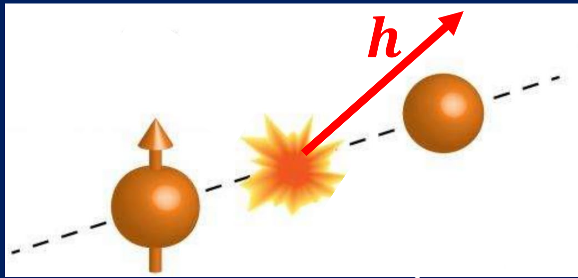


Transverse spin and transverse-momentum-dependent results from PHENIX

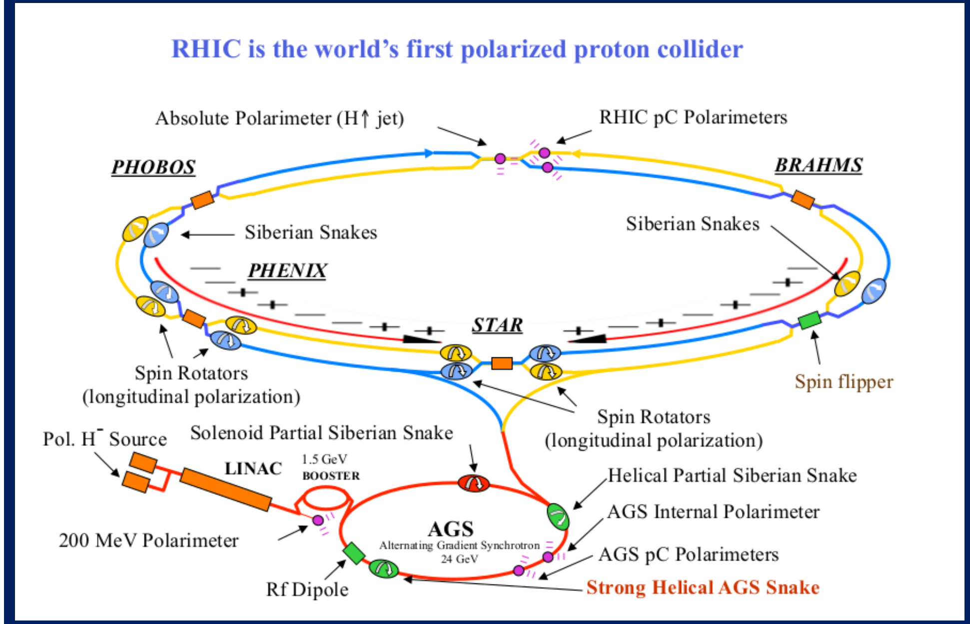
Christine A. Aidala
University of Michigan

Transversity 2022
Pavia, Italy
May 23-27, 2022

Polarized proton running at RHIC

Year	√s (GeV)	Recorded Luminosity for longitudinally / transverse polarized p+p STAR	Recorded Luminosity for longitudinally / transverse polarized p+p PHENIX	<P> in %
2006	62.4 200	-- pb^{-1} / 0.2 pb^{-1} 6.8 pb^{-1} / 8.5 pb^{-1}	0.08 pb^{-1} / 0.02 pb^{-1} 7.5 pb^{-1} / 2.7 pb^{-1}	48 57
2008	200	-- pb^{-1} / 7.8 pb^{-1}	-- pb^{-1} / 5.2 pb^{-1}	45
2009	200 500	25 pb^{-1} / -- pb^{-1} 10 pb^{-1} / -- pb^{-1}	16 pb^{-1} / -- pb^{-1} 14 pb^{-1} / -- pb^{-1}	55 39
2011	500	12 pb^{-1} / 25 pb^{-1}	18 pb^{-1} / -- pb^{-1}	48
2012	200 510	-- pb^{-1} / 22 pb^{-1} 82 pb^{-1} / -- pb^{-1}	-- pb^{-1} / 9.7 pb^{-1} 32 pb^{-1} / -- pb^{-1}	61/56 50/53
2013	510	300 pb^{-1} / -- pb^{-1}	155 pb^{-1}/ -- pb^{-1}	51/52
2015	200	52 pb^{-1} / 52 pb^{-1}	-- pb^{-1}/ 60 pb^{-1}	53/57
2015	200 *p* Au	total delivered Luminosity = 1.27 pb^{-1}		60
2015	200 *p* Al	total delivered Luminosity = 3.97 pb^{-1}		54

◯ = Transversely polarized



The PHENIX Detector

Central Arms: π^0, $\pi^{\pm}$, η, $e^{\pm}$, γ, …

- $|\eta| < 0.35$, $\Delta\phi = 0.5\pi$ per arm

Muon Arms: $\mu^{\pm}, h^{\pm}$

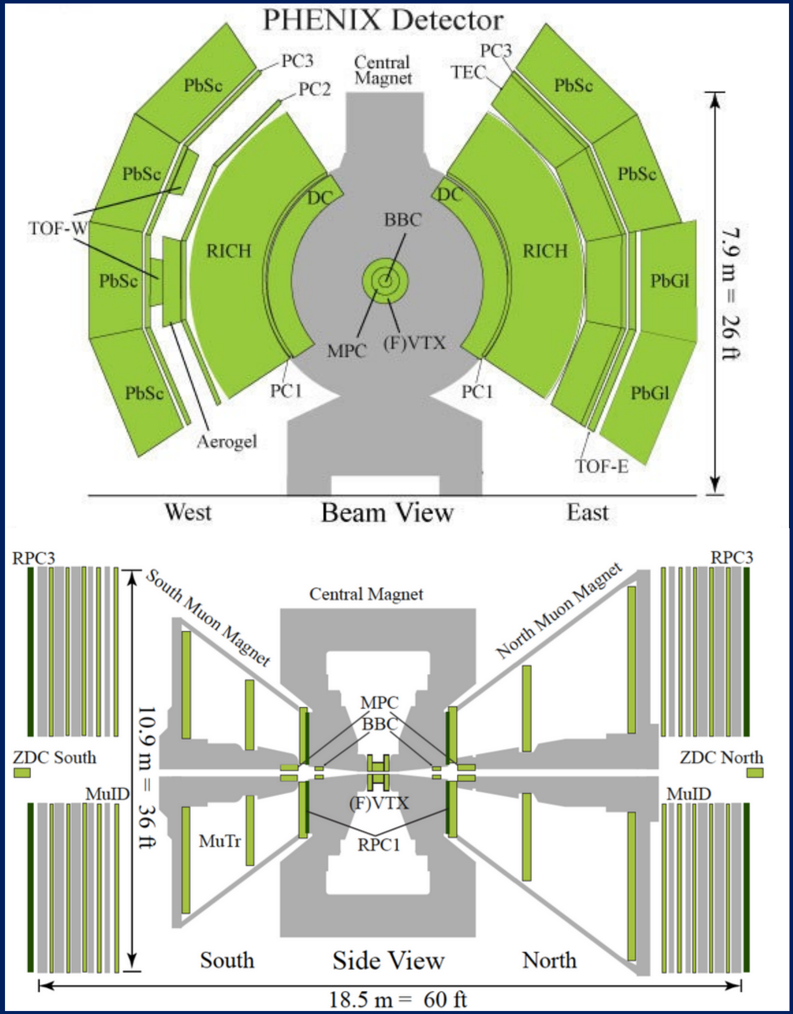
- $1.4 < |\eta| < 2.4$, $\Delta\phi = 2\pi$

Muon Piston Calorimeters: π^0, η

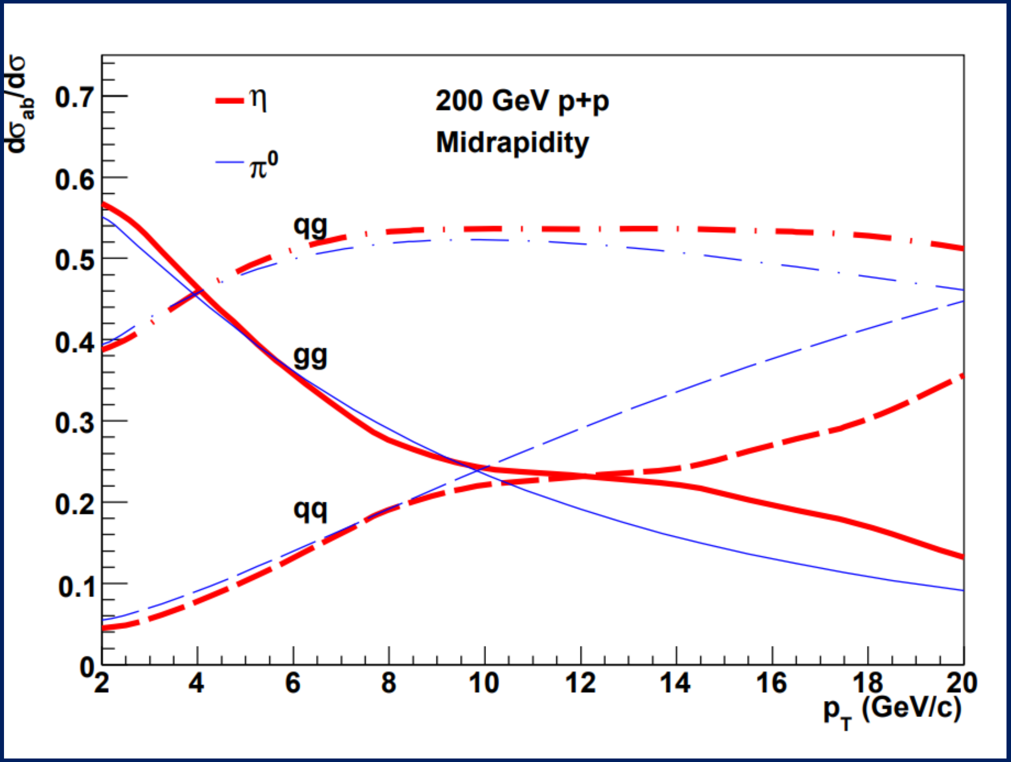
- $3.0 < |\eta| < 3.9$, $\Delta\phi = 2\pi$

