Fracture functions from back-to-back dihadron production

Timothy B. Hayward, Harut Avakian, Aram Kotzinian

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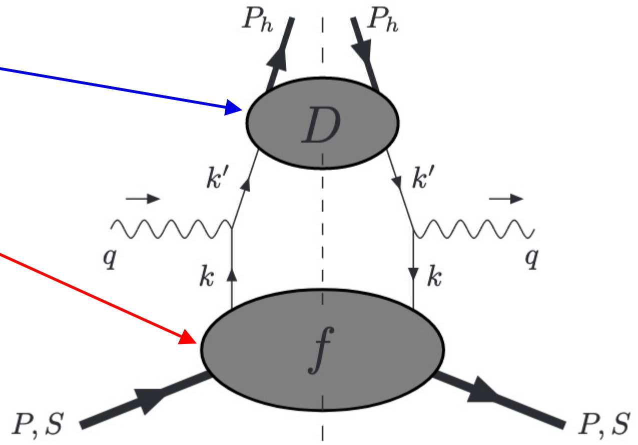
Traditional SIDIS measurements

- Decades of study have led to detailed mappings of the momentum distribution of partons in the nucleon in terms of 1-D and 3-D (TMD) parton distribution functions (PDFs).
- SIDIS measurements rely on the assumption that measured hadrons are produced in the CFR.
- Cross section factorized as a convolution of PDFs and Fragmentation Functions (FFs)[1].

$$\frac{d\sigma^{\mathrm{CFR}}}{dx_B\,dy\,dz_h} = \sum_a e_a^2 \boxed{f_a(x_B)} \frac{d\hat{\sigma}}{dy} \boxed{D_a(z_h)}$$

- PDFs
    - Confined motion of quarks and gluons inside the nucleus
    - Orbital motion of quarks, correlations between quarks and gluons
- Fragmentation Functions
    - Probability for a quark to form particular final state particles
    - Insight into transverse momenta and polarization



M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

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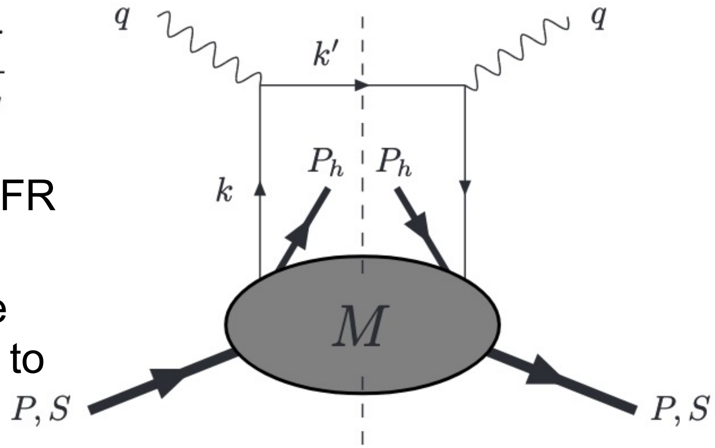
[1] A. Bacchetta et al., JHEP 02 (2007) 093 [hep-ph] 0611265,

The Neglected Hemisphere – Target Fragmentation

- Final state hadrons also form from the left-over target remnant (TFR) whose partonic structure is defined by “fracture functions”[1,2]: the probability for the target remnant to form a certain hadron given a particular ejected quark.
- In the TFR, factorization into x and z does not hold because it is not possible to separate quark emission from hadron production.

$$\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z} = \sum_a e_a^2\,(1-x_B)\,\boxed{M_a(x_B,(1-x_B)z)}\,\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}$$



M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

- Sometimes possible to kinematically separate CFR and TFR … but not always clear.
- Studying the TFR tests our complete understanding of the SIDIS production mechanism while also providing access to information not available in the CFR.

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1. L. Trentadue and G. Veneziano, Phys. Lett. B323 (1994) 201,
2. M. Anselmino et al., Phys. Lett. B. 699 (2011), 108-118, [hep-ph] 1102.4214

Categorizing Fracture Functions

- At leading twist 16 fracture functions exist that can be organized into tables of quark and nucleon polarizations just like the more familiar PDFs.

Quark polarization

Nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

CFR ↔ TFR

Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

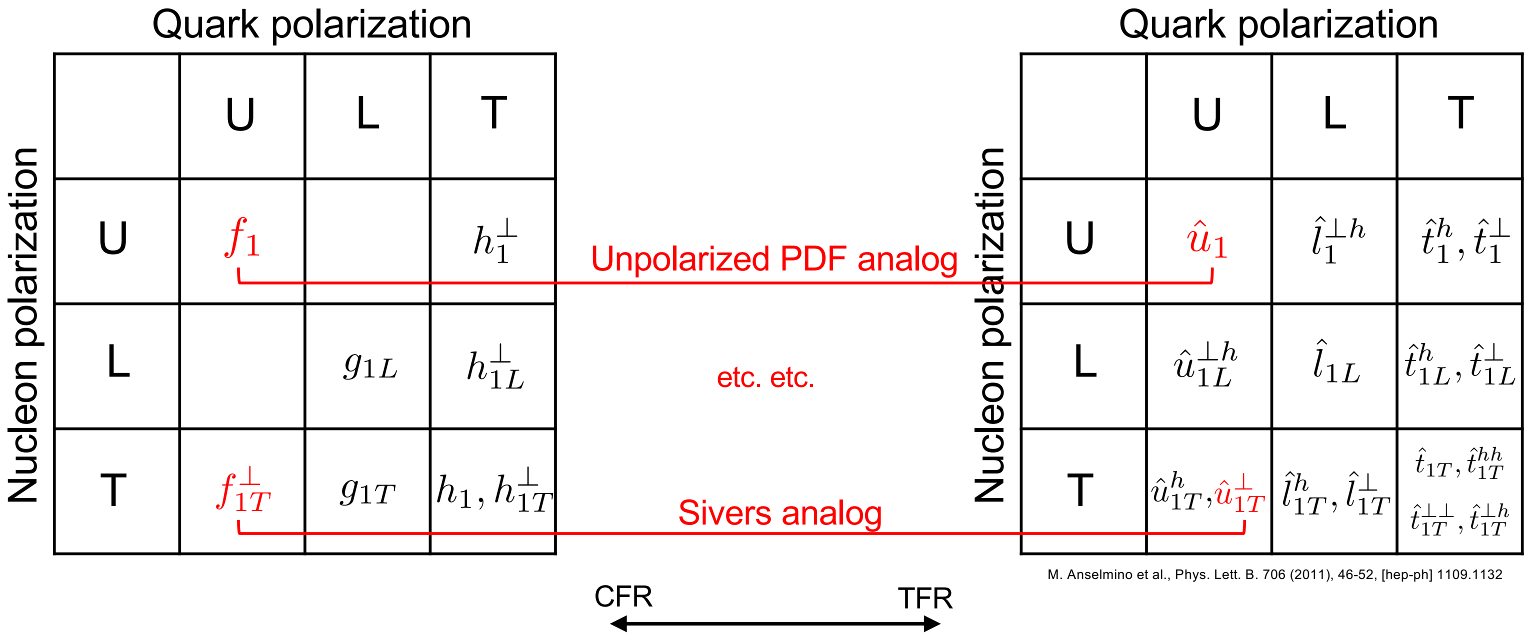
M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Analog to PDFs; Momentum Sum Rules

- A direct relationship exists to the eight leading twist PDFs after the fracture functions are integrated over the fractional longitudinal nucleon momentum, ζ.

$$\sum_h \int_0^{1-x} d\zeta \zeta M_a(x,\zeta) = (1-x) f_a(x)$$

M. Anselmino et al., Phys. Lett. B. 699 (2011), 108, [hep-ph] 1102.4214



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Unpolarized PDF analog

etc. etc.

Sivers analog

Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
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M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

CFR ↔ TFR

UCONN

Single hadron limitations

- FrFs describing transversely polarized quarks are chiral odd and inaccessible in single hadron production.
- Functions with double superscripts containing h and ⊥ have no analog and disappear after integration over either momentum.

Quark polarization

Nucleon polarization

	U	L	T
U	f_1	☹	$h_1^\perp$
L	☹	g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

CFR ⟷ TFR

Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
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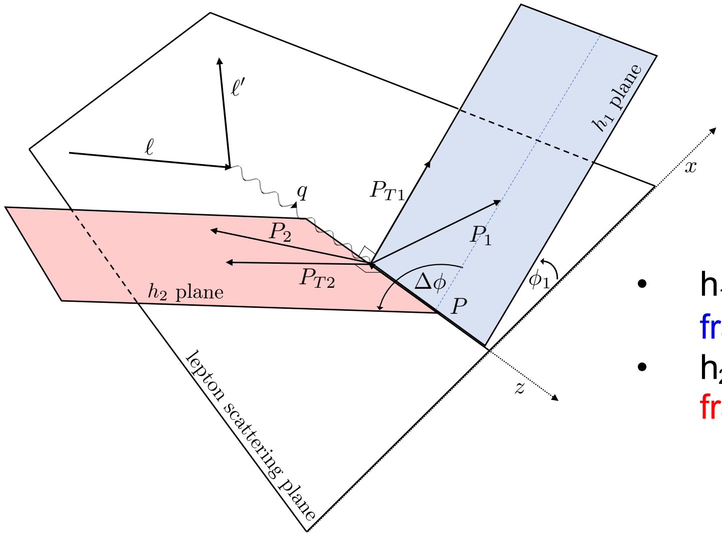
UCONN

Back-to-back Formalism

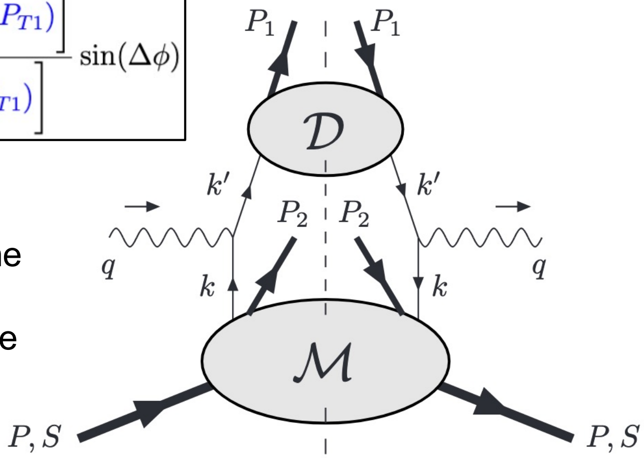
- When two hadrons are produced “back-to-back”[1,2] with one in the CFR and one in the TFR the structure function contains a convolution of a fracture function, $\hat{l}_1^{\perp h}$, and a fragmentation function, D_1.



