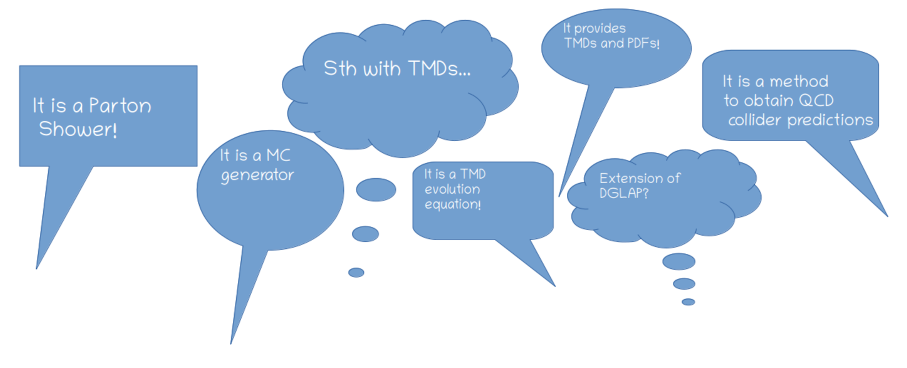
The Parton Branching method: a Monte Carlo approach based on TMDs

Transversity 2022

- **Ola Lelek** on behalf of the Parton Branching team

What is the Parton Branching method?



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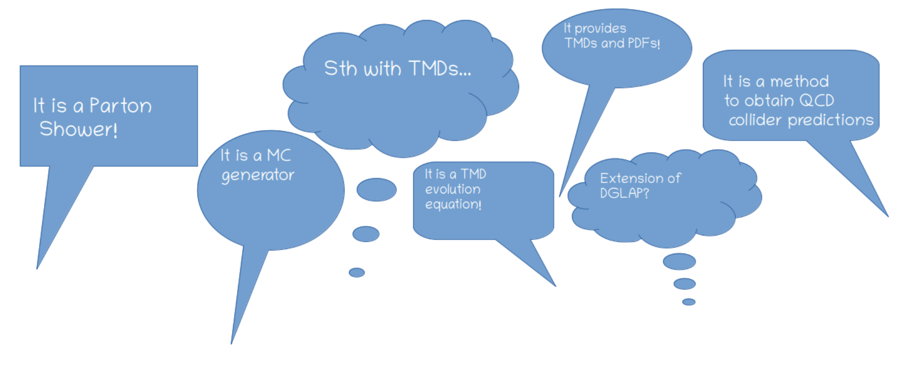
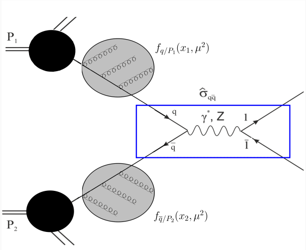


All this is true!

Factorization

Collinear factorization theorem

$$\sigma = \sum_{q\bar{q}} \int \mathrm{d}x_1 \mathrm{d}x_2 f_q(x_1, \mu^2) f_{\bar{q}}(x_2, \mu^2) \hat{\sigma}_{q\bar{q}}(x_1, x_2, \mu^2, Q^2)$$



Basis of many QCD calculations BUT

- proton structure in longitudinal direction only
- for some observables also the transverse degrees of freedom have to be taken into account
  $\rightarrow$ soft gluons need to be resummed

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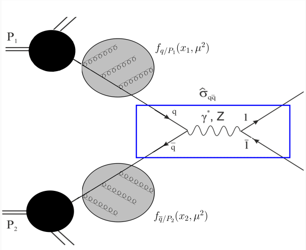

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$\rightarrow$Transverse Momentum Dependent (TMD) factorization theorems
low $q_\perp$ (Collins-Soper-Sterman CSS) or High energy ($k_\perp$-) factorization
For practical applications Monte Carlo approach needed: Parton Branching (PB) method:

$$\sigma = \sum_{q\overline{q}} \int \mathrm{d}^2 k_{\perp 1} \mathrm{d}^2 k_{\perp 2} \int \mathrm{d}x_1 \mathrm{d}x_2 A_q(x_1, k_{\perp 1}, \mu^2) A_{\overline{q}}(x_2, k_{\perp 2}, \mu^2) \hat{\sigma}_{q\overline{q}}(x_1, x_2, k_{\perp 1}, k_{\perp 2}, \mu^2, Q^2)$$

