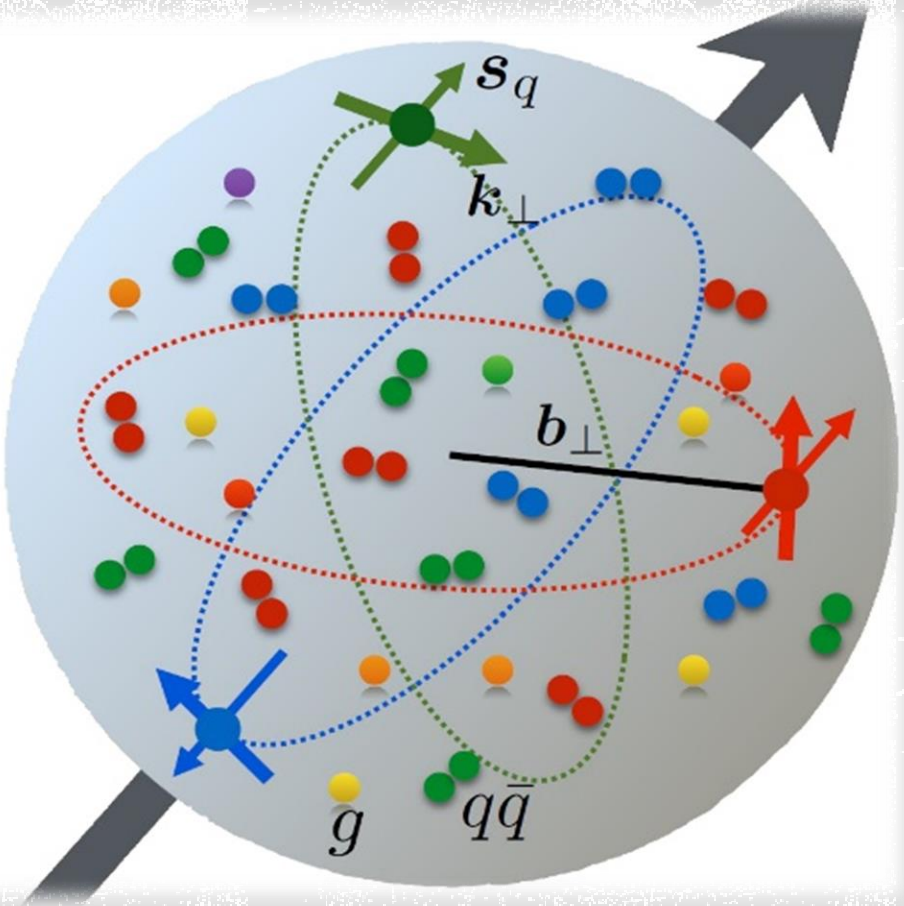

$cos2\phi$ AZIMUTHAL ASYMMETRY IN A BACK-TO-BACK J/ψ AND JET ELECTROPRODUCTION AT THE EIC

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arXiv:2203.13516

PLAN OF TALK

Gluon TMDs

Linearly Polarized Gluon TMDs $h_1^{\perp g}$

$\cos 2\phi_t$ azimuthal asymmetry in $J\psi - jet$ pair production in ep scattering at EIC

Numerical estimates

Conclusion

Gluon TMDs

Gluon-gluon correlator at leading twist

Gluon field tensor

$$\Gamma^{+i;+j}(x,\boldsymbol{k}_T;P,S)=\int\frac{d\xi^- d^2\xi_\perp}{(2\pi)^3}e^{ik\cdot\xi}\langle P,S|\mathrm{Tr}[F^{+i}(0)W_{[0,\xi]}F^{+j}(\xi)W_{[\xi,0]}]|P,S\rangle_{|\xi^+=0}$$

Gauge links

Parameterizations: unpolarized proton

$$\Gamma_U^{ij}(x,\boldsymbol{k}_T^2)=\frac{x}{2}\left\{-g_T^{ij}f_1^g(x,\boldsymbol{k}_T^2)+\left(\frac{k_T^i k_T^j}{M_P^2}+g_T^{ij}\frac{\boldsymbol{k}_T^2}{2M_P^2}\right)h_1^{\perp g}(x,\boldsymbol{k}_T^2)\right\}$$

f_1^g represents unpolarized gluon TMDs

$h_1^{\perp g}$ represents linearly polarized gluon TMDs in unpolarized proton

Hadron Pol.	Gluon Polarization: U	Circular	Linear	
U	f_1^g		$h_1^{\perp g}$	BOER-MULDERS
L		g_{1L}^g	$h_{1L}^{\perp g}$	KOTZINIAN-MULDERS
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$	

SIVERS HELICITY WORM-GEAR PRETZELOSITY

Mulders and Rodrigues PRD 63 094021(2001)

LINEARLY POLARIZED GLUON DISTRIBUTION FUNCTION (BOER-MULDERS) $h_1^{\perp g}(x, k_T^2)$

- Interpret $h_1^{\perp g}(x, k_T^2)$ as "azimuthal correlated" gluon distribution function.

- It affects the unpolarized cross section and cause azimuthal asymmetries: $\langle \cos(2\phi) \rangle$

- It's a time-reversal even function and in the small$-x$ domain, can be Weizsäcker-Williams(WW) or dipole distribution depending on type of Wilson line.

  - Wilson lines: $++$ or $--$ $\Longrightarrow$ WW distribution
  - Wilson lines: $+-$ or $-+$ $\Longrightarrow$ dipole distribution

Linearly polarized gluon distributions were first introduced in

Mulders and Rodrigues, PRD 63, 094021 (2001)

It can be probed in Drell-Yan and semi-inclusive deep inelastic scattering (SIDIS) processes. Though it has not been extracted from the data yet, but lot of theoretical studies has been done.

A BACK-TO-BACK $J/\psi - jet$ PRODUCTION IN ep SCATTERING

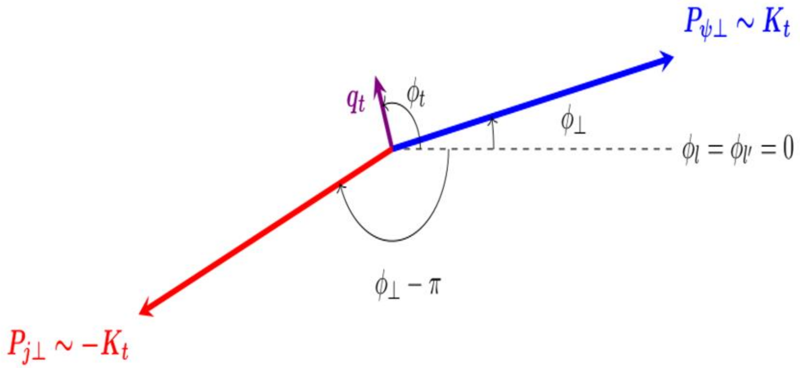
- Consider the electroproduction process: $\boldsymbol{e(l) + p(P) \to e(l') + J/\psi(P_\psi) + jet(P_j) + X}$

- Assume TMD factorization.

