# **Chiral-odd PDFs from dihadron observables** and the role of positivity constraints

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## **Transversity 2022**

Pavia May 23, 2022





Dirección General de Asuntos del Personal Académico



# Chiral-odd collinear parton distribution functions

Collinear PDF beyond inclusive DIS — this talk: dihadron observables. Access to twist-2 and twist-3 distributions.

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle PS | \bar{\psi}(0) i \sigma^{\nu\mu} \gamma_5 \psi(\lambda n) | PS \right\rangle$$

Transversity is the least known twist-2 PDF;

- reduced kinematics coverage wrt f<sub>1</sub> and g<sub>1</sub>;
- Service Error treatment must reflect
  - first principle constraints: <u>positivity</u> [large x]
  - integrability [small x]

$$\frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle PS | \bar{\psi}(0) \psi(\lambda n) | PS \right\rangle = M e(x)$$

- e(x) is the scalar (twist-3) PDF;
- sextremely reduced data sets;
- Point-by-point extraction so far
  - first principle constraints will matter





# PDF kinematics coverage: collinear PDFs



[Prog.Part.Nucl.Phys. 121 (2021) 103908]

Fixed Target DIS & SIDIS: M/Q is not so small

- Spurious contaminations
- Spin asymmetries can be defined to get sensitive to twist-3
- Present data: Hermes, COMPASS, JLab.









# Ingredients of a fit

- data:
  - first principles;
    - theoretical framework;
      - QCD-based assumptions;
        - parametrization
          - error treatment.

But, the implementation of the first-principle constraints might be data-driven, e.g. positivity for  $g_1$  and  $h_1$ .

## How the uncertainties/tensions between data and first principles are taken care of is a signature of the fit.

-> extensively studied for unpolarized PDFs, especially higher-order corrections and sources of uncertainties.





reconciles all previous points in an *elegant* and *physical* way.

Statistics: [Kovarik et al, Rev.Mod.Phys. 92 (2020)] Benchmarking: [PDF4LHC working group, 2203.05506]

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Transversity [Benel et al., EPJC80]

# Transversity PDF — constraints

## **Constraints based on first principles for the Transversity**

- Support, e.g.  $x \in [0,1]$
- Endpoint behavior, e.g.  $h_1(x=1)=0$
- Positivity bounds: Soffer bound

$$|h_1^q(x)| \le \frac{1}{2} \left( f_1^q(x) + g_1^q(x) \right)$$

Role of first principles on PDF analyses on the spotlight right now [see talk by Nobuo]

Sensible flexibility on the constraints might be the key [see, e.g. PDF4LHC working group, 2203.05506]

Pheno—positivity for polarized PDFs: [Benel et al., EPJC80] [d'Alesio et al., PLB803]

Theory—positivity for unpolarized PDFs: [Candido et al., JHEP11(2020)] [Collins et al, PRD105]

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# Transversity PDF — data

## **Characteristics of the transversity PDF**

- Can be assessed in semi-inclusive processes with a chiral-odd partner
- No constraining sum rule
- Its first Mellin moment gives the tensor charge
- Transversity is a non-singlet object

### Data $\rightarrow$ **Dihadron production**

- SIDIS @HERMES, COMPASS
- pp↑ @RHIC
- e+e- @Belle

## **Extended formalism for single hadron production**

- SIDIS @HERMES, COMPASS & JLab
- $pp\uparrow$  @RHIC, more DY and  $A_N$
- e+e- @Belle

<u>Global analysis</u>: Pavia18 [Radici & Bacchetta, PRL120]

Global analysis: JAM3D22 [Gamberg et al., 2205.00999]

[see talk by Daniel P.]

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# Imposing the positivity on the functional form

A crucial step and a physical statement

## **1.** Parametrization that obeys the positivity bounds.

$$|h_1^q(x)| \le \frac{1}{2} \left( f_1^q(x) + g_1^q(x) \right)$$

Its expression is based on the analyses of the unpolarized  $f_1$  and the helicity  $g_1$  PDFs.

$$x h_1(x) = \frac{F(x)}{|F(x)|} \times x$$
 Soffer Bound with  $F(x) \propto x^{\alpha} (1-x)^{\beta}$ 

Good fits are obrained, with a clear **saturation** of the bound.