- applicable in a wide kinematic range, for multiple processes and observables

$A\left(x, k_\perp, \mu^2\right)$ - TMD PDFs (TMDs)

Ingredients of the PB method

Determination of the TMD:

- TMD (forward) evolution equation (solved with MC methods)
  JHEP 1801 (2018) 070
- PDF fit procedure implemented within `xFitter`
  Phys. Rev. D 99, 074008 (2019)

Application to measurements:

- recipe on how to use PB TMDs in the hard process generation (LO, NLO) Phys.Rev.D 100 (2019) 7, 074027
- backward initial state PB Parton Shower (PS) implemented in Cascade MC generator Eur.Phys.J.C 81 (2021) 5, 425
- procedure to merge different jet multiplicities developed
  Phys. Lett. B 822 (2021) 136700

PB and standard TMD approaches

we would like to resum higher order emissions to DY process:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_\perp} \sim \int \mathrm{d}z_1 \mathrm{d}z_2 \mathrm{d}k_{\perp 1} \mathrm{d}k_{\perp 2} H(Q^2)\delta(q_\perp - k_{\perp 1} - k_{\perp 2}) F_1(z_1, k_{\perp 1}, \mathrm{scales}) F_2(z_2, k_{\perp 2}, \mathrm{scales})$$

Starting structure the same for PB and CSS

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$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_\perp} \sim \int \mathrm{d}^2 b \exp(ib \cdot q_\perp) \int \mathrm{d}z_1 \mathrm{d}z_2 H(Q^2)$$
$$\underbrace{\int \mathrm{d}k_{\perp 1} \exp(-ib \cdot k_{\perp 1}) F_1(z_1, k_{\perp 1}, \mathrm{scales})}_{F_1(z_1, b, \mathrm{scales})}$$
$$\underbrace{\mathrm{d}k_{\perp 2} \exp(-ib \cdot k_{\perp 1}) F_2(z_2, k_{\perp 2}, \mathrm{scales})}_{F_2(z_2, b, \mathrm{scales})}$$

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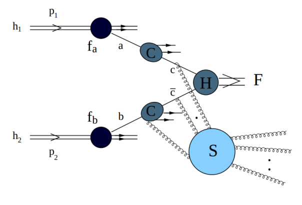
$$\frac{\mathrm{d}\sigma}{\mathrm{dq}_\perp} \sim \int \mathrm{dx}_1\mathrm{dx}_2\mathrm{dk}_{\perp 1}\mathrm{dk}_{\perp 2}\hat{\sigma}(\mathrm{x}_1, \mathrm{x}_2, \mathrm{k}_{\perp 1}, \mathrm{k}_{\perp 2}, \mu)$$
$$A_1(x_1, k_{\perp 1}, \mu)A_2(x_2, k_{\perp 2}, \mu)$$

where

$$\widetilde{A}_a\left(x, k_\perp, \mu^2\right) =$$
$$\Delta_a\left(\mu^2, \mu_0^2\right)\widetilde{A}_{a,0}\left(x, k_\perp, \mu_0^2\right) + \int K \otimes A(z, k'_\perp, \mu)$$

$F = f \otimes C \otimes \sqrt{S}$



Both methods:
- describe the evolution of partons
- use Sudakov form factors

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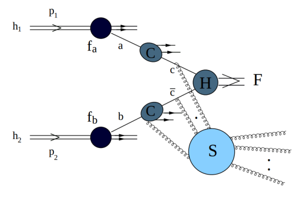

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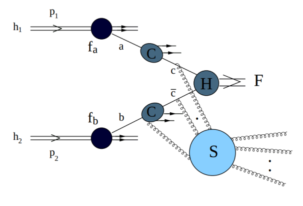

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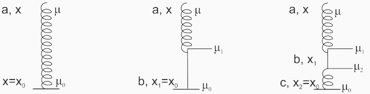

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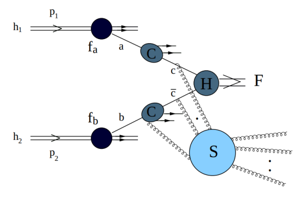

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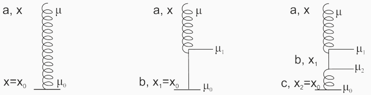

