Effective field theories for lepton dipole moments: updates and applications

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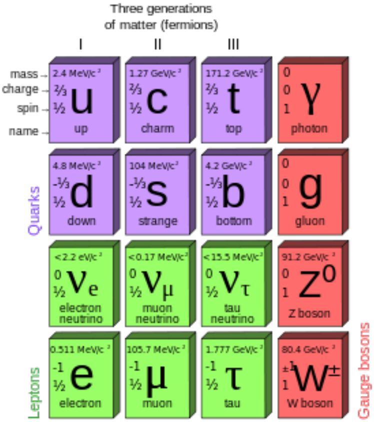
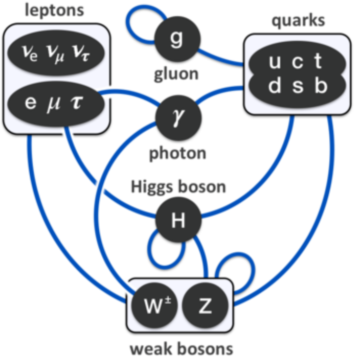
The Standard Model of particle physics





The Standard Model of particle physics is the theory describing three of the four known fundamental forces in the Universe.

Round the corner

High-energy physics is entering the ballistic age.

Low-energy physics is progressing towards unprecedented sensitivities on (beyond the) Standard Model observables.

Either new physics is weakly coupled with standard matter or we are in presence of a considerable scale-separation between the world that we know and what is beyond it.

Effective field theories are the best tools to describe a system with well-defined scale separations.

Lepton dipole moments

Dimension-six operators contribute to the Wilson coefficients C_{TL} and C_{TR} of the dipole interaction:

$$V^{\mu} = \frac{1}{\Lambda^2} i\sigma^{\mu\nu} \left(C_{TL}(p_\gamma^2)\,\omega_L + C_{TR}(p_\gamma^2)\,\omega_R \right) (p_\gamma)_\nu \,.$$

Anomalous magnetic and electric-dipole moments:

$$\begin{aligned} a_l &\propto \Re(C_{TR} + C_{TL})\big|_{p_\gamma^2 \to 0} && \text{CPC} \\ d_l &\propto \Im(C_{TR} - C_{TL})\big|_{p_\gamma^2 \to 0} && \text{CPV} \end{aligned}$$

In terms of effective coefficients:

$$a_l = \frac{2}{e} \frac{2^{1/4} m_l}{\sqrt{G_F}\Lambda^2} \Re \mathcal{C}_{e\gamma}^{ll}, \qquad d_l = \frac{2^{1/4}}{\sqrt{G_F}\Lambda^2} \Im \mathcal{C}_{e\gamma}^{ll}.$$

If flavour is not diagonal, then the momenta are “transitional”.

Low-energy Effective Field Theory

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QED} + \mathcal{L}_{\rm QCD} + \frac{1}{\Lambda^2}\sum_i C_i Q_i,$$

and the explicit structure of the operators is given by

Dipole			
$Q_{e\gamma}$	$e m_r (\bar{l}_p \sigma^{\mu\nu} P_L l_r) F_{\mu\nu} + \text{H.c.}$		
Scalar/Tensorial		**Vectorial**	
Q_S	$(\bar{l}_p P_L l_r)(\bar{l}_s P_L l_t) + \text{H.c.}$	Q_{VLL}	$(\bar{l}_p \gamma^\mu P_L l_r)(\bar{l}_s \gamma_\mu P_L l_t)$
		Q_{VLR}	$(\bar{l}_p \gamma^\mu P_L l_r)(\bar{l}_s \gamma_\mu P_R l_t)$
		Q_{VRR}	$(\bar{l}_p \gamma^\mu P_R l_r)(\bar{l}_s \gamma_\mu P_R l_t)$
$Q_{Slq(1)}$	$(\bar{l}_p P_L l_r)(\bar{q}_s P_L q_t) + \text{H.c.}$	Q_{VlqLL}	$(\bar{l}_p \gamma^\mu P_L l_r)(\bar{q}_s \gamma_\mu P_L q_t)$
$Q_{Slq(2)}$	$(\bar{l}_p P_L l_r)(\bar{q}_s P_R q_t) + \text{H.c.}$	Q_{VlqLR}	$(\bar{l}_p \gamma^\mu P_L l_r)(\bar{q}_s \gamma_\mu P_R q_t)$
Q_{Tlq}	$(\bar{l}_p \sigma^{\mu\nu} P_L l_r)(\bar{q}_s \sigma_{\mu\nu} P_L q_t) + \text{H.c.}$	Q_{VlqRL}	$(\bar{l}_p \gamma^\mu P_R l_r)(\bar{q}_s \gamma_\mu P_L q_t)$
		Q_{VlqRR}	$(\bar{l}_p \gamma^\mu P_R l_r)(\bar{q}_s \gamma_\mu P_R q_t)$

Muon LFV transitions below the EWSB scale

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QED} + \mathcal{L}_{\rm QCD}$$
$$+ \frac{1}{\Lambda^2}\Big\{ C_L^D O_L^D + \sum_{f=q,\ell}\left(C_{ff}^{V\,LL} O_{ff}^{V\,LL} + C_{ff}^{V\,LR} O_{ff}^{V\,LR} + C_{ff}^{S\,LL} O_{ff}^{S\,LL}\right)$$
$$+ \sum_{h=q,\tau}\left(C_{hh}^{T\,LL} O_{hh}^{T\,LL} + C_{hh}^{S\,LR}\, O_{hh}^{S\,LR}\right) + C_{gg}^L O_{gg}^L + L \leftrightarrow R \Big\} + \text{h.c.},$$

and the explicit structure of the operators is given by

$$\boxed{O_L^D} = e\, m_\mu\, (\bar e \sigma^{\mu\nu} P_L \mu)\, F_{\mu\nu},$$
$$O_{ff}^{V\,LL} = (\bar e \gamma^\mu P_L \mu)\left(\bar f \gamma_\mu P_L f\right),$$
$$O_{ff}^{V\,LR} = (\bar e \gamma^\mu P_L \mu)\left(\bar f \gamma_\mu P_R f\right),$$
$$\boxed{O_{ff}^{S\,LL}} = (\bar e P_L \mu)\left(\bar f P_L f\right),$$
$$O_{hh}^{S\,LR} = (\bar e P_L \mu)\left(\bar h P_R h\right),$$
$$O_{hh}^{T\,LL} = (\bar e \sigma_{\mu\nu} P_L \mu)\left(\bar h \sigma^{\mu\nu} P_L h\right),$$
$$O_{gg}^L = \alpha_s\, m_\mu G_F\, (\bar e P_L \mu)\, G_{\mu\nu}^a G_a^{\mu\nu}.$$

Low-energy LFV observables

Neutrinoless radiative decay

$$\mathrm{Br}\left(\mu \to e\gamma\right) = \frac{\alpha_e m_\mu^5}{\Lambda^4 \Gamma_\mu}\left(\left|C_L^D\right|^2 + \left|C_R^D\right|^2\right) .$$

Neutrinoless three-body decay

$$\begin{aligned}
\mathrm{Br}(\mu \to 3e) \quad &= \quad \frac{\alpha_e^2 m_\mu^5}{12\pi\Lambda^4\Gamma_\mu}\left(\left|C_L^D\right|^2 + \left|C_R^D\right|^2\right)\left(8\log\left[\frac{m_\mu}{m_e}\right] - 11\right) \\
&+ \quad \frac{m_\mu^5}{3(16\pi)^3\Lambda^4\Gamma_\mu}\left(\left|C_{ee}^{S\ LL}\right|^2 + 16\left|C_{ee}^{V\ LL}\right|^2 + 8\left|C_{ee}^{V\ LR}\right|^2\right. \\
&\qquad \left. + \left|C_{ee}^{S\ RR}\right|^2 + 16\left|C_{ee}^{V\ RR}\right|^2 + 8\left|C_{ee}^{V\ RL}\right|^2\right) .
\end{aligned}$$

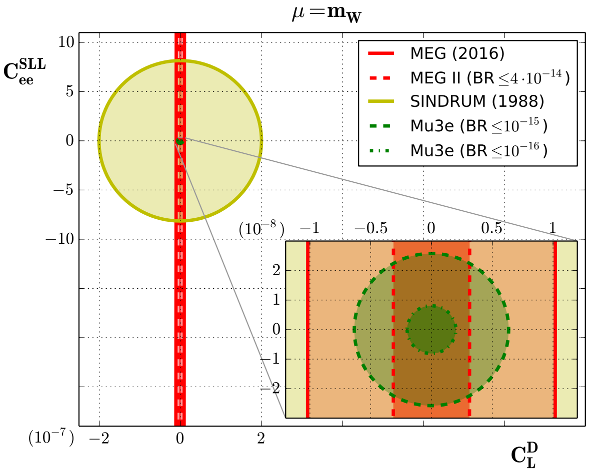
Coherent conversion in nuclei

$$\Gamma_{\mu\to e}^N = \frac{m_\mu^5}{4\Lambda^4}\left|e\,C_L^D\ D_N + 4\left(G_F m_\mu m_p \tilde{C}_{(p)}^{SL} S_N^{(p)} + \tilde{C}_{(p)}^{VR}\ V_N^{(p)} + p \to n\right)\right|^2 + L \leftrightarrow R.$$

