SCET sum rules for
$B \to P$ and $B \to V$ transition form factors

Fulvia De Fazio
INFN- Sezione di Bari

Super B Physics Workshop
Frascati, December 2009

Outline

- heavy to light decays: - general considerations
  - comparison with heavy-to-heavy case
- SCET- based LCSR: hadronic form factors
- summary and perspectives

Based on work in collaboration with T. Feldmann and T. Hurth
NPB B733 (2006) 1;
JHEP02 (2008) 031

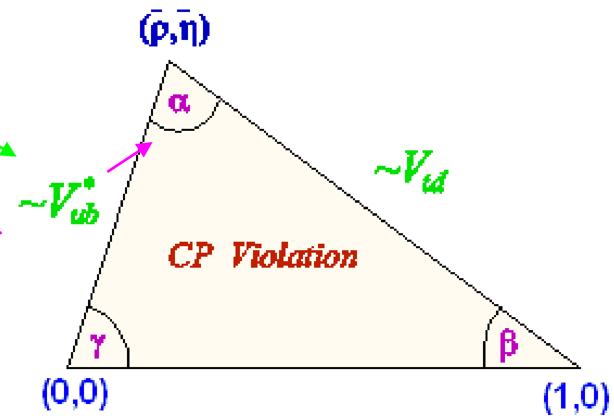
Heavy to light decays induced by b → u transition

$$B \to X_u \ell \nu \qquad\qquad B \to \pi \ell \nu, \; \rho \ell \nu \; \ldots$$

$$B \to \pi\pi, \; \rho\rho \ldots$$

- relevant for the determination of Vub

possibility to constrain one of the sides of the Unitarity Triangle



- form factors describing exclusive semileptonic modes enter also in the description of some non leptonic decays in the QCD factorization approach

possibility to constrain one of the angles of the Unitarity Traingle (α)

- description of rare B decays induced by b to s transition:

$$B \to K\pi, K^{(*)}\ell^+\ell^-, K^*\gamma \ldots$$

Heavy-to-heavy decays: heavy quark symmetries

Give the possibility to relate the various form factors describing exclusive processes induced by b $\to$ c transition

$$\langle D(v')|\, V_\mu\, |B(v)\rangle = \sqrt{m_B m_D}\left[h_+(w)\,(v+v')_\mu + h_-(w)\,(v-v')_\mu\right],$$

$$\langle D^*(v',\epsilon')|\, V_\mu\, |B(v)\rangle = i\sqrt{m_B m_{D^*}}\; h_V(w)\,\epsilon_{\mu\nu\alpha\beta}\,\epsilon'^{*\nu}\, v'^{\alpha}\, v^{\beta},$$

$$\langle D^*(v',\epsilon')|\, A_\mu\, |B(v)\rangle = \sqrt{m_B m_{D^*}}\left[h_{A_1}(w)\,(w+1)\,\epsilon'^*_\mu - h_{A_2}(w)\,\epsilon'^*\!\cdot v\, v_\mu - h_{A_3}(w)\,\epsilon'^*\!\cdot v\, v'_\mu\right],$$

$$\langle D^*(v',\epsilon')|\, V_\mu\, |B^*(v,\epsilon)\rangle = \sqrt{m_{B^*} m_{D^*}}\Big\{ -\epsilon\cdot\epsilon'^*\left[h_1(w)\,(v+v')_\mu + h_2(w)\,(v-v')_\mu\right] + h_3(w)\,\epsilon'^*\!\cdot v\,\epsilon_\mu + h_4(w)\,\epsilon\cdot v'\,\epsilon'^*_\mu - \epsilon\cdot v'\,\epsilon'^*\!\cdot v\left[h_5(w)\,v_\mu + h_6(w)\,v'_\mu\right]\Big\},$$

$$\langle D^*(v',\epsilon')|\, A_\mu\, |B^*(v,\epsilon)\rangle = i\sqrt{m_{B^*} m_{D^*}}\;\epsilon_{\mu\nu\alpha\beta}\,\epsilon^{\alpha}\,\epsilon'^{*\beta}\left[h_7(w)\,(v+v')^\nu + h_8\,(v-v')^\nu\right]$$

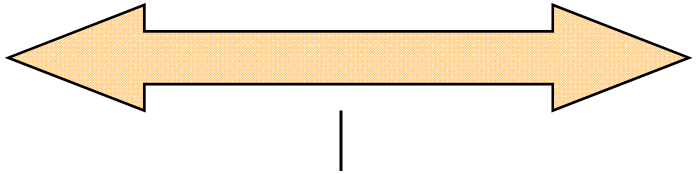
At leading order in the heavy quark expansion one finds that

$$h_+ = h_V = h_{A_1} = h_{A_3} = h_1 = h_3 = h_4 = h_7 = \xi \quad \Longrightarrow$$

all form factors are related to one Isgur-Wise function

Heavy-to-heavy decays

In the kinematic regime when **Q** interacts with the light antiquark only through soft gluon exchanges heavy quark symmetries arise **(HQET)**

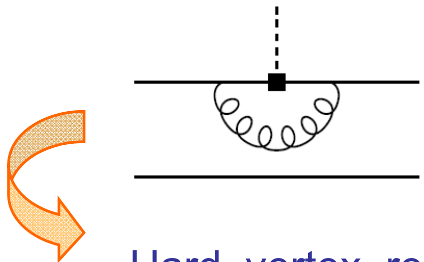


$\mu < m_b$ **m_b** $\mu > m_b$

HQET reproduces correctly the long distance physics of **QCD**

Short-distance corrections, to be computed in perturbation theory

$$\alpha_s(m_Q) \approx 0.2$$



Hard vertex renormalization

Heavy-to-light decays – Form factors

$$\langle P(p')|\bar{q}\gamma_\mu b|B(p)\rangle \quad \rightarrow \quad f_+(q^2),\ \ f_0(q^2)$$

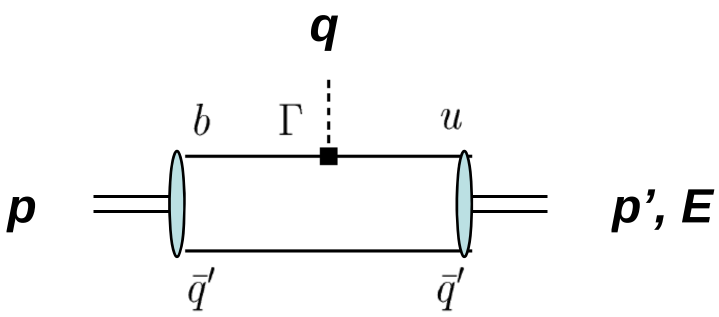
$$\langle P(p')|\bar{q}\sigma_{\mu\nu} b|B(p)\rangle \quad \rightarrow \quad f_T(q^2)$$

$$\langle V(p',\varepsilon)|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p)\rangle \quad \rightarrow \quad V(q^2),\ \ A_1(q^2),\ \ A_2(q^2),\ \ A_0(q^2)$$

$$\langle V(p',\varepsilon)|\bar{q}\sigma_{\mu\nu}(1-\gamma_5)b|B(p)\rangle \quad \rightarrow \quad T_1(q^2),\ \ T_2(q^2),\ \ T_3(q^2)$$

Are there symmetry relations also for heavy-to-light decays ?

Heavy-to-light form factors at large recoil



$$E = \frac{M^2 + m^2 - q^2}{2M}$$

Large energy of the light meson $E\text{-}\frac{M}{2} << M \quad \Leftrightarrow \quad q^2 << M^2$

If one assumes that the **b** and **u** still interact with the spectator only through "***soft***" gluon exchanges, symmetry relations can be derived for the heavy-to-light form factors

Charles et al. PRD 60 (99) 014001

Heavy-to-light form factors at large recoil

$$\left.\begin{array}{l}\langle P(p')|\bar{q}\gamma_\mu b|B(p)\rangle \\ \langle P(p')|\bar{q}\sigma_{\mu\nu} b|B(p)\rangle\end{array}\right\} \longrightarrow \xi_P(E)$$

$$\left.\begin{array}{l}\langle V(p',\varepsilon)|\bar{q}\gamma_\mu(1-\gamma_5) b|B(p)\rangle \\ \langle V(p',\varepsilon)|\bar{q}\sigma_{\mu\nu}(1-\gamma_5) b|B(p)\rangle\end{array}\right\} \longrightarrow \xi_\perp(E), \quad \xi_{||}(E)$$