Parton evolution in PB

Unitarity: evolution in terms of resolvable, real emission DGLAP splitting functions P^R_{ab} and non-resolvable/virtual contributions, included via Sudakov form factors Δ;
z_M - soft gluon resolution scale, separates resolvable ($z < z_M$) and non-resolvable ($z > z_M$) branchings
JHEP 1801 (2018) 070

$$\widetilde{A}_a\left(x, k_\perp, \mu^2\right) = \Delta_a\left(\mu^2, \mu_0^2\right)\widetilde{A}_a\left(x, k_\perp, \mu_0^2\right) + \sum_b \int \frac{\mathrm{d}\mu_1^2}{\mu_1^2} \int_0^{2\pi} \frac{d\phi}{2\pi} \Theta\left(\mu^2 - \mu_1^2\right)\Theta\left(\mu_1^2 - \mu_0^2\right)$$

$$\times \quad \Delta_a\left(\mu^2, \mu_1^2\right) \int_x^{z_M} \mathrm{d}z P^R_{ab}\left(z, \mu_1^2, \alpha_s((1-z)^2\mu_1^2)\right) \widetilde{A}_b\left(\frac{x}{z}, |\boldsymbol{k} + (1-z)\boldsymbol{\mu}_1|, \mu_0^2\right) \Delta_b(\mu_1^2, \mu_0^2) + \ldots$$

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k of the propagating parton is a sum of intrinsic transverse momentum and all emitted transverse momenta
$\mathbf{k} = \mathbf{k}_0 - \sum_i \mathbf{q}_i \rightarrow$ TMD from branchings!

$\widetilde{A} = xA$, $x = zx_1$

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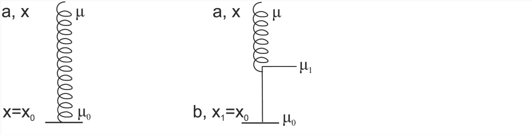

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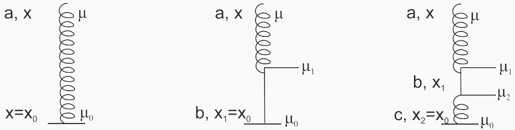

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Angular Ordering & Sudakov form factor

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- angles of emitted partons increase from the hadron side towards hard scattering
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notice: $\int_0^{1-\frac{q_\perp}{Q}} dz \left(\frac{1}{1-z}\right) = \frac{1}{2} \ln\left(\frac{Q^2}{q_\perp^2}\right)$

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CSS Sudakov form factor:

$$\sqrt{S} = \exp\left(-\frac{1}{2}\int_{c_0/b^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\left[A_i\left(\alpha_s(\mu^2)\right)\ln\left(\frac{Q^2}{\mu^2}\right) + B_i\left(\alpha_s(\mu^2)\right)\right]\right)$$

AO crucial for soft gluon resummation

We can compare: $k_a \Longleftrightarrow A$ and $d \Longleftrightarrow B$, order by order in α_s

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- LL (A_1), NLL (A_2, B_1) coefficients in Sudakov the same in PB and CSS

NNLL:

B_2:

difference between CSS and PB from renormalization group:

renormalization group equation: $\frac{\partial \ln H}{\partial \ln \mu^2} = \gamma(\alpha_s)$

solution: $H\left(\alpha_s(M^2)\right) = \exp\left(\int_{c_0/b^2}^{M^2} \frac{\mathrm{d}\mu'^2}{\mu'^2}\gamma\left(\alpha_s(\mu'^2)\right)\right) H\left(\alpha_s(\frac{c_0}{b^2})\right)$

This changes coefficient B in the Sudakov Nucl.Phys. B596 (2001) 299-312
$B(\alpha_s) \to B(\alpha_s) - \frac{\beta(\alpha_s)}{H(\alpha_s)} \frac{\partial H}{\partial \alpha_s}$

At $\mathcal{O}(\alpha_s^2)$: $B^2(\alpha_s) \to B^2(\alpha_s) + \pi \beta_0 H^1$

NNLL:

B_2:

difference between CSS and PB from renormalization group:

renormalization group equation: $\frac{\partial \ln H}{\partial \ln \mu^2} = \gamma(\alpha_s)$

solution: $H\left(\alpha_s(M^2)\right) = \exp\left(\int_{c_0/b^2}^{M^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \gamma\left(\alpha_s(\mu'^2)\right)\right) H\left(\alpha_s(\frac{c_0}{b^2})\right)$