Kinematic plane for b2b dihadron production.

$$A_{LU} = -k(\epsilon)\frac{P_{T1}P_{T2}}{m_1 m_2}\frac{\mathcal{C}\left[w_5\hat{l}_1^{\perp h}(x,\zeta_2,P_{T2})D_1(z_1,P_{T1})\right]}{\mathcal{C}\left[\hat{u}_1(x,\zeta_2,P_{T2})D_1(z_1,P_{T1})\right]}\sin(\Delta\phi)$$

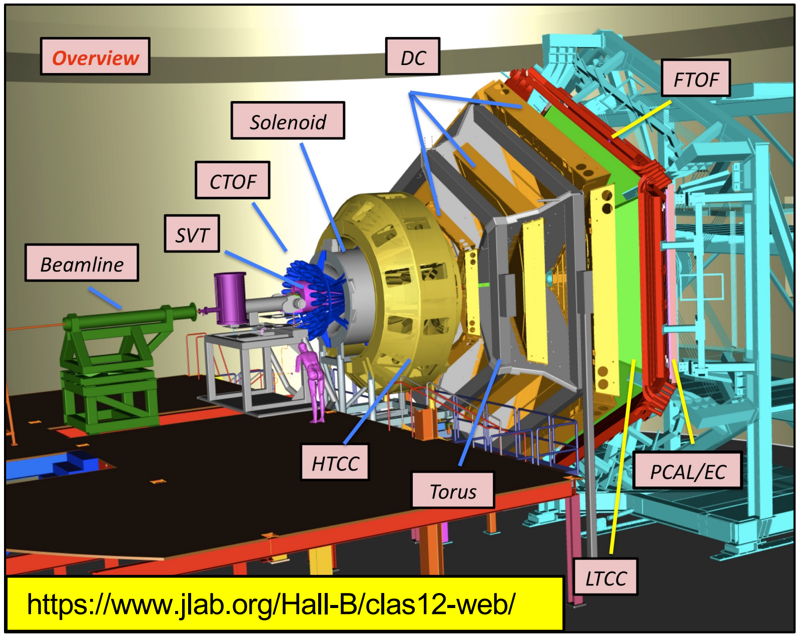
- h_1 in the CFR with production dictated by the fragmentation function
- h_2 in the TFR with production dictated by the fracture function



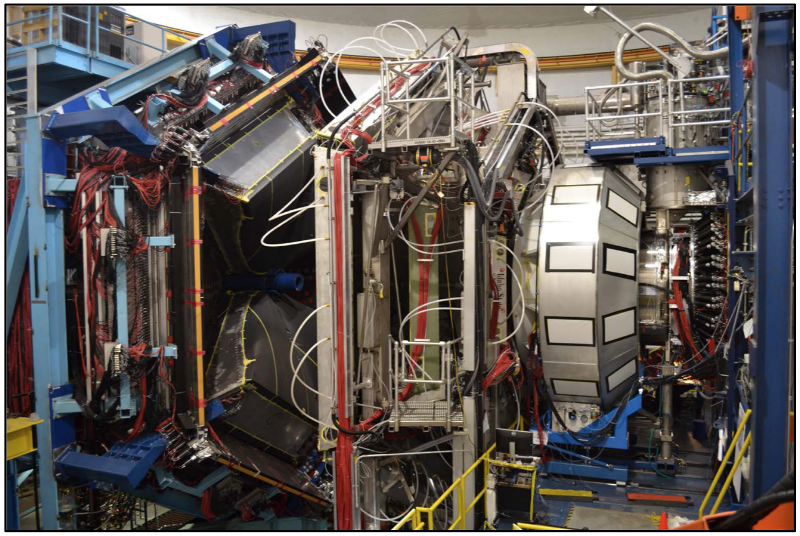
Handbag diagram for dihadron production; lower blob contains FrFs and upper blob contains the FFs.

1. M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132
2. M. Anselmino et al., Phys. Lett. B. 713 (2012), 317-320, [hep-ph] 1112.2604

CLAS12 Experimental Setup



https://www.jlab.org/Hall-B/clas12-web/

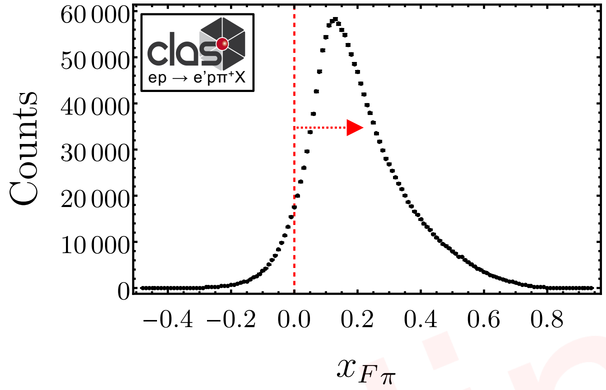


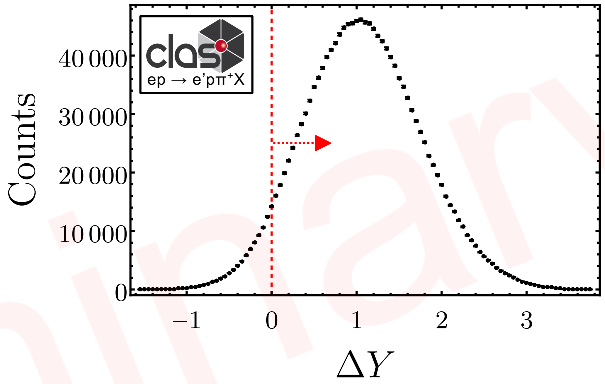
V. Burkert et al., Nucl. Instrum. Meth. A 959 (2020) 163419

- CLAS12: very high luminosity, wide acceptance, low Q^2 (higher twist measurements)
- Began data taking in Spring 2018 – many “run periods” now available.
- 10.6 (2018) and 10.2 (2019) GeV electron beam, longitudinally polarized beam, liquid H_2 target.

Selecting back-to-back events

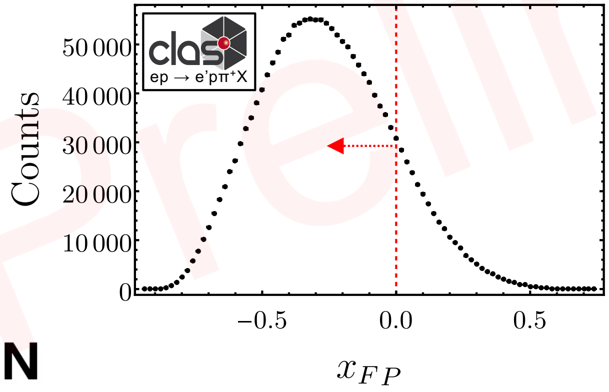
- A natural choice for a first analysis are events with a pion (CFR biased) and proton (TFR biased).

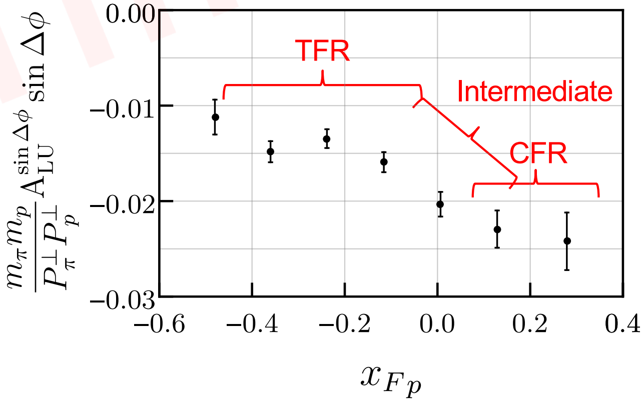




$$x_F = \frac{2p \cdot q}{|q|W}$$

$$Y = \frac{1}{2}\log\left[\frac{E_h + p_z}{E_h - p_z}\right]$$





Early signs of separate signatures in both interaction regions (see next talk!).