- $p - \gamma^*$ center of mass frame which move along z direction

- $\boldsymbol{P_{\psi\perp}}$ and $\boldsymbol{P_{j\perp}}$ are transverse momentum of J/ψ and jet respectively in the plane orthogonal to the proton momentum.

- We define sum and difference of transverse momenta



Transverse plane

$$\boldsymbol{q_t = P_{\psi\perp} + P_{j\perp}} \quad , \qquad \boldsymbol{K_t = \frac{P_{\psi\perp} - P_{j\perp}}{2}}$$

ϕ_t denotes azimuthal angle of $\boldsymbol{q_t}$

- In the case where $\boxed{\boldsymbol{|q_t| \ll |K_t|}}$, the J/ψ and jet are almost back-to-back in the transverse plane.

TMD Factorization

CROSS SECTION: $ep \rightarrow e + J\psi + jet + X$

$$d\sigma = \frac{1}{2s}\frac{d^3l'}{(2\pi)^3 2E_{l'}}\frac{d^3P_\psi}{(2\pi)^3 2E_\psi}\frac{d^3P_j}{(2\pi)^3 2E_j}\int dx d^2p_T (2\pi)^4\delta^4\left(q + p_g - P_\psi - P_j\right) \times \frac{1}{Q^4} L^{\mu\mu'}(l,q)\Phi_g^{\nu\nu'}\left(x, p_T^2\right) M_{\mu\nu}^{g\gamma^*\rightarrow J/\psi\, g} M_{\mu'\nu'}^{*g\gamma^*\rightarrow J/\psi\, g}$$

Lepton tensor: $L^{\mu\mu'}(l,q) = e^2\left(-g^{\mu\mu'}Q^2 + 2\left(l^\mu l'^{\mu'} + l^{\mu'} l^\mu\right)\right)$

Parameterization of gluon correlator for unpolarized proton target at 'Leading Twist'

$$\Phi_g^{\nu\nu'}(x, \boldsymbol{p}_T^2) = \frac{1}{2x}\left[-g_\perp^{\nu\nu'} f_1^g\left(x, \boldsymbol{p}_T^2\right) + \left(\frac{p_T^\nu p_T^{\nu'}}{M_p^2} + g_\perp^{\nu\nu'}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp g}\left(x, \boldsymbol{p}_T^2\right)\right]$$

Unpolarized gluon distribution

Linearly polarized gluon distribution

QUARKONIUM PRODUCTION

- Quarkonium is a bound state of heavy quark and anti-quark ($Q\bar{Q}$)

Describes conversion of $Q\bar{Q}[n]$ states into final quarkonium state.

NRQCD factorization

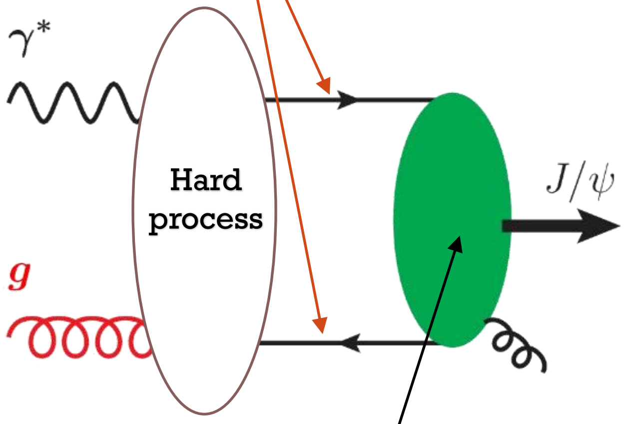
Non-perturbative, long distance matrix elements LDMEs

$$d\sigma^{ab\to J/\psi} = \sum_n d\hat{\sigma}[ab \to c\bar{c}(n)]\langle 0 \mid \mathcal{O}_n^{J/\psi} \mid 0\rangle$$

Perturbative short distance coefficient

Cross section in particular color, angular momentum and spin state "n" : ${}^{2S+1}L_J$, calculated by perturbative QCD

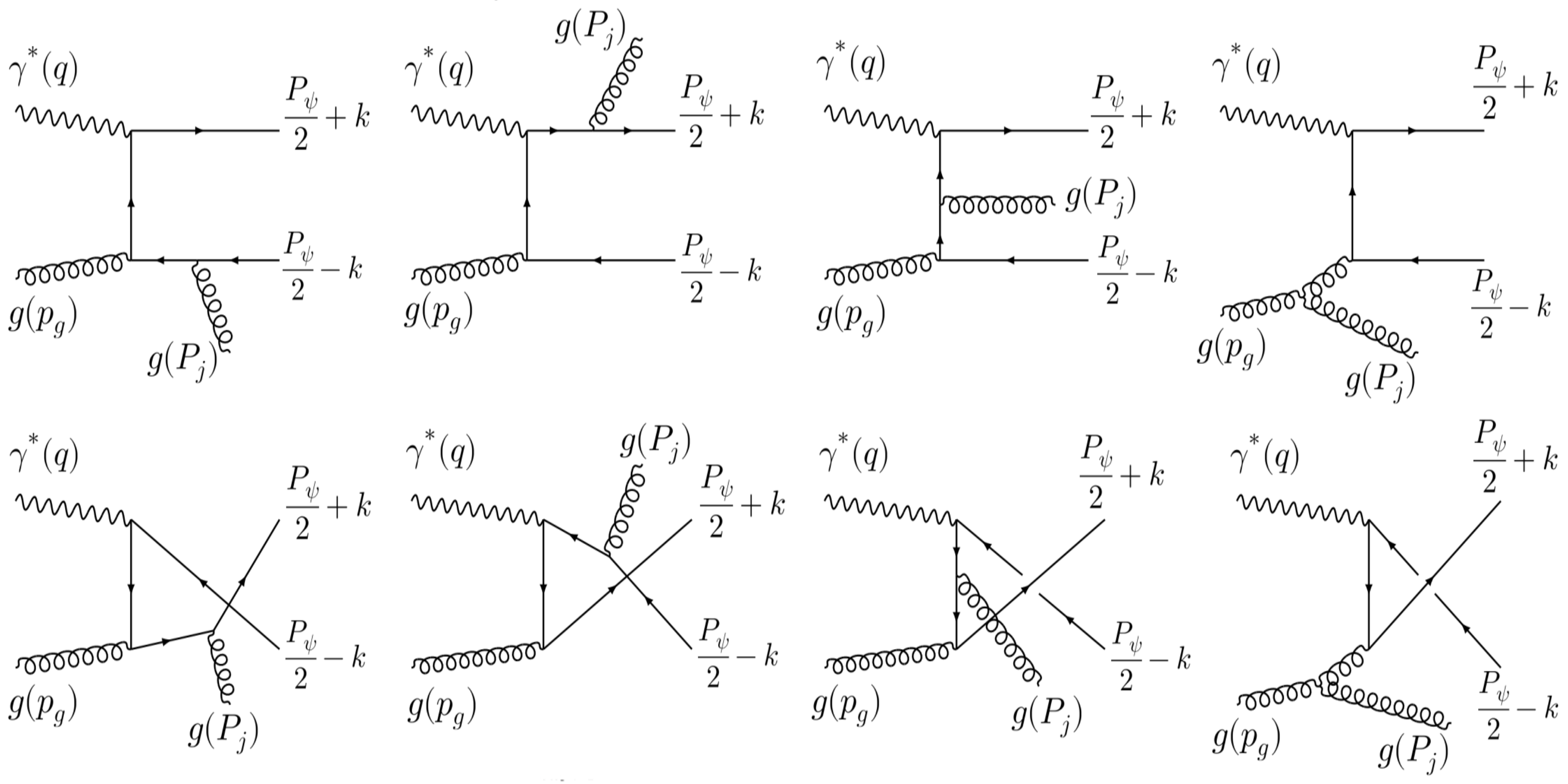
$Q\bar{Q}$ pair with $[{}^{2S+1}L_J^{(1,8)}]$ quantum number