Symmetry relations

$$f_+(q^2) = \frac{M}{2E} f_0(q^2) = \frac{M}{M+m_P} f_T(q^2) = \xi_P(E)$$

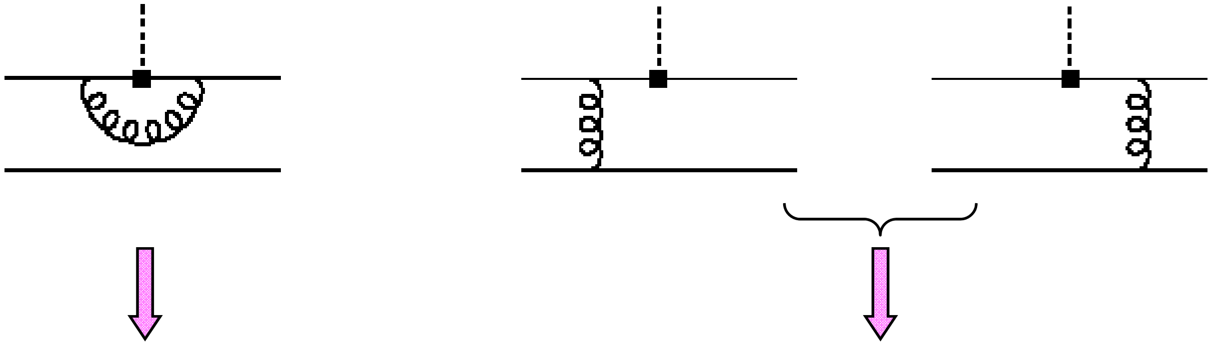
$$\frac{M}{M+m_V} V(q^2) = \frac{M+m_V}{2E} A_1(q^2) = T_1(q^2) = \frac{M}{2E} T_2(q^2) = \xi_\perp(E),$$

$$\frac{m_V}{E} A_0(q^2) = \frac{M+m_V}{2E} A_1(q^2) - \frac{M-m_V}{M} A_2(q^2) = \frac{M}{2E} T_2(q^2) - T_3(q^2) = \xi_{||}(E)$$

Symmetry relations

- Are valid for the soft contribution to the soft form factors at large recoil neglecting $1/m_b$ and α_s corrections

- Corrections stem from

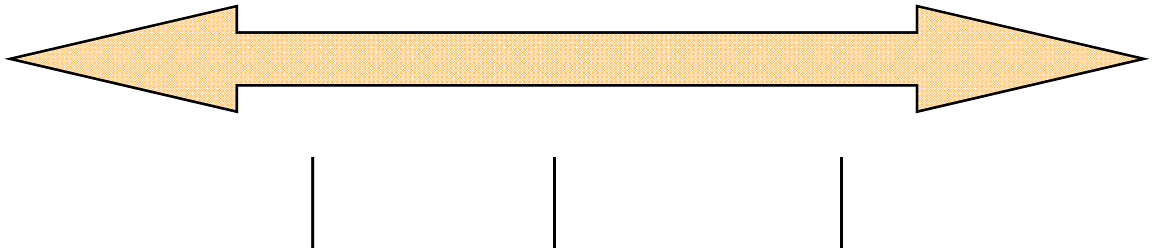


Hard vertex renormalization

factorizable spectator interaction

The inclusion of such corrections leads to a ***factorization theorem***

Energy scales



Λ_{QCD} $\quad\quad$ $\mu_{hc} = \sqrt{m_b \Lambda}$ $\quad\quad$ m_b

Soft **scale,**
typical of energies and momenta
of light degrees of freedom

Hard scale

Hard-collinear scale
in interactions between soft and energetic modes
in the initial and final state

Separation of scales is achieved in effective theories

Bauer, Fleming, Pirjol and Stewart
Beneke, Chapovsky, Diehl and Feldmann
Becher, Hill, Lange and Neubert

- $\mu<\mu_I$ $\quad$ $SCET_I$ $\quad$ (integrate out hard modes)
- $\mu<\mu_{II}$ $\quad$ $SCET_{II}$ $\quad$ (integrate out also hard-collinear modes)

Factorization theorem

Beneke and Feldmann, NPB 685 (04) 249
See also
Hill and Neubert NPB 657 (03) 229
Lange and Neubert NPB 690 (04) 249
Bauer, Pirjol and Stewart PRD 67 (03) 071502

Short-distance functions
Arise from integrating out hard modes: $\mu_I < m_b$

Light-cone distributions

$$f_i(q^2) = C_i(E,\mu_I)\xi(E,\mu_I) + T_i(E,\mu_{II}) \otimes \Phi_B(\mu_{II}) \otimes \Phi_P(\mu_{II}) + \ldots$$

Hard scattering kernel (perturbative series in α_s)
Contains the effects of both h and hc dynamics $\mu_{II} < \mu_{hc}$

Soft form factor
It is universal for each meson
and does not depend on the Dirac structure
of the current

Subleading terms in Λ / m_b

The second term is the symmetry breaking one.
The first enters already at tree level, while the second is $O(\alpha_s)$.
However, the first could be Sudakov suppressed.

⇨ **Their relative weight is a debated issue**

SCET – matching heavy-to-light currents

Vector current

Usual heavy quark field in HQET

$$\bar{q}\gamma_\mu b \to C_3(\mu)\,\left[\bar{\chi}\gamma_\mu^\perp h_v\right] + \left\{C_4(\mu)n_{-\mu} + C_5(\mu)v_\mu\right\}\,\left[\bar{\chi}h_v\right]$$

$\chi = W\xi$ → Hard-collinear light quark field

W → Wilson line

$$n_+^\mu = (1,0,0,-1)$$

$$n_-^\mu = (1,0,0,1)$$

At tree level $C_3(m_b)=C_4(m_b)=1$ $C_5(m_b)=0$

The running is obtained solving RGEs in SCET

$$\langle M(p)|\bar{q}\Gamma b|B(v)\rangle \to \sum_i C_i(\mu)\langle M(p)|\bar{\chi}\Gamma_i h_v|B(v)\rangle + \Delta F_\Gamma$$

- $C_i(\mu)$: Wilson coeff. in SCET
- $\langle M(p)|\bar{\chi}\Gamma_i h_v|B(v)\rangle$: Soft contribution
- ΔF_Γ: Hard, symmetry-breaking factorizable contribution

B→P

$$\langle P(p)|\overline{\chi} h_v|B(v)\rangle = 2E\xi(E)$$

$$\langle P(p)|\overline{\chi}\gamma_5 h_v|B(v)\rangle = 0$$

$$\langle P(p)|\overline{\chi}\gamma_\mu^\perp h_v|B(v)\rangle = 0$$

B→V

$$\langle V(p,\varepsilon)|\overline{\chi} h_v|B(v)\rangle = 0$$

$$\langle V(p,\varepsilon)|\overline{\chi}\gamma_5 h_v|B(v)\rangle = -2m_V\xi_\parallel(E)(v\cdot\varepsilon^*)$$

$$\langle V(p,\varepsilon)|\overline{\chi}_p\gamma_\mu^\perp h_v|B(v)\rangle = 2E\xi_\perp(E)i\varepsilon_\perp^{\mu\nu}\varepsilon_\nu^*$$

Form factors

SCET provides a theoretical framework to achieve factorization of short and long distance Physics

However, non perturbative quantities (such as form factors) should be determined through other approaches (lattice, QCD sum rules).

Light-cone QCD sum rules

Symmetry relations arise when the b quark decays to a highly energetic u quark and both interact with the spectator through “soft” gluon exchange

soft contribution to the form factors

The two quarks in the final light meson are in an asymmetric configuration: one of them takes almost all the meson’ momentum

end-point of the wave function

In conventional light-cone sum rules soft contribution is related to terms proportional to the wave function at the end point (in the limit $m_b \to \infty$).

a-posteriori identification of the soft form factors

Sum rules in SCET

The starting point is a correlation function

$$\Pi(p') = i\int d^4x \quad e^{ip'\cdot x} \quad \langle 0 | T[J_M(x) J_\Gamma(0)] | B(p_B = m_B \mathrm{v}) \rangle$$

The external state is the heavy meson in contrast to usual LCsr

Interpolating current for the light meson

Current inducing the transition

We work in the frame: $p'_\perp = \mathrm{v}_\perp = 0 \qquad n_+ \cdot \mathrm{v} = n_- \cdot \mathrm{v} = 1$

2 independent variables $\Rightarrow \quad n_+ \cdot p' \cong 2E = O(m_b) \qquad 0 > n_- \cdot p' \cong O(\Lambda)$

We work for fixed values of $(n_+ \cdot p')$, so that $\Pi = \Pi(n_- \cdot p')$

B ⟶ π

$$\begin{cases} J_M(x) = J_\pi(x) = -i\bar{\chi}(x) \not{n}_+ \gamma_5 \chi(x) - i(\bar{\chi}(x) \not{n}_+ \gamma_5 q_s(x) + h.c.) \\ J_\Gamma(0) = J_0(0) = \bar{\chi}(0) h_\mathrm{v}(0) \end{cases}$$

B ⟶ V

Correlators:

$$\Pi_{\|}(n_- \cdot p') = i \int d^4x\, e^{ip' \cdot x} \left\langle 0 \left| T[J_V^{\|}(x) J_0^{\|}(0)] \right| B(v) \right\rangle$$

$$\frac{1}{2} \varepsilon^{\mu_\perp \nu_\perp \sigma\tau} n_{+\sigma} n_{-\tau}\, \Pi_\perp(n_- \cdot p') = i \int d^4x\, e^{ip' \cdot x} \left\langle 0 \left| T[J_V^{\mu_\perp}(x) J_0^{\nu_\perp}(0)] \right| B(v) \right\rangle$$

Currents

$$J_V^{\|}(x) = -i\, \overline{\chi}(x) \not{n}_+ \chi(x) - i \left(\overline{\chi}(x) \not{n}_+ q_s(x) + h.c. \right)$$

$$i\, J_V^{\mu_\perp}(x) = \overline{\chi}(x)\, i\, \not{n}_+\, \gamma^{\mu_\perp} \chi(x) + \left(\overline{\chi}(x)\, i\, \not{n}_+\, \gamma^{\mu_\perp} q_s(x) + h.c. \right)$$

$$J_0^{\|} = \overline{\chi}(-\gamma_5) h_v$$

$$J_0^{\nu_\perp} = \overline{\chi}(\gamma^{\nu_\perp}) h_v$$

Matrix elements

$$\left\langle 0 \left| J_V^{\|} \right| V(p', \varepsilon) \right\rangle = m_V (n_+ \cdot \varepsilon)\, f_V^{\|}$$

$$\left\langle 0 \left| i J_V^{\mu_\perp} \right| V(p', \varepsilon) \right\rangle = (n_+ \cdot p')\, \varepsilon^{\mu_\perp} f_V^{\perp}$$

