Transverse spin distributions

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Transversity 2022, Pavia, Italy, May 23, 2022

Overview

- Transversity - status update
- Gluonic analogues
- Other transverse spin distributions (Sivers & QS)
- Transversity GPDs

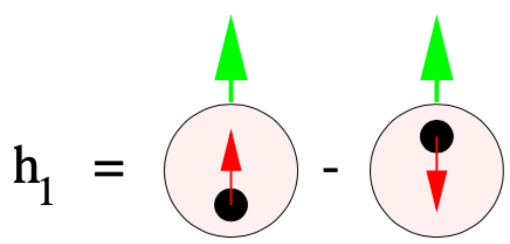
All in just 20 minutes, so this is going to be far from complete!

Transversity

Opening talk at Transversity 2008

Transversity Asymmetries

Daniël Boer
VU University Amsterdam

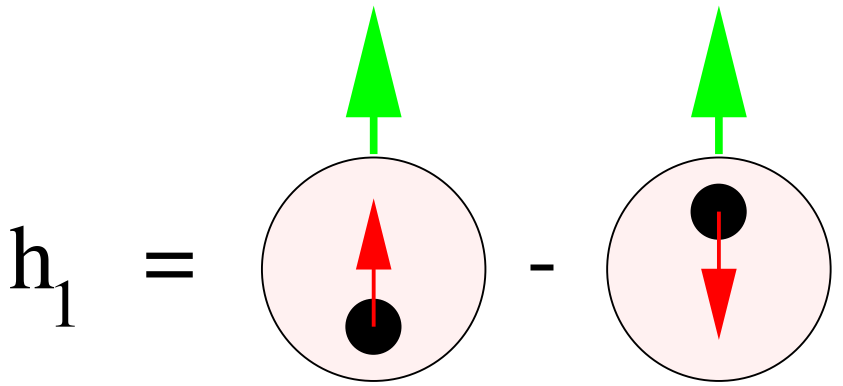


Outline

- Transversity 1978-2008
- Transversity asymmetries - 4 types
- Future transversity measurements

Transversity & the tensor charge

$h_1(x,Q^2)$: the distribution of transversely polarized quarks inside a transversely polarized hadron



An example of “Mulders’ traffic lights”

Transversity is the only known way to experimentally determine the **tensor charge**:

$$\delta q = \int_0^1 dx \left[h_1^q(x) - h_1^{\bar{q}}(x) \right]$$

Like the electric charge (coupling to photons) and axial charge (coupling to Z bosons), the tensor charge is a fundamental charge (but lacking a gauge boson)

It is scale dependent, like α_s (coupling to gluons)

Drell-Yan: the classic route to transversity

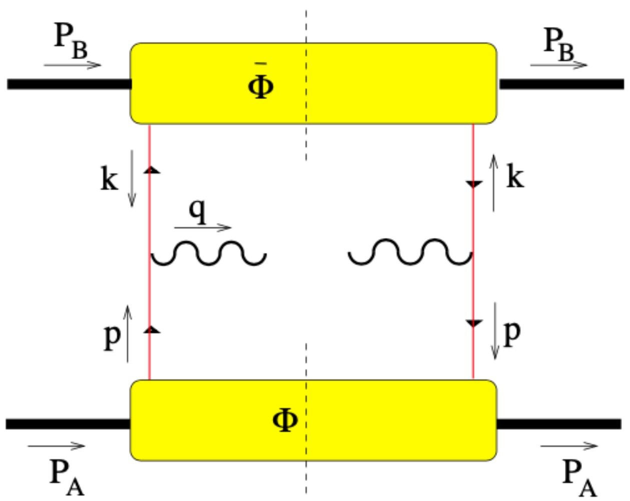
Transversity anno 1978

1978: birth of transversity *as a quark distribution*

Ralston & Soper, NPB 152 (1979) 109 (submitted: Nov 14, 1978)

The Drell-Yan Process

$H_1 + H_2 \to \ell + \bar{\ell} + X$



$$A_{TT} = \frac{\sigma(p^\uparrow p^\uparrow \to \ell \ell' X) - \sigma(p^\uparrow p^\downarrow \to \ell \ell' X)}{\sigma(p^\uparrow p^\uparrow \to \ell \ell' X) + \sigma(p^\uparrow p^\downarrow \to \ell \ell' X)} = \frac{\sin^2\theta \cos 2\phi_S^\ell}{1+\cos^2\theta} \frac{\sum_{a,\bar{a}} e_a^2\, h_1^a(x_1)\, \overline{h}_1^a(x_2)}{\sum_{a,\bar{a}} e_a^2\, f_1^a\, \overline{f}_1^a}$$

Artru, Mekhfi, ZPC 45 ('90) 669; Jaffe, Ji, NPB 375 ('92) 527; Cortes, Pire, Ralston, ZPC 55 ('92) 409

However, polarized Drell-Yan is very demanding, still not done...

Drell-Yan: likely too challenging

A_{TT} at RHIC

RHIC is at present the only place that can do double polarized hadron scattering

$$A_{TT} = \frac{\sigma(p^\uparrow p^\uparrow \to \ell\bar{\ell}X) - \sigma(p^\uparrow p^\downarrow \to \ell\bar{\ell}X)}{\sigma(p^\uparrow p^\uparrow \to \ell\bar{\ell}X) + \sigma(p^\uparrow p^\downarrow \to \ell\bar{\ell}X)} \propto \sum_q e_q^2\, h_1^q(x_1)\, h_1^{\bar{q}}(x_2)$$

This involves two unrelated functions, for which likely holds:

$$h_1^{\bar{q}} \ll h_1^q$$

An upper bound can be obtained by using Soffer's inequality,

$$|h_1(x)| \leq \frac{1}{2}\left[f_1(x) + g_1(x)\right]$$

The upper bound on A_{TT} was shown to be small at RHIC (percent level)

Martin, Schäfer, Stratmann & Vogelsang, PRD 60 (1999) 117502

Other analyzers of transversity

- Collins asymmetry in semi-inclusive DIS $\propto h_1 H_1^{\perp}$
- Collins-type asymmetry in di-hadron SIDIS or $pp^{\uparrow}$ $\propto h_1 H_1^{\sphericalangle}$
- $\pi\, p^{\uparrow}$ DY $\propto h_1 h_1^{\perp}$
- $\Lambda^{\uparrow}$ spin transfer (D_{NN}) in $ep^{\uparrow}$ or $pp^{\uparrow}$ $\propto h_1 H_1$

All single spin asymmetries, that need a separate extraction of yet another function (collinear fragmentation function or TMD)

Note that in the 1st & 3rd case this actually involves the transversity **TMD**

$$h_1(x, k_T^2)$$

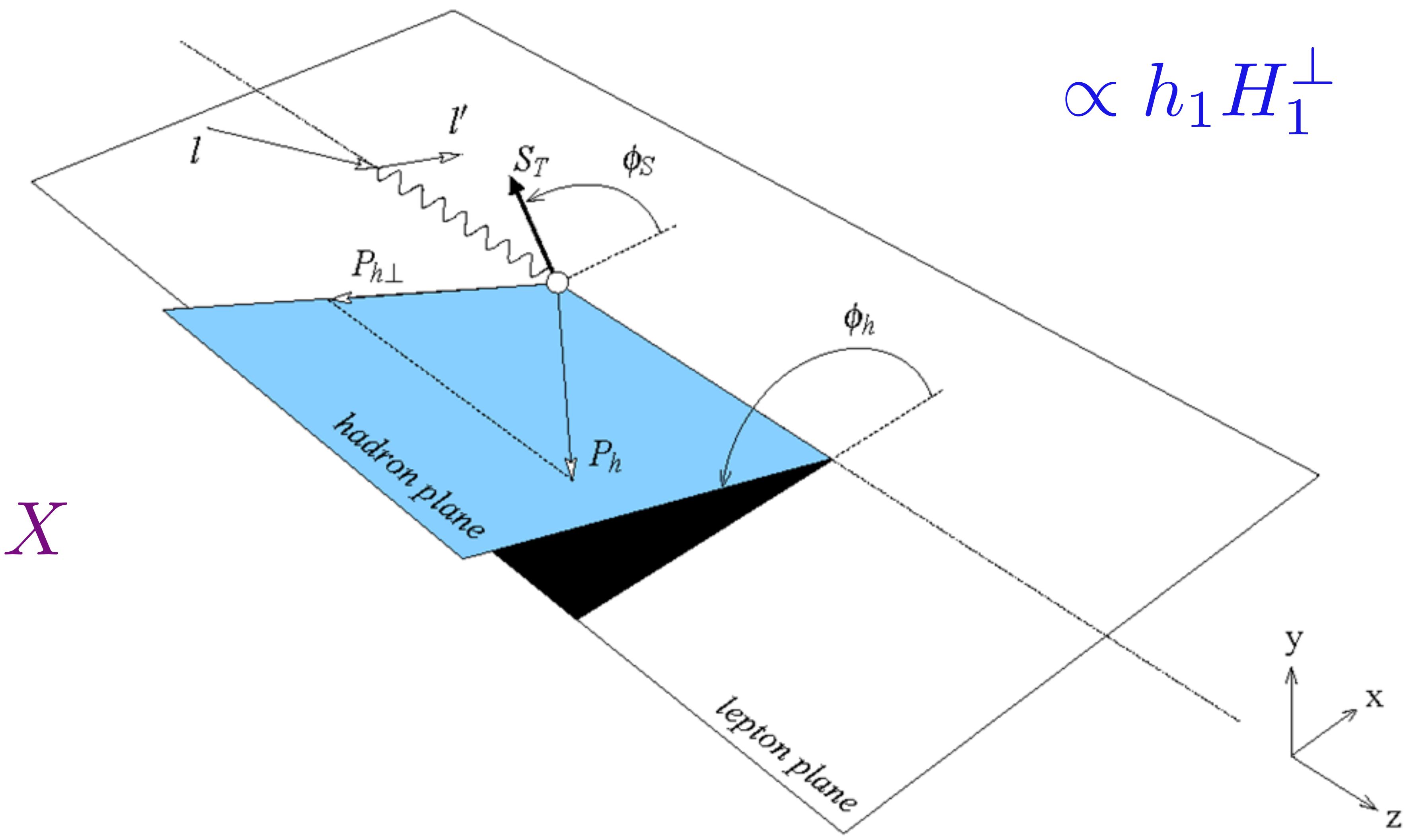
Collins effect in SIDIS

Collins effect leads to a $\sin(\phi_h + \phi_S)$ asymmetry in semi-inclusive DIS

Collins, 1993

$$\propto h_1 H_1^\perp$$

$$e\,p \to e'\,h\,X$$



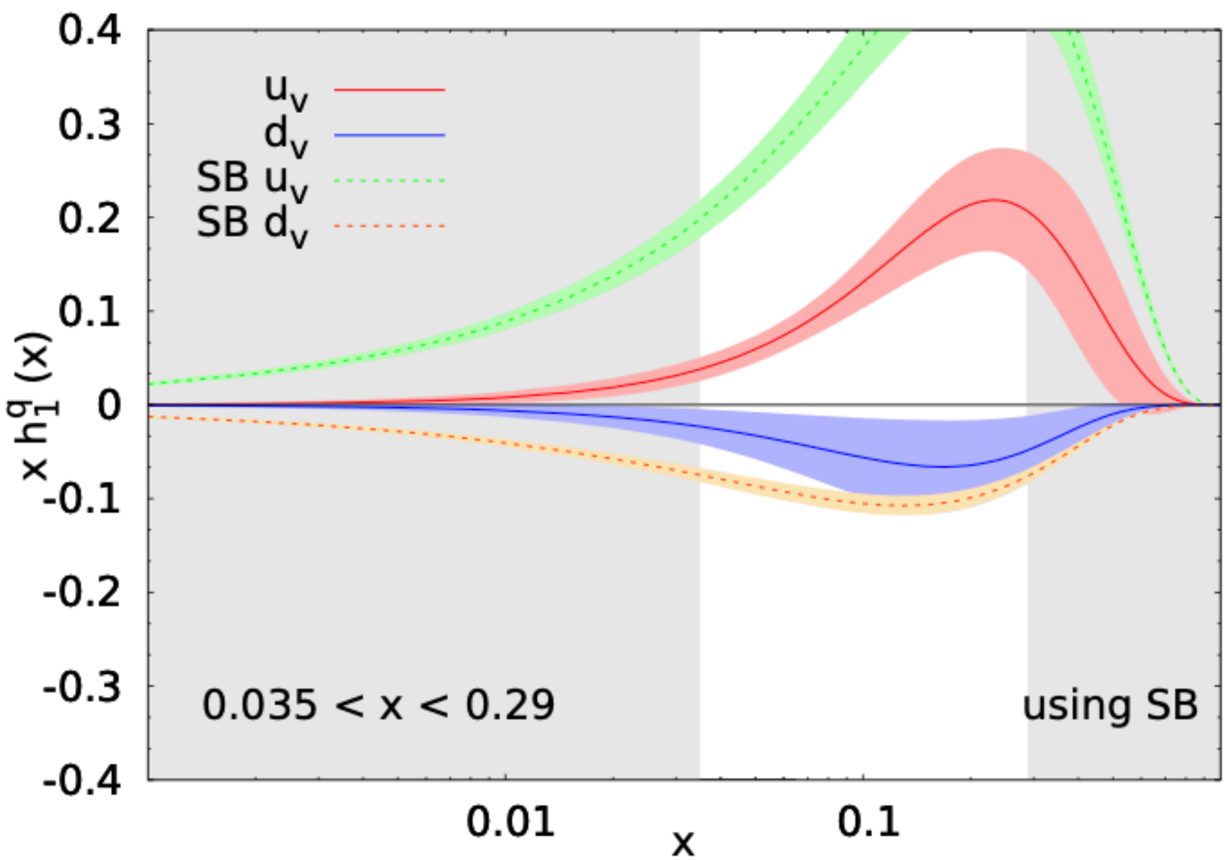
Clearly observed by HERMES and COMPASS

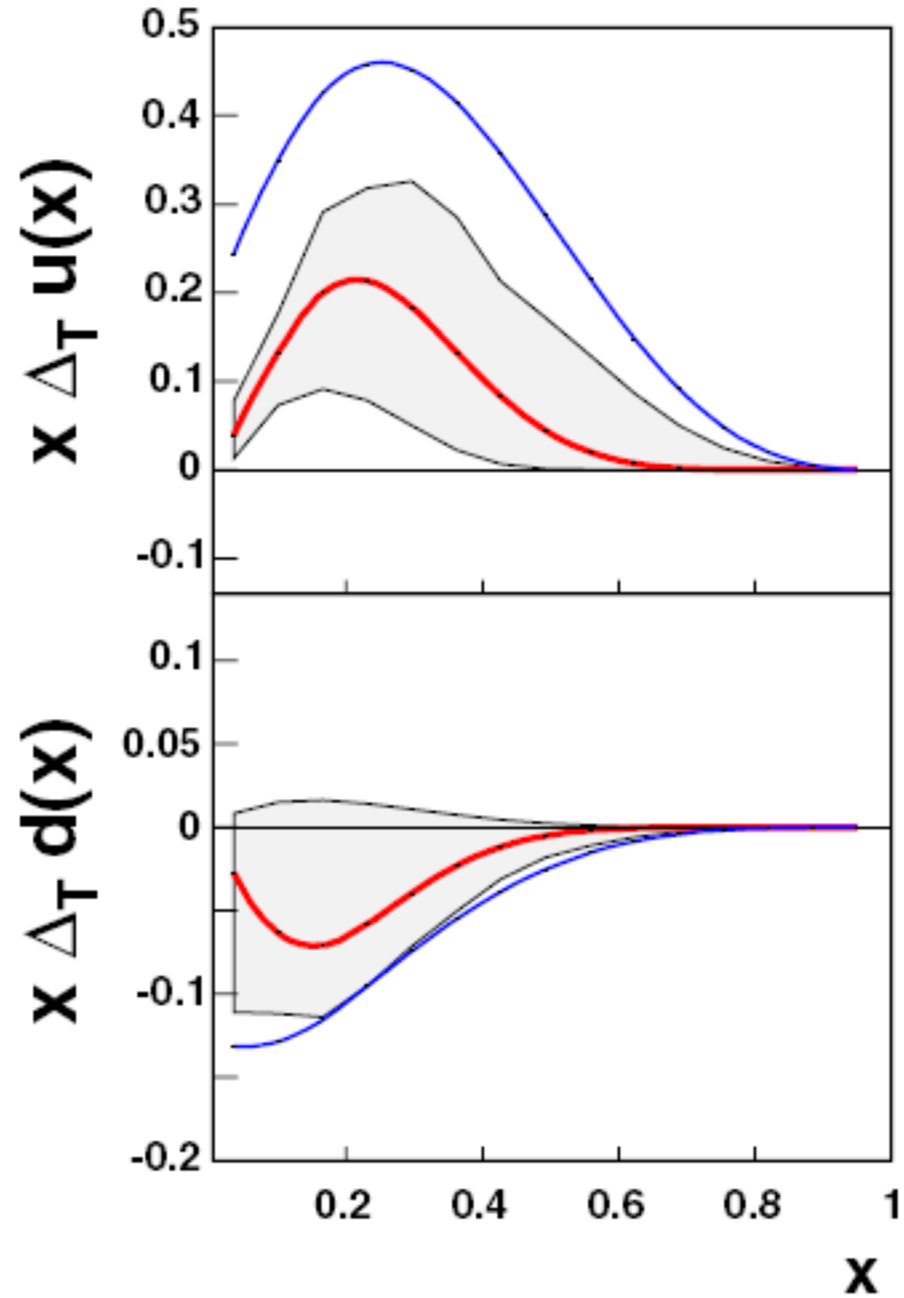
Double Collins effect [Boer, Jakob, Mulders, 1997] observed by BELLE

Transversity from Collins effect

Transversity functions for u_v (red) and d_v (blue) flavors from a global fit to SIDIS and e^+e^- data at $Q^2 = 4$ GeV^2

D'Alesio, Flore, Prokudin, 2020





Roughly $h_1(x) \approx f_1(x)/3$

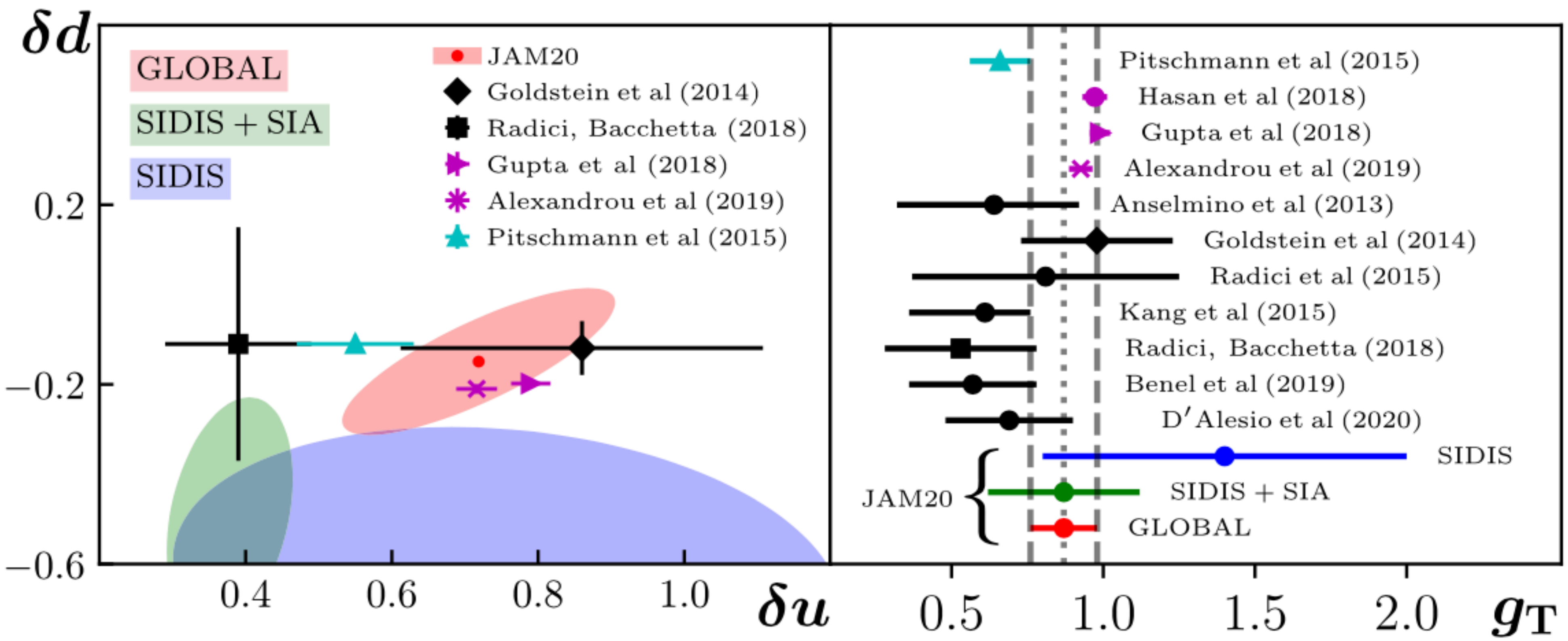
Anselmino et al., 2007

Tensor charge from Collins effect

$$g_T = \delta u_v - \delta d_v$$

	δu_v	δd_v	g_T
	$Q^2 = 4\,\mathrm{GeV}^2$		
using SB	0.42 ± 0.09	-0.15 ± 0.11	0.57 ± 0.13
no SB	0.40 ± 0.09	-0.29 ± 0.22	0.69 ± 0.21

D'Alesio, Flore, Prokudin, 2020



Cammarota et al., 2020

Transversity from lattice QCD

Large-Momentum Effective Theory (LaMET)

X. Ji, 2013

Liu et al., 2018 (LP^3 Collab.)