Recent global fit shows that pp data slightly release that saturation [Bacchetta & Radici, PRL 120 (2018)], **PAV18** 







[Bacchetta, A.C., Radici, JHEP1303] [Radici, Bacchetta, A.C., Guagnelli, JHEP1505] **PAV15** 

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# Imposing the positivity by the fitting procedure

## 2. Implicitly guide the positivity bounds at large x

A. Weight by the probability of the positivity bound given the data

$$\chi_r^2\left(\{p^I\}\right) = \sum_j w_j \frac{\left[x_j h_{1\,\text{theo}}^{p/D}\left(x_j; \{p^I\}\right) - \left(x h_1^{p/D}(x)\right)_{j,r\,\text{data}}\right]^2}{\sigma_j^2}$$

B. Parametrize the x-dependence ; repeat N times (Bootstrap method)

$$x h_{1,i}^{q_v} \left( x; p_{i,k}^q \right) = x^{1.25} \sum_{k = \{\kappa_q\}} \sum_{k = \{\kappa_$$

C.Constrain the falloff of the PDF at large-x with Lagrange multipliers

$$C_{i,j}^{d_{V}}(p_{i,k}^{d\,II}) = h_{1,i}^{q_{V}}(x_{j}; p_{i,k}^{d\,II})$$
$$C_{i,j}^{d_{V}}(p_{i,k}^{d\,II}) = h_{1,i}^{q_{V}}(x_{j}; p_{i,k}^{d\,II})$$

$$p_{i,k}^{q} B_{k,n_{i}} \left(g(x)\right)$$

$$<\epsilon_j$$
  
 $>-\epsilon_j$ 





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**Balanced interplay between constraints and data.** 

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# Tensor charge from constrained fit

Stacked histogram of contribution from the four functional forms



Can we learn from the distribution of the chi square and  $g_T$ ?



e(x)



# Beam spin asymmetry at CLAS and CLAS12

dihadron SIDIS on proton target – sensitive to  $e^P \equiv \frac{1}{\Omega} (4 e^{u_V} - e^{d_V});$ 

> [Bacchetta & Radici, PRD69 (2004)] [Courtoy, 1405.7659]

non-vanishing twist-3 effects at CLAS12;

- projections of beam spin asymmetries on  $(x, z, M_h; Q^2, y)$ 
  - $\Rightarrow$  (x, z, M<sub>h</sub>) triptych from the parton distribution and fragmentation function.

Road map for e(x) extraction and (global) analysis.

### see talk by Christopher Dilks



[CLAS Collaboration, PRL126 (2021) 6, 062002]

CLAS12:  $1.5 < Q^2 < 5.7 \,\text{GeV}^2$ 



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[CLAS Collaboration, PRL126 (2021) 152501]



$$A_{LU}^{\sin\phi_R}(x,z,m_{\pi\pi}) \propto \frac{M}{Q} \frac{\sum_q e_q^2 \left[xe^q(x) H\right]}{\sum_q e_q^2} \frac{1}{\sum_q e_q^2} \left[xe^q(x) H\right]}{\sum_q e_q^2} \frac{1}{\sum_q e_q^2} \frac{1}{\sum_q$$

1.0

### **Twist-2 Dihadron Fragmentation Functions**

Phenomenologically tested for the twist-2 transversity PDF — see above we get the ratio R that is believed to be universal (portable) up to evolution effects

$$R(z, M_h) = \frac{|\mathbf{R}|}{M_h} \frac{H_1^{\triangleleft u}(z, M_h; Q_0^2)}{D_1^u(z, M_h; Q_0^2)}$$



# Extraction of e

### **Twist-3 Dihadron Fragme**

- Unknown phenomenologically;
- Solutions for genuine twist-3 DiFF:  $\tilde{D}^{\triangleleft}$  [Luo et al., PRD100],  $\tilde{G}^{\triangleleft}$  [Yang et al., PRD99]
- Setimate of Interference FF through the asymmetries on longitudinally-polarized target at COMPASS [Sirtl, PhD thesis, 2017]

$$\begin{split} A_{UL}^{\sin(\phi_R)} &= -\frac{M}{Q} \frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 \left[ x h_L^q(x) H_1^{\angle q, sp}(z, M_h^2) + \frac{M_h}{Mz} g_1^q(x) \tilde{G}^{\angle q, sp}(z, M_h^2) \right]}{\sum_q e_q^2 f_1^q(x) D_1^{q, ss + pp}(z, M_h^2)} \\ A_{LL}^{\cos(\phi_R)} &= -\frac{M}{Q} \frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 \left[ x e_L^q(x) H_1^{\angle q, sp}(z, M_h^2) - \frac{M_h}{Mz} g_1^q(x) \tilde{D}^{\angle q, sp}(z, M_h^2) \right]}{\sum_q e_q^2 f_1^q(x) D_1^{q, ss + pp}(z, M_h^2)}. \end{split}$$

$$A_{UL}^{\sin(\phi_R)} = 0.0050 \pm 0.0010(\text{stat}) \pm 0.0007(\text{sys})$$
$$A_{LL}^{\cos(\phi_R)} = -0.0135 \pm 0.0064(\text{stat}) \pm 0.0046(\text{sys})$$