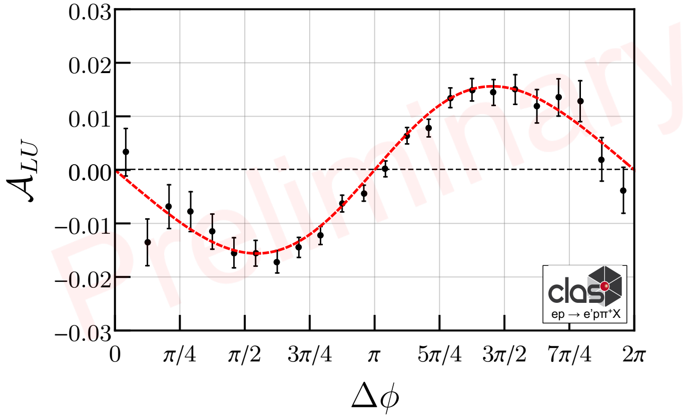
Extracting A_{LU}

- Select $ep \rightarrow e'P\,\pi^+ + X$.
- Consider all possible hadron pairs.
- Amplitudes are extracted simultaneously via maximizing a likelihood function.
- Unbinned maximum likelihood method:

$$-\ln \mathcal{L}_{ML}(A) = N - \sum_{i}^{N} \ln \left[1 + h_i P_i \left(A_1 \sin \Delta\phi_i + A_2 \sin 2\Delta\phi_i \right) \right]$$

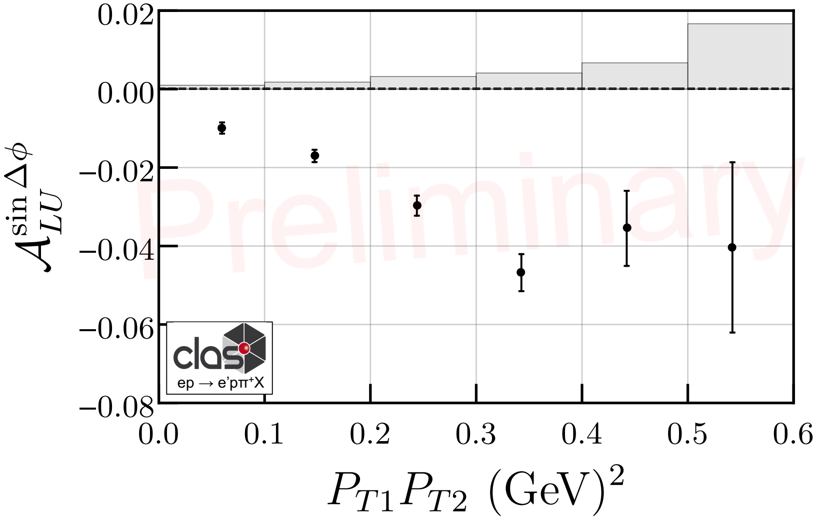
- Include relevant beam polarization (~85% at JLab).

Channel selection
- Q^2>1.0 GeV^2
- W>2.0 GeV
- M_x>0.90 GeV
- y<0.75
- ΔY > 0
- x_{f1} > 0
- x_{f2} < 0
- z_1 > 0.2
- M_h > 1.5 GeV



Initial Observation

- Observed linear dependence on the product of transverse momenta is consistent with expectations (linear, goes to zero at zero transverse momenta, etc.)
- Non-zero asymmetries are the first experimental observation of possible spin-orbit correlations between hadrons produced simultaneously in the CFR and TFR.



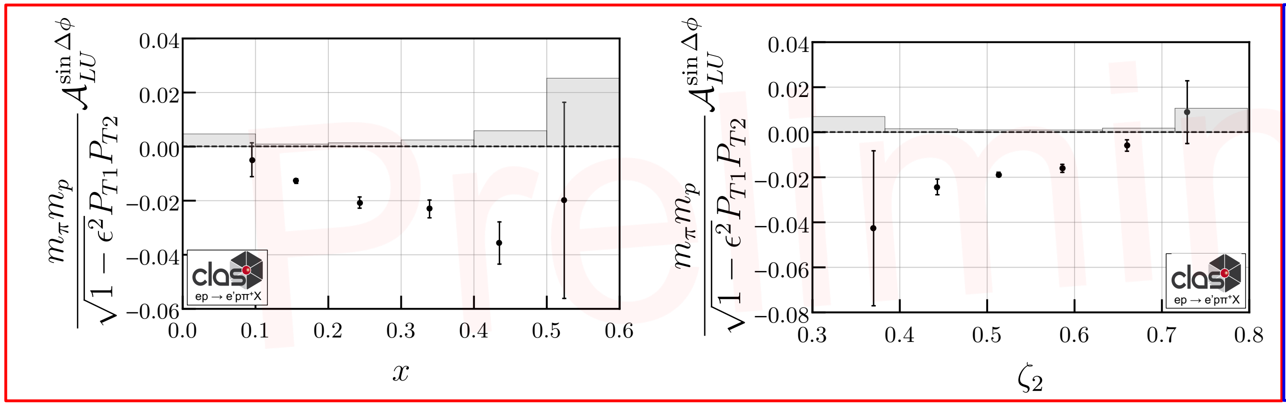
$$\mathcal{A}_{LU} = -\boxed{\sqrt{1-\epsilon^2}\,\frac{|\vec{P}_{T1}||\vec{P}_{T2}|}{m_N\, m_2}}\,\frac{\mathcal{C}[w_5\, \hat{l}_1^{\perp h} D_1]}{\mathcal{C}[\hat{u}_1 D_1]}\sin\Delta\phi$$

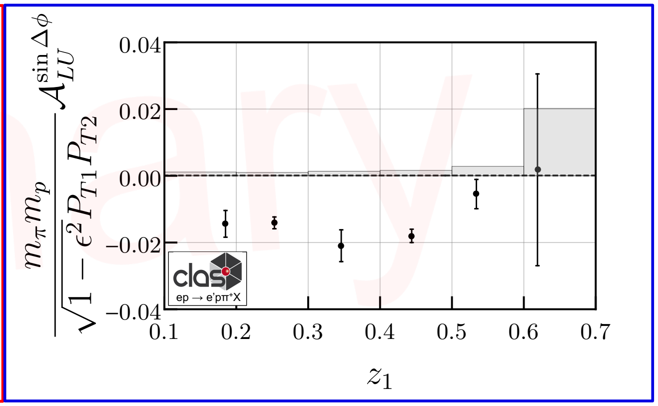
Divide out the kinematic factors for clearer description of fracture and fragmentation function dependence…

Access to unmeasured fracture functions

- x-dependence increases in magnitude in the valance quark region.
- ζ_2-dependence shows decreasing amplitude with increasing momenta. Possibly due to correlations with x.
- Relatively flat as a function of z_1, likely due to cancellation of fragmentation functions.

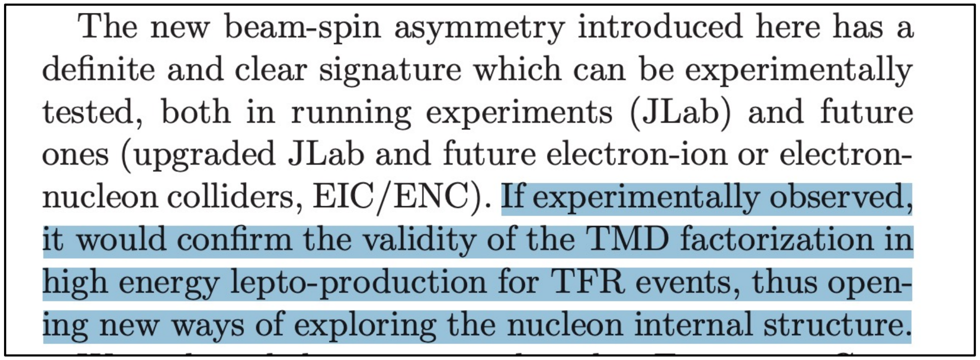
$$A_{LU} \propto \frac{\mathcal{C}\left[w_5 \hat{l}_1^{\perp h}(x,\zeta_2,P_{T2}) D_1(z_1,P_{T1})\right]}{\mathcal{C}\left[\hat{u}_1(x,\zeta_2,P_{T2}) D_1(z_1,P_{T1})\right]}$$





Conclusions

- Kinematic dependencies of SSAs in back-to-back dihadron production observed **for the first time**.
- SSAs can be interpreted in a framework using FrFs, specifically the leading twist FrF $\hat{l}_1^{\perp h}$, describing the conditional probability of the target remnant forming a proton after the emission of a longitudinally polarized quark from an unpolarized proton.
- Future work: deuterium target, separate pion flavors, polarized targets ($\hat{u}_{1L}^{\perp h}$), etc.



The new beam-spin asymmetry introduced here has a definite and clear signature which can be experimentally tested, both in running experiments (JLab) and future ones (upgraded JLab and future electron-ion or electron-nucleon colliders, EIC/ENC). If experimentally observed, it would confirm the validity of the TMD factorization in high energy lepto-production for TFR events, thus opening new ways of exploring the nucleon internal structure.

