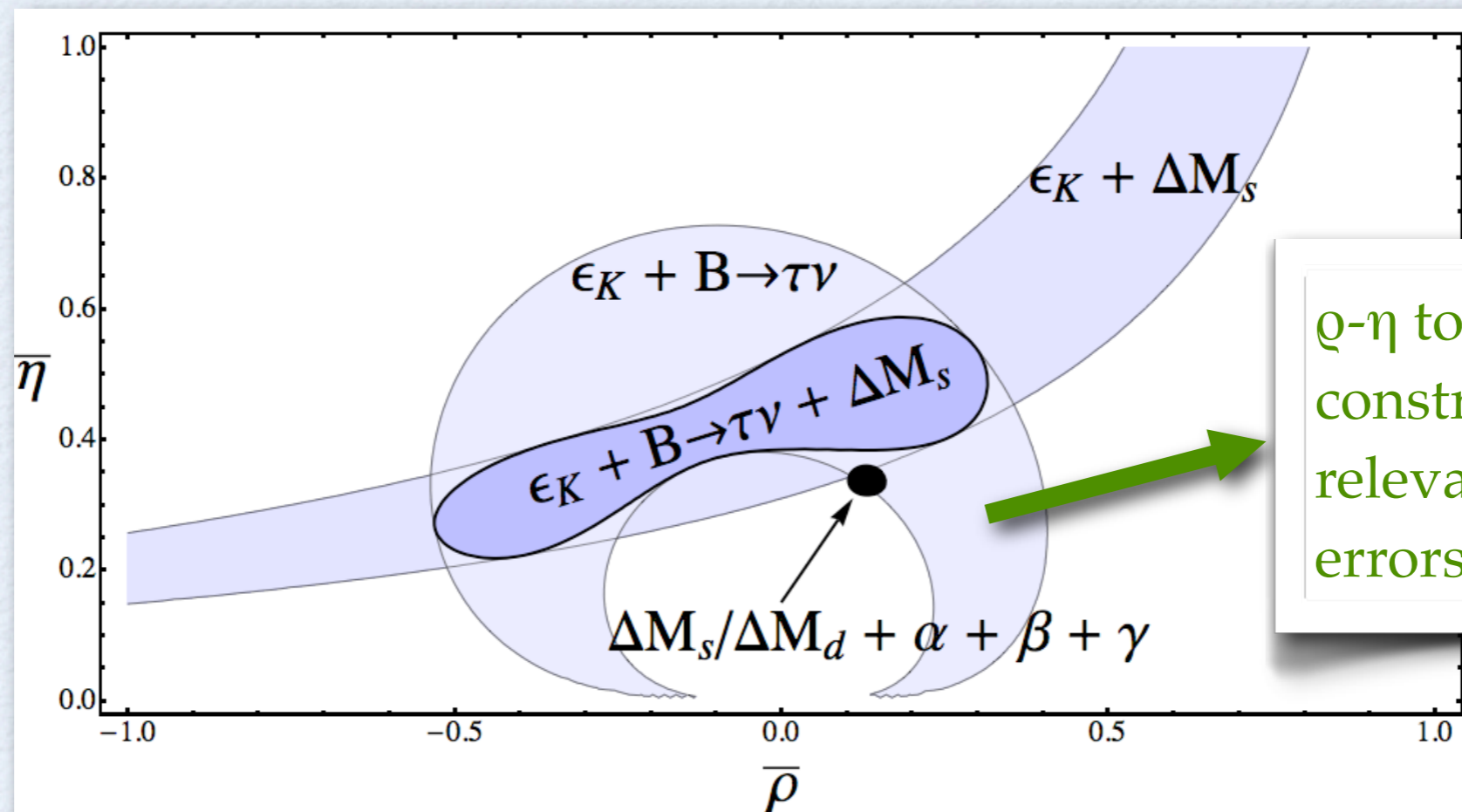
$$|\varepsilon_K| = 2\chi_\varepsilon \hat{B}_K \kappa_\varepsilon \eta \lambda^6 \left( A^4 \lambda^4 (\rho - 1) \eta_2 S_0(x_t) + A^2 (\eta_3 S_0(x_c, x_t) - \eta_1 S_0(x_c)) \right)$$

$$\text{BR}(B \rightarrow \tau\nu) = \chi_\tau f_B^2 A^2 \lambda^6 (\rho^2 + \eta^2)$$

- The interplay of these constraints allows to drop  $V_{cb}$ , while still constraining new physics in K mixing:

$$|\varepsilon_K| \propto \hat{B}_K (f_{B_s} \hat{B}_s^{1/2})^{-4} f(\rho, \eta)$$

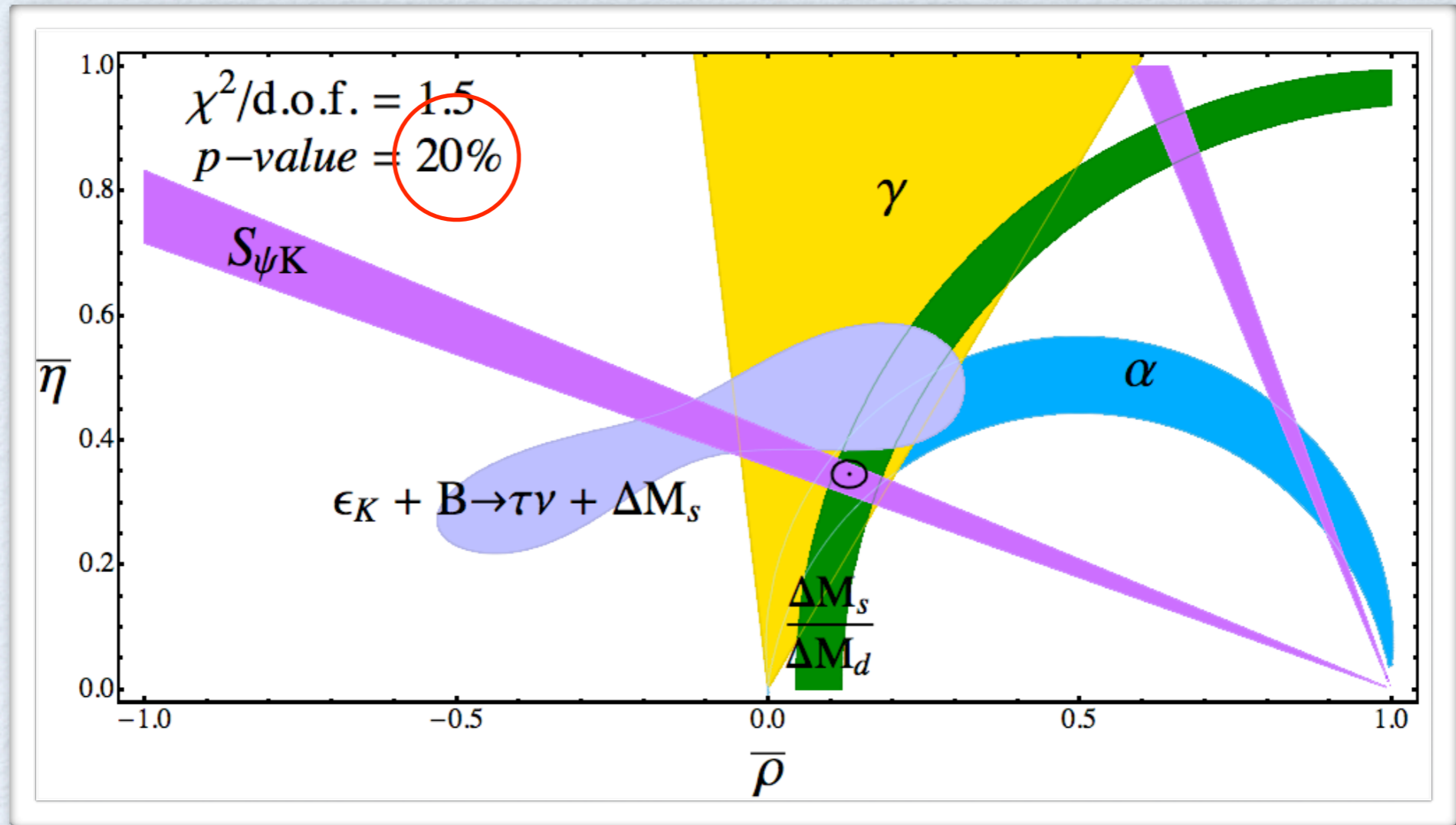
$$|\varepsilon_K| \propto \hat{B}_K \text{BR}(B \rightarrow \tau\nu)^2 f_B^{-4} g(\rho, \eta)$$

REMOVING  $V_{XB}$ 

$\varrho$ - $\eta$  topology of the constraint makes it relevant despite large errors on  $B \rightarrow \tau\nu$

$X :$	$\widehat{B}_K$	$ V_{cb} $	$f_{B_s} \widehat{B}_s^{1/2}$	$\text{BR}(B \rightarrow \tau\nu)$	$f_B$
$\delta X :$	4%	2.5%	5%	26%	5%
$\delta\epsilon_K :$	4%	10%	20%	52%	20%

# FIT W/OOUT SEMILEPTONIC DECAYS



$$\begin{aligned}
 |\epsilon_K^{\text{NP}}| &= C_\epsilon |\epsilon_K^{\text{SM}}| \\
 M_{12}^{d,\text{NP}} &= e^{i\theta_d} M_{12}^{d,\text{SM}} \\
 \text{BR}(B \rightarrow \tau \nu)^{\text{NP}} &= r_H \text{BR}(B \rightarrow \tau \nu)^{\text{SM}}
 \end{aligned}$$

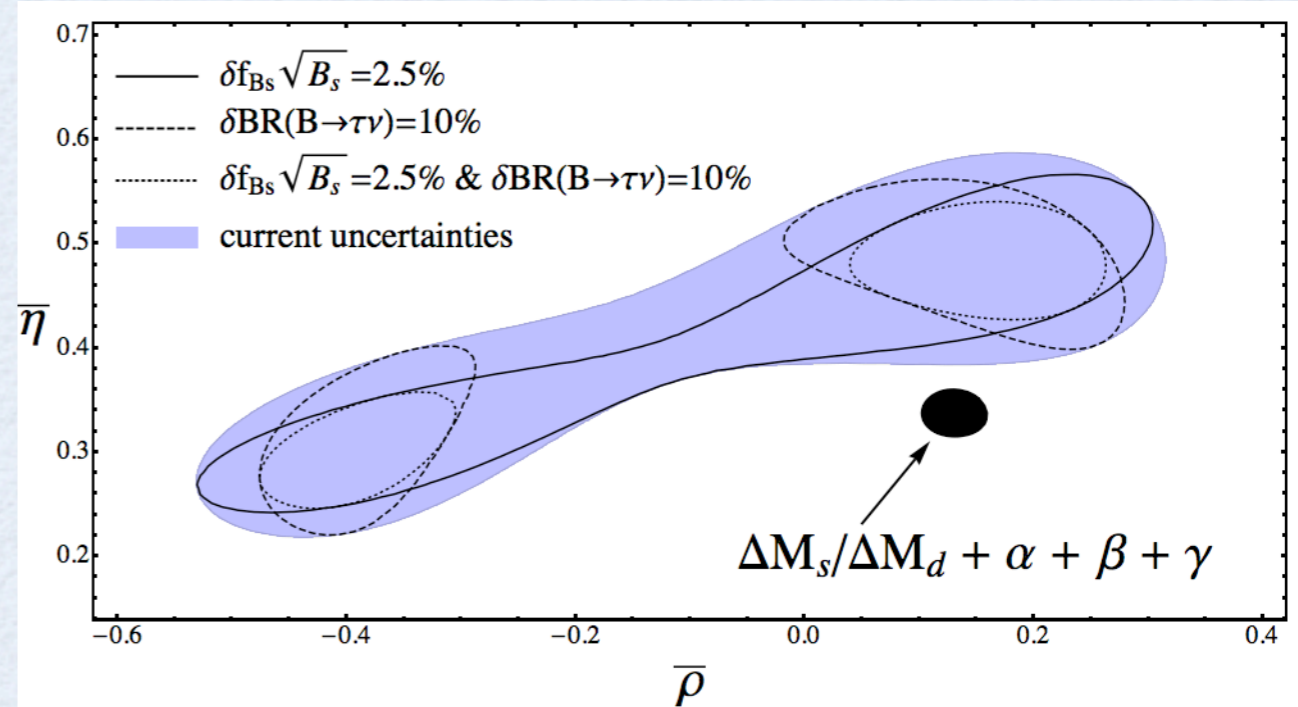


$$\begin{aligned}
 C_\epsilon^{\text{no}V_{qb}} &= 1.30 \pm 0.23 \Rightarrow (1.3\sigma, p = 40\%) \\
 \theta_d^{\text{no}V_{qb}} &= -(8.9 \pm 4.2)^\circ \Rightarrow (2.1\sigma, p = 89\%) \\
 r_H^{\text{no}V_{qb}} &= 1.7 \pm 0.5 \Rightarrow (1.4\sigma, p = 43\%)
 \end{aligned}$$

# SUPERB EXPECTATIONS

$$\delta_\tau = \delta\text{BR}(B \rightarrow \tau\nu) \quad \delta_s = \delta(f_{B_s} \sqrt{B_s})$$

