Measuring quark and gluon Orbital angular momentum: twist three GPDs

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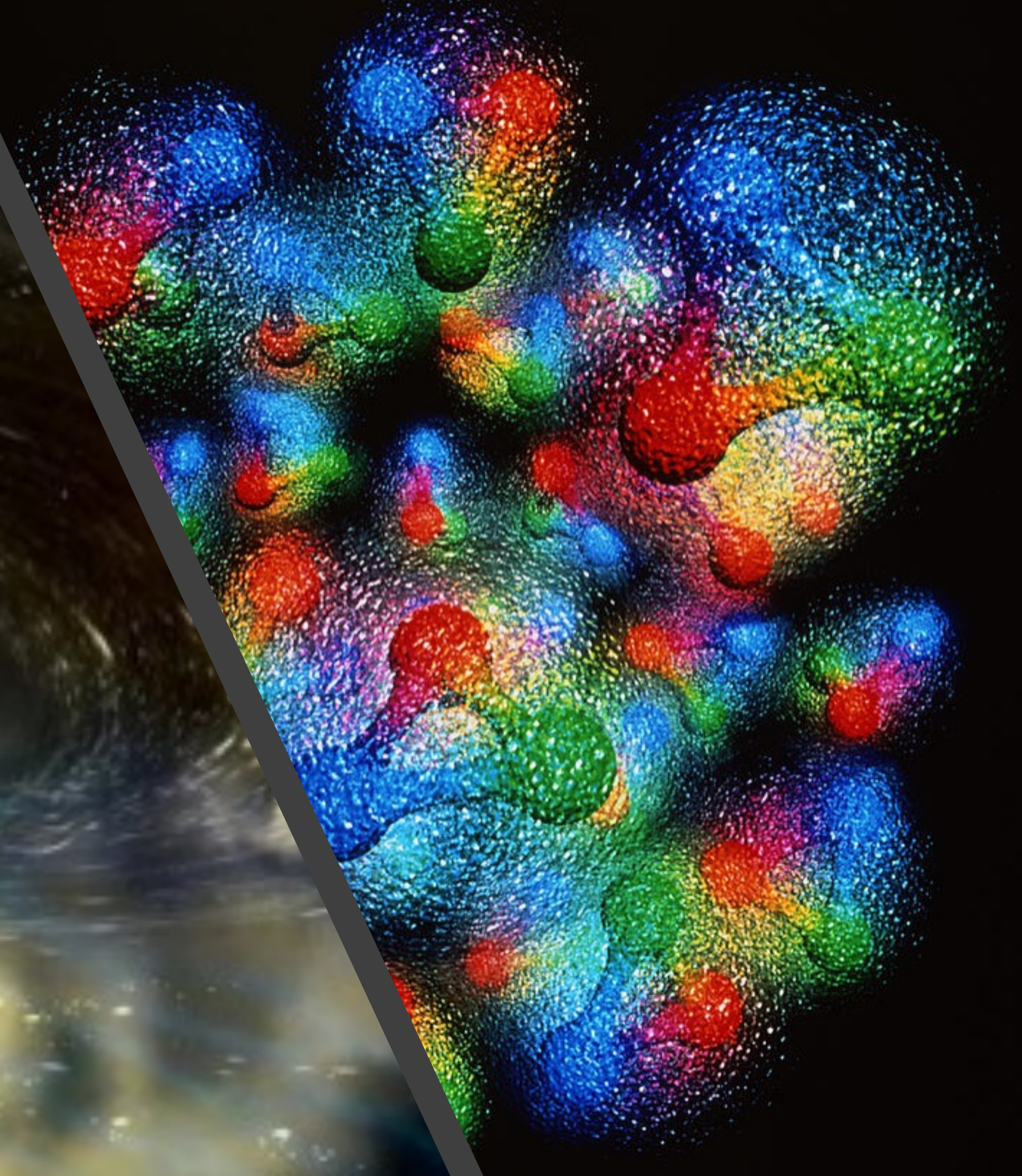
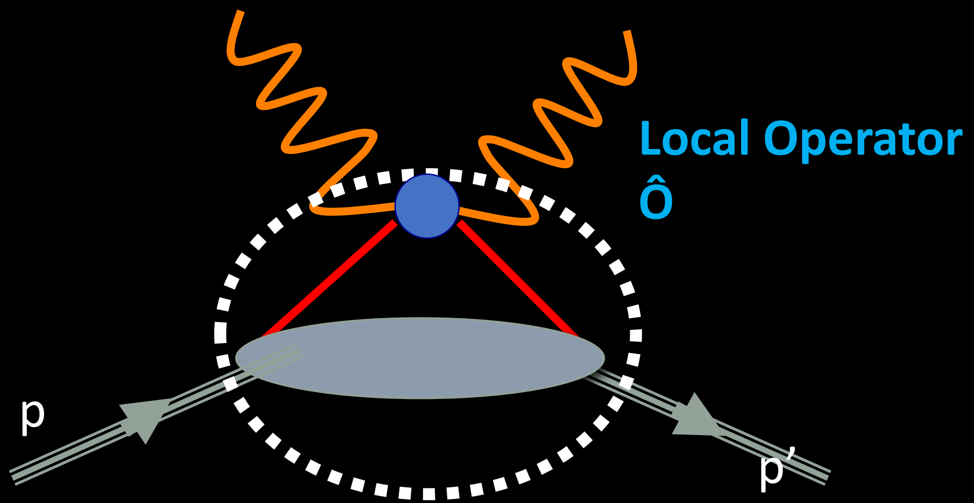
Transversity 2022
May 22-27, 2022



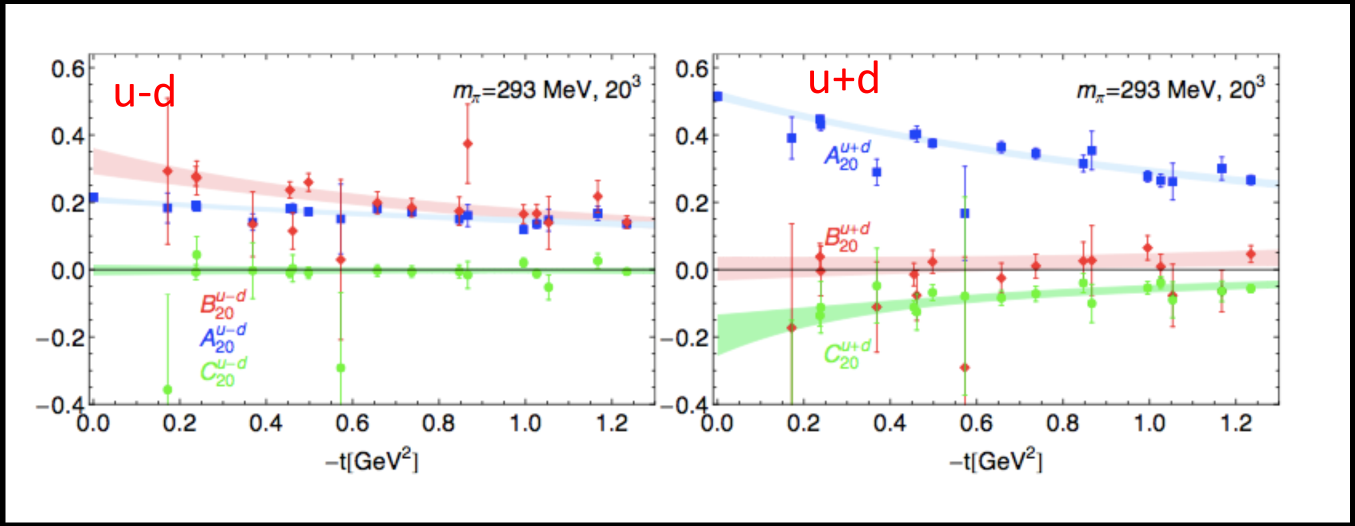
Quark longitudinal angular momentum: J_z^q