Zero Degree Calorimeters: Neutrons

- $|\eta| > 6.8$, $\Delta\phi = 2\pi$



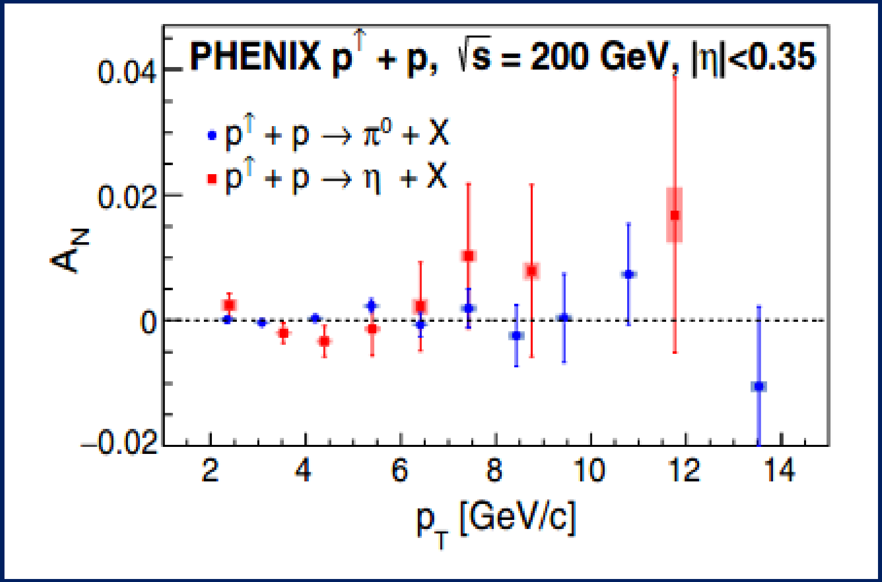
Partonic contributions for midrapidity π and η meson production

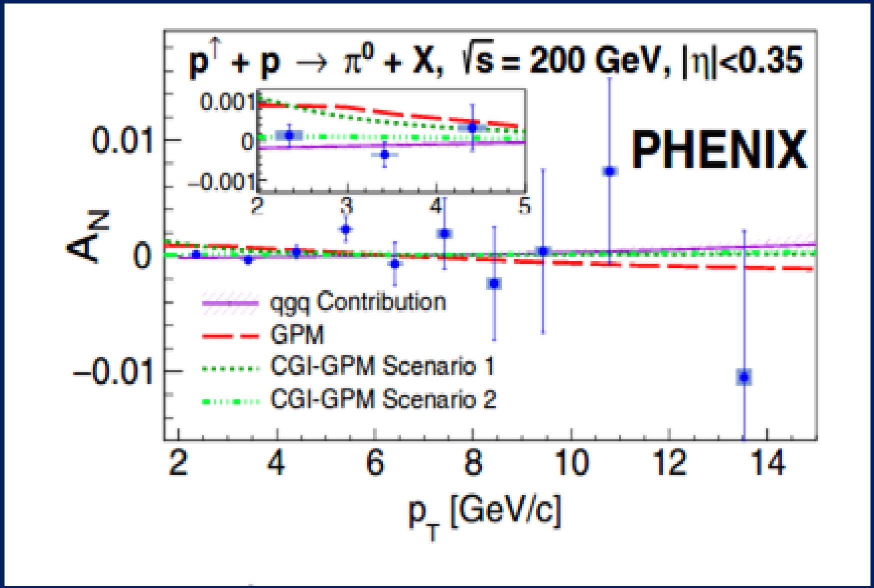


PRD 83, 032001 (2011)

- Midrapidity: Combination of quarks and gluons coming from the polarized proton
- (Forward rapidity: Primarily quarks coming from the polarized proton)

Midrapidity neutral pion and eta meson A_N at $\sqrt{s}$ = 200 GeV





Phys. Rev. D 103, 052009 (2021)

π^0 result

- Higher reach in p_T and factor of 3 increase in statistical precision compared to previous PHENIX result
- Useful in constraining twist-3 trigluon correlation functions Phys. Rev. D 89, 034029 (2014)
- Useful in constraining the gluon Sivers TMD function in the Generalized Parton Model (GPM) Phys. Rev. D 99, 036013 (2019)

***η* result**

- Higher reach in p_T and factor of 3 increase in statistical precision compared to previous PHENIX result
- Sensitive to strangeness effects in twist-3 correlation functions

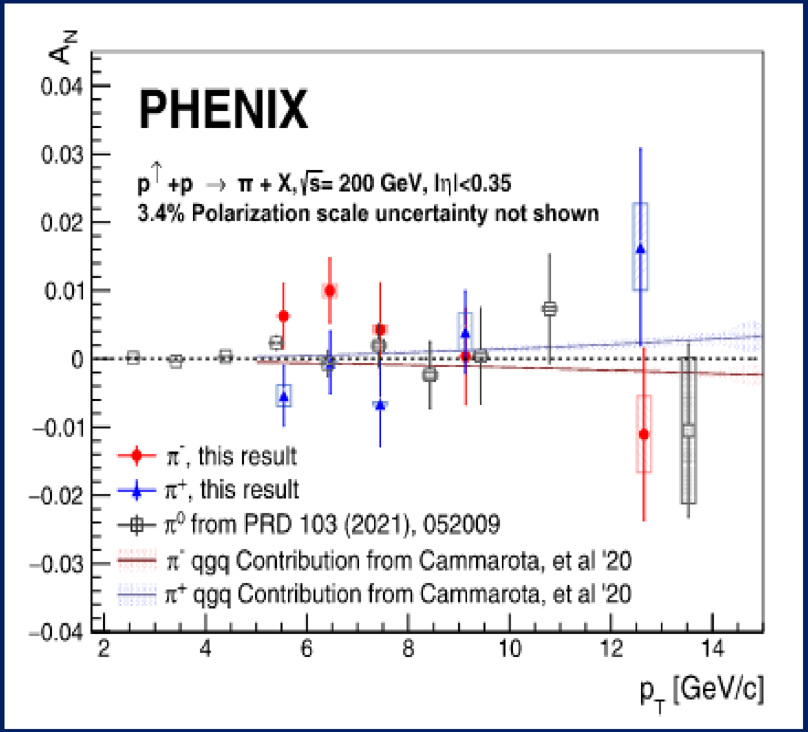
Midrapidity charged pion A_N at $\sqrt{s}$ = 200 GeV

First results of midrapidity charged pion A_N from PHENIX

Compared with π^0 A_N from PRD 103, 052009 (2021)

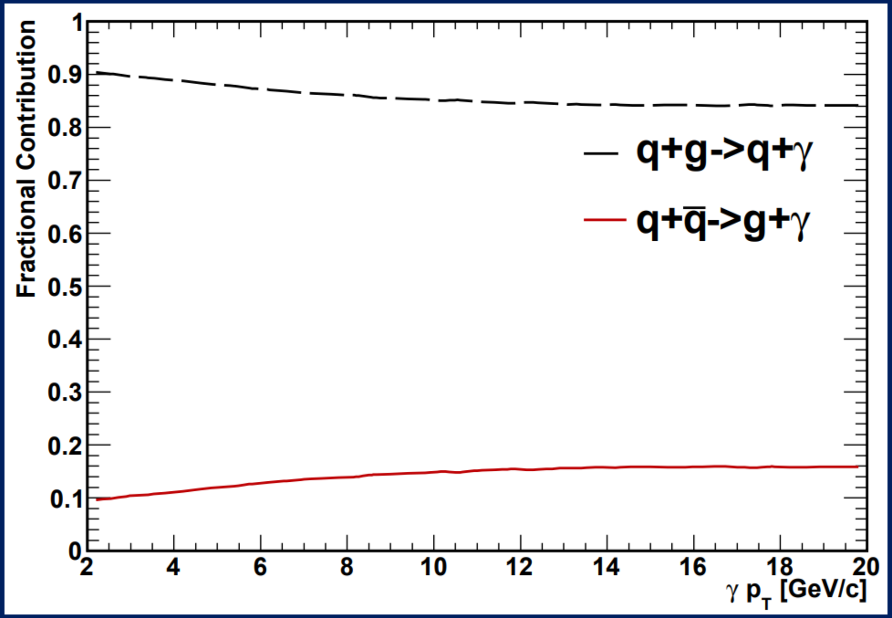
$\pi^{+/-}$ A_N consistent with zero and theoretical predictions in measured range, but there is an indication that $\pi^{+/-}$ behave differently (potential flavor dependence)

- Flavor dependence can be seen in the *qgq* theory calculations at higher p_T

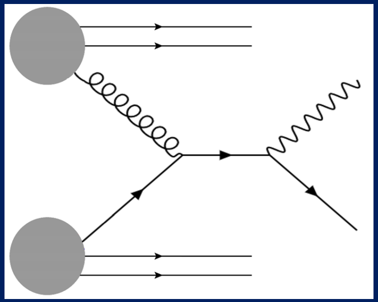


Phys. Rev. D 105, 032003 (2022)

Midrapidity direct photons



Fractional contribution of parton scattering to midrapidity inclusive direct photon production at leading order for p+p collisions at $\sqrt{s} = 200$ GeV. PHENIX PRD82, 072001 (2010)



- Only sensitive to initial-state proton structure
  - With isolation cut, NLO fragmentation photon contribution < 15% (PHENIX PRD 82, 072001 (2010))
- Strong sensitivity to gluons in the proton

Direct photon selection

- Tagging cut - subtract out photons that are tagged as coming from $\pi^0 \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma$ decays
- Isolation cut – eliminate decay photons and next-to-leading-order fragmentation photons