$\delta_\tau$	$\delta_s$	$p_{\text{SM}}$	$\theta_d \pm \delta\theta_d$	$p_{\theta_d}$	$\theta_d/\delta\theta_d$
*26%	*5%	20%	$-(8.9 \pm 4.2)^\circ$	89%	$2.1\sigma$
*26%	2.5%	3.5%	$-(9.6 \pm 3.5)^\circ$	88%	$2.7\sigma$
*26%	1%	0.1%	$-(10.1 \pm 2.9)^\circ$	87%	$3.4\sigma$
10%	*5%	1%	$-(8.8 \pm 2.7)^\circ$	89%	$3.3\sigma$
3%	*5%	0.04%	$-(8.8 \pm 2.2)^\circ$	89%	$4.0\sigma$
10%	2.5%	0.1%	$-(9.2 \pm 2.5)^\circ$	88%	$3.7\sigma$
10%	1%	0.004%	$-(9.6 \pm 2.2)^\circ$	86%	$4.4\sigma$
3%	2.5%	0.004%	$-(9.1 \pm 2.1)^\circ$	88%	$4.4\sigma$
3%	1%	0.0001%	$-(9.4 \pm 1.9)^\circ$	86%	$5.0\sigma$



- Even modest improvements on  $B \rightarrow \tau\nu$  have tremendous impact on the UT fit ( $10 \text{ ab}^{-1} \rightarrow 10\%$ ;  $50 \text{ ab}^{-1} \rightarrow 3\%$ )
- Interplay with reduced errors on  $B_s$  mixing can produce a  $5\sigma$  effect
- **Fit is completely clean**

# CONCLUSIONS

## (1) Inclusive $b \rightarrow sll$ decays

- Calculations are approaching the “end-of-the-road”
- Electromagnetic corrections: effect of BaBar & Belle treatment of soft and collinear photons *seems* to have very large impact (7-13%)
- QED effects on  $H_T$  and  $H_L$  ( $\Gamma = H_T + H_L$ ) are at the top of the TODO list

## (2) UT fit without semileptonic decays

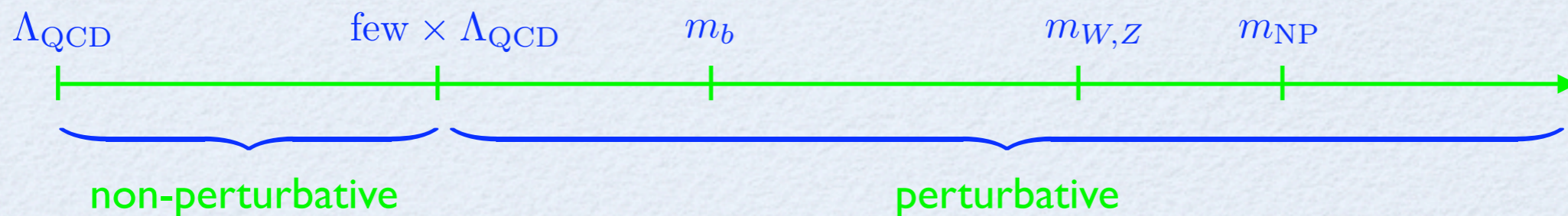
- As long as  $V_{xb}$  determinations remain problematic, removing semileptonic decays allows to cast the UT fit as a clean & high-precision tool to identify new physics
- Super-B level precision on  $B \rightarrow \tau\nu$  coupled with improvements on the lattice determination of  $f_{B_s} \hat{B}_s^{1/2}$  can test the SM at the  $5\sigma$  level



BACKUP SLIDES

# EFFECTIVE LAGRANGIAN

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \left[ \sum_{i=1}^{10} C_i Q_i + \frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*} \sum_{i=1}^2 C_i (Q_i - Q_i^u) + \underbrace{\sum_{i=3}^6 C_{iQ} Q_{iQ} + C_b Q_b}_{\text{for QED corrections}} \right]$$



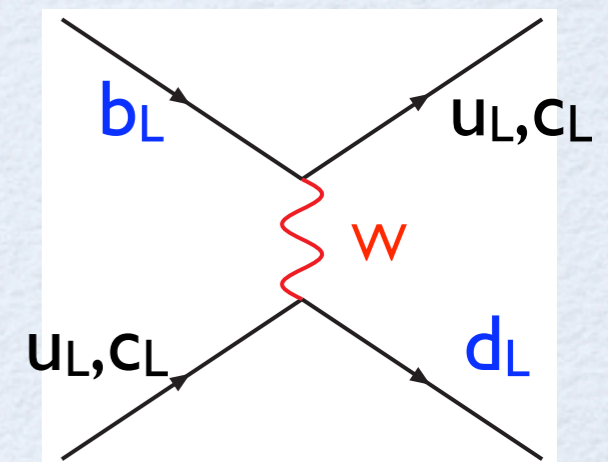
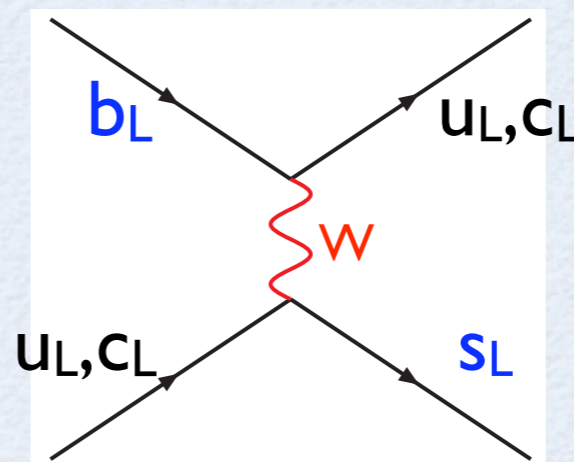
- Current-current:

$$Q_1 = (\bar{q}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L)$$

$$Q_2 = (\bar{q}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$$

$$Q_1^u = (\bar{q}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma^\mu T^a b_L)$$

$$Q_2^u = (\bar{q}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L)$$

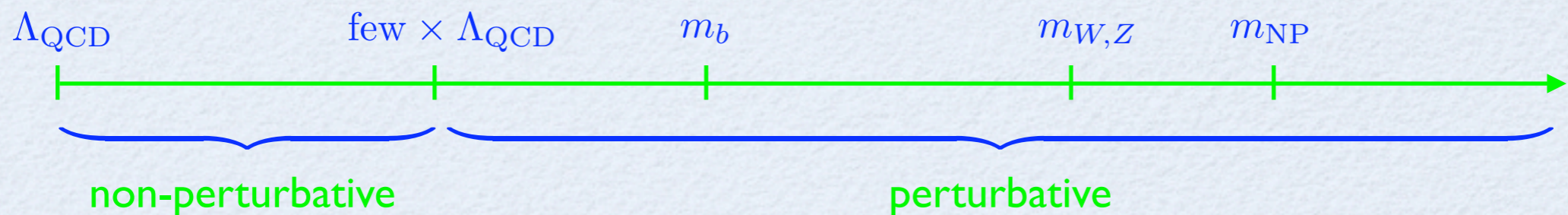


$$\left| \frac{C_2}{C_9} \frac{V_{ub} V_{us}}{V_{tb} V_{ts}} \right| \sim 0.5 \%$$

$$\left| \frac{C_2}{C_9} \frac{V_{ub} V_{ud}}{V_{tb} V_{td}} \right| \sim -10 \%$$

# EFFECTIVE LAGRANGIAN

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \left[ \sum_{i=1}^{10} C_i Q_i + \frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*} \sum_{i=1}^2 C_i (Q_i - Q_i^u) + \underbrace{\sum_{i=3}^6 C_{iQ} Q_{iQ} + C_b Q_b}_{\text{for QED corrections}} \right]$$



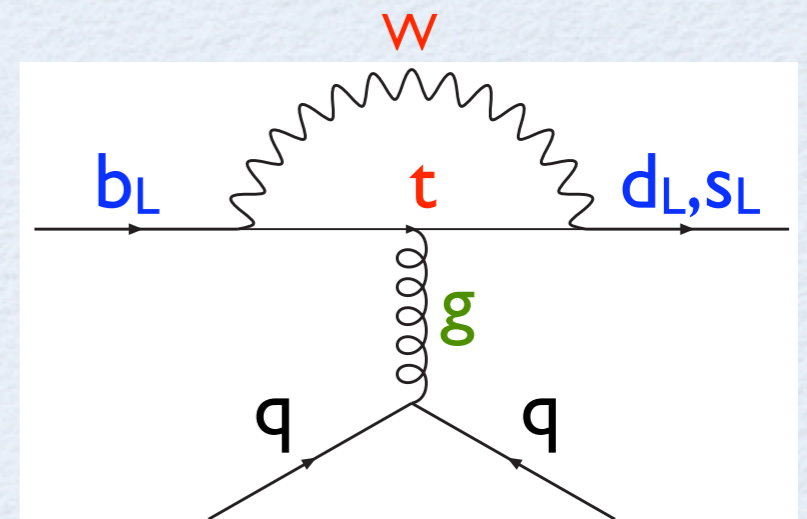
- QCD penguins:

$$Q_3 = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{q} \gamma^\mu q)$$

$$Q_4 = (\bar{q}_L \gamma_\mu T^a b_L) \sum (\bar{q} \gamma^\mu T^a q)$$

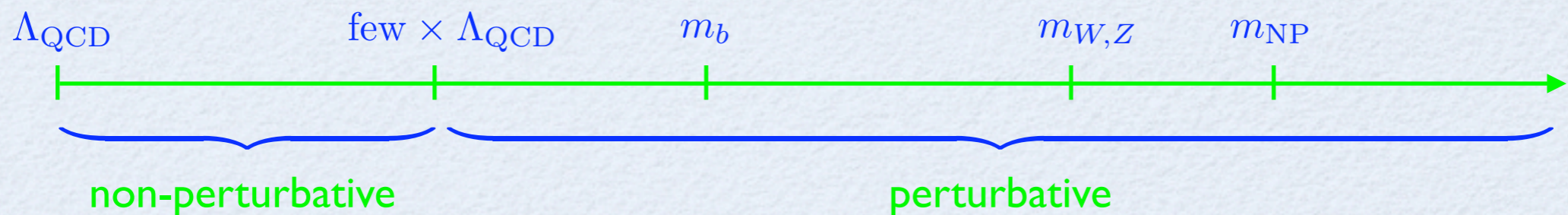
$$Q_5 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q)$$

$$Q_6 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q)$$



# EFFECTIVE LAGRANGIAN

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \left[ \sum_{i=1}^{10} C_i Q_i + \frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*} \sum_{i=1}^2 C_i (Q_i - Q_i^u) + \underbrace{\sum_{i=3}^6 C_{iQ} Q_{iQ} + C_b Q_b}_{\text{for QED corrections}} \right]$$



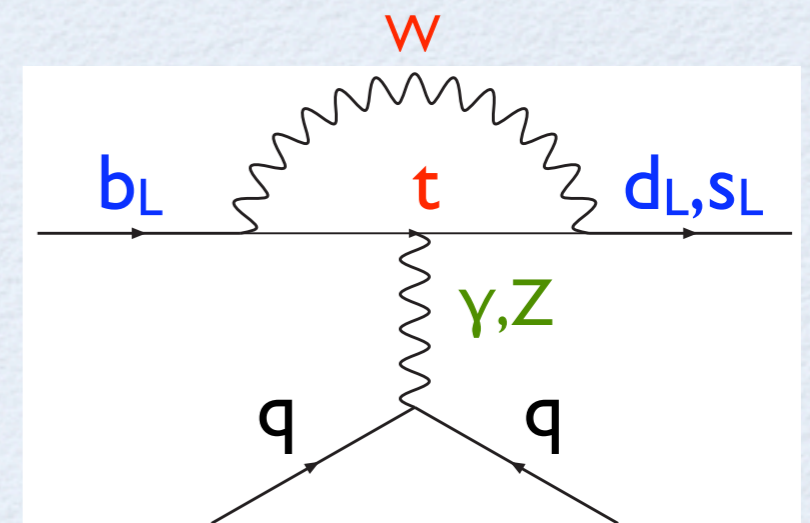
- EW penguins:

$$Q_{3Q} = (\bar{q}_L \gamma_\mu b_L) \sum Q_q (\bar{q} \gamma^\mu q)$$

$$Q_{4Q} = (\bar{q}_L \gamma_\mu T^a b_L) \sum Q_q (\bar{q} \gamma^\mu T^a q)$$

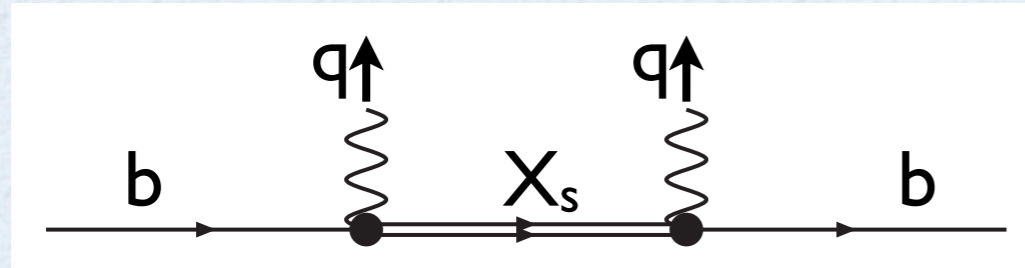
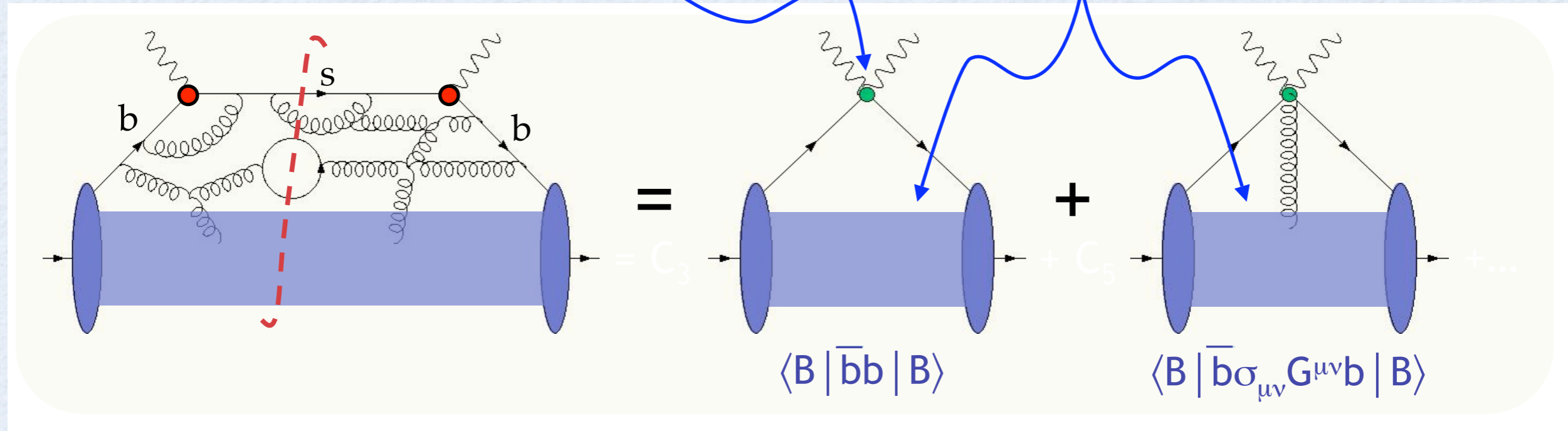
$$Q_{5Q} = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum Q_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q)$$

$$Q_{6Q} = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum Q_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q)$$



