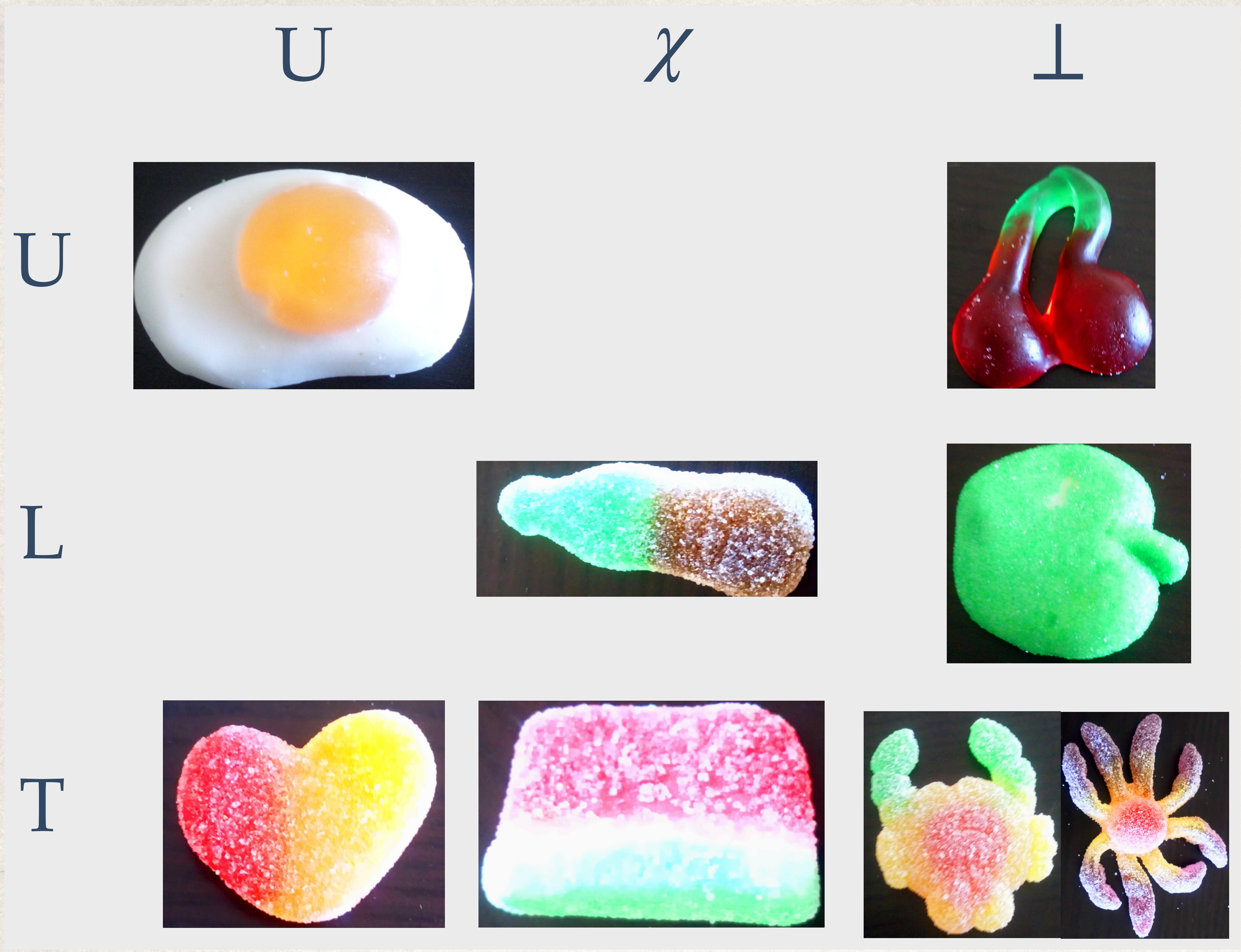


TMDs and jets's observables

Ignazio Scimemi

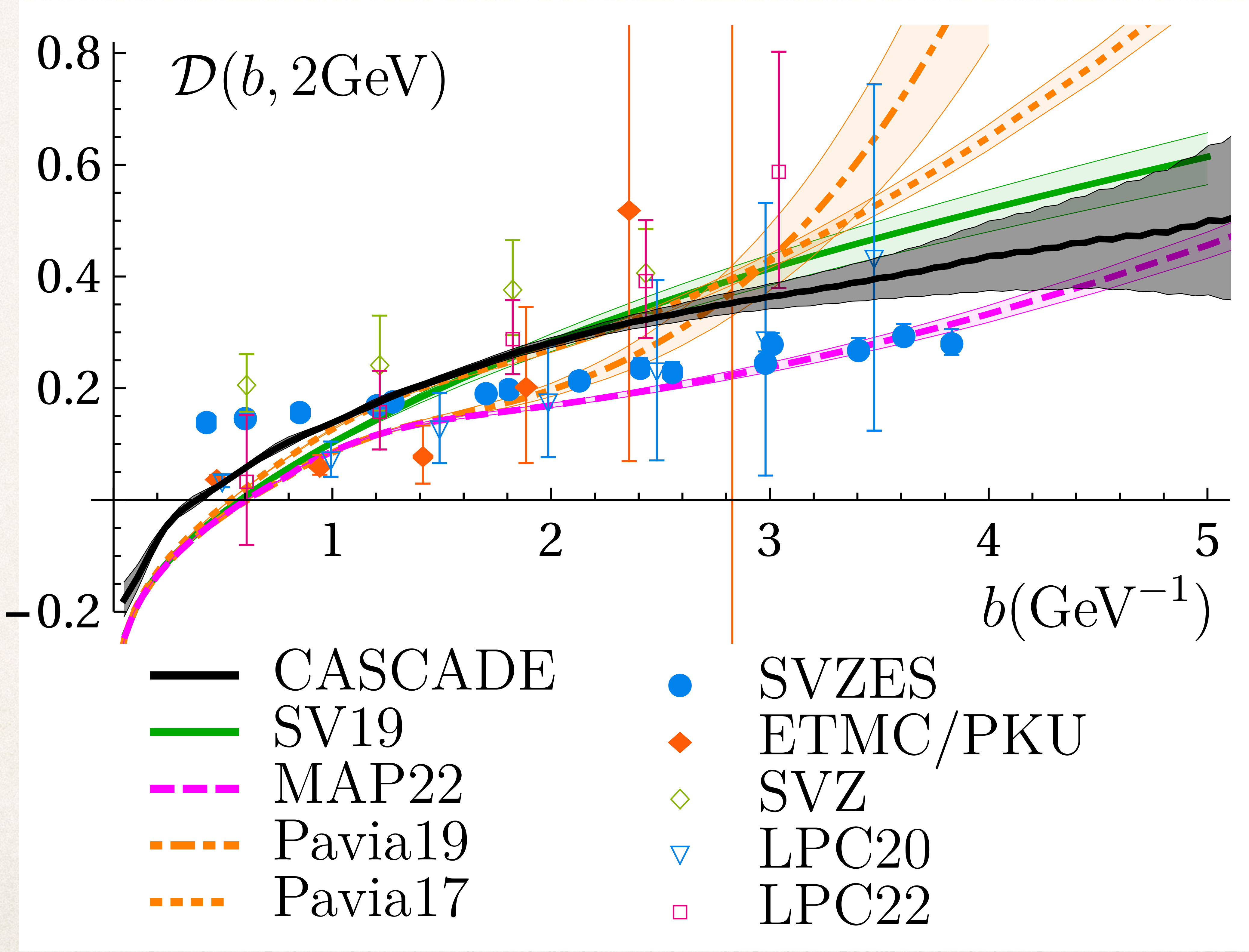
23-27/05/2022 Transversity, Pavia

Why jets

- Jets are constructed as IR-safe final state distributions: The jet perturbative expansion is highly convergent
- Several important nuances can be considered to improve convergence or phenomenological analysis (Standard Axis, WTA, grooming, thrust..)
- One can use jets in combination with hadronic distributions (hadrons inside jets, hadron + jets, vector boson + jets,..)
- It is possible to encounter new hadronic distributions that have never been classified