$$E_\gamma * 10\% > E_{cone}$$

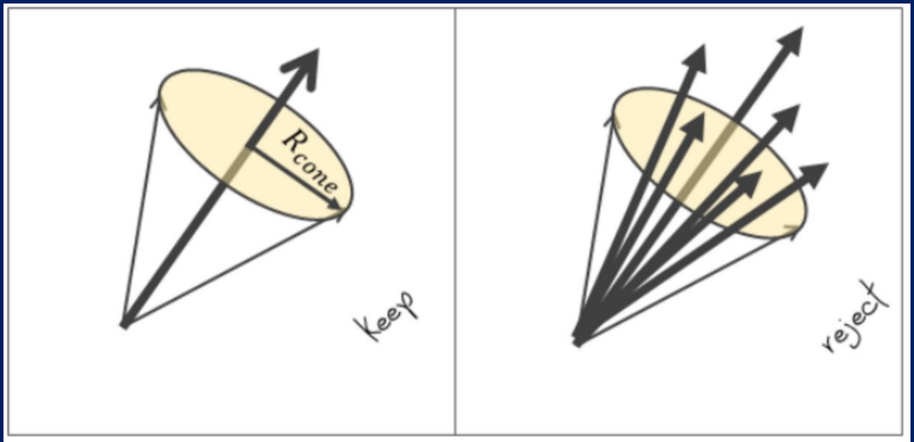
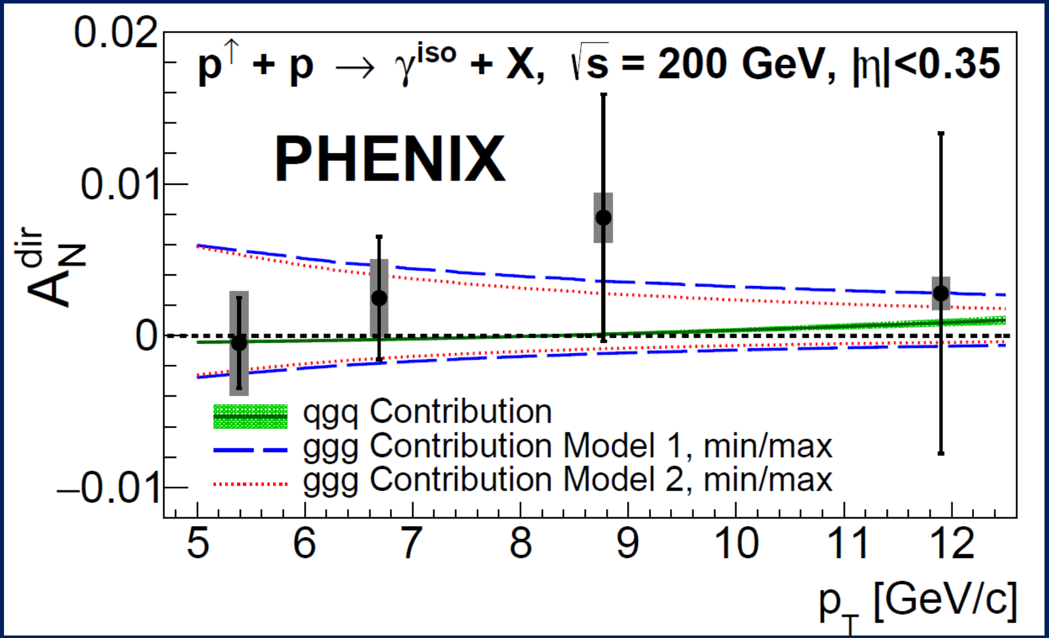


Figure from J.D. Osborn

Midrapidity direct photon A_N

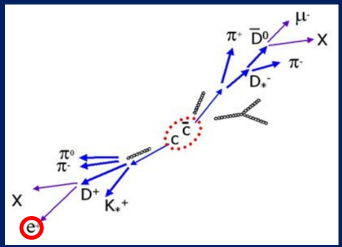
- Measured for the first time at RHIC
- Consistent with zero to within $\sim 2 \times 10^{-2}$
- Will help constrain the trigluon correlators
  - In particular in conjunction with new PHENIX midrapidity open heavy flavor electron results
- *qgq c*ontribution – Kanazawa, Koike, Metz, Pitonyak, PRD 91, 014013 (2015)
  - Already constrained by other measurements to be very small
- *ggg* contribution – Koike and Yoshida, PRD 85, 034030 (2012)



PRL 127, 162001 (2021)

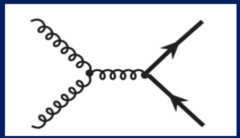
Open heavy flavor production

Open charm production is dominant contribution

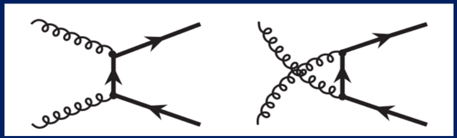


**

gg→QQ



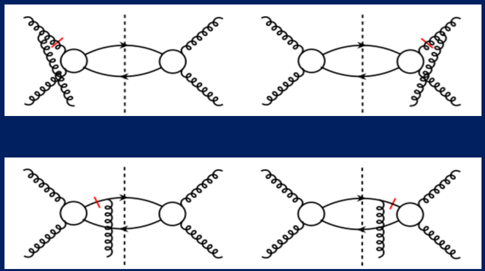
*



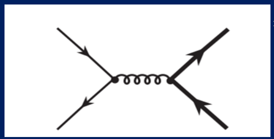
Dominant contribution @ 200 GeV midrapidity. ggg correlator **not** well constrained from previous measurements

ggg (trigluon) correlators

*



q$\bar{q}$→QQ

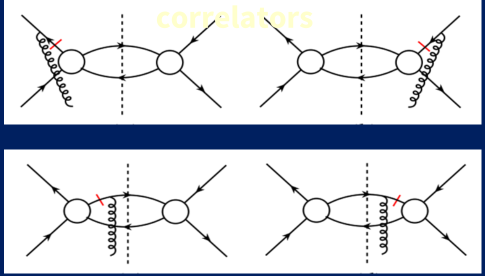


*

Small contribution @ 200 GeV midrapidity. qgq correlator somewhat constrained from previous measurements

qgq (Efremov-Teryaev-Qiu-Sterman) correlators

*



*Kang, Qiu, Vogelsang, Yuan, PRD78, 114013

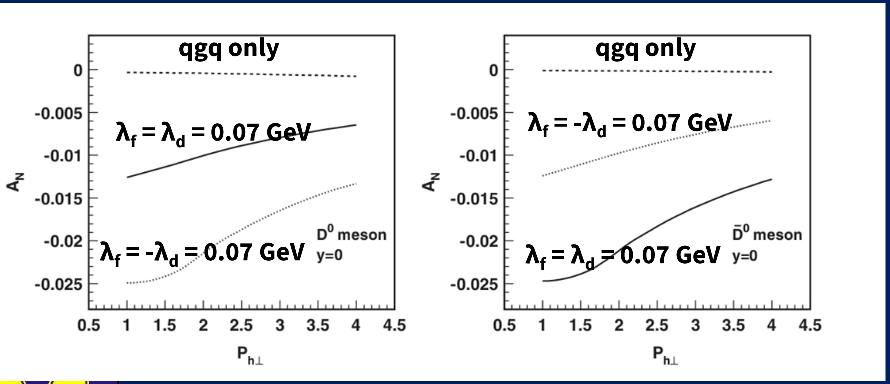
**S. Sakai, The Azimuthal Anisotropy of Electrons from Heavy Flavor Decays in sqrt(s) = 200 GeV Au-Au Collisions at PHENIX, March 26, 2000

Open charm TSSA model calculations

PRD78, 114013 (Z. Kang, J. Qiu, W. Vogelsang, F. Yuan)

- Contributions from twist-3 qgq correlators as well as antisymmetric and symmetric ggg correlators considered
- Authors provided TSSA calculations for $A_N^{D0}(p_T)$, and $A_N^{\bar{D}0}(p_T)$
- Trigluon (ggg) correlators written as $T_G^{(f)}(x_1,x_2)$ (antisymmetric) and $T_G^{(d)}(x_1,x_2)$ (symmetric)
  - Model calculations rely on normalizing to the unpolarized gluon PDF

$$T_G^{(f)}(x,x) = \lambda_f G(x), \qquad T_G^{(d)}(x,x) = \lambda_d G(x)$$



PRD84, 014026 (Y. Koike, S. Yoshida)

- Contributions from twist-3 antisymmetric and symmetric ggg correlators considered
- Authors provided TSSA calculations for $A_N^{D0}(p_T)$, $A_N^{D+}(p_T)$, $A_N^{\bar{D}0}(p_T)$, and $A_N^{D-}(p_T)$
- Trigluon (ggg) correlators written as $N(x_1,x_2)$ (antisymmetric) and $O(x_1,x_2)$ (symmetric)
  - 4 independent contributions from these functions: {N(x,x), N(x,0), O(x,x), O(x,0)}
  - In $\sqrt{s}$ = 200 GeV $p^{\uparrow}+p$ collisions, m_c is negligible, and TSSAs depend on effective trigluon correlators N(x,x) - N(x,0) and O(x,x) + O(x,0)
  - Model calculations rely on normalizing to the unpolarized gluon PDF

$$O(x,x) = O(x,0) = N(x,x) = -N(x,0)$$
$$\text{[Model1]} \quad O(x,x) = K_G x G(x)$$
$$\text{[Model2]} \quad O(x,x) = K'_G \sqrt{x} G(x)$$

Open charm TSSA model calculations

PRD78, 114013 (Z. Kang, J. Qiu, W. Vogelsang, F. Yuan)

$$T_G^{(f)}(x,x) = \lambda_f G(x), \qquad T_G^{(d)}(x,x) = \lambda_d G(x)$$

PRD84, 014026 (Y. Koike, S. Yoshida)

$$O(x,x) = O(x,0) = N(x,x) = -N(x,0)$$

$$\text{[Model1]} \quad O(x,x) = K_G x G(x)$$

$$\text{[Model2]} \quad O(x,x) = K'_G \sqrt{x} G(x)$$

Effective trigluon correlators N(x,x) - N(x,0) and O(x,x) + O(x,0) are explicitly related to $T_G^{(f,d)}(x,x) = T_G^{(+,-)}(x,x)$ in PRD82, 054005

$$\frac{xg}{2\pi} T_G^{(+)}(x,x) = -4M_N \left(N(x,x) - N(x,0)\right)$$

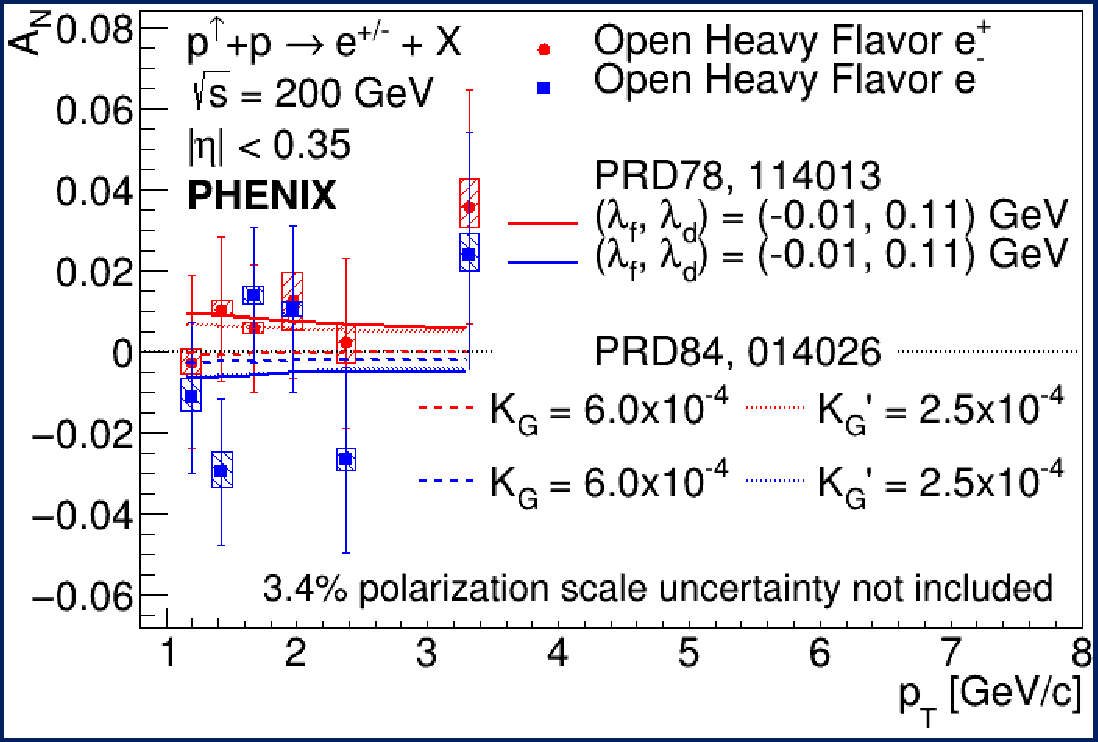
$$\frac{xg}{2\pi} T_G^{(-)}(x,x) = -4M_N \left(O(x,x) + O(x,0)\right)$$