# POWER CORRECTIONS

$$\Gamma [\bar{B} \rightarrow X_s \ell^+ \ell^-] = \underbrace{\Gamma [\bar{b} \rightarrow X_s \ell^+ \ell^-]}_{C_3} + O \left( \underbrace{\frac{\Lambda_{QCD}^2}{m_b^2}, \frac{\Lambda_{QCD}^3}{m_b^3}}_{C_5}, \frac{\Lambda_{QCD}^2}{m_c^2}, \dots \right)$$



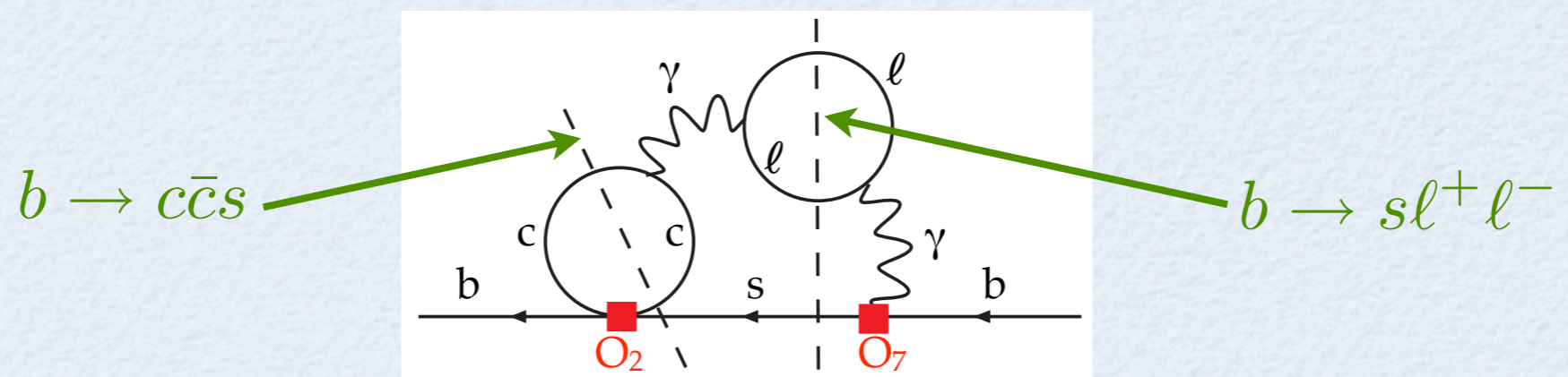
$$p_{X_s}^2 = (p_b - q)^2 = m_b^2 + q^2 - 2m_b q_0$$

$$< m_b^2 + q^2 - 2m_b \sqrt{q^2} = (m_b - \sqrt{q^2})^2$$

OPE is an expansion in  $\Lambda_{QCD}/(m_b - \sqrt{q^2})$  and breaks down at  $q^2 \sim m_b^2$

# Q<sup>2</sup> CUTS

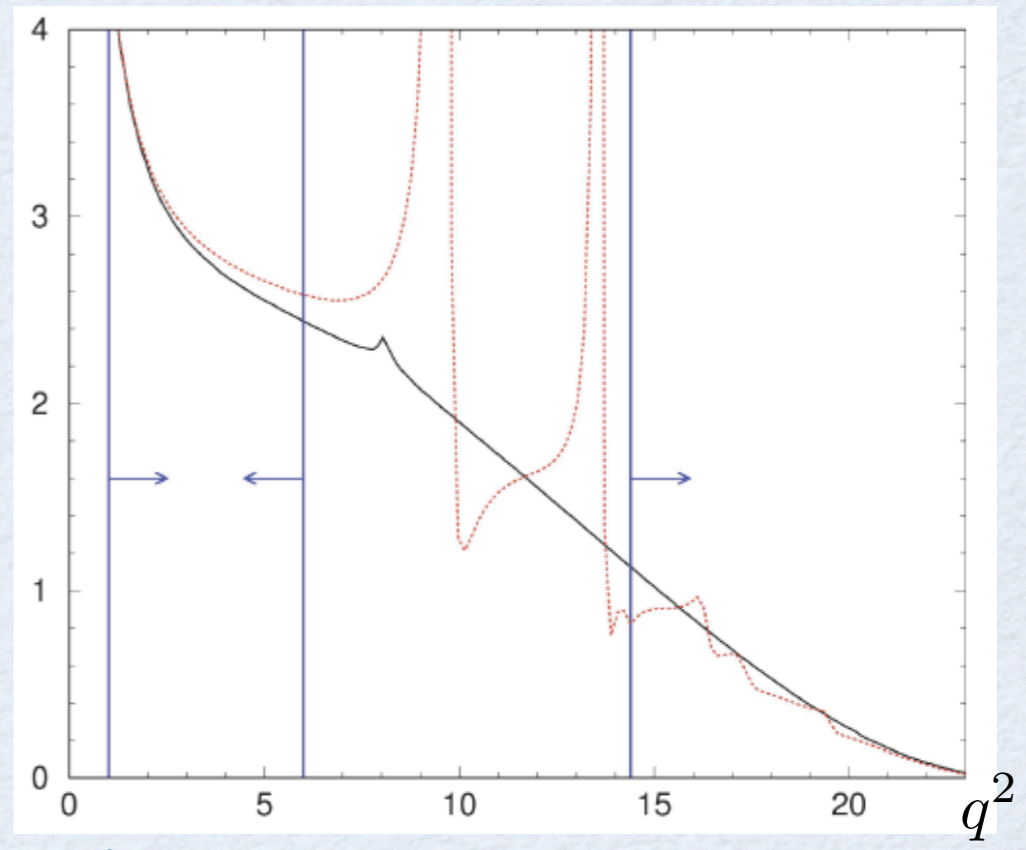
- Quark-hadron duality breaks down when the rate is dominated by charmonium resonances:



- Three regions:
  - $0.04 \text{ GeV}^2 < q^2 < 1 \text{ GeV}^2$
  - $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$
  - $q^2 > 14.4 \text{ GeV}^2$
 dominated by the photon pole ( $b \rightarrow s\gamma$ )

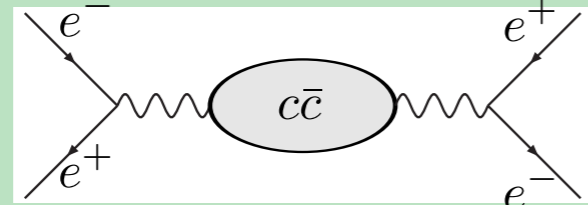
- Resonances model using data:
  - ★ Krüger-Sehgal ( $e+e-$  data)
  - ★ Simple Breit-Wigner

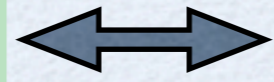
- Away from resonances expansion in  $1/m_c^2$  is performed

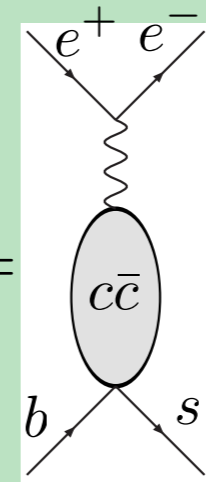


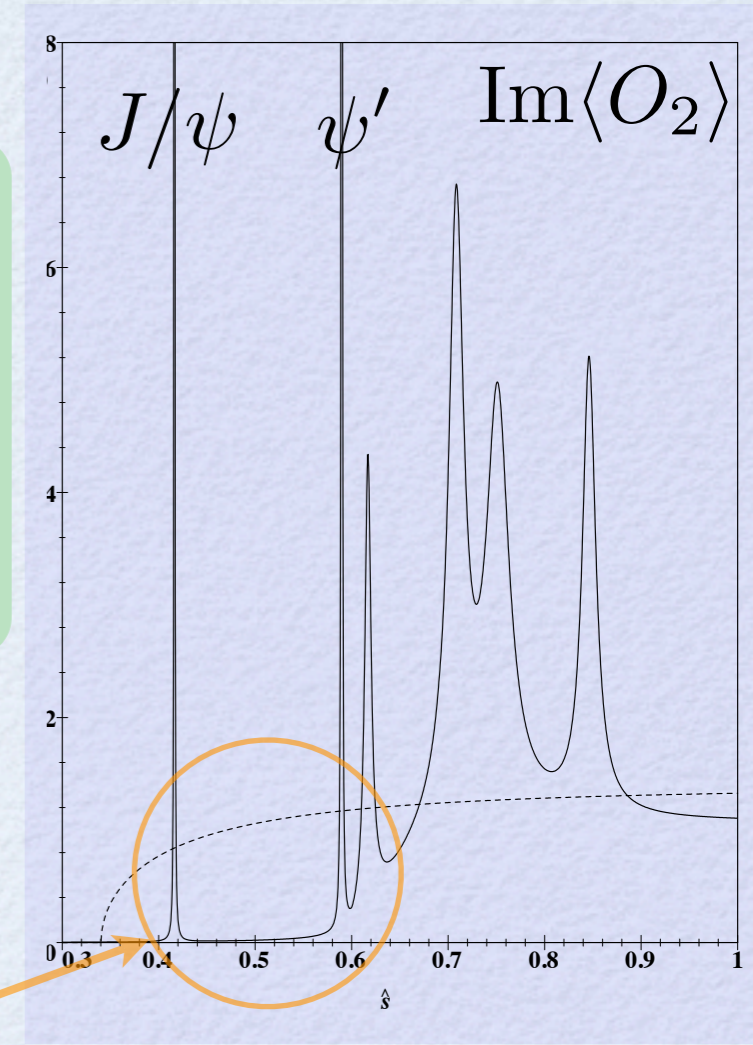
# Q<sup>2</sup> CUTS

- Kruger-Sehgal mechanism:

$$R_{\text{had}}^{c\bar{c}} = \frac{\sigma(e^+e^- \rightarrow c\bar{c} \text{ hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$




$$\langle O_2 \rangle =$$




$$\text{Im}\langle O_2 \rangle \rightarrow \langle O_9 \rangle_{\text{tree}} \left( \frac{\pi}{3} R_{\text{had}}^{c\bar{c}}(\hat{s}) \right)$$