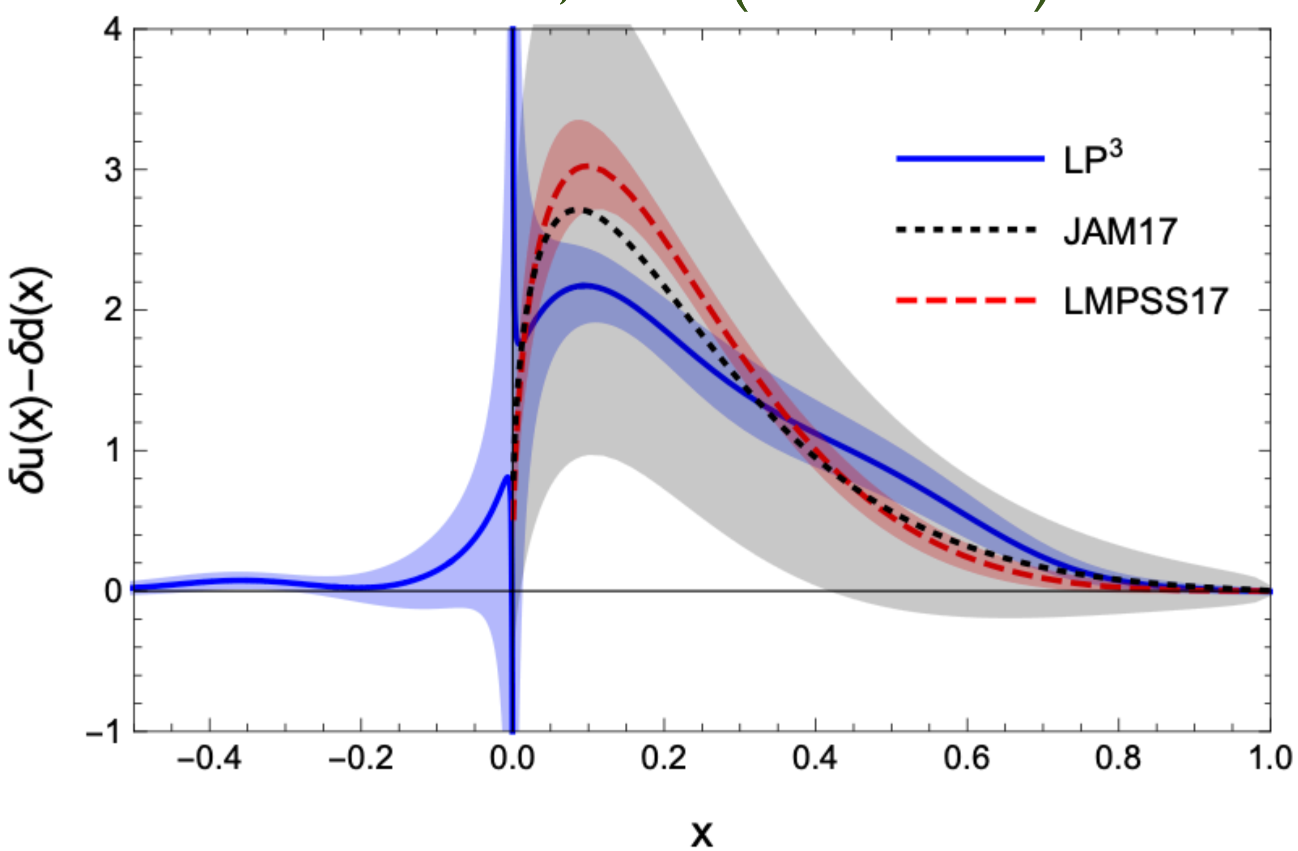
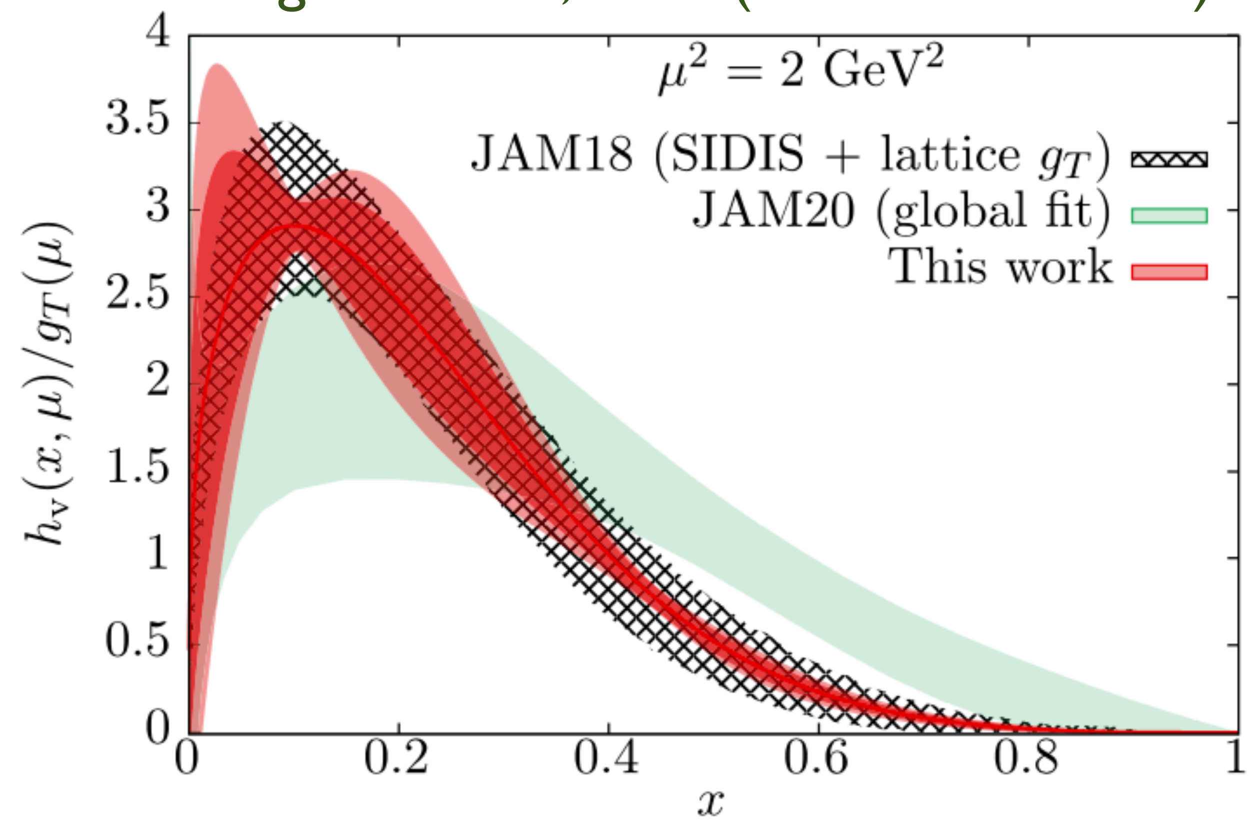


FIG. 4. Our final proton isovector transversity PDF at renormalization scale $\mu = \sqrt{2}$ GeV ($\overline{\text{MS}}$ scheme), extracted from lattice QCD and LaMET at $P_z = 3$ GeV, compared with global fits by JAM17 and LMPSS17 [12]. The blue error band includes statistical errors (which fold in the excited-state uncertainty) and systematics.

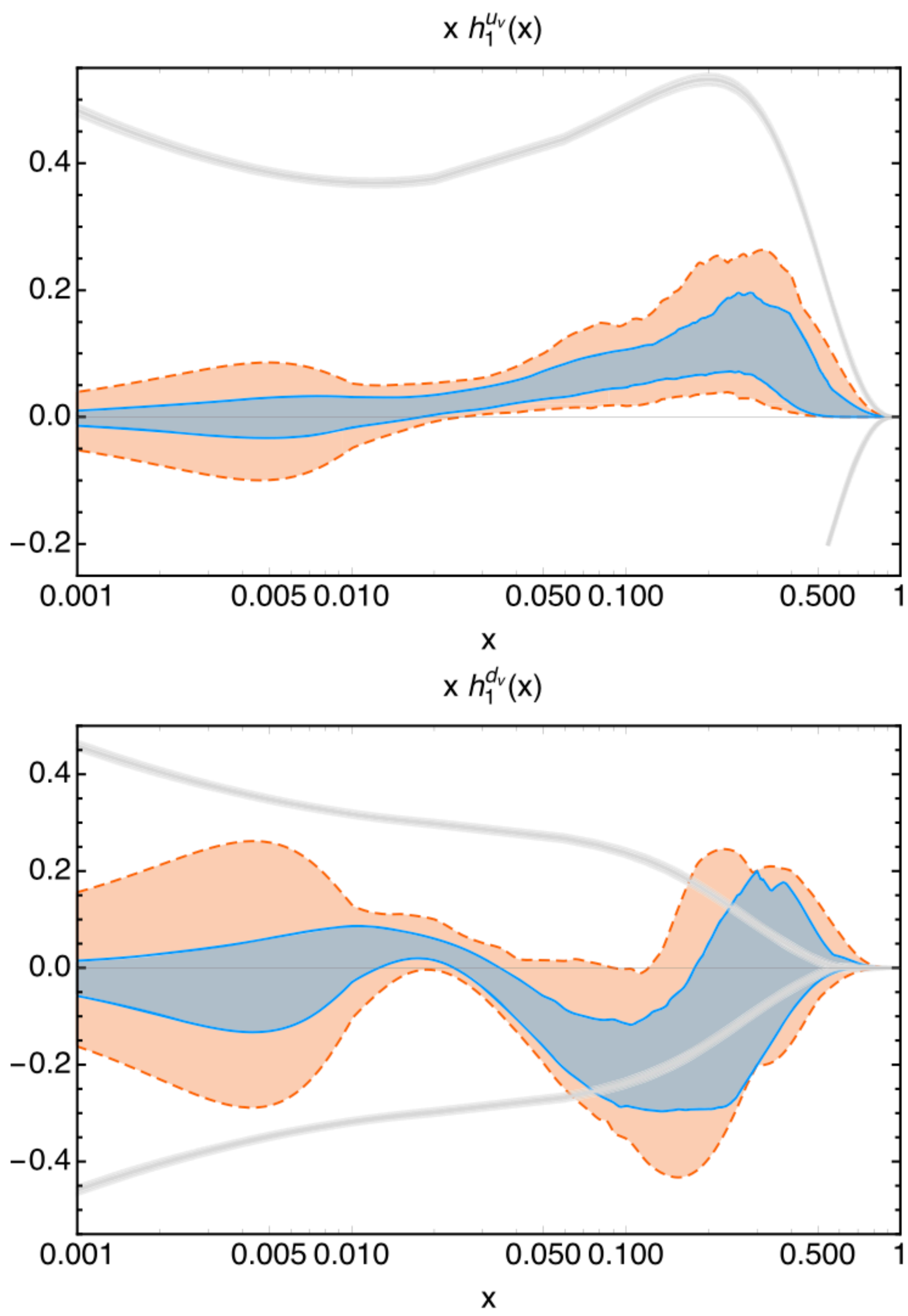
Egerer et al., 2022 (HadStruc Collab.)

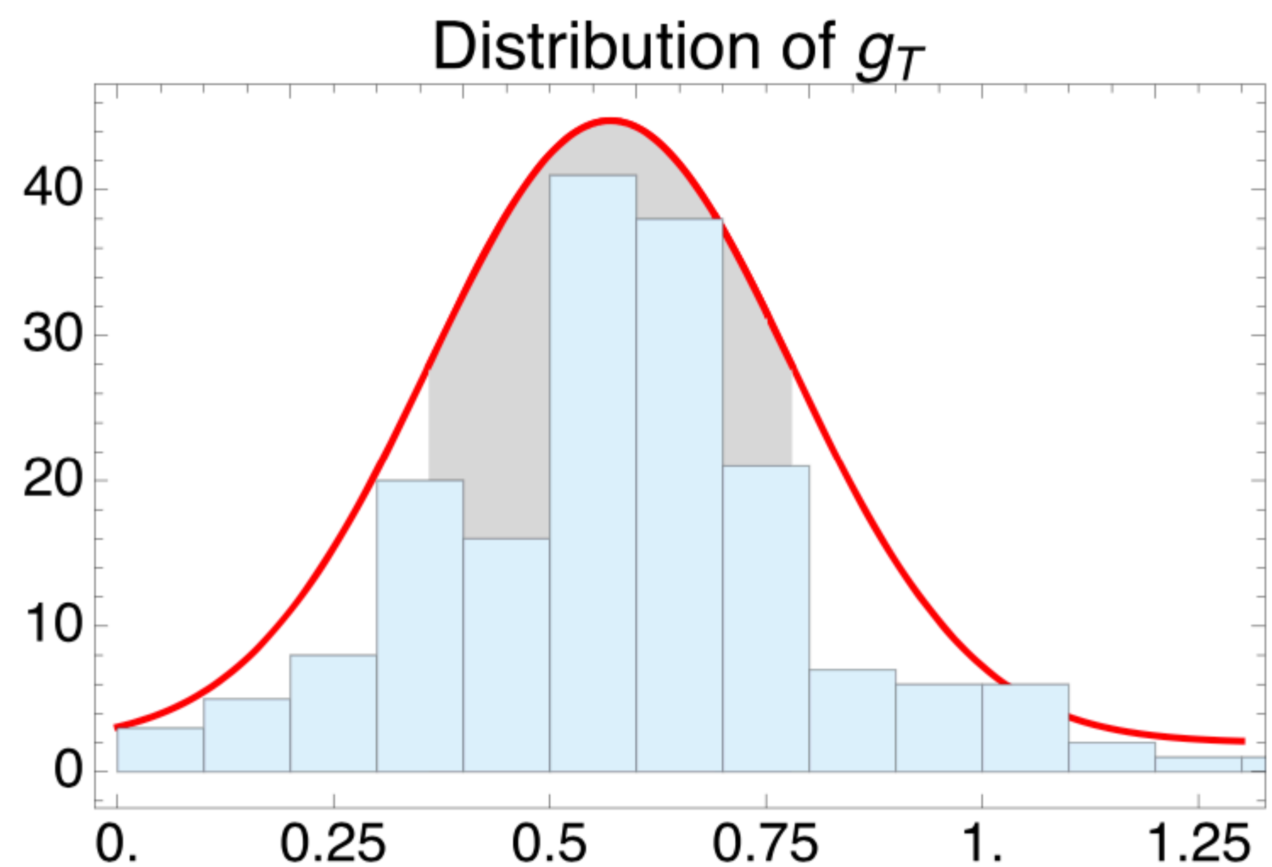


P_z up to 2.46 GeV

Results by various other groups are available

Transversity & g_T from di-hadron production





Again fitted to HERMES & COMPASS SIDIS data and BELLE's e^+e^- data

Benel, Courtoy, Ferro-Hernandez, 2020

Earlier results:

Radici, Courtoy, Bacchetta, Guagnellia, 2015

Original idea: Jaffe, Jin, Tang, 1998

Transversity from a global analysis of ep and pp data

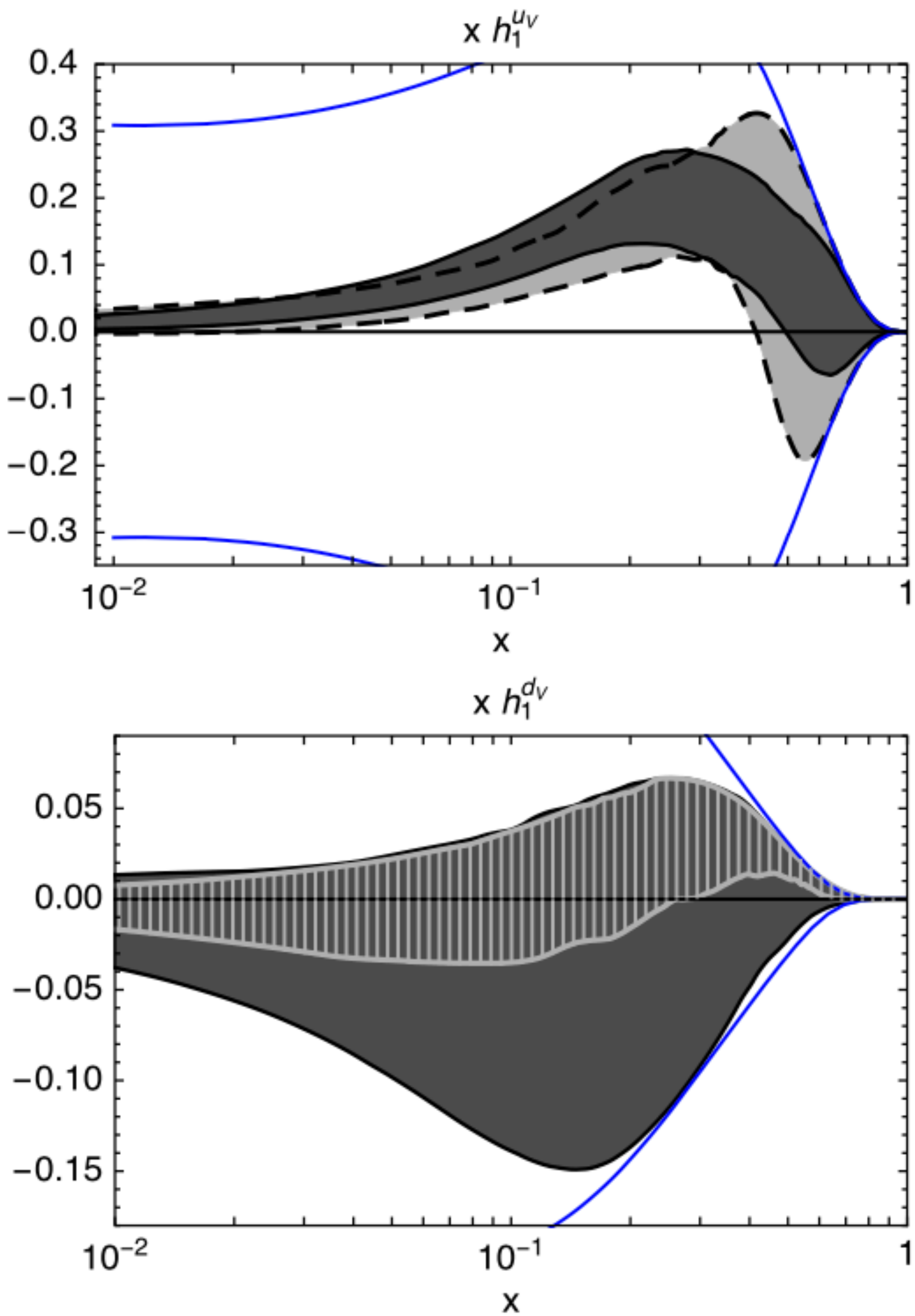


TABLE I. The tensor charge δq, truncated tensor charge $\delta\tilde{q}$, and isovector tensor charge g_T at 90% confidence level (see text).