The results presented here and in arXiv:2204.12899 [PHENIX] explicitly constrain parameters (λ_f,λ_d) from PRD78, 114013 as well as K_G and K_G' from PRD84, 014026

Open heavy flavor electron and positron A_N

arXiv:2204.12899

- Plotted alongside theoretical calculations from PRD78, 114013 (Z. Kang, J. Qiu, W. Vogelsang, F. Yuan) and PRD84, 014026 (Y. Koike, S. Yoshida) for parameters λ_f, λ_d, K_G, and K_G’ that best fit the data
- In models with (λ_f, λ_d), and K_G’ the best-fit parameters yield predictions for A_N with opposite signs for the separate charges



Open heavy flavor: Results from (λ_f,λ_d) scan

arXiv:2204.12899

- -0.2 GeV < λ_f < 0.2 GeV
- -0.1 GeV < λ_d < 0.3 GeV
- 41 steps per parameter
- Calculate A_N for D^0 and $\bar{D}^0$
- Simulate $D^0 \rightarrow e^+$ and $\bar{D}^0 \rightarrow e^-$ decay with PYTHIA6
- Calculate $A_N^{D0\rightarrow e+}$ and $A_N^{\bar{D}0\rightarrow e-}$
- Calculate $\Delta\chi^2_{(+,-)}(\lambda_f,\lambda_d) = \chi^2_{(+,-)}(\lambda_f,\lambda_d) - \chi^2_{(+,-)min}$

a_0, b_0, a_1, a_2 parameterizations provided by Z.B. Kang, J.W. Qiu, W. Vogelsang, F. Yuan (**PRD78, 114013**)

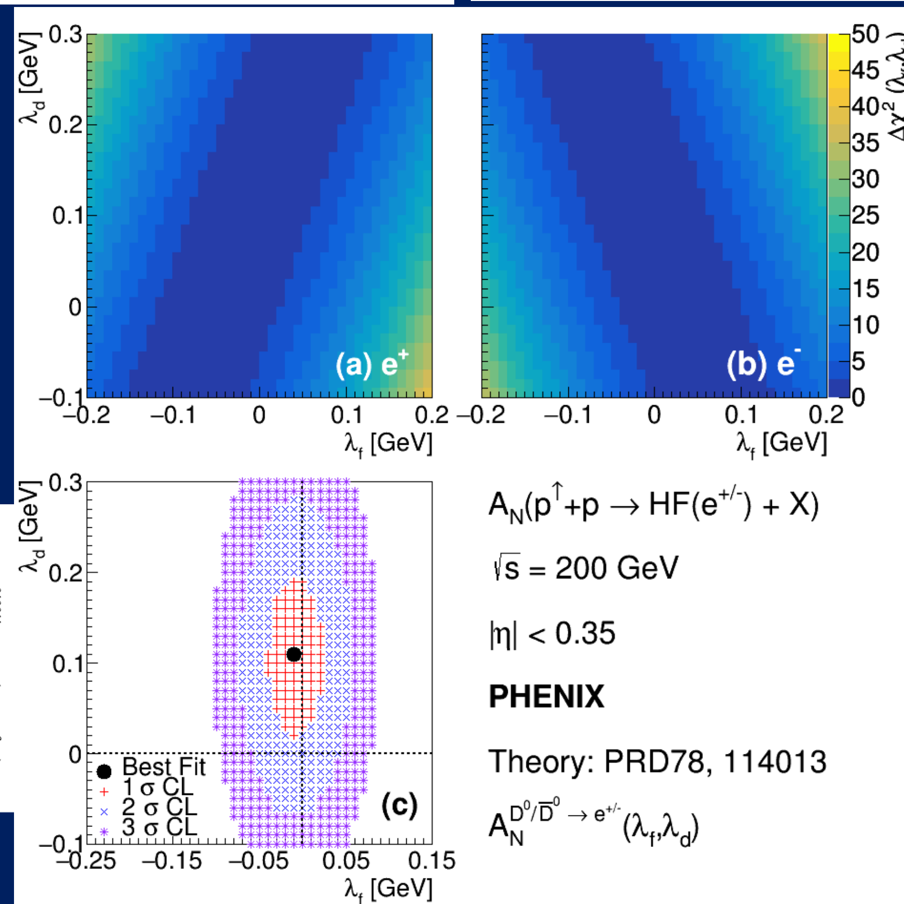
- a_0 and b_0: contributions from qgq correlators
- a_1 and a_2: contributions from trigluon correlators

$$\chi^2_{(+)}(\lambda_f,\lambda_d) = \sum_i^{nbins} \frac{\left(A_N^{(+)data} - A_N^{(+)theory}(\lambda_f,\lambda_d)\right)^2}{\sigma^2_{(+)data}}$$

$$\chi^2_{(-)}(\lambda_f,\lambda_d) = \sum_i^{nbins} \frac{\left(A_N^{(-)data} - A_N^{(-)theory}(\lambda_f,\lambda_d)\right)^2}{\sigma^2_{(-)data}}$$

$$\chi^2(\lambda_f,\lambda_d) = \chi^2_{(+)}(\lambda_f,\lambda_d) + \chi^2_{(-)}(\lambda_f,\lambda_d)$$

$$\chi^2(\lambda_f,\lambda_d) - \chi^2_{min} < n^2$$



$$A_N^{D^0} = a_0 + \lambda_f a_1 + \lambda_d a_2$$
$$A_N^{\bar{D}^0} = b_0 + \lambda_f a_1 - \lambda_d a_2$$

Open heavy flavor: Results from (λ_f,λ_d) scan

arXiv:2204.12899

- -0.2 GeV < λ_f < 0.2 GeV
- -0.1 GeV < λ_d < 0.3 GeV
- 41 steps per parameter
- Calculate A_N for D^0 and $\bar{D}^0$
- Simulate $D^0 \rightarrow e^+$ and $\bar{D}^0 \rightarrow e^-$ decay with PYTHIA6
- Calculate $A_N^{D0\rightarrow e+}$ and $A_N^{\bar{D}0\rightarrow e-}$
- Calculate $\Delta\chi^2_{(+,-)}(\lambda_f,\lambda_d) = \chi^2_{(+,-)}(\lambda_f,\lambda_d) - \chi^2_{(+,-)min}$

a_0, b_0, a_1, a_2 parameterizations provided by Z.B. Kang, J.W. Qiu, W. Vogelsang, F. Yuan (**PRD78, 114013**)

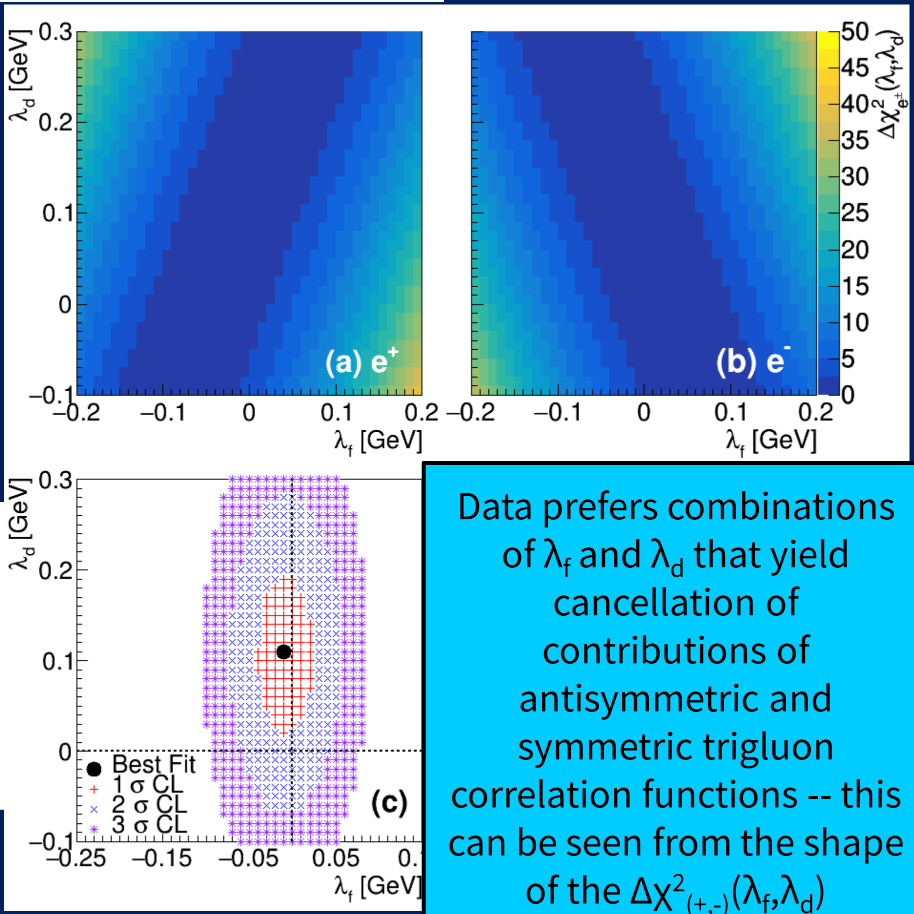
- a_0 and b_0: contributions from qgq correlators
- a_1 and a_2: contributions from trigluon correlators

$$\chi^2_{(+)}(\lambda_f,\lambda_d) = \sum_i^{nbins} \frac{\left(A_N^{(+)data} - A_N^{(+)theory}(\lambda_f,\lambda_d)\right)^2}{\sigma^2_{(+)data}}$$

$$\chi^2_{(-)}(\lambda_f,\lambda_d) = \sum_i^{nbins} \frac{\left(A_N^{(-)data} - A_N^{(-)theory}(\lambda_f,\lambda_d)\right)^2}{\sigma^2_{(-)data}}$$

$$\chi^2(\lambda_f,\lambda_d) = \chi^2_{(+)}(\lambda_f,\lambda_d) + \chi^2_{(-)}(\lambda_f,\lambda_d)$$

$$\chi^2(\lambda_f,\lambda_d) - \chi^2_{min} < n^2$$



Data prefers combinations of λ_f and λ_d that yield cancellation of contributions of antisymmetric and symmetric trigluon correlation functions -- this can be seen from the shape of the $\Delta\chi^2_{(+,-)}(\lambda_f,\lambda_d)$ distributions

$$A_N^{D^0} = a_0 + \lambda_f a_1 + \lambda_d a_2$$
$$A_N^{\bar{D}^0} = b_0 + \lambda_f a_1 - \lambda_d a_2$$

