Model independent analysis of MeV scale dark matter

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Contents

Direct Detection - May be light DM???

Why MeV DM

- Direct detection experiments are usually sensitive to WIMP in mass range GeV to few TeVs.
- Recent results from several direct detection experiments have imposed severe constraints on the multi-GeV mass window for various dark matter(DM) models.

However, many of these experiments are not sensitive to MeV scale DM as the corresponding recoil energies are much below the detector thresholds. For higher mass, the sensitivity drops because the number density of DM drops.

Calculation of Relic Abundance of Light DM

Effective operators

$$
\begin{aligned}
\mathcal{O}_s^f &= \frac{\mathcal{C}_s^f}{\Lambda}\phi^\dagger\phi\,\bar{f}f \\
\mathcal{O}_p^f &= \frac{\mathcal{C}_p^f}{\Lambda}\phi^\dagger\phi\,\bar{f}\gamma^5 f \\
\mathcal{O}_v^f &= \frac{\mathcal{C}_v^f}{\Lambda^2}i\left(\phi^\dagger\partial_\mu\phi - \partial_\mu\phi^\dagger\phi\right)\bar{f}\gamma^\mu f \\
\mathcal{O}_a^f &= \frac{\mathcal{C}_a^f}{\Lambda^2}i\left(\phi^\dagger\partial_\mu\phi - \partial_\mu\phi^\dagger\phi\right)\bar{f}\gamma^\mu\gamma^5 f \\
\mathcal{O}_\gamma &= \frac{\mathcal{C}_\gamma}{\Lambda^2}(\phi^\dagger\phi)F_{\mu\nu}F^{\mu\nu}.
\end{aligned}
$$

ϕ is scalar DM,
f is SM fermion,
Λ is scale of new physics and
$\mathcal{C}$'s are the dimensionless Wilson coefficients.

Caveat in Relic calculation

- For DM masses between about 100 MeV and a few GeV, it is not a good approximation to assume that DM annihilates into a pair of free quarks.
- With weak interaction strengths, such a DM would freeze out only around the QCD phase transition temperature.
- Around this temperature, the relative momentum between the quark-antiquark pair is small and a bound state would ensue.
- Therefore, it becomes necessary to determine the effective couplings of DM to mesons and baryons and calculate the annihilation rate into these final states.

DM annihilation to bound states

- For arbitrary bound states $\mathcal{B}_{1,2}$, the matrix element for the process $\varphi + \varphi^\dagger \to \mathcal{B}_1 + \mathcal{B}_2$, driven by an operator $\mathcal{O}_I$, is given by

$$\mathcal{M} = \langle \mathcal{B}_1\, \mathcal{B}_2 | \mathcal{O}_I | \varphi\, \varphi \rangle = \frac{\mathcal{C}_I^f}{\Lambda} \langle \mathcal{B}_1\, \mathcal{B}_2 | J_I^{\bar{f}f} | 0 \rangle \langle 0 | J_\varphi^I | \varphi\, \varphi \rangle \ .$$

Hence, our main aim is to calculate

$$\boxed{\langle \mathcal{B}_1\, \mathcal{B}_2 | J_I^{\bar{f}f} | 0 \rangle}$$

- $\langle \mathcal{B}_1\, \mathcal{B}_2 | J_I^{\bar{f}f} | 0 \rangle$ can be parametrized in terms of form factors multiplying momentum- dependent structures dictated by Lorentz symmetry. For example:

$$\boxed{\begin{aligned} \langle 0 | \bar{q} q | \pi^-(p_1) \pi^+(p_2) \rangle &= F_s(Q^2), \\ \langle 0 | \bar{q} \gamma^\sigma q | \pi^-(p_1) \pi^+(p_2) \rangle &= F_v(Q^2)(p_1 - p_2)^\sigma \end{aligned}}$$

Chiral Perturbation theory

- In the simplest version, i.e for two massless quarks (u, d), QCD admits an exact $SU(2) \otimes SU(2)$ chiral symmetry, and the corresponding χPT lagrangian is described by