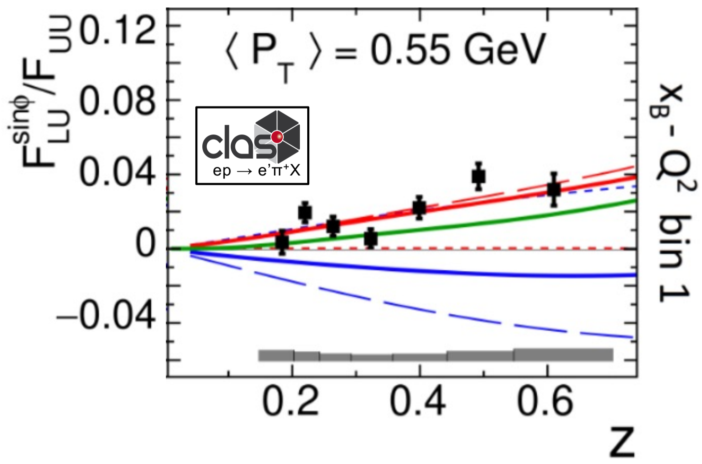
M. Anselmino et al., Phys. Lett. B. 713 (2012), 317-320, [hep-ph] 1112.2604

Backup Slides

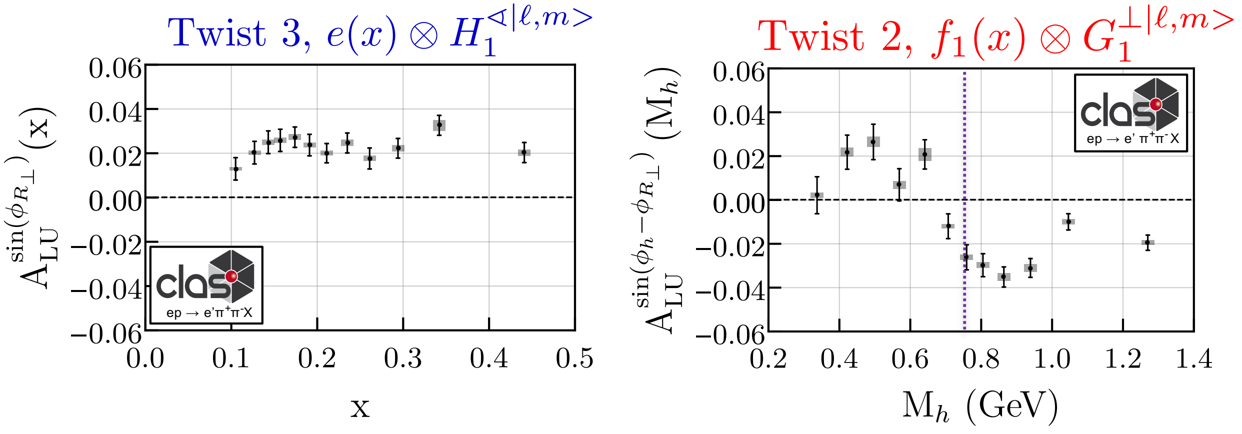
PDF Sensitive CLAS12 Measurements

- Measurements traditionally focus on factorization theorems and assumption that **hadrons are produced in current fragmentation**.

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\hat{h}\cdot k_T}{M_h}\left(xeH_1^{\perp} + \frac{M_h}{M}f_1\frac{\tilde{G}^{\perp}}{z}\right) + \frac{\hat{h}\cdot p_T}{M}\left(xg^{\perp}D_1 + \frac{M_h}{M}h_1^{\perp}\frac{\tilde{E}}{z}\right)\right]$$



S. Diehl et al., *Phys. Rev. Lett.*, 128, 062005, (2022), [hep:ex] 2101.03544



T. B. Hayward et al., *Phys. Rev. Lett.*, 126, 152501, (2021), [hep:ex] 2101.04842

Ambiguous Modulations

$$\begin{aligned}\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}\zeta\,\mathrm{d}^2\boldsymbol{P}_{h\perp}\,\mathrm{d}\phi_S} = \frac{2\alpha_{\mathrm{em}}^2}{Q^2 y}\Bigg\{&\left(1-y+\frac{y^2}{2}\right)\\ &\times\sum_a e_a^2\left[\hat{u}_1(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)-|\boldsymbol{S}_\perp|\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\,\hat{u}_{1T}^{\perp}(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\,\boxed{\sin(\phi_h-\phi_S)}\right]\\ &+\lambda_l\,y\left(1-\frac{y}{2}\right)\sum_a e_a^2\Bigg[S_\parallel\;\hat{l}_{1L}\,(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\\ &+\;|\boldsymbol{S}_\perp|\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\;\hat{l}_{1T}^{\perp}\;(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\,\boxed{\cos(\phi_h-\phi_S)}\Bigg]\Bigg\}\,.\end{aligned}$$

M. Anselmino et al., Phys. Lett. B. 699 (2011), 108-118, [hep-ph] 1102.4214

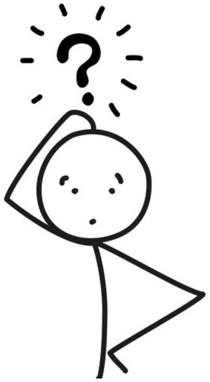
$$\left[F_{UT,T}^{\sin(\phi_h-\phi_S)}\right]_{\mathrm{TFR}} = -\sum_a e_a^2\,x_B\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\hat{u}_{1T}^{\perp}(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)$$

$$\left[F_{UT,T}^{\sin(\phi_h-\phi_S)}\right]_{\mathrm{CFR}} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{m_N}\,f_{1T}^{\perp}\,D_1\right]$$

$$\left[F_{LT}^{\cos(\phi_h-\phi_S)}\right]_{\mathrm{TFR}} = \sum_a e_a^2\,x_B\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\;\hat{l}_{1T}^{\perp}\;(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)$$

$$\left[F_{LT}^{\cos(\phi_h-\phi_S)}\right]_{\mathrm{CFR}} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{m_N}\,g_{1T}\,D_1\right]$$

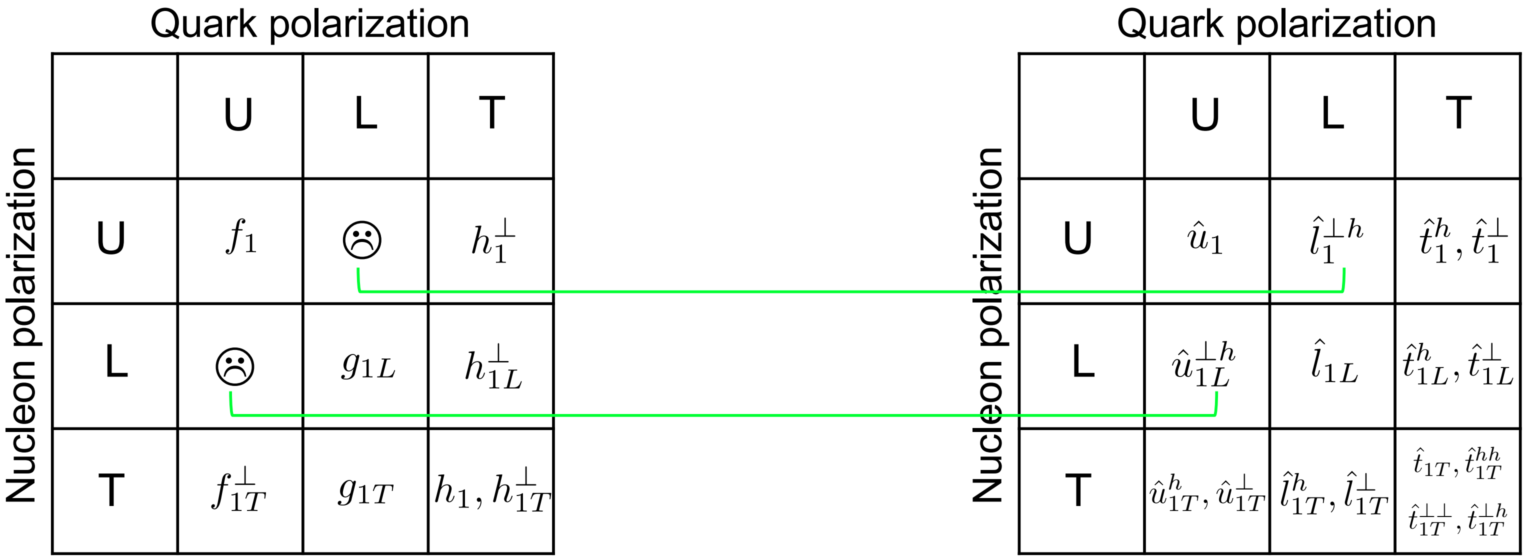
The same azimuthal asymmetries can appear in both the CFR and TFR complicating their interpretation…



… six more azimuthal asymmetries appear in the CFR at leading twist which are absent in the TFR.

Accessing longitudinal polarization

- TFR studies provide unique access to longitudinally polarized quarks in unpolarized nucleons and unpolarized quarks in longitudinally polarized nucleons which ordinarily disappear after integration over either momentum.