Tree level sum rule: B →π

A similar procedure developed in parallel in
Khodjamirian, Mannel, Offen PLB 620 (05) 52
PRD 75 (07) 054013

The procedure consists in writing the correlator in two different ways (as with all QCD sum rule calculations)

Hadronic side

$$\Pi^{\rm HAD}(n_- p') = \Pi(n_- p')\Big|_{\rm res.} + \Pi(n_- p')\Big|_{\rm cont.}$$

contribution of the pion

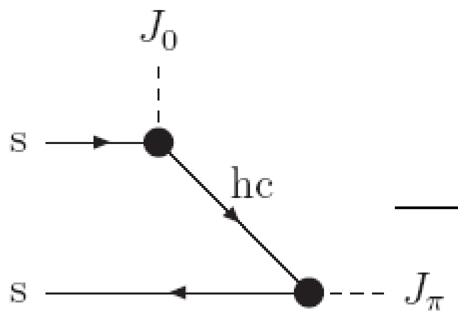
Higher states and continuum;
starts above an effective threshold ω_s

Using $\langle 0|J_\pi|\pi(p')\rangle = (n_+\cdot p')f_\pi$ $\quad \langle \pi(p')|J_0(0)|B(m_B\mathrm{v})\rangle = (n_+\cdot p')\xi_\pi(n_+\cdot p',\mu_I)$

⇒

$$\Pi(n_-\cdot p')\Big|_{res} = -\frac{(n_+\cdot p')\xi_\pi(n_+\cdot p')f_\pi}{n_-\cdot p'}$$

SCET side



Hard-collinear quark propagator $S_F^{\rm hc} = \frac{i}{n_- p' - \omega + i\eta} \frac{\not n_-}{2}$

$\omega = n_- \cdot k$

k^μ being the momentum of the soft light quark that ends up as spectator in the B

Result:

$$\Pi(n_- p') = f_B m_B \int_0^\infty d\omega \, \frac{\phi_-^B(\omega)}{\omega - n_- p' - i\eta}$$

where the B light-cone distribution amplitude enters through

$$\langle 0 | \bar{q}_s(x_-) \gamma_5 \frac{\not n_+ \not n_-}{2} h_v(0) | B(p_B) \rangle = -i f_B m_B \int d\omega' e^{-i\omega' \frac{n_+ \cdot x}{2}} \phi_-^B(\omega')$$

Has already the form of a dispersion relation in $n_- \cdot p'$

$$\Pi(n_- p') = \frac{1}{\pi} \int_0^\infty d\omega' \, \frac{\mathrm{Im}[\Pi(\omega')]}{\omega' - n_- p' - i\eta}$$

with $$\frac{1}{\pi} \mathrm{Im}[\Pi(\omega')] = f_B m_B \phi_-^B(\omega')$$

Also $\Pi(n_- \cdot p')\big|_{cont.}$ can be written as a dispersion relation. Assuming quark-hadron duality, we identify the spectral density with the expression obtained in SCET

$$\Pi(n_- \cdot p')\big|_{cont.} = f_B m_B \int_{\omega_s}^{\infty} d\omega \frac{\phi_-^B(\omega)}{\omega - n_- \cdot p' - i\eta}$$

Sum rule obtained equating the two representations:

$$-\frac{(n_+ \cdot p')\xi_\pi(n_+ \cdot p')f_\pi}{n_- \cdot p'} = f_B m_B \int_0^{\omega_s} d\omega \frac{\phi_-^B(\omega)}{\omega - n_- \cdot p' - i\eta}$$

Borel transformation $\hat{B}(\omega_M)\frac{1}{\omega - n_- \cdot p'} = \frac{1}{\omega_M} e^{-\omega/\omega_M}$

Final sum rule – tree level

$$\xi_\pi(n_+ \cdot p') = \frac{f_B m_B}{(n_+ \cdot p')f_\pi} \int_0^{\omega_s} d\omega \ e^{-\omega/\omega_M} \phi_-^B(\omega)$$

Tree level sum rule: $B \to V_{\parallel}, V_{\perp}$

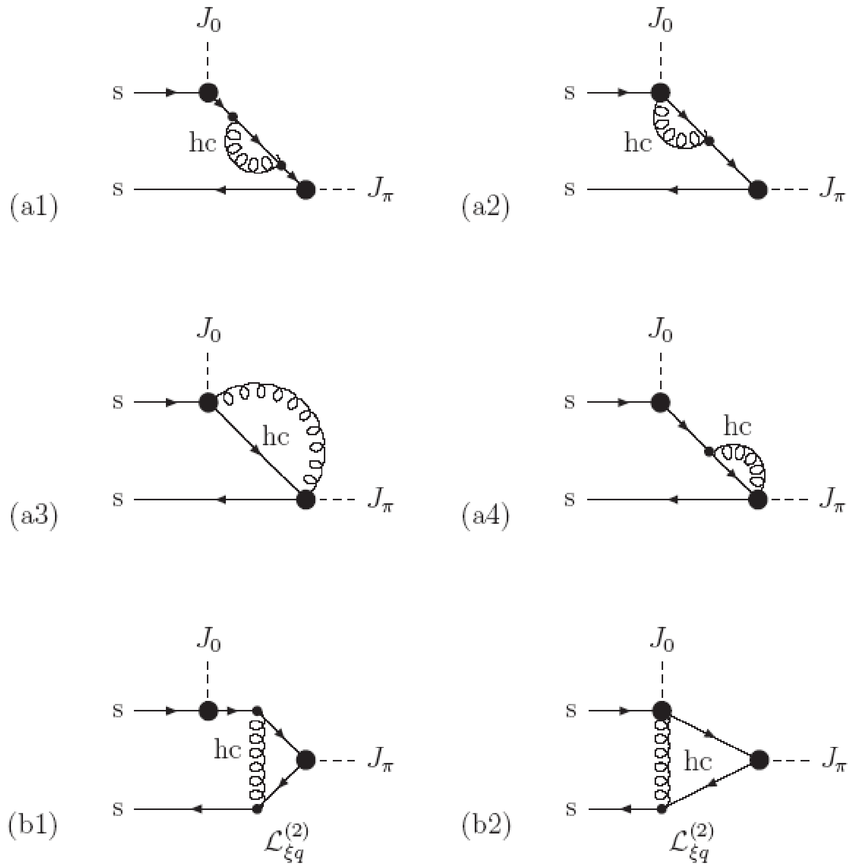
$$\hat{\xi}_{\parallel}(n_+p') \equiv \frac{n_+p'}{2m_V}\,\xi_{\parallel}(n_+p')$$

$$= \frac{f_B m_B}{f_V^{\parallel}\,(n_+p')}\,\exp\left[\frac{m_V^2}{(n_+p')\,\omega_M}\right]\int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\,\phi_B^-(\omega')$$

Identical to the pion case except that now $m_V^2 \neq 0, \quad f_\pi \to f_V^{\parallel}$

$$\xi_{\perp}(n_+p') = \frac{f_B m_B}{f_V^{\perp}\,(n_+p')}\,\exp\left[\frac{m_V^2}{(n_+p')\,\omega_M}\right]\int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\,\phi_B^-(\omega')$$

Radiative corrections from hard-collinear loops



$$\Pi(n_- \cdot p') = f_B m_B \int_0^\infty d\omega \frac{\phi_-^B(\omega)}{\omega - n_- \cdot p' - i\eta} \left\{ 1 + \frac{\alpha_s C_F}{4\pi} \left[(a1 - a4) + (b1 - b2) \right] \right\}$$

Results

$$\hat{\xi}_{\|}(\xi_{\perp}) = \frac{f_B m_B}{f_V^{\|(\perp)}(n+p')} \exp\left[\frac{m_V^2}{(n+p')\omega_M}\right] \int_0^{\omega_s} d\omega' e^{-\omega'/\omega M} \phi_{\|(\perp)}^{eff}(\omega')$$

$$\phi_{\|,\perp}^{\text{eff}}(\omega', n_+p', \mu) \equiv \left\{ -\int_{\omega'}^{\infty} d\omega\, f_{\|,\perp}(\omega,\omega',\mu)\, \frac{d\phi_B^-(\omega,\mu)}{d\omega} + \int_0^{\omega'} d\omega \left[\frac{g_{\|,\perp}(\omega,\omega',\mu)}{\omega-\omega'}\right]_+ \phi_B^-(\omega,\mu) \right\}$$

μ-dependence through $L_0 = \ln\left[\frac{\mu^2}{(n_+p')\omega'}\right]$

Renormalization scale dependence

The renormalization scale dependence of the form factors should cancel against that of the Wilson coefficients:

$$\frac{d}{d\ln\mu} C_i(\mu) = -\frac{\alpha_s C_F}{4\pi}\left(\Gamma^{(1)}_{\rm cusp}\,\ln\frac{\mu}{m_b} + 5\right) C_i(\mu) + \dots$$

This can be checked knowing the scale dependence of ϕ^-_B

$$\frac{d}{d\ln\mu}\,\phi^-_B(\omega;\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\tilde\omega\;\gamma^{(1)}_-(\omega,\tilde\omega;\mu)\;\phi^-_B(\tilde\omega;\mu) + \dots$$

↓ anomalous dimension

$$\gamma^{(1)}_-(\omega,\tilde\omega;\mu) = \left(\Gamma^{(1)}_{\rm cusp}\,\ln\frac{\mu}{\omega} - 2\right)\delta(\omega-\tilde\omega) - \Gamma^{(1)}_{\rm cusp}\,\frac{\theta(\tilde\omega-\omega)}{\tilde\omega} - \Gamma^{(1)}_{\rm cusp}\,\omega\left[\frac{\theta(\tilde\omega-\omega)}{\tilde\omega(\tilde\omega-\omega)}\right]_+ - \Gamma^{(1)}_{\rm cusp}\,\omega\left[\frac{\theta(\omega-\tilde\omega)}{\omega(\omega-\tilde\omega)}\right]_+$$

→ T. Feldmann, G. Bell JHEP 0804:061,2008.