The TMD evolution kernel and jets

The TMD evolution is per-se a relevant hadronic distribution that can be extracted from data, lattice and also appears in factorization theorem with jets:



$$R(b, Q, \mu) = \left(\frac{Q^2}{\zeta_\mu[\mathcal{D}(b,\mu)]}\right)^{-2\mathcal{D}(b,\mu)}$$

$$\mathcal{D}(b,\mu) = \mathcal{D}_{\text{pert}}(b,\mu) + d_{\text{NP}}(b)$$

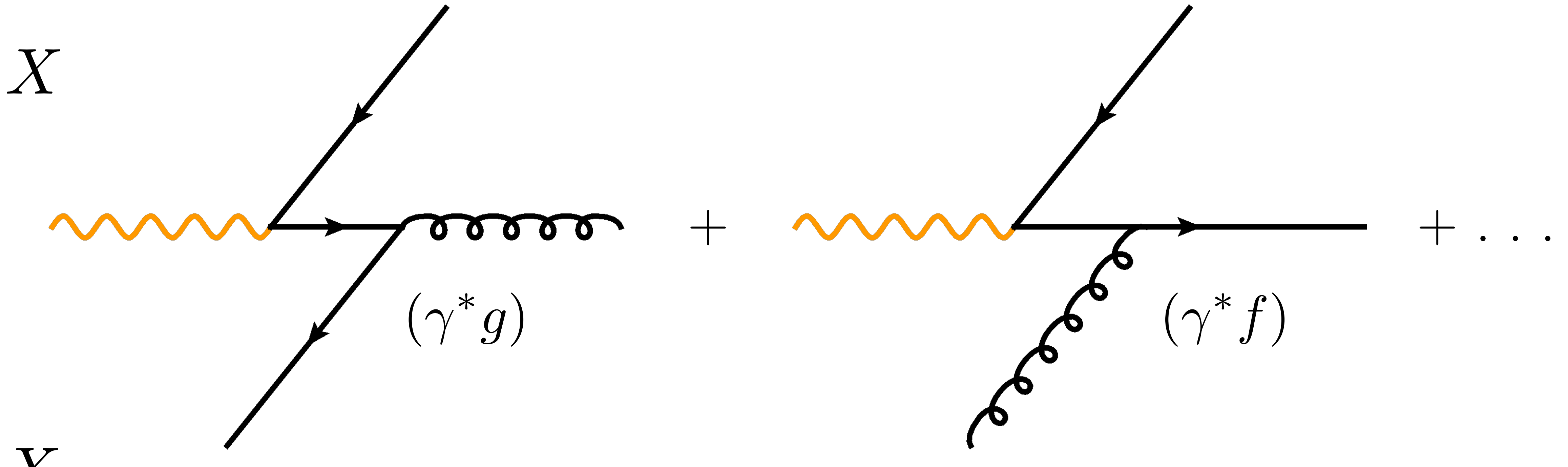
How many jets?

- At EIC/Belle there is probably an energy limitation of 2 jets for a single process …
- At LHC one can achieve processes with many more jets
- Can one build a factorization theorem for differential distributions for any number of jets (when some energy scale becomes very small?)?

The di-jet (DJ) and heavy hadron pair (HHP)

$\ell + h \to \ell' + J_1 + J_2 + X$

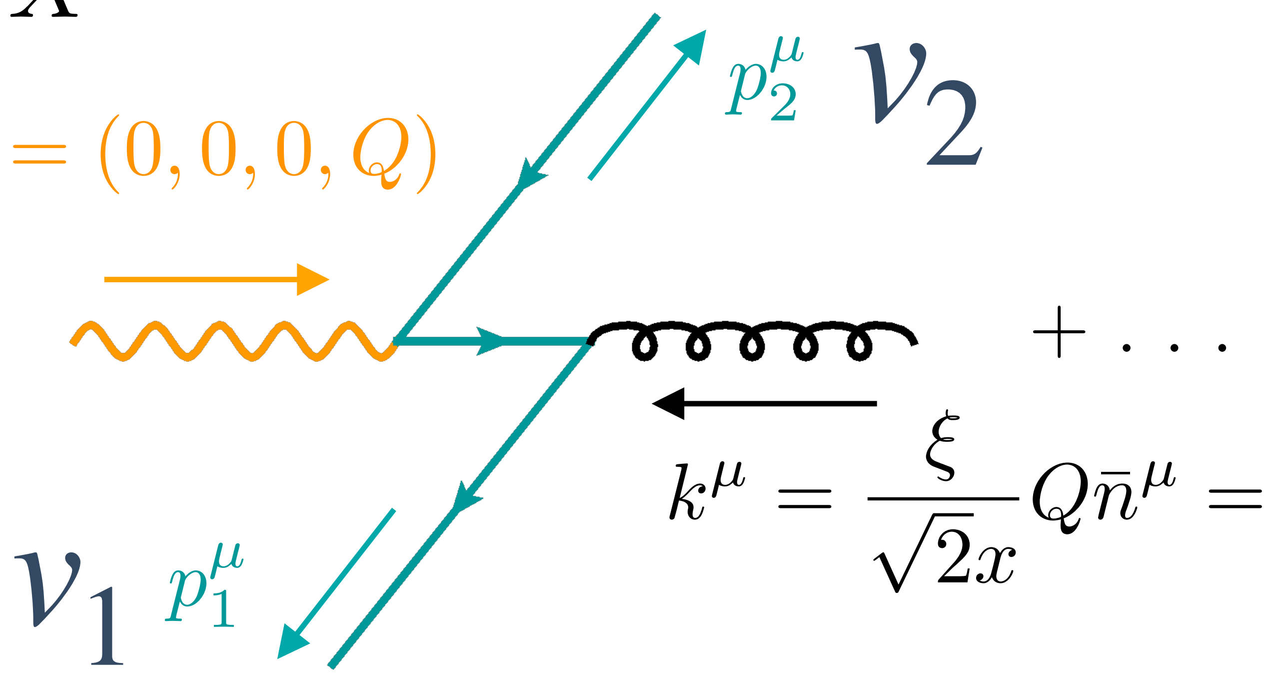
dijet LO process:



$\ell + h \to \ell' + H + \bar{H} + X$

$q^\mu = \frac{Q}{\sqrt{2}}(n^\mu - \bar{n}^\mu) = (0,0,0,Q)$

heavy meson pair at LO:

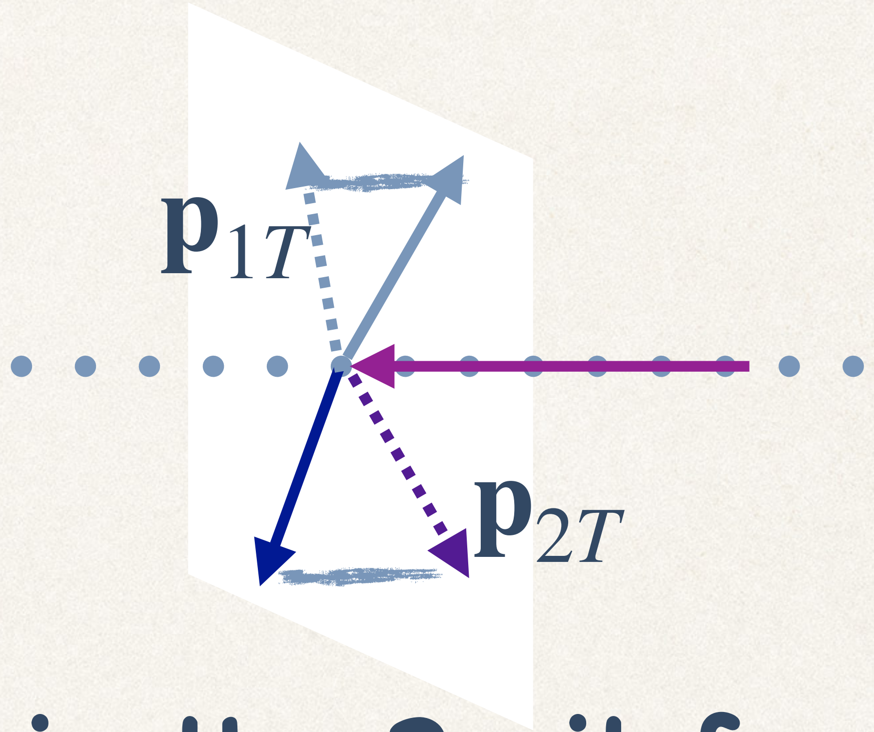


$k^\mu = \frac{\xi}{\sqrt{2}x} Q\bar{n}^\mu = \frac{\xi}{2x}(Q,0,0,-Q)$

EIC: $p_T \in [5, 40]$, central rapidity

Observables

$$\frac{d\sigma}{dx\,d\eta_1\,d\eta_2\,dp_T\,d\boldsymbol{r}_T}$$



All transverse quantities are referred to the beam axis in the Breit frame

$$\boldsymbol{r}_T = \boldsymbol{p}_{1T} + \boldsymbol{p}_{2T}, \qquad p_T = \frac{|\boldsymbol{p}_{1T}| + |\boldsymbol{p}_{2T}|}{2}$$