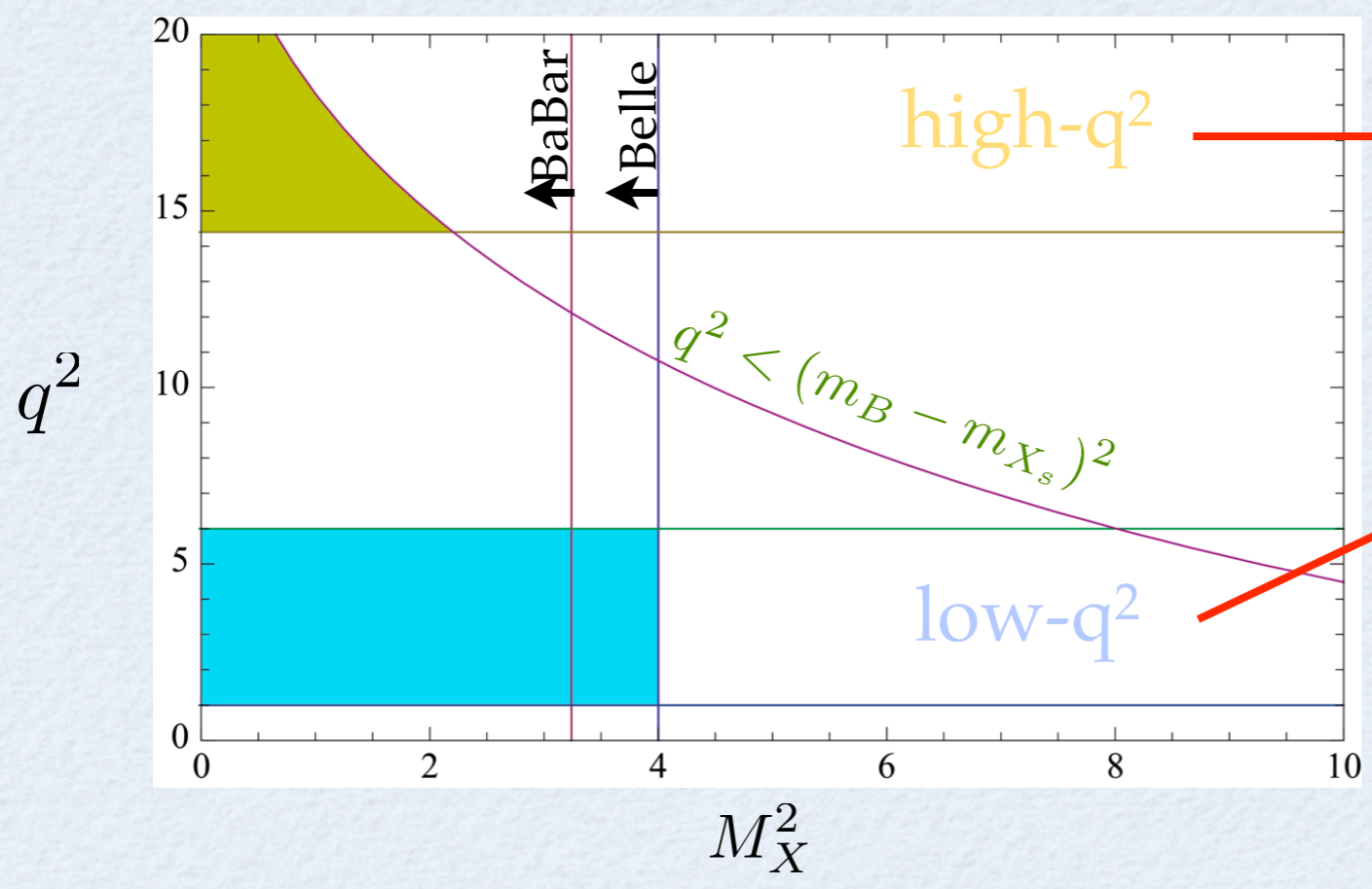
$$\text{Re}\langle O_2 \rangle \rightarrow \langle O_9 \rangle_{\text{tree}} \left( -\frac{8}{9} \log m_c/m_b - \frac{4}{9} + \frac{\hat{s}}{3} P \int_{4\hat{m}_D^2}^{\infty} \frac{R_{\text{had}}^{c\bar{c}}(\hat{s}')}{\hat{s}'(\hat{s}' - \hat{s})} d\hat{s}' \right)$$

- Alternatively use a Breit-Wigner ansatz to parametrize  $\langle O_2 \rangle$
- The two approaches agree well above and below the resonances **but not in between**
- The impact in the low  $q^2$  region is **+1.8%**, in the high  $q^2$  region is **-10%**

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# $X_s$ CUT

- $M_X$  cuts required to suppress the  $b \rightarrow c l^- \nu \rightarrow s l^- l^+ \nu \nu$  background



unaffected

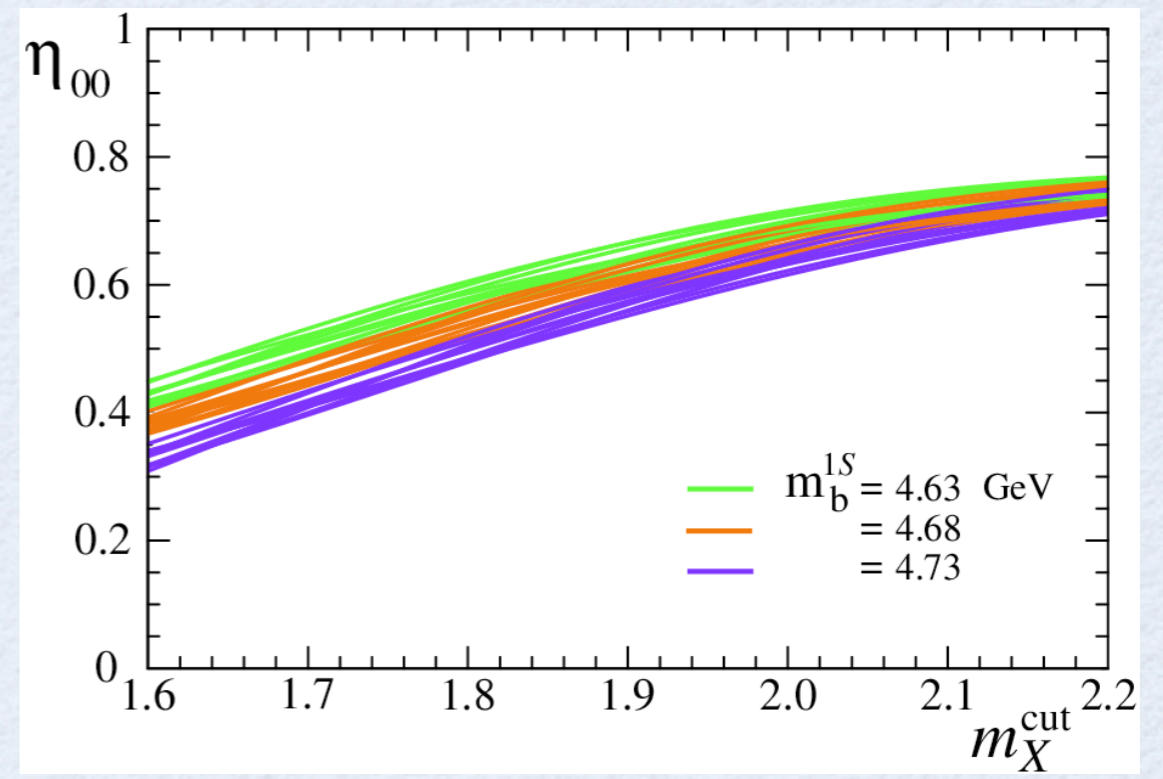
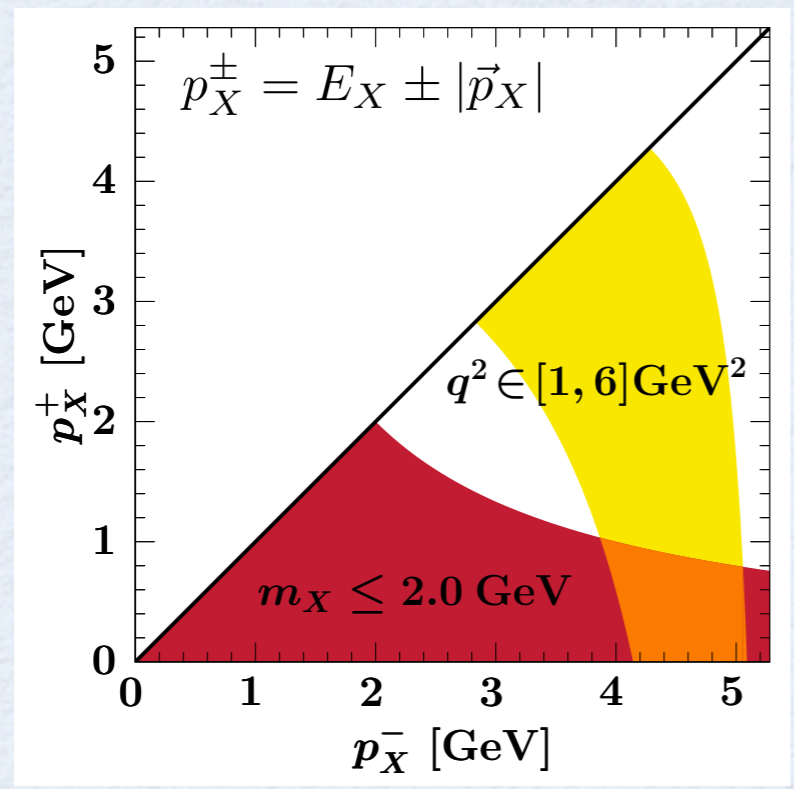
parton level at LO:  
 $M_{X_s} = m_s$   
 bremsstrahlung:  
 $m_s < M_{X_s} < m_b$   
 non-perturbative effects:  
 phase space ( $M_B - m_b = \Lambda$ )  
 Fermi motion [Ali, Hiller]

- Correction factor added in experimental results
- Framework: Fermi motion, SCET



# $X_S$ CUT

- New idea: use SCET to describe the  $X_S$  system



$$p_X^+ \ll p_X^- \implies m_X^2 \ll E_X^2$$

$X_S$  is a hard-collinear mode:

$$\Lambda^2 \ll p_{X_S}^2 \sim \Lambda m_b \ll m_b^2$$

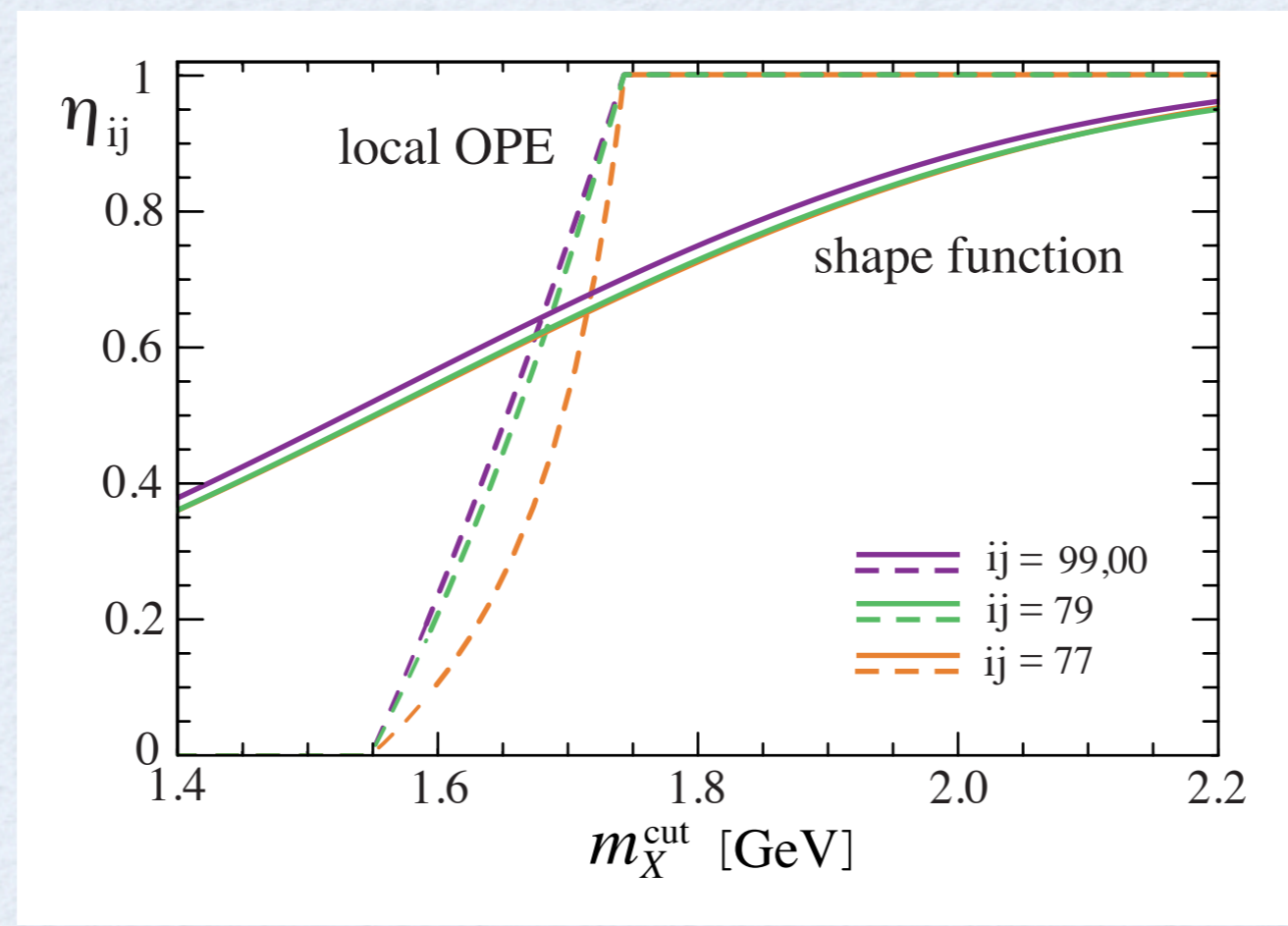
$$\eta_{ij} = \frac{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 \int_0^{m_X^{\text{cut}}} dm_X^2 \frac{d\Gamma_{ij}}{dq^2 dm_X^2}}{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 \frac{d\Gamma_{ij}}{dq^2}}$$

$ij$ :  $C_9^2$  and  $C_{10}^2$ ,  $C_7 C_9$ ,  $C_7^2$

- The effect seems very large (power corrections?)