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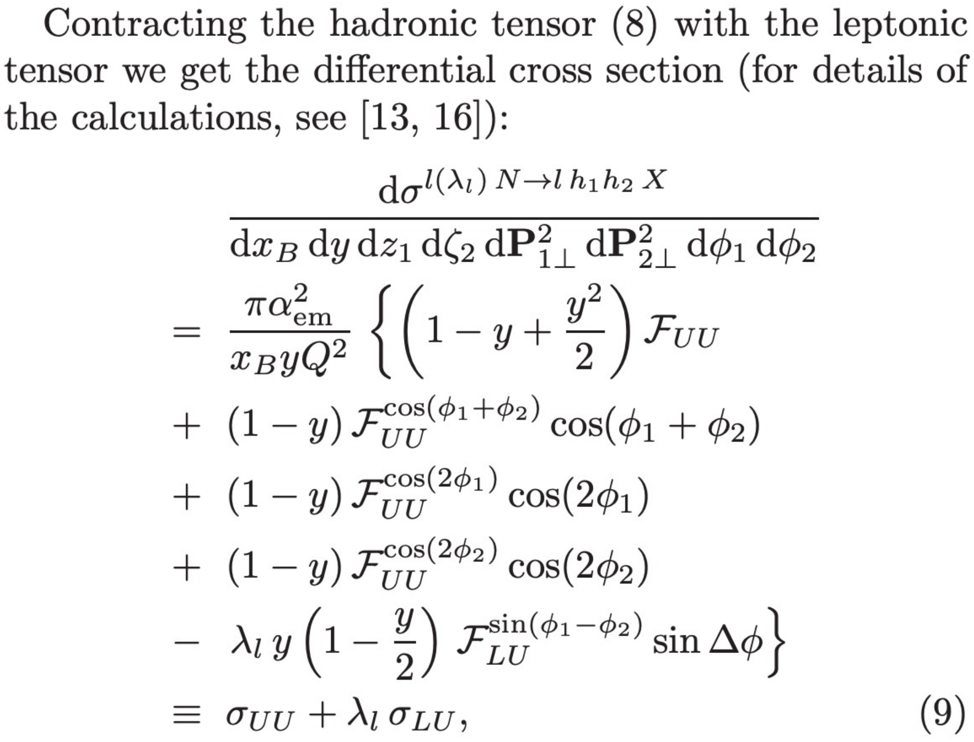
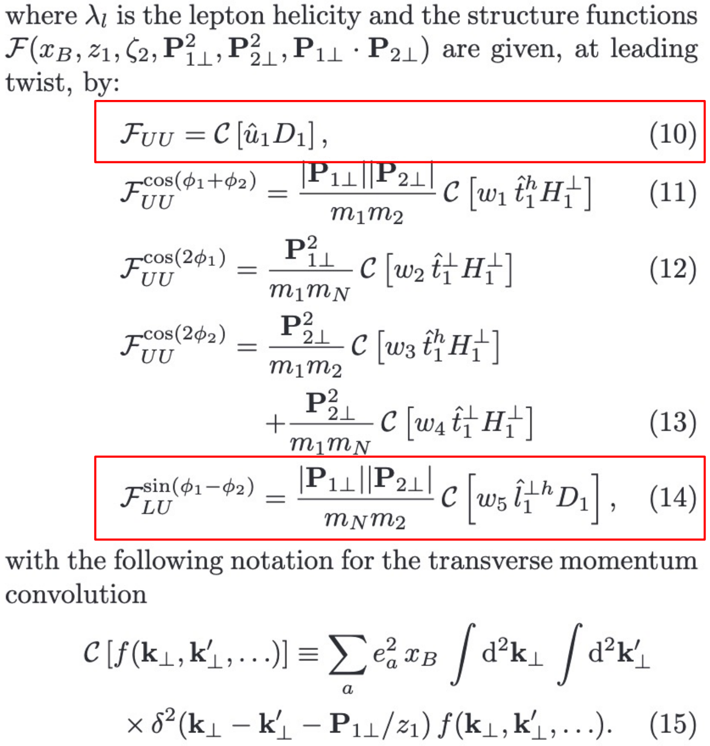
Differential Cross Section



Contracting the hadronic tensor (8) with the leptonic tensor we get the differential cross section (for details of the calculations, see [13, 16]):

$$
\begin{aligned}
&\frac{\mathrm{d}\sigma^{l(\lambda_l)\,N\to l\,h_1 h_2\,X}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z_1\,\mathrm{d}\zeta_2\,\mathrm{d}\mathbf{P}_{1\perp}^2\,\mathrm{d}\mathbf{P}_{2\perp}^2\,\mathrm{d}\phi_1\,\mathrm{d}\phi_2} \\
&= \frac{\pi\alpha_{\rm em}^2}{x_B y Q^2}\left\{\left(1-y+\frac{y^2}{2}\right)\mathcal{F}_{UU}\right. \\
&+ (1-y)\,\mathcal{F}_{UU}^{\cos(\phi_1+\phi_2)}\cos(\phi_1+\phi_2) \\
&+ (1-y)\,\mathcal{F}_{UU}^{\cos(2\phi_1)}\cos(2\phi_1) \\
&+ (1-y)\,\mathcal{F}_{UU}^{\cos(2\phi_2)}\cos(2\phi_2) \\
&- \left.\lambda_l\,y\left(1-\frac{y}{2}\right)\mathcal{F}_{LU}^{\sin(\phi_1-\phi_2)}\sin\Delta\phi\right\} \\
&\equiv \sigma_{UU}+\lambda_l\,\sigma_{LU}, \qquad (9)
\end{aligned}
$$



where λ_l is the lepton helicity and the structure functions $\mathcal{F}(x_B, z_1, \zeta_2, \mathbf{P}_{1\perp}^2, \mathbf{P}_{2\perp}^2, \mathbf{P}_{1\perp}\cdot\mathbf{P}_{2\perp})$ are given, at leading twist, by:

$$\mathcal{F}_{UU} = \mathcal{C}\left[\hat{u}_1 D_1\right], \qquad (10)$$

$$\mathcal{F}_{UU}^{\cos(\phi_1+\phi_2)} = \frac{|\mathbf{P}_{1\perp}||\mathbf{P}_{2\perp}|}{m_1 m_2}\,\mathcal{C}\left[w_1\,\hat{t}_1^h H_1^\perp\right] \qquad (11)$$

$$\mathcal{F}_{UU}^{\cos(2\phi_1)} = \frac{\mathbf{P}_{1\perp}^2}{m_1 m_N}\,\mathcal{C}\left[w_2\,\hat{t}_1^\perp H_1^\perp\right] \qquad (12)$$

$$
\begin{aligned}
\mathcal{F}_{UU}^{\cos(2\phi_2)} &= \frac{\mathbf{P}_{2\perp}^2}{m_1 m_2}\,\mathcal{C}\left[w_3\,\hat{t}_1^h H_1^\perp\right] \\
&+ \frac{\mathbf{P}_{2\perp}^2}{m_1 m_N}\,\mathcal{C}\left[w_4\,\hat{t}_1^\perp H_1^\perp\right] \qquad (13)
\end{aligned}
$$

$$\mathcal{F}_{LU}^{\sin(\phi_1-\phi_2)} = \frac{|\mathbf{P}_{1\perp}||\mathbf{P}_{2\perp}|}{m_N m_2}\,\mathcal{C}\left[w_5\,\hat{l}_1^{\perp h} D_1\right], \qquad (14)$$

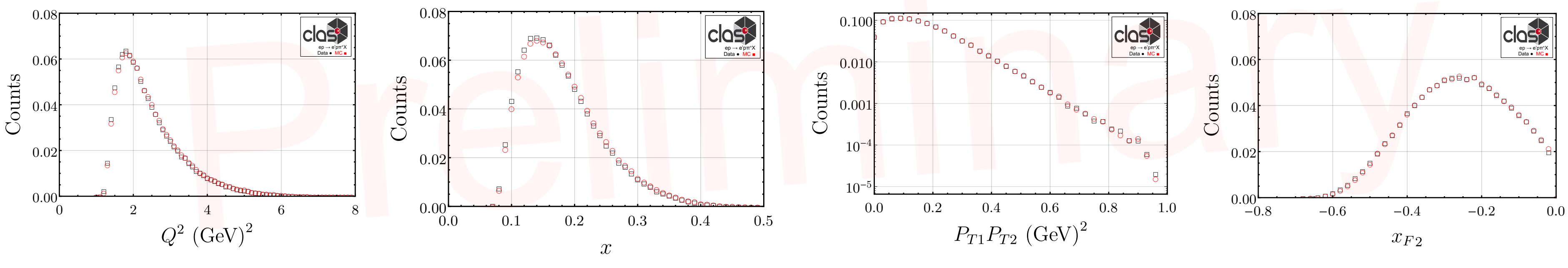
with the following notation for the transverse momentum convolution

$$
\begin{aligned}
\mathcal{C}\left[f(\mathbf{k}_\perp, \mathbf{k}'_\perp, \ldots)\right] &\equiv \sum_a e_a^2\, x_B \int \mathrm{d}^2\mathbf{k}_\perp \int \mathrm{d}^2\mathbf{k}'_\perp \\
&\times \delta^2(\mathbf{k}_\perp - \mathbf{k}'_\perp - \mathbf{P}_{1\perp}/z_1)\, f(\mathbf{k}_\perp, \mathbf{k}'_\perp, \ldots). \qquad (15)
\end{aligned}
$$

M. Anselmino et al., Phys. Lett. B. 713 (2012), 317-320, [hep-ph] 1112.2604

Monte Carlo

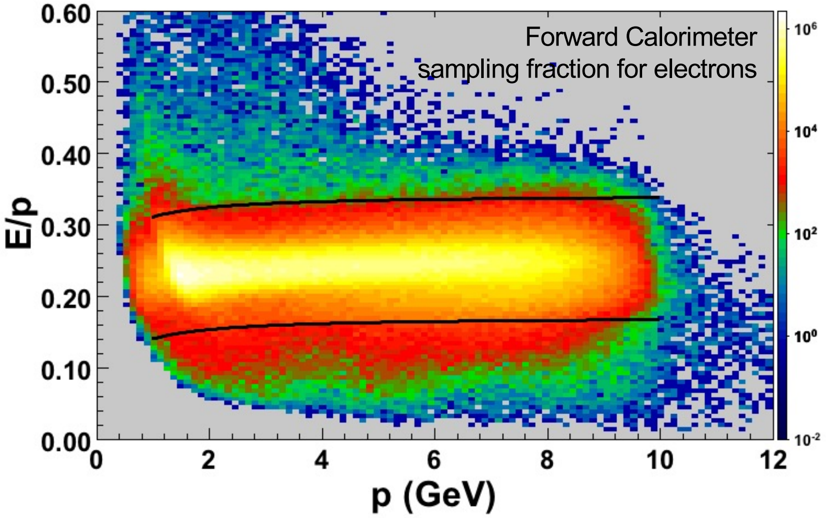
- SIDIS MC “clasdis”[1] based on PEPSI[2] generator, the polarized version of the well-known LEPTO[3] generator.
- Parameters changed to reproduce observed distributions include average transverse momentum, fraction of spin-1 light mesons and fraction of spin-1 strange mesons.
- CLAS12 detector system described in “GEMC”[4], a detailed GEANT4 simulation package.
- Excellent agreement between data and MC!