Small transverse momentum condition on the imbalance momentum

$$|\boldsymbol{r}_T| \ll p_T$$

Kinematics in the Breit frame

$$Q^2 = -q^2; \quad x = \frac{Q^2}{2P \cdot q}; \quad q^\mu = (0, 0, 0, Q); \quad P^\mu = \frac{1}{2x}(Q, 0, 0, -Q);$$

$$\xi = \frac{k^+}{P^+} \; \frac{\text{parton}}{\text{target hadron}}$$

This variable enters TMD

$$\eta_\pm = \frac{\eta_1 \pm \eta_2}{2}$$

$$Q = 2p_T \cosh(\eta_-) \exp(\eta_+); \quad \xi = 2x \cosh(\eta_+) \exp(-\eta_+)$$

Mandelstam variables

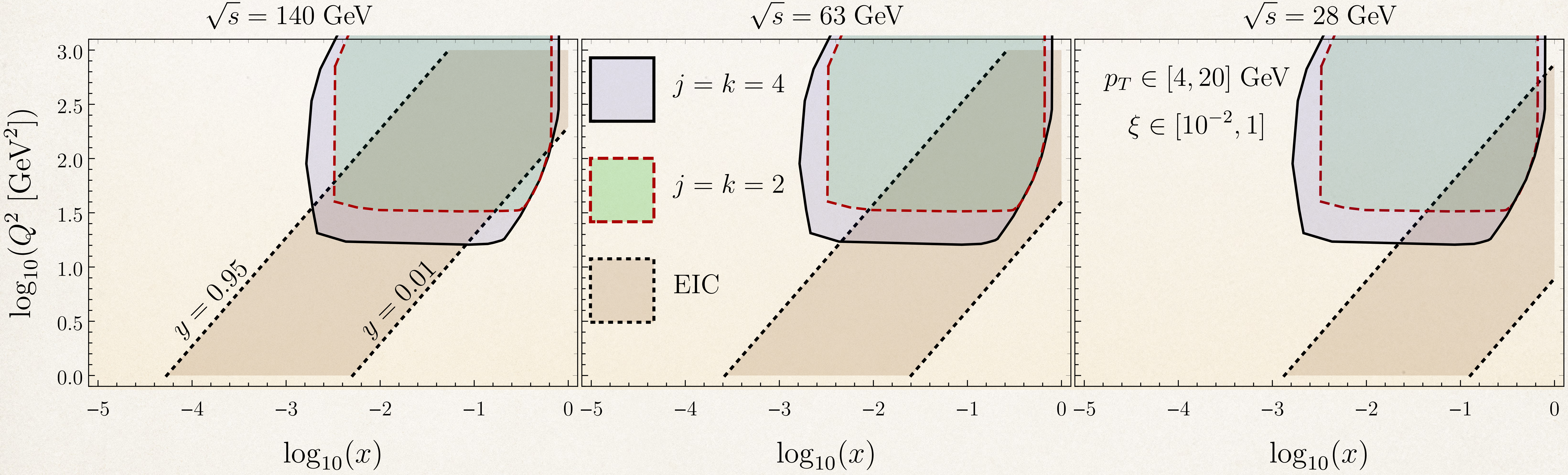
$$\hat{s} = (q + k)^2 = +4p_T^2 \cosh^2(\eta_-) \,,$$

$$\hat{t} = (q - p_2)^2 = -4p_T^2 \cosh(\eta_-) \cosh(\eta_+) \exp(\eta_1) \,,$$

$$\hat{u} = (q - p_1)^2 = -4p_T^2 \cosh(\eta_-) \cosh(\eta_+) \exp(\eta_2) \,,$$

$$\hat{s} + \hat{t} + \hat{u} = -Q^2$$

EIC coverage



Phenomenology: di-jet case

For the moment we consider the phenomenology of

$$d\sigma^L = d\sigma^L(\gamma^* g)$$

$$\frac{d\sigma}{d\Pi dr_T} = r_T \int_0^\infty \frac{b\,db}{2\pi} J_0(r_T b) \int_{-\pi}^{+\pi} d\phi_b \left[\frac{d\tilde{\sigma}^U(\mathbf{b})}{d\Pi d\mathbf{b}} - \frac{\cos 2\phi_b}{2} \frac{d\tilde{\sigma}^L(\mathbf{b})}{d\Pi d\mathbf{b}} \right]$$

$$d\Pi = dx d\eta_1 d\eta_2 dp_T$$

$$d\sigma^U = d\sigma^U(\gamma^* f) + d\sigma^U(\gamma^* g)$$

Unpolarized and linearly polarized gluons can contribute

Factorization

TMD

$$\frac{d\sigma^U(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sigma_0^{gU} \sum_f H^U_{\gamma^* g \to f\bar{f}}(\hat{s},\hat{t},\hat{u},\mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, f_1^g(\xi,\boldsymbol{b},\mu,\zeta_1)$$
$$\times S_{\gamma g}(\boldsymbol{b},\eta_1,\eta_2,\mu,\zeta_2)\left(\mathcal{C}_f(\boldsymbol{b},R,\mu)J_f(p_T,R,\mu)\right)\left(\mathcal{C}_{\bar{f}}(\boldsymbol{b},R,\mu)J_{\bar{f}}(p_T,R,\mu)\right)$$

NEW SOFT FUNCTION

JET: COLLINEAR SOFT AND JET FUNCTIONS

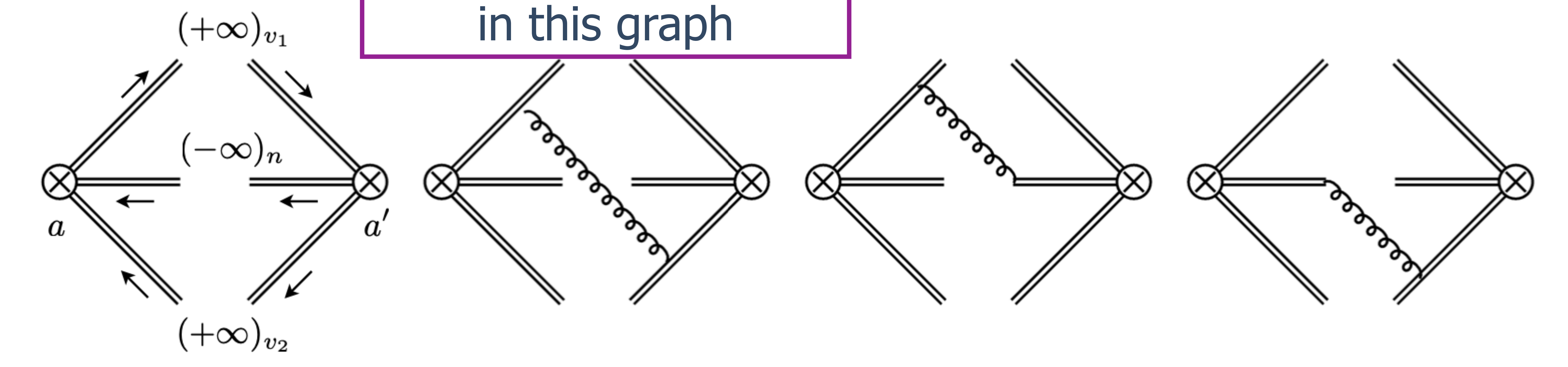
$$\frac{d\sigma^U(\gamma^* f)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sigma_0^{f} \sum_f H^U_{\gamma^* f \to g\bar{f}}(\hat{s},\hat{t},\hat{u},\mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, f_1^f(\xi,\boldsymbol{b},\mu,\zeta_1)$$
$$\times S_{\gamma f}(\boldsymbol{b},\eta_1,\eta_2,\mu,\zeta_2)\left(\mathcal{C}_g(\boldsymbol{b},R,\mu)J_g(p_T,R,\mu)\right)\left(\mathcal{C}_{\bar{f}}(\boldsymbol{b},R,\mu)J_{\bar{f}}(p_T,R,\mu)\right)$$

A NEW SOFT FUNCTION APPEARS

A new (measureable?) soft function

$$\hat{S}_{\gamma g}(\boldsymbol{b}) = \frac{1}{C_F C_A} \langle 0 | \mathcal{S}_n^\dagger(\boldsymbol{b}, -\infty)_{ca'} \mathrm{Tr}\left[S_{v_2}(+\infty, \boldsymbol{b}) T^{a'} S_{v_1}^\dagger(+\infty, \boldsymbol{b}) S_{v_1}(+\infty, 0) T^a S_{v_2}^\dagger(+\infty, 0) \right] \mathcal{S}_n(0, -\infty)_{ac} | 0 \rangle .$$

No rapidity divergences in this graph



(a) (b) (c) (d)

Only one rapidity regulator needed because we measure only momenta transverse to *n*

Re-organization, zero-bin, ..