# $X_s$ CUT

- At leading power and at order  $\alpha_s$ , these corrections are a universal multiplicative factor:



- Reduce non-perturbative effects by considering: [Lee, Ligeti, Stewart, Tackmann]

$$\Gamma^{\text{cut}}(B \rightarrow X_s \ell^+ \ell^-) / \Gamma^{\text{cut}}(B \rightarrow X_u \ell \bar{\nu}) \quad [\text{same } M_X \text{ cut}]$$

1

# INPUTS FOR $B \rightarrow SLL$

$$\alpha_s(M_Z) = 0.1189 \pm 0.0010 \text{ [40]}$$

$$\alpha_e(M_Z) = 1/127.918$$

$$s_W^2 \equiv \sin^2 \theta_W = 0.2312$$

$$|V_{ts}V_{tb}/V_{cb}|^2 = 0.962 \pm 0.002 \text{ [41]}$$

$$|V_{ts}V_{tb}/V_{ub}|^2 = (1.28 \pm 0.12) \times 10^2 \text{ [41]}$$

$$BR(B \rightarrow X_c e \bar{\nu})_{\text{exp}} = 0.1061 \pm 0.0017 \text{ [43]}$$

$$M_Z = 91.1876 \text{ GeV}$$

$$M_W = 80.426 \text{ GeV}$$

$$\lambda_2^{\text{eff}} = (0.12 \pm 0.02) \text{ GeV}^2$$

$$\lambda_1^{\text{eff}} = (-0.243 \pm 0.055) \text{ GeV}^2 \text{ [42]}$$

$$f_u^0 - f_s = (0 \pm 0.04) \text{ GeV}^3 \text{ [24]}$$

$$m_e = 0.51099892 \text{ MeV}$$

$$m_\mu = 105.658369 \text{ MeV}$$

$$m_\tau = 1.77699 \text{ GeV}$$

$$m_c(m_c) = (1.224 \pm 0.017 \pm 0.054) \text{ GeV [42]}$$

$$m_b^{1S} = (4.68 \pm 0.03) \text{ GeV [31]}$$

$$m_{t,\text{pole}} = (170.9 \pm 1.8) \text{ GeV [44]}$$

$$m_B = 5.2794 \text{ GeV}$$

$$C = 0.58 \pm 0.01 \text{ [31]} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma(\bar{B} \rightarrow X_c e \bar{\nu})}{\Gamma(\bar{B} \rightarrow X_u e \bar{\nu})}$$

$$\rho_1 = (0.06 \pm 0.06) \text{ GeV}^3 \text{ [31]}$$

$$f_u^0 + f_s = (0 \pm 0.2) \text{ GeV}^3 \text{ [24]}$$

$$f_u^\pm = (0 \pm 0.4) \text{ GeV}^3 \text{ [24]}$$

# BRANCHING RATIO

- Theory [Huber,Lunghi,Misiak,Wyler; Huber,Hurth,Lunghi]:

LOW

$$\mathcal{B}_{\mu\mu}^{\text{low}} = \left[ 1.59 \pm 0.08_{\text{scale}} \pm 0.06_{m_t} \pm 0.024_{C,m_c} \pm 0.015_{m_b} \pm 0.02_{\alpha_s(M_Z)} \right. \\ \left. \pm 0.015_{\text{CKM}} \pm 0.026_{\text{BR}_{sl}} \pm 0.08_{\alpha_s/m_b} \right] \times 10^{-6} = (1.59 \pm 0.14) \times 10^{-6}$$

$$\mathcal{B}_{ee}^{\text{low}} = (1.64 \pm 0.14) \times 10^{-6}$$

HIGH

$$\mathcal{B}_{\mu\mu}^{\text{high}} = 2.40 \times 10^{-7} \left( 1 + \begin{bmatrix} +0.01 \\ -0.02 \end{bmatrix}_{\mu_0} + \begin{bmatrix} +0.14 \\ -0.06 \end{bmatrix}_{\mu_b} \pm 0.02_{m_t} + \begin{bmatrix} +0.006 \\ -0.003 \end{bmatrix}_{C,m_c} \pm 0.05_{m_b} \right. \\ \left. + \begin{bmatrix} +0.0002 \\ -0.001 \end{bmatrix}_{\alpha_s} \pm 0.002_{\text{CKM}} \pm 0.02_{\text{BR}_{sl}} \pm 0.05_{\lambda_2} \pm 0.19_{\rho_1} \pm 0.14_{f_s} \pm 0.02_{f_u} \pm 0.05_{\alpha_s/m_b} \right)$$

$$= (2.40 \pm 0.7) \times 10^{-7}$$

$$\mathcal{B}_{ee}^{\text{high}} = (2.1 \pm 0.6) \times 10^{-7}$$



Breakdown of the OPE results in large power corrections over which we have a poor control

- Experiment [BaBar and Belle]:

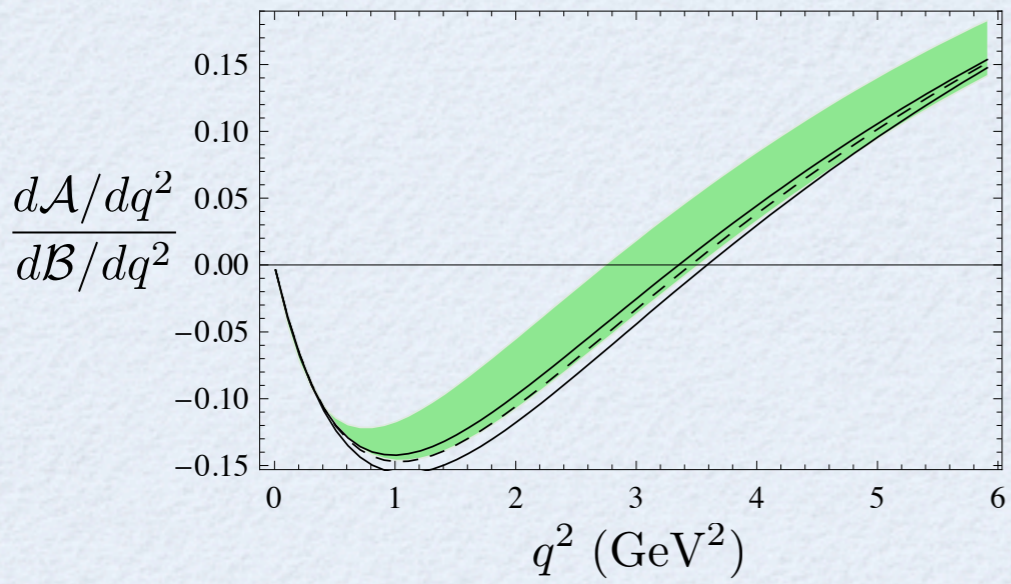
$$\mathcal{B}_{ll}^{\text{low}} = (1.60 \pm 0.51) \times 10^{-6}$$

$$\mathcal{B}_{ll}^{\text{high}} = (4.4 \pm 1.2) \times 10^{-7}$$

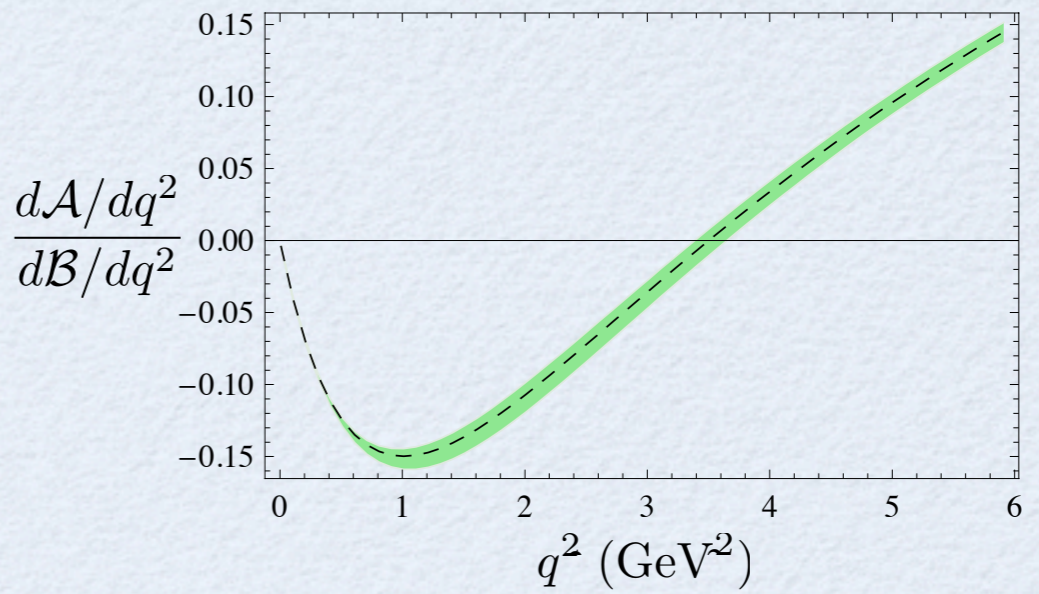
# LOW- $Q^2$ : FBA

[Huber,Hurth,Lunghi]

NNLO vs NLO



NNLO + QED



$$(q_0^2)_{\mu\mu} = \left[ 3.50 \pm 0.10_{\text{scale}} \pm 0.002_{m_t} \pm 0.04_{m_c, C} \pm 0.05_{m_b} \pm 0.03_{\alpha_s(M_Z)} \pm 0.001_{\lambda_1} \pm 0.01_{\lambda_2} \right] \text{GeV}^2$$

$$= (3.50 \pm 0.12) \text{GeV}^2$$

$$(q_0^2)_{ee} = (3.38 \pm 0.11) \text{GeV}^2$$

• Integrated observables:

Bin 1 ( $q^2 \in [1, 3.5] \text{GeV}^2$ )	Bin 2 ( $q^2 \in [3.5, 6] \text{GeV}^2$ )
$(\bar{\mathcal{A}}_{\mu\mu})_{\text{bin1}} = [-9.1 \pm 0.9]\%$	$(\bar{\mathcal{A}}_{\mu\mu})_{\text{bin2}} = [7.8 \pm 0.8]\%$
$(\bar{\mathcal{A}}_{ee})_{\text{bin1}} = [-8.1 \pm 0.9]\%$	$(\bar{\mathcal{A}}_{\mu\mu})_{\text{bin2}} = [8.3 \pm 0.6]\%$

low - $q^2$ ( $q^2 \in [1, 6] \text{GeV}^2$ )
$(\bar{\mathcal{A}}_{\mu\mu})_{\text{low}} = [-1.5 \pm 0.9]\%$
$(\bar{\mathcal{A}}_{\mu\mu})_{\text{low}} = [-0.9 \pm 0.9]\%$

1

# HIGH- $Q^2$ : REDUCING THE ERRORS

- New idea: normalize the decay width to the semileptonic  $B \rightarrow X_u l \nu$  rate with the *same dilepton invariant mass cut*:

$$\mathcal{R}(s_0) = \frac{\int_{\hat{s}_0}^1 d\hat{s} \frac{d\Gamma(\bar{B} \rightarrow X_s l^+ l^-)}{d\hat{s}}}{\int_{\hat{s}_0}^1 d\hat{s} \frac{d\Gamma(\bar{B}^0 \rightarrow X_u l \nu)}{d\hat{s}}} \quad [\text{Ligeti, Tackmann}]$$