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\partial_\mu U \partial^\mu U^\dagger + \frac{Bf_\pi^2}{2}\mathrm{Tr}(M^\dagger U + U^\dagger M)$$

where, f_π is the pion decay constant, and $U = e^{i\pi(x)/f_\pi}$, with

$$\pi(x) \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} \therefore \mathrm{Tr}\partial_\mu U \partial^\mu U^\dagger = (m_u + m_d)\pi^+\pi^- + (m_u + m_d)\frac{{\pi^0}^2}{2}$$

- The operator $\bar{q}q$ can be given as $\dfrac{\partial \mathcal{H}_{\mathcal{QCD}}}{\partial m_q}$ and therefore

$$\boxed{\langle 0|\bar{q}q|\pi^-\pi^+\rangle = F_s(Q^2) = \frac{m_\pi^2}{(m_u + m_d)}}$$

Caveat

These form factors are defined within the lowest order χPT.

The NLO terms in the Lagrangian contains terms that are $\mathcal{O}(p^4)$ or, in other words, suppressed by further factors of $\mathcal{O}(p^2/\Lambda^2_{\rm QCD})$. With $\Lambda_{\rm QCD} \sim 200\,{\rm MeV}$, a perturbative calculation of the higher-order effects is valid only for small momentum exchanges. In the present context, this translates to a limit on the dark matter mass, viz. $m_\varphi \lesssim 300\,{\rm MeV}$.

R. S Chivukula et al, PLB, 1989

Contribution from Higher order

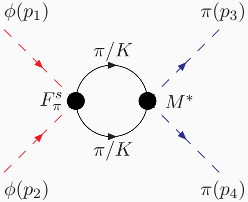

Figure 1: *Typical diagrams that contribute to $\varphi\varphi \to \pi\pi$ at the next to leading order.*

- With the hadron-hadron interactions being strong, there is no a priori compelling reason to limit ourselves to only one-loop results.
- To write down the S-matrix for such a system, we need to include contributions from channels like $\pi\pi \to \pi\pi$, $\pi\pi \to K^+ K^-$, $\pi\pi \to 4\pi$, $\pi\pi \to \eta\,\eta$ etc.
- But direct calculation of the loops is a very difficult task.
- Instead, we determined the imaginary part using the Cutkosky rules and, subsequently, calculating the real part using dispersion relations.

Method

Consider $\varphi\varphi^* \to \pi\pi$. The only allowed final state rescattering is $\pi\pi \to \pi\pi$, and, in the limit of identical masses, is an entirely elastic process, so, the in- and out-states only differ in phase. The S-matrix and form factor, using Cutkosky rule and Unitarity, are given as

$$\boxed{F_s^{\pi} = S_{\pi\pi} F_s^{*\pi} \ , \qquad S_{\pi\pi} = \exp\left[2\,\pi\, i\, \delta_{\pi}(s)\right],}$$

where $\delta_{\pi}(s)$ is the $I = 0$, $J = 0$ pion-scattering phase shift.

If δ_π is known, the problem is to find all such function which obey all these properties. It has been shown that if phase tends to a finite value as $s \to \infty$, the solution is given by Omnes fuction:

$$\boxed{F_s(s) = P(s)\Omega(s) = exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}\frac{\delta(s)}{s - s_o - i\epsilon}\right)}$$

where $\delta(s)$ is phase shift, $P(s)$ is a polynomial which is fixed by behaviour of $F_s(s)$ and $\Omega(s)$.

These methods are taken from J. F. Donoghue, J. Gasser and H. Leutwyler, Nucl. Phys B, 1990 and B. Moussallam, EPJC, 2000

Form factors for 2-channel process

The S-matrix for two channel process can be parametrized as

$$\boxed{S_{\mathrm{total}} = \begin{pmatrix} \cos\theta\, e^{2i\delta_\pi} & i\,\sin\theta\, e^{i(\delta_\pi+\delta_K)} \\ i\,\sin\theta\, e^{i(\delta_\pi+\delta_K)} & \cos\theta\, e^{2i\delta_K} \end{pmatrix}}$$

and the consequent form factors through

$$\boxed{\begin{pmatrix} F^\pi(s) \\ \frac{2}{\sqrt{3}} F^K(s) \end{pmatrix} = \begin{pmatrix} \Omega_1^1 & \Omega_2^1 \\ \Omega_1^2 & \Omega_2^2 \end{pmatrix} \begin{pmatrix} F^\pi(0) \\ \frac{2}{\sqrt{3}} F^K(0) \end{pmatrix}}$$

where the Clebsch-Gordan coefficient occurring in the projection of the $\pi\pi$ state with $I = 0$ is shifted to F^K