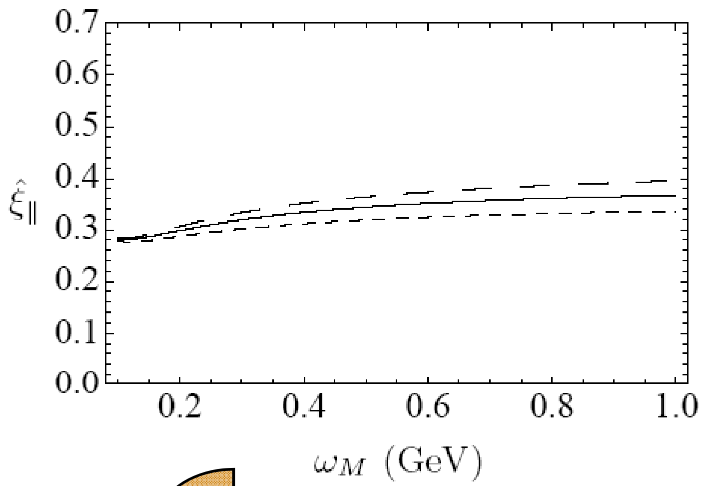
Complete cancellation of μ-dependent terms

Numerical results I: Soft form factor for longitudinal vector mesons

V=ρ

$$n_+ \cdot p' = m_B$$

$$\mu = 1 \quad \text{GeV}$$



$$\Rightarrow \omega_s = \frac{m_S^2}{m_B^2} = \{0.35, 0.4, 0.45\}$$

next vector resonance being ρ(1450)

Stability requirement: $$D = \frac{\omega_M}{\xi_\parallel}\frac{\partial \xi_\parallel}{\partial \omega_M} < 25\%$$

→ Lower bound

Further constraint:
continuum contribution not too large:

$$R = \frac{\int_0^{\omega_s} d\omega' e^{-\omega'/\omega_M} \phi_\parallel^{eff}(\omega')}{\int_0^{\infty} d\omega' e^{-\omega'/\omega_M} \phi_\parallel^{eff}(\omega')} > 50\%$$

→ Upper bound on ω_M

Numerical results I: Soft form factor for longitudinal vector mesons

$$\hat{\xi}_{\parallel}(n_+ \cdot p' = m_B) = 0.33 \pm^{0.02}_{0.02}\Big|_{\omega_s} \pm^{0.03}_{0.06}\Big|_{\omega_M} \pm^{0.03}_{0.02}\Big|_{\omega_0} \pm 0.05\Big|_{f_B}$$

dependence on the B meson wave function

$$\phi_B^-(\omega) = \frac{1}{\omega_0} e^{-\omega/\omega_0} \qquad \omega_0(1\ \mathrm{GeV}) = (0.48 \pm 0.05)\ \mathrm{GeV}$$

Lee, Neubert

Adding errors in quadrature:

$$\hat{\xi}_{\parallel}(n_+ \cdot p' = m_B) = 0.33 \pm^{0.07}_{0.09}$$

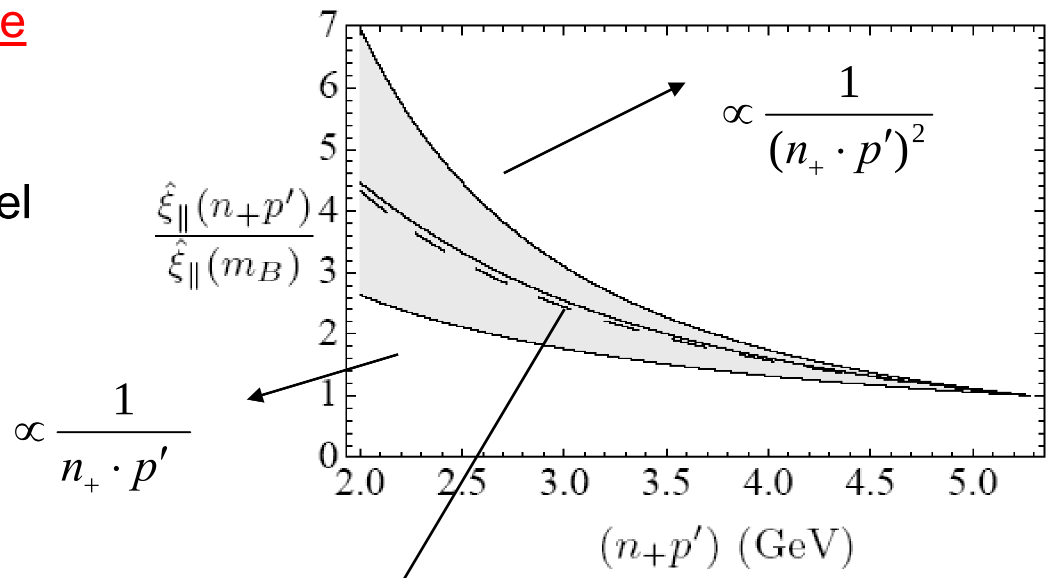
Compatible with the value obtained from traditional sum rules

Ball, Braun
Ball, Zwicky
Beneke, Feldmann, Seidel

Numerical results I: Soft form factor for longitudinal vector mesons

Energy dependence

- - - - - Tree level



Fitted curve:

$$\hat{\xi}_\parallel(n_+ \cdot p') = \hat{\xi}_\parallel(mB)\left(-a + \frac{b}{(n_+ \cdot p')} + c\,(n_+ \cdot p')\right)$$

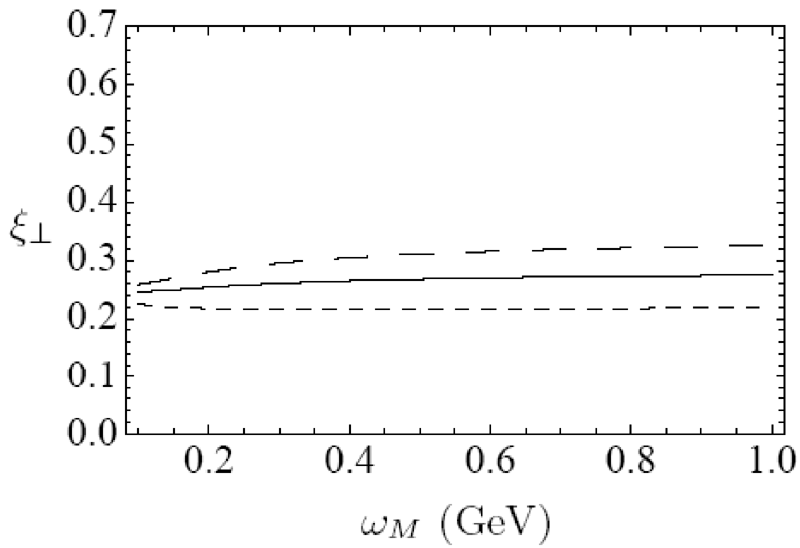
$$a = 1.92 \pm^{0.42}_{0.17} \qquad b = 12.25 \pm^{2.65}_{1.12} \qquad c = 0.114 \pm^{0.024}_{0.01}$$