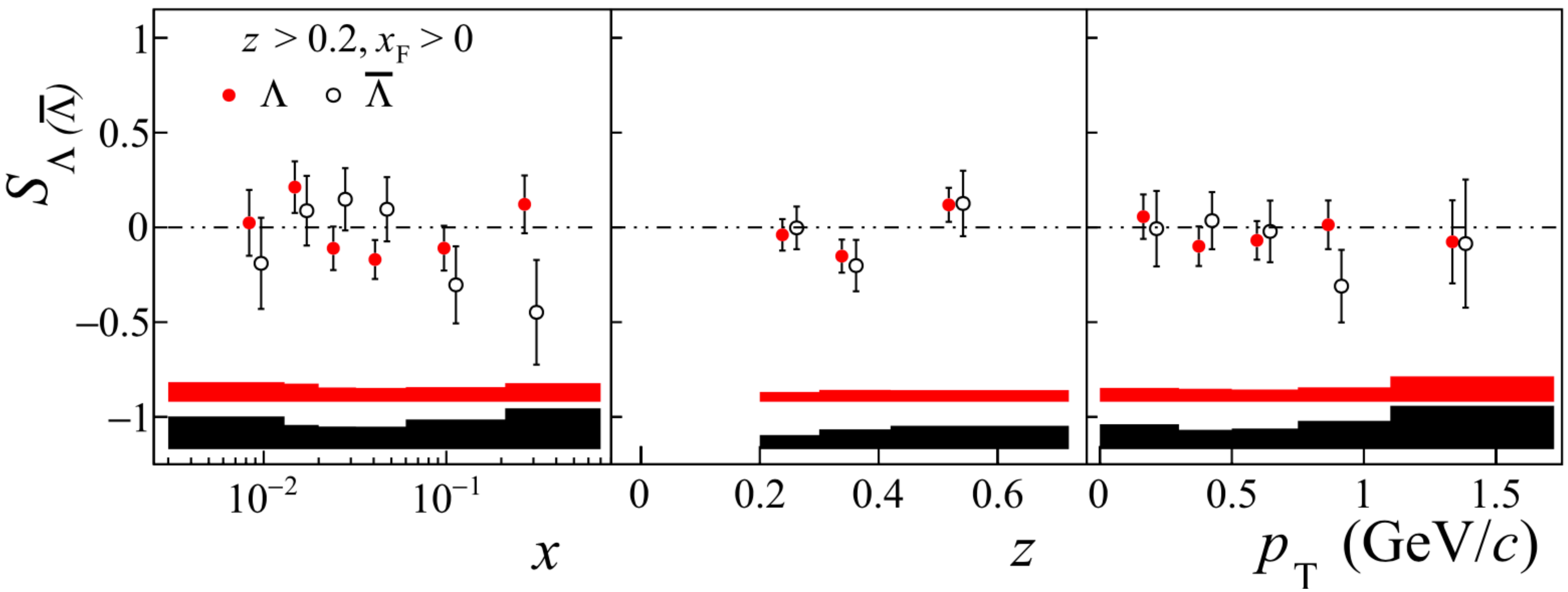
	δq	δq	$\delta\tilde{q}$	g_T
(Q^2 [GeV^2])	$Q_0^2 = 1$	$Q^2 = 4$	$Q^2 = 10$	$Q^2 = 4$
Up	0.43(11)	0.39(10)	0.32(8)	0.53(25)
Down	−0.12(28)	−0.11(26)	−0.10(22)	

Adding STAR data on pion pairs

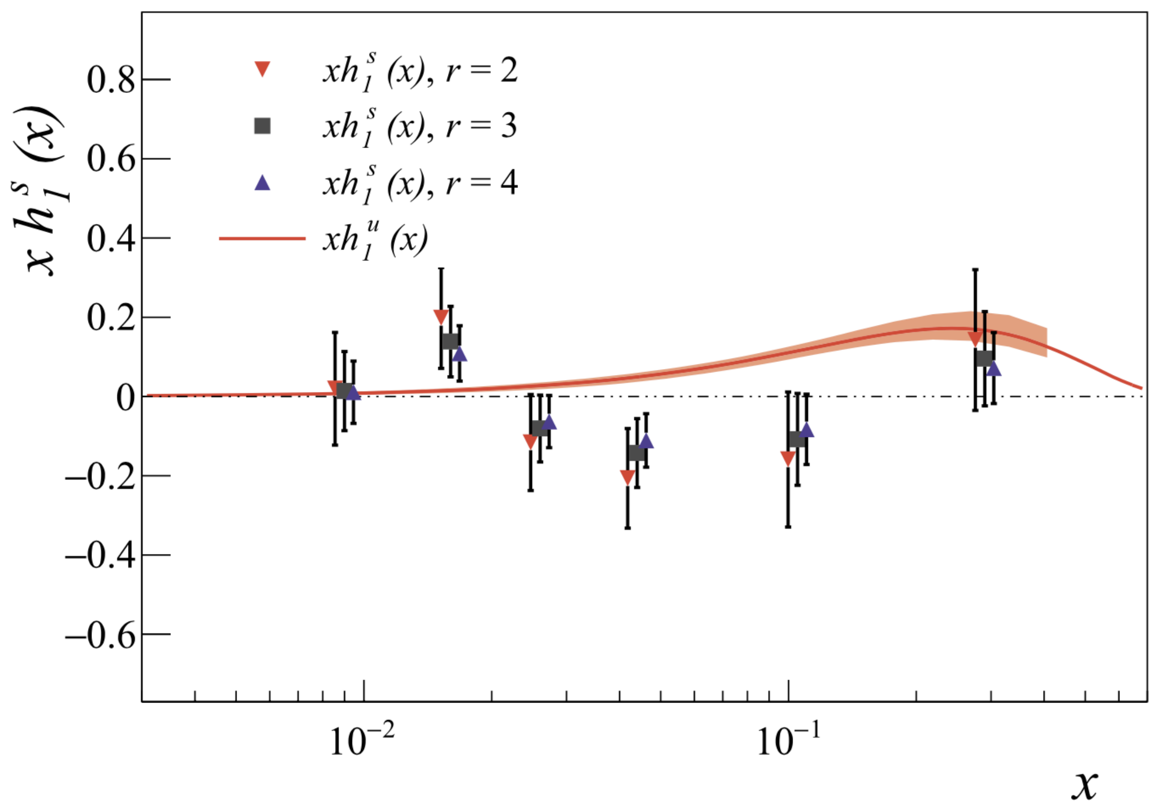
Radici, Bacchetta, 2018

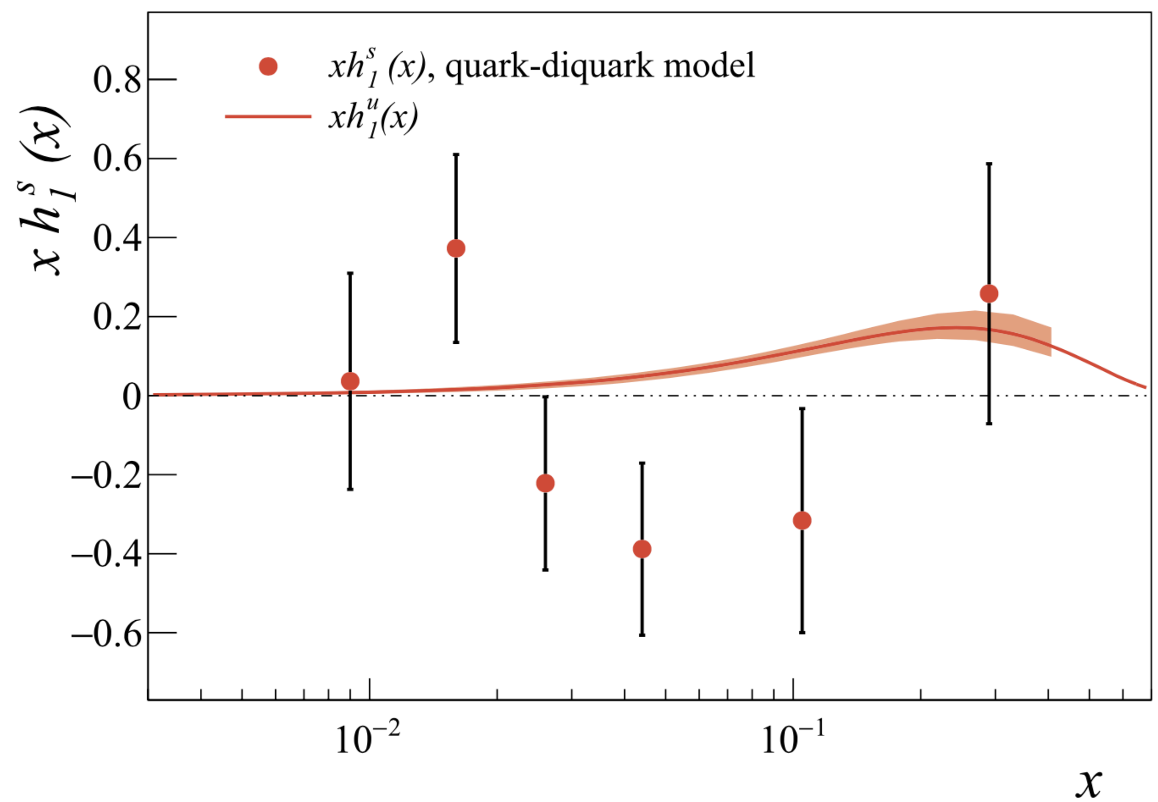
FIG. 1. The transversity xh_1 as a function of x at $Q^2 = 2.4$ GeV^2. Dark (blue) lines represent the Soffer bounds. Dark bands with solid borders for the global fit of this work including all options $D_1^g(Q_0^2) = 0$, $D_1^u(Q_0^2)/4$, and $D_1^u(Q_0^2)$. (Top) For valence up quark: comparison with our previous fit in Ref. [29] (lighter band with dashed borders). (Bottom) For valence down quark: comparison with this global fit with only $D_1^g(Q_0^2) = 0$ (hatched area with lighter borders).

Transversity from spin transfer



COMPASS, 2021





Gluons

What about gluons?

At leading twist there is no gluon transversity distribution for the proton

For spin-1 hadrons, such as the deuteron, there is such a distribution:

In the transverse tensor polarization case
there is a contribution solely from gluons

Jaffe, Manohar, 1989

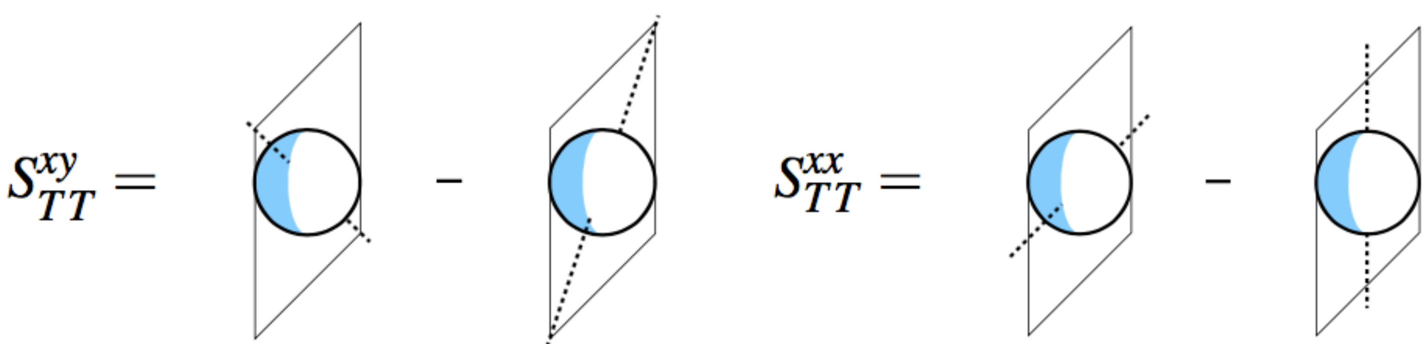
Artru, Mekhfi, 1990

$$\Delta(x)$$

$$\Delta_2 G(x)$$

$$h_{1TT}(x)$$

$$h^g_{1TT}(x)$$



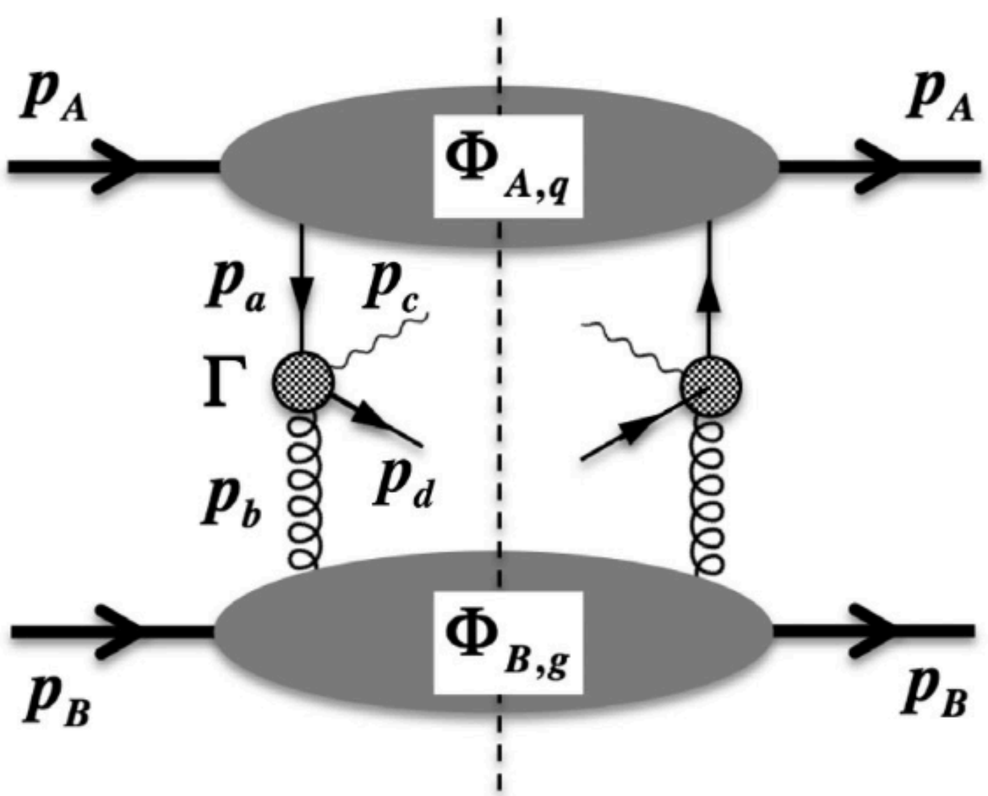
Bacchetta, Mulders, 2000

not yet measured - an objective of EIC

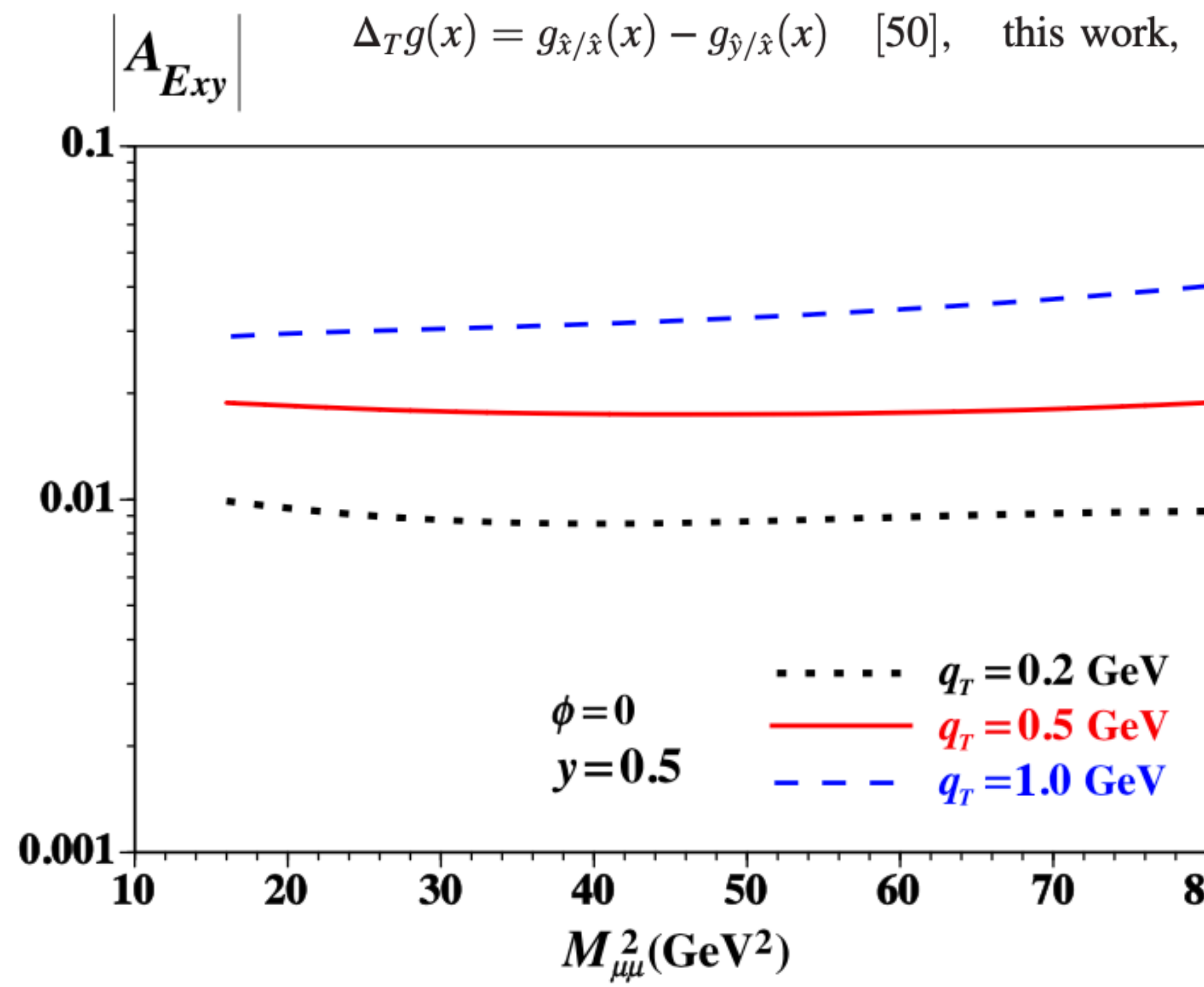
What about gluons?

Use proton-deuteron DY

Kumano, Song, 2020



$$
\begin{aligned}
\Delta_2 G(x) &= g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \quad [13,47],\\
a(x) &= g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \quad [25,27],\\
\Delta_L g(x) &= g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \quad [20],\\
\delta G(x) &= -g_{\hat{x}/\hat{x}}(x) + g_{\hat{y}/\hat{x}}(x) \quad [28,48],\\
h_{1\mathrm{TT},g}(x) &= -g_{\hat{x}/\hat{x}}(x) + g_{\hat{y}/\hat{x}}(x) \quad [39,41,49],\\
\Delta_T g(x) &= g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \quad [50], \quad \text{this work},
\end{aligned}
$$



Referred to as “gluon transversity” (spin-1), not the same as h_1^g (spin-1/2)

What about gluons?

$$\Phi_U(x,\boldsymbol{k}) = \frac{1}{2}\left[\bar{\not{n}}\, f_1(x,\boldsymbol{k}^2) + \frac{\sigma_{\mu\nu}k_T^\mu \bar{n}^\nu}{M}\, h_1^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Phi_L(x,\boldsymbol{k}) = \frac{1}{2}\left[\gamma^5 \bar{\not{n}}\, S_L\, g_1(x,\boldsymbol{k}^2) + \frac{i\sigma_{\mu\nu}\gamma^5 \bar{n}^\mu k_T^\nu S_L}{M}\, h_{1L}^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Phi_T(x,\boldsymbol{k}) = \frac{1}{2}\left[\frac{\bar{\not{n}}\,\epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}^2) + \frac{\gamma^5 \bar{\not{n}}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}^2) + i\sigma_{\mu\nu}\gamma^5 \bar{n}^\mu S_T^\nu\, h_1(x,\boldsymbol{k}^2) - \frac{i\sigma_{\mu\nu}\gamma^5 \bar{n}^\mu k_T^{\nu\rho} S_{T\rho}}{M^2}\, h_{1T}^\perp(x,\boldsymbol{k}^2)\right]$$

For quarks h_1 is k_T-even, T-even and chiral-odd

$$\Gamma_U^{ij}(x,\boldsymbol{k}) = x\left[\delta_T^{ij}\, f_1(x,\boldsymbol{k}^2) + \frac{k_T^{ij}}{M^2}\, h_1^\perp(x,\boldsymbol{k}^2)\right],$$

Mulders, Rodrigues, 2001
(with different notation)

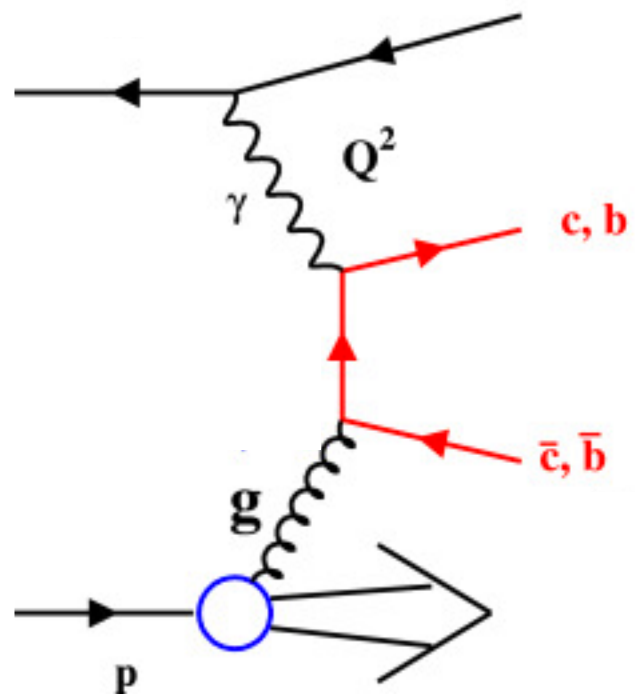
$$\Gamma_L^{ij}(x,\boldsymbol{k}) = x\left[i\epsilon_T^{ij} S_L\, g_1(x,\boldsymbol{k}^2) + \frac{\epsilon_T^{\{i}{}_\alpha k_T^{j\}\alpha} S_L}{2M^2}\, h_{1L}^\perp(x,\boldsymbol{k}^2)\right],$$

For gluons h_1 is k_T-odd, T-odd and unrelated to chirality

$$\Gamma_T^{ij}(x,\boldsymbol{k}) = x\left[\frac{\delta_T^{ij}\,\epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}^2) + \frac{i\epsilon_T^{ij}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{k_T\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\, h_1(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{\{i}{}_\alpha k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^\perp(x,\boldsymbol{k}^2)\right]$$

Why on earth give it the same name then?