Form factor

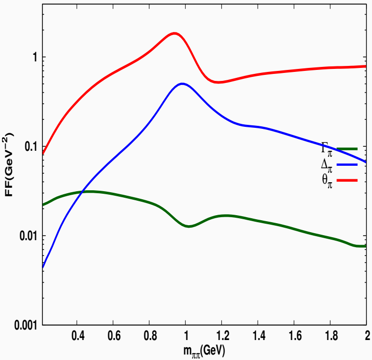

Figure 2: *The $s\,(= 4m_{\pi\pi}^2)$-dependence of various form factors associated with the pion.*

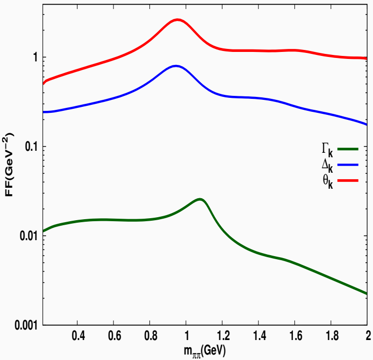

Figure 3: *The $s\,(= 4m_{\pi\pi}^2)$-dependence of various form factors associated with the kaon.*

Results

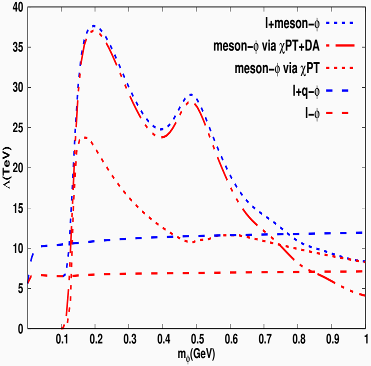

Figure 4: *Contours in the m_φ-Λ plane for dimension-5 operators satisfying $\Omega_\varphi h^2 = 0.1199 \pm 0.0022$, obtained using form factors determined by χPT and dispersion analysis(DA)*

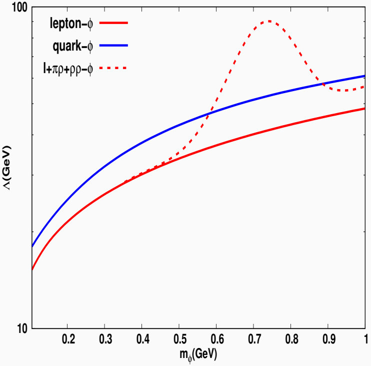

Figure 5: *Comparison of contours in the $m_\varphi - \Lambda$ plane for the dimension-6 operators satisfying* $\Omega_\varphi h^2 = 0.1199 \pm 0.0022$

Discussion

Implications from Astrophysical and Cosmological Observation

Effective Relativistic degrees of freedom

Energy injection from DM annihilation in the early universe can alter the effective number of relativistic degrees of freedom $N_{\rm eff}$. Indeed, MeV-scale DM is especially constrained by these observations. If the DM freezes out after the neutrinos have decoupled (at $T = T_\nu^{\rm decoup}$), its annihilation will result in heating the e^-–γ plasma relative to the neutrinos, thereby reducing the ratio of the neutrino and photon temperatures (T_ν/T_γ). This results in a reduction of $N_{\rm eff}$ as $N_{\rm eff} \propto (T_\nu/T_\gamma)^4$. From standard cosmology results, $N_{\rm eff} = 3.046$(PLANCK 2015)

$$\boxed{m_\varphi < 6\,\mathrm{MeV}}$$

is tightly constrained by N_{eff}.