$$J_z^q = \int_0^1 dx\, x\left(H_q + E_q\right)$$

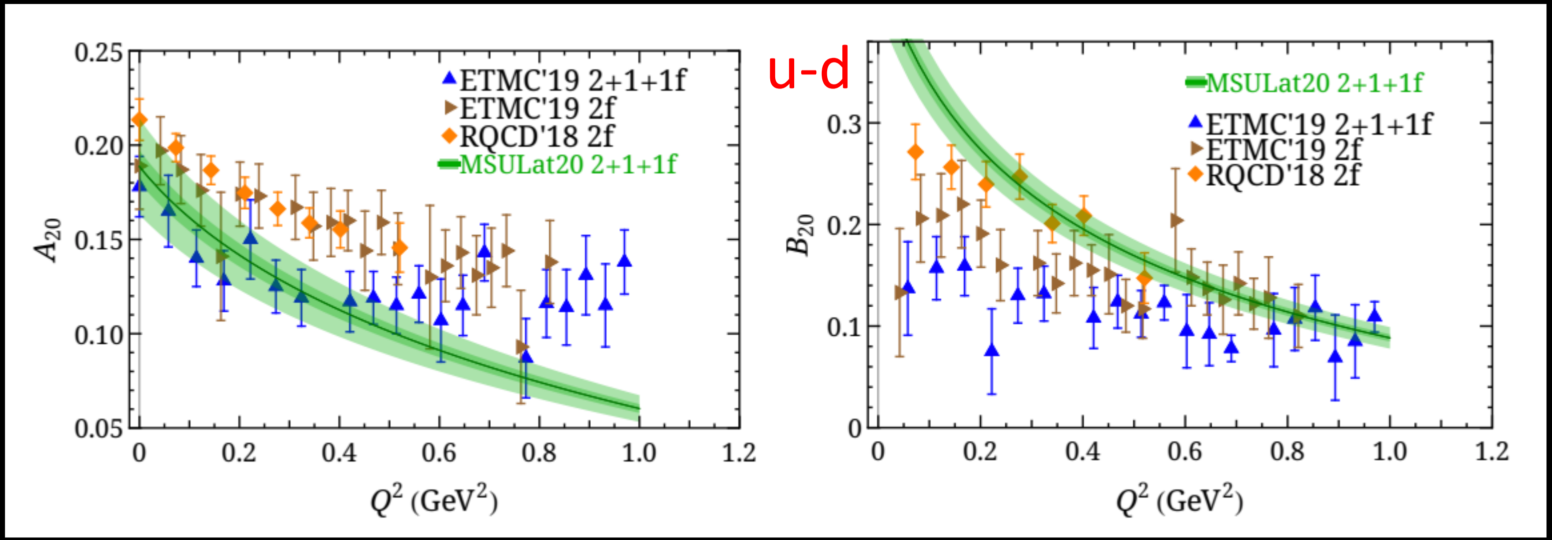


GPD Moments → EMT Form Factors

Calculable on lattice…



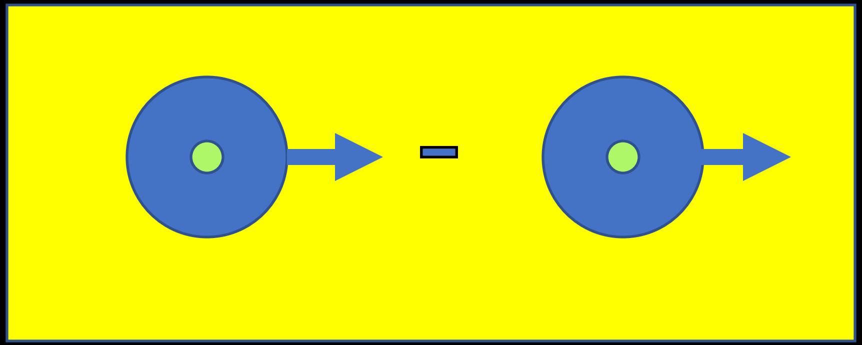
Ph. Haegler, JoP: **295** (2011) 012009



H-W Lin, *Phys.Rev.Lett.* 127 (2021)

Quark longitudinal orbital angular momentum: L_z^q

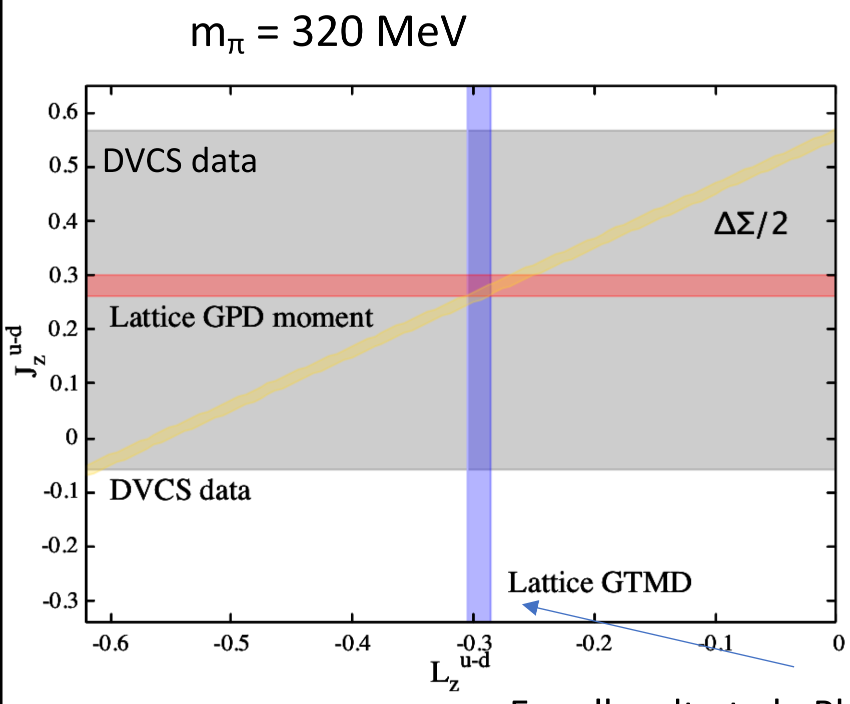
F_{14}



GTMD UL correlation

$$L_z^q = \int_0^1 dx \int d^2k_T \; k_T^2 F_{14}(x, 0,0, k_T)$$

What we know from measurements and lattice



Engelhardt et al., Phys. Rev. D102, (2020)

q=u-d

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

Do we need to measure a GTMD to learn about OAM?