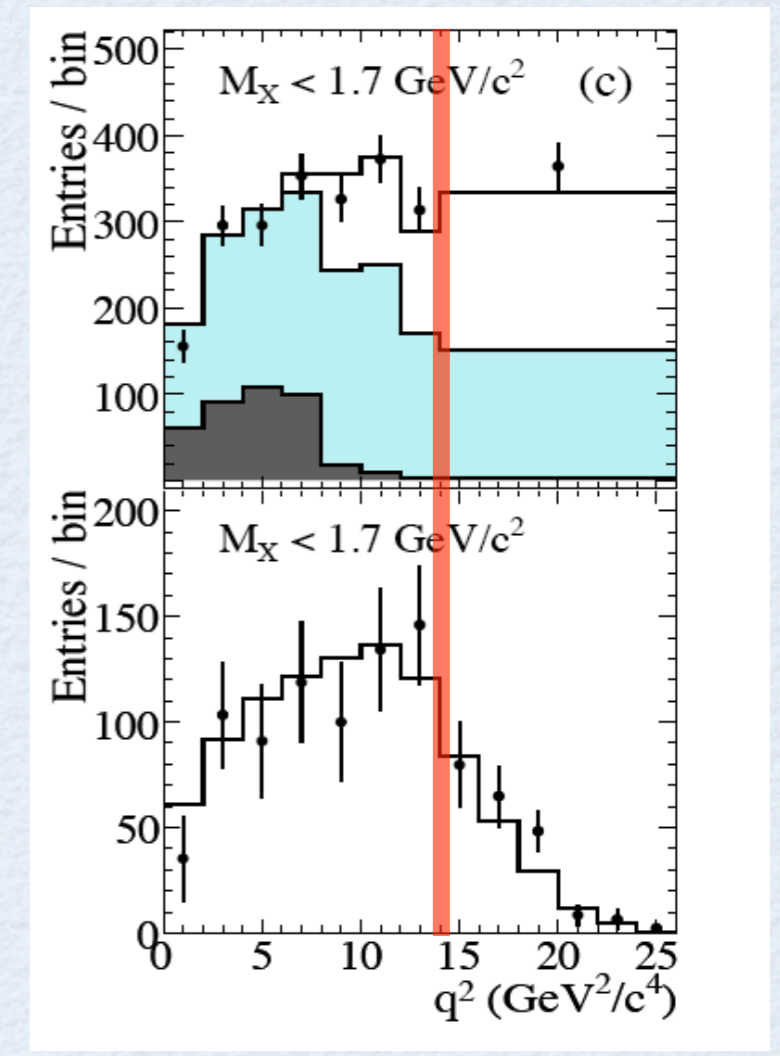
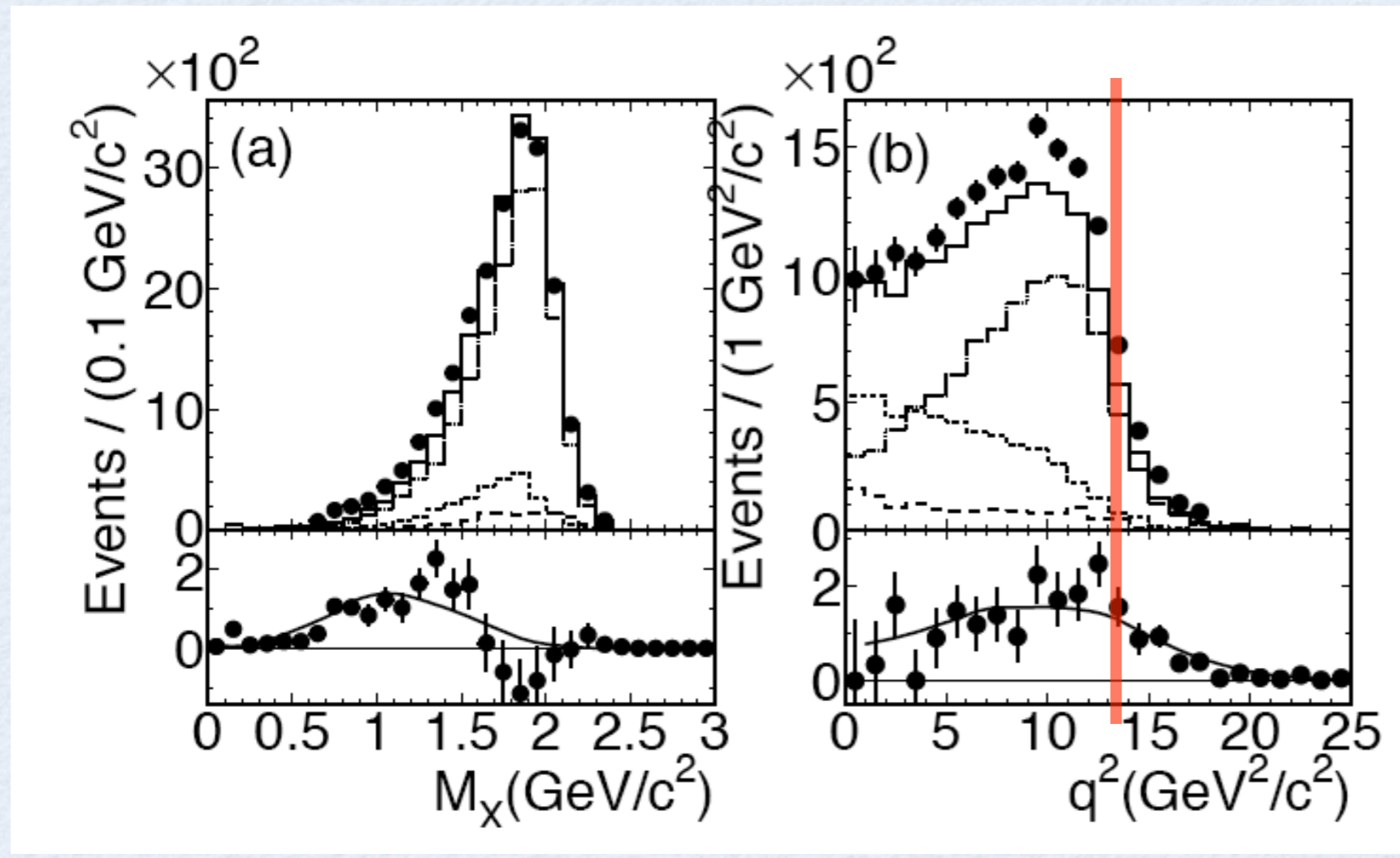
- *Impact of non-perturbative  $1/m_b^2$  and  $1/m_b^3$  power corrections drastically reduced*

- In the high- $q^2$  region we find:

$$\begin{aligned} \mathcal{R}(14.4\text{GeV}^2) &= 2.29 \times 10^{-3} \left( 1 \pm 0.04_{\text{scale}} \pm 0.02_{m_t} \pm 0.01_{C, m_c} \pm 0.006_{m_b} \pm 0.005_{\alpha_s} \pm 0.09_{\text{CKM}} \right. \\ &\quad \left. \pm 0.003_{\lambda_2} \pm 0.05_{\rho_1} \pm 0.03_{f_u^0 + f_s} \pm 0.05_{f_u^0 - f_s} \right) \\ &= 2.29 \times 10^{-3} (1 \pm 0.13) \end{aligned} \quad [\text{Huber, Hurth, Lunghi}]$$

- *The largest source of uncertainty is  $V_{ub}$*

# HIGH- $Q^2$ : REDUCING THE ERRORS



[Belle, 87 fb<sup>-1</sup>, hep-ex/0311048]

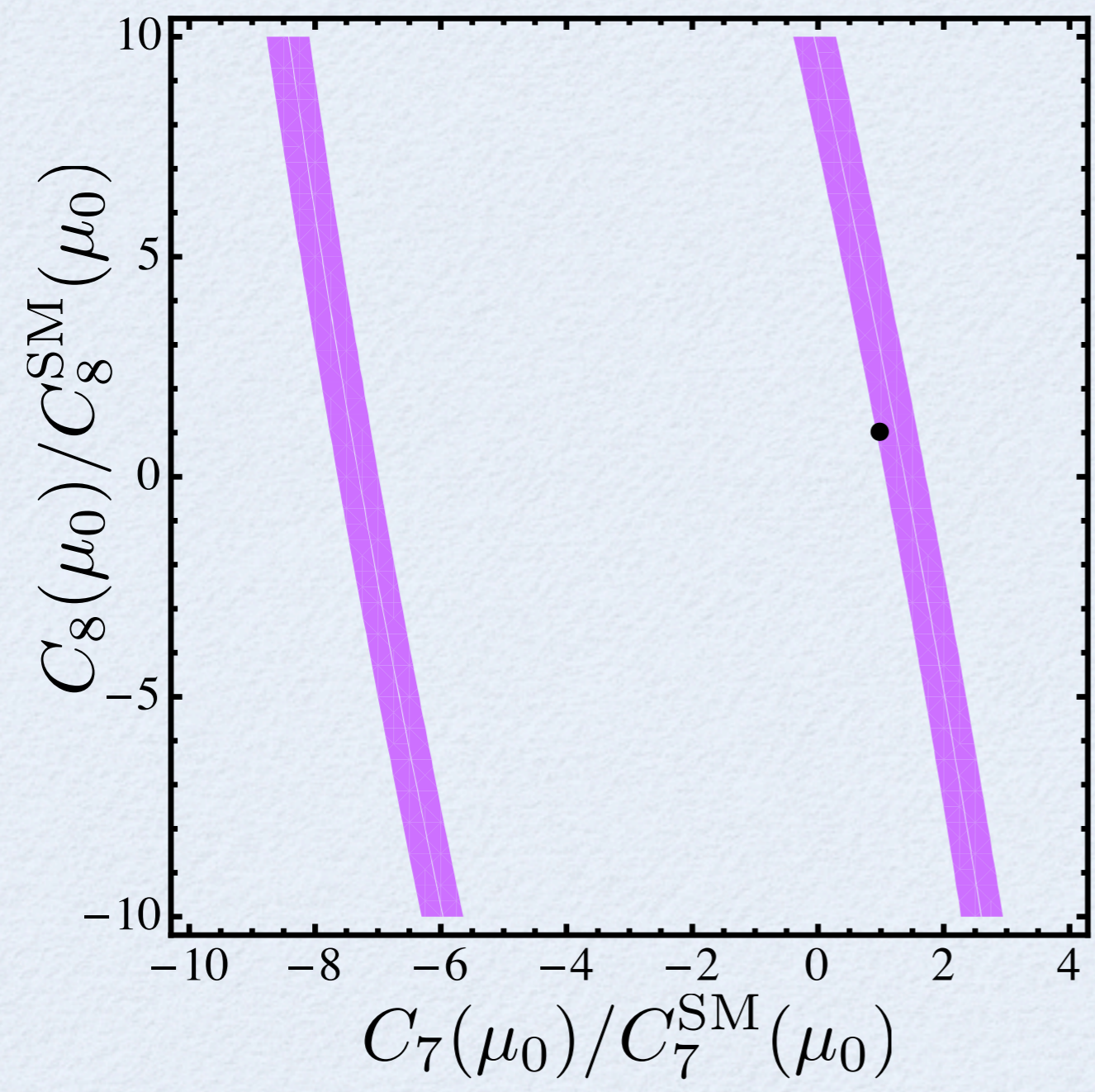
[BaBar, 383 m Y, arXiv:0708.3702]

- Experiments already positioned to measure B→X<sub>u</sub>lv with a q<sup>2</sup> cut
- Separation of B<sup>0</sup> and B<sup>+</sup> is important to control WA contributions

1

# MODEL INDEPENDENT ANALYSIS

- Use  $B \rightarrow X_s \gamma$  to constrain  $C_7$  and  $C_8$ :



Theory:

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}$$

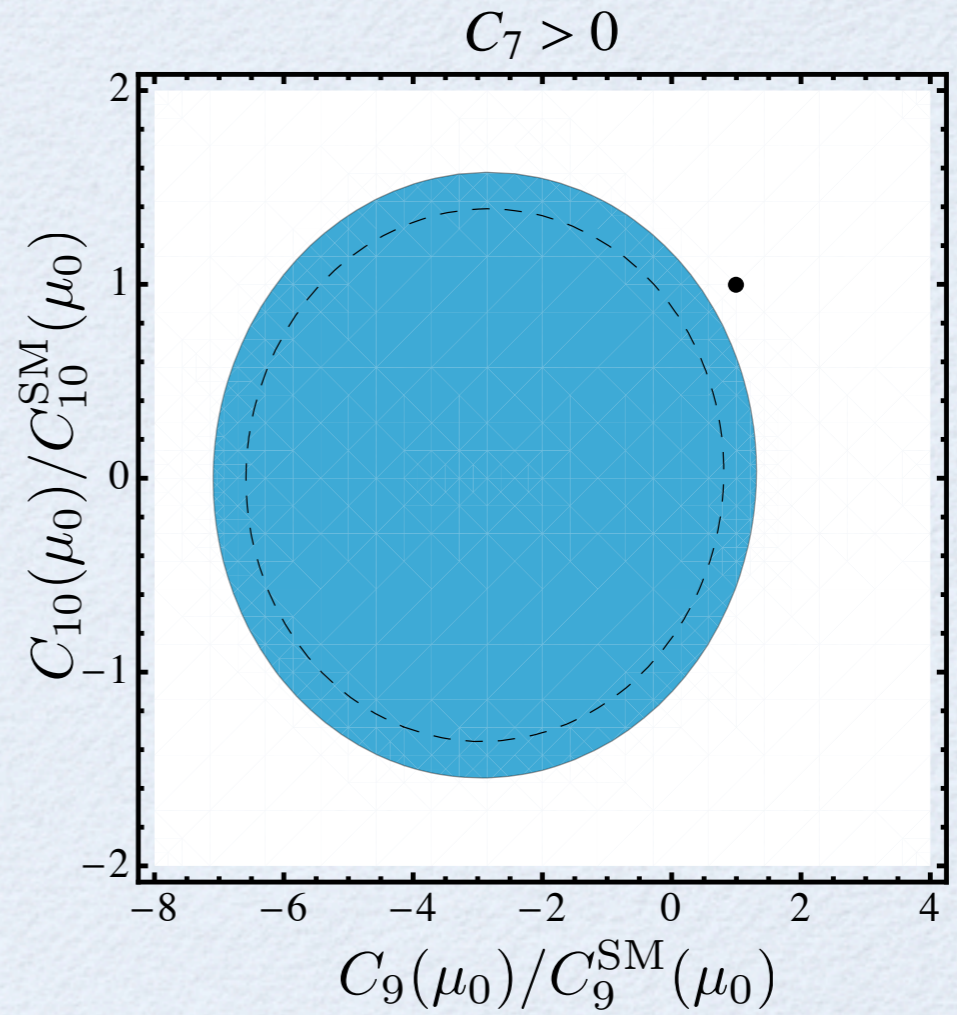
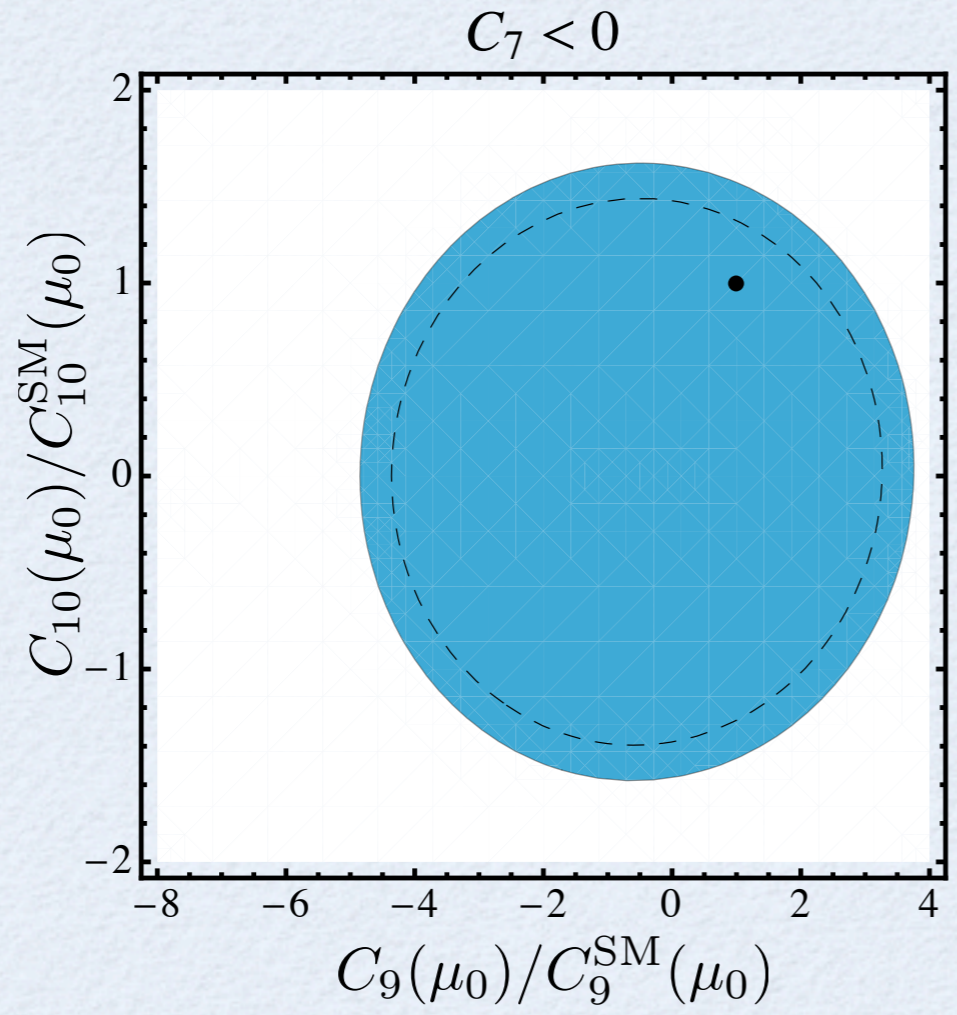
Experiment:

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{\text{exp}} = (3.52 \pm 0.25) \times 10^{-4}$$



# MODEL INDEPENDENT ANALYSIS

- Use  $C_7$  and  $C_8$  from  $B \rightarrow X_s \gamma$  to constrain  $C_9$  and  $C_{10}$

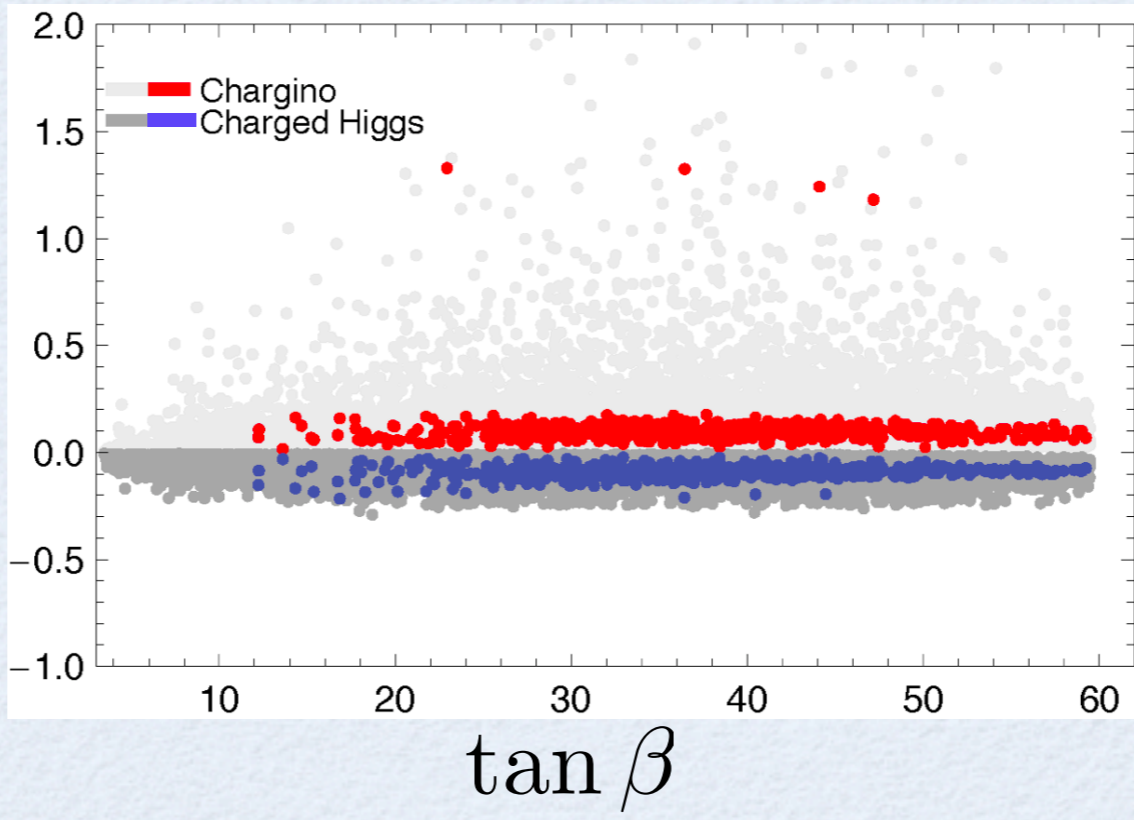


- $C_7 > 0$  requires sizable contributions to  $C_9$  and  $C_{10}$
- Reversing the sign of  $C_7$  we obtain  $\mathcal{B}(\bar{B} \rightarrow X_s \ell^+ \ell^-) = (3.11 \pm 0.22) \times 10^{-6}$  hence the SM sign is favored at the  $2.7\sigma$  level  
[Gambino,Haisch,Misiak]

# MFV SUSY

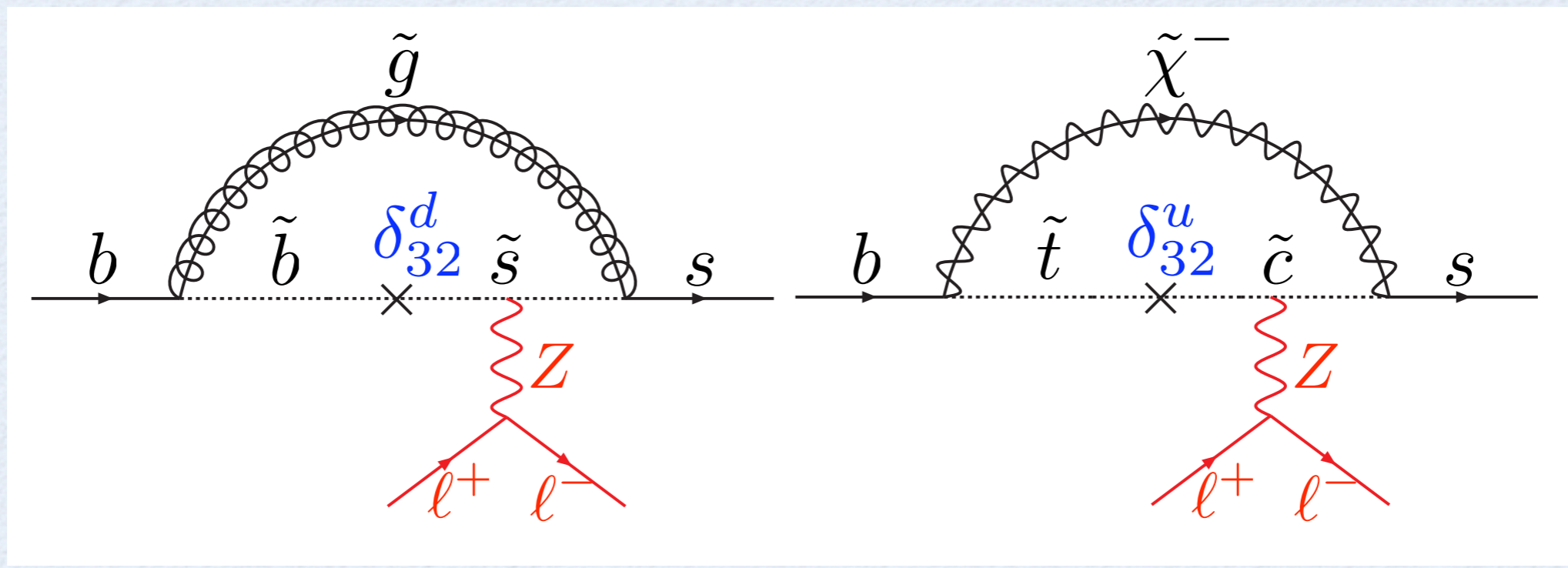
- Computing aid: *Spheno* for the RGE of the MSSM and *MicrOMEGAs* for the relic dark matter density
- Effects on C9 and C10 are tiny:  $|C_{9,10}(\mu_0)/C_{9,10}^{\text{SM}}(\mu_0)| < 0.1$
- $b \rightarrow s\gamma$  shapes the surviving parameter space:

$C_7(\mu_b)$



# SUSY: MIA ANALYSIS

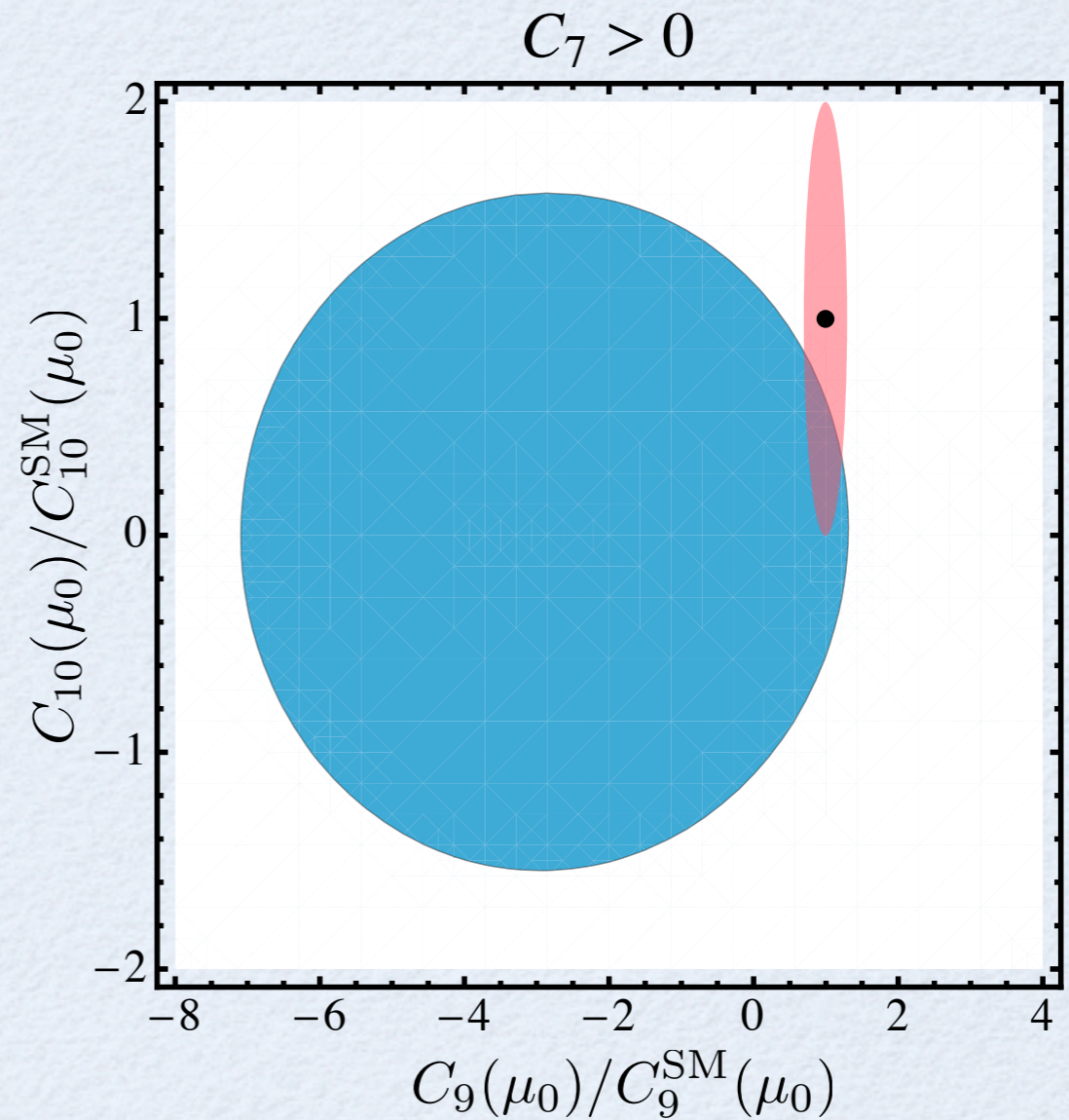
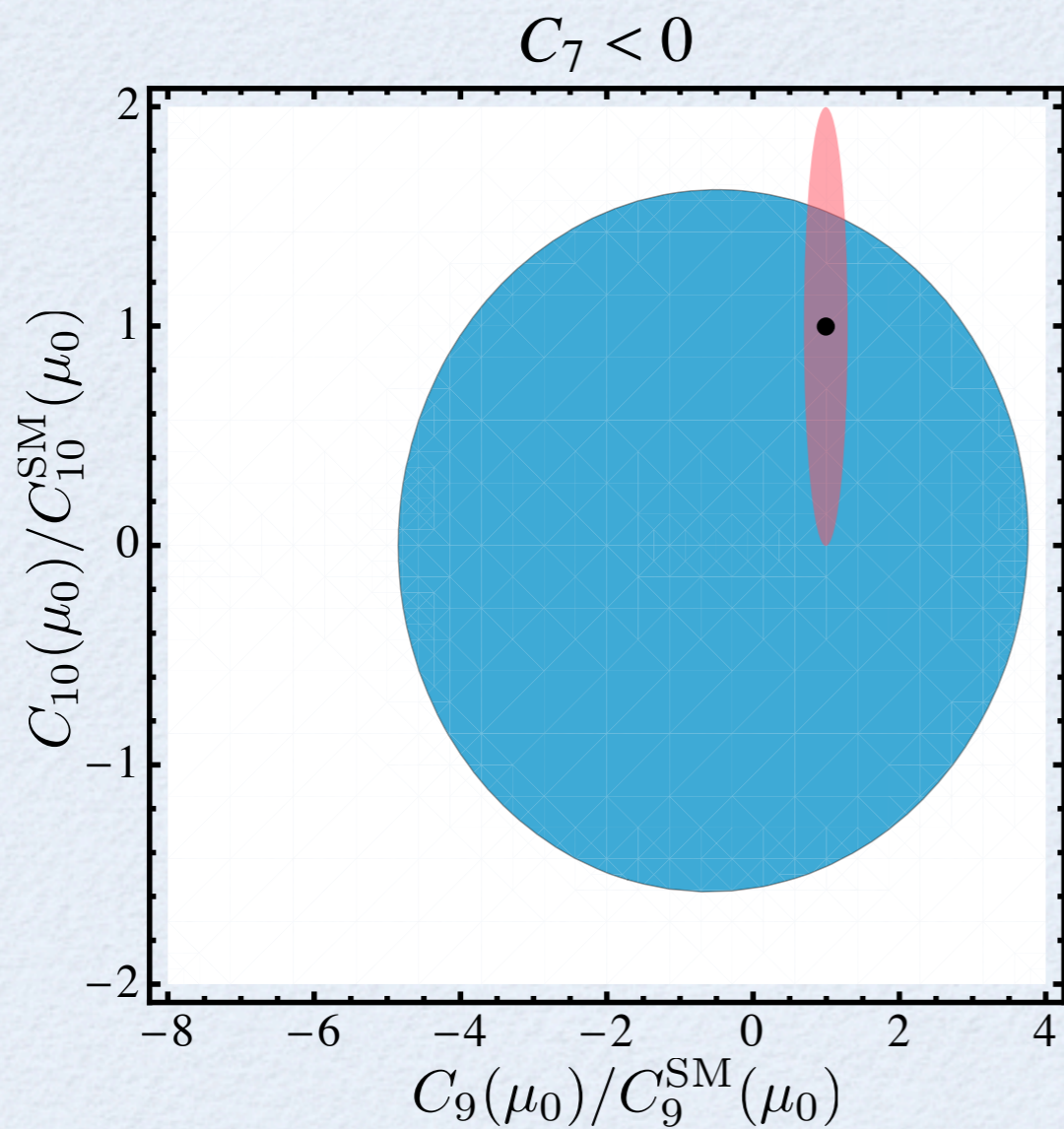
- In the most general MSSM, gluino and chargino diagrams can lead to huge contributions to the semileptonic operators:



$$0.7 < C_9(\mu_0) / C_9^{\text{SM}}(\mu_0) < 1.3$$

$$0 < C_{10}(\mu_0) / C_{10}^{\text{SM}}(\mu_0) < 2$$

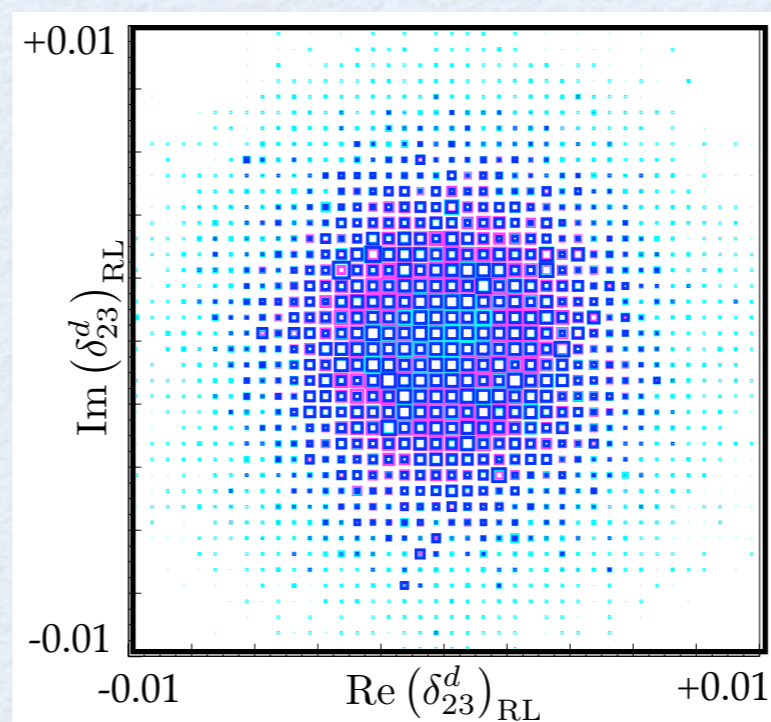
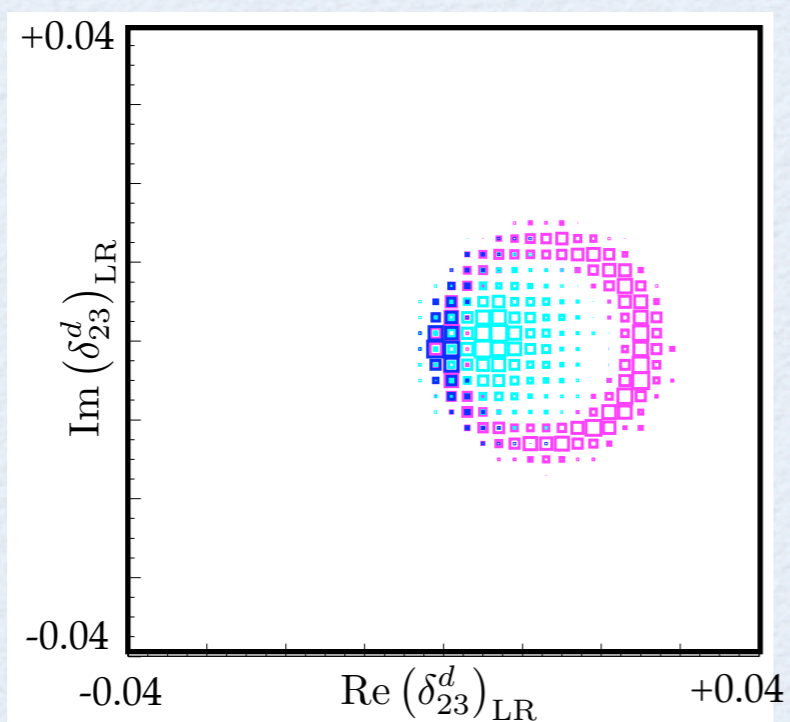
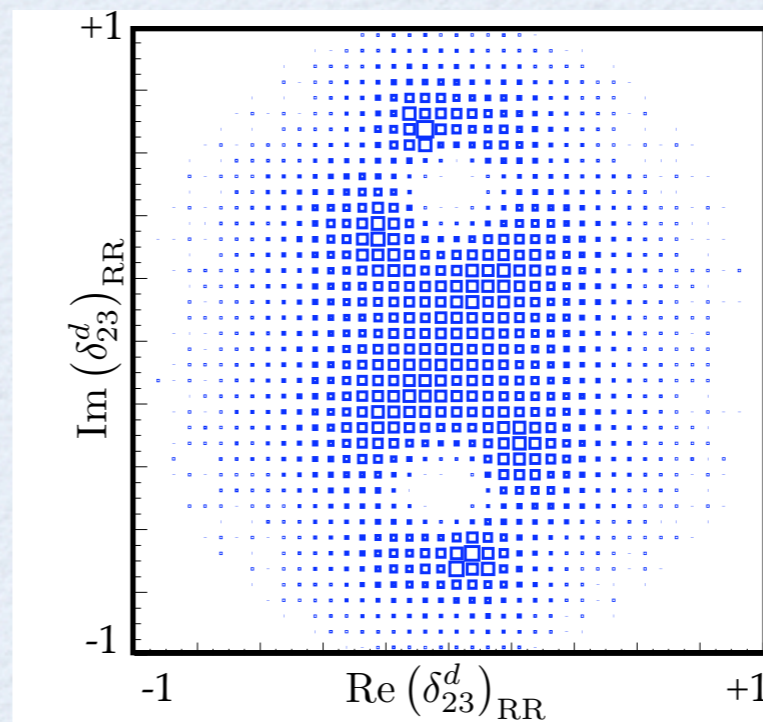
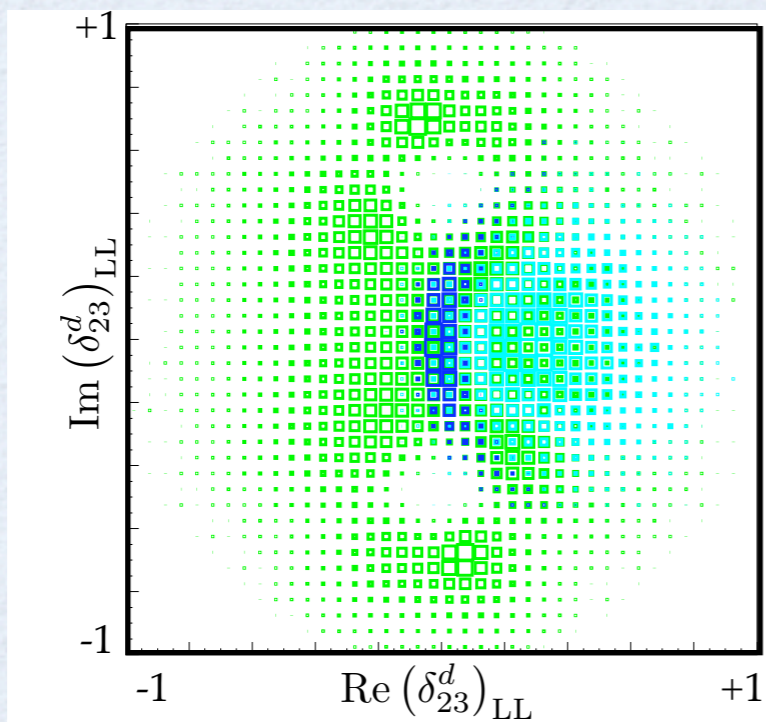
# SUSY: MIA ANALYSIS



- The  $C_7 > 0$  scenario is viable (with some degree of fine tuning)
- More than one mass insertion present at the same time

# SUSY: MIA ANALYSIS

- Constraints on (23) mass insertions in the down sector  
[Ciuchini,Silvestrini]



# INPUTS TO THE UT FIT

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$ V_{cb} _{\text{excl}} = (38.6 \pm 1.2)10^{-3}$	$\eta_1 = 1.51 \pm 0.24$
$ V_{cb} _{\text{incl}} = (41.31 \pm 0.76)10^{-3}$	$\eta_2 = 0.5765 \pm 0.0065$
$ V_{cb} _{\text{incl+excl}} = (40.3 \pm 1.0)10^{-3}$	$\eta_3 = 0.47 \pm 0.04$
$ V_{ub} _{\text{excl}} = (3.42 \pm 0.37)10^{-3}$	$\eta_B = 0.551 \pm 0.007$
$ V_{ub} _{\text{incl}} = (4.03 \pm 0.15_{-0.25}^{+0.20})10^{-3}$	$\xi = 1.243 \pm 0.028$
$\Delta m_{B_d} = (0.507 \pm 0.005) \text{ ps}^{-1}$	$\alpha = (89.5 \pm 4.3)^\circ$
$\Delta m_{B_s} = (17.77 \pm 0.12) \text{ ps}^{-1}$	$S_{\psi K_S} = 0.672 \pm 0.024$
$\varepsilon_K = (2.229 \pm 0.012) \times 10^{-3}$	$\gamma = (78 \pm 12)^\circ$
$m_c(m_c) = (1.268 \pm 0.009) \text{ GeV}$	$\widehat{B}_K = 0.725 \pm 0.026$
$m_{t,pole} = (172.4 \pm 1.2) \text{ GeV}$	$\kappa_\varepsilon = 0.92 \pm 0.01$
$f_K = (155.8 \pm 1.7) \text{ MeV}$	$f_B = (192.8 \pm 9.9) \text{ MeV}$
$f_{B_s} \sqrt{\widehat{B}_s} = (275 \pm 13) \text{ MeV}$	$\lambda = 0.2255 \pm 0.0007$
$\text{BR}(B \rightarrow \tau\nu) = (1.43 \pm 0.37)10^{-4} [17]$	

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