Implications from Astrophysical and Cosmological Observation

CMB observations and Indirect detection

The Cosmic Microwave Background Radiation encodes information about the thermal his- tory of the early universe, and is well described by SM physics. On the other hand, DM annihilation, at early times, into high energy photons or charged particles can not only heat the gas, but can also lead to atomic excitations and even its ionization. This increase in the amount of the ionized fraction causes an increase in the width of the last scattering surface, thereby affecting the power spectrum of the CMB.

M. S. Madhavacheril, N. Sehgal and T. R. Slatyer, PRD, 2014

Implications from Astrophysical and Cosmological Observation

CMB observations and Indirect detection

The rate of energy deposited, into the CMB, by DM pair annihilation per unit time per unit volume is given by:

$$\frac{dE}{dt\,dV} = \rho_c^2\,\Omega_\varphi^2\,(1+z)^6\,P_{\rm ann}(z)$$

where z is the redshift of the epoch, and $\rho_c(\Omega_\varphi)$ is the critical density of the universe (DM relic abundance) today, *i.e.*, at $z=0$. The factor $(1+z)^6$ just encapsulates the standard evolution of the dark matter number density.

CMB observations(PLANCK 2015) constrain

$$\boxed{P_{\rm ann} \sim \frac{\langle \sigma v\rangle_{\rm CMB}}{m_\varphi} < 4.1\times 10^{-28} cm^3 s^{-1}\,{\rm GeV}^{-1}}$$

Cosmological constraints - Results

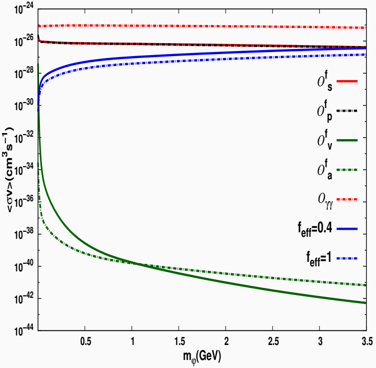

Figure 6: $\langle \sigma v \rangle (\varphi\varphi \to e^- e^+) - m_\varphi$ *plane obtained using those value of* Λ *that satisfy* $\Omega_\varphi h^2 = 0.1199 \pm 0.0022$ *for the case when DM is allowed to annihilate into leptons and free quarks.*

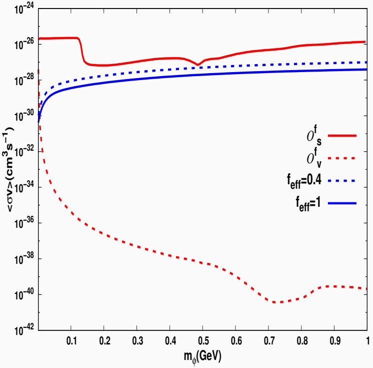

Figure 7: $\langle \sigma v \rangle (\varphi\varphi \to e^- e^+) - m_\varphi$ *plane obtained using those value of* Λ *that satisfy* $\Omega_\varphi h^2 = 0.1199 \pm 0.0022$ *for the case when DM is allowed to leptons and final states*

Direct detection experiments

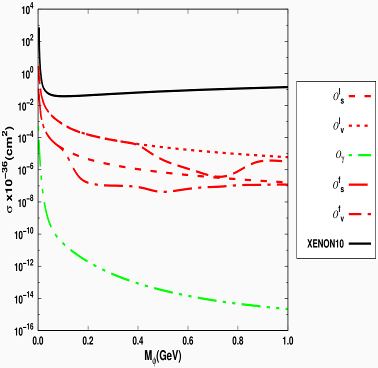

Figure 8: *Exclusion plots from XENON10 at 90% C.L. for the case of DM-electron scattering are also shown.*

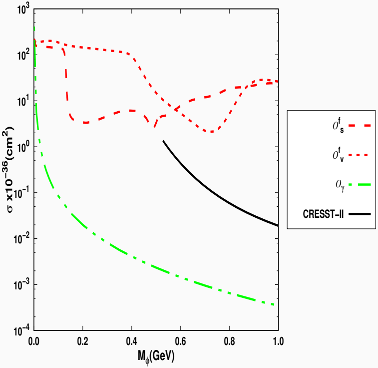

Figure 9: *Exclusion plots from CRESST-II at 90% C.L. for the case of DM-nucleon scattering are also shown.*

Summary

As already stated, for DM masses below a couple of GeVs, it is not a good approximation to assume that a pair of DM particles may annihilate into a pair of free quarks as DM of such a mass would freeze out only around the QCD phase transition temperature, leaving them with very little overall energy. Consequently, we should, instead, re frame the analysis in terms of bound states.

