TMD distributions @ next-to-leading power

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Outline

TMD distributions of genuine twist-3

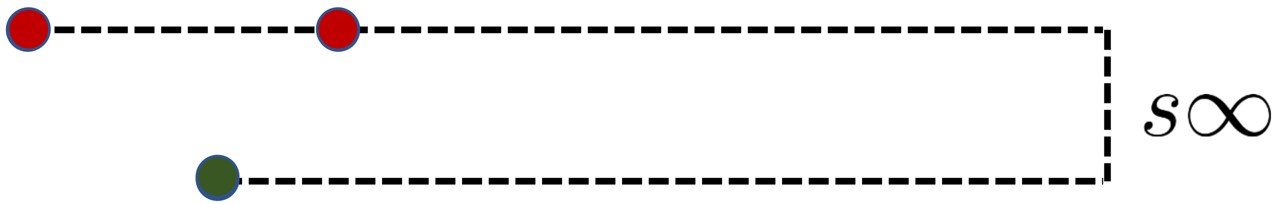
UV-evolution equations

Genuine twist-3 distributions of definite T-parity

Bi-quark correlator of twist-3 and
Large-Nc evolution equations

TMD distributions of genuine twist-3

$$\Gamma \in \{\gamma^+, \gamma^+\gamma_5, i\sigma^{\alpha+}\gamma_5\}$$



$$\widetilde{\Phi}^{[\Gamma],\mu}_{21,\text{bare}} = g\langle p,s|\bar{q}(z_1 n+b)F^{\mu+}(z_2 n+b)[z_2 n+b, s\infty n+b]\frac{\Gamma}{2}[s\infty n, z_3 n]q(z_3 n)|p,s\rangle$$

$$\widetilde{\Phi}^{[\Gamma],\mu}_{12,\text{bare}} = g\langle p,s|\bar{q}(z_1 n+b)[z_1 n+b, s\infty n+b]\frac{\Gamma}{2}[s\infty n, z_2 n]F^{\mu+}(z_2 n)q(z_3 n)|p,s\rangle.$$

Semi-compact operators (since include Wilson line to infinity) of twist 1 and 2

Process dependence through ‘s’ = +1 (SIDIS-like), = -1 (DY-like)

We can renormalize the semi-compact operators separately (!)

$$\Phi_{21,bare} = Z_1 Z_2 R\left(b^2\right)\Phi_{21}$$

$R(b^2)$ is the standard rapidity divergence factor.

From Z1 and Z2 we get the anomalous dimensions => UV evolution equations

$$\frac{d}{d\log\mu^2}\widetilde{\Phi}_{21} = (\widetilde{\gamma}_2(z_1,z_2)+\widetilde{\gamma}_1(z_3))\widetilde{\Phi}_{21}$$

We can consider the Fourier transformed distributions

$$\widetilde{\Phi}^{[\Gamma]}_{\mu,\bullet}(z_1,z_2,z_3,b) = (p^+)^2\int[dx]e^{-i(x_1z_1+x_2z_2+x_3z_3)p^+}\,\Phi^{[\Gamma]}_{\mu,\bullet}(x_1,x_2,x_3,b)$$

Time-reversal maps twist-1+2 to twist-2+1 and *vice versa*!

$$\mathscr{P}\mathscr{T}\,\Phi^{[\Gamma]}_{\mu,12}(x_1,x_2,x_3,b;s)(\mathscr{P}\mathscr{T})^{-1} = -\Phi^{[\gamma^0 T\Gamma^* T^{-1}\gamma^0]}_{\mu,21}(-x_3,-x_2,-x_1,-b;-s)$$

$$\mathscr{P}\mathscr{T}\,\Phi^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b;s)(\mathscr{P}\mathscr{T})^{-1} = -\Phi^{[\gamma^0 T\Gamma^* T^{-1}\gamma^0]}_{\mu,12}(-x_3,-x_2,-x_1,-b;-s)$$

Moreover, they are complex-valued functions (!)