G. T. Bodwin et al, PRD51 (1995), Lepage 95

FEYNMAN DIAGRAMS

Gluon initiated hard process: $\gamma^* g \to Q\bar{Q}\ g$, contributes significantly over the quark(anti-quark) initiated hard process: $\gamma^* q(\bar{q}) \to Q\bar{Q}\ q(\bar{q})$, in the small$-x$ domain.



Tree level Feynman diagrams for the hard process: $\gamma^* + g \to c + \bar{c} + g$

AMPLITUDE CALCULATIONS USING NRQCD

The amplitude can be written as

$$M(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}](P_\psi) + g[jet])$$
$$= \sum_{L_z S_z} \int \frac{d^3k}{(2\pi)^3} \Psi_{LL_z}(k) \langle LL_z; SS_z | JJ_z \rangle \mathrm{Tr}[\mathcal{O}(q,p,P_\psi,k)\mathcal{P}_{SS_z}(P_\psi,k)]$$

D. Boer and C. Pisano (2012)

$\mathcal{O}(q,p,P_\psi,k)$: amplitude for production of $Q\bar{Q}$ pair.

$$\mathcal{O}(q,p,P_\psi,k) = \sum_{m=1}^{8} C_m \mathcal{O}_m(q,p,P_\psi,k)$$

The spin projection operator, $\mathcal{P}_{SS_z}(P_\psi,k)$, projects the spin triplet and spin singlet states of $Q\bar{Q}$ pair

$$\mathcal{P}_{SS_z}(P_\psi,k) = \sum_{s_1 s_2} \left\langle \frac{1}{2} s_1; \frac{1}{2} s_2 \middle| SS_z \right\rangle v\left(\frac{P_\psi}{2} - k, s_1\right) \bar{u}\left(\frac{P_\psi}{2} + k, s_2\right)$$
$$= \frac{1}{4M_\psi^{3/2}} (-\not{P}_\psi + 2\not{k} + M_\psi)\Pi_{SS_z}(\not{P}_\psi + 2\not{k} + M_\psi) + O(k^2)$$

$\Pi_{SS_z} = \gamma^5$ for spin singlet ($S = 0$)

$\Pi_{SS_z} = \epsilon^\mu_{S_z}(P_\psi)\gamma_\mu$ for spin triplet ($S = 1$)

Amplitude Calculations

Since, $k \ll P_h$, amplitude expanded in Taylor series about $k = 0$

First term in the expansion gives the S-states $(L = 0, J = 0,1)$. The linear term in k gives the P-states $(L = 1, J = 0,1,2)$.

The S-states amplitude :

$$M[{}^{2S+1}S_J^{(1,8)}](P_\psi, k) = \frac{1}{\sqrt{4\pi}} R_0(0) \mathrm{Tr}[\mathcal{O}(q,p,P_\psi,k) \mathcal{P}_{SS_z}(P_\psi,k)\Big|_{k=0}$$

The P-states amplitude :

$$M[{}^{2S+1}P_J^{(1,8)}](P_\psi, k) = -i\sqrt{\frac{3}{4\pi}} R_1'(0) \sum_L \epsilon_{L_z}^{\alpha}(P_\psi) \langle LL_z; SS_z | JJ_z \rangle \mathrm{Tr}[\mathcal{O}_\alpha(0) \mathcal{P}_{SS_z}(0) + \mathcal{O}(0) \mathcal{P}_{SS_z\alpha}(0)]$$

$$\mathcal{O}_\alpha(0) = \frac{\partial}{\partial k^\alpha} \mathcal{O}(q,p,P_\psi,k)\Big|_{k=0} \qquad \mathcal{P}_{SS_z\alpha}(0) = \frac{\partial}{\partial k^\alpha} \mathcal{P}_{SS_z}(q,p,P_\psi,k)\Big|_{k=0}$$

Contributions: ${}^3S_1^{(1)}$, ${}^3S_1^{(8)}$, ${}^1S_0^{(8)}$, ${}^3P_0^{(8)}$, ${}^3P_1^{(8)}$, ${}^3P_2^{(8)}$

R_0 and R_1' are related with the LDMEs

ASYMMETRY CALCULATIONS

Final expression of the unpolarized differential cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}^2\boldsymbol{q}_t\,\mathrm{d}^2\boldsymbol{K}_t} = \frac{1}{(2\pi)^4}\frac{1}{16sz(1-z)Q^4}\{(\mathbb{A}_0 + \mathbb{A}_1\cos\phi_\perp + \mathbb{A}_2\cos 2\phi_\perp)f_1^g(x,\boldsymbol{q}_t^2)$$
$$+\frac{\boldsymbol{q}_t^2}{M_P^2}h_1^{\perp g}(x,\boldsymbol{q}_t^2)\big(\mathbb{B}_0\cos 2\phi_t + \mathbb{B}_1\cos(2\phi_t-\phi_\perp) + \mathbb{B}_2\cos 2(\phi_t-\phi_\perp) + \mathbb{B}_3\cos(2\phi_t-3\phi_\perp)$$
$$+\mathbb{B}_4\cos(2\phi_t-4\phi_\perp)\big)\}$$