Interplay between $\mu \to e\gamma$ and $\mu \to 3e$

A. Crivellin, S. Davidson, GMP and A. Signer, arXiv:1611.03409 [hep-ph].

Below the EW scale, four-fermion vs dipole:



Dipole evolution below the EWSB scale

At the two-loop level, in the tHV scheme:

$$\dot{C}_L^D = 16\,\alpha_e\, Q_l^2 \boxed{C_L^D} - \frac{Q_l}{(4\pi)}\frac{m_e}{m_\mu}\boxed{C_{ee}^{S\,LL}} - \frac{Q_l}{(4\pi)}\boxed{C_{\mu\mu}^{S\,LL}}$$
$$+ \sum_h \frac{8Q_h}{(4\pi)}\frac{m_h}{m_\mu} N_{c,h}\boxed{C_{hh}^{T\,LL}}\Theta(\mu - m_h)$$
$$- \frac{\alpha_e Q_l^3}{(4\pi)^2}\left(\frac{116}{9}\boxed{C_{ee}^{V\,RR}} + \frac{116}{9}\boxed{C_{\mu\mu}^{V\,RR}} - \frac{122}{9}\boxed{C_{\mu\mu}^{V\,RL}} - \left(\frac{50}{9} + 8\,\frac{m_e}{m_\mu}\right)\boxed{C_{ee}^{V\,RL}}\right)$$
$$- \sum_h \frac{\alpha_e}{(4\pi)^2}\left(6Q_h^2 Q_l + \frac{4Q_h Q_l^2}{9}\right) N_{c,h}\boxed{C_{hh}^{V\,RR}}\Theta(\mu - m_h)$$
$$- \sum_h \frac{\alpha_e}{(4\pi)^2}\left(-6Q_h^2 Q_l + \frac{4Q_h Q_l^2}{9}\right) N_{c,h}\boxed{C_{hh}^{V\,RL}}\Theta(\mu - m_h)$$
$$- \sum_h \frac{\alpha_e}{(4\pi)^2}\,4Q_h^2 Q_l N_{c,h}\frac{m_h}{m_\mu}\boxed{C_{hh}^{S\,LR}}\Theta(\mu - m_h) + [\ldots]\,.$$

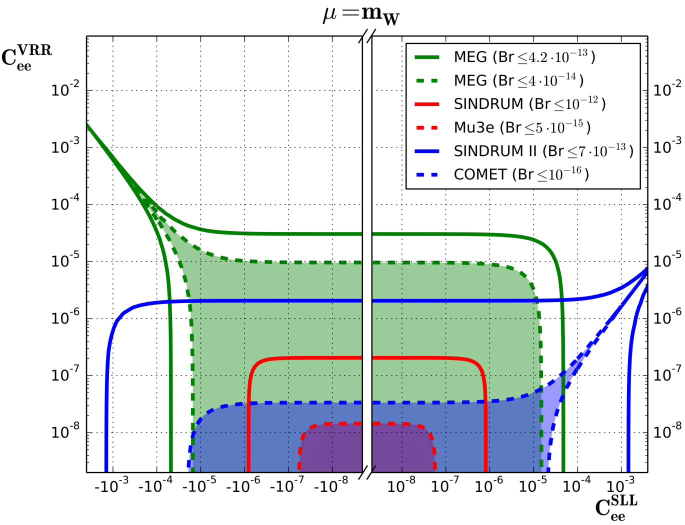
A. Crivellin, S. Davidson, GMP and A. Signer, JHEP **1705** (2017) 117.

In absence of interplay at the EWSB scale

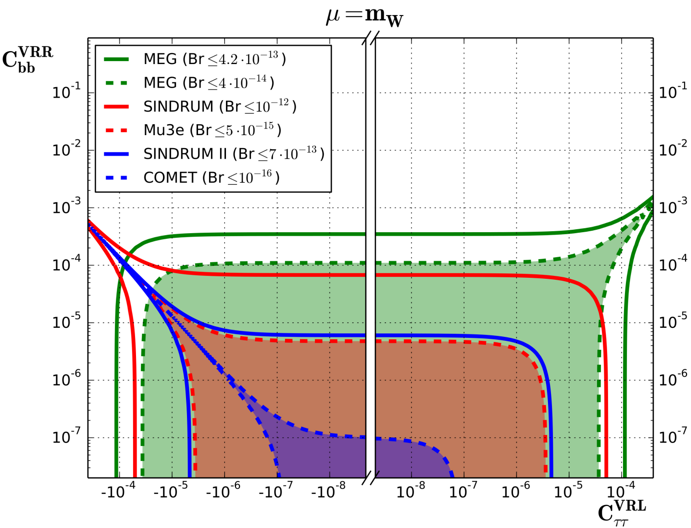
	$\mathrm{Br}(\mu^+ \to e^+\gamma)$		$\mathrm{Br}(\mu^+ \to e^+e^-e^+)$		$\mathrm{Br}^{\mathrm{Au/Al}}_{\mu\to e}$	
	$4.2\cdot10^{-13}$	$4.0\cdot10^{-14}$	$1.0\cdot10^{-12}$	$5.0\cdot10^{-15}$	$7.0\cdot10^{-13}$	$1.0\cdot10^{-16}$
C_L^D	$1.0\cdot10^{-8}$	$3.1\cdot10^{-9}$	$2.0\cdot10^{-7}$	$1.4\cdot10^{-8}$	$2.0\cdot10^{-7}$	$2.9\cdot10^{-9}$
$C_{ee}^{S\,LL}$	$4.8\cdot10^{-5}$	$1.5\cdot10^{-5}$	$8.1\cdot10^{-7}$	$5.8\cdot10^{-8}$	$1.4\cdot10^{-3}$	$2.1\cdot10^{-5}$
$C_{\mu\mu}^{S\,LL}$	$2.3\cdot10^{-7}$	$7.2\cdot10^{-8}$	$4.6\cdot10^{-6}$	$3.3\cdot10^{-7}$	$7.1\cdot10^{-6}$	$1.0\cdot10^{-7}$
$C_{\tau\tau}^{S\,LL}$	$1.2\cdot10^{-6}$	$3.7\cdot10^{-7}$	$2.4\cdot10^{-5}$	$1.7\cdot10^{-6}$	$2.4\cdot10^{-5}$	$3.5\cdot10^{-7}$
$C_{\tau\tau}^{T\,LL}$	$2.9\cdot10^{-9}$	$9.0\cdot10^{-10}$	$5.7\cdot10^{-8}$	$4.1\cdot10^{-9}$	$5.9\cdot10^{-8}$	$8.5\cdot10^{-10}$
$C_{bb}^{S\,LL}$	$2.8\cdot10^{-6}$	$8.6\cdot10^{-7}$	$5.4\cdot10^{-5}$	$3.8\cdot10^{-6}$	$9.0\cdot10^{-7}$	$1.2\cdot10^{-8}$
$C_{bb}^{T\,LL}$	$2.1\cdot10^{-9}$	$6.4\cdot10^{-10}$	$4.1\cdot10^{-8}$	$2.9\cdot10^{-9}$	$4.2\cdot10^{-8}$	$6.0\cdot10^{-10}$
$C_{ee}^{V\,RR}$	$3.0\cdot10^{-5}$	$9.4\cdot10^{-6}$	$2.1\cdot10^{-7}$	$1.5\cdot10^{-8}$	$2.1\cdot10^{-6}$	$3.5\cdot10^{-8}$
$C_{\mu\mu}^{V\,RR}$	$3.0\cdot10^{-5}$	$9.4\cdot10^{-6}$	$1.6\cdot10^{-5}$	$1.1\cdot10^{-6}$	$2.1\cdot10^{-6}$	$3.5\cdot10^{-8}$
$C_{\tau\tau}^{V\,RR}$	$1.0\cdot10^{-4}$	$3.2\cdot10^{-5}$	$5.3\cdot10^{-5}$	$3.8\cdot10^{-6}$	$4.8\cdot10^{-6}$	$7.9\cdot10^{-8}$
$C_{bb}^{V\,RR}$	$3.5\cdot10^{-4}$	$1.1\cdot10^{-4}$	$6.7\cdot10^{-5}$	$4.8\cdot10^{-6}$	$6.0\cdot10^{-6}$	$1.0\cdot10^{-7}$
C_{bb}^{RA}	$4.2\cdot10^{-4}$	$1.3\cdot10^{-4}$	$6.5\cdot10^{-3}$	$4.6\cdot10^{-4}$	$1.3\cdot10^{-3}$	$2.2\cdot10^{-5}$
C_{bb}^{RV}	$2.1\cdot10^{-3}$	$6.4\cdot10^{-4}$	$6.7\cdot10^{-5}$	$4.7\cdot10^{-6}$	$6.0\cdot10^{-6}$	$1.0\cdot10^{-7}$

Limits on the various coefficients $C_i(m_W)$ from current and future experimental constraints, assuming that (at the high scale m_W) only one coefficient at a time is non-vanishing and not including operator-dependent efficiency corrections.