Parallels between SIDIS and HQ pair production



A heavy quark pair will not be exactly back-to-back in the transverse plane:

$$K_\perp = (K_{Q\perp} - K_{\bar{Q}\perp})/2$$

$$q_T = K_{Q\perp} + K_{\bar{Q}\perp}$$

$$|q_T| \ll |K_\perp|$$

ϕ_T is the angle of the sum momentum

There is a "Collins" sin($\phi_T + \phi_S$) asymmetry (EIC) that arises from h_1^g

$$A_N^{\sin(\phi_S+\phi_T)} = \frac{2(1-y)\,\mathcal{B}_{0T}^{\gamma^* g\to Q\bar{Q}}}{[1+(1-y)^2]\,\mathcal{A}_{U+L}^{\gamma^* g\to Q\bar{Q}} - y^2\mathcal{A}_L^{\gamma^* g\to Q\bar{Q}}}\,\frac{|\boldsymbol{q}_T|}{M_p}\,\frac{h_1^g(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

Boer, Pisano, Mulders, J. Zhou, 2016

SIDIS - Fragmentation functions

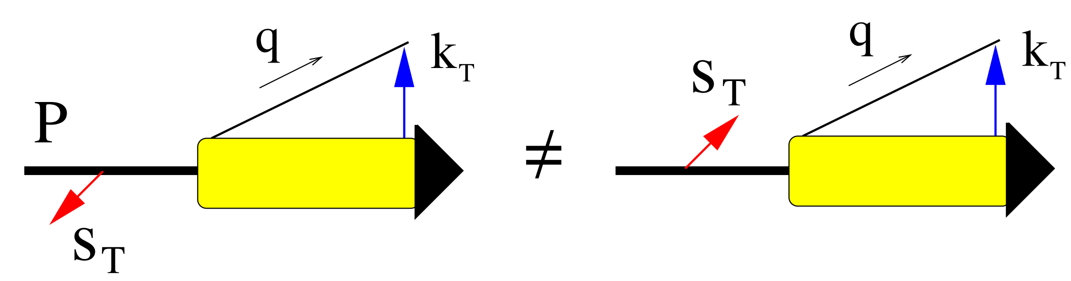
HQ pairs - calculable amplitudes

Quark Sivers & QS

What else is inside a transversely polarized proton?

$$\Phi_T(x,\boldsymbol{k}) = \frac{1}{2}\Bigg[\frac{\not{n}\,\epsilon_T^{S_T k_T}}{M} f_{1T}^{\perp}(x,\boldsymbol{k}^2) + \frac{\gamma^5 \not{n}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M} g_{1T}(x,\boldsymbol{k}^2) + i\sigma_{\mu\nu}\gamma^5 \bar{n}^{\mu} S_T^{\nu}\, h_1(x,\boldsymbol{k}^2) - \frac{i\sigma_{\mu\nu}\gamma^5 \bar{n}^{\mu} k_T^{\nu\rho} S_{T\rho}}{M^2} h_{1T}^{\perp}(x,\boldsymbol{k}^2)\Bigg]$$

$$\Gamma_T^{ij}(x,\boldsymbol{k}) = x\Bigg[\frac{\delta_T^{ij}\,\epsilon_T^{S_T k_T}}{M} f_{1T}^{\perp}(x,\boldsymbol{k}^2) + \frac{i\epsilon_T^{ij}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M} g_{1T}(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{k_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M} h_1(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha S_T}}{2M^3} h_{1T}^{\perp}(x,\boldsymbol{k}^2)\Bigg]$$



Quark and gluon Sivers TMDs

Sivers, 1989/90

They are the main candidates to explain the single spin left-right asymmetries A_N

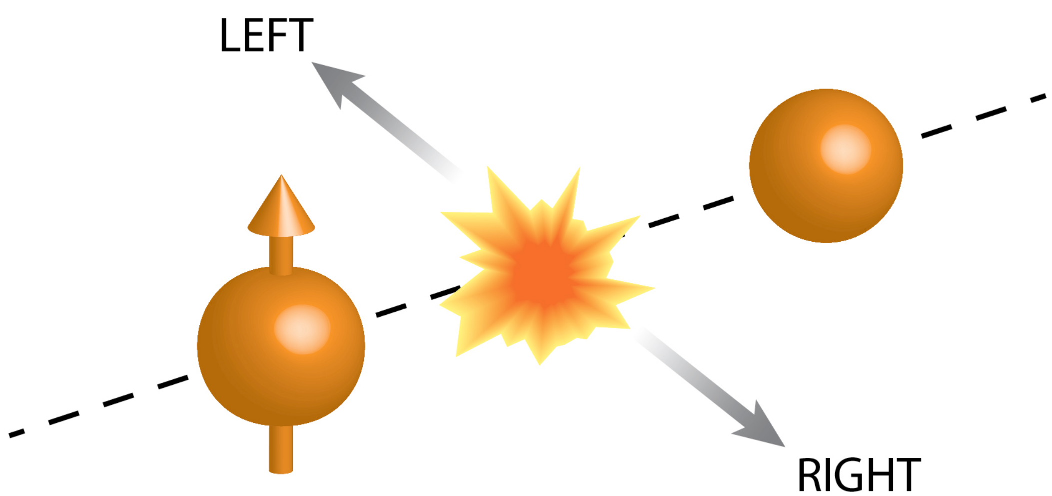
Another candidate is the twist-3 Qiu-Sterman function $T_F(x,x)$:

$$T_F(x,x) \overset{A^+=0}{\propto} \text{F.T.}\,\langle P|\,\overline{\psi}(0)\int d\eta^-\; F^{+\alpha}(\eta^-)\,\gamma^+\,\psi(\xi^-)\,|P\rangle$$

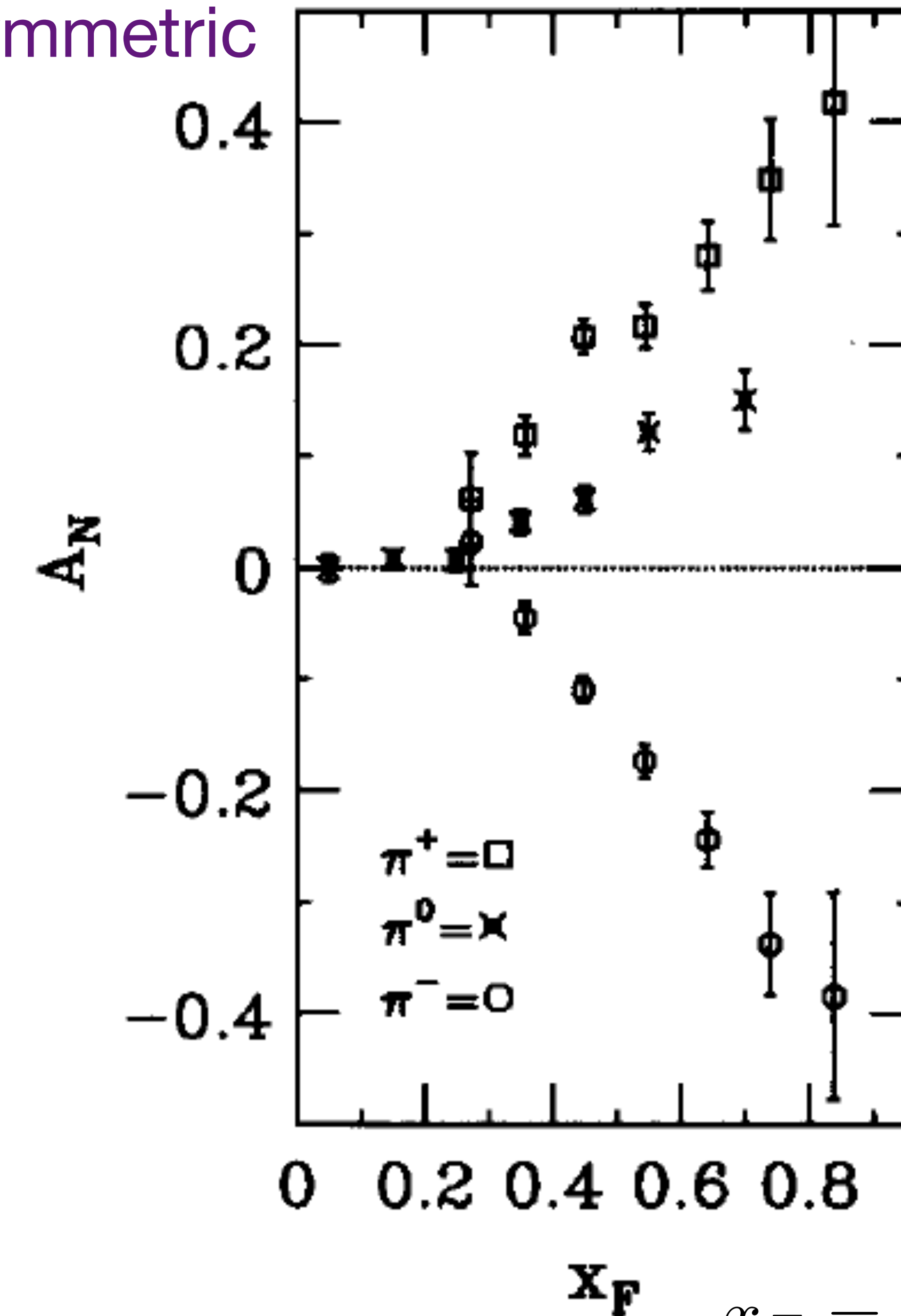
Qiu, Sterman, 1991

Transverse spin asymmetries

Distribution of produced particles highly asymmetric



$$A_N = \frac{\sigma(p^{\uparrow}\,p \to \pi\,X) - \sigma(p^{\downarrow}\,p \to \pi\,X)}{\sigma(p^{\uparrow}\,p \to \pi\,X) + \sigma(p^{\downarrow}\,p \to \pi\,X)}$$



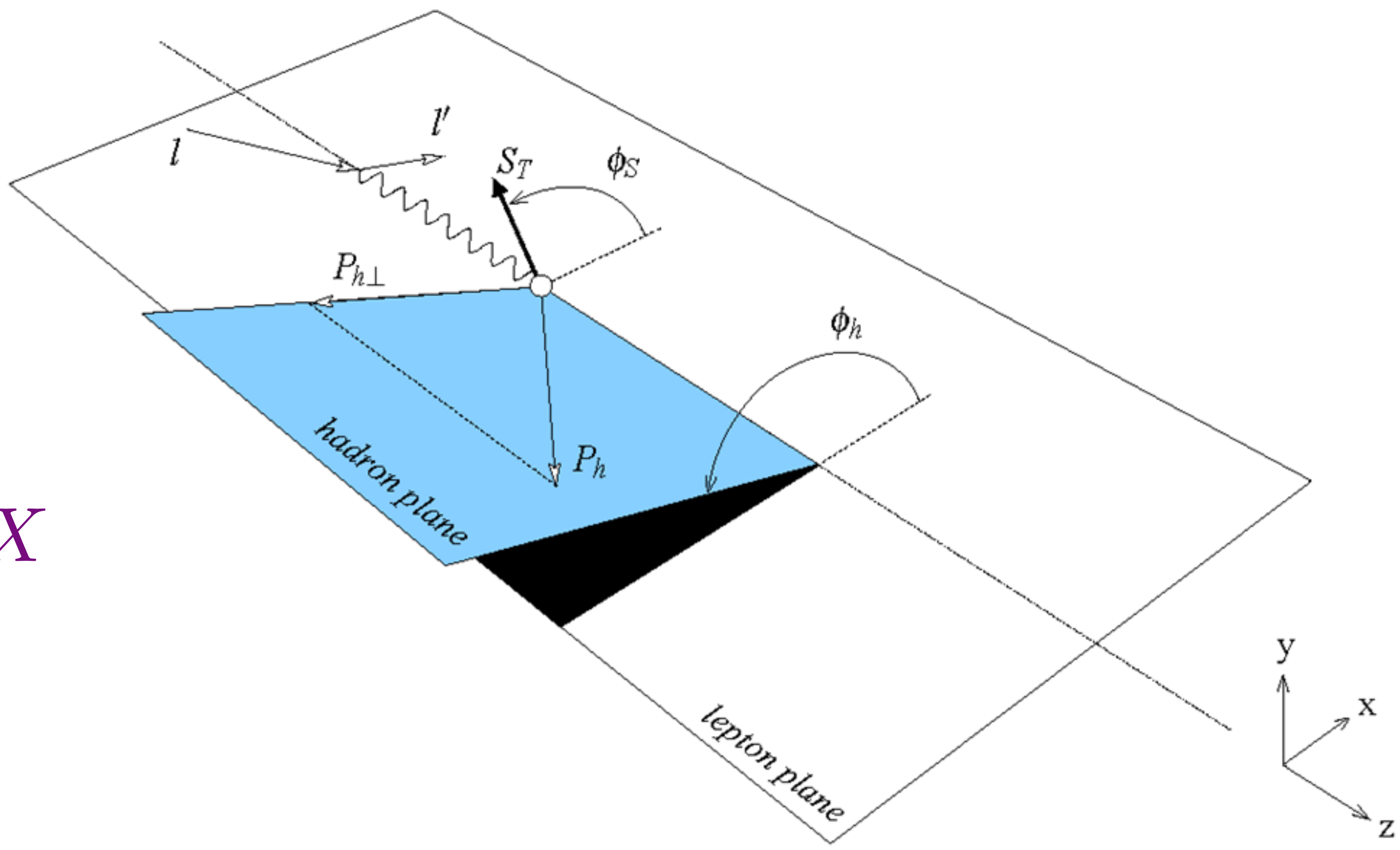
$$x_F = \frac{2p_z}{\sqrt{s}}$$

[Fermilab: E704 ('91) & BNL: AGS ('99); STAR ('02); BRAHMS ('05); PHENIX ('13)]

Sivers effect in SIDIS

Sivers effect should lead to a sin(ϕ_h - ϕ_S) asymmetry in semi-inclusive DIS

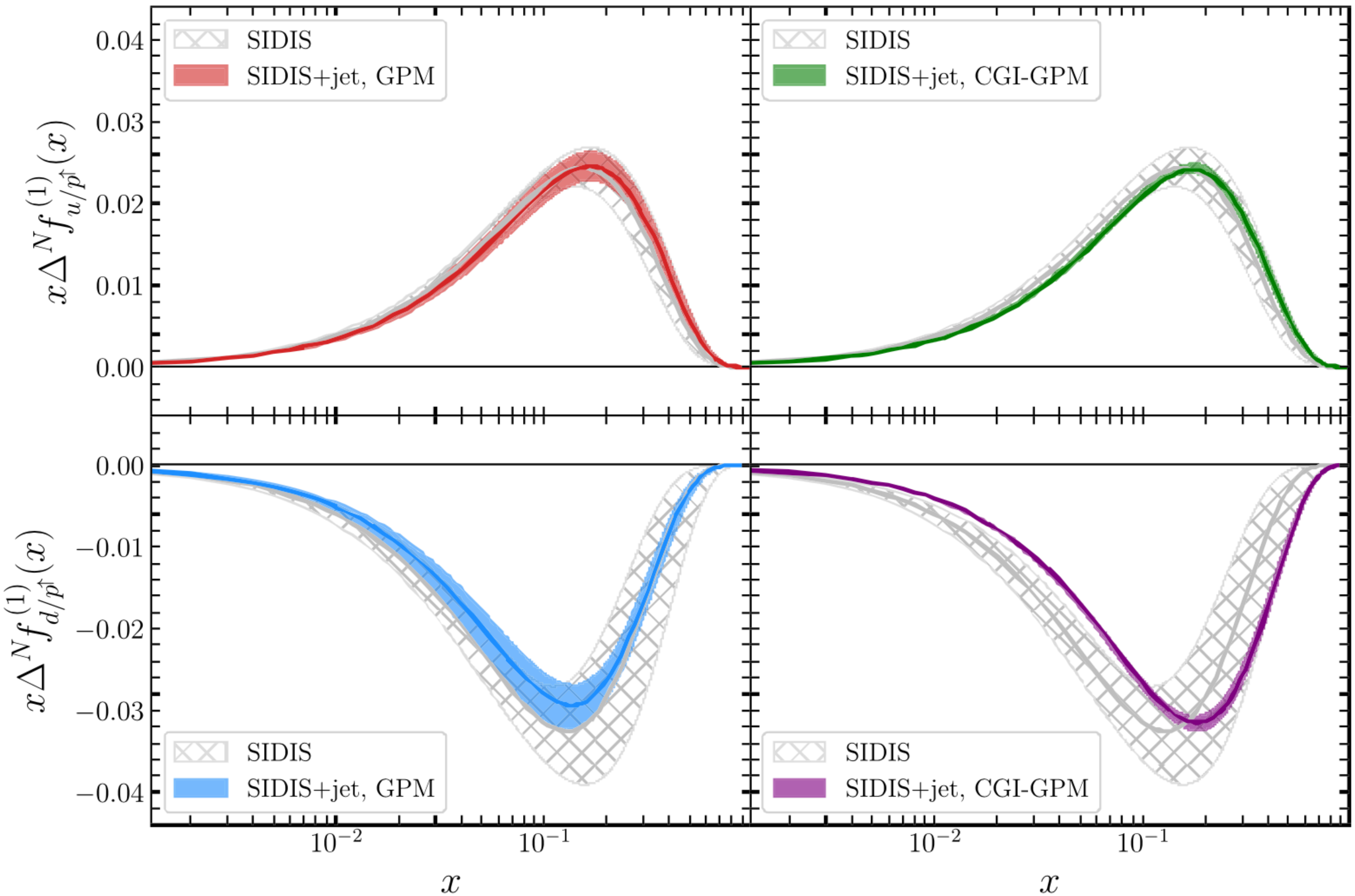
Boer, Mulders, 1998



$$e\,p \to e'\,h\,X$$

Clearly observed by HERMES (2009) and COMPASS (2010)

Sivers from a global analysis of ep and pp data



Includes STAR jet data

moderate impact of process dependence

Boglione et al., 2021

First transverse moment:

Kotzinian, Mulders, 1995/6

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) = \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{4M_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \equiv -f_{1T}^{\perp(1)q}(x)$$

Relation between Sivers and Qiu-Sterman functions

Conventional transverse moment:

$$f_{1T}^{\perp(1)}(x) \equiv \int d^2k_T \, \frac{k_T^2}{2M^2} \, f_{1T}^{\perp}(x, k_T^2)$$

Without including QCD corrections one has the (gauge invariant) relation:

$$f_{1T}^{\perp(1)}(x) = -\frac{g}{2M} T(x, S_T)$$

Boer, Mulders, Pijlman, 2003

$T(x,S_T)$ is the collinear twist-3 Qiu-Sterman function $T_F(x,x)$ Qiu, Sterman, 1991

Beyond LO this relation becomes ambiguous

Conventional transverse moment has convergence issues & the scale dependence is unclear

Bessel moments

TMD factorization involves TMDs Fourier transformed to b space

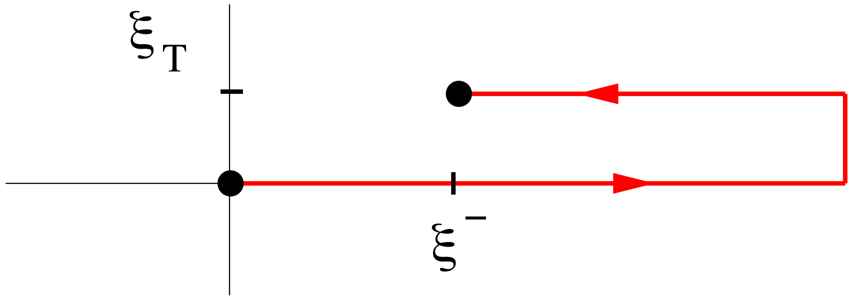
$$\tilde{f}^{[\mathcal{U}]}(x, b_T^2; \zeta, \mu)$$