$$[\Phi^{[\Gamma]}_{\mu,12}(x_1,x_2,x_3,b)]^* = \Phi^{[\gamma^0\Gamma^\dagger\gamma^0]}_{\mu,21}(-x_3,-x_2,-x_1,-b)$$

$$[\Phi^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b)]^* = \Phi^{[\gamma^0\Gamma^\dagger\gamma^0]}_{\mu,12}(-x_3,-x_2,-x_1,-b)$$

The UV-evolution equations in momentum space are complex and process-dependent!

$$\mu^2 \frac{d}{d\mu^2} \Phi^{[\Gamma]}_{\mu,21} = \left(\frac{\Gamma_{\text{cusp}}}{2} \ln\left(\frac{\mu^2}{\zeta}\right) + \Upsilon_{x_1 x_2 x_3} + \boxed{2\pi i\, s\, \Theta_{x_1 x_2 x_3}} \right) \Phi^{[\Gamma]}_{\mu,21} + \mathbb{P}^A_{x_2 x_1} \otimes \Phi^{[\gamma^\nu \gamma^\mu \Gamma]}_{\nu,21} + \mathbb{P}^B_{x_2 x_1} \otimes \Phi^{[\gamma^\mu \gamma^\nu \Gamma]}_{\nu,21}$$

$$\mu^2 \frac{d}{d\mu^2} \Phi^{[\Gamma]}_{\mu,12} = \left(\frac{\Gamma_{\text{cusp}}}{2} \ln\left(\frac{\mu^2}{\zeta}\right) + \Upsilon_{x_3 x_2 x_1} + \boxed{2\pi i\, s\, \Theta_{x_3 x_2 x_1}} \right) \Phi^{[\Gamma]}_{\mu,12} + \mathbb{P}^A_{x_2 x_3} \otimes \Phi^{[\Gamma \gamma^\mu \gamma^\nu]}_{\nu,12} + \mathbb{P}^B_{x_2 x_3} \otimes \Phi^{[\Gamma \gamma^\nu \gamma^\mu]}_{\nu,12}$$

First observation of process-dependent evolution equations

not a problem, since $\Phi_{12,21}$ are complex-valued

Let us introduce combinations with definite complexity and T-parity

$$\Phi^{[\Gamma]}_{\mu,\oplus}(x_1,x_2,x_3,b)=\frac{\Phi^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b)+\Phi^{[\Gamma]}_{\mu,12}(-x_3,-x_2,-x_1,b)}{2},$$

$$\Phi^{[\Gamma]}_{\mu,\ominus}(x_1,x_2,x_3,b)=i\frac{\Phi^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b)-\Phi^{[\Gamma]}_{\mu,12}(-x_3,-x_2,-x_1,b)}{2},$$

and parametrize them

	U	L	$\mathrm{T}_{J=0}$	$\mathrm{T}_{J=1}$	$\mathrm{T}_{J=2}$
U	$f_\bullet^\perp$	$g_\bullet^\perp$		$h_\bullet$	$h_\bullet^\perp$
L	$f_{\bullet L}^\perp$	$g_{\bullet L}^\perp$	$h_{\bullet L}$		$h_{\bullet L}^\perp$
T	$f_{\bullet T}$, $f_{\bullet T}^\perp$	$g_{\bullet T}$, $g_{\bullet T}^\perp$	$h_{\bullet T}^{D\perp}$	$h_{\bullet T}^{A\perp}$	$h_{\bullet T}^{S\perp}$, $h_{\bullet T}^{T\perp}$

In total we have 32 distributions, since $\bullet=\oplus,\ominus$

The price to pay is the mixing under UV evolutions of $\oplus,\ominus$ distributions

Bi-quark correlator <-> TMD distributions of generic twist-3

$$\Phi^{[\Gamma]}_{\bar{q}q}(x,b) = F.T.\left(\langle p,s|\bar{q}(zn+b)[zn+b,s\infty n+b]\frac{\Gamma}{2}[s\infty n,0]q(0)|p,s\rangle\right)$$

$$\Gamma \in \{\mathbb{1}, i\gamma_5, \gamma_T^\alpha, \gamma_T^\alpha\gamma_5, i\sigma^{\alpha\beta}\gamma_5, i\sigma^{+-}\gamma_5\}$$