The other way that OAM is known

Polyakov Kiptily(2004), Hatta(2012)

$$\int_0^1 dx\, x G_2 = -\frac{1}{2}\int_0^1 dx\, x(H+E) + \frac{1}{2}\int_0^1 dx\, \tilde{H}$$

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

A generalized integrated Wandzura Wilczek relation obtained using OPE for twist 2 and twist 3 operators for the off-forward matrix elements

Using QCD Equations of Motion and Lorentz Invariance Relations

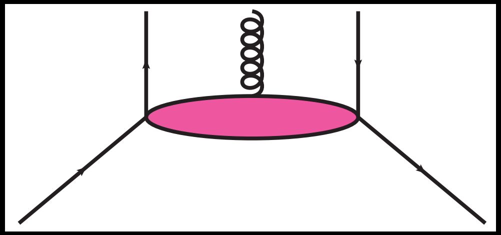
- A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
- A. Rajan, M. Engelhardt, S.L., PRD (2018)

$$
\begin{aligned}
J_L \quad &= \quad L_L \quad + \quad S_L \\
\frac{1}{2}\int dx\, x(H+E) &= \int dx\, x(\widetilde{E}_{2T} + H + E) + \frac{1}{2}\int dx\, \widetilde{H} \\
&= -\int dx\, F_{14}^{(1)} + \frac{1}{2}\int dx\, \widetilde{H}
\end{aligned}
$$

Wandzura Wilczek relation for OAM

Straight gauge link

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$



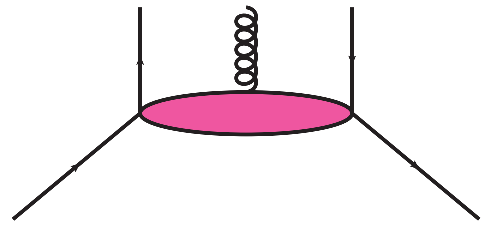
genuine twist 3

- A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
- A. Rajan, M. Engelhardt, S.L., PRD (2018)

Wandzura Wilczek relation for OAM

Staple link

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{F_{14}}$$



LIR violating term

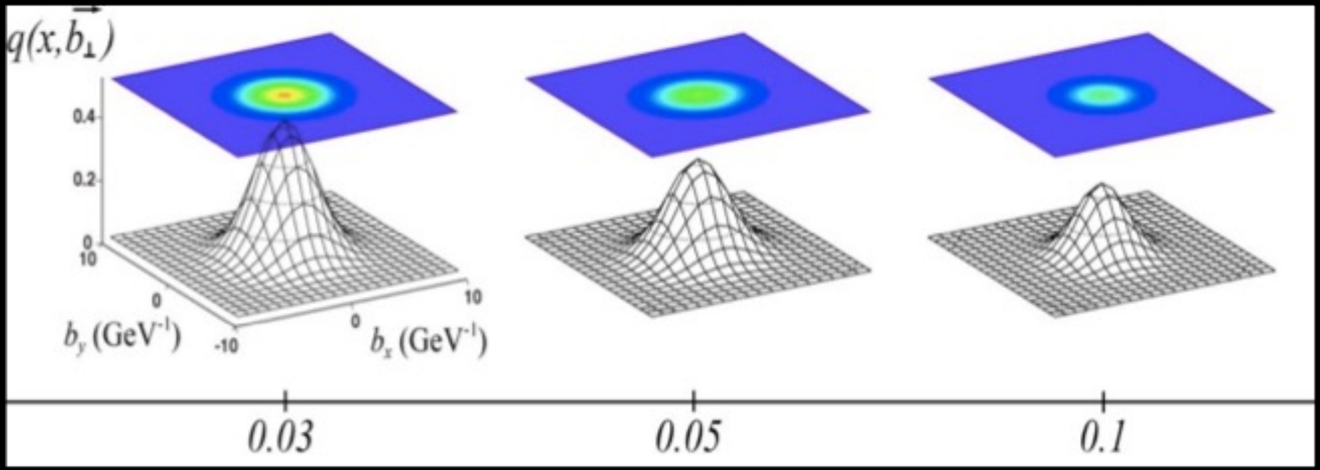
- A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
- A. Rajan, M. Engelhardt, S.L., PRD (2018)