Collins, 2011

$$\lim_{b_T^2 \to 0} \tilde{f}^{[\mathcal{U}]}(x, b_T^2; \zeta, \mu) = \int d^2k_T \, f^{[\mathcal{U}]}(x, k_T^2; \zeta, \mu) \stackrel{?}{=} f(x; \mu)$$

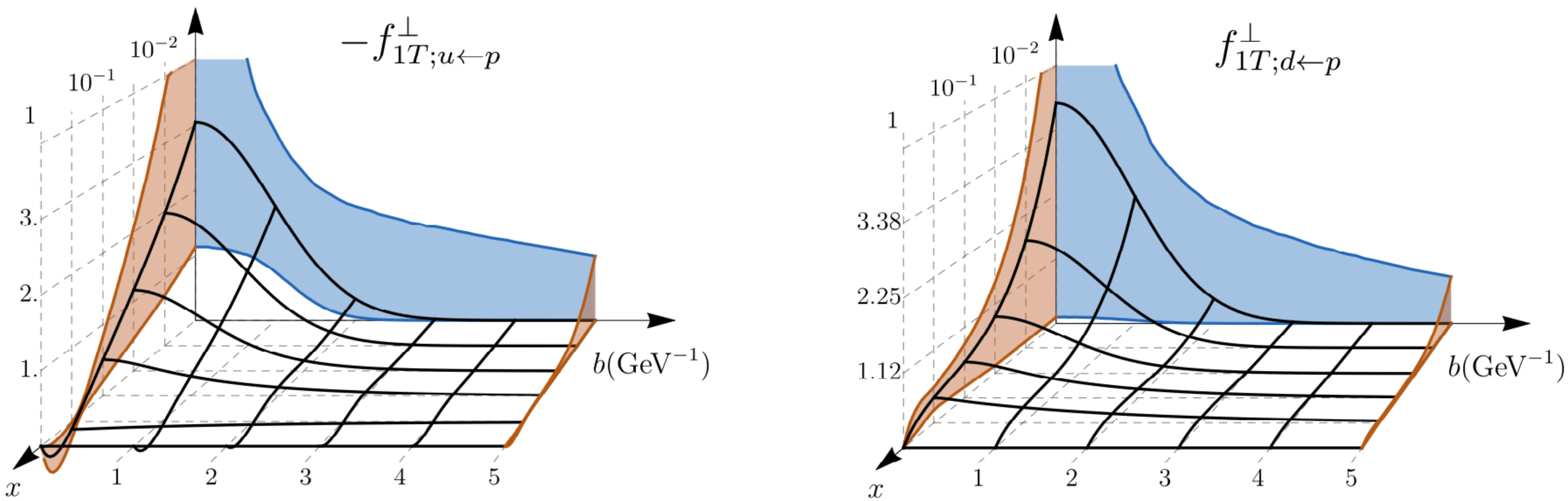
Idem for the Sivers function (its first Bessel moment):

$$\lim_{b_T^2 \to 0} \left(-\frac{2}{M^2} \partial_{b_T^2} \right) \tilde{f}_{1T}^{\perp [+]}(x, b_T^2; \zeta, \mu) \stackrel{?}{=} \frac{T_F(x, x; \mu)}{2M}$$



Boer, Gamberg, Musch, Prokudin, 2011

N^{3}LO extraction of quark Sivers TMD from SIDIS/DY/W/Z



Bury, Prokudin, Vladimirov, 2020

The perturbative small-b expression of the Sivers TMD in terms of the QS function in the so-called ζ-prescription can be inverted

Scimemi, Vladimirov, 2018; Scimemi, Tarasov, Vladimirov, 2019

$$T_q(-x,0,x;\mu_b) = -\frac{1}{\pi} f^{\perp}_{1T;q\leftarrow h}(x,b) - \frac{\alpha_s(\mu_b)}{4\pi^2}\int_x^1 \frac{dy}{y}\left[\frac{\bar{y}}{N_c} f^{\perp}_{1T;q\leftarrow h}\left(\frac{x}{y},b\right) + \frac{3y^2\bar{y}}{2x} G^{(+)}\left(-\frac{x}{y},0,\frac{x}{y};\mu_b\right)\right] + \mathcal{O}(a_s^2,b^2)$$

Bury, Prokudin, Vladimirov, 2020

$G^{(+)}$ is the gluon QS function

Qiu-Sterman function

$$T_q(-x,0,x;\mu_b) = -\frac{1}{\pi} f_{1T;q\leftarrow h}^{\perp}(x,b) - \frac{\alpha_s(\mu_b)}{4\pi^2}\int_x^1 \frac{dy}{y}\left[\frac{\bar{y}}{N_c} f_{1T;q\leftarrow h}^{\perp}\left(\frac{x}{y},b\right) + \frac{3y^2\bar{y}}{2x} G^{(+)}\left(-\frac{x}{y},0,\frac{x}{y};\mu_b\right)\right] + \mathcal{O}(a_s^2,b^2)$$

The gluon QS function $G^{(+)}$ is mostly relevant at small x

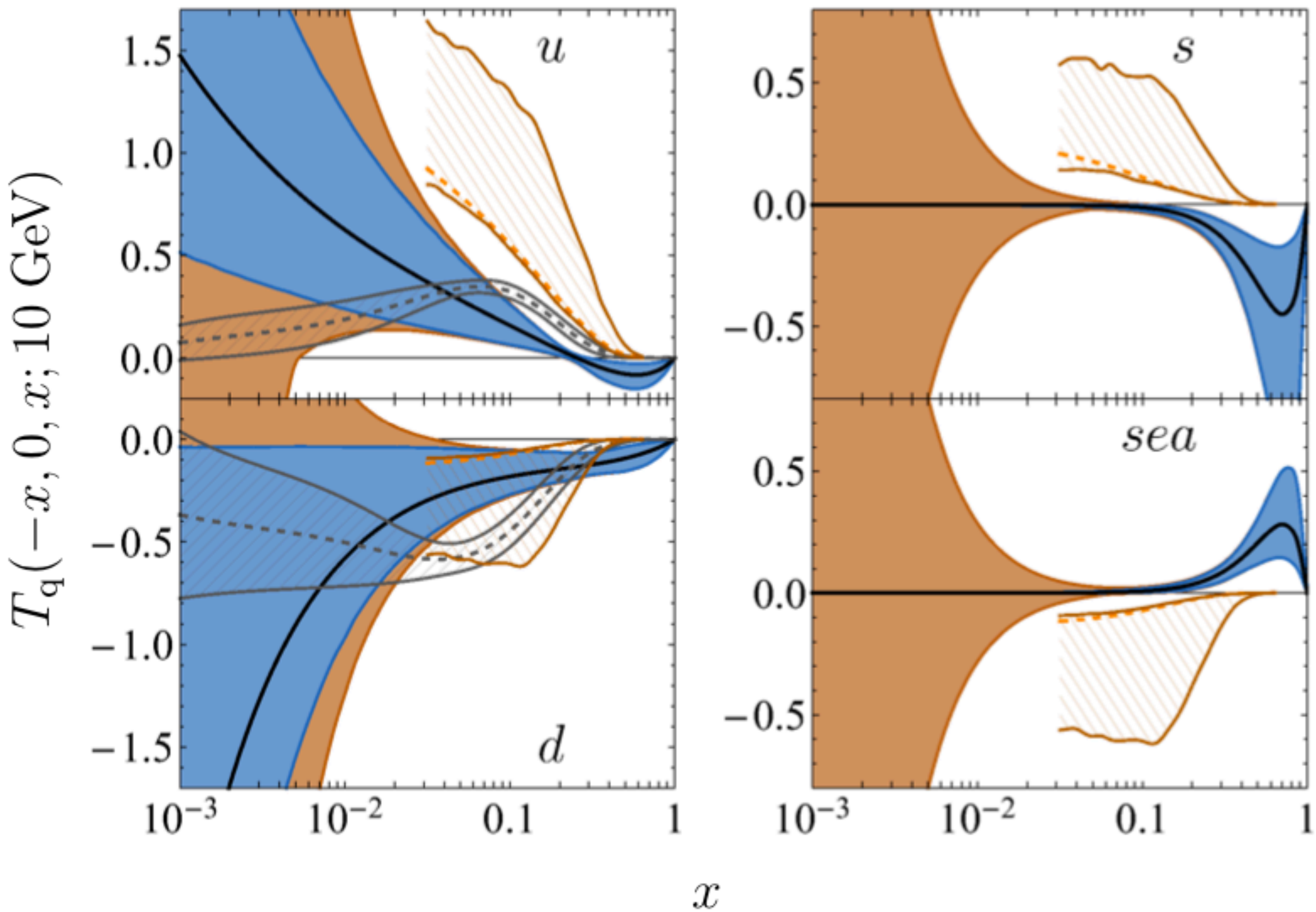


FIG. 3. Qiu-Sterman function at $\mu = 10$ GeV for different quark flavors, derived from the Sivers function via Eq. (13). The black line shows the CF value and blue band shows 68%CI. The brown band shows the band obtained by adding the gluon contribution $G^{(+)}$. We compare our results to JAM20 [35] (gray dashed lines) and ETK20 [34] (orange dashed lines).

Bury, Prokudin, Vladimirov, 2020

Gluon Sivers TMD

Process dependence of gluon TMDs

Gluon TMD correlators involve 2 gauge links:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

$$\mathcal{U}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right) \qquad \xi = [0^+,\xi^-,\boldsymbol{\xi}_T]$$

For most cases there are 2 link combinations of interest: [+,+] & [+,–]

[–,–] & [–,+] are related to them by parity and time reversal

The Sivers function $f_{1T}^{\perp g\,[+,-]}$ at small x is especially of interest

Gauge loop correlator

The [+,–] gluon TMD correlator becomes in the small-x limit:

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \xrightarrow{x\to 0} \frac{k_T^i k_T^j}{2\pi L}\Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

a single gauge loop matrix element

$$U^{[\Box]} = U^{[+]}_{[0,y]}U^{[-]}_{[y,0]}$$

The gluon Sivers function $f_{1T}^{\perp\,g\,[+,-]}$ at small x is part of:

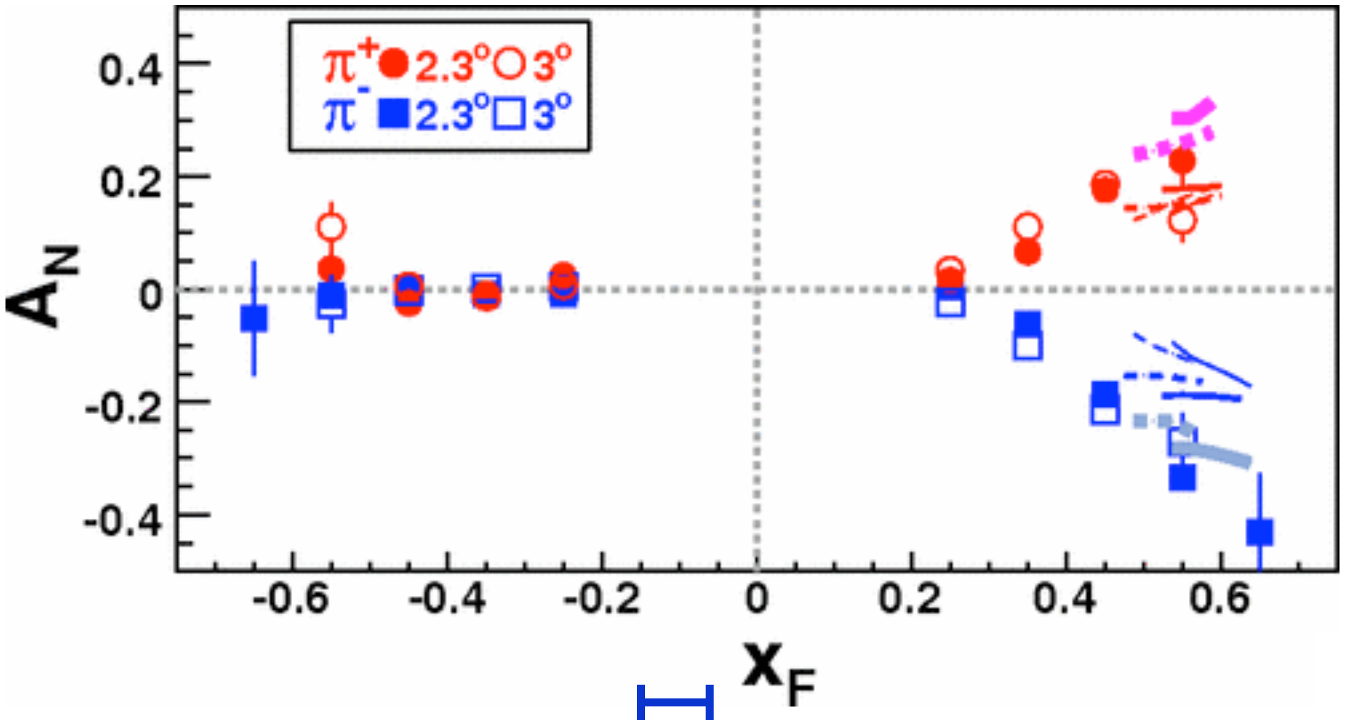
$$\left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \text{F.T.}\,\langle P, S_T|\text{Tr}\left[U^{[\Box]}(0_T,y_T) - U^{[\Box]\dagger}(0_T,y_T)\right]|P,S_T\rangle$$

Boer, Echevarria, Mulders, J. Zhou, 2016

This can be identified with the *spin-dependent odderon*   J. Zhou, 2013

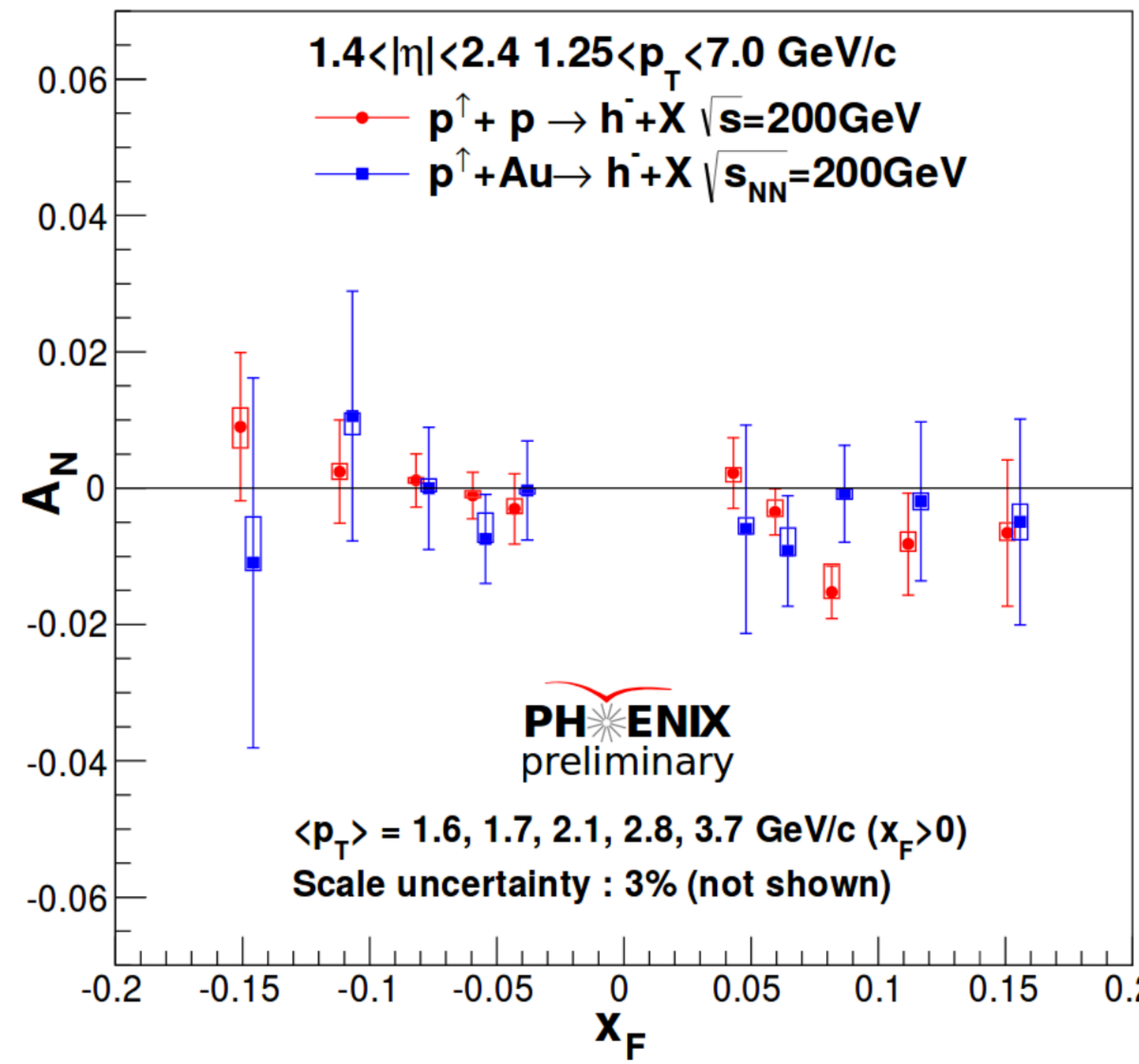
It is the only relevant contribution to A_N in backward ($x_F < 0$) charged hadron production in $p^\uparrow p$ or $p^\uparrow A$ (in contrast to the many contributions at $x_F > 0$)

$p^{\uparrow}p \rightarrow h^{\pm} X$ at $x_F < 0$



BRAHMS, 2008 $\sqrt{s}$ = 62.4 GeV
low p_T, up to roughly 1.2 GeV
where gg channel dominates

PHENIX, 2017
$\sqrt{s}$ = 200 GeV
p_T between 1.25 and 7 GeV



Data at larger negative x_F needed

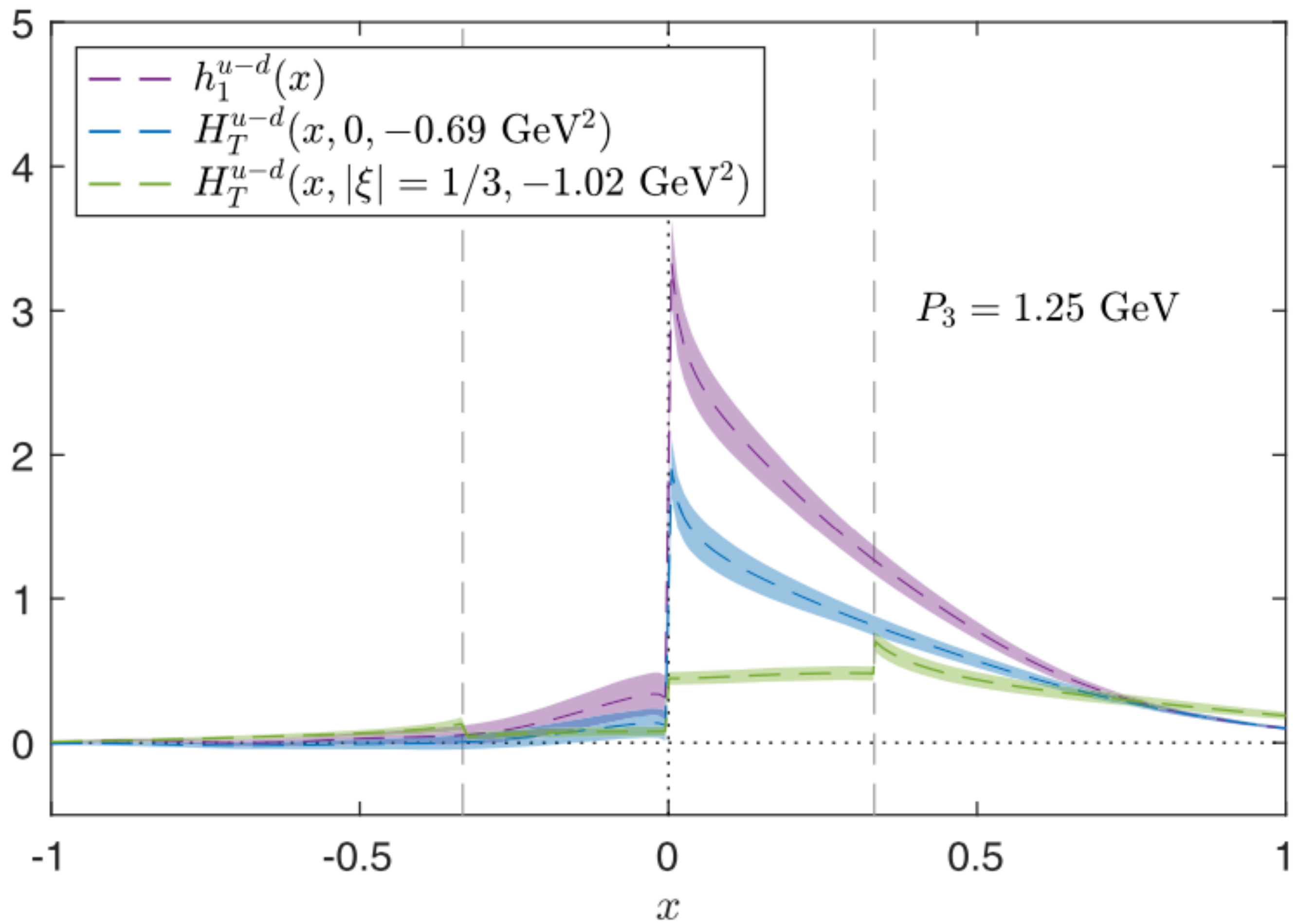
Possible at RHIC & NICA?