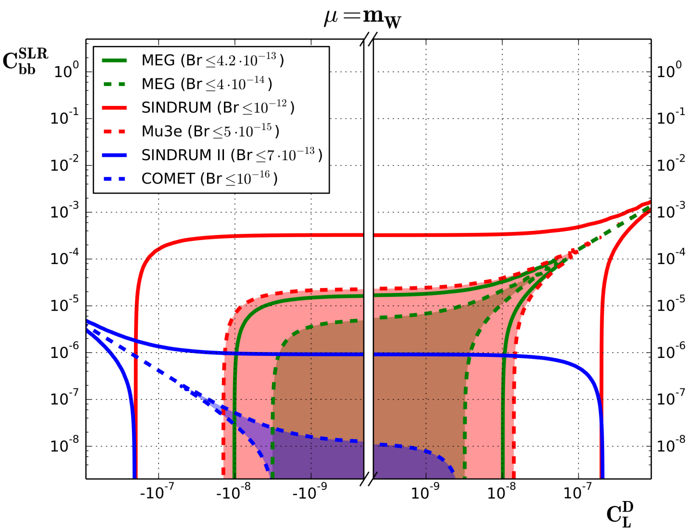
Interplay at the EWSB scale
Mu3e money plot



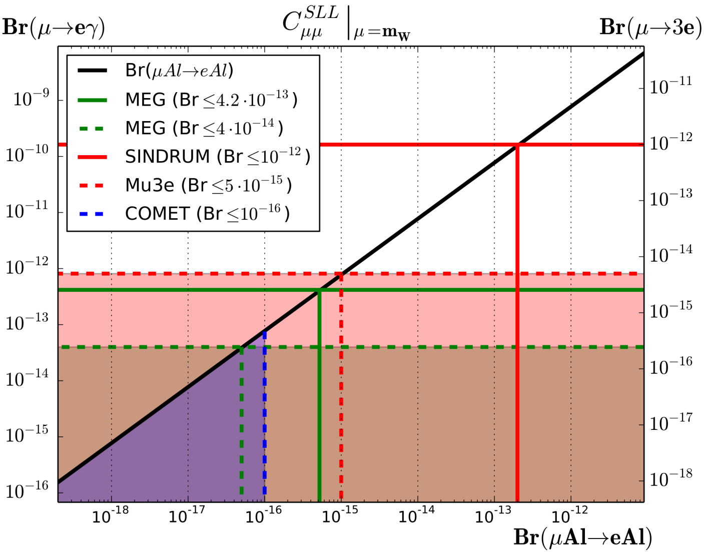
Interplay at the EWSB scale
COMET/Mu2e money plot (1)



Interplay at the EWSB scale
COMET/Mu2e money plot (2)



MEG/MEG-II money plot



The doubly Charged $SU(2)_L$-singlet scalar

Minimal model for neutrino mass generation

SM + 1 $SU(2)_L$-singlet doubly charged scalar: $S_R^{\pm\pm}$

It couples only with right-handed charged leptons:

$$\begin{aligned}\Delta\mathcal{L} &= \left(D_\mu S^{++}\right)^\dagger \left(D^\mu S^{++}\right) + \left(\lambda_{ab}\,\overline{(\ell_R)^c_a}\ell_{Rb}\,S^{++} + \text{h.c.}\right) \\ &+ \lambda_2\left(H^\dagger H\right)\left(S^{--}S^{++}\right) + \lambda_4\left(S^{--}S^{++}\right)^2 + [\text{inv.}]\end{aligned}$$

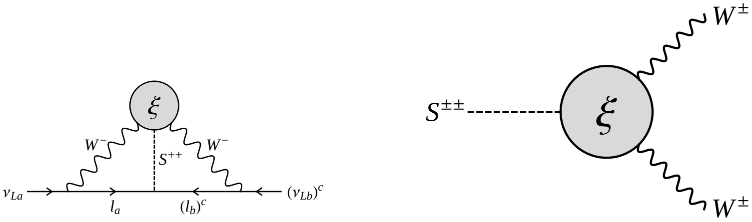
λ_{ab} consists of six independent complex parameters.

Lepton Flavour Violation

S. F. King, A. Merle and L. Panizzi, JHEP 1411 (2014) 124

The doubly charged $SU(2)_L$-singlet scalar

Neutrino masses are generated at the three-loop level:



Effective Field Theory:

$$\frac{\xi}{\Lambda^3} S^{--} \left[H^+ H^+ \left(D_\mu H^0\right)\left(D^\mu H^0\right) - 2 H^+ H^0 \left(D_\mu H^+\right)\left(D^\mu H^0\right) + H^0 H^0 \left(D_\mu H^+\right)\left(D^\mu H^+\right)\right] + \text{h.c.}$$

S. F. King, A. Merle and L. Panizzi, JHEP 1411 (2014) 124

Current low-energy experimental limits

$$\begin{aligned}
\mathrm{Br}\left[\tau^\mp \to e^\mp e^\pm e^\mp\right] &\leq 1.4\times10^{-8}\\
\mathrm{Br}\left[\tau^\mp \to \mu^\mp \mu^\pm \mu^\mp\right] &\leq 1.2\times10^{-8}\\
\mathrm{Br}\left[\tau^\mp \to e^\mp \mu^\pm \mu^\mp\right] &\leq 1.6\times10^{-8}\\
\mathrm{Br}\left[\tau^\mp \to \mu^\mp e^\pm \mu^\mp\right] &\leq 9.8\times10^{-9}\\
\mathrm{Br}\left[\tau^\mp \to \mu^\mp e^\pm e^\mp\right] &\leq 1.1\times10^{-8}\\
\mathrm{Br}\left[\tau^\mp \to e^\mp \mu^\pm e^\mp\right] &\leq 8.4\times10^{-8}\\
\mathrm{Br}\left[\mu^\mp \to e^\mp e^\pm e^\mp\right] &\leq 1.0\times10^{-12}
\end{aligned}$$

$$\mathcal{P}\left(\bar{M}-M\right) = 2.4\times10^{-10}$$

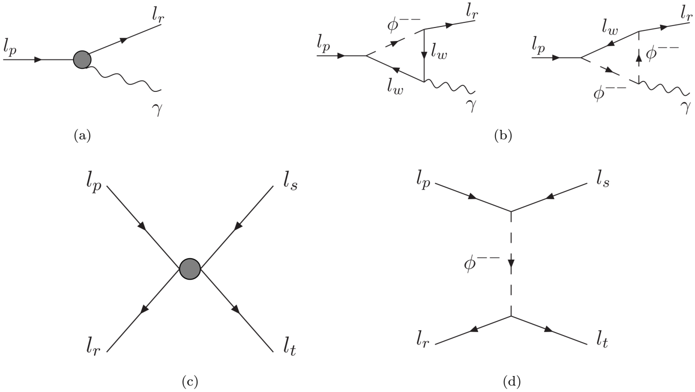
(for right-handed currents)

$$\mathrm{Br}^{\mathrm{Au}}_{\mu\to e} \leq 7\times10^{-13}$$

$$\begin{aligned}
\mathrm{Br}\left[\tau \to e\gamma\right] &\leq 3.3\times10^{-8}\\
\mathrm{Br}\left[\tau \to \mu\gamma\right] &\leq 4.4\times10^{-8}\\
\mathrm{Br}\left[\mu \to e\gamma\right] &\leq 4.2\times10^{-13}
\end{aligned}$$

SINDRUM Collaboration, Nucl.Phys. B299 (1988) 1-6
MEG Collaboration, Eur.Phys.J. C76 (2016) no.8, 434
HFLAV Collaboration, Eur.Phys.J. C77 (2017) no.12, 895
BaBar Collaboration, Phys.Rev.Lett. 104 (2010) 021802