Open heavy flavor: Results from K_G and K_G' scans

arXiv:2204.12899

- $-0.005 < K_G < 0.005$
- $-0.00025 < K_G' < 0.00075$
- 101 steps per parameter
- Calculate A_N for D^0, D^+, $\bar{D}^0$, and D^-
- Simulate $D^0 \rightarrow e^+$, $D^+ \rightarrow e^+$, $\bar{D}^0 \rightarrow e^-$, and $D^- \rightarrow e^-$ decay with PYTHIA6
- Calculate $A_N^{D0 \rightarrow e+}$, $A_N^{D+ \rightarrow e+}$, $A_N^{\bar{D}0 \rightarrow e-}$, and $A_N^{D- \rightarrow e-}$
- Calculate $\Delta\chi^2_{(+,-)}(K_G) = \chi^2_{(+,-)}(K_G) - \chi^2_{(+,-)min}$ and $\Delta\chi^2_{(+,-)}(K_G') = \chi^2_{(+,-)}(K_G') - \chi^2_{(+,-)min}$

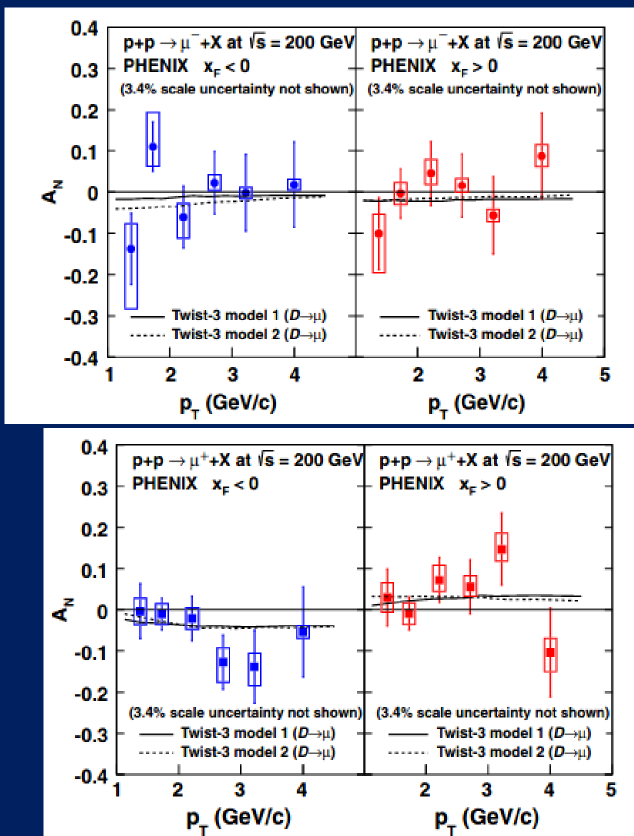
1σ Confidence Intervals:

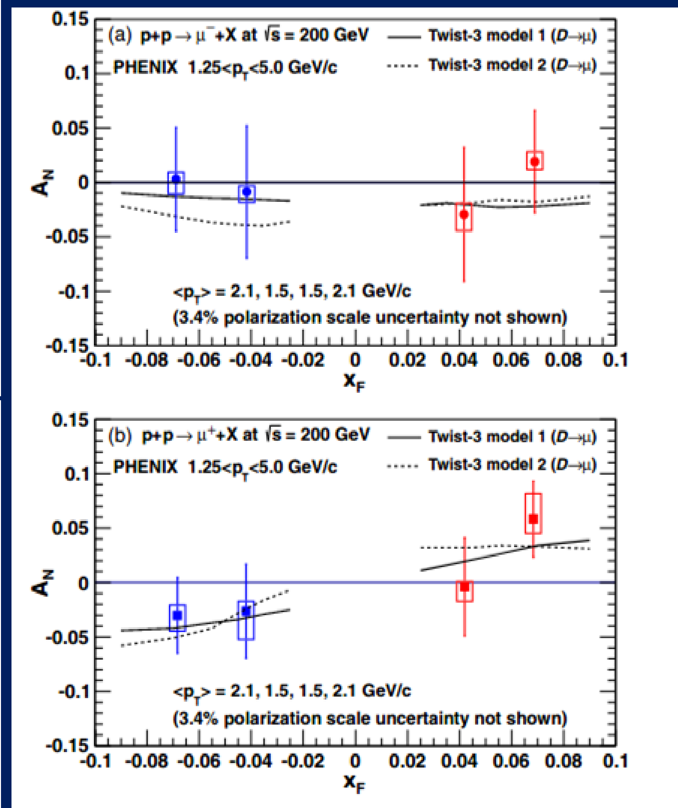
$K_G = 6.0\times10^{-4}$ (+0.0014 -0.0017)

$K_G' = 2.5\times10^{-4}$ (± 0.00025)

Consistent with modest upper bound on K_G and K_G' derived in PRD84, 014026 (2011)

Previous open heavy flavor results: Forward muon A_N at $\sqrt{s}$ = 200 GeV

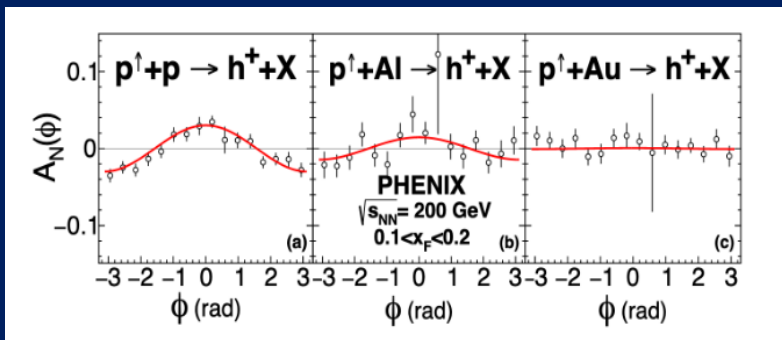


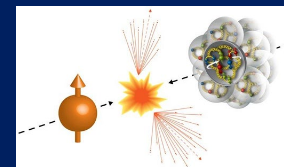


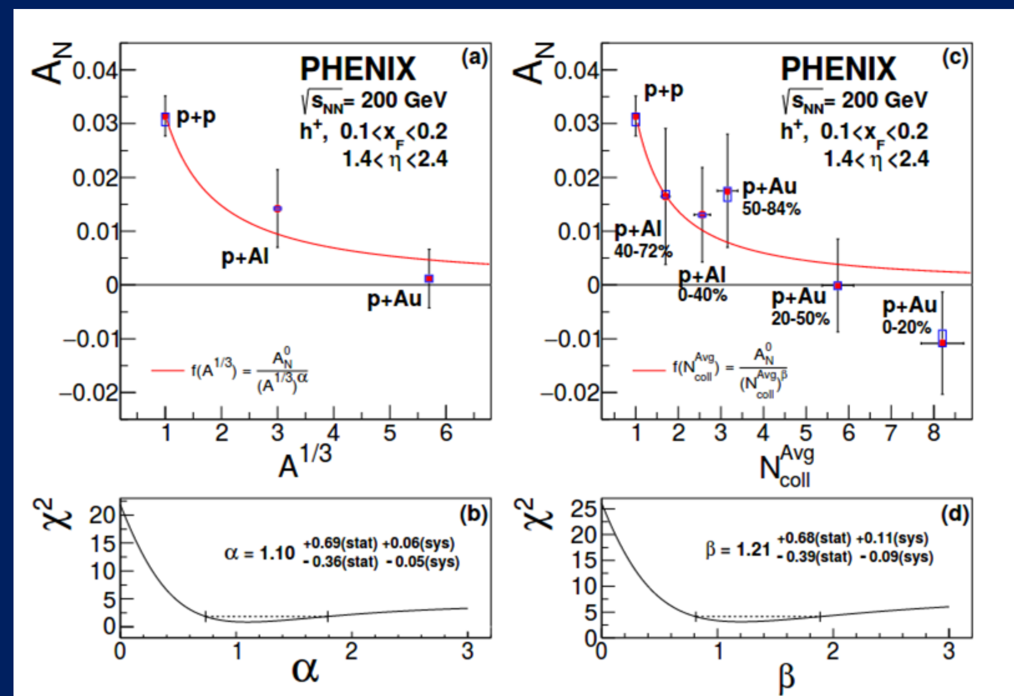
Asymmetries consistent with 0 and theoretical predictions taking into account contributions from trigluon correlation functions from Phys. Rev. D 84, 014026 (2011)

Phys. Rev. D 95, 112001 (2017)

Nuclear effects: Forward hadron A_N at $\sqrt{s_{NN}}$ = 200 GeV ($p^{\uparrow}$+p, $p^{\uparrow}$+Al, $p^{\uparrow}$+Au)