TMDs:

$$\mathbb{1} \to e, e_T^\perp$$
$$i\gamma_5 \to e_L, e_T$$
$$\gamma_T^\alpha \to f_T, f_L^\perp, f^\perp, f_T^\perp$$
$$\gamma_T^\alpha\gamma_5 \to g_T, g_L^\perp, g^\perp, g_T^\perp$$
$$i\sigma^{\alpha\beta}\gamma_5 \to h_T^\perp, h$$
$$i\sigma^{+-}\gamma_5 \to h_L^\perp, h_T$$

Isolate one ‘good’ and one ‘bad’ quark component, we must use quark EOM to express everything in terms of good components, i.e. operators of definite twist

Example

$$x\, f_T = \quad \mathbf{f}^{(0)}_{\ominus,T} - \mathbf{g}^{(0)}_{\oplus,T} - f^{\perp}_{1T} - b^2 \frac{\partial f^{\perp}_{1T}}{\partial b^2}$$

Generic twist-3

Twist-2

Derivative of twist-2

Genuine twist-1+2,2+1

So far, not so bad, but…

evolution equations are NOT closed

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} F_+ \\ F_- \end{pmatrix} = \left(\frac{\Gamma_{\text{cusp}}}{2} \ln\left(\frac{\mu^2}{\zeta}\right) + \gamma_1 \right) \begin{pmatrix} F_+ \\ F_- \end{pmatrix} - \left(\gamma_1 + \frac{\gamma_V}{2}\right) \frac{1}{x} \begin{pmatrix} f_+ \\ f_- \end{pmatrix}$$
$$+ \frac{1}{x} \int \frac{[dx]}{x_2} \begin{pmatrix} 2\mathbb{P} & 2\pi s \Theta \\ -2\pi s \Theta & 2\mathbb{P} \end{pmatrix} \begin{pmatrix} \Phi_+ \\ \Phi_- \end{pmatrix}$$

$$\{F_+, F_-\} = \{f, g\} \text{ or } \{h, e\}$$

$$\{f_+, f_-\} = \text{twsit-2 of} \{F_+, F_-\}$$

The large Nc limit closes the evolution equations

$$\mathbb{P}, \Theta \sim \mathcal{O}\left(\frac{\alpha_s}{N_c}\right)$$

$$\mu^2 \frac{d}{d\mu^2}\left(\begin{array}{c} F_+ \\ F_- \end{array}\right) = \left(\frac{\Gamma_{\rm cusp}}{2}\ln\left(\frac{\mu^2}{\zeta}\right) + \gamma_1\right)\left(\begin{array}{c} F_+ \\ F_- \end{array}\right) - \left(\gamma_1 + \frac{\gamma_V}{2}\right)\frac{1}{x}\left(\begin{array}{c} f_+ \\ f_- \end{array}\right)$$
$$+\frac{1}{x}\int \frac{[dx]}{x_2}\left(\begin{array}{cc} 2\mathbb{P} & 2\pi s\Theta \\ -2\pi s\Theta & 2\mathbb{P} \end{array}\right)\left(\begin{array}{c} \Phi_+ \\ \Phi_- \end{array}\right)$$

Examples

$$\mu^2 \frac{d}{d\mu^2} f_T = \left(\frac{\Gamma_{\rm cusp}}{2}\ln\left(\frac{\mu^2}{\zeta}\right) + a_s C_F\right) f_T - \frac{2a_s C_F}{x}\left(f_{1T}^{\perp} + b^2 \frac{\partial f_{1T}^{\perp}}{\partial b^2}\right)$$

$$\mu^2 \frac{d}{d\mu^2} e = \left(\frac{\Gamma_{\rm cusp}}{2}\ln\left(\frac{\mu^2}{\zeta}\right) + a_s C_F\right) e$$

Conclusions

- 32 genuine TMD distributions of twist-three, 16 are T-odd

- The UV evolution mix T-even and T-odd distributions.
  The mixture is proportional to a process-dependent sign factor, such that each function preserves its T-parity and in-between-process universality.

- The evolution of bi-quark twist-three TMD distributions is autonomous only in the large-Nc limit.

What I did not cover in this talk:
Special rapidity divergences and
their cancellation in physical observables