Low-energy effective Lagrangian and the matching



- Diagrams in Fig. (b) match into the diagram in Fig. (a)
- Diagram in Fig. (d) matches into the diagram in Fig. (c)

Low-energy effective Lagrangian and the matching

Dipole	
$Q_{e\gamma}$	$e m_r (\bar{l}_p \sigma^{\mu\nu} P_L l_r) F_{\mu\nu} + \text{H.c.}$

Scalar/Tensorial		Vectorial	
Q_S	$(\bar{l}_p P_L l_r)(\bar{l}_s P_L l_t) + \text{H.c.}$	Q_{VLL}	$(\bar{l}_p \gamma^\mu P_L l_r)(\bar{l}_s \gamma_\mu P_L l_t)$
		Q_{VLR}	$(\bar{l}_p \gamma^\mu P_L l_r)(\bar{l}_s \gamma_\mu P_R l_t)$
		Q_{VRR}	$(\bar{l}_p \gamma^\mu P_R l_r)(\bar{l}_s \gamma_\mu P_R l_t)$
$Q_{Slq(1)}$	$(\bar{l}_p P_L l_r)(\bar{q}_s P_L q_t) + \text{H.c.}$	Q_{VlqLL}	$(\bar{l}_p \gamma^\mu P_L l_r)(\bar{q}_s \gamma_\mu P_L q_t)$
$Q_{Slq(2)}$	$(\bar{l}_p P_L l_r)(\bar{q}_s P_R q_t) + \text{H.c.}$	Q_{VlqLR}	$(\bar{l}_p \gamma^\mu P_L l_r)(\bar{q}_s \gamma_\mu P_R q_t)$
Q_{Tlq}	$(\bar{l}_p \sigma^{\mu\nu} P_L l_r)(\bar{q}_s \sigma_{\mu\nu} P_L q_t) + \text{H.c.}$	Q_{VlqRL}	$(\bar{l}_p \gamma^\mu P_R l_r)(\bar{q}_s \gamma_\mu P_L q_t)$
		Q_{VlqRR}	$(\bar{l}_p \gamma^\mu P_R l_r)(\bar{q}_s \gamma_\mu P_R q_t)$

$$C_{VRR}^{prst}(m_W) = \frac{\lambda_{rt}\lambda_{ps}^*}{2} \qquad C_{e\gamma}^{pr}(m_W) = \frac{1}{24\pi^2}\sum_{w=1}^{3}(\lambda_{rw}\lambda_{pw}^*)$$

A. Crivellin, M. Ghezzi, L. Panizzi, GMP and A. Signer, arXiv:1807.10224

Low-energy effective Lagrangian and the matching

Branching ratios at the physical scale:

$$\mathrm{BR}(l_p^\pm \to l_r^\pm \gamma) \simeq \frac{\alpha m_p^5}{(24\pi^2)^2 m_S^4 \Gamma_p} \left| \sum_{w=1}^{3} \lambda_{pw} \lambda_{rw}^* \right|^2$$

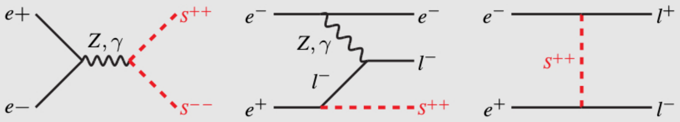
$$\mathrm{BR}[l_p^\pm \to l_r^\pm l_s^\mp l_t^\pm] = \simeq \frac{m_p^5 |\lambda_{ps}|^2 |\lambda_{rt}|^2}{s_{rt} 6 (4\pi)^3 m_S^4 \Gamma_p}$$

$$\Gamma^N_{\mu\to e} = \frac{m_\mu^5 \alpha^2}{(12\pi)^2 m_S^4} \left(\frac{D_N}{e} + 32 V_N^{(p)} \log\left(\frac{m_\tau}{m_W}\right)\right)^2 \left| \sum_{w=1}^{3} \lambda_{2w} \lambda_{1w}^* \right|^2 + \frac{m_\mu^5 \alpha^2}{(12\pi)^2 m_S^4} \left(32 V_N^{(p)} \log\left(\frac{m_\mu}{m_\tau}\right)\right)^2 \left| \sum_{w=1}^{2} \lambda_{2w} \lambda_{1w}^* \right|^2$$

Signatures at CLIC

	Stage I		Stage II	Stage III
$\sqrt{s}$	380 GeV	350 GeV	1.5 TeV	3 TeV
$\mathcal{L}$	0.9/ab	0.1/ab	2.5/ab	5/ab

Topologies

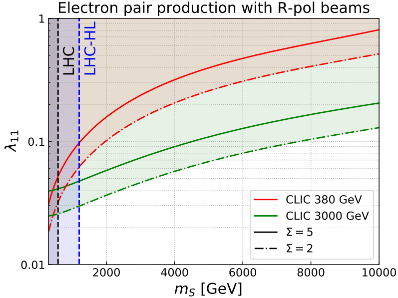


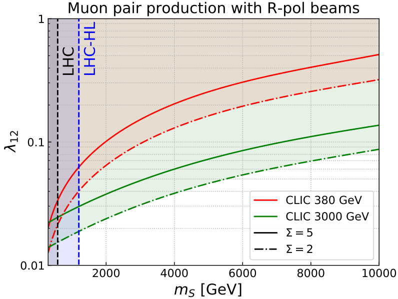
Pair production — Single production — t-channel

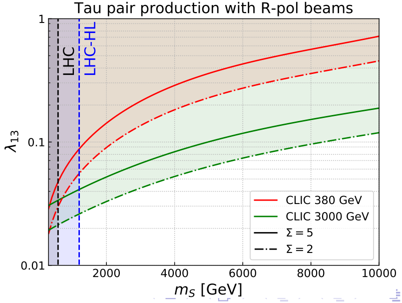
- Pair production limited by phase space to probe $M_S < 1500$ GeV
- Single production can probe twice as much (in principle)
- The t-channel probes the dependence of mass-Yukawa up to any mass, but requires S^{++} to interact with electrons

t-channel

- Simulation with CALCHEP including ISR and beamstrahlung
- Standard acceptance cuts: $E(l) \geq$ 10 GeV and $|\cos(\theta)| \leq$ 0.95 for μ and τ or 0.5 for e
- For τ_h final states, assuming a reconstruction efficiency of 70%
- Significance without systematic errors: $S/\sqrt{S+B}$

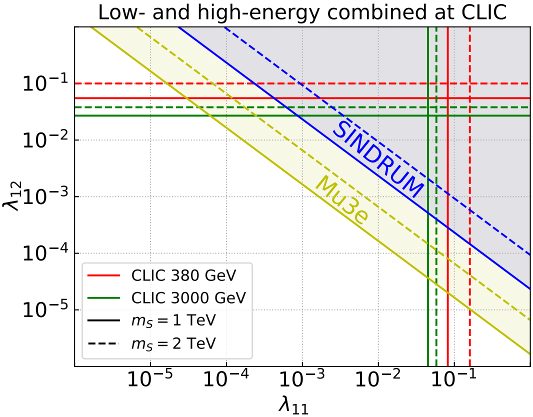






Complementarity with low energy

Yukawa couplings with electrons and muons



- At CLIC the two Yukawas are mostly explored independently
- Through $\mu \to 3e$ the product of the Yukawa is constrained: $BR(l_p \to l_r l_s l_t) \simeq \frac{m_p^5 |\lambda_{ps}|^2 |\lambda_{rt}|^2}{s_{rt} 6(4\pi)^3 m_S^4 \Gamma_p}$

$BR(\mu \to 3e)_{SINDRUM} < 10^{-12}$ and $BR(\mu \to 3e)_{Mu3e} < 5 \times 10^{-15}$

An exercise in SMEFT: lepton EDMs

Assumptions: SM is merely an effective theory, valid up to some scale Λ. It can be extended to a field theory that satisfies the following requirements:

- its gauge group should contain $SU(3)_C \times SU(2)_L \times U(1)_Y$;
- all the SM degrees of freedom must be incorporated;
- at low energies (i.e. when $\Lambda \to \infty$), it should reduce to SM.