Azimuthal modulation: $A^{W(\phi_S,\phi_T,\phi_\perp)} = 2\frac{\int d\phi_S\,d\phi_T\,d\phi_\perp W(\phi_S,\phi_T,\phi_\perp)d\sigma}{\int d\phi_S\,d\phi_T\,d\phi_\perp d\sigma}$

U. D'Alesio (2019)

$\cos 2\phi_t$ azimuthal asymmetry as function of z, x_B, y and K_t:

$$\langle\cos 2\phi_t\rangle \equiv A^{\cos 2\phi_t} = \frac{\int dq_t\,q_t\,\frac{q_t^2}{M_P^2}\mathbb{B}_0 h_1^{\perp g}(x,q_t^2)}{\int dq_t\,q_t\,\mathbb{A}_0 f_1^g(x,q_t^2)}$$

TMD EVOLUTION

Gluon TMDs at any probing scale Q_f can be obtained by solving the Collins-Soper evolution equation and renormalization group equation. In impact parameter space, Gluon TMDs can be expressed as

$$\widehat{f_1^g}(x, \boldsymbol{b}_t^2, Q_f^2) = \frac{1}{2\pi} \sum_{p=q,\bar{q},g} \left(C_{g/p} \otimes f_1^p\right)(x, Q_i^2) e^{-\frac{1}{2} S_A(\boldsymbol{b}_t^2, Q_f^2, Q_i^2)} e^{-S_{np}(\boldsymbol{b}_t^2, Q_f^2)}$$

Aybat and Rogers (2011)

$C_{g/p}(x, Q_i^2)$: perturbative quantity, can be written as series of α_s

$S_A(\boldsymbol{b}_t^2, Q_f^2, Q_i^2)$ and $S_{np}(\boldsymbol{b}_t^2, Q_f^2)$ are perturbative and non-perturbative part, respectively, of the Sudakov factor.

At leading order, perturbative Sudakov factor, which valid in the limit $|\boldsymbol{b}_t| \ll 1/\Lambda_{QCD}$, can be given as

$$S_A(b_t^2, Q_f^2, Q_i^2) = \frac{C_A \alpha_s}{\pi} \left(\frac{1}{2} \ln^2 \frac{Q_f^2}{Q_i^2} - \frac{11 - 2n_f/C_A}{6} \ln \frac{Q_f^2}{Q_i^2} \right)$$

$f_1^p(x, Q_i^2)$ are collinear PDFs which evolve using DGLAP equation from a scale Q_f down to $Q_i < Q_f$.

TMD EVOLUTION

By taking the Fourier transform of $\widehat{f_1^g}(x, q_t^2, Q_f^2)$, we obtain the TMDs in momentum space in the region $|\boldsymbol{q}_t| \gg \Lambda_{QCD}$

$$f_1^g(x, \boldsymbol{q}_t^2, Q_f^2) = \frac{1}{2\pi}\int_0^\infty b_t \mathrm{d}b_t J_0(b_t q_t)\,\{f_1^g(x, Q_f^2) - \frac{\alpha_s}{2\pi}[\left(\frac{C_A}{2}\log^2\frac{Q_f^2}{Q_i^2} - \frac{11C_A - 2n_f}{6}\log\frac{Q_f^2}{Q_i^2}\right) f_1^g(x, Q_f^2)$$

$$+ (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, Q_f^2)\log\frac{Q_f^2}{Q_i^2} - 2f_1^g(x, Q_f^2)]\} \times e^{-S_{np}(\boldsymbol{b}_t^2)}$$

Perturbative tail of $h_1^{\perp g}$ follows f_1^g, but its expression begins at $\mathcal{O}(\alpha_s)$.

At leading order, $h_1^{\perp g}$ in terms of the unpolarized collinear PDFs is given as

D.Boer, U. D'Alesio (2020)

$$\frac{\boldsymbol{q}_t^2}{M_P^2} h_1^{\perp g}(x, \boldsymbol{q}_t^2) = \frac{\alpha_S}{\pi^2}\int_0^\infty \mathrm{d}b_t b_t J_2(b_t q_t)\left[C_A \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x} - 1\right) f_1^g(\hat{x}, Q_f^2) + c_F \sum_{p=q,\bar{q}} \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x} - 1\right) f_1^p(\hat{x}, Q_f^2)\right] \times e^{-S_{np}(\boldsymbol{b}_t^2)}$$

s_{np} is introduced to suppress the large b_t region (non-perturbative),
$e^{-S_{np}(b_t^2)} \to 1$ as $b_t \to 0$ and vanishes in large b_t.

F.Scarpa, D.Boer (2020)

We consider, $S_{np} = \frac{A}{2}\log\frac{Q_f}{Q_{np}}\, b_c^2$ where $b_c(b_t) = \sqrt{b_t^2 + \left(\frac{2\gamma_E}{Q_f}\right)^2}$, $Q_{np} = 1\text{GeV}$ and $\gamma_E = 0.577$.

SPECTATOR MODEL (SM)

A. Bacchetta, F. G. Celiberto, M. Radici, and P. Taels (2020)

Nucleon is assumed to emit a gluon and the remaining is treated as a single on-shell particle called spectator particle.

Mass of the spectator particle is allowed to take a continuous values described by a spectral function, $\rho_X(M_X)$

Gluon TMDs: $F^g\left(x, q_t^2\right) = \int_M^\infty \mathrm{d}M_X \rho_X(M_X) \hat{F}^g(x, q_t^2; M_X)$

$$\rho_X(M_X) = \mu^{2a}\left[\frac{A}{B+\mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X-D)^2}{\sigma^2}}\right]$$

where, A, B, C, D, a, b, σ are free parameters

At leading-twist, T-even unpolarized and linearly polarized gluon TMDs can be written as

$$\hat{f}_1^g(x, \boldsymbol{q}_t^2; M_X) = \left[(2Mxg_1 - x(M+M_X)g_2)^2\left[\left(M_X - M(1-x)\right)^2 + \boldsymbol{q}_t^2\right] + 2\boldsymbol{q}_t^2\left(\boldsymbol{q}_t^2 + xM_X^2\right)g_2^2 + 2\boldsymbol{q}_t^2 M^2(1-x)\left(4g_1^2 - xg_2^2\right)\right] \times \left[(2\pi)^3 4xM^2\left(L_X^2(0) + \boldsymbol{q}_t^2\right)^2\right]^{-1}$$

$$\hat{h}_1^{\perp g}(x, \boldsymbol{q}_t^2; M_X) = \left[4M^2(1-x)g_1^2 + \left(L_X^2(0) + \boldsymbol{q}_t^2\right)g_2^2\right] \times \left[(2\pi)^3 x\left(L_X^2(0) + \boldsymbol{q}_t^2\right)^2\right]^{-1}$$