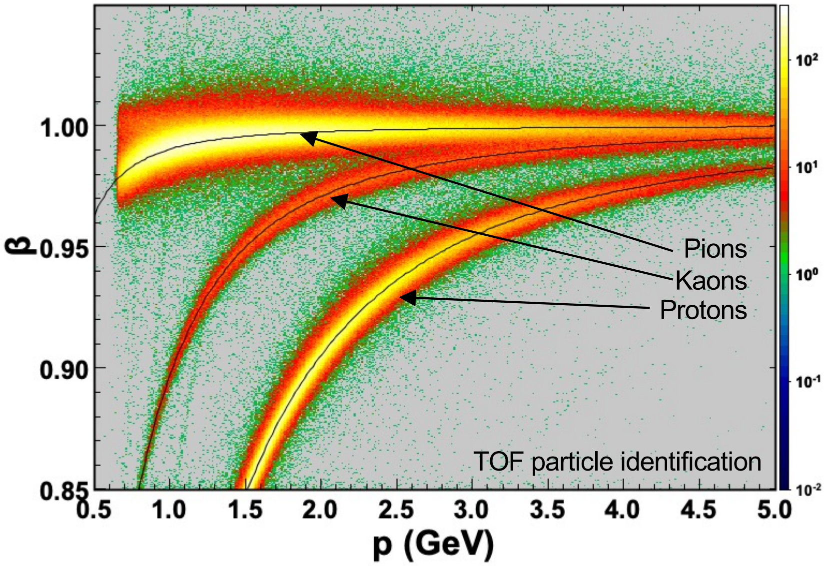
1. H. Avakian, “clasdis.” https://github.com/JeffersonLab/clasdis, 2020.
2. L. Mankiewicz, A. Schafer, and M. Veltri, “Pepsi: A monte carlo generator for polarized leptoproduction,” Comput. Phys. Commun., vol. 71, pp. 305–318, 1992.
3. G. Ingelman, A. Edin, and J. Rathsman, “LEPTO 6.5: A Monte Carlo generator for deep inelastic lepton - nucleon scattering,” Comput. Phys. Commun., vol. 101, pp. 108–134, 1997.
4. M. Ungaro et al., “The CLAS12 Geant4 simulation,” Nucl. Instrum. Meth. A, vol. 959, p. 163422, 2020.

Particle ID

- Electron
  - Electromagnetic calorimeter.
  - Cherenkov detector.
  - Vertex and fiducial cuts.

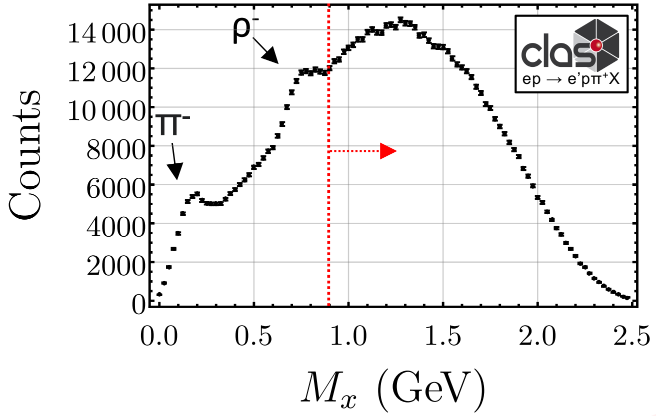


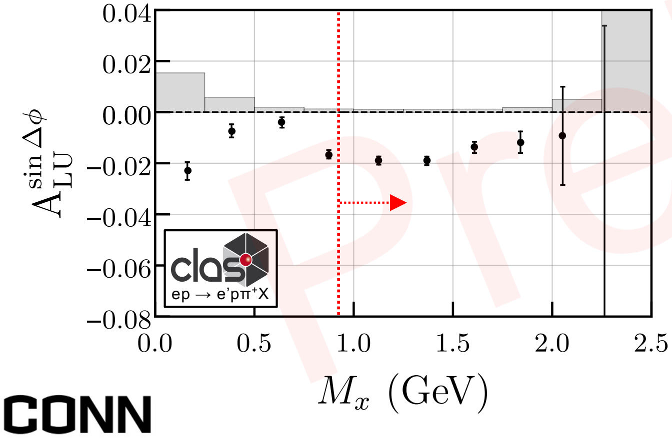
- Hadron
  - β vs p comparison between vertex timing and event start time.
  - Vertex and fiducial cuts.
  - Pion momentum limited to < 4 GeV.



UCONN

Removing background

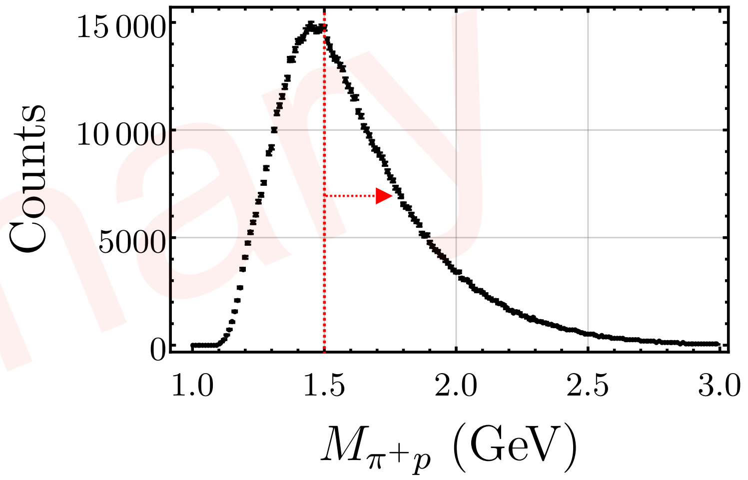




- Exclusive pion and rho production clearly visible.

- Different amplitudes at low M_x are generated from separate physics than our signal.

$\Delta^{++} \rightarrow p\pi^+$ Contamination?



- Little sign of Δs; cut on mass > 1.5 GeV for safety.
- Estimate remaining contribution from MC.

Additional Modulations

$$\mathcal{F}_{LU} = \frac{|p_\pi^\perp||p_P^\perp|}{m_p m_\pi}\mathcal{C}\left[w_5 \hat{l}_1^{\perp h} D_1\right] \longrightarrow$$

Phys. Lett. B. 713 (2012), 317-320, [hep-ph] 1112.2604

Structure functions carry a dependence on $|p_\pi^\perp||p_P^\perp|$ which introduces a dependence on $\cos\Delta\phi$.

If the correlations are assumed small, the fracture functions can be expanded in powers of $k^\perp \cdot p_P^\perp$. →

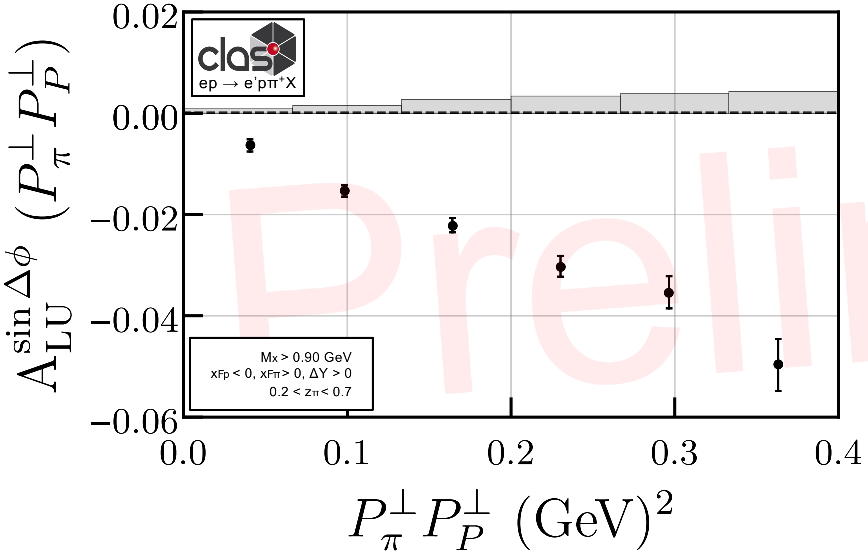
$$\begin{aligned}&\hat{l}_1^{\perp h}(x,\zeta,{\mathbf{k}^\perp}^2,{\mathbf{p}_\mathbf{P}^\perp}^2,\mathbf{k}^\perp\cdot\mathbf{p}_\mathbf{P}^\perp)\\ &\quad\approx a(x,\zeta,{\mathbf{k}^\perp}^2,{\mathbf{p}_\mathbf{P}^\perp}^2)\\ &\qquad+b(x,\zeta,{\mathbf{k}^\perp}^2,{\mathbf{p}_\mathbf{P}^\perp}^2)\mathbf{k}^\perp\cdot{\mathbf{p}_\mathbf{P}}^\perp\end{aligned}$$