The zero-bin subtracted TMD $\hat{B}_i(\xi, \boldsymbol{b}, \mu, k^- \delta_+) = \dfrac{B_i^{\text{un.}}(\xi, \boldsymbol{b}, \mu, k^-/\delta^-)}{S(\boldsymbol{b}, \mu, \sqrt{\delta^+\delta^-})}$

DY/SIDIS soft-function as a product $S(\boldsymbol{b}, \mu, \sqrt{\delta^+\delta^-}) = S^{\frac{1}{2}}(\boldsymbol{b}, \mu, \delta^+\nu) S^{\frac{1}{2}}(\boldsymbol{b}, \mu, \delta^-/\nu)$

Re-organization of the product of beam function and soft function in TMD and subtracted soft-function

Re-organization, zero-bin, ..

Re-organization of the product of beam function and soft function in TMD and subtracted soft-function

$$\hat{B}_i(\xi, \boldsymbol{b}, \mu, k^- \delta_+)\hat{S}_{\gamma i}(\boldsymbol{b}, \mu, \sqrt{A_n}\,\delta_+) = f^i(\xi, \boldsymbol{b}, \mu, \zeta_1)\, S_{\gamma i}(\boldsymbol{b}, \mu, \zeta_2)$$

We have a condition similar to DY case

$$\zeta_1\,\zeta_2 = \frac{\hat{u}\,\hat{t}}{\hat{s}} = p_T^2 \to \zeta_1 = p_T^2;\ \zeta_2 = 1$$

$$f^i(\xi, \boldsymbol{b}, \mu, \zeta_1) = \left.\frac{B_i^{\text{un.}}(\xi, \boldsymbol{b}, \mu, k^-/\delta^-)}{S^{\frac{1}{2}}(\boldsymbol{b}, \mu, \delta^-/\nu)}\right|_{\sqrt{2}\,k^-/\nu \to \sqrt{\zeta_1}}$$

$$S_{\gamma i}(\boldsymbol{b}, \mu, \zeta_2) = \left.\frac{\hat{S}_{\gamma i}(\boldsymbol{b}, \mu, \sqrt{A_n}\,\delta^+)}{S^{\frac{1}{2}}(\boldsymbol{b}, \mu, \delta^+\nu)}\right|_{\nu/\sqrt{2A_n} \to \sqrt{\zeta_2}}$$

$$A_n = \frac{v_1 \cdot v_2}{2(n \cdot v_1)(n \cdot v_2)}$$

Evolution

The anomalous dimensions now depend non-trivially on angles and develop imaginary parts: the two facts are connected

$$\frac{d}{d\ln\mu} S_{\gamma i}(\mathbf{b},\zeta,\mu) = \gamma_{S_{\gamma i}}(\mathbf{b},\mu,\zeta) S_{\gamma i}(\mathbf{b},\zeta,\mu),$$
$$\frac{d}{d\ln\zeta} S_{\gamma i}(\mathbf{b},\zeta,\mu) = -\mathcal{D}_i(\mu,b) S_{\gamma i}(\mathbf{b},\zeta,\mu),$$

$$v_j\cdot\mathbf{b} = v_j b\, c_{\mathbf{b}}\ \text{, with } c_{\mathbf{b}} \equiv \cos\phi_b$$

$$\gamma_i(\mathbf{b},\mu) = \gamma_{\text{cusp}}[\alpha_s]\ (c_i\, 2\ln|\cos\phi_b| - c_i'\, i\pi\Theta(\phi_b)) + \text{other } \phi_b \text{ independent terms}$$

$$\sum_i c_i = \sum_i c_i' = 0$$

$$\Theta(\phi_b) = \begin{cases} +1 & : -\pi/2 < \phi_b < \pi/2 \\ -1 & : \text{otherwise} \end{cases}$$

New AD property

$$\gamma^{[1]}_{S_{\gamma g}} = 4\left\{-C_A \ln\zeta_2 + 2C_F\left[\ln(B\mu^2\, e^{2\gamma_E}) - \ln\hat{s} + \ln p_T^2 + \ln(4c_{\boldsymbol{b}}^2)\right]\right\}$$

$$\gamma^{[1]}_{S_{\gamma f}} = 4\left\{(C_F + C_A)\left[\ln(B\mu^2 e^{2\gamma_E}) - \ln\hat{s} + \ln p_T^2 + \ln(4c_{\boldsymbol{b}}^2)\right] + (C_F - C_A)\left[\ln\left(\frac{\hat{t}}{\hat{u}}\right) - i\pi\right] - C_F\ln\zeta_2\right\}$$

$$\gamma^{[1]}_{\mathcal{C}_i} = 4C_i\left[-\ln\left(B\mu^2\, e^{2\gamma_E}\right) + \ln R^2 - \ln(4c_{\boldsymbol{b}}^2) + i\pi c_{\boldsymbol{b}}\right]$$

The sum of all anomalous dimensions cancel as usual but
Imaginary parts and angle dependence in the anomalous dimensions must be treated carefully.

Evolution: ζ-prescription (for new soft matrix element)

The objective of ζ-prescription is to define scale-independent matrix elements, re-absorbing the evolution in a factor. Here we need a further step because of angular dependence

$$S_{\gamma i}\left(\mathbf{b},\mu_f,\zeta_{2,f}\right) = \exp\left[\int_{\mu_0}^{\mu_f}\left(\gamma^{\phi}_{S_{\gamma i}}(\phi)d\ln\mu\right)\right]\exp\left[\int_P\left(\bar{\gamma}_{S_{\gamma i}}(b,\mu,\zeta_2)d\ln\mu - \mathcal{D}_i(\mu,b)d\ln\zeta_2\right)\right]S_{\gamma i}\left(\mathbf{b},\mu_0,\zeta_{2,0}\right)$$

$$= \mathcal{R}^{\phi}_{S_{\gamma i}}\ \mathcal{R}_{S_{\gamma i}}\ S_{\gamma i}\left(\mathbf{b},\mu_0,\zeta_{2,0}\right)$$

ζ and ϕ_b do not mix in AD's

$$\frac{d}{d\ln\mu}\mathcal{R}^{\phi}_{S_{\gamma i}} = \gamma^{\phi}_S(\phi)R^{\phi}_{S_{\gamma i}},\qquad \frac{d}{d\ln\zeta}\mathcal{R}^{\phi}_{S_{\gamma i}} = 0$$

$$\frac{d}{d\ln\mu}R_{S_{\gamma i}} = \bar{\gamma}_S R_{S_{\gamma i}},\qquad \frac{d}{d\ln\zeta}R_{S_{\gamma i}} = -\mathcal{D}$$

ζ-prescription is independent from ϕ_b integration

Evolution and angular integrations

Fixed order

$$d\tilde{\sigma}(\mathbf{b}) \sim |\cos\phi_b|^{2\mathcal{A}} \left(\cos(\mathcal{B}\pi) - i\Theta(\phi_b)\sin(\mathcal{B}\pi)\right)\mathcal{R}(\{\mu_k\} \to \mu)\left[1 + \sum_{k\in\{h,j,s,cs\}} a_s(\mu_k) f_k^{[1]}(b, \cos\phi_b)\right]$$