Transverse Angular Momentum Sum Rule

O. Alkassasbeh, M. Engelhardt, SL and A. Rajan, soon on arXiv

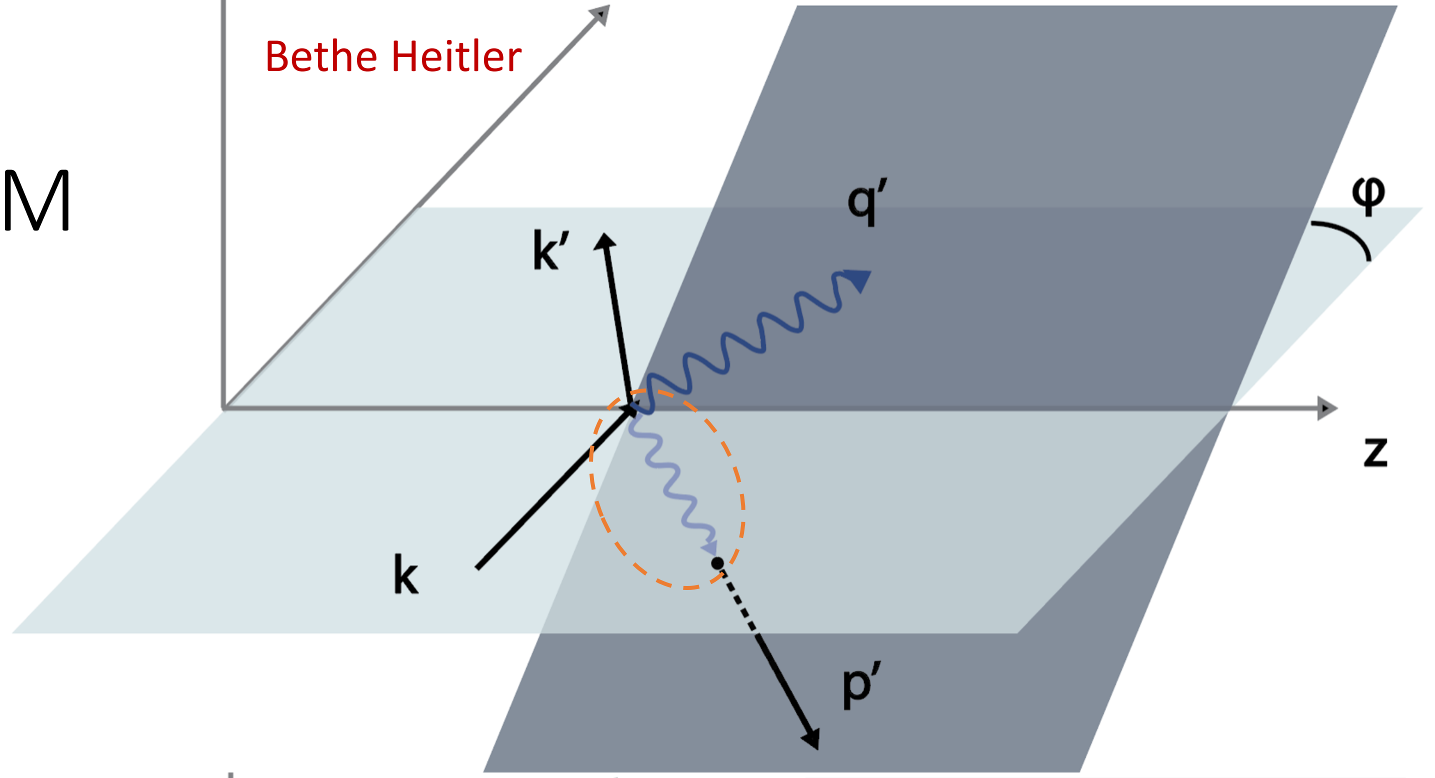
$$\underbrace{\frac{1}{2}\int dx\,x(H+E) - \frac{1}{2}\int dx\,\mathcal{M}_T}_{J_T} = \underbrace{\int dx\,x\left(\widetilde{E}_{2T} + H + E + \frac{H_{2T}}{\xi}\right)}_{L_T} + \underbrace{\frac{1}{2}\int dx\,g_T - \frac{1}{2}\int dx\,x\mathcal{A}_T}_{S_T}$$