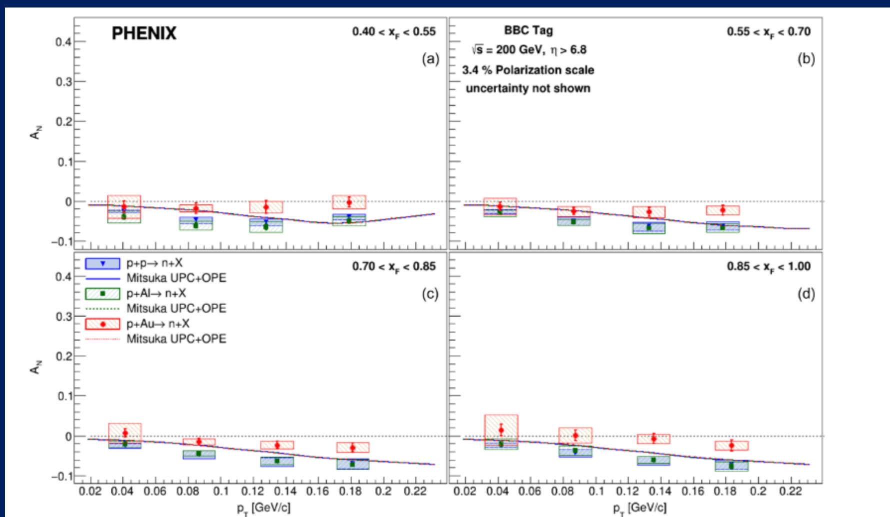


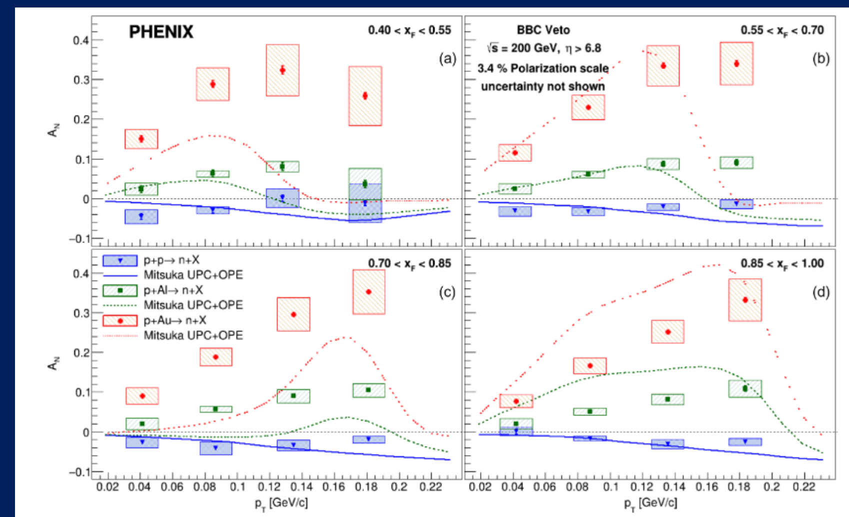


- Inclusive positively charged forward hadron TSSAs
  - *$\pi^+/K^+/p$ fractions: 0.45/0.47/0.05*
- Clear suppression of A_N in p+A observed

Phys. Rev. Lett. 123, 122001 (2019)

Nuclear effects: Forward neutron A_N at $\sqrt{s_{NN}} = 200$ GeV ($p^{\uparrow}+p$, $p^{\uparrow}+Al$, $p^{\uparrow}+Au$)





- Very forward neutron asymmetries are shown for *$p^{\uparrow}$+p, $p^{\uparrow}$+Al, $p^{\uparrow}$+Au* collision systems for both beam-beam-counter (BBC) tagged events, dominated by hadronic interactions, and BBC vetoed events (with little activity in the BBC) that show an enhancement of ultra peripheral collisions (UPC)
- The asymmetries qualitatively agree with the UPC + one-pion-exchange (OPE) theory predictions

Phys. Rev. D 105, 032004 (2022)

See also detailed p_T-dependent p+p results in Phys. Rev. D 103, 032007 (2021)

Conclusions

- A number of recent transverse-single-spin asymmetry results from PHENIX : midrapidity $\pi^0, \pi^\pm$, η, direct γ, open heavy flavor electrons, forward $h^\pm$ and very forward neutrons in $p^\uparrow + p, p^\uparrow +$ Al, $p^\uparrow +$ Au

- New midrapidity measurements particularly valuable for constraining gluon spin-momentum correlations in transversely polarized protons

- Striking differences observed between $p^\uparrow + p$ and collisions between polarized protons and nuclei

- Stay tuned for more results to come from 2012 and 2015 data!

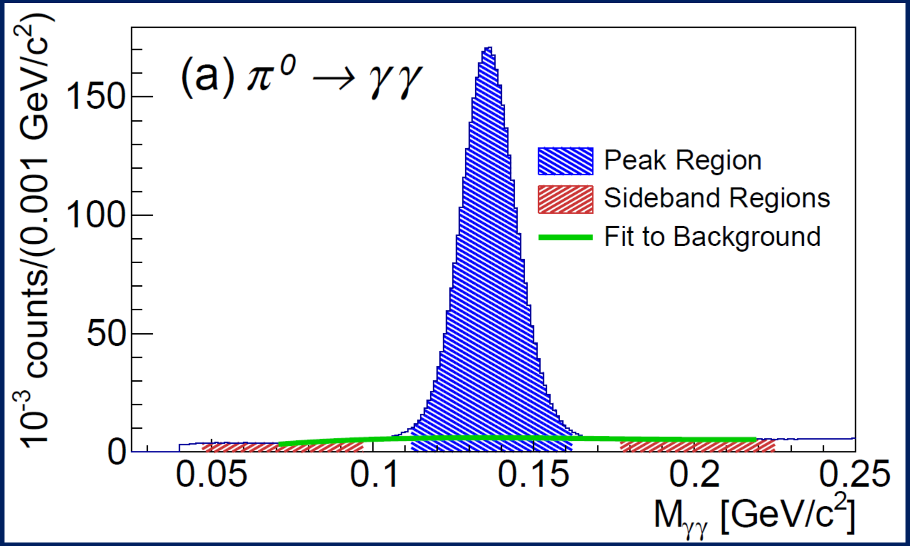
Extra

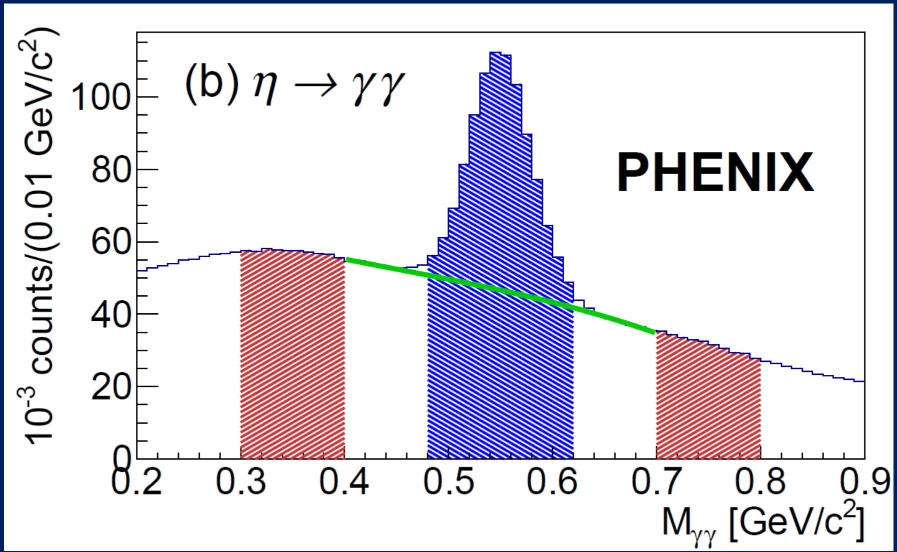
Asymmetry background subtraction

$$A_N = \frac{A_N^{peak} - r A_N^{bg}}{1-r}$$

Where $r = \frac{N_{bg}}{N_{sig}+N_{bg}}$ is measured from a fit in the invariant mass peak region

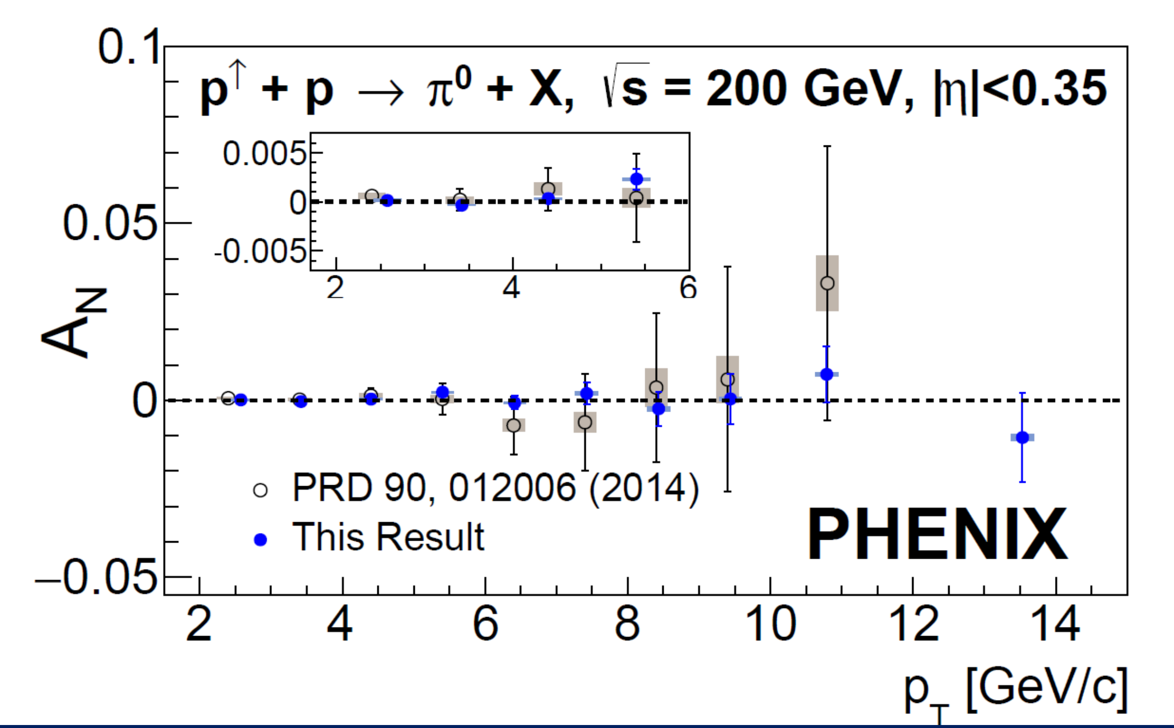
Example invariant mass spectra for photon pairs in the West Arm with $4 < p_T < 5$ GeV/c





PRD103, 052009 (2021)