Where $g_{1,2}(p^2)$ are model-dependent form factors, given as: $g_{1,2}(p^2) = \kappa_{1,2}\frac{p^2(1-x)^2}{(q_t^2 + L_X^2(\Lambda_X^2))^2}$

p^2 is gluon momentum, $\kappa_{1,2}$ and Λ_X are normalization and cut-off parameters respectively, and $L_X^2(\Lambda_X^2) = xM_X^2 + (1-x)\Lambda_X^2 - x(1-x)M^2$

GAUSSIAN PARAMETERIZATION (GP)

$f_1^g(x, \boldsymbol{q}_t^2)$ and $h_1^{\perp g}(x, \boldsymbol{q}_t^2)$ are assumed to be factorized as function of x, i.e. collinear PDFs and a Gaussina function of the transverse momentum q_t.

$$f_1^g(x, \boldsymbol{q}_t^2) = f_1^g(x, \mu)\frac{1}{\pi\langle q_t^2\rangle^2} e^{-\frac{q_t^2}{\langle q_t^2\rangle}}$$

$$h_1^{\perp g}(x, \boldsymbol{q}_t^2) = \frac{M_P^2 f_1^g(x, \mu)}{\pi\langle q_t^2\rangle^2}\frac{2(1-r)}{r} e^{1-\frac{q_t^2}{r\langle q_t^2\rangle}}$$

r $(0 < r < 1)$ and $\langle q_t^2\rangle$ are parameters

We took, $r = 1/3$ and $\langle q_t^2\rangle = 0.25$

Linearly polarized gluon TMD satisfies a positivity bound

$$\frac{\boldsymbol{q}_t^2}{2M_P}\left|h_1^{\perp g}(x, \boldsymbol{q}_t^2)\right| \le f_1^g(x, \boldsymbol{q}_t^2)$$

D. Boer and C. Pisano (2012)

RESULTS

RK,A.Mukerjee,A.Pawar,M.Siddiqah ,arXiv:2203.13516

$\cos 2\phi_t$ azimuthal asymmetry in
(A) TMD evolution
(B) Gaussian Parameterization
(C) Spectator Model

Kinematics: $\sqrt{s} = 140$ GeV
$0.1 < y < 1, \;\; 0 < q_t < 1$ GeV

$$Q = \sqrt{M_\psi^2 + K_t^2}$$

Plots as function of K_t and y are at fixed $z = 0.7$

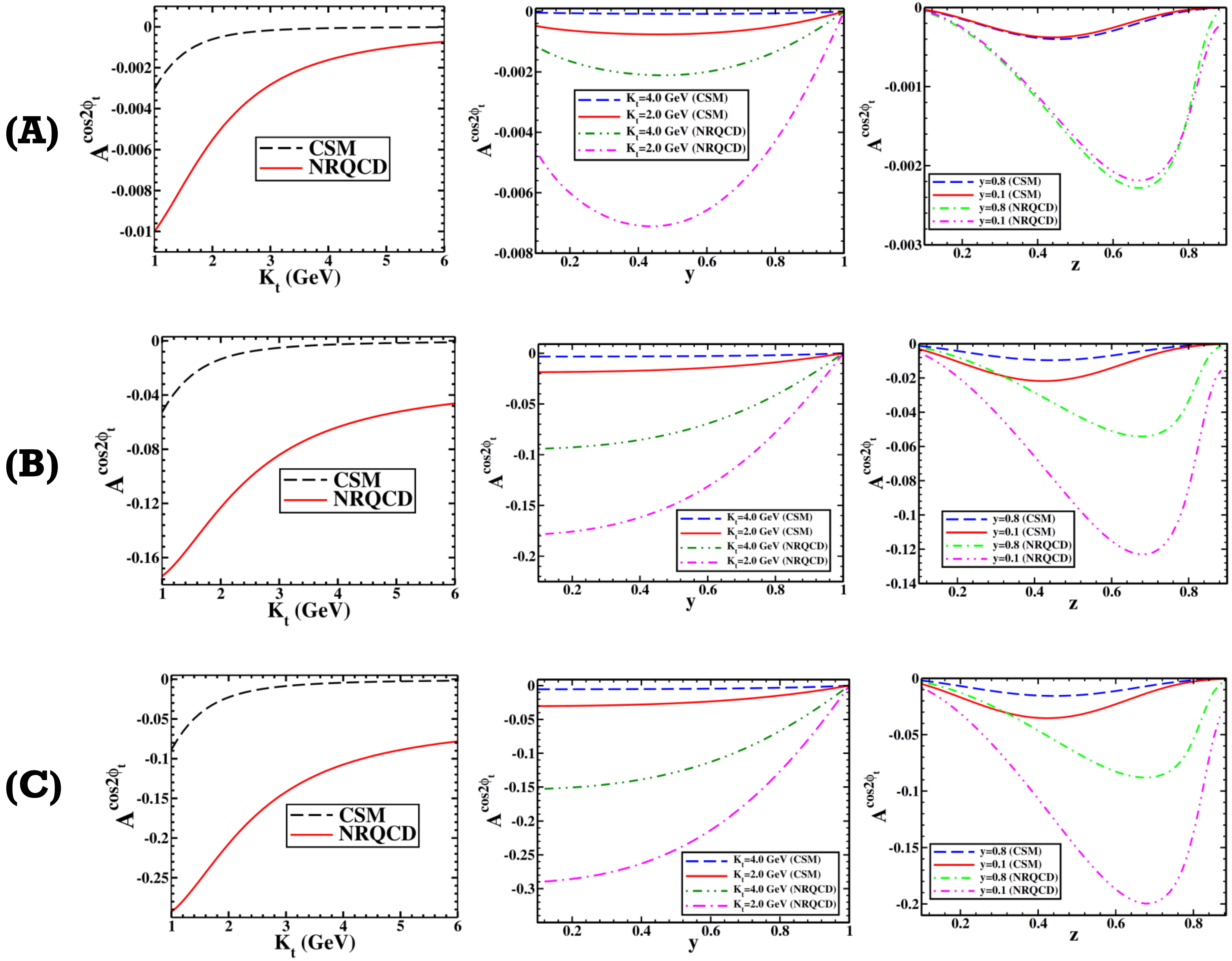
These kinematics can be accessible at the EIC

Significant contribution to $A^{\cos 2\phi_t}$ coming from color octet states.