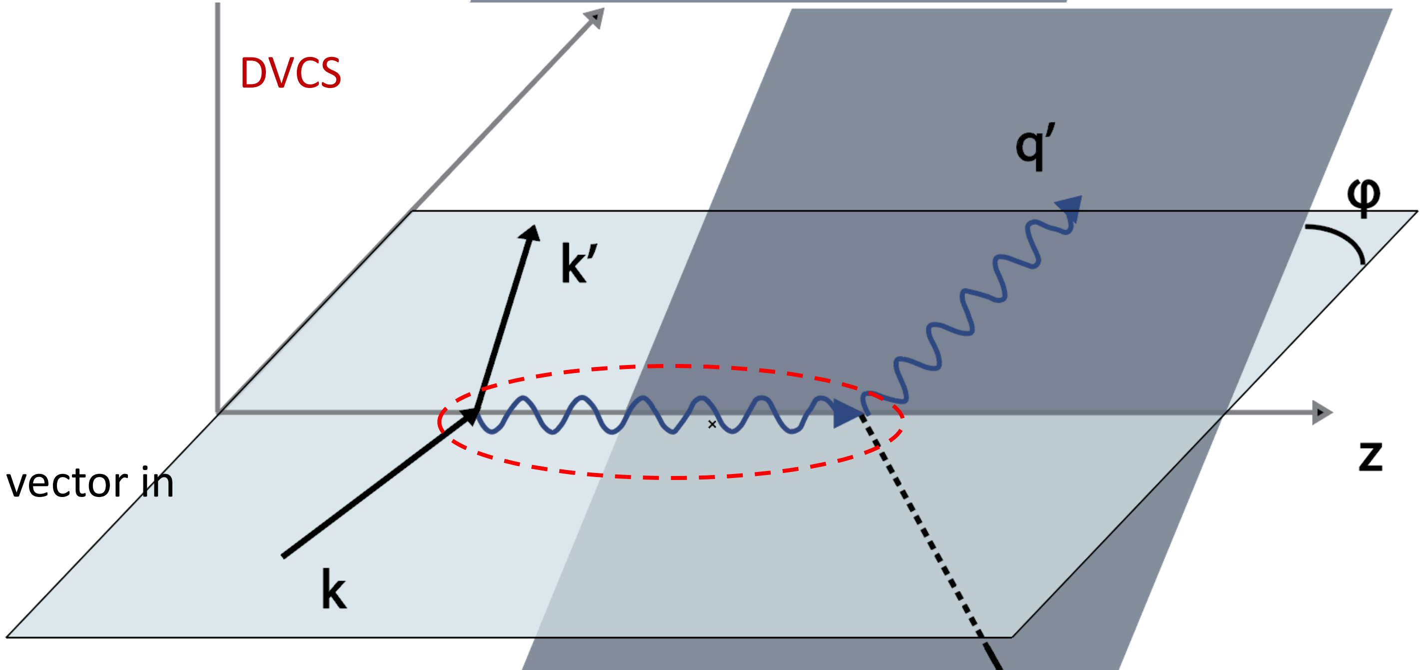
Measuring Angular Momentum



graph from M. Defurne

Demystification of harmonics formalism: BKM missed a phase

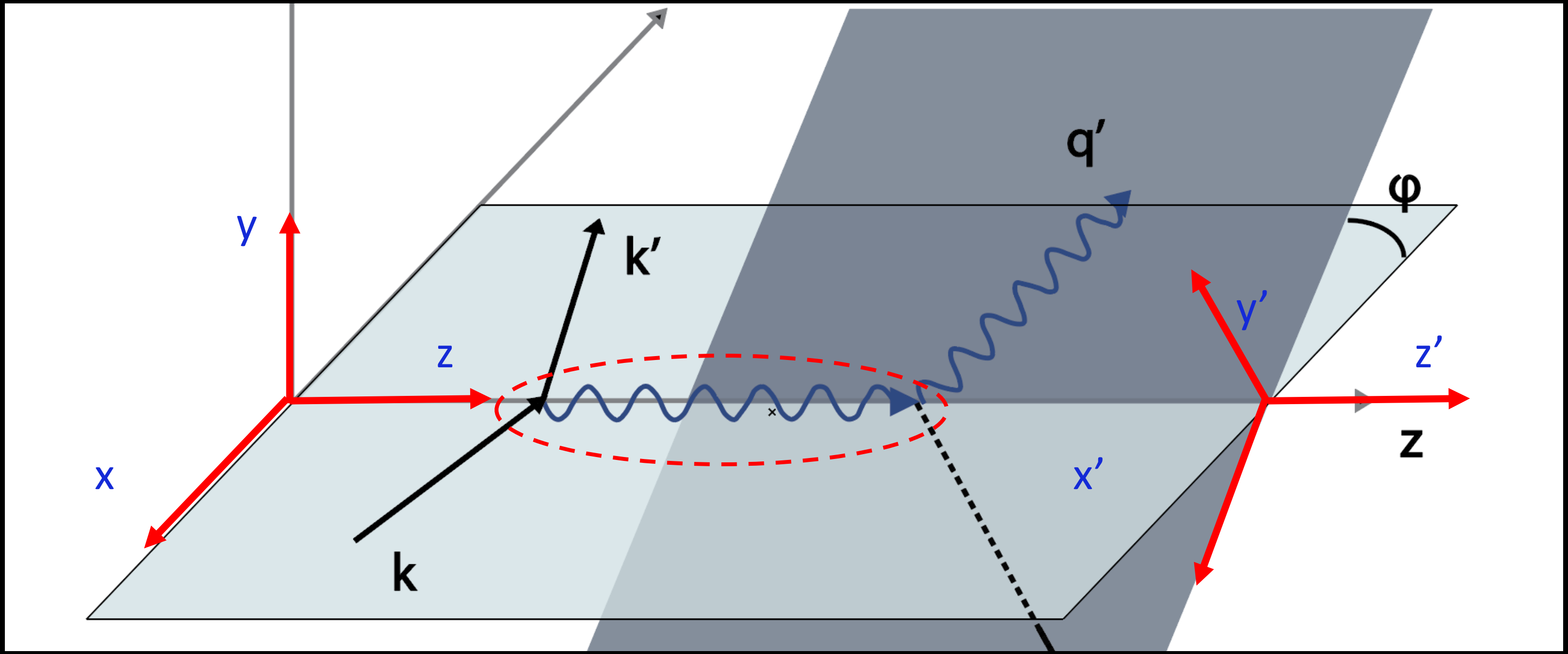




- In DVCS the virtual photon is along the z axis: φ dependence from usual rotation of polarization vector in helicity amp

To understand the cross section we need to understand the ϕ dependence

DVCS



The hadronic tensor is evaluated in the rotated frame

BH

$$\frac{d^5\sigma^{BH}_{unpol}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} \equiv \frac{\Gamma}{t}F^{BH}_{UU} = \frac{\Gamma}{t}\left[A(y,x_{Bj},t,Q^2,\phi)\left(F_1^2+\tau F_2^2\right) + B(y,x_{Bj},t,Q^2,\phi)\tau G_M^2(t)\right]$$

$$A = \frac{16\,M^2}{t(k\,q')(k'\,q')}\left[4\tau\left((k\,P)^2+(k'\,P)^2\right) - (\tau+1)\left((k\,\Delta)^2+(k'\,\Delta)^2\right)\right]$$

$$B = \frac{32\,M^2}{t(k\,q')(k'\,q')}\left[(k\,\Delta)^2+(k'\,\Delta)^2\right],$$

$$\epsilon_{BH} = \left(1+\frac{B}{A}(1+\tau)\right)^{-1}$$

…compared to ELASTIC SCATTERING

10/21/21

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{\epsilon (G_E^N)^2 + \tau (G_M^N)^2}{\epsilon (1+\tau)},$$

where $N = p$ for a proton and $N = n$ for a neutron, (
the recoil-corrected relativistic point-particle (Mott)
and τ, ϵ are dimensionless kinematic variables:

$$\tau = \frac{Q^2}{4m_N^2}, \quad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1},$$

J. Arrington, G. Cates, S. Riordan, Z. Ye, B. Wojsetowski, A. Puckett …

…compared to BKM, NPB (2001)

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6}{x_{\rm B}^2 y^2 (1+\epsilon^2)^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \times \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) + s_1^{\rm BH} \sin(\phi) \right\},$$

$$\begin{aligned} c_{0,\rm unp}^{\rm BH} = 8K^2 \Big\{ & (2+3\epsilon^2)\frac{Q^2}{\Delta^2}\left(F_1^2 - \frac{\Delta^2}{4M^2}F_2^2\right) + 2x_{\rm B}^2 (F_1+F_2)^2 \Big\} \\ & + (2-y)^2 \Big\{ (2+\epsilon^2)\left[\frac{4x_{\rm B}^2 M^2}{\Delta^2}\left(1+\frac{\Delta^2}{Q^2}\right)^2 \right. \\ & \left. + 4(1-x_{\rm B})\left(1+x_{\rm B}\frac{\Delta^2}{Q^2}\right)\right]\left(F_1^2 - \frac{\Delta^2}{4M^2}F_2^2\right) \\ & + 4x_{\rm B}^2\left[x_{\rm B} + \left(1-x_{\rm B}+\frac{\epsilon^2}{2}\right)\left(1-\frac{\Delta^2}{Q^2}\right)^2 \right. \\ & \left. - x_{\rm B}(1-2x_{\rm B})\frac{\Delta^4}{Q^4}\right](F_1+F_2)^2 \Big\} \\ & + 8\left(1+\epsilon^2\right)\left(1-y-\frac{\epsilon^2 y^2}{4}\right) \\ & \times \left\{ 2\epsilon^2\left(1-\frac{\Delta^2}{4M^2}\right)\left(F_1^2 - \frac{\Delta^2}{4M^2}F_2^2\right) - x_{\rm B}^2\left(1-\frac{\Delta^2}{Q^2}\right)^2 (F_1+F_2)^2 \right\}, \end{aligned}$$