All imaginary parts cancel AFTER b-angle integrations

Caveat: Scale choice must respect the integration condition $2\mathcal{A} > -1$

Scale prescriptions: di-jet

$$\frac{d\sigma^U(\gamma^* g)}{d\Pi\, dr_T} = \sum_f \sigma_0^{gU} H^U_{\gamma^* g \to f\bar{f}}(\hat{s}, \hat{t}, \hat{u}, \mu = p_T) J_f(p_T, R, \mu_J) J_{\bar{f}}(p_T, R, \mu_J)$$

$$\times \int_0^{+\infty} bdb\, J_0(br_T) f_1(\xi, \mathbf{b}) \mathcal{R}_g\left((\{\mu_k\}, \zeta_{1,0}, \zeta_{2,0}) \to (p_T, p_T^2, 1)\right) \hat{\sigma}_g^U(b, R, \{\mu_i\})$$

Scale independent gluon TMD
Defined on non-perturbative saddle point in scale space

$$\mathcal{R}_g\left((\{\mu_k\}, \zeta_{1,0}, \zeta_{2,0}) \to (p_T, p_T^2, 1)\right) = \mathcal{R}_{J_f}(\mu_J \to p_T)^2 \mathcal{R}_{\mathcal{C}_f}(\mu_{\mathcal{C}} \to p_T)^2 \mathcal{R}_F^g\left((\mu_0, \zeta_{1,0}) \to (p_T, p_T^2)\right) \mathcal{R}_S^q\left((\mu_0, \zeta_{2,0})\right.$$

Scale choices and non-perturbative models

	$\mathcal{C}$	$\mathcal{J}$	S
B^i_{NP} (GeV^{-1})	2.5	2.5	2.5

	$\mathcal{C}$	$\mathcal{J}$
$b_{\max}$ (GeV^{-1})	0.5	0.3

$$f_i^{\mathrm{NP}}(b) = \exp\left(-\frac{b^2}{(B^i_{NP})^2}\right), \qquad i = \mathcal{J}, \mathcal{C}, S$$

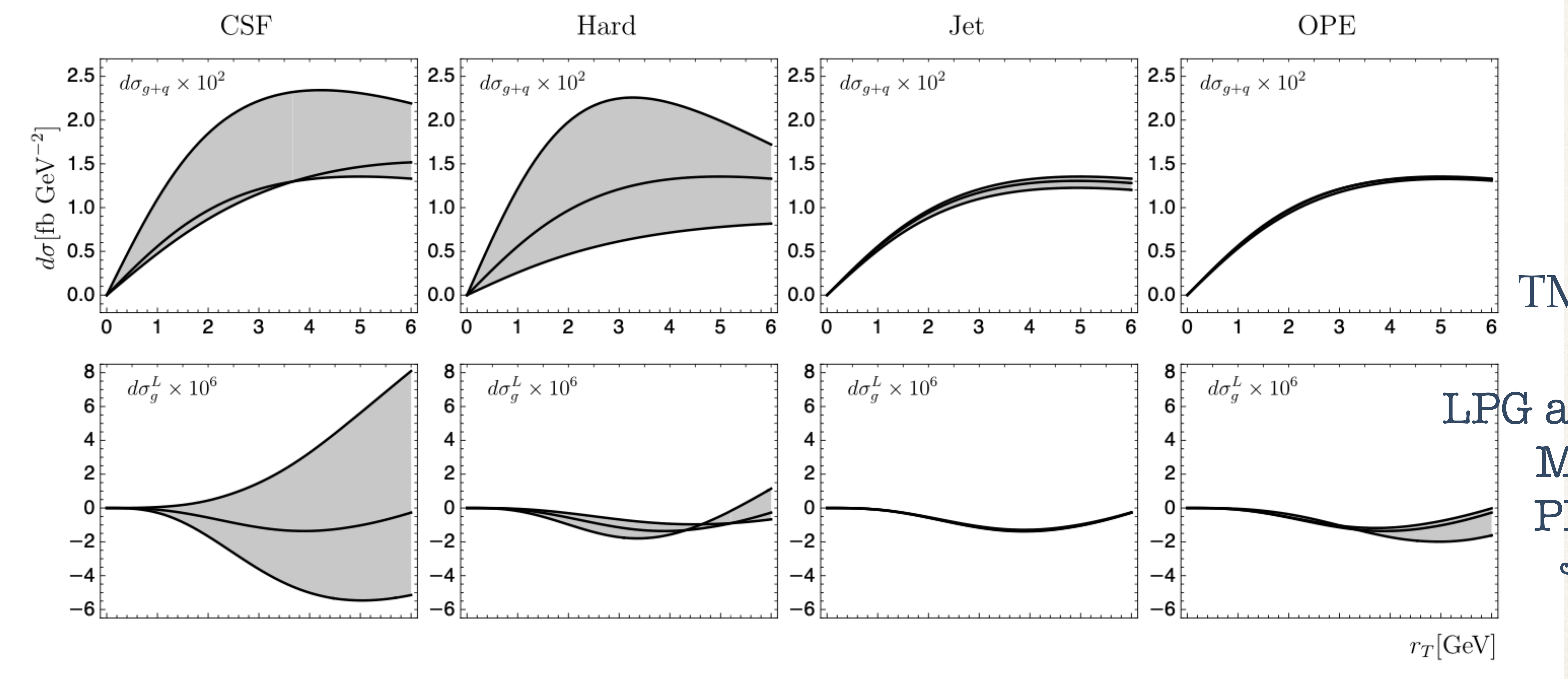
$$\mu_J = p_T R$$

$$\mu_{\mathcal{C}} = 2e^{-\gamma_E}\left(\frac{1}{b} + \frac{1}{b_{\max}}\right)$$

$$\mathcal{C}(b, R; p_T) = \mathcal{R}_{\mathcal{C}}(b, R; p_T, \mu_{\mathcal{C}})\mathcal{C}^{pert}(b, R; \mu_{\mathcal{C}}) f_{\mathcal{C}}^{\mathrm{NP}}(b, R),$$

$$S_{\gamma i}(b; p_T, 1) = \mathcal{R}_S(\{\mu_0, \zeta_0\} \to \{p_T, 1\}) S_{\gamma i}^{pert}(b; \mu_0, \zeta_0) f_{\mathcal{S}}^{\mathrm{NP}}(b)$$