Asymmetry hardly change with $\sqrt{s}$

We used CSMWZ set of LDME

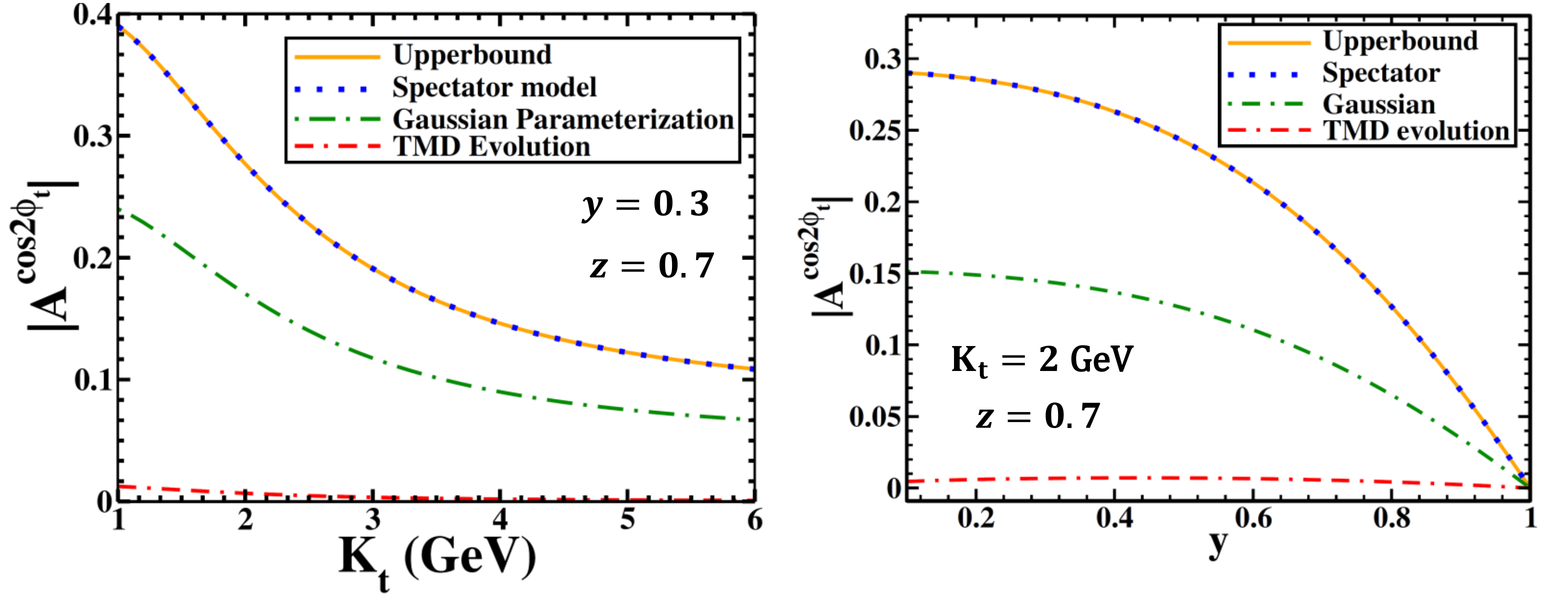
Chao (2012)



RESULTS

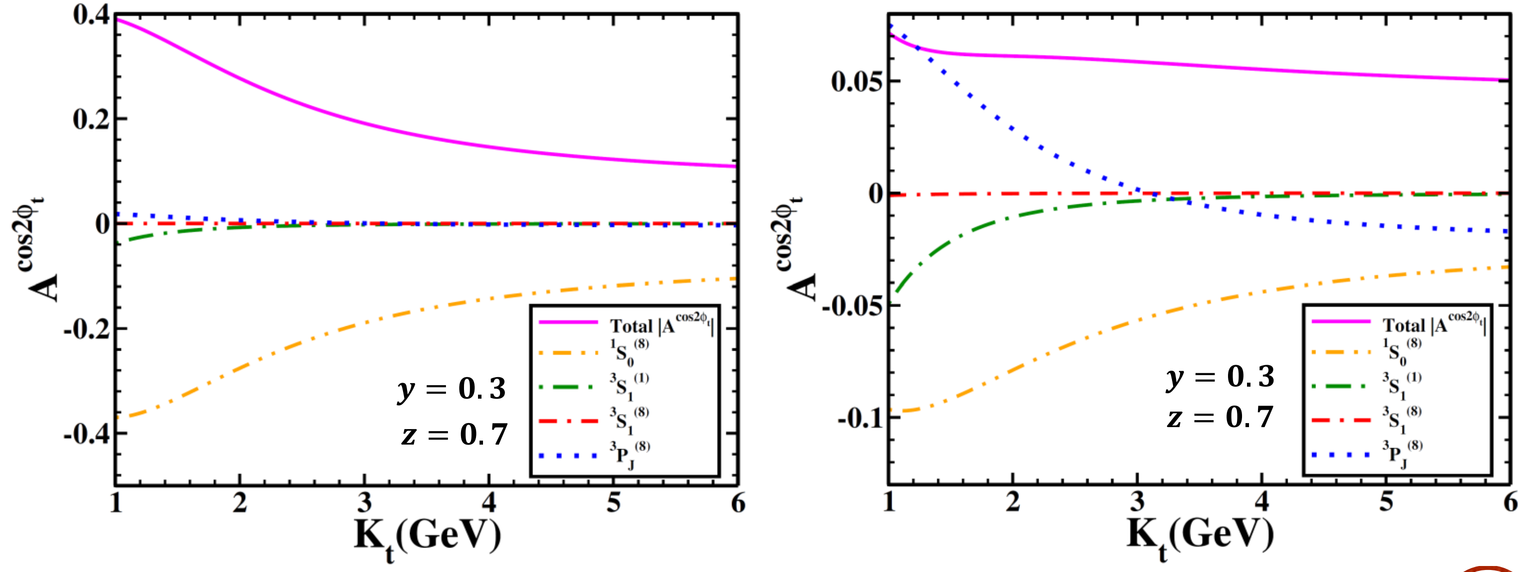
(A) Comparing $|A^{\cos 2\phi_t}|$ calculated in SM, GP and TMD evolution with the upper bound on the asymmetry.

(A)



(B) Contribution to $|A^{\cos 2\phi_t}|$ from all the color singlet and color octet states for two sets of LDMEs, CMSWZ (left) and SV (right).