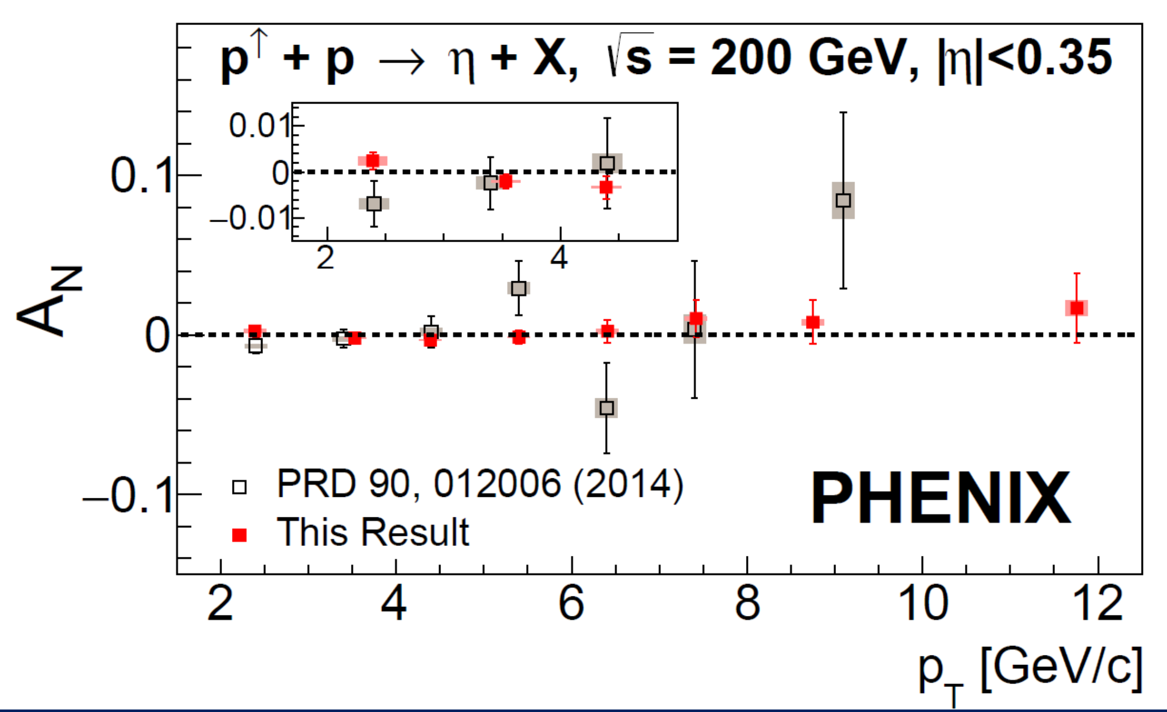
Midrapidity neutral pion A_N



PRD103, 052009 (2021)

- Consistent with zero to within 10^{-4} at low p_T
- Definitive results from PHENIX, from final $p^{\uparrow}+p$ data taken in 2015
- Factor of 3 improvement in statistical uncertainty with respect to previous result, and higher reach in p_T

Midrapidity η meson A_N



PRD103, 052009 (2021)

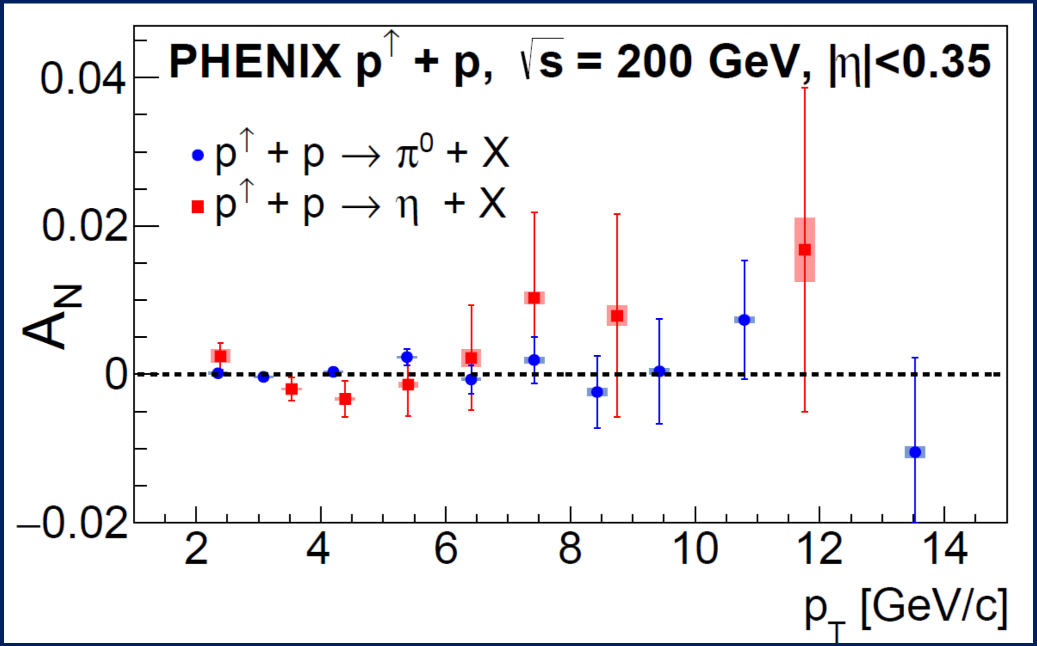
- Consistent with zero to within 5×10^{-3} at low p_T
- Also a factor of 3 improvement in statistical uncertainty with respect to previous result, and higher reach in p_T

Comparing midrapidity π^0 and η results

- Any differences in π^0 and η A_N could be due to effects of strangeness or isospin in hadronization:

$$\pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

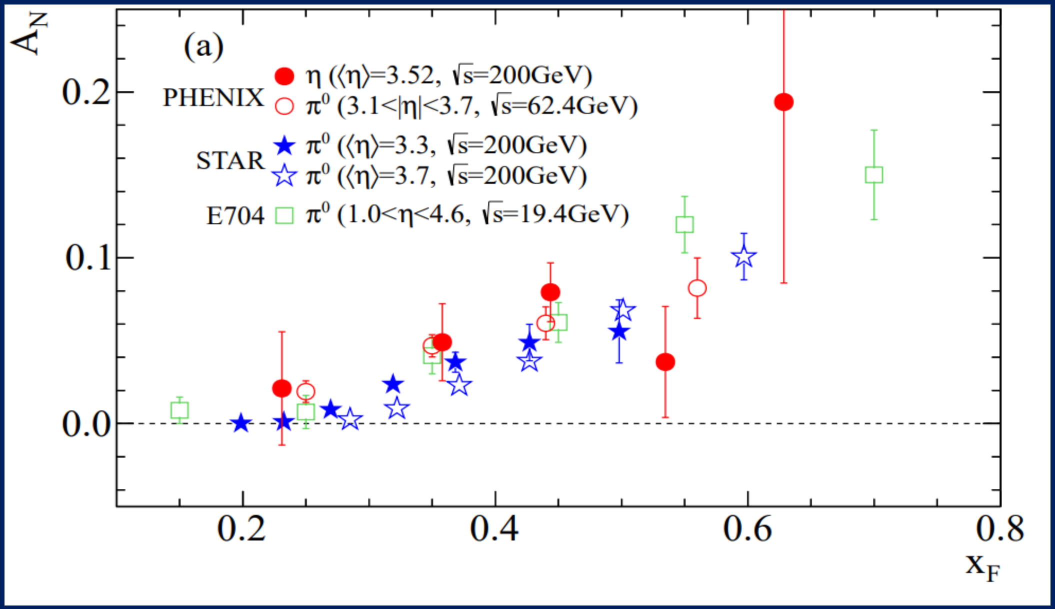
$$\eta = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$



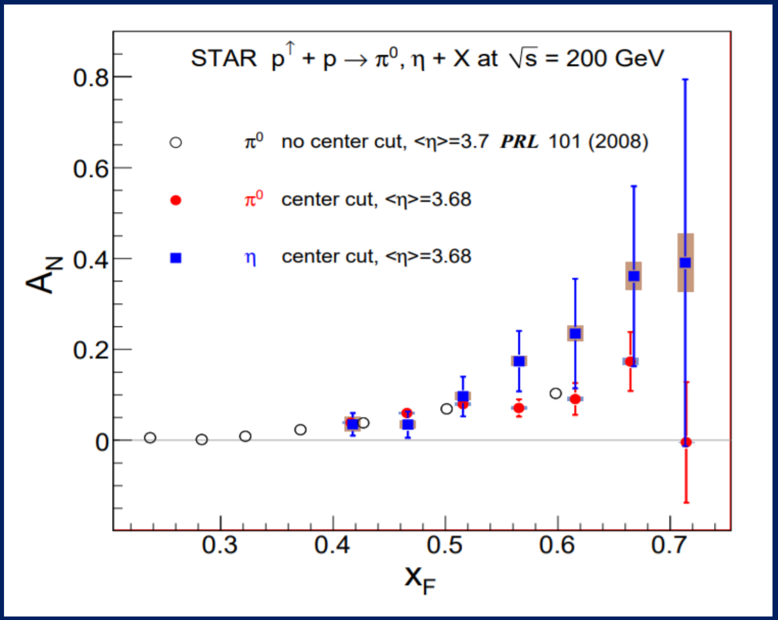
PRD103, 052009 (2021)

- Or to mass effects in hadronization:
  $m_{\pi^0} \approx 135$ MeV/c^2 $m_{\eta} \approx 548$ MeV/c^2

Comparing forward π^0 and η results



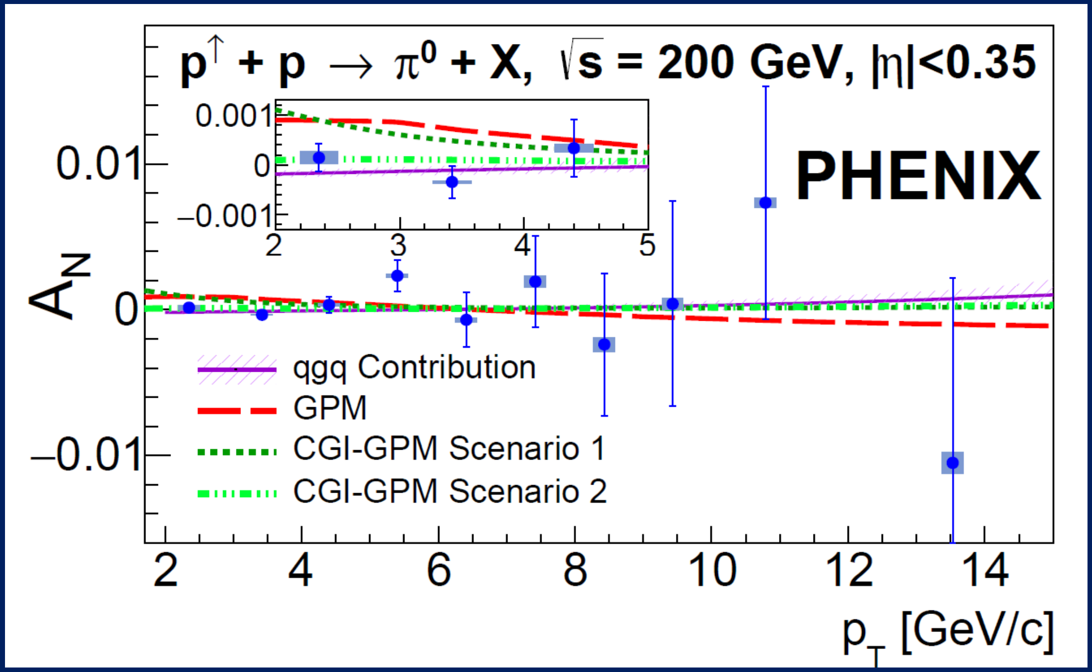
PHENIX, PRD 90, 072008 (2014)



STAR, PRD 86, 051101(R) (2012)

- Forward rapidity: Large contribution of quarks coming from the polarized proton
- Results hint that A_N^{η} may be larger than $A_N^{\pi^0}$ at forward rapidity. Further studies required.