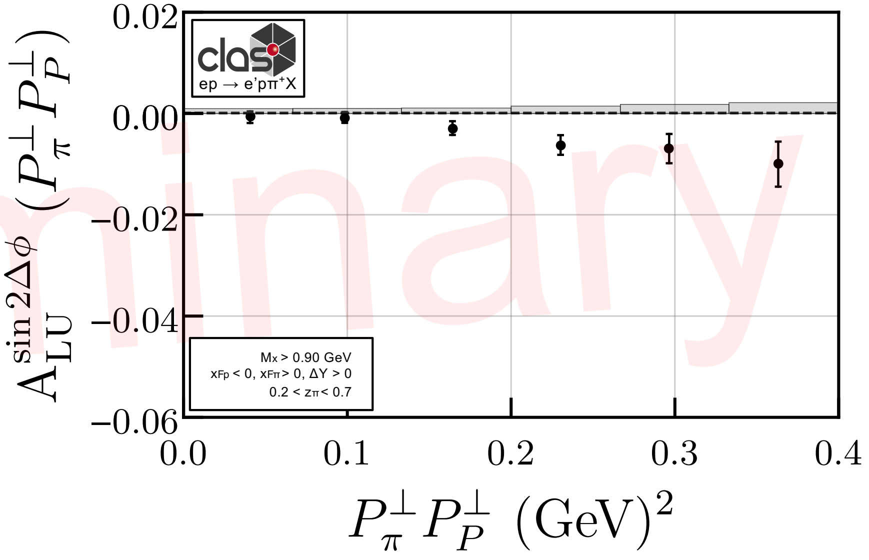
The term linear in $k^\perp \cdot p_P^\perp$ yields a $\cos\Delta\phi$ which when combined with the already existing $\sin\Delta\phi$ term results in a $\sin 2\Delta\phi$.

$$\boxed{\mathcal{A}_{LU}(x,\zeta,{\mathbf{k}^\perp}^2,{\mathbf{p}_\mathbf{P}^\perp}^2,\Delta\phi) = A(x,\zeta,{\mathbf{k}^\perp}^2,{\mathbf{p}_\mathbf{P}^\perp}^2)\sin\Delta\phi + B(x,\zeta,{\mathbf{k}^\perp}^2,{\mathbf{p}_\mathbf{P}^\perp}^2)\sin(2\Delta\phi)}$$

Additional modulation is small

- The sin(2Δφ) is mostly (coincidentally) very small.
- Still important to extract simultaneously.

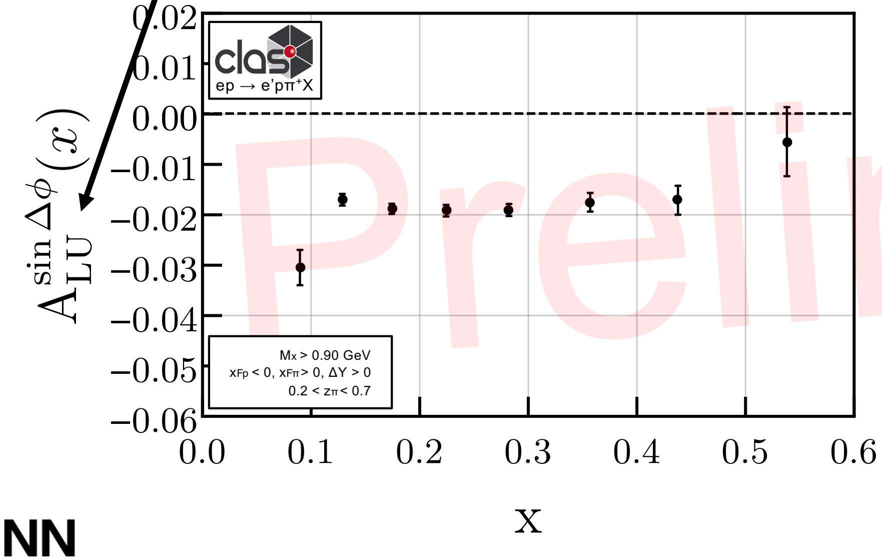


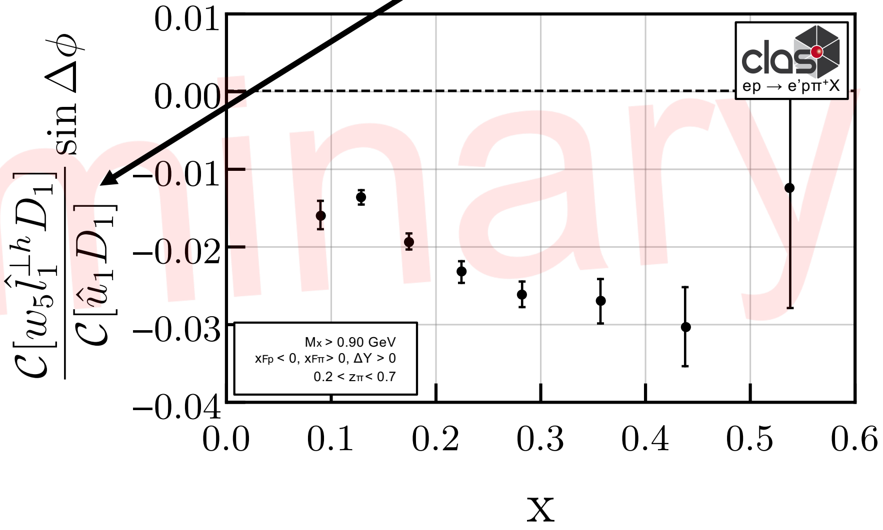


Directly plotting FrFs and FFs

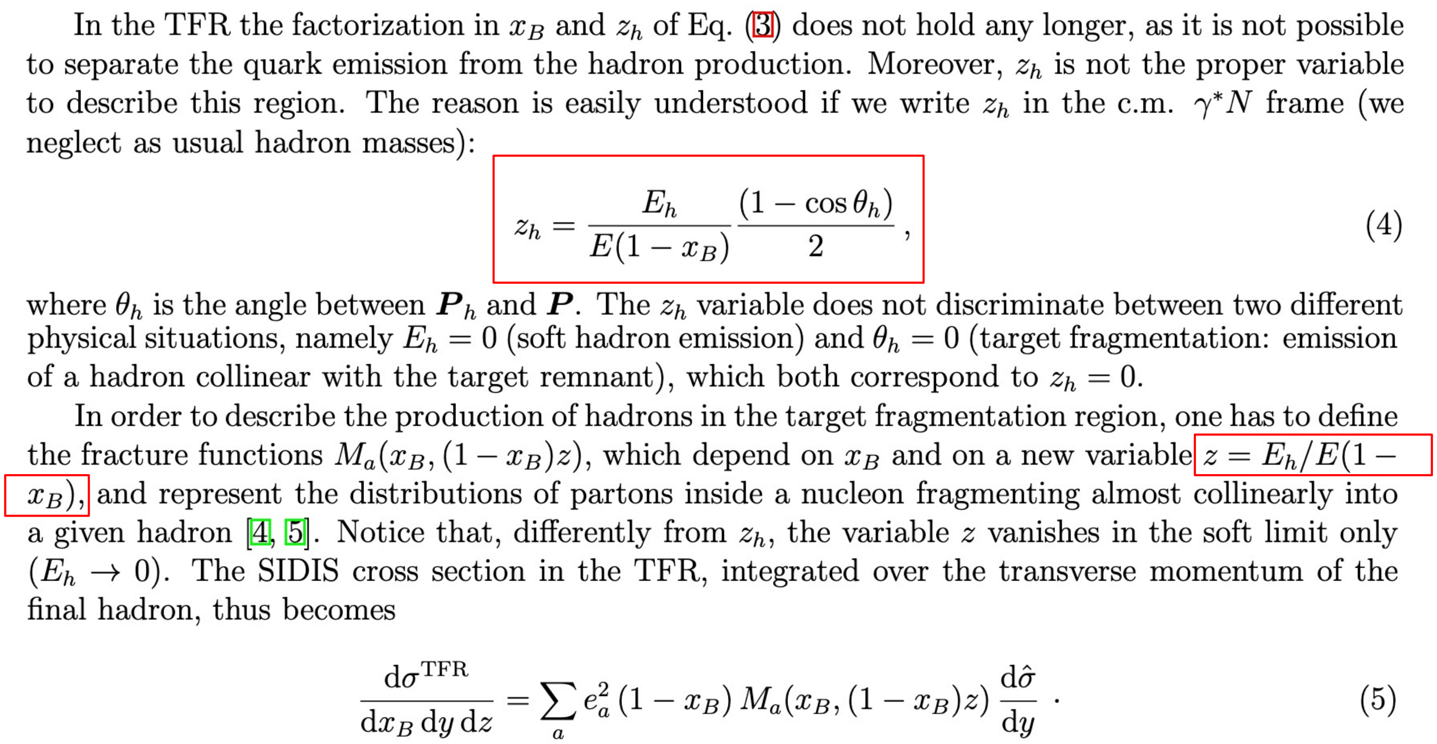
Depolarization Structure Functions Depolarization Structure Functions

$$\mathcal{A}_{LU} = -\boxed{\frac{1-\frac{y}{2}}{1-y+\frac{y^2}{2}}}\,\boxed{\frac{\mathcal{F}_{LU}^{\sin\Delta\phi}}{\mathcal{F}_{UU}}}\sin\Delta\phi, = -\boxed{\frac{y\left(1-\frac{y}{2}\right)}{\left(1-y+\frac{y^2}{2}\right)}}\,\boxed{\frac{|\boldsymbol{P}_{1\perp}||\boldsymbol{P}_{2\perp}|}{m_N\,m_2}\,\frac{\mathcal{C}[w_5\,\hat{l}_1^{\perp h}D_1]}{\mathcal{C}[\hat{u}_1 D_1]}}\sin\Delta\phi\,.$$