→ Including only the uncertainty on ω_M

Numerical results II: Soft form factor for transverse vector mesons

$n_+ \cdot p' = m_B$

V=ρ



→ $\omega_s = \{0.20, 0.25, 0.30\}$

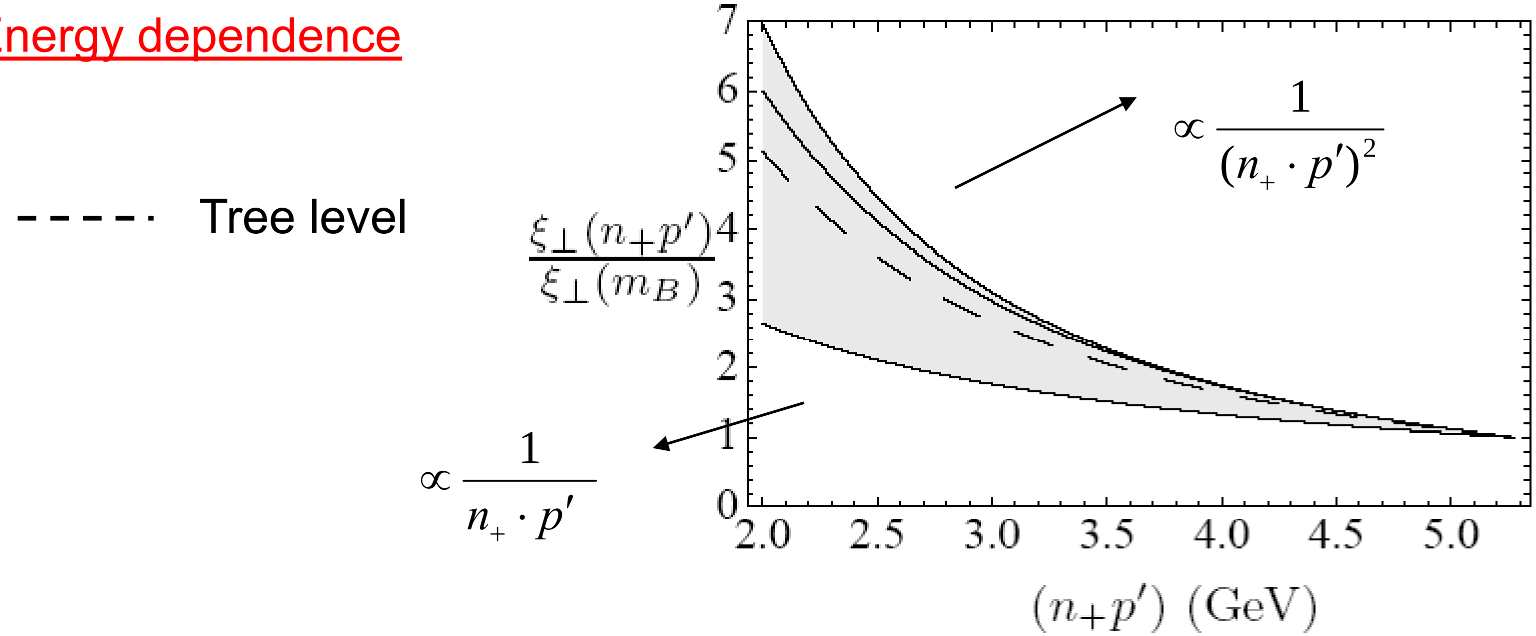
Lower values in order to get rid of $b_1(1235)$

$$\xi_\perp(n_+ \cdot p' = m_B) = 0.26 \pm^{0.03}_{0.04}\Big|_{\omega_s} \pm^{0.01}_{0.01}\Big|_{\omega_M} \pm 0.03\Big|_{\omega_0} \pm 0.04\Big|_{f_B}$$

$$\xi_\perp(n_+ \cdot p' = m_B) = 0.26 \pm^{0.06}_{0.07}$$

Numerical results II: Soft form factor for transverse vector mesons

Energy dependence



Fitted curve:

$$\xi_\perp(n_+ \cdot p') = \xi_\perp(mB)\left(-a + \frac{b}{(n_+ \cdot p')} + \frac{c}{(n_+ \cdot p')^2}\right)$$

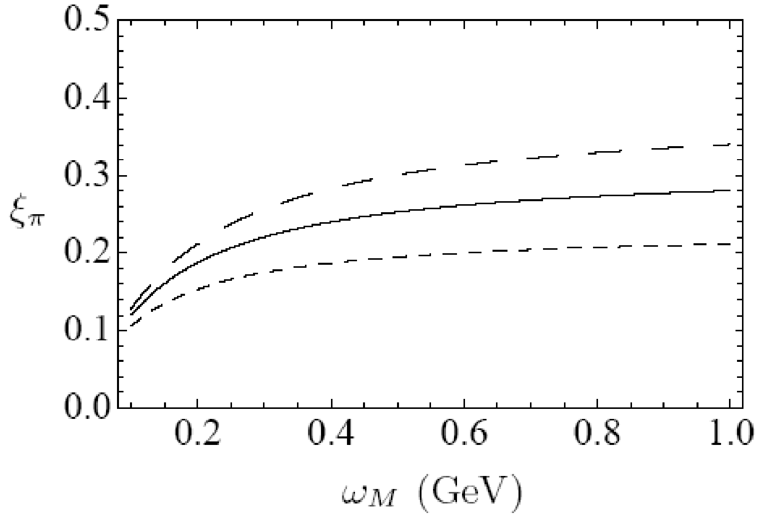
$$a = 0.26 \pm^{0.03}_{0.15} \qquad b = 2.49 \pm^{0.69}_{2.63} \qquad c = 21.76 \pm^{9.81}_{2.58}$$

→ Including only the uncertainty on ω_M

Numerical results III: Soft form factor for pseudoscalar mesons

$n_+ \cdot p' = m_B$

P=π



$\longrightarrow \omega_s = \{0.15, 0.20, 0.25\}$

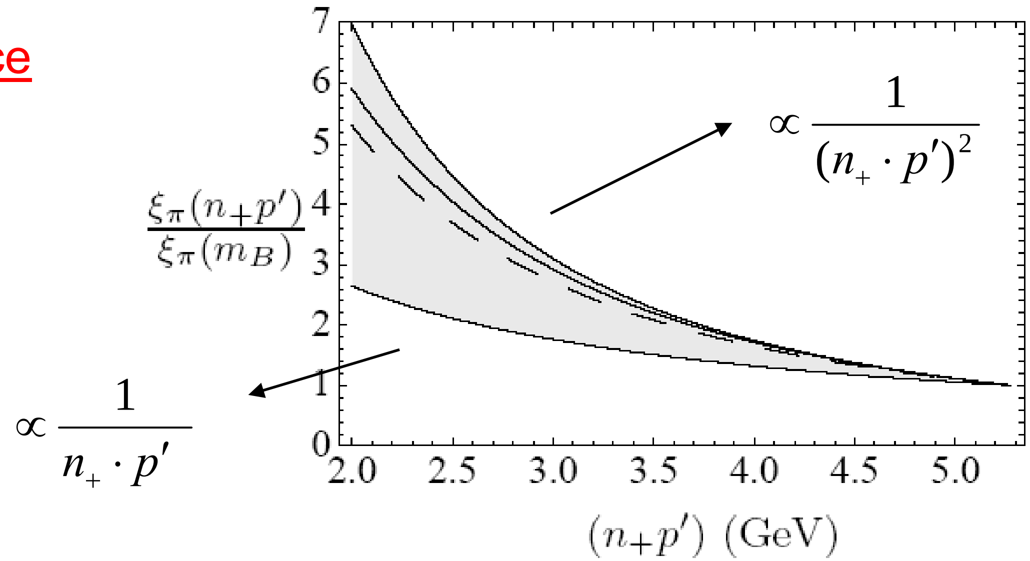
$$\xi_\pi(n_+ \cdot p' = m_B) = 0.25 \pm^{0.05}_{0.06}\Big|_{\omega_s} \pm^{0.02}_{0.03}\Big|_{\omega_M} \pm^{0.03}_{0.02}\Big|_{\omega_0} \pm 0.04\Big|_{f_B}$$

$$\xi_\pi(n_+ \cdot p' = m_B) = 0.25 \pm^{0.07}_{0.08}$$

Numerical results III: Soft form factor for pseudoscalar mesons

Energy dependence

- - - - Tree level



Fitted curve:

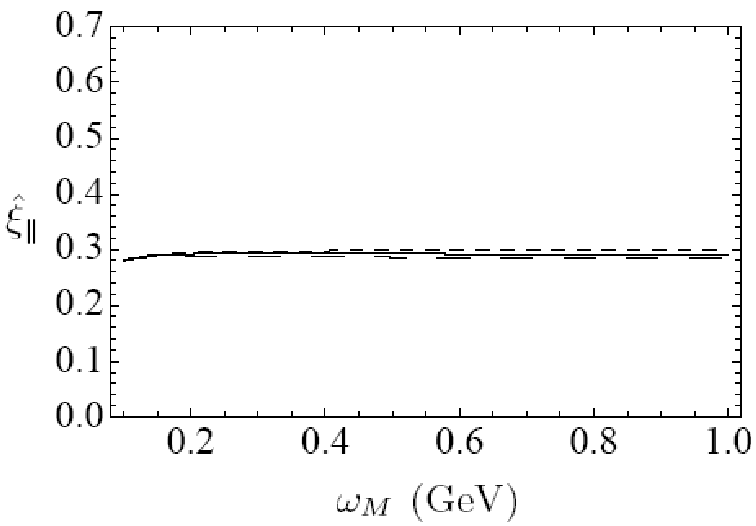
$$\xi_\pi(n_+ \cdot p') = \xi_\pi(mB)\left(-a + \frac{b}{(n_+ \cdot p')} + \frac{c}{(n_+ \cdot p')^2}\right)$$

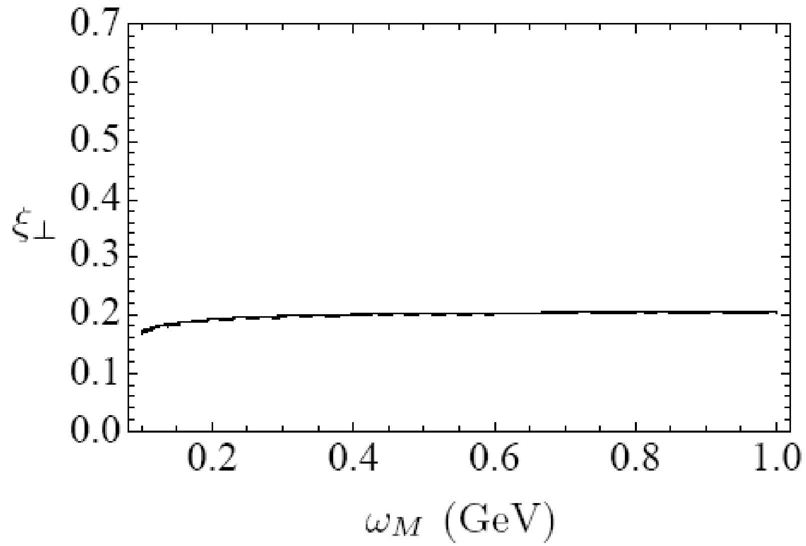
$$a = 0.23 \pm^{0.06}_{0.08} \qquad b = 2.53 \pm^{0.94}_{1.27} \qquad c = 21.00 \pm^{4.71}_{3.46}$$