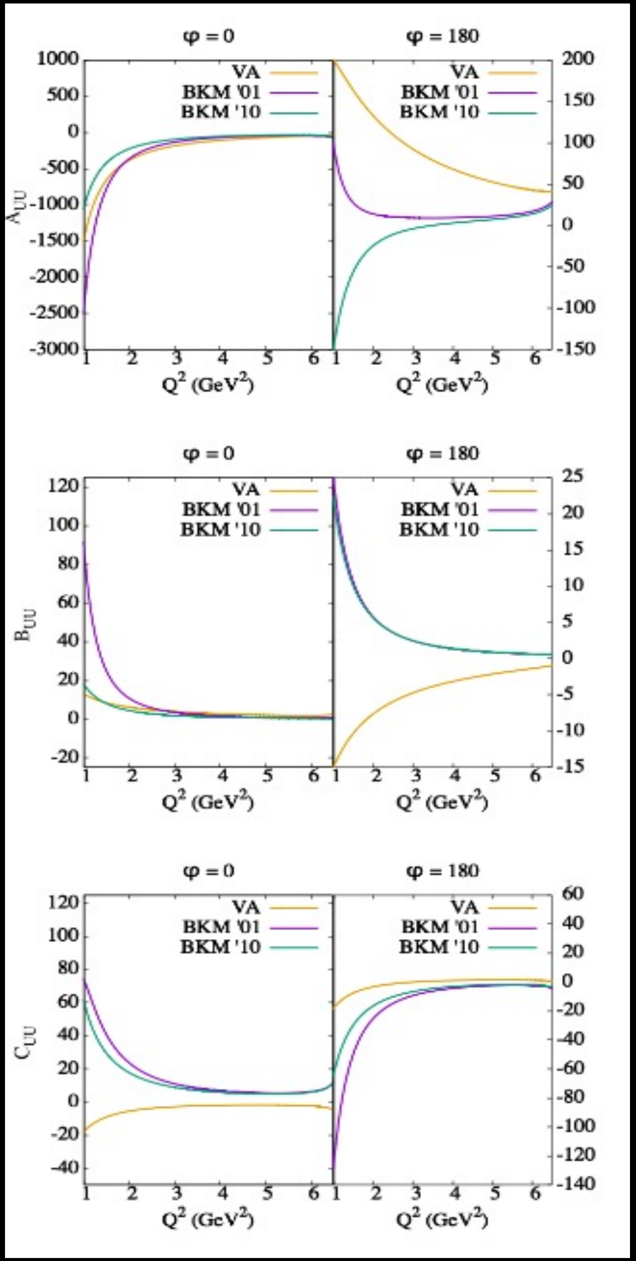
$$\begin{aligned} c_{1,\rm unp}^{\rm BH} = 8K(2-y)\Big\{ & \left(\frac{4x_{\rm B}^2 M^2}{\Delta^2} - 2x_{\rm B} - \epsilon^2\right)\left(F_1^2 - \frac{\Delta^2}{4M^2}F_2^2\right) \\ & + 2x_{\rm B}^2\left(1-(1-2x_{\rm B})\frac{\Delta^2}{Q^2}\right)(F_1+F_2)^2 \Big\}, \end{aligned}$$

$$c_{2,\rm unp}^{\rm BH} = 8x_{\rm B}^2 K^2 \left\{ \frac{4M^2}{\Delta^2}\left(F_1^2 - \frac{\Delta^2}{4M^2}F_2^2\right) + 2(F_1+F_2)^2 \right\}.$$

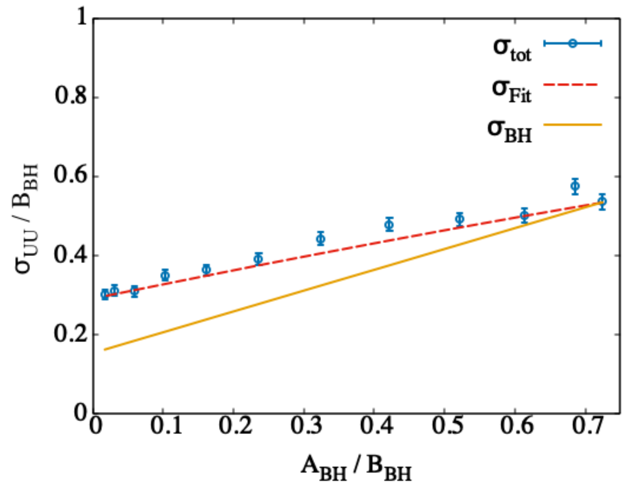
BH-DVCS interference

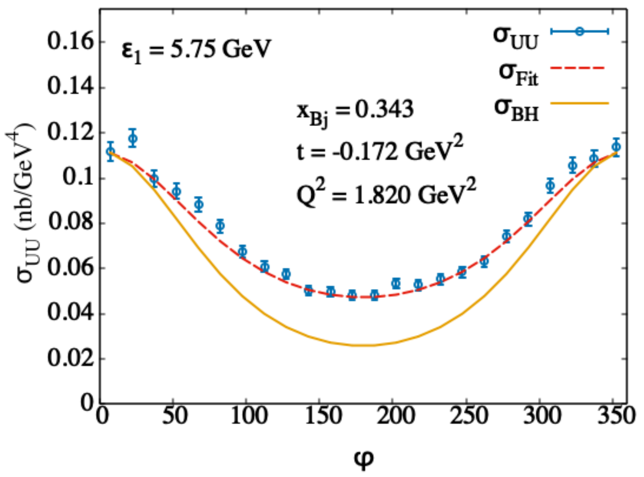
$$F_{UU}^{\mathcal{I},tw2} = A_{UU}^{\mathcal{I}} \Re e \left(F_1 \mathcal{H} + \tau F_2 \mathcal{E}\right) + B_{UU}^{\mathcal{I}} G_M \Re e \left(\mathcal{H} + \mathcal{E}\right) + C_{UU}^{\mathcal{I}} G_M \Re e \widetilde{\mathcal{H}}$$