Assuming that such reduction proceeds via decoupling of New Physics (NP), the Appelquist-Carazzone theorem allows us to write such theory in the form:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right).$$

Dimension-six operators

$$\boxed{Q_{eW}} = (\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu};$$

$$\boxed{Q_{eB}} = (\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}.$$

$$Q^{(1)}_{\varphi l} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$$

$$Q^{(3)}_{\varphi l} = (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$$

$$Q_{\varphi e} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$$

$$Q_{e\varphi} = (\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$$

$$Q_{ll} = (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$$

$$Q_{ee} = (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$$

$$Q_{le} = (\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$$

$$\boxed{Q_{\varphi \widetilde{B}}} = (\varphi^\dagger \varphi)(B_{\mu\nu} \widetilde{B}^{\mu\nu})$$

$$\boxed{Q_{\varphi \widetilde{W}}} = (\varphi^\dagger \varphi)(W^I_{\mu\nu} \widetilde{B}^{\mu\nu}_I)$$

$$\boxed{Q_{\varphi \widetilde{W} B}} = (\varphi^\dagger \tau^I \varphi)(B_{\mu\nu} \widetilde{W}^{\mu\nu}_I)$$

$$Q^{(1)}_{lq} = (\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$$

$$Q^{(3)}_{lq} = (\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$$

$$Q_{eu} = (\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$$

$$Q_{ed} = (\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$$

$$Q_{lu} = (\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$$

$$Q_{ld} = (\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$$

$$Q_{qe} = (\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$$

$$\boxed{Q_{ledq}} = (\bar{l}^j_p e_r)(\bar{d}_s q^j_t)$$

$$\boxed{Q^{(1)}_{lequ}} = (\bar{l}^j_p e_r)\varepsilon_{jk}(\bar{q}^k_s u_t)$$

$$\boxed{Q^{(3)}_{lequ}} = (\bar{l}^j_p \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}^k_s \sigma^{\mu\nu} u_t)$$

Leptonic tensorial current at the tree level

One dimension-six operator can produce tensorial current:

B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010** (2010) 085

Working in the physical basis, we consider:

$$C_{eB} \to C_{e\gamma} c_W - C_{eZ} s_W ,$$
$$C_{eW} \to -C_{e\gamma} s_W - C_{eZ} c_W ,$$

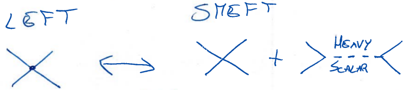
where $s_W = \sin(\theta_W)$ and $c_W = \cos(\theta_W)$ are the sine and cosine of the weak mixing angle.

$$\mathcal{L}_{e\gamma} \equiv \frac{C_{e\gamma}}{\Lambda^2} Q_{e\gamma} + \text{h.c.} = \frac{C_{e\gamma}^{pr}}{\Lambda^2} (\bar{l}_p \sigma^{\mu\nu} e_r) \varphi F_{\mu\nu} + \text{h.c.}$$

Writing on the back of an envelope…

Matching LEFT and SMEFT

AT THE TREE LEVEL, THE OPERATORS MATCH IN THE FOLLOWING WAY:



$$\mathrm{Im}\{C^{ijkl}_{VLR}\} : \mathrm{Im}\{C^{ijkl}_{\ell e} + \frac{v^2}{4m_H^2}\left(y_{ik} C^{jl}_{e\varphi}\delta_{jk} - y_{il} C^{jk}_{e\varphi}\delta_{il}\right)\}$$

$$\mathrm{Im}\{C^{ijkl}_{S}\} : \mathrm{Im}\{-\frac{v^2}{4m_H^2}\left(y_{kl} C^{ij}_{e\varphi}\delta_{kl} + y_{ij} C^{kl}_{e\varphi}\delta_{ij}\right)\}$$

$$\mathrm{Im}\{C^{ijkl}_{Seu(1)}\} : \mathrm{Im}\{-C^{ijkl}_{\ell equ(1)} - \frac{v^2}{2m_H^2} y_{kl} C^{ij}_{e\varphi}\delta_{kl}\}$$

$$\mathrm{Im}\{C^{ijkl}_{Seu(2)}\} : \mathrm{Im}\{-\frac{v^2}{2m_H^2} y_{kl} C^{ij}_{e\varphi}\delta_{kl}\}$$

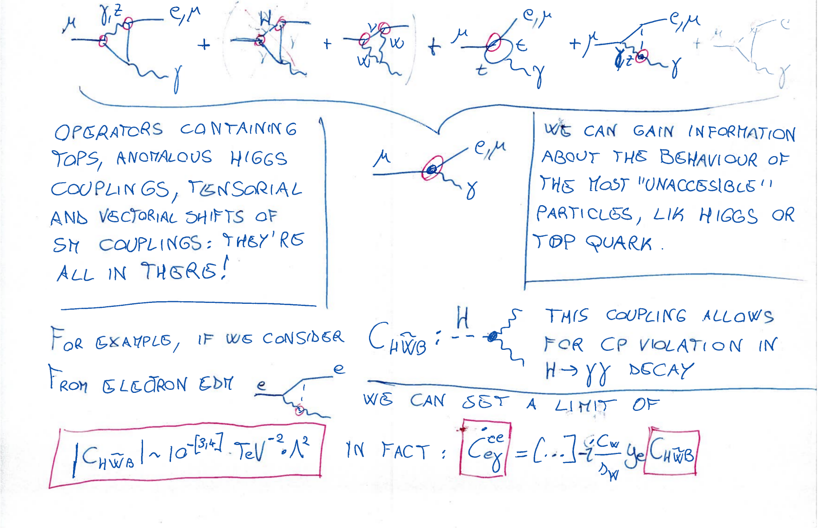
$$\mathrm{Im}\{C^{ijkl}_{T\ell u}\} : \mathrm{Im}\{-C^{ijkl}_{\ell equ(3)}\}$$

$$\mathrm{Im}\{C^{ijkl}_{Sed(1)}\} : \mathrm{Im}\{-\frac{v^2}{2m_H^2} y_{kl} C^{ij}_{e\varphi}\delta_{kl}\}$$

$$\mathrm{Im}\{C^{ijkl}_{Sed(2)}\} : \mathrm{Im}\{\Sigma_p\left(V^{pl}_{CKM} C^{ijkp}_{\ell edq}\right) - \frac{v^2}{2m_H^2} y_{kl} C^{ij}_{e\varphi}\delta_{kl}\}$$

EVEN IN ABSENCE OF EXPLICIT FOUR-FERMION CONTRIBUTIONS IN SMEFT THE ANOMALOUS HIGGS COUPLING $C_{e\varphi} = (H^\dagger H)(\bar{\ell} H e)$ WILL PRODUCE 6 CLASSES OF FOUR-FERMION OPERATORS IN THE LOW-ENERGY REGIME. THIS IS VERY IMPORTANT: MASTERING FOUR-FERMION OPERATORS AT LOW ENERGIES IS EQUIVALENT TO MASTER SCALAR INTERACTIONS AT HE

“Hard” Matching into dipoles



R. Alonso, E. E. Jenkins, A. V. Manohar and M. Trott, JHEP **1404** (2014) 159

"Hard" Matching into dipoles at one loop

TREE-LEVEL

$$\mathrm{Im}\{\hat{C}^{ij}_{e\gamma}\} : \mathrm{Im}\{C^{ij}_{e\gamma}\} \quad \text{WITH} \quad \hat{Q}^{LEFT}_{e\gamma} : (2\sqrt{2}G_F)^{-1/2}(\bar{\ell}_p \sigma^{\mu\nu} P_R \ell_r) F_{\mu\nu} + h.c.$$

ONE LOOP:

$$C^{ij}_{e\varphi} \to \frac{e v^2}{384\pi^2 m_H^2}\left(-4y_i^2 - y_i y_j - 4y_j^2 + [\ldots]\right)\left(\mathrm{Im}\{C^{ij}_{e\varphi} + C^{ji}_{e\varphi}\}\right)$$

$$C^{ij}_{eZ} \to \frac{\alpha}{8\pi c_W s_W}\left(-3c_W^2 + 3s_W^2 + [\ldots]\right)\mathrm{Im}\{C^{ij}_{eZ}\}$$

$$C_{\varphi\tilde{B}} \to \frac{e y_{ij}}{64\pi^2}\left(-9 + [\ldots]\right) C_{\varphi\tilde{B}}\, \delta_{ij}$$

$$C_{\varphi\tilde{W}} \to \frac{e y_{ij}}{64\pi^2}\left(-3 + [\ldots]\right) C_{\varphi\tilde{W}}\, \delta_{ij}$$

$$C_{\varphi\tilde{W}B} \to \frac{e y_{ij}}{128\pi^2 c_W s_W}\left(9 + [\ldots]\right) C_{\varphi\tilde{W}B}\, \delta_{ij}$$

THE $[\ldots]$ IS HIDING SMALL LOGS POST-$\overline{MS}$ RENORMALISATION

THE INSERTION OF THE EXPLICIT ANTISYMMETRIC ε TENSOR IN $C_{\varphi\tilde{B}}, C_{\varphi\tilde{W}}, C_{\varphi\tilde{W}B}$ ($\tilde{V}^{\mu\nu} = \varepsilon^{\alpha\beta\mu\nu} V_{\alpha\beta}$) IN OBSERVABLES INVOLVING CHIRAL FIELDS PRODUCES AN IMAGINARY PART IN THE DIPOLE OPERATOR, EVEN WITH REAL OVERALL COEFFICIENTS!