Plot for di-jet production

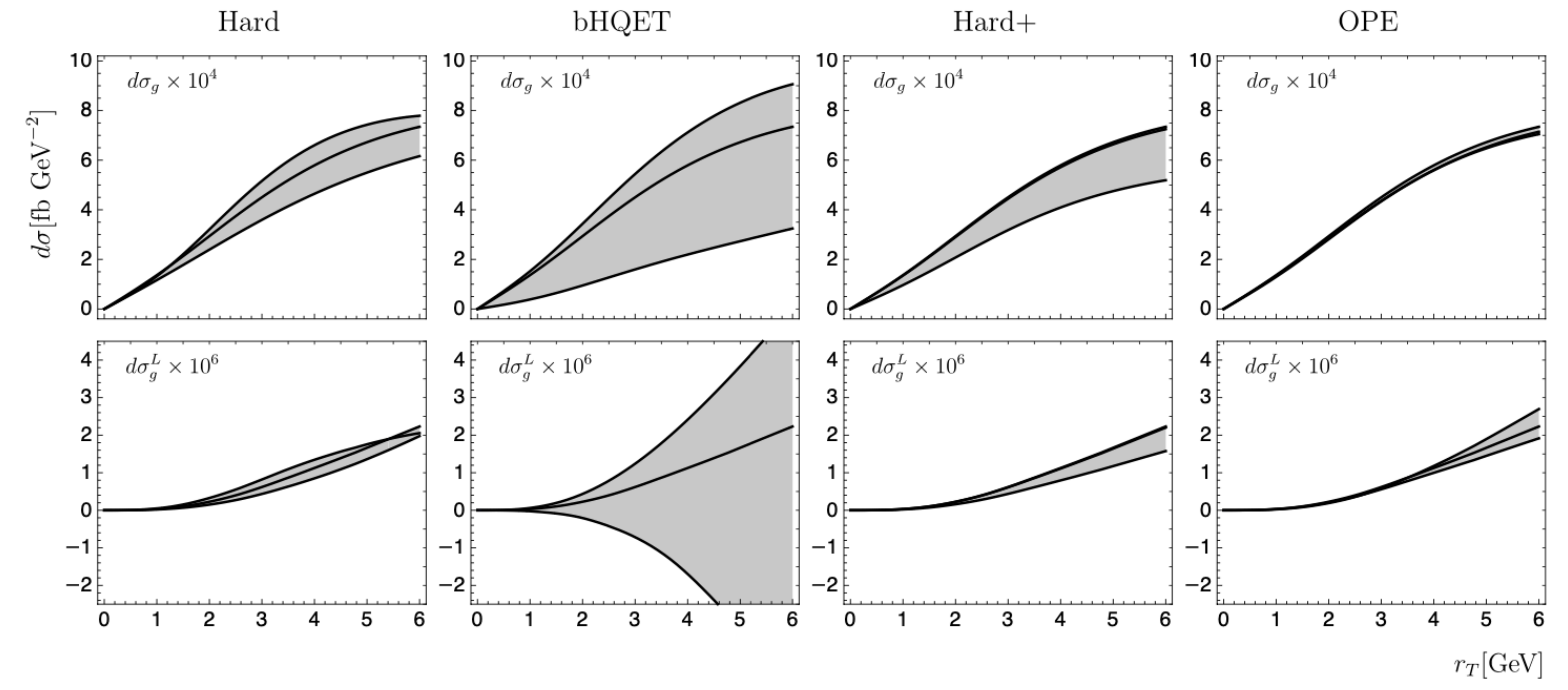


TMD from Artemide:

LPG are negligible because Matching to gluon PDF is suppressed

JHEP 11 (2019)121)

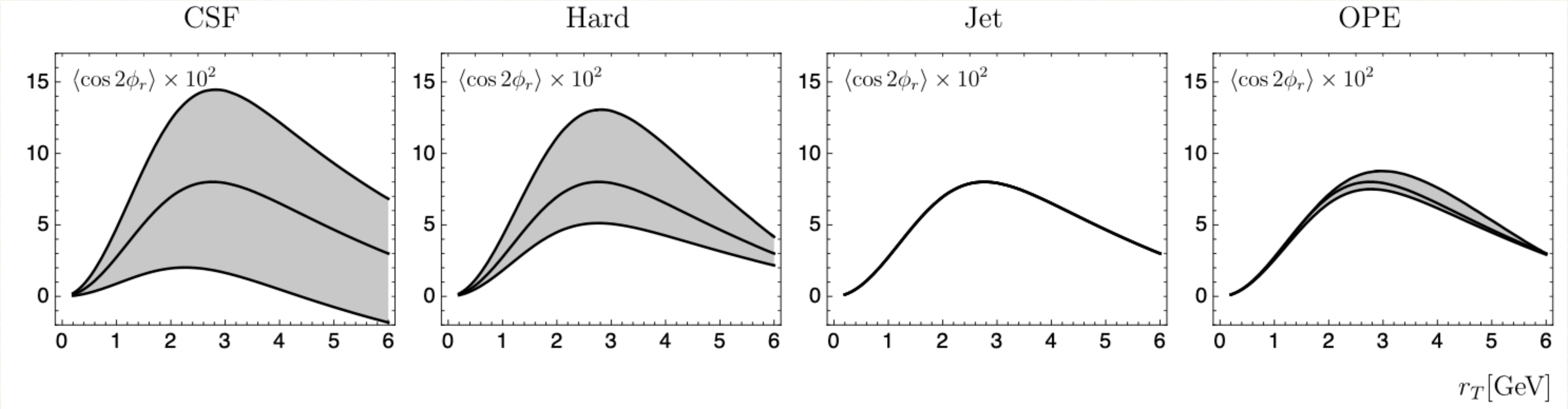
Plot for heavy di-hadron production



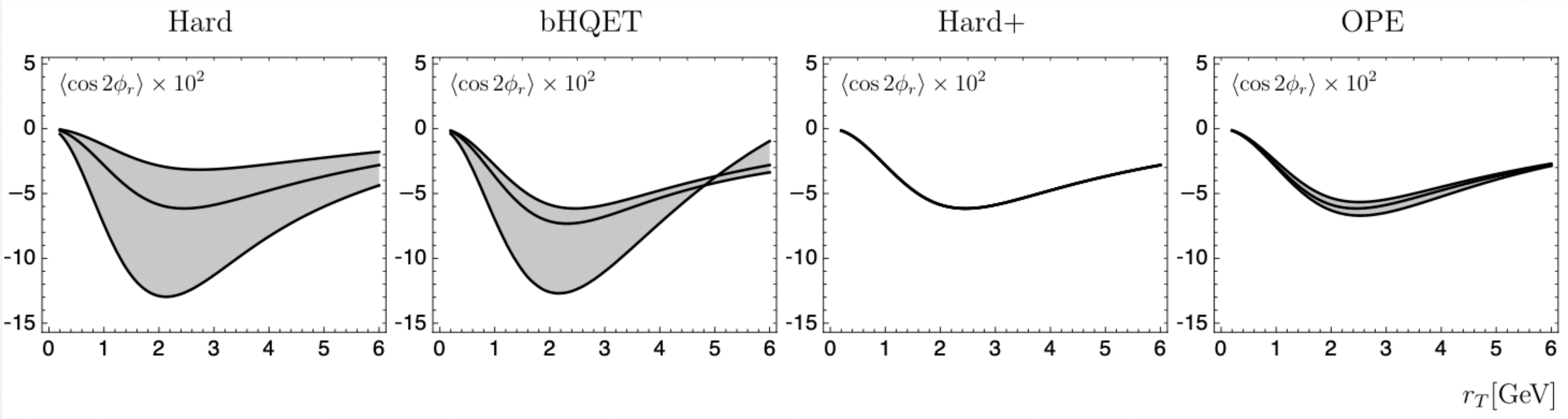
TMD from Artemide:

LPG are negligible because Matching to gluon PDF is suppressed
JHEP 11 (2019)121)

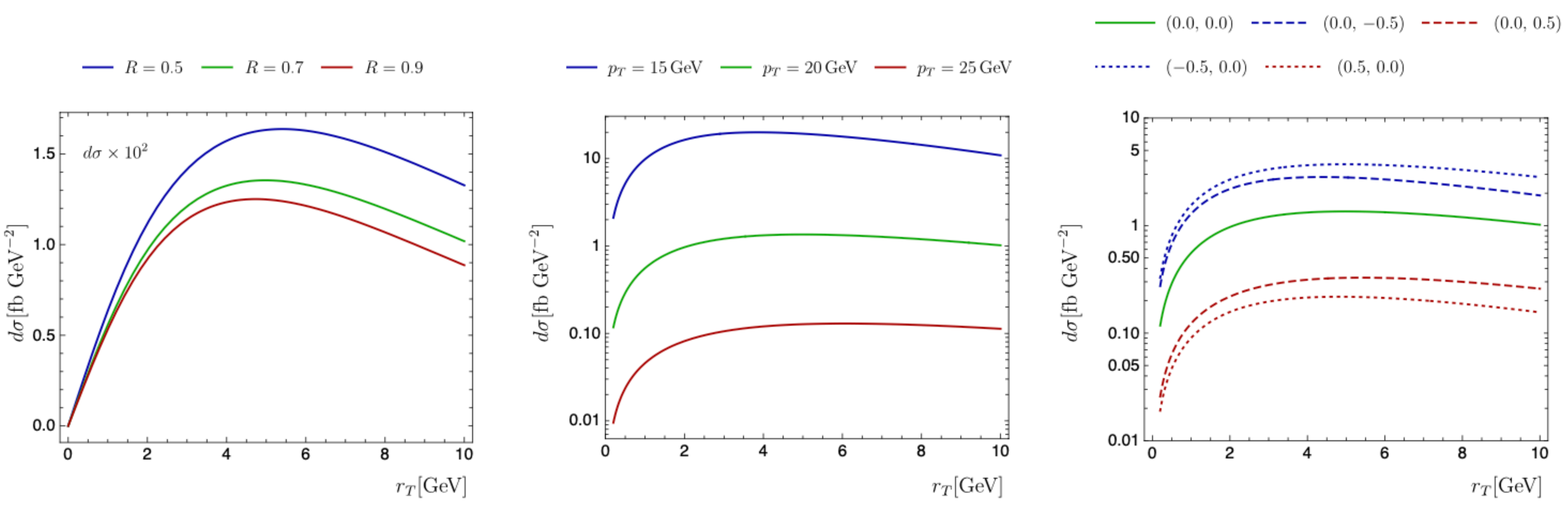
$\cos(2\phi)$ angular modulation at LP for di-jet



$\cos(2\phi)$ angular modulation at LP for di-hadron



Several features of di-jets



(a) R variation (b) p_T variation (c) η variation (η_1, η_2)

Conclusion

- We have shown the factorization of two processes involving gluon TMD. A new soft function appears, whose properties has been studied. This is a vacuum-to-vacuum matrix element that cannot be reabsorbed in other functions. Is this problematic?
- We have solved a problem of resummation with angular dependent complex anomalous dimensions. All functions coded in Artemide.
- Unpolarized and linearly polarized gluons appear together, but the latter has a suppressed matching coefficient, and they are negligible
- Angular modulation asymmetries around 5%
- Testing on H1?Similar processes?Higher perturbative orders?