A_{UU}^{I} B_{UU}^{I} C_{UU}^{I} are φ dependent coefficients

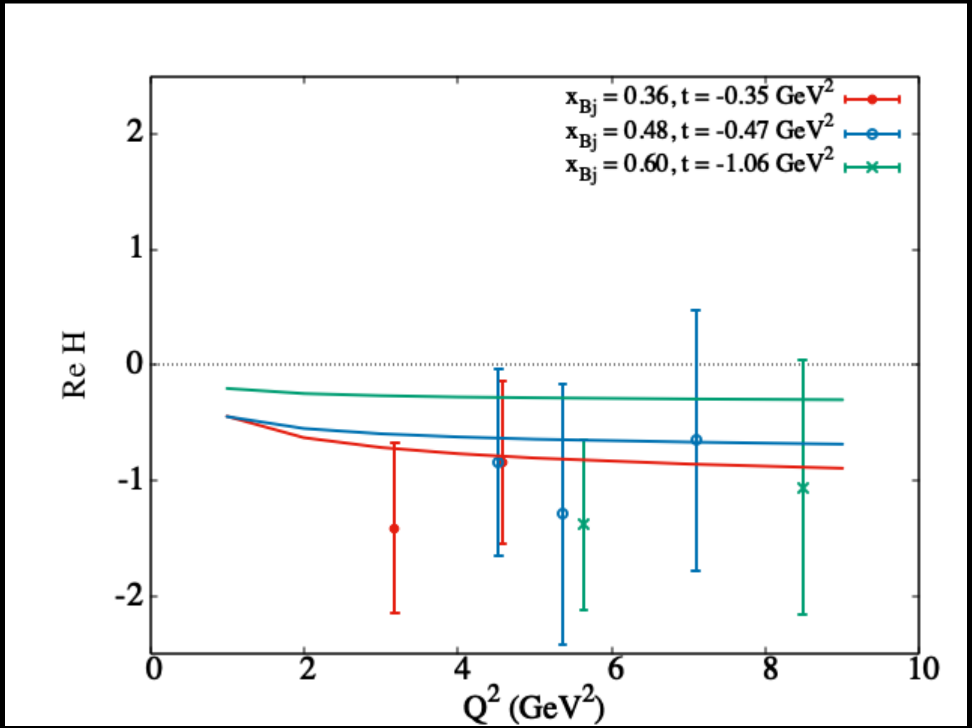


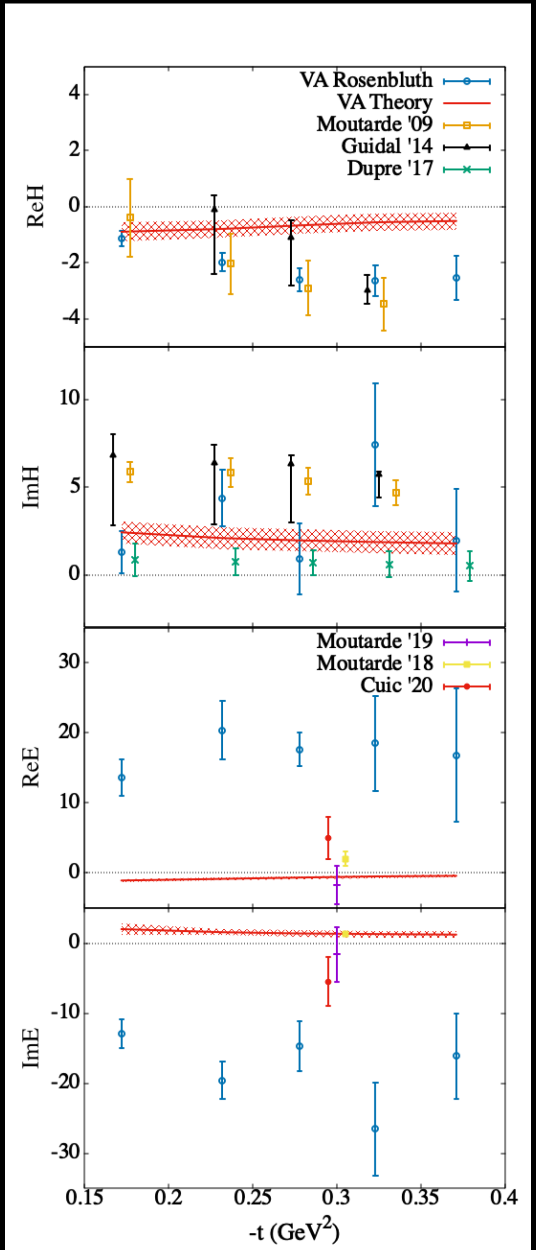
- Rosenbluth Separated BH-DVCS interference data





Compton Form Factor Extraction





Q^2 dependence

Twist 3 BH-DVCS interference

$$F_{UU}^{\mathcal{I}} = F_{UU}^{\mathcal{I},tw2} + \frac{K}{\sqrt{Q^2}} F_{UU}^{\mathcal{I},tw3}$$

$$F_{UU}^{\mathcal{I},tw3} = A_{UU}^{(3)\mathcal{I}} \left[F_1 \left(\Re e(2\widetilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T}) - \Re e(2\widetilde{\mathcal{H}}'_{2T} + \mathcal{E}'_{2T}) \right) + F_2 \left(\Re e(\mathcal{H}_{2T} + \tau \widetilde{\mathcal{H}}_{2T}) - \Re e(\mathcal{H}'_{2T} + \tau \widetilde{\mathcal{H}}'_{2T}) \right) \right]$$
$$+ B_{UU}^{(3)\mathcal{I}} G_M \left(\Re e \widetilde{\mathcal{E}}_{2T} - \Re e \widetilde{\mathcal{E}}'_{2T} \right)$$
$$+ C_{UU}^{(3)\mathcal{I}} G_M \left[2\xi (\Re e \mathcal{H}_{2T} - \Re e \mathcal{H}'_{2T}) - \tau \left(\Re e(\widetilde{\mathcal{E}}_{2T} - \xi \mathcal{E}_{2T}) - \Re e\left(\widetilde{\mathcal{E}}'_{2T} - \xi \mathcal{E}'_{2T}\right) \right) \right]$$

Orbital Angular Momentum

Twist 3 GPDs Physical Interpretation

GPD	$P_q P_p$	TMD	Ref. 1
$H^{\perp}$	UU	$f^{\perp}$	$2\widetilde{H}_{2T} + E_{2T}$
$\widetilde{H}_L^{\perp}$	LL	$g_L^{\perp}$	$2\widetilde{H}'_{2T} + E'_{2T}$
$H_L^{\perp}$	UL	$f_L^{\perp\,(*)}$	$\widetilde{E}_{2T} - \xi E_{2T}$
$\widetilde{H}^{\perp}$	LU	$g^{\perp\,(*)}$	$\widetilde{E}'_{2T} - \xi E'_{2T}$
$H_T^{(3)}$	UT	$f_T^{(*)}$	$H_{2T} + \tau \widetilde{H}_{2T}$
$\widetilde{H}_T^{(3)}$	LT	g'_T	$H'_{2T} + \tau \widetilde{H}'_{2T}$