 $\Rightarrow |A_{LL}^{\cos\phi_R}| >> A_{UL}^{\sin\phi_R}$ 











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[Radici, AC, Bacchetta, JHEP 05 (2015)]





# Our ansatz for the twist-3 DiFF contribution

- <u>CLAS12</u>: split invariant-mass regions  $M_h > \text{ or } < 0.63 \text{ GeV}$  to pinpoint vector meson contributions Ð We assume the trend of all interference DiFFs in the invariant mass is similar for  $M_h > 0.63$  GeV (up to overall sign)  $\Rightarrow$  supported by model evaluation of  $\tilde{D}^{\triangleleft}$  and  $\tilde{G}^{\triangleleft}$
- Solution Reproducing  $A_{LL}^{\cos \phi_R}$  in that range sets our upper bound to  $\kappa \Rightarrow \kappa_{M_h}$  $\kappa_{M_k}$  reproduces the order of magnitude for  $A_{III}^{\sin \phi_R}$  adequately



 $\Rightarrow$  invariant-mass behavior is key, twist-2 DiFF alone not enough to interpret all  $M_h$  -projected twist-3 asymmetries.

# Point-by-point e(x) from CLAS data

Scenario I: Wandzura-Wilczek approximation

$$\frac{e^{V}(x)}{f_{1}^{\Sigma}(x)}\frac{\tilde{H}_{1}^{\triangleleft}}{D_{1}} \propto \frac{Q}{M} A_{LU}^{\sin\phi_{R}}$$

Scenario II: beyond WW approximation

$$\frac{e^V(x)}{f_1^{\Sigma}(x)}\frac{\tilde{H}_1^{\sphericalangle}}{D_1} \propto \frac{Q}{M} A_{LU}^{\sin\phi_R} \pm \kappa \frac{f_1^V(x)}{f_1^{\Sigma}(x)}\frac{\tilde{H}_1^{\sphericalangle}}{D_1}$$

Combining the uncertainty at 90% CL⇒





Sign of twist-3 DiFFs undetermined



# What is the probability for $e^{P}(x)$ to be non-zero?

Probability that the proton combination is <u>greater</u> than zero — not exactly "how incompatible with zero is it?" — is a useful information from the point-by-point extraction of a collinear twist-3 PDF with a minimum set of approximations.



Mostly far from zero!

# Conclusions

We have discussed the extraction of chiral-odd collinear PDF through the dihadron framework.

We have discussed an alternative way to account for relaxed positivity constraints on the transversity fit. We have presented a truly updated extraction of the scalar PDF, e(x). It is non-zero to more than 75% probability.

Studies of chiral-odd PDFs are important:

- Precision imaging of nucleons.
- Emergence of hadronic mass from the scalar PDF.
- Possible tensor and scalar interactions beyond the standard model.
- Collinear-to-PDF interpretations.
- Understanding of the mid-to-high energy transitions.

Could benefit from the experience gained from the unpolarized PDFs — statistical analyses-wise.

# Backup

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## Higher-twist corrections

From spurious contaminations...

CJ15 global analysis includes lower cuts on W<sup>2</sup>. [Accardi et al., PRD93]

$$F_2(x, Q^2) = F_2^{\text{LT}}(x, Q^2) \left(1 + \frac{C_{\text{HT}}(x)}{Q^2}\right)$$

HT's role in fulfilling duality [e.g. Melnitchouk et al., Phys.Rept.406]

### …to genuine effects

JAM analysis of the helicity PDF  $g_1$  extends to  $g_T$ , with  $g_T=g_1+g_2$ . [Sato et al., PRD93]

$$g_2^{(\tau 3)}(x, Q^2) = D(x, Q^2) - \int_x^1 \frac{dz}{z} D(z, Q^2)$$





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# Twist-3 PDF in observables

Only twist-3 PDF accessible through inclusive DIS: g⊤

[e.g., Tangerman & Mulders, eprint/9408305]

Experimentally accessed at SLAC E142 & E155x and JLab-Hall A E99-117 & E01-012.