$[\ldots]$ ARE FUNCTIONS DEPENDING ON SMALL LOGS, $\log\left(\frac{m_W, m_Z, m_H, m_t}{\Lambda_{EW}}\right) \sim 0$

THIS ALL MEANS THAT FOR $\Lambda \gg \Lambda_{EW}$ ALSO THE RGEs ARE IMPORTANT.

RG flow below the EWSB scale

THE EXPLICIT COMPUTATION OF THE LEFT ANOMALOUS DIMENSIONS THAT GIVES AN EVOLUTION OF THE DIPOLE TELLS US:

$$16\pi^2 \left(2^{-3/4} G_F^{-1/2}\right) \dot{C}_{e\gamma}^{ii} \simeq +2 e \sum_r m_r C_S^{riir} + 16 e \sum_c m_c C_{T\ell u}^{iicc} + [\dots]$$

ONE-LOOP

AT THE LEPTON SCALE, THIS EVOLUTION TRANSFORM $C_{e\gamma}$ INTO

$$C_{e\gamma}^{ii}(m_i) \simeq C_{e\gamma}^{ii}(m_{EW}) + \left(\frac{2^{3/4} e \sqrt{G_F}}{\pi^2} \sum_c m_c C_{T\ell u(m_{EW})}^{iicc} + \frac{e \sqrt{G_F}}{4 \cdot 2^{1/4} \pi^2} \sum_r m_r C_{S(m_{EW})}^{riir} \right) \log\left(\frac{m_\mu}{m_W}\right)$$

WITH THE ESTABLISHED MATCHING AT THE EW SCALE, WE CAN WRITE:

$$\mathrm{Im}\{C_{e\gamma}^{ii}(m_i)\} = \underbrace{-\frac{3e}{64\pi^2} \frac{v^2}{m_H^2} y_{ii}^2 \,\mathrm{Im}\{C_{e\varphi}^{ii}\}}_{\text{HM}} + \frac{3\alpha}{8\pi} \frac{s_W^2 - c_W^2}{s_W c_W} \mathrm{Im}\{C_{eZ}^{ii}\} + \mathrm{Im}\{C_{e\gamma}^{ii}\}$$

$$+ \frac{e y_{ii}}{64\pi^2} \left[-9 C_{\varphi\tilde{B}} - 3 C_{\varphi\tilde{W}} + \frac{9}{2 s_W c_W} C_{\varphi\tilde{W}B} \right] +$$

$$\left(-\frac{e}{\pi^2} \sum_c y_{cc} \,\mathrm{Im}\{C_{eequ(3)}^{iicc}\} + \underbrace{\frac{e}{8\pi^2} \frac{v^2}{4 m_H^2} y_{ii}^2 \,\mathrm{Im}\{C_{e\varphi}^{ii}\}}_{\text{RGE-LEFT}} \right) \log\left(\frac{m_i}{m_W}\right)$$

WE FOUND $\mathrm{Im}\{C_{e\gamma}\}$. ARE THERE OTHER QUALITATIVE NEW CONTRIBUTIONS ABOVE THE EW SCALE, I.E. WHEN $\Lambda \gg EW$? SMEFT RGE...

RG flow above the EWSB scale

ABOVE THE EW SCALE, THE DIPOLE OPERATOR EVOLVES LIKE

$$\underbrace{\mathrm{Im}\{\dot{C}_{e\gamma}^{ii}\}}_{16\pi^2 \dot{C}} \simeq 12\cot(2\theta_W)\, e^2\, \mathrm{Im}\{C_{eZ}^{ii}\} - \cot\theta_W\, y^{ii} C_{H\tilde{W}B} + 4e^2 y^{ii}\left[\frac{1}{g_2^2}C_{H\tilde{W}} + \frac{1}{g_1^2}C_{H\tilde{B}} - \frac{1}{g_1 g_2}C_{H\tilde{W}B}\right]$$

$$+ e^2(\cot\theta_W - 3\tan\theta_W)\, y^{ii}\left[\frac{1}{g_1 g_2}C_{H\tilde{W}} - \frac{1}{g_1 g_2}C_{H\tilde{B}} - \left(\frac{1}{2g_1^2} - \frac{1}{2g_2^2}\right)C_{H\tilde{W}B}\right] + 16\, y_u^{jj}\, \mathrm{Im}\{C_{lequ(3)}^{iijj}\}$$

FOR SIMPLICITY, WE ASSUME THAT Λ IS NOT MUCH BIGGER THAN EWSB: THE LOG PRODUCED BY EVOLUTION WILL NOT ALTER THE PRE-EXISTING CONTRIBUTIONS. HOWEVER WE WILL HAVE A NEW QUALITATIVE EFFECT FROM $\mathrm{Im}\{C_{lequ}^{ii33}\} \equiv$

, THEN

$$\mathrm{Im}\{C_{e\gamma}^{ii}\} \simeq [\ldots] + \frac{y_t}{\pi^2} C_{lequ}^{ii33} \log\left(\frac{\Lambda}{m_{EW}}\right).$$

WE HAVE ALL INGREDIENTS TO APPLY THE LEPTON EDM LIMITS AND EXPRESS THEM INTO BOUNDS ON THE EFFECTIVE COEFFICIENTS

WE SUMMARISE THESE NEW AND INTERESTING RESULTS IN THE NEXT SLIDE ...

Results

$$|d_e| = \frac{2^{1/4}}{\sqrt{G_F}\,\Lambda_{UV}^2}\cdot|\mathrm{Im}\{C_{e\gamma}\}| \leq 10^{-29}\, e\,\mathrm{cm}\ (\text{ACME } 2018) \simeq 2\cdot 10^{-17}\ \mathrm{GeV}^{-1}$$

FOR A NEW PHYSICS SCALE OF 1 TeV:

$$\mathrm{Im}\{C_{e\gamma}^{11}\} < 5\cdot 10^{-14}$$

$$\mathrm{Im}\{C_{eZ}^{11}\} < 5\cdot 10^{-11}$$

$$\mathrm{Im}\{C_{\varphi\tilde{W}B}\} < 4\cdot 10^{-6}$$

$$\mathrm{Im}\{C_{\varphi\tilde{B}}\} < 4\cdot 10^{-6}$$

$$\mathrm{Im}\{C_{\varphi\tilde{W}}\} < 1\cdot 10^{-5}$$

$$\mathrm{Im}\{C_{\ell equ(3)}^{1133}\} < 2\cdot 10^{-13}$$

$$\mathrm{Im}\{C_{\ell equ(3)}^{1122}\} < 2\cdot 10^{-11}$$

$$\mathrm{Im}\{C_{\ell equ(3)}^{1111}\} < 1.3\cdot 10^{-8}$$

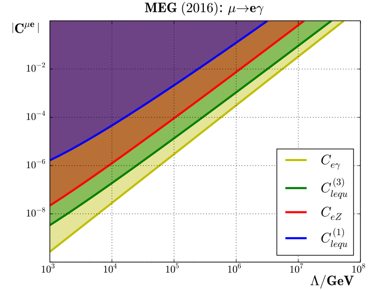
TERRIFIC SET OF LIMITS FROM THE ELECTRON EDM. NOT ONLY DIPOLES, BUT ALSO ANOMALOUS HIGGS-GAUGE BOSONS COUPLINGS AND FOUR-FERMION COUPLINGS INVOLVING U-TYPE QUARKS.

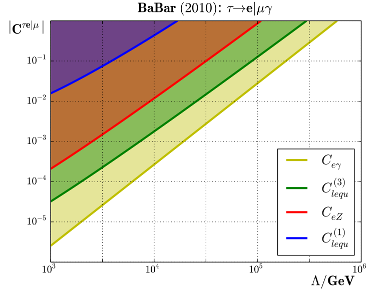
THE SAME EXERCISE CAN BE CARRIED OUT FOR μEDM, AND WITH A LOT MORE OF CARE FOR THE nEDM. OF COURSE, DIRECT HIGH-ENERGY SEARCHES CAN NOT COMPARE WITH THIS VALUES.