→ 1/Q correction to H

→ 1/Q correction to $\widetilde{\mathrm{H}}$

NEW!! Orbital Angular Momentum L

NEW!! Spin Orbit correlation L ·S

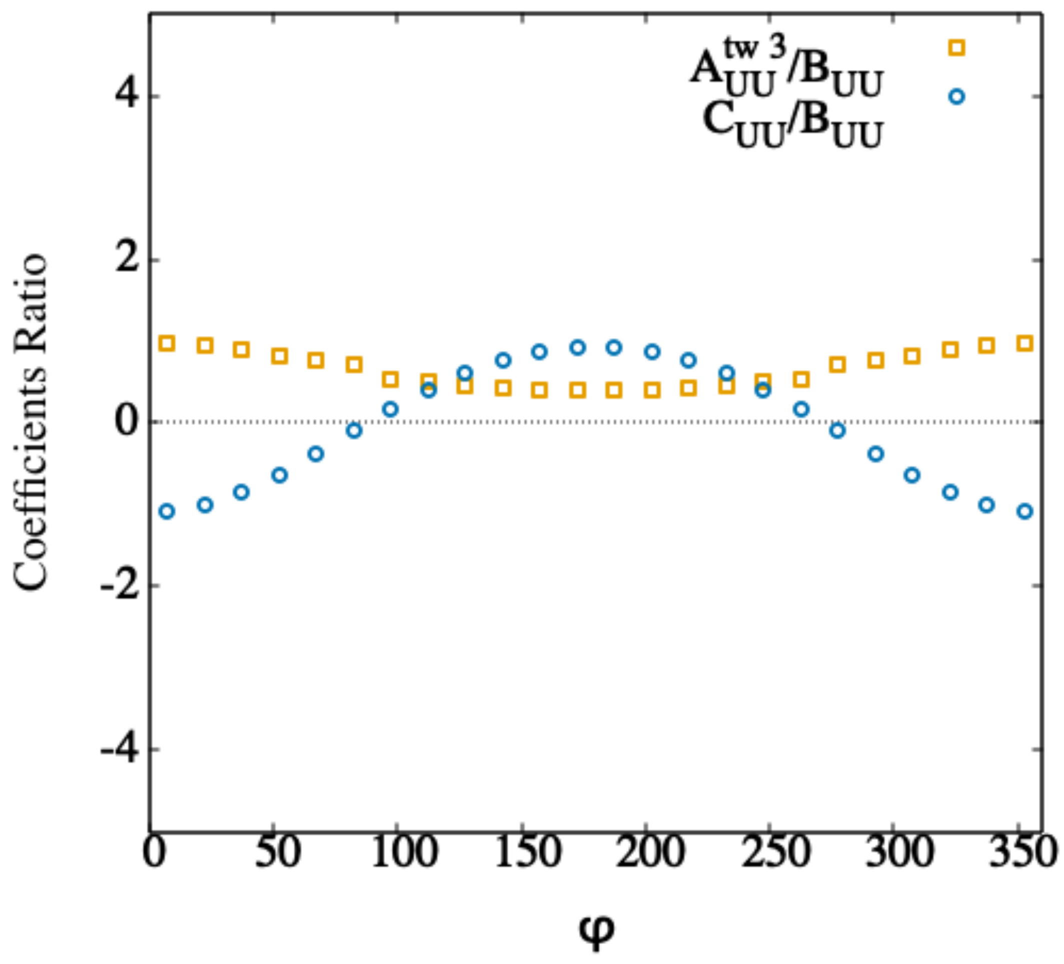
→ 1/Q correction to E

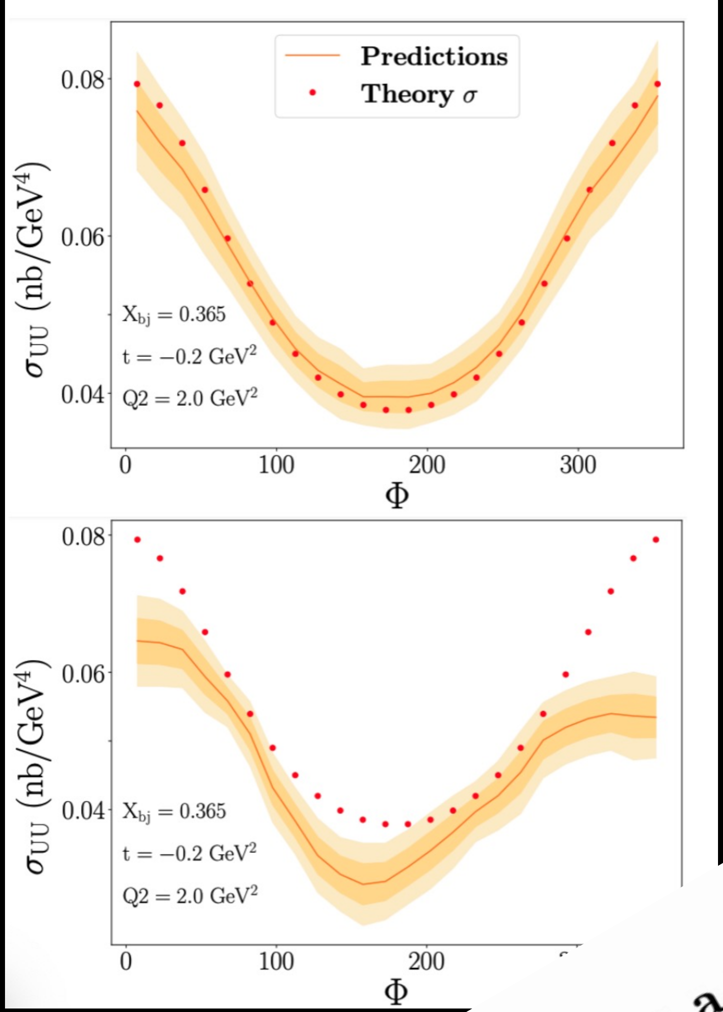
→ 1/Q correction to $\widetilde{\mathrm{E}}$

(*) T-odd

[1] Meissner, Metz and Schlegel, JHEP(2009)

Twist 3 seems small



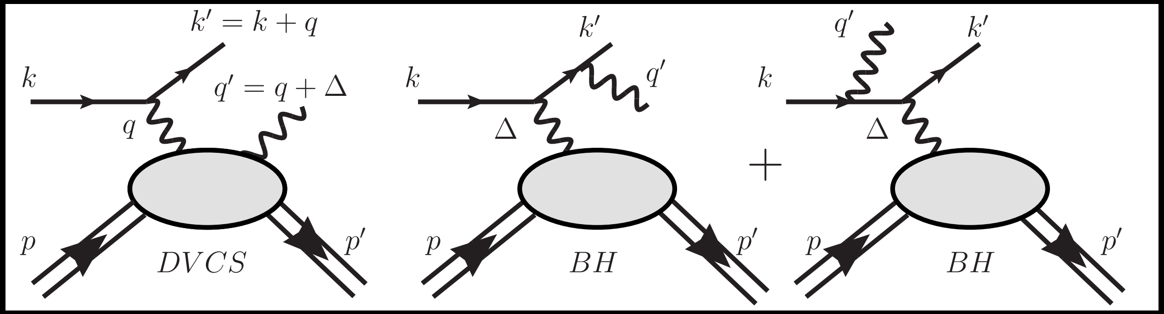


Benchmarks for a Global Extraction of Information from Deeply Virtual Exclusive Scattering Experiments

Manal Almaeen,[1,*] Jake Grigsby,[2,†] Joshua Hoskins,[3,‡] Brandon Kriesten,[4,§] Yaohang Li,[1,¶] Huey-Wen Lin,[5,6,**] and Simonetta Liuti[3,††]

… to appear soon…

We need a robust framework for DVES processes cross section, where kinematic limits are under control



- To observe, evaluate and interpret GPDs and Wigner distributions at the subatomic level requires stepping up data analyses from the standard methods and developing new numerical/analytic/quantum computing methods

DVCS formalism

- B. Kriesten et al, *Phys.Rev. D* 101 (2020)
- B. Kriesten and S. Liuti, *Phys.Rev. D105 (2022),* arXiv 2004.08890
- B. Kriesten and S. Liuti, Phys. Lett. B829 (2022), arXiv:2011.04484

ML

- J. Grigsby, B. Kriesten, J. Hoskins, S. Liuti, P. Alonzi and M. Burkardt, Phys. Rev. D104 (2021)

GPD Parametrization for global analysis

- B. Kriesten, P. Velie, E. Yeats, F. Y. Lopez and S. Liuti,

Phys. Rev D 105 (2022), arXiv:2101.01826