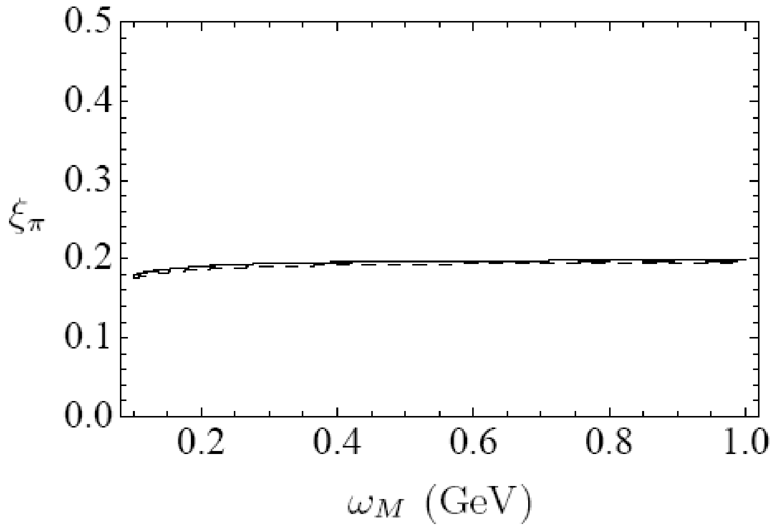
→ Including only the uncertainty on ω_S

Numerical results : modifying the sum rules

Possibility to decrease the sensitivity to sum rule parameters:
Dividing for the sum rules relative to the decay constants





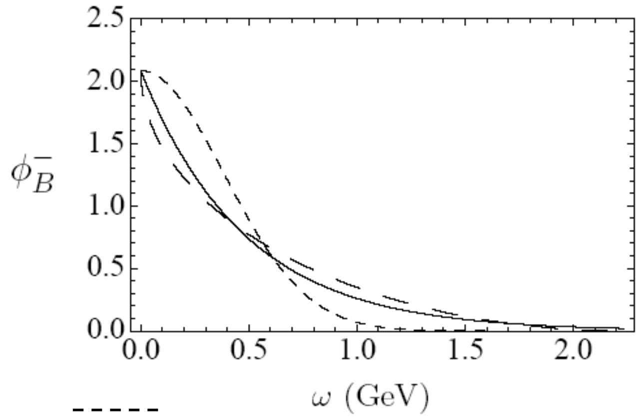


Reduction of the uncertainties due to ω_s, ω_M
but introduction of an unknown systematic error:
Are the sum rules really correlated?

Dependence on the shape of the B meson wave function

Default model:

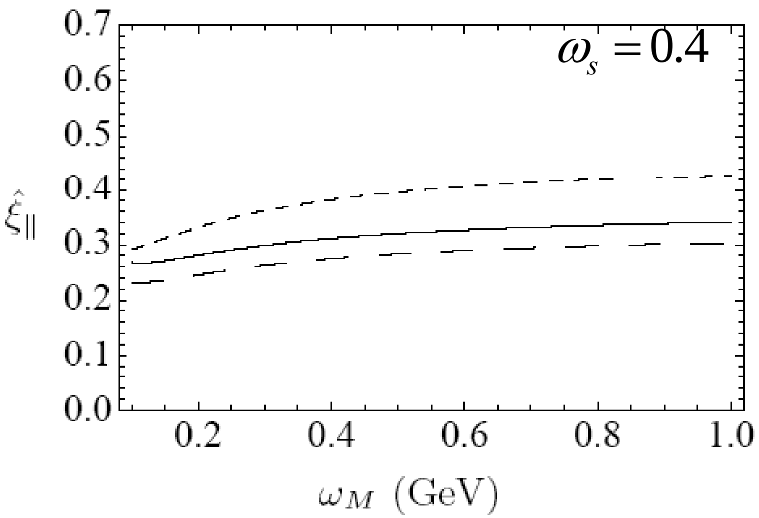
$$\phi_B^-(\omega) = \frac{1}{\omega_0}\, e^{-\omega/\omega_0} \qquad \omega_0(1\ \mathrm{GeV}) = (0.48 \pm 0.05)\ \mathrm{GeV}$$



Alternatives:

$$\phi_B^-(\omega) = \frac{1}{\omega_0}\, \exp\left[-\left(\frac{\omega}{\omega_1}\right)^2\right], \qquad \omega_1 = \frac{2\,\omega_0}{\sqrt{\pi}};$$

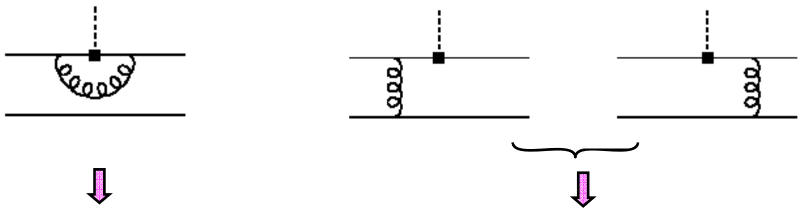
$$\phi_B^-(\omega) = \frac{1}{\omega_0}\left(1 - \sqrt{\left(2 - \frac{\omega}{\omega_2}\right)\frac{\omega}{\omega_2}}\right)\theta(\omega_2 - \omega), \qquad \omega_2 = \frac{4\,\omega_0}{4 - \pi}$$



Sizable dependence
More information on ϕ^-_B required

Symmetry breaking corrections

Stem from:



Hard vertex renormalization

Hard-collinear spectator interaction

Explicit calculation in B →π case gives

Beneke and Feldmann, NPB 592 (01) 3

$$\frac{f_0}{f_+} = \frac{2E}{M}\left(1 + \frac{\alpha_s C_F}{4\pi}[2-2L] + \frac{\alpha_s C_F}{4\pi}\frac{M(M-2E)}{(2E)^2}\frac{\Delta F_\pi}{\xi_\pi}\right)$$

Where ΔF_π parametrizes the factorizable form factor contribution:

$$\Delta F_\pi = \frac{8\pi^2 f_B f_\pi}{N_C M}\left(\int dl_+ \frac{\phi_+^B(l_+)}{l_+}\right)\left(\int du \frac{\phi_\pi(u)}{1-u}\right)$$

Factorizable form factor: B→π case

The spectator scattering terms can be identified by comparing $B \to \pi$ form factors for different Dirac structures

Tree level matching (in the light-cone gauge) gives:

$$\bar{\psi} Q \to J_0 - \frac{1}{n_+ \cdot p'} J_1 \qquad \bar{\psi} \not{n}_+ Q \to 2J_0 - \frac{2}{m_b} J_1$$

$$J_0 = \bar{\chi} h_v \qquad J_1 = \bar{\chi} g \not{A}_{hc}^{\perp} h_v$$

Using:

$$\langle \pi(p') | \bar{q} \gamma_\mu b | B(p) \rangle = f_+(q^2) \left[p_\mu + p'_\mu - \frac{m_B^2}{q^2} q_\mu \right] + f_0(q^2) \frac{m_B^2}{q^2} q_\mu$$

$$\langle \pi(p') | \bar{q} b | B(p) \rangle = \frac{m_B^2}{m_b} f_0(q^2)$$

and $q^2 = m_B (m_B - n_+ \cdot p')$

$$m_B f_0 = \langle \pi | J_0 | B \rangle - \frac{1}{n_+ p'} \langle \pi | J_1 | B \rangle \,,$$

$$(n_+ p') f_+ + m_B f_0 = 2 \langle \pi | J_0 | B \rangle - \frac{2}{m_B} \langle \pi | J_1 | B \rangle \,.$$

$$f_0 / f_+ = \frac{n_+ p'}{m_B} \left(1 - \frac{2q^2}{m_B^3} \frac{\langle \pi | J_1 | B \rangle}{\langle \pi | J_0 | B \rangle} + \mathcal{O}(\alpha_s^2) \right)$$

Factorizable form factor: B→π case

Comparing the two expressions for the ratio $\frac{f_0}{f_+}$

$$\frac{\alpha_s C_F}{4\pi}\Delta F_\pi = -\frac{2}{m_B^2}\langle\pi|J_1|B\rangle + \dots$$

In SCET-QCD-sum rules we need to consider a correlator involving the current J_1

Factorizable form factor

$$\Pi(p') = i\int d^4x\; e^{ip'\cdot x}\; \langle 0|T[J_\pi(x)J_1(0)]|B(p_B)\rangle \qquad J_1 = \overline{\chi}\; g\mathcal{A}_{hc}^{\perp} h_v$$

Hadronic side

$$\hat{B}[\Pi_1](\omega_M)\,|_{had} = -\frac{\alpha_s C_F}{4\pi}\frac{f_\pi m_B^2}{2\omega_M}\Delta F_\pi e^{-m_\pi^2/(n_+\cdot p')\omega_M}$$