The inclusion of bound states may change the interpretation of the results in the direct and indirect detection experiments

Back up

Scalar form factor

$$\begin{aligned}
\Gamma_\pi &= m_\pi^2 \left(\Omega_1^1 + \frac{1}{\sqrt{3}} \Omega_2^1 \right) \\
\Delta_\pi &= \frac{2}{\sqrt{3}} \left(m_K^2 - \frac{m_\pi^2}{2} \right) \Omega_2^1 \\
\theta_\pi &= \left(2m_\pi^2 + p_1 s \right) \Omega_1^1 + \frac{2}{\sqrt{3}} \left(2m_K^2 + p_2 s \right) \Omega_1^1 \\
\Gamma_K &= \frac{m_\pi^2}{2} \left(\sqrt{3}\, \Omega_1^2 + \Omega_2^2 \right) \\
\Delta_K &= \left(m_K^2 - \frac{m_\pi^2}{2} \right) \Omega_2^2 \\
\theta_K &= \frac{\sqrt{3}}{2} \left(2m_\pi^2 + p_1 s \right) \Omega_1^2 + \left(2m_K^2 + p_2 s \right) \Omega_2^2 .
\end{aligned} \tag{1}$$

Vector form factor

For $\pi - \rho$ system, we will have

$$\sum_q \langle 0|\bar{q}\gamma_\mu q|\pi(p_1)\,\rho(p_2,\lambda)\rangle = 6\,F_{\rm em}(q^2)\,\varepsilon_{\mu\nu\sigma\omega}\,p_1^\nu\,p_2^\sigma\,\epsilon^{*\omega}(\lambda),$$

where λ denotes the polarization state of the ρ-meson and $F_{\rm em}(q^2)$ is the electromagnetic form factor.

To determine the expressions and behaviour of above-said form factors, we make use of the **vector meson dominance model**,wherein, for $Q^2 < 4\,\mathrm{GeV}^2$, the major contributions accrue from the $\omega(782)$, $\phi(1020)$ and $\omega(1420)$. We obtained the following expressions

$$\boxed{F_{\rm em}(q^2) = \sum_{V=\varphi,\omega} \frac{m_V^2}{g_v}\,\frac{1}{q^2 - m_V^2 + i m_V \Gamma_V}\,g_{V\pi\rho}.}$$

Vector form factor and Results

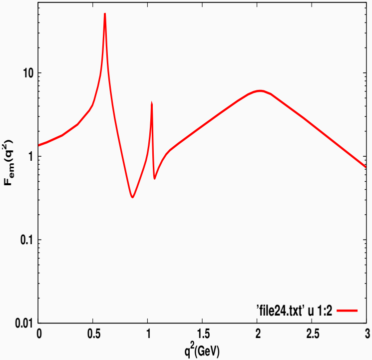

Figure 10: *The time like(TL) electromagnetic form factor estimated using the Vector Meson Dominance model.*

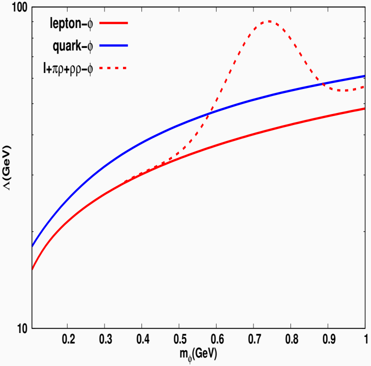

Figure 11: *Comparison of contours in the $m_\varphi - \Lambda$ plane for the dimension-6 operators satisfying* $\Omega_\varphi h^2 = 0.1199 \pm 0.0022$