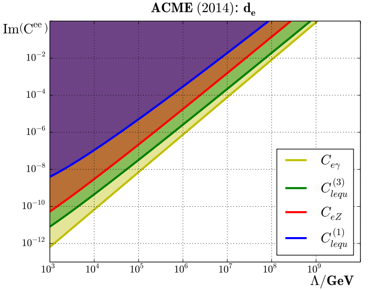
THIS CONCLUDES OUR JOURNEY ACROSS EFFECTIVE FIELDS

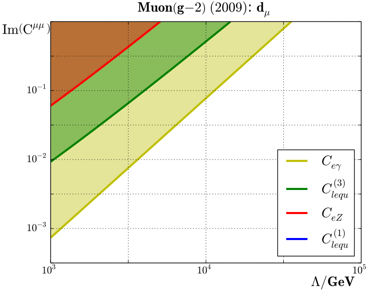
See also A. Crivellin *et al.*, Phys. Rev. D **98** (2018) no.11, 113002

Experimental limits “reinterpreted” at the EW scale







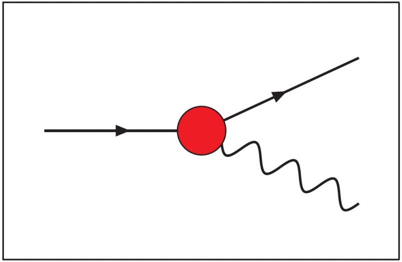


The two-loop frontier

We have to set the stage: computation of two-loop anomalous dimensions for lepton dipole moments in QED.

We are working in a chiral theory, therefore we will have issues with anti-symmetric tensors: for illustrative purposes we adopt

- conventional dimensional regularisation and
- assume that we can anti-commute γ^5 all over the place.



Feynman diagrams

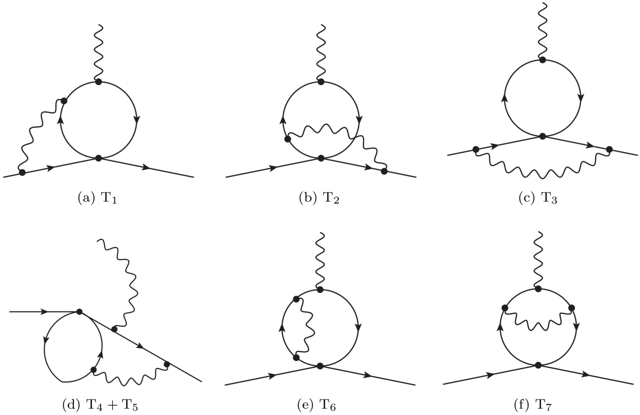


FIG Two-loop Feynman diagrams for the mixing of the four-fermion operators into the dipole operator.

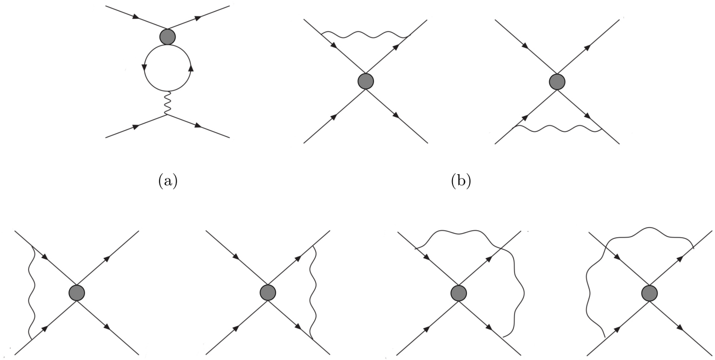
Defining suitable projectors

Off-shell external momenta will allow us to get rid of spurious infra-red divergences and potential subdivergencies arising from manipulations.

But we end up with 24 possible structures for the lepton current:

$$
\begin{aligned}
\Delta C_{e\gamma} \simeq \mathrm{Tr}\Big[& \left(\not{p}_l - \not{p}_\gamma + m_l\right) \Gamma^\rho \left(\not{p}_l + m_l\right) \\
& (m_l P_L \gamma^\sigma + m_l P_R \gamma^\sigma \\
& + m_l / p_\gamma^2 P_R \not{p}_\gamma \gamma^\sigma \not{p}_l \\
& - (m_l^2 - 2p_l^2)/p_\gamma^2 P_L \gamma^\sigma \not{p}_\gamma \\
& + m_l / p_\gamma^2 P_L \not{p}_\gamma \gamma^\sigma \not{p}_l + m_l^2 / p_\gamma^2 P_R \gamma^\sigma \not{p}_\gamma \\
& + [\ldots]^\sigma)(-g_{\rho\sigma} + (p_\gamma)_\rho (p_\gamma)_\sigma / p_\gamma^2)\Big]
\end{aligned}
$$

Feynman rules for four-fermion operators



(a) (b) (c)

Suitable projectors for four-fermion operators

We need to define projectors for the four-fermion structures.

This will automatically identify both the one-loop counter-terms and the structures that are "invisible" to these projectors.

We give a name to these structures: "evanescent operators"

For example:

$$E_{VLR}^{ijji} \propto \left(\bar{l}_i P_L \gamma^\mu \gamma^\nu \gamma^\rho l_j\right) \left(\bar{l}_j P_R \gamma^\mu \gamma^\nu \gamma^\rho l_i\right) - (-4 - (-6 + D)D) \left(\bar{l}_i P_L \gamma^\mu l_j\right) \left(\bar{l}_j P_R \gamma^\mu l_i\right)$$

$$\mathcal{E}_{VLR}^{ijji} = \frac{-e^2 Q_l^2 Q_{VLR}^{ijji}}{16(-4 + D)\Lambda^2 \pi^2}$$

At the one-loop level

By calculating the one-loop insertions, including counter-terms and evanescent operators one ends up with:

$$\begin{aligned} C_{e\gamma} =& e^3\Lambda^2\left(\mathrm{QED}\times 1\mathrm{LCT}_{\mathrm{EFT}}\right)+\\ & \sum_n e m_X C^{(n)}\times\left(1\mathrm{LQED}+\mathrm{MIXING}\right)+\\ & \sum_n e^3 m_X \varepsilon^{(n)} \end{aligned}$$

In general, these objects are regularisation- and renormalisation-scheme dependent.

Is there a sort of “convenient/best” systematic choice?

Calculating the bare two-loop diagrams

Many methods in literature, mostly based on a rearrangement of the propagators:

$$\frac{1}{(r-p)^2 - m^2} = \frac{1}{r^2 - m^2} + \frac{1}{(r^2 - m^2)} \frac{\left[-p^2 \Delta^{-2} + 2\,(r \cdot p)\,\Delta^{-1}\right]}{\left[(r-p)^2 - m^2\right]}$$

where $\Delta = 1$ is a flag to count the overall degree of divergence.

After these rearrangement, we are left with tadpoles two-loop integrals only: they are all very well known and under control.

If we are dealing with massless propagators we can adopt an infrared regularisator (to deal with great care).