(B)



Chao (2012)

Sharma (2013)

RK,A.Mukerjee,A.Pawar,M.Siddiqah ,arXiv:2203.13516

CONCLUSION

We calculated the $\cos 2\phi_t$ azimuthal asymmetry in a J/ψ and a jet electroproduction where the $J/\psi - jet$ pair produced in a almost back-to-back in the transverse plane.

We consider the full NRQCD framework for J/ψ production

We show the numerical estimates of the $A^{\cos 2\phi_t}$ using the parameterizations of the TMDs in the Spectator model and the Gaussian parameterizations.

We also show the effect of TMD evolution on the asymmetry, where we see that the magnitude of the asymmetry is small as compare with the asymmetries calculated using the TMD parameterizations.

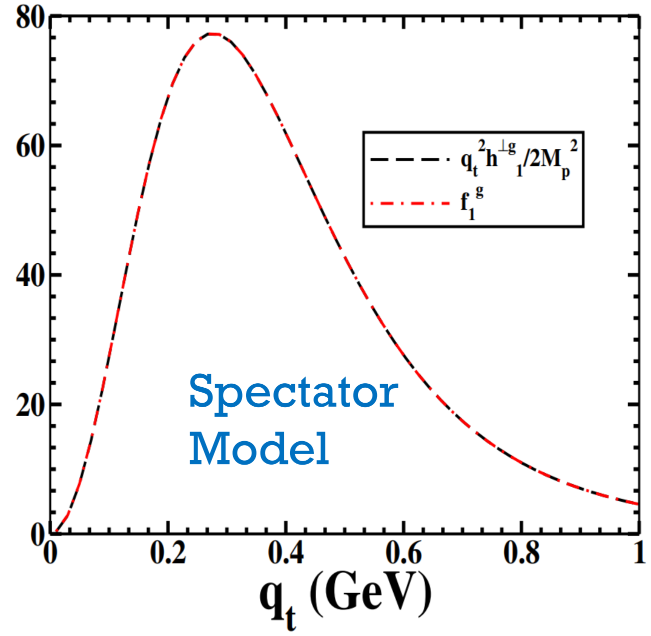
We obtained a significant asymmetry both in the Spectator model and Gaussian parameterizations of TMDs.

Back-to-back J/ψ and jet electroproduction could be a promising channel to probe poorly known linearly polarized gluon TMDs at the future proposed EIC.

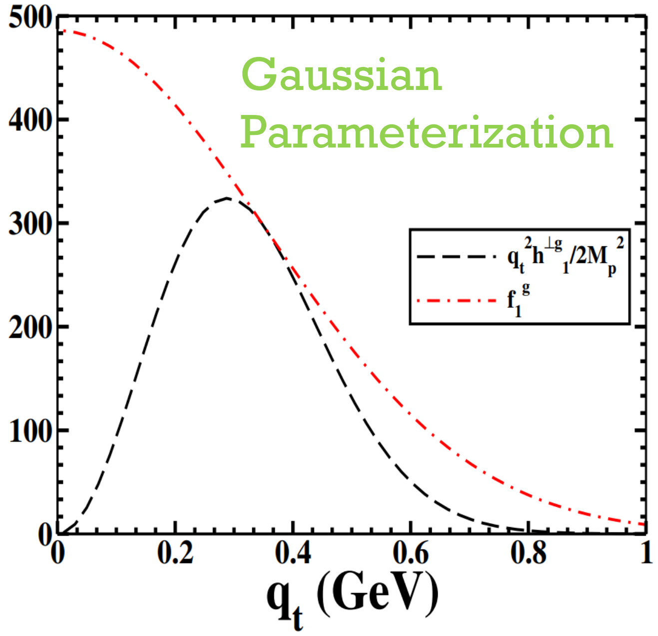
Thank You

BACKUP SLIDES…

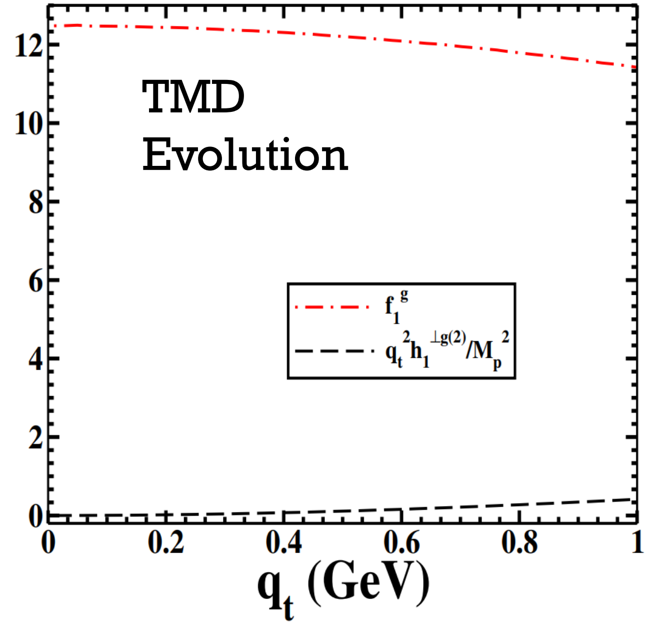
(a)



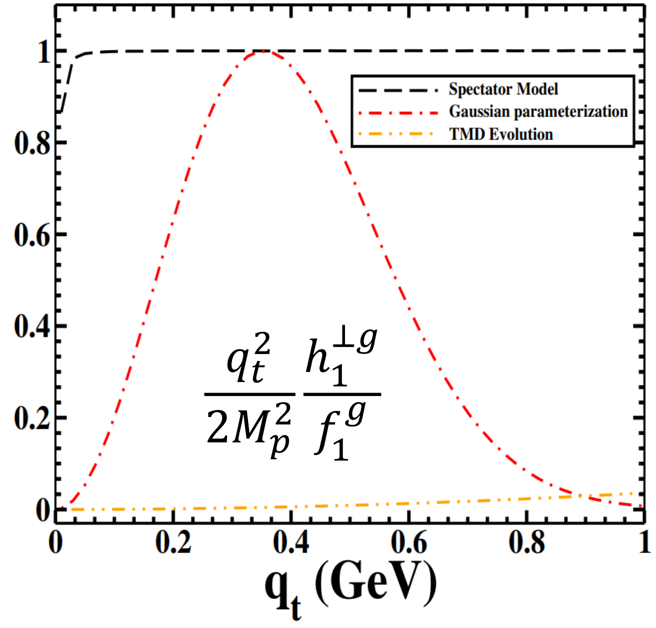
(b)



(c)