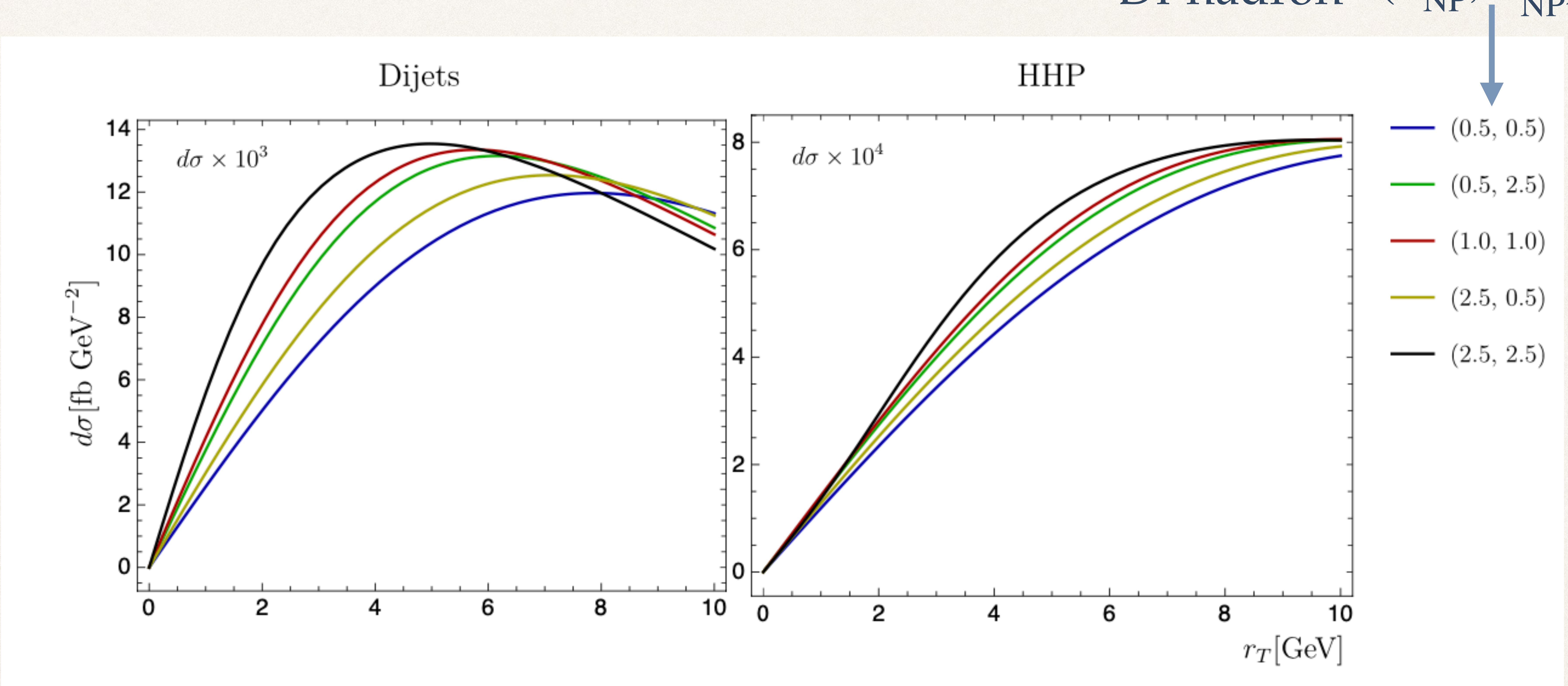
R. F. Del Castillo, M.G. Echevarría, Y. Makris, I.S., JHEP 03 (2022) 047, JHEP 01 (2021) 088

BACK UP SLIDES

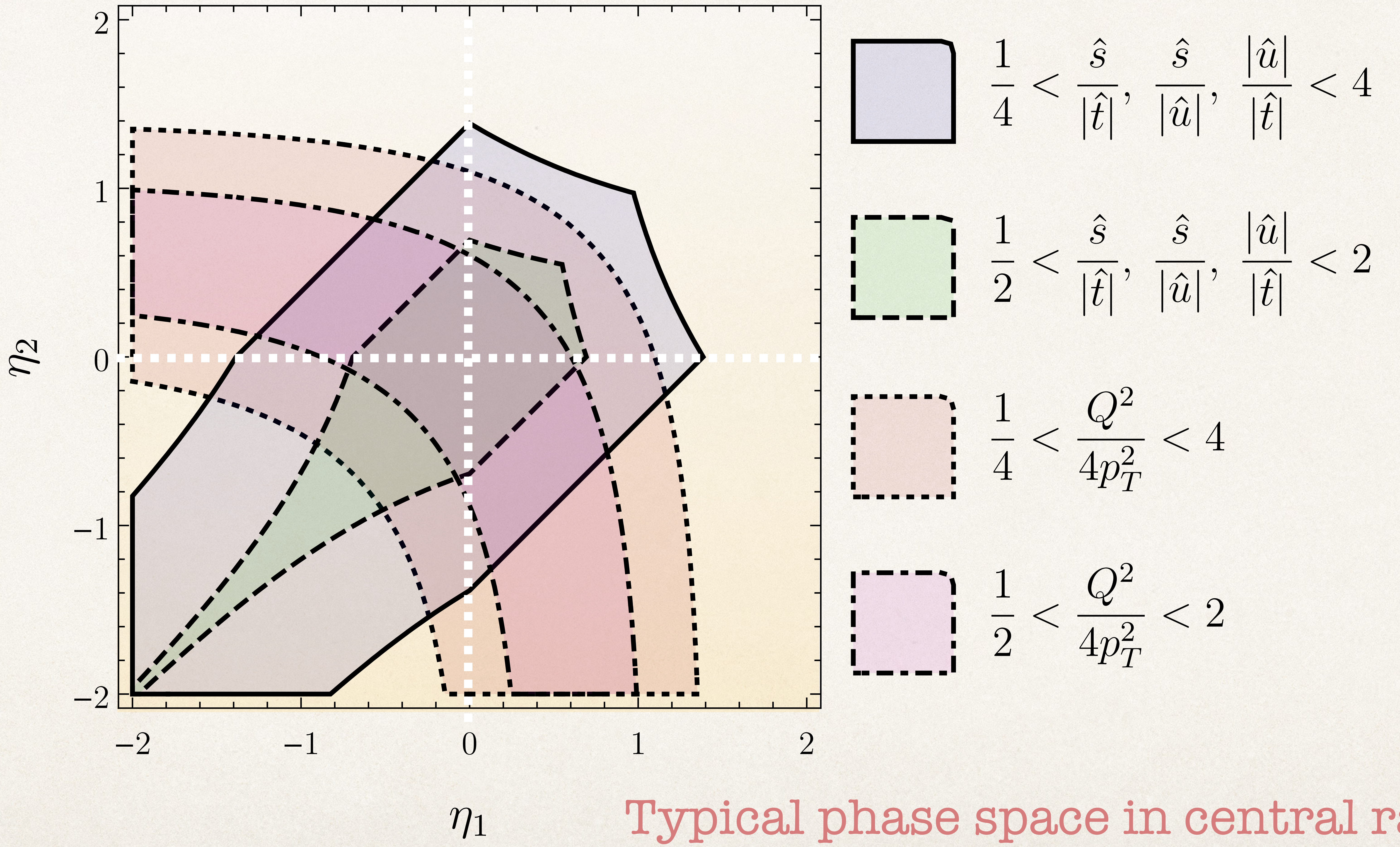
B_{NP} (model) dependance

Di-jet $(B^{\mathcal{S}}_{\mathrm{NP}}, B^{\mathcal{C}}_{\mathrm{NP}})$

Di-hadron $(B^{\mathcal{S}}_{\mathrm{NP}}, B^{\mathcal{J}}_{\mathrm{NP}})$



Phase space at EIC



Typical phase space in central rapidity region

The gluon TMD quest

Gluon Polarization

Nucleon Polarization

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

See Feng Yuan talk!