Result

Bare diagrams

Operator	T1 + T2	T3	T4 +T5	T6	T7
$\mathcal{Q}_S^{iijj}$	$-\frac{5}{2}m_jQ_l^3$	0	0	0	0
$(\mathcal{Q}_S^{iijj})^\dagger$	$-\frac{1}{2}m_jQ_l^3$	0	0	0	0
$\mathcal{Q}_{VLL}^{iijj}$	$3m_iQ_l^3$	0	$\frac{8}{9}m_iQ_l^3$	0	0
$\mathcal{Q}_{VLR}^{jjii}$	$-\frac{3}{4}m_iQ_l^3$	0	$\frac{2}{9}m_iQ_l^3$	0	0
$\mathcal{Q}_{VLR}^{iijj}$	$-\frac{3}{4}m_iQ_l^3$	0	$\frac{2}{9}m_iQ_l^3$	0	0
$\mathcal{Q}_{VLR}^{ijji}$	$m_jQ_l^3$	$\frac{1-\xi_\gamma}{2}m_jQ_l^3$	$m_jQ_l^3$	$-\frac{3-2\xi_\gamma}{4}m_jQ_l^3$	$\frac{1-2\xi_\gamma}{4}m_jQ_l^3$
$\mathcal{Q}_{VRR}^{iijj}$	$3m_iQ_l^3$	0	$\frac{8}{9}m_iQ_l^3$	0	0
$\mathcal{Q}_{Slu(1)}^{iijj}$	$-\frac{15}{4}m_jQ_lQ_u^2$	0	0	0	0
$\left(\mathcal{Q}_{Slu(1)}^{iijj}\right)^\dagger$	$-\frac{3}{4}m_jQ_lQ_u^2$	0	0	0	0
$\mathcal{Q}_{Slu(2)}^{iijj}$	$\frac{9}{4}m_jQ_lQ_u^2$	0	0	0	0
$\left(\mathcal{Q}_{Slu(2)}^{iijj}\right)^\dagger$	$\frac{3}{4}m_jQ_lQ_u^2$	0	0	0	0
$\mathcal{Q}_{VluLL}^{iijj}$	$\frac{9}{4}m_iQ_lQ_u^2$	0	$\frac{2}{3}m_iQ_l^2Q_u$	0	0
$\mathcal{Q}_{VluLR}^{iijj}$	$-\frac{9}{4}m_iQ_lQ_u^2$	0	$\frac{2}{3}m_iQ_l^2Q_u$	0	0
$\mathcal{Q}_{VluRL}^{iijj}$	$-\frac{9}{4}m_iQ_lQ_u^2$	0	$\frac{2}{3}m_iQ_l^2Q_u$	0	0
$\mathcal{Q}_{VluRR}^{iijj}$	$\frac{9}{4}m_iQ_lQ_u^2$	0	$\frac{2}{3}m_iQ_l^2Q_u$	0	0
$\mathcal{Q}_{Sld(1)}^{iijj}$	$-\frac{15}{4}m_jQ_lQ_d^2$	0	0	0	0
$\left(\mathcal{Q}_{Sld(1)}^{iijj}\right)^\dagger$	$-\frac{3}{4}m_jQ_lQ_d^2$	0	0	0	0
$\mathcal{Q}_{Sld(2)}^{iijj}$	$\frac{9}{4}m_jQ_lQ_d^2$	0	0	0	0
$\left(\mathcal{Q}_{Sld(2)}^{iijj}\right)^\dagger$	$\frac{3}{4}m_jQ_lQ_d^2$	0	0	0	0
$\mathcal{Q}_{VldLL}^{iijj}$	$\frac{9}{4}m_iQ_lQ_d^2$	0	$\frac{2}{3}m_iQ_l^2Q_d$	0	0
$\mathcal{Q}_{VldLR}^{iijj}$	$-\frac{9}{4}m_iQ_lQ_d^2$	0	$\frac{2}{3}m_iQ_l^2Q_d$	0	0
$\mathcal{Q}_{VldRL}^{iijj}$	$-\frac{9}{4}m_iQ_lQ_d^2$	0	$\frac{2}{3}m_iQ_l^2Q_d$	0	0
$\mathcal{Q}_{VldRR}^{iijj}$	$\frac{9}{4}m_iQ_lQ_d^2$	0	$\frac{2}{3}m_iQ_l^2Q_d$	0	0

TABLE Single-pole contribution to $\mathcal{Q}_{e\gamma}^{ii}$ in units of $(e^3\sqrt{Gf})/(64\times 2^{1/4}\pi^4)$

Counterterms

Operator	Four-fermion	Fermion WFR	Propagator Ren.	Penguin
$\mathcal{Q}_{VLL}^{iijj}$	0	0	0	$-m_iQ_l^3$
$\mathcal{Q}_{VLR}^{jjii}$	0	0	0	$-\frac{1}{4}m_iQ_l^3$
$\mathcal{Q}_{VLR}^{iijj}$	0	0	0	$-\frac{1}{4}m_iQ_l^3$
$\mathcal{Q}_{VLR}^{ijji}$	$3m_jQ_l^3$	$\frac{\xi_\gamma}{2}m_jQ_l^3$	$\frac{3}{2}m_jQ_l^3$	0
$\mathcal{Q}_{VRR}^{iijj}$	0	0	0	$-m_iQ_l^3$
$\mathcal{Q}_{Slu(1)}^{iijj}$	$\frac{3}{2}m_jQ_lQ_u^2$	0	0	0
$\left(\mathcal{Q}_{Slu(1)}^{iijj}\right)^\dagger$	$\frac{3}{2}m_jQ_lQ_u^2$	0	0	0
$\mathcal{Q}_{VluLL}^{iijj}$	0	0	0	$-\frac{3}{4}m_iQ_l^2Q_u$
$\mathcal{Q}_{VluLR}^{iijj}$	0	0	0	$-\frac{3}{4}m_iQ_l^2Q_u$
$\mathcal{Q}_{VluRL}^{iijj}$	0	0	0	$-\frac{3}{4}m_iQ_l^2Q_u$
$\mathcal{Q}_{VluRR}^{iijj}$	0	0	0	$-\frac{3}{4}m_iQ_l^2Q_u$
$\mathcal{Q}_{Sld(1)}^{iijj}$	$\frac{3}{2}m_jQ_lQ_d^2$	0	0	0
$\left(\mathcal{Q}_{Sld(1)}^{iijj}\right)^\dagger$	$\frac{3}{2}m_jQ_lQ_d^2$	0	0	0
$\mathcal{Q}_{VldLL}^{iijj}$	0	0	0	$-\frac{3}{4}m_iQ_l^2Q_d$
$\mathcal{Q}_{VldLR}^{iijj}$	0	0	0	$-\frac{3}{4}m_iQ_l^2Q_d$
$\mathcal{Q}_{VldRL}^{iijj}$	0	0	0	$-\frac{3}{4}m_iQ_l^2Q_d$
$\mathcal{Q}_{VldRR}^{iijj}$	0	0	0	$-\frac{3}{4}m_iQ_l^2Q_d$

Evanescent

Operator	One-particle irreducible	Penguin
$\mathcal{Q}_S^{iijj}$	$m_jQ_l^3$	0
$(\mathcal{Q}_S^{iijj})^\dagger$	$m_jQ_l^3$	0
$\mathcal{Q}_{VLL}^{iijj}$	0	$-\frac{1}{3}m_iQ_l^3$
$\mathcal{Q}_{VLR}^{jjii}$	0	$-\frac{1}{12}m_iQ_l^3$
$\mathcal{Q}_{VLR}^{iijj}$	0	$-\frac{1}{12}m_iQ_l^3$
$\mathcal{Q}_{VLR}^{ijji}$	$-8m_jQ_l^3$	0
$\mathcal{Q}_{VRR}^{iijj}$	0	$-\frac{1}{3}m_iQ_l^3$
$\mathcal{Q}_{Slu(2)}^{iijj}$	$-\frac{3}{2}m_jQ_lQ_u^2$	0
$\left(\mathcal{Q}_{Slu(2)}^{iijj}\right)^\dagger$	$-\frac{3}{2}m_jQ_lQ_u^2$	0
$\mathcal{Q}_{VluLL}^{iijj}$	0	$-\frac{1}{4}m_iQ_l^2Q_u$
$\mathcal{Q}_{VluLR}^{iijj}$	0	$-\frac{1}{4}m_iQ_l^2Q_u$
$\mathcal{Q}_{VluRL}^{iijj}$	0	$-\frac{1}{4}m_iQ_l^2Q_u$
$\mathcal{Q}_{VluRR}^{iijj}$	0	$-\frac{1}{4}m_iQ_l^2Q_u$
$\mathcal{Q}_{Sld(2)}^{iijj}$	$-\frac{3}{2}m_jQ_lQ_d^2$	0
$\left(\mathcal{Q}_{Sld(2)}^{iijj}\right)^\dagger$	$-\frac{3}{2}m_jQ_lQ_d^2$	0
$\mathcal{Q}_{VldLL}^{iijj}$	0	$-\frac{1}{4}m_iQ_l^2Q_d$
$\mathcal{Q}_{VldLR}^{iijj}$	0	$-\frac{1}{4}m_iQ_l^2Q_d$
$\mathcal{Q}_{VldRL}^{iijj}$	0	$-\frac{1}{4}m_iQ_l^2Q_d$
$\mathcal{Q}_{VldRR}^{iijj}$	0	$-\frac{1}{4}m_iQ_l^2Q_d$

What did we learn?

A four-dimensional Lagrangian implementation was not enough: we had to understand the D-dimensional evanescent structure and to implement a new set of operators accordingly.

The chain FeynRules+FeynArts+FormCalc was adopted, a form file was produced and then heavily manipulated.

Not very efficient at the status of the art.

Two-loop example in LEFT (QED+EFT) was delivered by assuming CDR and naive anticommuting γ^5, everything works but nothing guarantees that this approach is stable in SMEFT.

What are the "convenient/best" choices? Let's discuss...

Conclusion

- $\surd$ Standard Model is an effective field theory
- $\surd$ Accidental symmetries are broken by the EFT expansion
- $\surd$ Lepton dipole moments (and low energy observables) are perfect places to look for departures from the SM
- $\surd$ If NP lives at very high energy, then consistent EFT techniques can be adopted to extract information on NP at high scales
- $\surd$ It is possible to gain information on the parameter space of possible UV-complete BSM theories from LFV and EDMs
- $\surd$ Precise limits on low-energy observables can have a stronger impact than direct tests at high energies...
- $\surd$ ...And, in general, they provide important complementary information on realistic NP parameter space