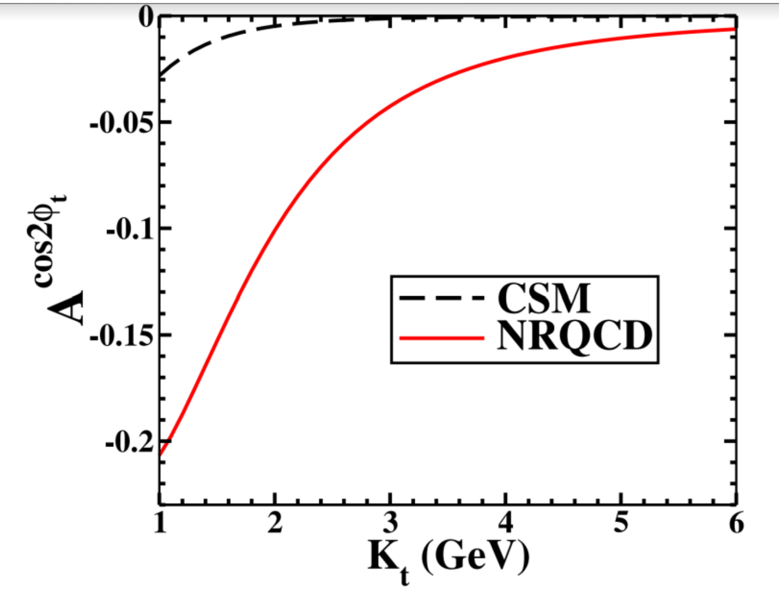
(d)

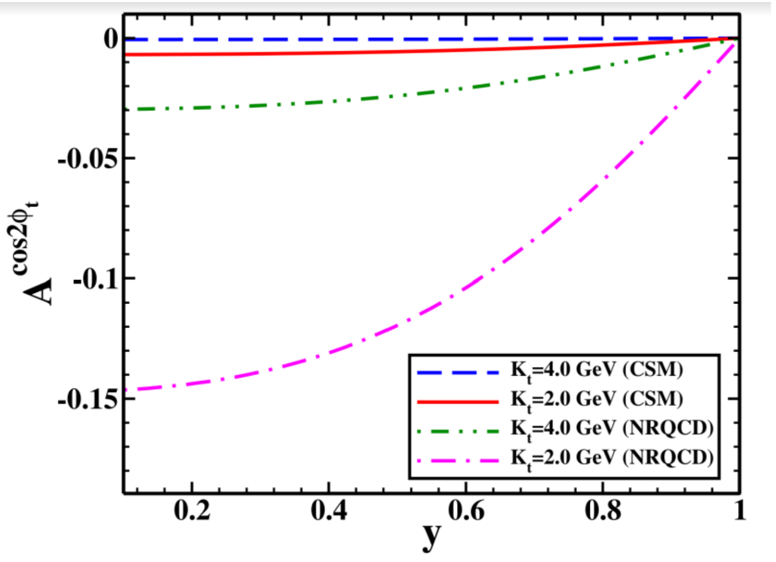


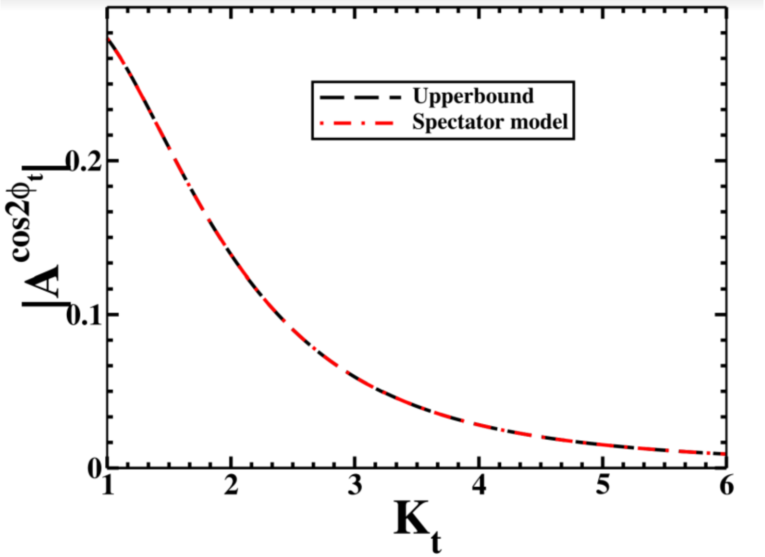
BACKUP SLIDES...

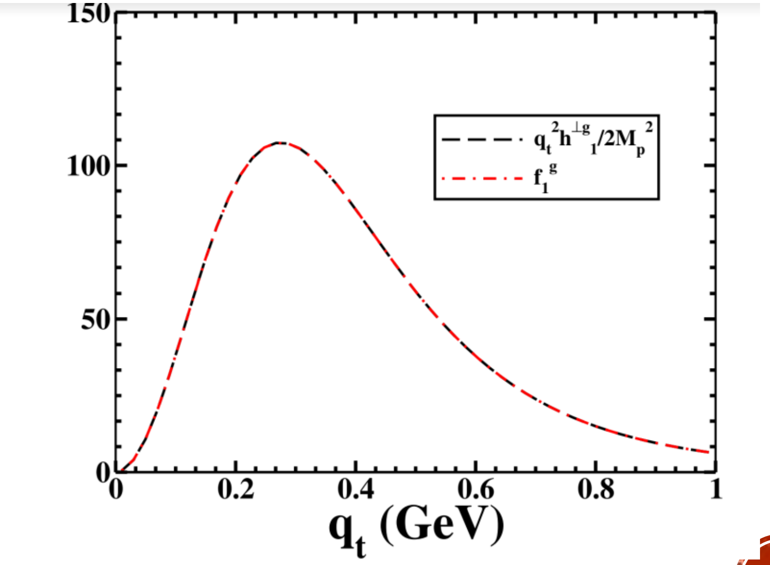
Spectator Model

$Q = 1.64$ GeV









AZIMUTHAL ASYMMETRY IN J/ψ PRODUCTION IN ep SCATTERING: TO PROBE $h_1^{\perp g}(x, k_T^2)$

Initial state interactions and final state interactions may affect the generalized factorization. Such effects are less complicated in ep compared with pp and pA.

The leading order process contributing to the $\cos(2\phi)$ asymmetry is $\gamma^* + g \to c + \bar{c}$

A. Mukherjee and S. Rajesh, EPJC 77, 854 (2017)

Contributes at $z = 1$, where z is energy fraction of γ^* carried by J/ψ in proton rest frame.

We extended it to the kinematical region $z < 1$. With the heavy quark pair produced in the hard process: $\gamma^* + g \to \mathrm{c} + \bar{\mathrm{c}} + \mathrm{g}$. We used full NRQCD framework for J/ψ production.

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