Exploratory studies suggest that <u>quark-gluon-quark</u> correlations are non-zero.

$$\Delta_{\rm ex}(x_B, Q^2) = g_2^{\rm ex}(x_B, Q^2) - g_2^{\rm WW}(x_B, Q^2)$$



[Accardi et al, JHEP11 (2009)]

# **Comparison with lattice**



It is possible to get an **agreement with lattice g\_T** with more flexibility in the functional form. δu is the most problematic and yet most determined.

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# Scalar PDF

The composition of the scalar PDF is worked out through the EoM of QCD:

$$e^{q}(x) = e^{q}_{loc}(x) + e^{q}_{gen}$$

$$\text{"local" term} \qquad \text{genuine qg}$$
originates from singularity originates from
$$e^{q}_{loc}(x) = \frac{1}{2M} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | \bar{\psi}_{q}(0) \psi_{q}(0) | P \rangle = \frac{\delta(x)}{2M} \langle P | \bar{\psi}_{q}(0) \psi_{q}(0) | P \rangle$$

Only observable-related contribution to the proton mass: the singularity of e(x) is proportional to the pion-nucleon sigma term through sum rules [e.g. Kodaira & Tanaka, PTP, Vol. 101]

### Lots of interests for that function in the past years

[Schweitzer and Efremov, JHEP08006] [Burkardt & Koike, NPB632] [Ji, NPB960] [Lorcé, Pasquini, Schweitzer, JHEP01 (2015)] [Pasquini & Rodini, PLB788] [Hatta & Zhao, PRD102] [Bhattacharya et al., PRD102]

 $(x) + e^q_{\text{mass}}(x)$ 

gq correlation

rom covariant derivative

quark mass term

originates from kinetic+mass

 $(0)|P\rangle$ 

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# Proton mass decomposition

### **QCD** mass decomposition



### Sigma terms

$$\langle P|m_u \bar{u}u + m_d \bar{d}d|P \rangle = \sigma_{\pi N}$$
 &  $\langle P|T$ 

9 have been determined from theoretical analysis of  $\pi N$  data [Meissner et al.]

- have been evaluated on the lattice [Constantinou et al.]
- are related to the twist-3 PDF e(x) via sum rules [e.g. Kodaira & Tanaka, PTP, Vol. 101] ٢

[Ji, PRL 74; Ji, PRD 52] [Lorcé, EPJC78; Lorcé et al, 2109.11785]

 $|m_s \bar{s}s|P\rangle = \sigma_s$ 

 $\langle P|\bar{q}q|P
angle$  $dxe_q(x)$ 2M

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# Universality of non-perturbative functions

**Dihadron fragmentation functions** 

- ➡ DiFF extracted in e<sup>+</sup>e<sup>-</sup>, to be tested against SIDIS multiplicities
- Consistency check on SIDIS  $(z, M_h)$  dependence at CLAS & CLAS12
- Determination of the integral of  $e^{P}(x)$  from reconstruction:  $n_{r}$

- Twist-2 and -3 PDFs
  - → Universality of transversity in pp and SIDIS [Radici et al, PRD94]
  - Global analysis of the transversity possible [Radici & Bacchetta, PRL120; JAM Coll., PRD102]
  - Are twist-3 PDFs universal?

Yet to be answered.

Examples through TMD and dynamical twist-3 relations (e.g. Sivers and Qiu-Sterman)



Data

Chiral-odd PDFs

# Consequences of the extraction

- 1. Are twist-3 PDFs non-zero? Yes, to a certain CL.
- 2. Can we access qgq correlations and more non-perturbative information? Let's take the example of e(x).



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Schematic models for illustration purpose only!

Moments will matter.

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Other nonperturbative effects at not so small x



# Can we study qgq correlation at the EIC?

### EIC Yellow Report [2103.05419]



EIC will cover low- to mid-Q<sup>2</sup> and smallish x values

Projections for IR2@EIC White Paper

Proposal for a 2nd interaction region — IR2@EIC Discuss the complementarity with present data.