Midrapidity $A_N^{\pi^0}$: Theoretical predictions



PRD103, 052009 (2021)

- Very small qgq correlator contribution predicted
  - JAM Collaboration, PRD 102, 054002 (2020)
- Results can help constrain gluon spin-momentum correlations
  - Twist-3 trigluon correlators (Beppu, Kanazawa, Koike, Yoshida, PRD 89, 034029 (2014))
  - Gluon Sivers function – in the Generalized Parton Model (GPM) (D'Alesio Flore, Murgia, Pisano, Taels, PRD 99, 036013 (2019))

Direct photon background:
Hadron decay photons with missing partner

Sometimes only one of the photons from a $h \to \gamma\gamma$ decay is measured and the second photon is missed

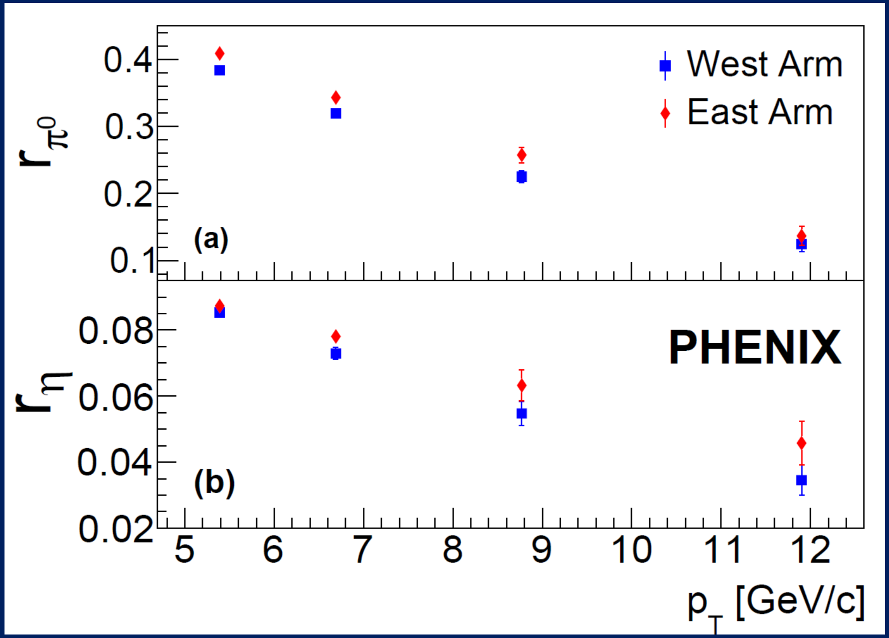
- η decays tend to be more asymmetric because they are ~4 times heavier than π^0s

Estimate number missed using ones that were tagged:

From data: Ratio of number isolated decay photons to direct photon candidate sample

$$r_{miss} = \frac{N_{bg}}{N_{sig} + N_{bg}} = R\,\frac{N_{tag}^{iso,h}}{N^{iso}}$$

From simulation: Converts between tagged decay photons to missed decay photons



PRL 127, 162001 (2021)

Direct photon background: Merging of π^0 decay photons

Photon merging - sometimes the two photons from a $\pi^0 \rightarrow \gamma\gamma$ decay are so close together that the EMCal cannot resolve them as separate photons

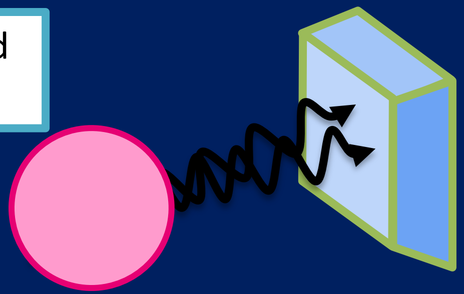
Similar to the background fraction due to missing one of the decay photons

Found to be negligible after a cluster shower shape cut

From Data: Ratio of number isolated decay photons to direct photon sample

$$r_{merge} = \frac{N_{bg}}{N_{sig} + N_{bg}} = \frac{N_{merge}}{N_{tag}} \frac{N_{tag}^{iso,h}}{N^{iso}}$$

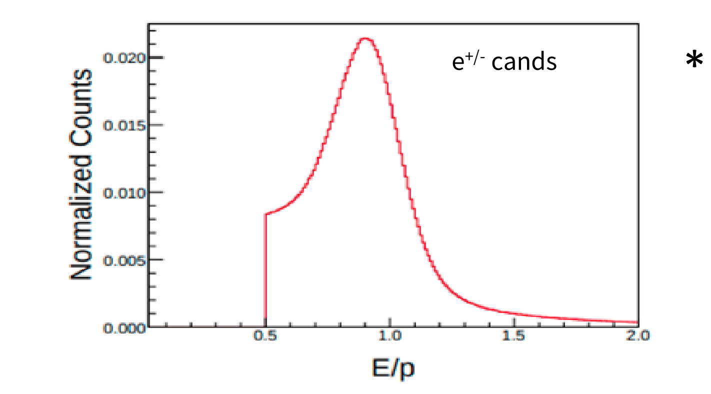
From Simulation: Converts between tagged decay photons and merged π^0 photons

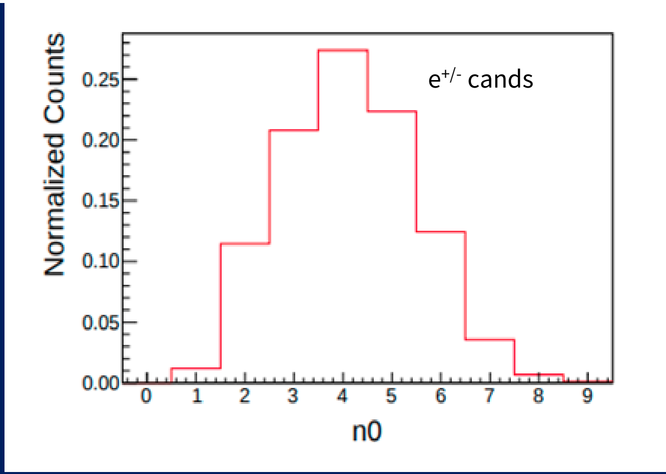


Open heavy flavor electron analysis procedure 1

$e^{+/-}$ identification at PHENIX

- $|(E/p - \langle E/p \rangle)/\sigma_{E/p}| < 2$ -- ($\langle E/p \rangle \sim 1$)
- Track matching to EMCal energy deposits and RICH shower ring center
- >1 photomultiplier firing in RICH -- $p_e > 20$ MeV/c
- EM shower shape probability > 0.01
- Hit requirement in inner 2 layers of VTX
- Conversion veto cut on opening angle of nearby $e^{+/-}$ candidates





*Esha, Roli. (2020, September 15). Electron Identification in PHENIX

Open heavy flavor electron analysis procedure 2

TSSA Observable

A_N is calculated using the Relative Luminosity formula, integrating over the φ ranges of the east and west arms

$$A_N = \frac{1}{\langle|\cos\phi|\rangle}\frac{1}{P}\frac{N_L^{\uparrow} - R\cdot N_L^{\downarrow}}{N_L^{\uparrow} + R\cdot N_L^{\downarrow}} \quad \text{where } R = \frac{\mathcal{L}^{\uparrow}}{\mathcal{L}^{\downarrow}}$$

Background Sources (Heavy Flavor $e^{+/-}$)

- Photonic: π^0, η, γ: Asymmetries measured to be 0 → treated as dilution
  - π^0 and η (**PRD 103, 052009**)
  - γ (**PRL 127, 162001**)
- Nonphotonic: J/ψ, Ke3
  - Ke3 is a negligible fraction
  - J/ψ A_N taken from **PRD 82, 112008**, large source of statistical uncertainty
- Hadron contamination: $h^{+/-}$
  - $h^{+/-}$ A_N taken from **PRL 95, 202001**

Cross checks and systematic studies (Heavy Flavor $e^{+/-}$)

- Square Root formula: A_N^{sqrt} - A_N^{Lumi} taken as systematic
- cosφ modulation fit: 3 φ bins per arm

$$A_N \cdot \cos\phi_s = \frac{1}{P}\frac{N^{\uparrow}(\phi_s) - R\cdot N^{\downarrow}(\phi_s)}{N^{\uparrow}(\phi_s) + R\cdot N^{\downarrow}(\phi_s)}$$

- Bunch shuffling: Randomize polarization direction, measure A_N/σ_{AN}
- Propagation of systematics on background fractions through background correction formula

$$A_N^{OHF\to e} = \frac{A_N^e - f_{h^\pm}A_N^{h^\pm} - f_{J/\psi\to e}A_N^{J/\psi\to e}}{1 - f_{h^\pm} - f_{J/\psi\to e} - f_{\pi^0\to e} - f_{\eta\to e} - f_{\gamma\to e}}$$

$$\sigma_{A_N^{OHF\to e}} = \frac{\sqrt{(\sigma_{A_N^e})^2 + (f_{h^\pm}\sigma_{A_N^{h^\pm}})^2 + (f_{J/\psi\to e}\sigma_{A_N^{J/\psi\to e}})^2}}{1 - f_{h^\pm} - f_{J/\psi\to e} - f_{\pi^0\to e} - f_{\eta\to e} - f_{\gamma\to e}}$$