- Much effort in using unpolarized targets, unpolarized and linearly polarized gluons appear together
- The only color neutral particle available for this search is the Higgs
- Perturbative calculations at NNLO for unpolarized nucleon distributions, D. Gutierrez-Reyes, S. Leal-Gomez, et al. JHEP 1911 (2019) 121, M.-X Luo, et al. JHEP 2001 (2020) 040

HHP $\ell + p \to \ell + H + \bar{H} + X$

In principle all very similar (we consider only the charm case for EIC)

Momentum imbalance $\boldsymbol{r}_T = \boldsymbol{p}_T^H + \boldsymbol{p}_T^{\bar{H}}$

Di-hadron Momentum $p_T = \dfrac{|\boldsymbol{p}_T^H| + |\boldsymbol{p}_T^{\bar{H}}|}{2}$

$|\boldsymbol{r}_T|,\ m_H \ll p_T^{H,\bar{H}}$

$$\frac{d\sigma(\gamma^* g)}{dx\, d\eta_H\, d\eta_{\bar{H}}\, dp_T\, d\boldsymbol{r}_T} = H^{\mu\nu}_{\gamma^* g \to Q\bar{Q}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, F_{g,\mu\nu}(\xi, \boldsymbol{b}, \mu, \zeta_1)$$
$$\times\, S_{\gamma g}(\boldsymbol{b}, \mu, \zeta_2)\, J_{Q\to H}(\boldsymbol{b}, p_T, m_Q, \mu)\, J_{\bar{Q}\to\bar{H}}(\boldsymbol{b}, p_T, m_Q, \mu)$$

Only gluons in initial state

$$F_g^{\mu\nu}(\xi, \boldsymbol{b}) = f_1(\xi, \boldsymbol{b}) \frac{g_T^{\mu\nu}}{d-2} + h_1^{\perp}(\xi, \boldsymbol{b}) \left(\frac{g_T^{\mu\nu}}{d-2} + \frac{b^\mu b^\nu}{\boldsymbol{b}^2} \right)$$

Scales and bHQET

New scales appear,
whose logs should be resummed

$$\mu_+ = m_Q; \quad \mu_{\mathcal{J}} = m_Q \frac{r_T}{p_T}$$

bHQET

$$p_Q^\mu\Big|_{\text{rest frame}} = m_Q \beta^\mu + k_s^\mu\,, \qquad k_s^\mu \sim \Lambda_{\text{QCD}}(1,1,1)_v, \quad \Longrightarrow$$

$$p_Q^\mu\Big|_{\text{boosted frame}} \simeq \left(2E_H, \frac{m_H^2}{2E_H}, \Lambda_{\text{QCD}}\right)_v, \qquad k_{uc}^\mu \sim \Lambda_{\text{QCD}}\left(\frac{2E_H}{m_H}, \frac{m_H}{2E_H}, 1\right).$$

Typical scaling

$$r_T \sim \Lambda_{\text{QCD}} \frac{2p_T}{m_H}$$

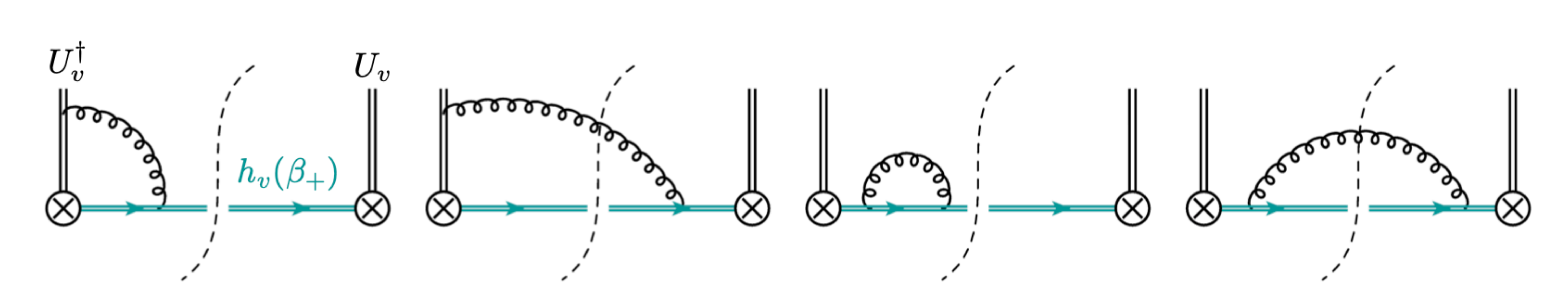
Matching bHQET to massive SCET

βμ is the collinear velocity of the heavy hadron and vμ is the lightlike vector along the direction of the boosted quark.

$$W_v^\dagger \xi_v \to C_+(m_Q, \mu) W_v^\dagger h_{v\beta_+}$$

$$J_{Q\to H}(\boldsymbol{b}, p_T, m_Q, \mu) = |C_+(m_Q, \mu)|^2 \mathcal{J}_{Q\to H}\left(\boldsymbol{b}, \frac{m_Q}{p_T}, \mu\right)$$

Shape function in b-space



Fragmentation Shape function/bHQET Jet function connection

$$\mathcal{J}_{Q\to H}\left(\boldsymbol{b}, \frac{m_Q}{p_T}, \mu\right) = \int d\boldsymbol{r} \exp(i\boldsymbol{b}\cdot\boldsymbol{r})\mathcal{J}_{Q\to H}(\boldsymbol{r})$$

$$\mathcal{J}_{Q\to H}(\boldsymbol{r}) = \frac{1}{2\,p_H^-\,N_C}\sum_X \langle 0|\delta^{(2)}\left(\boldsymbol{r} - i\boldsymbol{v}\left(\bar{v}\cdot\partial\right)\right) W_v^\dagger h_{v\beta_+}|XH\rangle\langle XH|\bar{h}_{v,\beta_+} W_v \,\bar{\not{v}}|0\rangle$$

$$S_{Q\to H}(\omega) = \frac{1}{2N_c}\sum_X \langle 0|\delta(\omega - i\sqrt{2}\,\bar{v}\cdot\partial)W_v^\dagger h_{v\beta_+}|H_\beta X\rangle\langle H_\beta X|\bar{h}_{v,\beta_+} W_v \frac{\bar{\not{v}}}{\sqrt{2}}|0\rangle$$

$$\tilde{S}_{Q\to H}(\tau) = \int d\omega \exp(i\omega\tau)S_{Q\to H}(\omega)$$

→

$$\mathcal{J}_{Q\to H}(\boldsymbol{b}) = \frac{m_H}{\sqrt{2}\,p_H^-}\tilde{S}_{Q\to H}\left(\tau \to \frac{\boldsymbol{v}\cdot\boldsymbol{b}}{\sqrt{2}}\right)$$

Soft function AD up to 3-loops

$$\gamma_{S_{\gamma g}} = -\left(\gamma_{H_{\gamma g}} + \gamma_{F_g} + \gamma_\alpha + \gamma_{\mathcal{J}}(\boldsymbol{v}_1) + \gamma_{\mathcal{J}}(\boldsymbol{v}_2) + 2\gamma_+\right)$$

$$\gamma_{S_{\gamma g}} = \gamma_{\text{cusp}}\left[2C_F \ln\left(\frac{B\mu^2 e^{2\gamma_E} 4p_T^2 c_{\boldsymbol{b}}^2}{\hat{s}}\right) - C_A \ln \zeta_2\right] + \delta\gamma_{S_{\gamma g}}$$

$$\delta\gamma^{[1]}_{S_{\gamma g}} = 0$$

$$\delta\gamma^{[2]}_{S_{\gamma g}} = C_F\left[C_A\left(\frac{1616}{27} - \frac{22}{9}\pi^2 - 56\,\zeta_3\right) + n_f T_F\left(-\frac{448}{27} + \frac{8}{9}\pi^2\right)\right]$$

$$\delta\gamma^{[3]}_{S_{\gamma g}} = ..$$


Thanks
M
D