Transversity GPDs

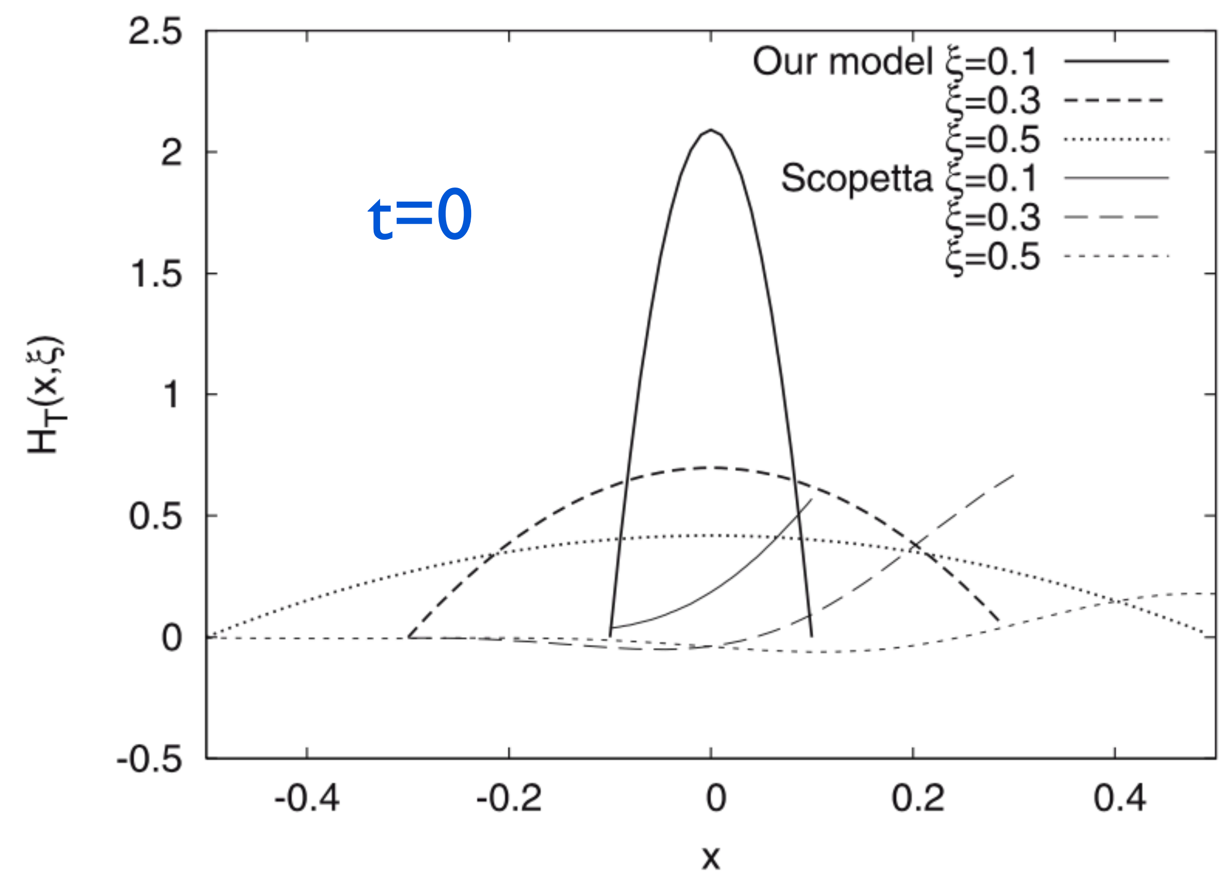
Transversity GPD from lattice QCD and models

Large-Momentum Effective Theory (LaMET)

Alexandrou et al., 2022 $H_T^{u-d}(x, \xi, t)$

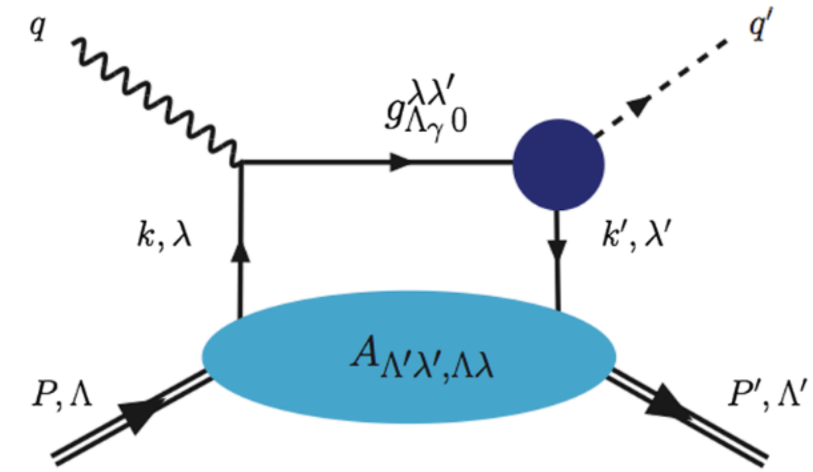


Can be compared to EIC data and to models, e.g.



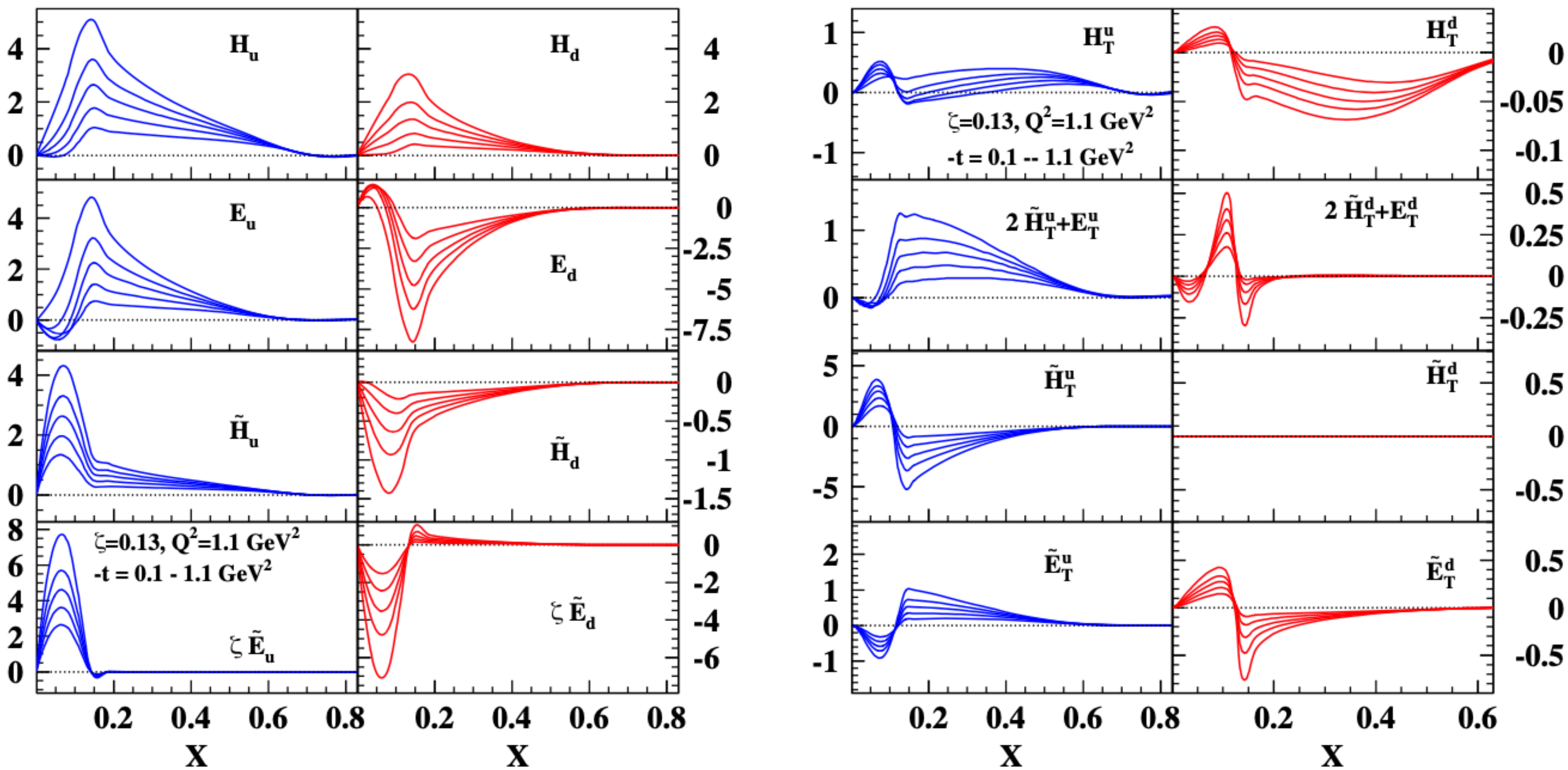
Enberg, Pire, Szymanowski, 2006

Transversity GPDs in exclusive processes



Hard exclusive vector meson pair production, pseudoscalar production, …

Enberg, Pire, Szymanowski, 2006; Ahmad, Goldstein, Liuti, 2009; Goloskokov, Kroll, 2011 & 2014; Goldstein, Gonzalez Hernandez, Liuti, 2015; …



chiral-even GPDs

chiral-odd GPDs

Conclusions

Conclusions

- The transversity distribution and tensor charge are reasonably known, even without the classic Drell-Yan observable
- Gluonic analogues of transversity for the deuteron, but even for the proton can be investigated at the EIC; neither is "gluon transversity" though
- Quark Sivers and even Qiu-Sterman functions have been extracted using new methods based on TMD factorization
- The gluon Sivers function $f_{1T}^{\perp\, g\, [+,-]}$ at small x (Wilson loop) is related to the spin dependent odderon and can be probed in A_N for $h^{\pm}$ at $x_F<0$
- Transversity and chiral-odd GPDs can be extracted from the lattice using LaMET; comparisons to extractions from EIC data is eagerly awaited

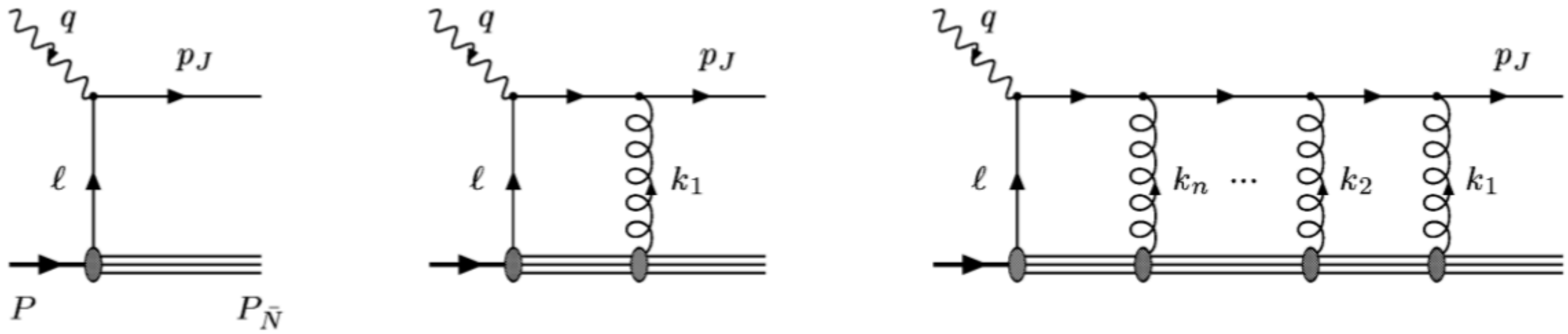
Back-up slides

Sivers TMD definition

$$P\cdot(\mathbf{k}_T \times \mathbf{S}_T)\, f_{1T}^{\perp[\mathcal{C}]}(x, \mathbf{k}_T^2) \propto \ \mathrm{F.T.}\, \langle P, S_T | \bar{\psi}(0)\, \mathcal{L}_{\mathcal{C}[0,\xi]}\, \gamma^+\, \psi(\xi) | P, S_T \rangle \big|_{\xi=(\xi^-, 0^+, \xi_T)}$$

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P} \exp\left(-ig \int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s) \right)$$

Summation of all gluon exchanges leads to gauge links (path-ordered exponentials) in the operators



The path C depends on whether the color interactions are with an incoming or outgoing color charge, yielding different paths for different processes

Collins & Soper, 1983; Boer & Mulders, 2000

Process dependence

Gauge invariant definition of TMDs in semi-inclusive DIS contains a future pointing gauge link (+ link), whereas in Drell-Yan (DY) it is past pointing (– link)

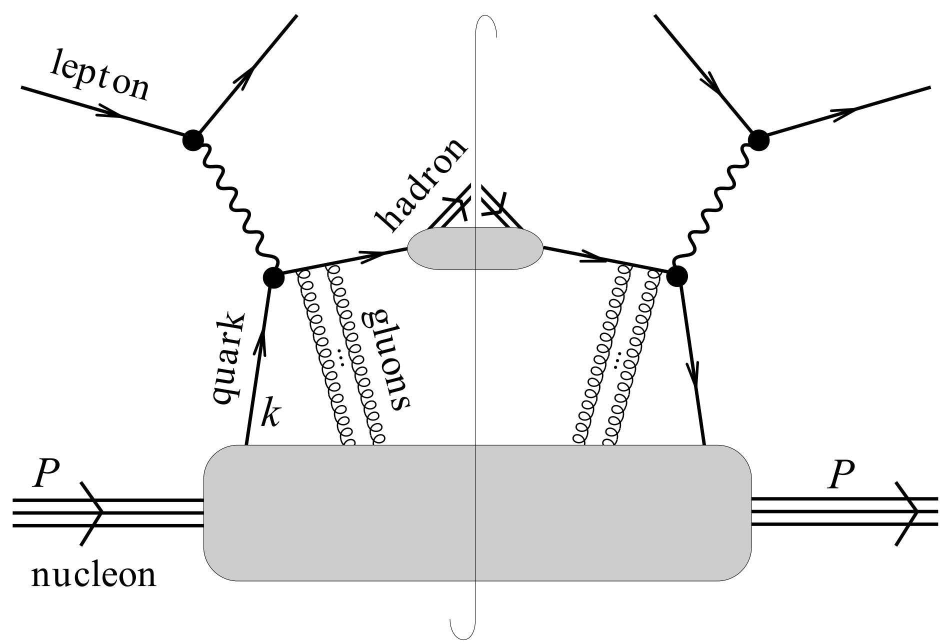
Brodsky, Hwang & Schmidt '02; Collins '02; Belitsky, Ji & Yuan '03

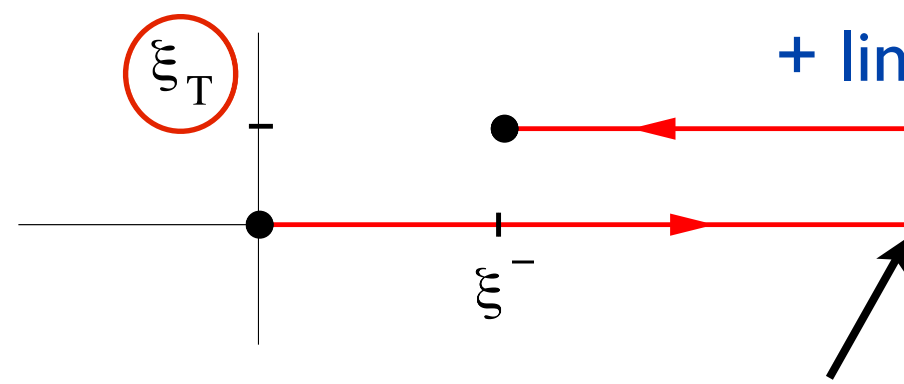
semi-inclusive DIS

$$e\,p \to e'\,h\,X$$

$$k \approx xP + k_T$$

$$P^\mu \approx P^+$$



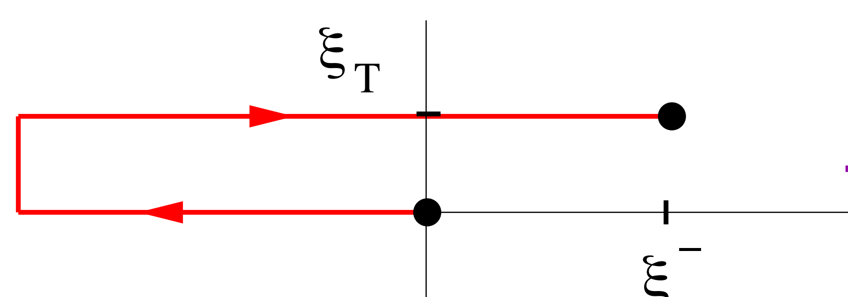


+ link

lightcone infinity ∞^-

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$$

$pp \to \gamma^* X$ Drell-Yan: $-\infty^-$



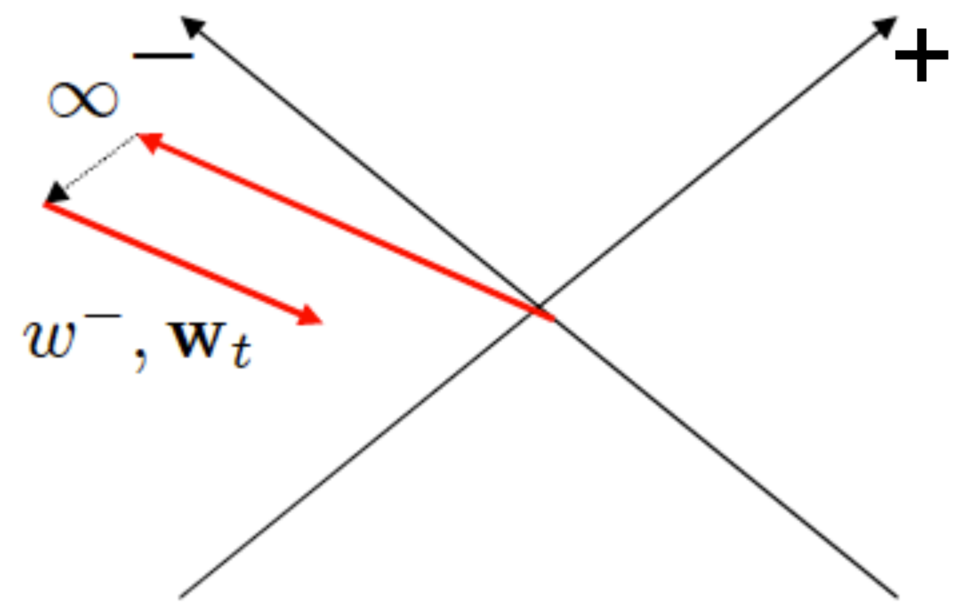
– link

Leads to observable effects (sign change relation) : $f_{1T}^{\perp[\mathrm{SIDIS}]} = -f_{1T}^{\perp[\mathrm{DY}]}$

Scale dependence of TMDs

As a regularization of LC divergences, in JCC's TMD factorization the path is taken off the lightfront, the variation in rapidity determines the change with ζ

$$\tilde{f}^{[\mathcal{U}]}(x, b_T^2; \zeta, \mu)$$



Two important consequences:

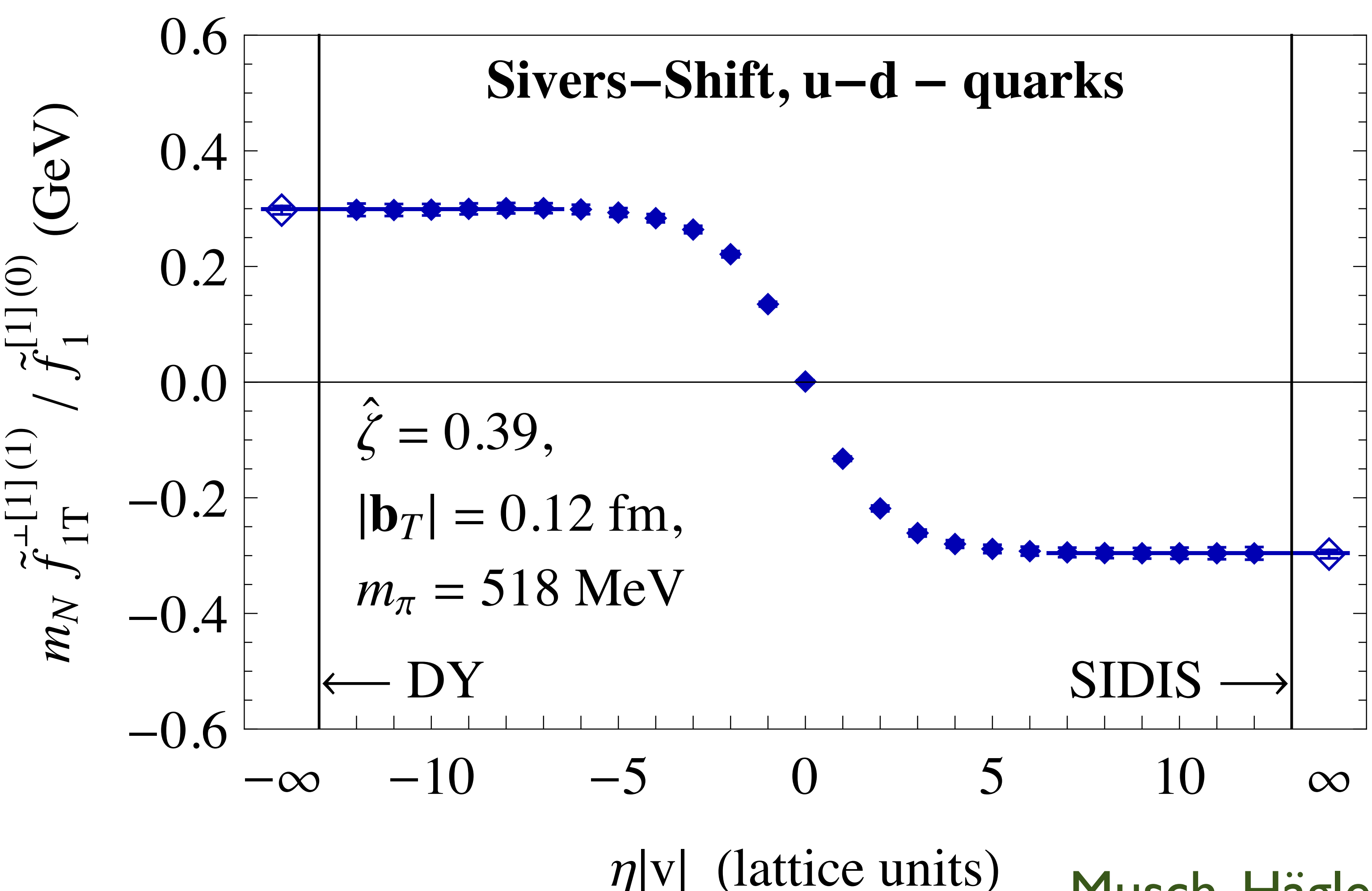
- yields scale evolution of TMD observables
- allows for calculation of the Sivers effect on the lattice (quite unexpectedly)

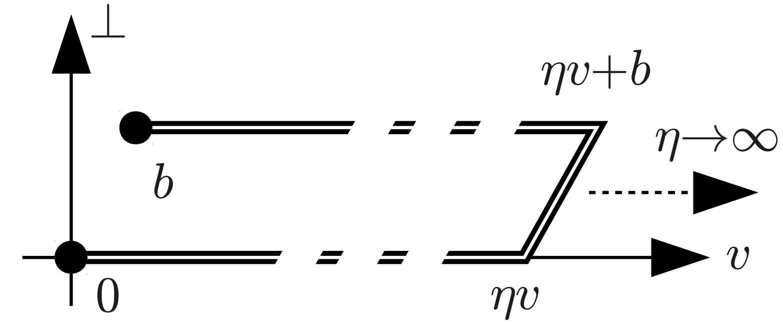
Musch, Hägler, Engelhardt, Negele & Schäfer, 2012

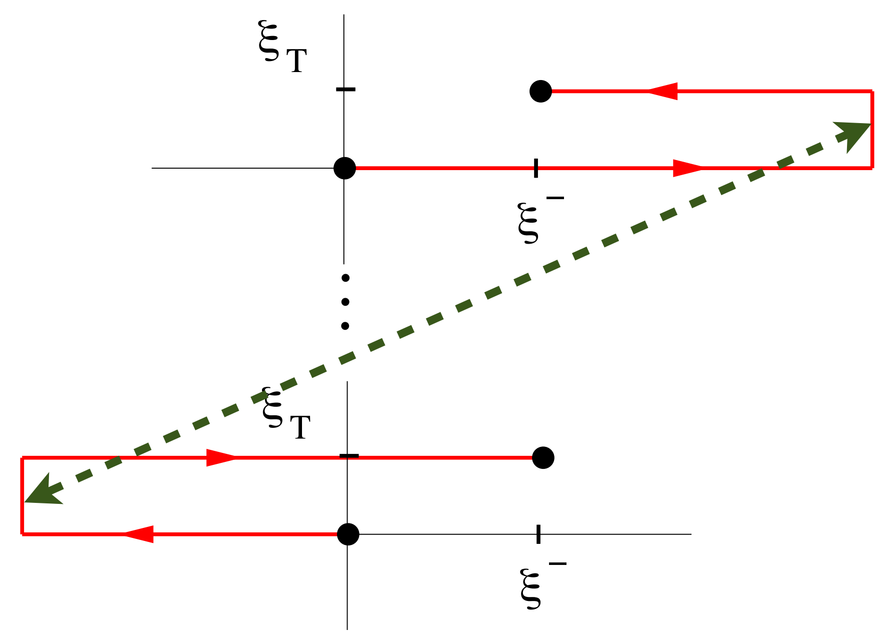
Sivers effect on the lattice

The "Sivers shift" <k_T x S_T> (the average transverse momentum shift orthogonal to transverse spin S_T) can be calculated on the lattice

Boer, Gamberg, Musch, Prokudin, 2011







Musch, Hägler, Engelhardt, Negele & Schäfer, 2012

This is the first first-principle demonstration in QCD that the Sivers function is nonzero for staple-like links. It clearly displays the sign change relation

Relation between TMDs and collinear pdfs

Collinear pdfs are not simply integrals of TMDs

$$\int d\boldsymbol{k}_T f_1(x, \boldsymbol{k}_T; \mu, \zeta) \stackrel{?}{=} f_1(x; \mu)$$

Collins, 2011

Large transverse momentum tail of TMDs are determined by collinear pdfs

$$f_1(x, \boldsymbol{p}_T^2) \stackrel{\boldsymbol{p}_T^2 \gg M^2}{\sim} \alpha_s \frac{1}{\boldsymbol{p}_T^2} \left(K \otimes f_1\right)(x)$$

Similarly, the perturbative tail of Sivers function determined by QS function

$$f_{1T}^{\perp}(x, \boldsymbol{p}_T^2) \stackrel{\boldsymbol{p}_T^2 \gg M^2}{\sim} \alpha_s \frac{M^2}{\boldsymbol{p}_T^4} \left(K' \otimes T_F\right)(x)$$

Ji, Qiu, Vogelsang, Yuan, 2006; Koike, Vogelsang, Yuan, 2008

Bessel moments

To avoid the convergence issue one can consider Bessel moments:

$$\tilde{f}^{(n)}(x,\boldsymbol{b}_T^2) = n!\left(-\frac{2}{M^2}\partial_{\boldsymbol{b}_T^2}\right)^n \tilde{f}(x,\boldsymbol{b}_T^2)$$

Boer, Gamberg, Musch, Prokudin, 2011

Generalization of the conventional transverse moments

$$\tilde{f}^{(n)}(x,\boldsymbol{b}_T^2) = n!\left(-\frac{2}{M^2}\partial_{\boldsymbol{b}_T^2}\right)^n \tilde{f}(x,\boldsymbol{b}_T^2) \xrightarrow{\boldsymbol{b}_T^2\to 0} f^{(n)}(x)$$

The limit should be considered with care

Collinear pdfs are not simply integrals of TMDs

$$\int d\boldsymbol{k}_T f_1(x,\boldsymbol{k}_T;\mu,\zeta) \stackrel{?}{=} f_1(x;\mu)$$

Collins, 2011

The limit $b_T \to 0$ of Sivers shift tells us about the Qiu-Sterman function

$$\lim_{b_T \to 0} \tilde{f}_{1T}^{(1)[+]}(x, b_T^2; \mu, \zeta) \stackrel{?}{=} \frac{T_F(x, x; \mu)}{2M}$$

This is especially promising if the limits $b_T \to 0$ and large ζ are constant/flat

$$\hat{\zeta} = \frac{\zeta}{2m_N} = \frac{\vec{v} \cdot \vec{P}}{\sqrt{|\vec{v}^{\,2}|}\sqrt{P^2}} = \sinh(y_P - y_v) \gg \frac{\Lambda_{\mathrm{QCD}}}{2m_N} \approx 0.1$$

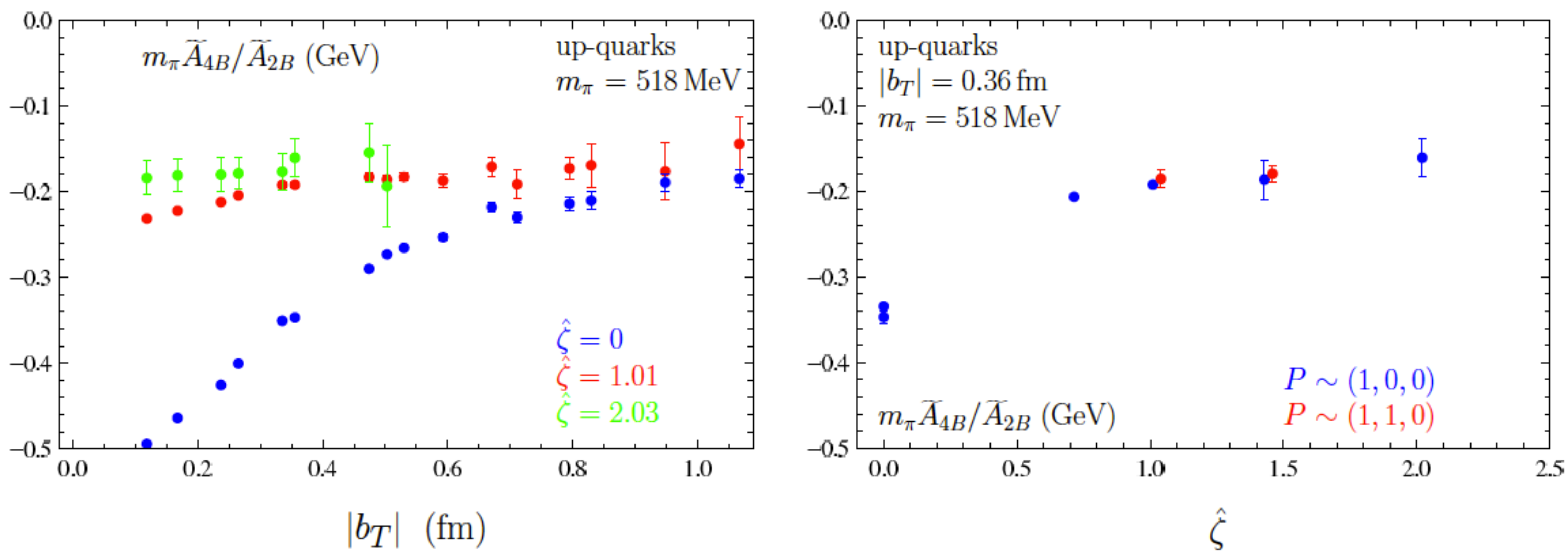


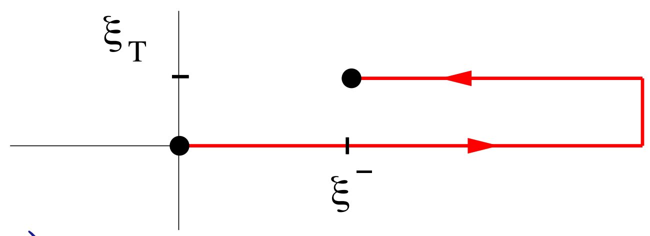
Figure 3: Generalized Boer-Mulders shift in the $\eta \to \infty$ SIDIS limit as a function of $|b_T|$ (left) and $\hat{\zeta}$ (right). In the left panel, the data in the region below $|b_T| \approx 0.25$ fm may be significantly affected by finite lattice cutoff effects. In the right panel, the congruence of the data obtained for P in different directions exhibits the good rotational properties of the calculation.

Engelhardt, 2013

Relation between Sivers and Qiu-Sterman functions

First Bessel moment of the Sivers TMD:

$$\tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2) \xrightarrow{\boldsymbol{b}_T^2 \to 0} f_{1T}^{\perp(1)}(x)$$



The limit should be considered with care

$$\lim_{b_T \to 0} \tilde{f}_{1T}^{(1)[+]}(x, b_T^2; \mu, \zeta) \stackrel{?}{=} \frac{T_F(x, x; \mu)}{2M}$$

Recent result: use the perturbative small-b expression and invert it in the so-called ζ-prescription

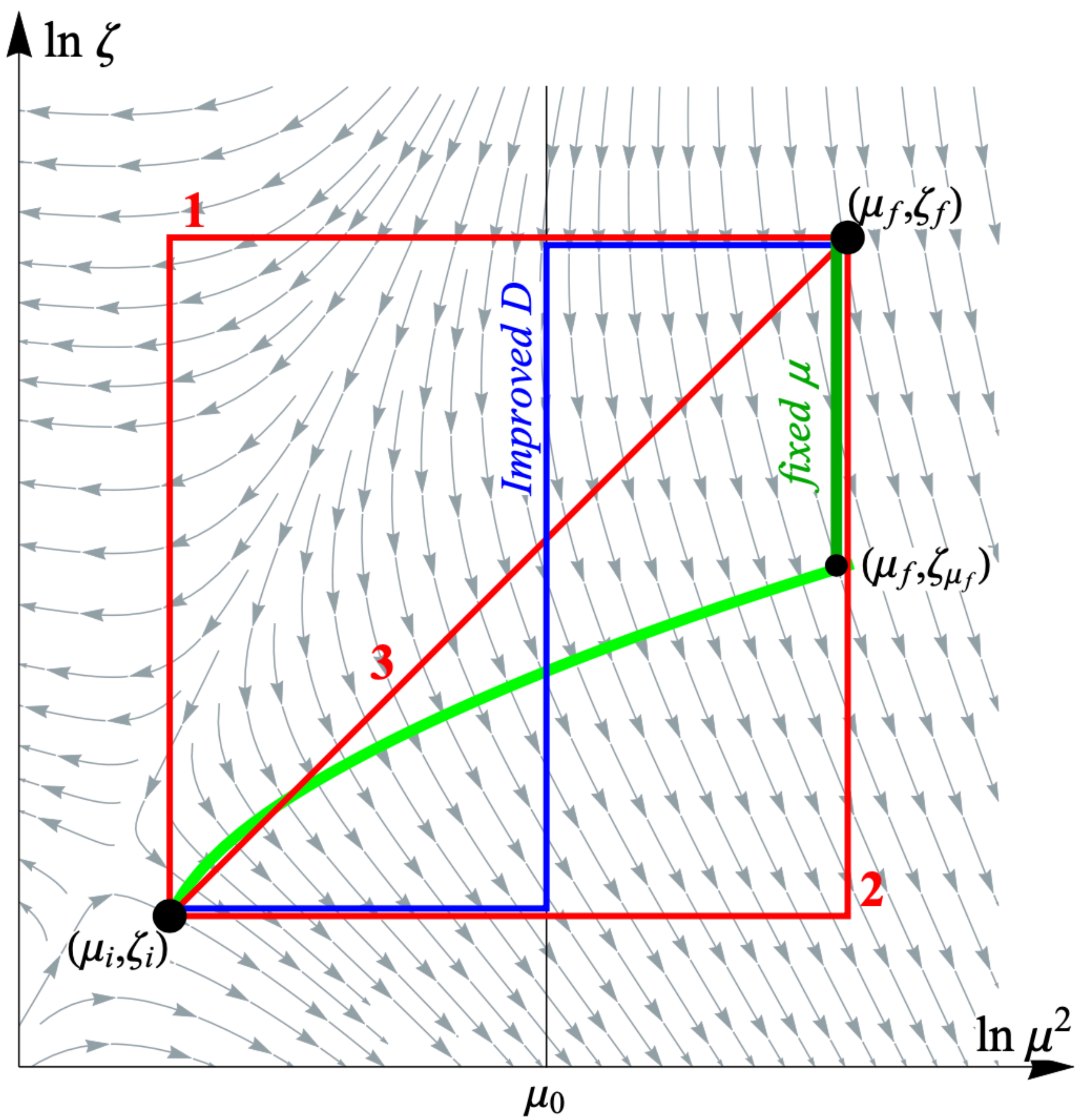
Scimemi, Vladimirov, 2018
Scimemi, Tarasov, Vladimirov, 2019

$$T_q(-x, 0, x; \mu_b) = -\frac{1}{\pi} f_{1T;q\leftarrow h}^{\perp}(x, b) - \frac{\alpha_s(\mu_b)}{4\pi^2} \int_x^1 \frac{dy}{y} \left[\frac{\bar{y}}{N_c} f_{1T;q\leftarrow h}^{\perp}\left(\frac{x}{y}, b\right) + \frac{3y^2\bar{y}}{2x} G^{(+)}\left(-\frac{x}{y}, 0, \frac{x}{y}; \mu_b\right) \right] + \mathcal{O}(a_s^2, b^2)$$

Bury, Prokudin, Vladimirov, 2020

$G^{(+)}$ is the gluon QS function

ζ-prescription



Scimemi, Vladimirov, 2018
Scimemi, Tarasov, Vladimirov, 2019

TMD factorization requires evolution of TMDs from large (μ_f,ζ_f) to small (μ_i,ζ_i)

This leads appearance of the Sudakov factor or evolutor term

However, one can evolve along a null-evolution curve ($\zeta_i = \zeta(\mu_i)$) and only a fixed-μ evolutor will appear

One recovers one scale evolution

Allows to connect express small b "tail" in terms of collinear function (e.g. QS function) and allows to invert expression (along this line coefficient function simplifies)

Gluons TMDs

The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{-g_T^{\mu\nu} f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)\right\}$$

unpolarized gluon TMD

linearly polarized gluon TMD

[Mulders, Rodrigues '01]

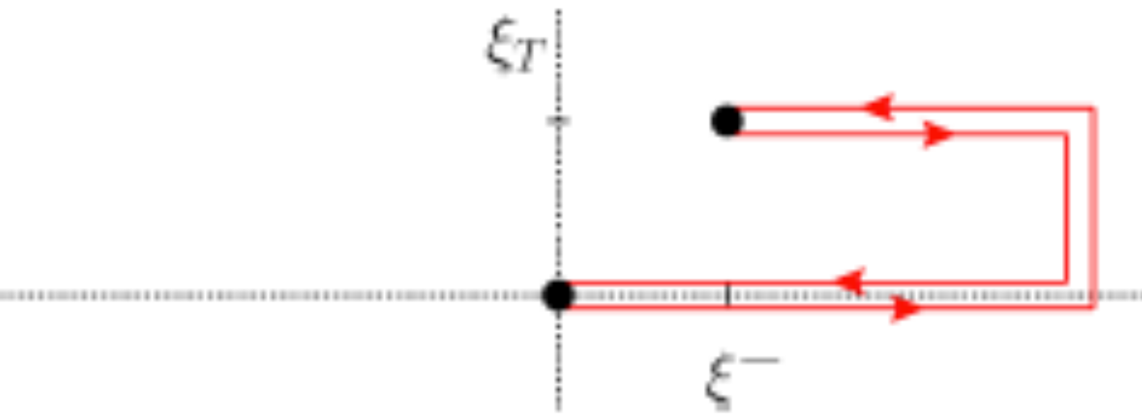
Gluons inside *unpolarized* protons can be polarized!