Subtly different z defintions



In the TFR the factorization in x_B and z_h of Eq. (3) does not hold any longer, as it is not possible to separate the quark emission from the hadron production. Moreover, z_h is not the proper variable to describe this region. The reason is easily understood if we write z_h in the c.m. $\gamma^* N$ frame (we neglect as usual hadron masses):

$$\boxed{z_h = \frac{E_h}{E(1-x_B)} \, \frac{(1-\cos\theta_h)}{2} \, ,} \tag{4}$$

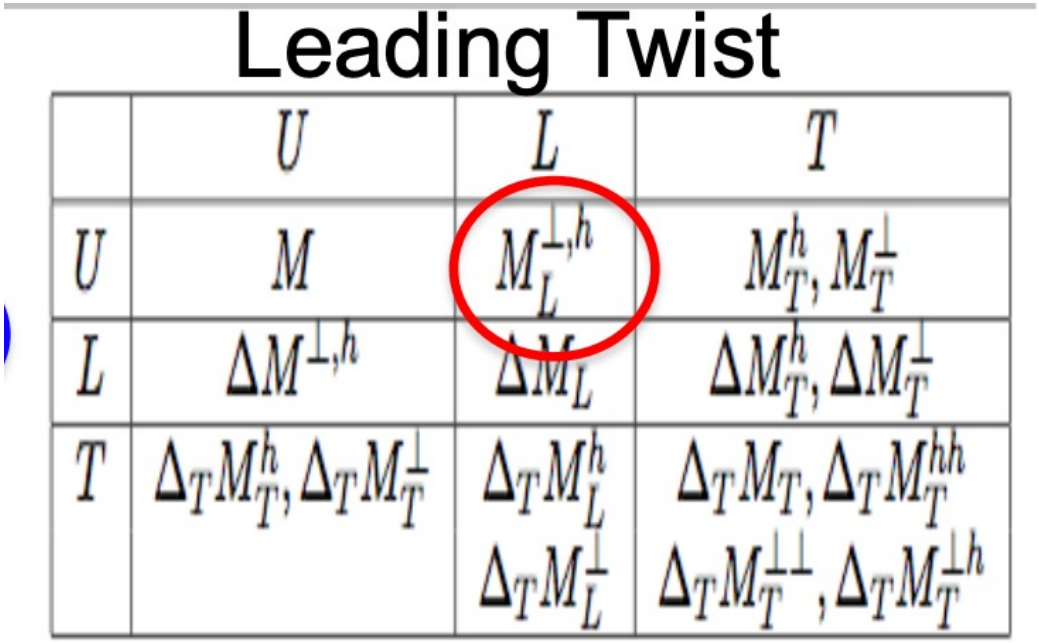
where θ_h is the angle between $\boldsymbol{P}_h$ and $\boldsymbol{P}$. The z_h variable does not discriminate between two different physical situations, namely $E_h = 0$ (soft hadron emission) and $\theta_h = 0$ (target fragmentation: emission of a hadron collinear with the target remnant), which both correspond to $z_h = 0$.

In order to describe the production of hadrons in the target fragmentation region, one has to define the fracture functions $M_a(x_B, (1-x_B)z)$, which depend on x_B and on a new variable $\boxed{z = E_h/E(1-x_B)}$, and represent the distributions of partons inside a nucleon fragmenting almost collinearly into a given hadron [4, 5]. Notice that, differently from z_h, the variable z vanishes in the soft limit only ($E_h \to 0$). The SIDIS cross section in the TFR, integrated over the transverse momentum of the final hadron, thus becomes

$$\frac{d\sigma^{\rm TFR}}{dx_B \, dy \, dz} = \sum_a e_a^2 \, (1-x_B) \, M_a(x_B, (1-x_B)z) \, \frac{d\hat{\sigma}}{dy} \, . \tag{5}$$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Fracture Functions Renamed



Leading Twist

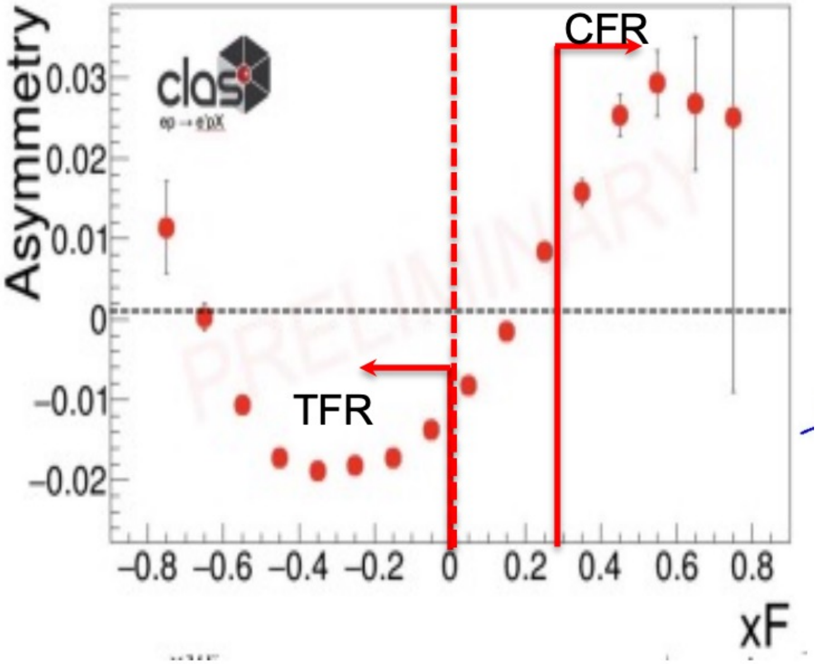
	U	L	T
U	M	$M_L^{\perp,h}$	$M_T^h, M_T^\perp$
L	$\Delta M^{\perp,h}$	ΔM_L	$\Delta M_T^h, \Delta M_T^\perp$
T	$\Delta_T M_T^h, \Delta_T M_T^\perp$	$\Delta_T M_L^h$ $\Delta_T M_L^\perp$	$\Delta_T M_T, \Delta_T M_T^{hh}$ $\Delta_T M_T^{\perp\perp}, \Delta_T M_T^{\perp h}$

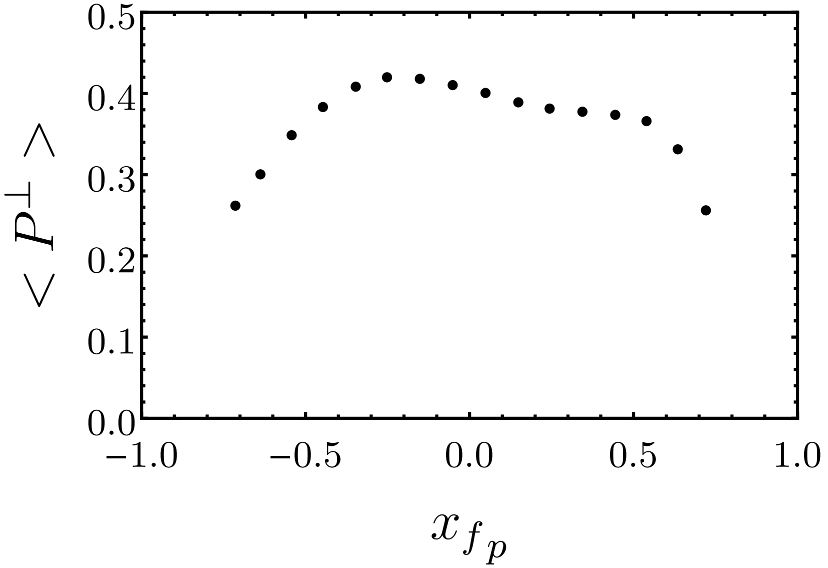
Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

ALU(xFp)


clas
ep → e'pX
PRELIMINARY
CFR
TFR
Asymmetry
0.03
0.02
0.01
0
−0.01
−0.02
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
xF


$< P^{\perp} >$
0.5
0.4
0.3
0.2
0.1
0.0
−1.0
−0.5
0.0
0.5
1.0
x_{f_p}