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# Twist-3 in semi-inclusive DIS

## By detecting a hadronization product, more Lorentz structures become available: <u>semi-inclusive DIS</u>



- Not all small.
- We have the distributions are very intertwined and render extraction very hypothesis-dependent.

$$S = \frac{M}{P^{+}} [\text{twist-3 PDFs}]$$

$$(Boer, Mulders \& Pijlman, NPB 667]$$

$$(Bacchetta et al, JHEP02 (2007)]$$

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HERMES and CLAS collaborations have measured various twist-3 TMD distributions through asymmetries.

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# Extraction of e(x) from CLAS data

 $A_{LU}^{\sin\phi_R}(x,z,m_{\pi\pi}) \propto \frac{M}{Q} \frac{\sum_q e_q^2 \left[ x e^q(x) H_{1,sp}^{\triangleleft,q}(z,m_{\pi\pi}) + \frac{m_{\pi\pi}}{zM} f_1^q(x) \tilde{G}_{sp}^{\triangleleft,q}(z,m_{\pi\pi}) \right]}{\sum_q e_q^2 f_1^q(x) D_{1,ss+nn}^q(z,m_{\pi\pi})}$ 



### **Twist-2 Dihadron Fragmentation Functions**

- Phenomenologically tested for the twist-2 transversity PDF [Bacchetta, AC & Radici, PRL107 and follow-ups]
- Sextracted in e+e- at Belle [A.C., Bacchetta, Radici & Bianconi, PRD85]
- we get the ratio R that is believed to be universal (portable) up to evolution effects

$$R(z, M_h) = \frac{|\mathbf{R}|}{M_h} \frac{H_1^{\triangleleft u}(z, M_h; Q_0^2)}{D_1^u(z, M_h; Q_0^2)}$$

0.6 0.4 0.8 1.0 Z  $A_{LU}^{\sin\phi_R}(x,z,m_{\pi\pi}) \propto \frac{M}{Q} \frac{\sum_q e_q^2 \left[ x e^q(x) H_{1,sp}^{\triangleleft,q}(z,m_{\pi\pi}) + \frac{m_{\pi\pi}}{zM} f_1^q(x) \tilde{G}_{sp}^{\triangleleft,q}(z,m_{\pi\pi}) \right]}{\sum_q e_q^2 f_1^q(x) D_{1,ss+pp}^q(z,m_{\pi\pi})}$ 

leading-twist DiFFs

chiral-odd DiFF unpolarized DiFF \_Chiral-odd PDFs\_ Transversity\_2022

# Scalar PDF and the proton mass

## **QCD** mass decomposition



<u>Sigma terms</u>

$$\left\langle P|m_u\bar{u}u + m_d\bar{d}d|P\right\rangle = \sigma_{\pi N}$$

<sup>a</sup> have been determined from theoretical analysis of  $\pi N$  data [Meissner et al.]

- have been evaluated on the lattice [Constantinou et al.]
- <sup>(a)</sup> pheno analysis of e(x) could pave the way towards another possible determination



quark and gluon energy  $\propto \langle x \rangle_{q,g}$ 

[Ji, PRL 74; Ji, PRD 52] [Lorcé, EPJC78; Lorcé et al, 2109.11785]

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# Twist-3 in SIDIS dihadron production



- modulations of spin asymmetries single out:
- Scalar PDF from the beam spin asymmetry

$$A_{LU}^{\sin\phi_R}(x,z,m_{\pi\pi}) \propto \frac{M}{Q} \frac{\sum_q e_q^2 \left[ x e^q(x) H_{1,sp}^{\triangleleft,q}(z,m_{\pi\pi}) + \frac{m_{\pi\pi}}{zM} f_1^q(x) \tilde{G}_{sp}^{\triangleleft,q}(z,m_{\pi\pi}) \right]}{\sum_q e_q^2 f_1^q(x) D_{1,ss+pp}^q(z,m_{\pi\pi})}$$



# EIC coverage

- EIC error projections (from transversity studies) 0
- Proton target shown, but need for neutron
- Models × DiFFs predictions 0
  - LC model [Pasquini & Rodini, PLB 788]
  - made-up mass-term contribution with m<sub>q</sub>=300MeV
- Non-negligible for lowest beam configurations 0



Archetype of observables for IR2@EIC

EIC Yellow Report [2103.05419]



# QCD and twist-3 PDFs

### Underlying and omitted in all this presentation: Q<sup>2</sup>-evolution!



- Evolution equations for genuine qgq twist-3 known in most cases;
- Understanding of the various contributions to twist-3 PDFs;
- Especially "hot" for TMD studies.
- Require a second interaction region @EIC.



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