ΔF_π gives access to the factorizable term

The final sum rule has the same structure in the three cases:

$$\Delta F_X(\mu, n_+p') = \frac{2f_B\omega_M(n_+p')}{m_B f_X}\, e^{m_X^2/(n_+p'\omega_M)} \times \int_0^\infty \frac{d\omega}{\omega}\,\phi_B^+(\omega,\mu)\left(1 - e^{-\omega_s/\omega_M}\,\theta(\omega-\omega_s) - e^{-\omega/\omega_M}\,\theta(\omega_s-\omega)\right)$$

with $X = \pi, \rho_\parallel, \rho_\perp$

the parameters depending on each of the three cases

Factorizable form factor

Inserting the leading order sum rule for the decay constants:

$$4\pi^2 f_X^2 \simeq M^2\, e^{m_X^2/M^2}\left(1 - e^{-s_0/M^2}\right)$$

$$M^2 = \omega_M (n_+ \cdot p'), \quad s_0 = \omega_s (n_+ \cdot p')$$

$$\Delta F_X(\mu, n_+ p') = \frac{8\pi^2 f_B f_X}{m_B} \int_0^\infty \frac{d\omega}{\omega}\, \phi_B^+(\omega,\mu) \times$$

$$\times \frac{1 - e^{-\frac{s_0}{M^2}}\, \theta\left(\omega - \frac{s_0}{n_+ p'}\right) - e^{-\omega n_+ p'/M^2}\, \theta\left(\frac{s_0}{n_+ p'} - \omega\right)}{1 - e^{-\frac{s_0}{M^2}}}$$

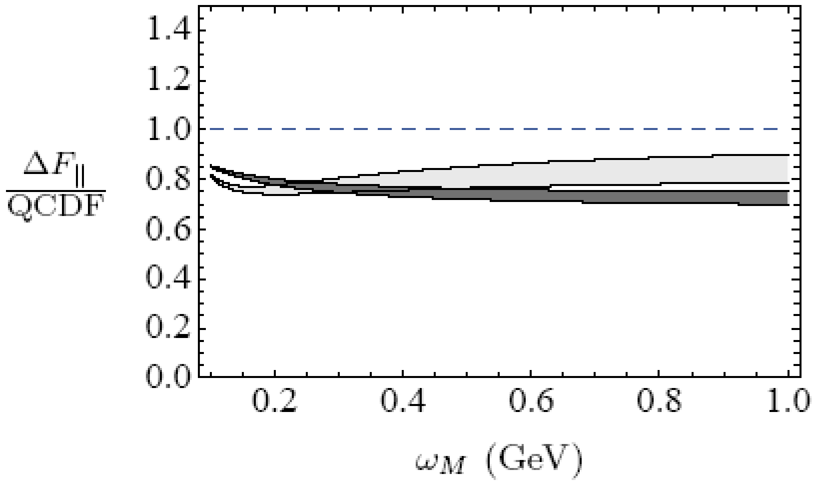
In the limit $s_0 << \omega(n_+ \cdot p')$ the QCD factorization result is recovered

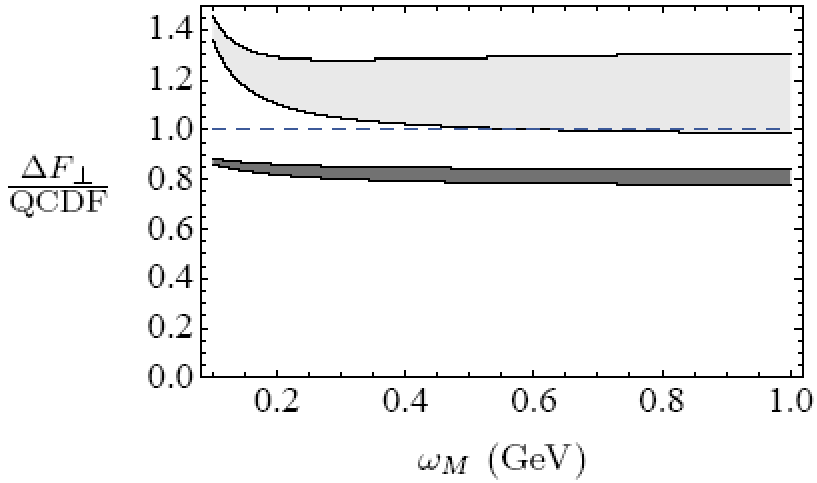
$$\Delta F_X(\mu)\Big|_{\rm QCDF,asympt.} = \frac{8\pi^2 f_B f_X}{m_B} \int_0^\infty \frac{d\omega}{\omega}\, \phi_B^+(\omega,\mu)$$

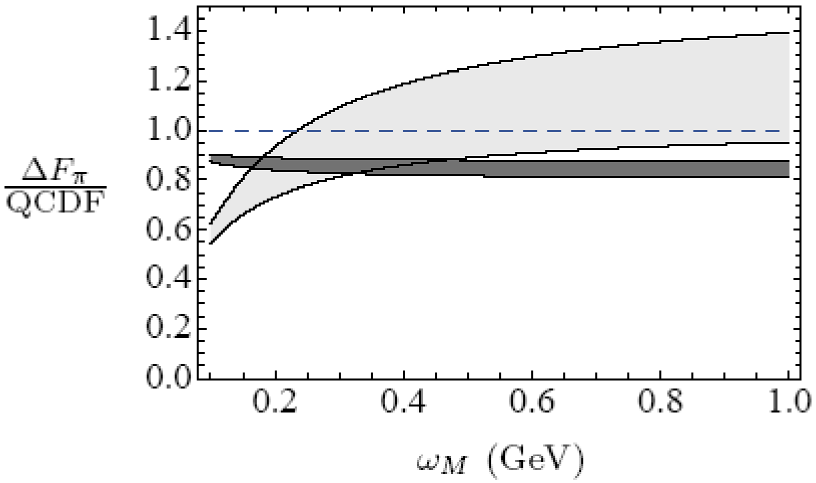
Corrections to symmetry relations: numerics

original sum rule

modified sum rule







- the dependence on ω_M is milder for the modified sum rule

- qualitative agreement at this level of accuracy with the QCDF result i.e. no enhancement of the symmetry breaking term

A SU(3) related case: $B_s \to \eta(\eta')$ form factors

Assuming quark-flavour basis $|\eta_q\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle) \qquad |\eta_s\rangle = |\bar{s}s\rangle$

to describe η-η' mixing

$$|\eta\rangle = \cos\varphi|\eta_q\rangle - \sin\varphi|\eta_s\rangle$$

$$|\eta'\rangle = \sin\varphi|\eta_q\rangle + \cos\varphi|\eta_s\rangle$$

and SU(3) to relate $B_s \to \eta(\eta')$ to $B \to K$ form factors, we obtain :

$$F_1^{B_s\to\eta} = -\sin\varphi\,\xi^{B\to K} \quad F_0^{B_s\to\eta} = -\sin\varphi\frac{2E'}{M_{B_s}}\xi^{B\to K} \quad F_T^{B_s\to\eta} = -\sin\varphi\frac{M_{B_s}+M_\eta}{M_{B_s}}\xi^{B\to K}$$

$$F_1^{B_s\to\eta'} = \cos\varphi\,\xi^{B\to K} \quad F_0^{B_s\to\eta'} = \cos\varphi\frac{2E'}{M_{B_s}}\xi^{B\to K} \quad F_T^{B_s\to\eta'} = \cos\varphi\frac{M_{B_s}+M_{\eta'}}{M_{B_s}}\xi^{B\to K}$$

with

$$\xi^{B\to K}(n_+\cdot p') = \xi^{B\to K}(m_B)\left[-a + \frac{b}{(n_+\cdot p')} + c(n_+\cdot p')\right]$$

$$\xi^{B\to K}(m_B) = 0.335^{+0.078}_{\ 0.094} \qquad a = 2.418$$

$$b = 13.765 \qquad c = 0.154.$$

Predictions for rare B_s decays:

M.V. Carlucci, P. Colangelo, FDF
Phys Rev D80 (09) 055023

$$BR(B_s \to \eta \ell^+ \ell^-) = \begin{cases} (1.2 \pm 0.3) \times 10^{-7} & \text{set A} \\ (2.6 \pm 0.7) \times 10^{-7} & \text{set B} \\ (3.4 \pm 1.8) \times 10^{-7} & \text{set C} \end{cases}$$

- set A → 3pt sum rules
- set B → Usual LC sum rules
- set C → SCET-based LC sum rules

$$BR(B_s \to \eta' \ell^+ \ell^-) = \begin{cases} (1.1 \pm 0.3) \times 10^{-7} & \text{set A} \\ (2.2 \pm 0.6) \times 10^{-7} & \text{set B} \\ (2.8 \pm 1.5) \times 10^{-7} & \text{set C} \end{cases}$$

$$BR(B_s \to \eta \tau^+ \tau^-) = \begin{cases} (3 \pm 0.5) \times 10^{-8} & \text{set A} \\ (8 \pm 1.5) \times 10^{-8} & \text{set B} \\ (10 \pm 5.5) \times 10^{-8} & \text{set C} \end{cases}$$

$$BR(B_s \to \eta' \tau^+ \tau^-) = \begin{cases} (1.55 \pm 0.3) \times 10^{-8} & \text{set A} \\ (3.85 \pm 0.75) \times 10^{-8} & \text{set B} \\ (4.7 \pm 2.5) \times 10^{-8} & \text{set C.} \end{cases}$$