For transversely polarized protons:

gluon Sivers TMD

$$\Gamma_T^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma}\,p_{T\rho}\,S_{T\sigma}}{M_p} f_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2) + \ldots\right\}$$

Sign change relation for gluon Sivers TMD



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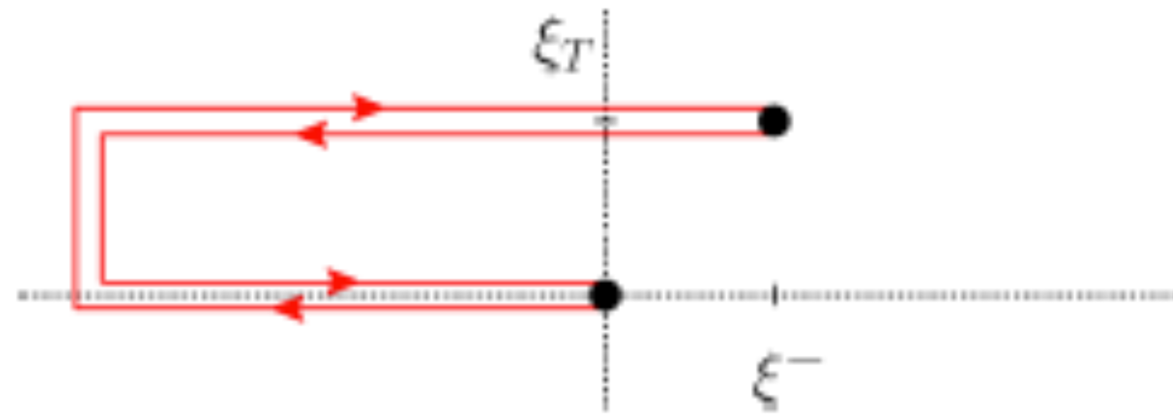
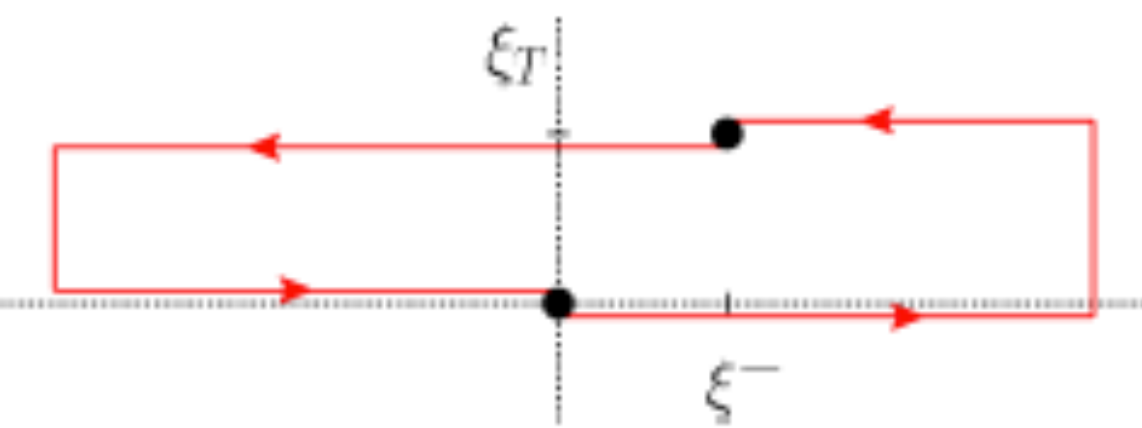


$$f_{1T}^{\perp\, g\, [e\, p^\uparrow \to e'\, Q\, \overline{Q}\, X]}(x, p_T^2) = -f_{1T}^{\perp\, g\, [p^\uparrow\, p \to \gamma\, \gamma\, X]}(x, p_T^2)$$

EIC

RHIC

D.B., Mulders, Pisano, J. Zhou, 2016



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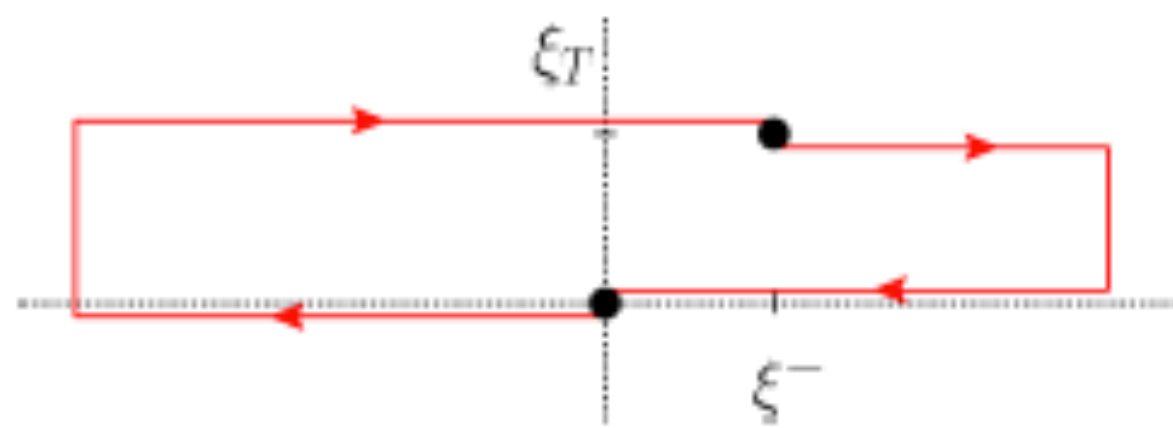


There is a distinct, *independent* gluon Sivers function with [+,−] links

Gluon Sivers TMDs for [+,+] & [+,−] are related to the f^{abc} & d^{abc} color structures

Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

Dipole versus WW distributions

For most processes of interest there are 2 relevant unpolarized gluon TMDs

$$xG^{(1)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \quad [+,+]$$

$$xG^{(2)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \quad [+,-]$$

For unpolarized gluons [+,+] = [–,–] and [+,–] = [–,+]

At small x the two correspond to the Weizsäcker-Williams (WW) and dipole (DP) distributions, which are generally different in magnitude and width:

$$xG^{(1)}(x,k_\perp) = -\frac{2}{\alpha_S}\int \frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2}\, e^{-ik_\perp\cdot(v-v')}\left\langle \mathrm{Tr}\left[\partial_i U(v)\right]U^\dagger(v')\left[\partial_i U(v')\right]U^\dagger(v)\right\rangle_{x_g} \quad \text{WW}$$

$$xG^{(2)}(x,q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-iq_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle \mathrm{Tr} U(0)U^\dagger(r_\perp)\right\rangle_{x_g} \quad \text{DP}$$

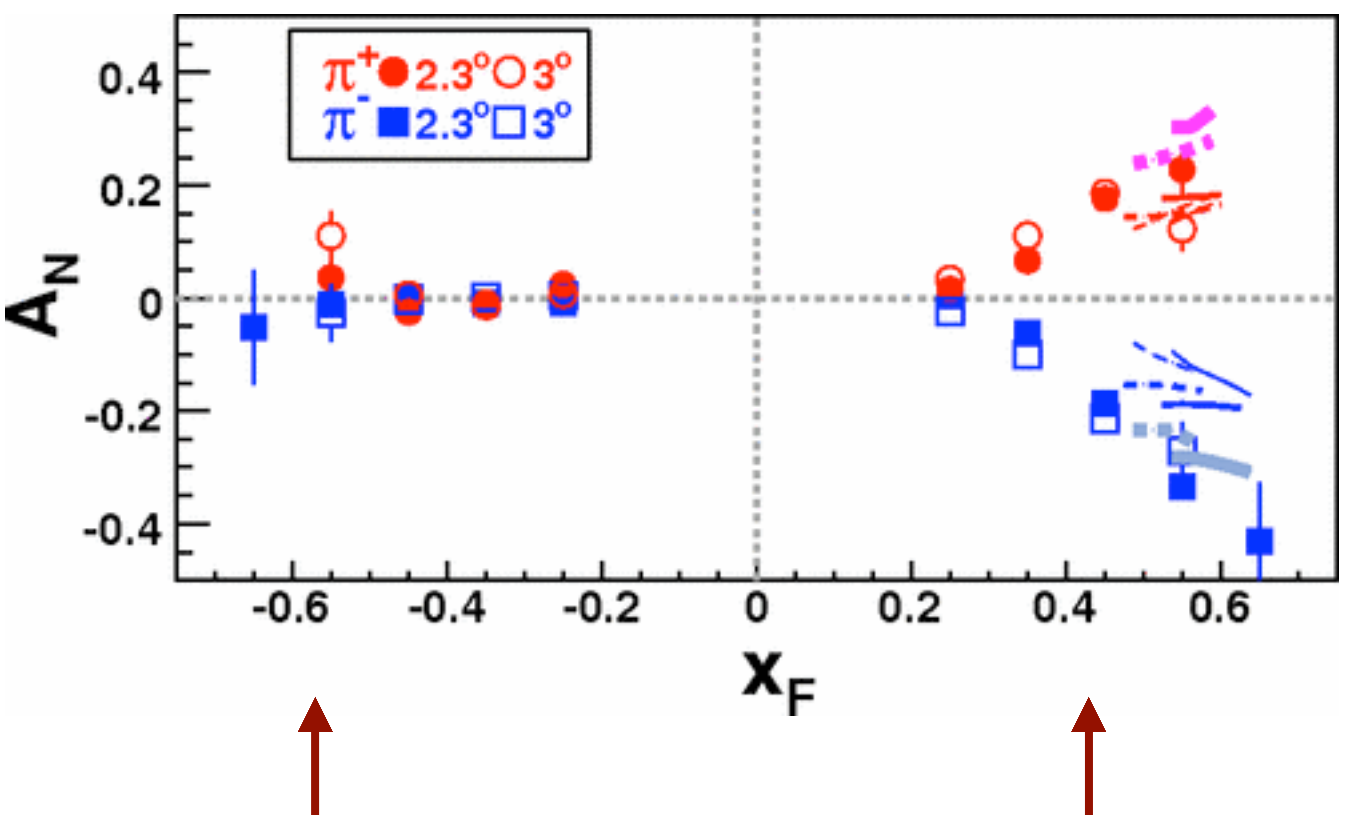
Dominguez, Marquet, Xiao, Yuan, 2011

$p^{\uparrow}p \to h^{\pm} X$ at $x_F < 0$

It is the only relevant contribution to A_N in backward ($x_F < 0$) charged hadron production in $p^{\uparrow}p$ or $p^{\uparrow}A$ (in contrast to the many contributions at $x_F > 0$)

As the odderon is C-odd, for gg-dominated scattering one should select final states that are not C-even, hence charged hadron production (as opposed to jets or π^0)

Backward charged hadron production at RHIC



BRAHMS, 2008 $\sqrt{s}$ = 62.4 GeV
low p_T, up to roughly 1.2 GeV
where gg channel dominates

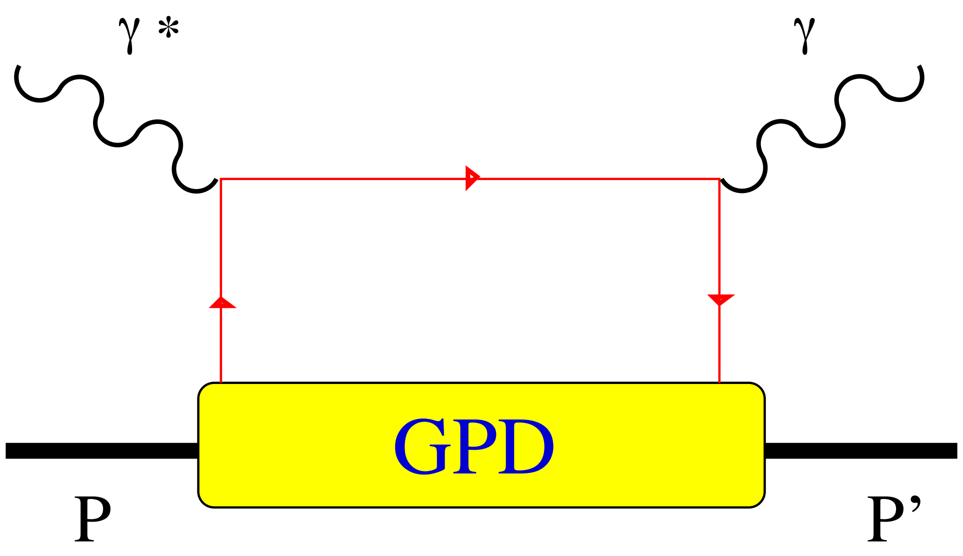
$$x_F = \frac{2p_z}{\sqrt{s}}$$

gluon dominated quark dominated

The asymmetry in the gluon dominated region is smaller and needs more precision

GPDs

Deeply Virtual Compton Scattering (DVCS):



Theoretical description involves Generalized Parton Distributions (GPDs)

GPDs are off-forward matrix elements (P' ≠ P)

This describes the spatial distribution of quarks inside nucleons

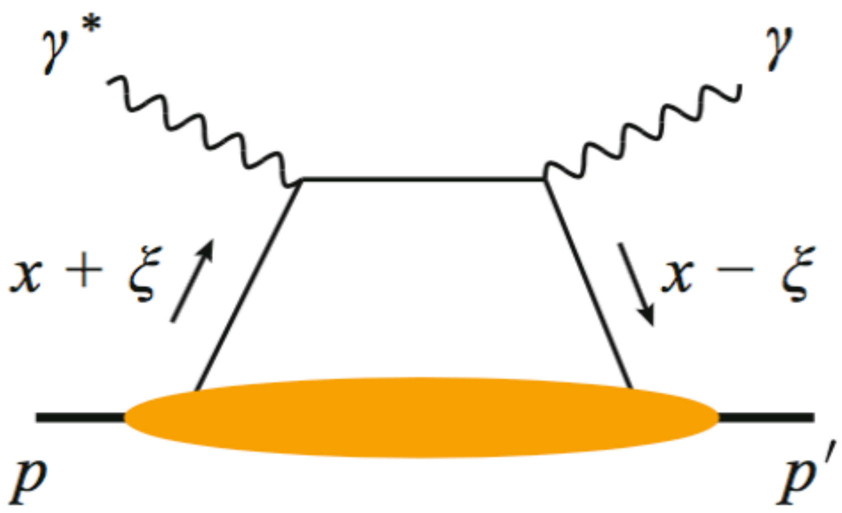
b_T is *not* the Fourier conjugate of k_T

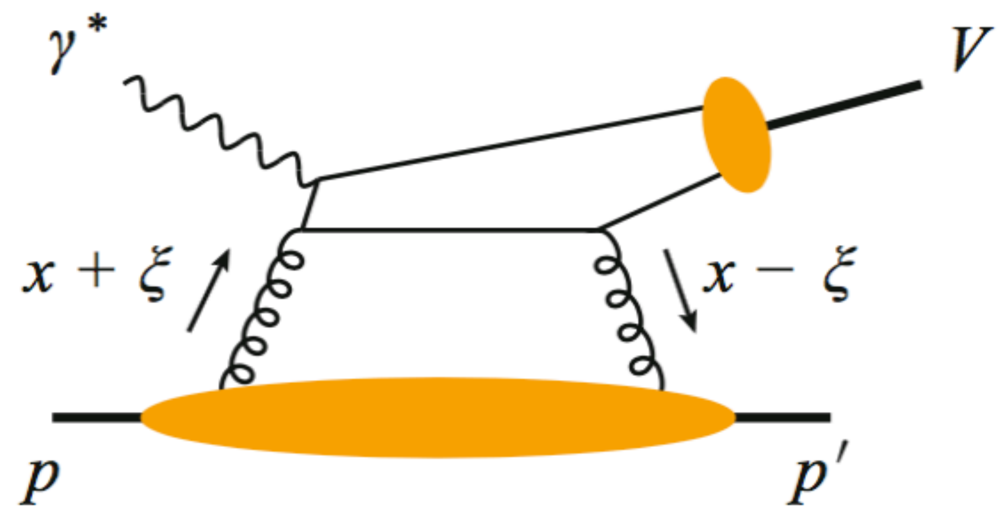
$b_\perp$ = transverse spatial distance w.r.t. the "center" of the proton

The transverse center of longitudinal momentum: $$\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{\perp i}$$

[Burkardt 2000; Soper 1977]

GPDs

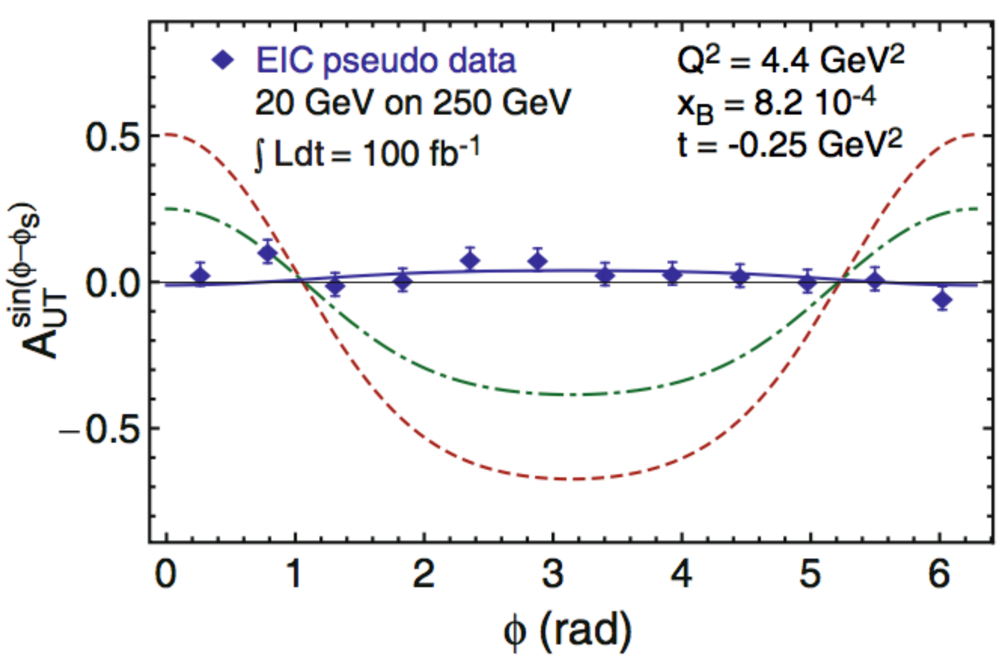


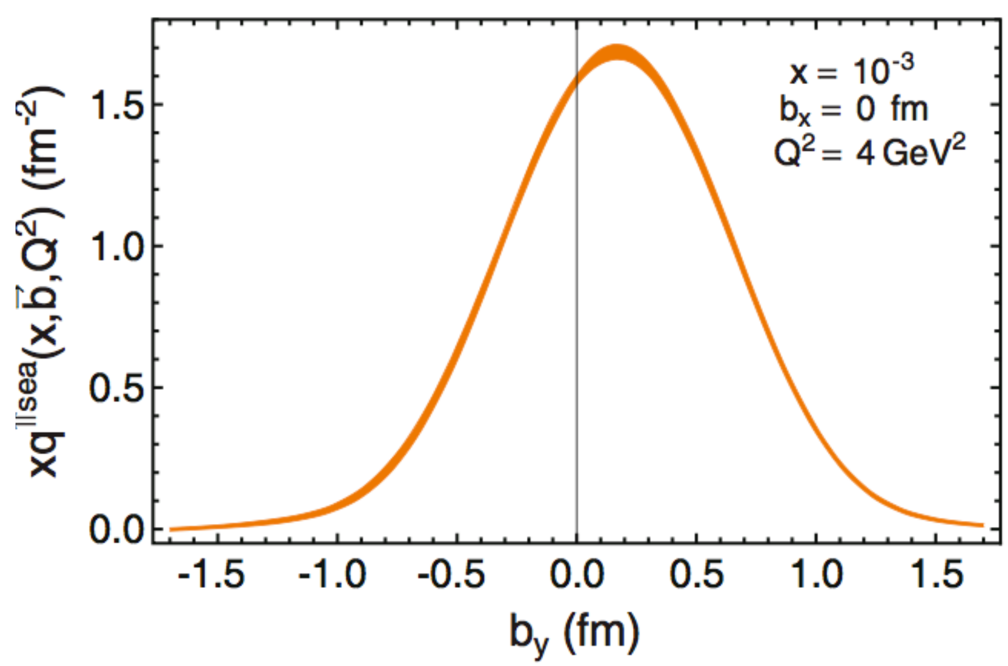


At EIC quark GPDs will be extracted in order to study quark OAM

$$J^q = \frac{1}{2}\int \mathrm{d}x\, x\, [H^q(x,\xi,t=0) + E^q(x,\xi,t=0)]$$

Sivers-like distortions ($b_T \times S_T$) give rise to transverse spin asymmetries





See Boer et al., arXiv:1108.1713; Accardi *et al.*, Understanding the glue that binds us all, EPJA (2016)