SCET sum rules provide the results affected by the largest uncertainty at present.
This conservative estimate shows that these modes could be accessible in the future

Conclusions

- light cone sum rules for exclusive B decays at large recoil can be derived within SCET

- non perturbative ingredients:
  - light cone wave functions of the B meson
  - sum rule parameters

- outcome consistent with full QCD results

- main result: possibility to separate from the very beginning soft - non factorizable term from the factorizable form factor

- hierarchy among soft/hard contributions in agreement with QCD factorization

SCET

Separation of scales is achieved in effective theories

Bauer, Fleming, Pirjol and Stewart
Beneke, Chapovsky, Diehl and Feldmann
Becher, Hill, Lange and Neubert

- $\mu<\mu_I$ SCET$_I$ (integrate out hard modes)
- $\mu<\mu_{II}$ SCET$_{II}$ (integrate out also hard-collinear modes)

SCET applies to B decays to hadrons with energies much larger than their masses assuming that their constituents carry momenta collinear to the hadron momentum P^μ

$$n_+^\mu = (1,0,0,-1)$$
$$n_-^\mu = (1,0,0,1)$$

$$\Rightarrow \quad P^\mu = \underbrace{\frac{(n_+ \cdot P)}{2} n_-^\mu}_{P_-^\mu} + \underbrace{\frac{(n_- \cdot P)}{2} n_+^\mu}_{P_+^\mu} + P_\perp^\mu$$

In SCET one realizes an expansion in powers of $\lambda \approx \frac{\Lambda_{QCD}}{E}$

Momenta are classified according to the scaling of their LC coordinates $(P_+, P_-, P_\perp)$

$(P_+, P_-, P_\perp) \approx (1,1,1)$	$\rightarrow$	hard	(h)
$(P_+, P_-, P_\perp) \approx (\lambda, 1, \lambda^{1/2})$	$\rightarrow$	hard-collinear	(hc)
$(P_+, P_-, P_\perp) \approx (\lambda^2, 1, \lambda)$	$\rightarrow$	collinear	(c)
$(P_+, P_-, P_\perp) \approx (\lambda, \lambda, \lambda)$	$\rightarrow$	soft	(s)
$(P_+, P_-, P_\perp) \approx (\lambda^2, \lambda, \lambda^{3/2})$	$\rightarrow$	soft-collinear	(sc)
$(P_+, P_-, P_\perp) \approx (\lambda^2, \lambda^2, \lambda^2)$	$\rightarrow$	ultrasoft	(us)

SCET

Let us consider a light energetic hadron with P^μ in the z-direction and $P^2 = m_H^2 << E^2$

Since $|P| \cong E - \frac{m_H^2}{2E}$

One can decompose: $$P^\mu = \underbrace{\left(E - \frac{m_H^2}{4E} + \ldots \right) n_-^\mu}_{P_-^\mu} + \underbrace{\left(\frac{m_H^2}{4E} + \ldots \right) n_+^\mu}_{P_+^\mu}$$

P_+ is subleading with respect to P_-

SCET – effective fields, lagrangian, Feynmann rules

One writes $p = \tilde{p} + k$ with $\tilde{p} = p_\perp + p_-$ and $k = p_+$ $\quad k^2 << \tilde{p}^2$

Then defines $\psi = \sum_{\tilde{p}} e^{-i\tilde{p}x} \psi_p(x)$

Large and small components are separated by using projection operators

$$\xi_p = \frac{\not{n}_- \not{n}_+}{4} \psi_p(x) \qquad \eta_p = \frac{\not{n}_+ \not{n}_-}{4} \psi_p(x)$$

with

$$\frac{\not{n}_- \not{n}_+}{4} \xi_p = \xi_p \qquad \not{n}_- \xi_p = 0$$

$$\frac{\not{n}_+ \not{n}_-}{4} \eta_p = \eta_p \qquad \not{n}_+ \eta_p = 0$$

→ η_p is removed using the equation of motion

The gluon field is splitted into *collinear* and *soft* components $A^\mu = A_c^\mu + A_s^\mu$
and again one redefines $A_c(x) \rightarrow e^{-i\tilde{q}x} A_q(x)$

SCET – effective fields, lagrangian, Feynmann rules

Effective Lagrangian (D contains only the soft gluon field)

$$L = \bar{\xi}_p \left(i n_- \cdot D + \frac{p_\perp^2}{n_+ \cdot p} \right) \frac{\not{n}_+}{2} \xi_p +$$
$$\bar{\xi}_{p+q} \left[g n \cdot A_q + g \not{A}_q^\perp \frac{\not{p}_\perp}{n_+ \cdot p} + \frac{\not{p}_\perp + \not{q}_\perp}{n_+ \cdot (p+q)} g \not{A}_q^\perp - \frac{\not{p}_\perp + \not{q}_\perp}{n_+ \cdot (p+q)} g n_+ \cdot A_q \frac{\not{p}_\perp}{n_+ \cdot p} \right] \frac{\not{n}_+}{2} \xi_p + \ldots O(\lambda)$$

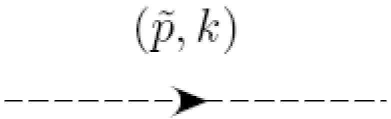
To be supplemented by the usual HQET lagrangian for the heavy quark

$$L_{HQET} = \bar{h}_v i v \cdot D h_v$$

SCET – effective fields, lagrangian, Feynmann rules

Bauer, Fleming, Pirjol and Stewart, PRD 63 (01) 114020

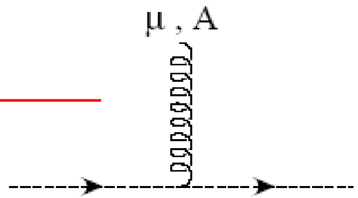
Feynmann rules



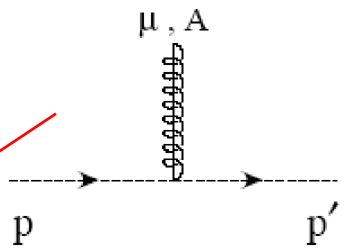
$$= i\,\frac{\not n}{2}\,\frac{\bar n\cdot p}{n\cdot k\,\bar n\cdot p + p_\perp^2 + i\epsilon}$$

Collinear quark propagator

soft gluon

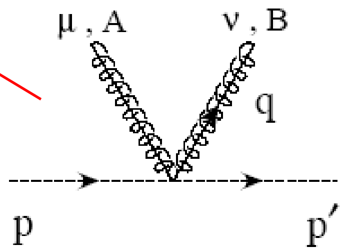


$$= ig\,T^A\,n_\mu\,\frac{\not{\bar n}}{2}$$



$$= ig\,T^A\left[n_\mu + \frac{\gamma_\mu^\perp \not p_\perp}{\bar n\cdot p} + \frac{\not p'_\perp \gamma_\mu^\perp}{\bar n\cdot p'} - \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu\right]\frac{\not{\bar n}}{2}$$

collinear gluons



$$= \frac{ig^2\,T^A\,T^B}{\bar n\cdot(p-q)}\left[\gamma_\mu^\perp\gamma_\nu^\perp - \frac{\gamma_\mu^\perp \not p_\perp}{\bar n\cdot p}\bar n_\nu - \frac{\not p'_\perp \gamma_\nu^\perp}{\bar n\cdot p'}\bar n_\mu + \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \bar n_\nu\right]\frac{\not{\bar n}}{2}$$

$$+ \frac{ig^2\,T^B\,T^A}{\bar n\cdot(q+p')}\left[\gamma_\nu^\perp\gamma_\mu^\perp - \frac{\gamma_\nu^\perp \not p_\perp}{\bar n\cdot p}\bar n_\mu - \frac{\not p'_\perp \gamma_\mu^\perp}{\bar n\cdot p'}\bar n_\nu + \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \bar n_\nu\right]\frac{\not{\bar n}}{2}$$

At next to leading order

$$f_+(q^2) = \xi(E)\left[C_4 + \frac{E}{m_B}C_5\right]$$

$$f_0(q^2) = \xi(E)2\frac{E}{m_B}\left[C_4 + \left(1 - \frac{E}{m_B}\right)C_5\right]$$

$$f_T(q^2) = \xi(E)C_{11}$$

$$A_1(q^2) = \xi_\perp(E)2\frac{E}{m_B}C_3$$

$$A_2(q^2) = \xi_\perp(E)C_3$$

$$V(q^2) = \xi_\perp(E)C_3$$

$$A_0(q^2) = \xi_\parallel(E)\left[C_4 + C_5\left(1 - \frac{E}{m_B}\right)\right]$$

$$T_1(q^2) = \xi_\perp(E)C_9$$

$$T_2(q^2) = \xi_\perp(E)2\frac{E}{m_B}C_9$$

$$T_3(q^2) = \xi_\perp(E)C_9$$

Form factor ratios

ratio:	$(\xi_\pi/f_\pi) : (\hat{\xi}_\rho^{\parallel}/f_\rho^{\parallel})$	$(\xi_\rho^{\perp}/f_\rho^{\perp}) : (\hat{\xi}_\rho^{\parallel}/f_\rho^{\parallel})$
original sum rule	$1.18^{+0.37}_{-0.32}$	$1.02^{+0.28}_{-0.21}$
modified sum rule	$1.05^{+0.06}_{-0.04} \pm (?)_{\text{syst.}}$	$0.87^{+0.06}_{-0.12} \pm (?)_{\text{syst.}}$