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A_3:

double logarithmic part in PB: cusp anomalous dimension K is used at LO and NLO:
$P_{ii} = \frac{1}{1-z} K_i + \ldots$ (part of the DGLAP splitting function)

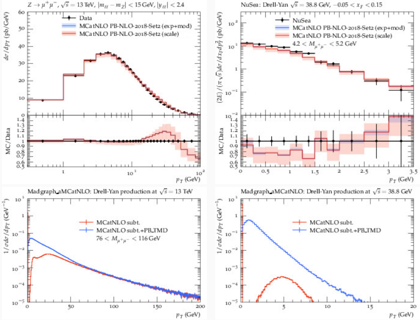
Resummation:
To go to higher orders: effective soft gluon coupling JHEP 01 (2019) 083, Eur.Phys.J.C 79 (2019) 8, 685:

$$\alpha_s^{\mathrm{eff}} = \alpha_s \left(1 + \sum_n \left(\frac{\alpha_s}{2\pi}\right)^n A^{(n)}\right)$$

A and K are equal up to $\mathcal{O}(\alpha_s^2)$

- CSS: A^3 is included by using α_s^{eff} for $n = 2$
- PB: work in progress to implement effective soft gluon coupling for A^3

Drell-Yan from fixed-target up to LHC energies



Eur.Phys.J.C 80 (2020) 7, 598

- Low and middle $p_\perp$ spectrum well described
- At higher $p_\perp$ from Z+ jets important $\to$ see later
- Good description of DY from experiments in different kinematic ranges: NuSea, R209, Phenix, Tevatron and LHC

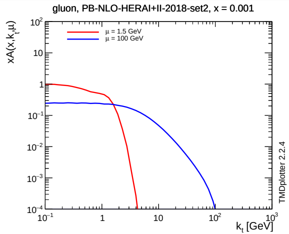
Madgraph MCatNLO ME matched with PB TMD in 2 steps (bottom plots):
subtracted collinear NLO ME generated by MCatNLO using iTMD
ME supplemented with $k_\perp$ by CASCADE using TMD corresponding to the iTMD used in MCatNLO.

Literature "low $q_\perp$ crisis" Phys. Rev. D 100, 014018 (2019): perturbative fixed order calculations in collinear factorization not able to describe DY p_T spectra at fixed target experiments for $p_T/m_{DY} \sim 1 \to$ we confirm this:

- at larger masses and LHC energies the contribution from soft gluons in the region of $p_\perp/m_{DY} \sim 1$ is small and the spectrum driven by hard real emission.
- at low DY mass and low $\sqrt{s}$ even in the region of $p_\perp/m_{DY} \sim 1$ the contribution of soft gluon emissions essential

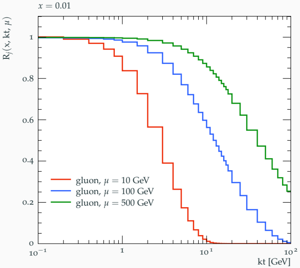
TMD effects at high $p_\perp$

It is commonly known that TMD effects play a role at scales $\mathcal{O}(\text{few GeV})$
Can TMDs also play a role at higher scales?



PB TMD: at $\mu \sim \mathcal{O}(1\ \text{GeV})$ TMD is a gaussian with $\Lambda_{QCD} < \sigma < \mathcal{O}(1\ \text{GeV})$. Effect of the evolution: $k_\perp$ accumulated in each step $\rightarrow$ TMD broadening

in PB: iTMDs (=PDFs) from TMD: $\widetilde{f}_a(x,\mu^2) = \int \mathrm{d}k_\perp^2\, \widetilde{A}_a(x,k_\perp,\mu^2)$



What is the contribution to the emission of an extra jet of $p_\perp < \mu$ from the $k_\perp$-broadening of the TMD?

$$R_j(x,k_\perp,\mu^2) = \frac{\int_{k_\perp^2}^{\infty} \mathrm{d}k_\perp'^2 \widetilde{A}_j(x,k_\perp',\mu^2)}{\int \mathrm{d}k_\perp'^2 \widetilde{A}_j(x,k_\perp',\mu^2)}$$

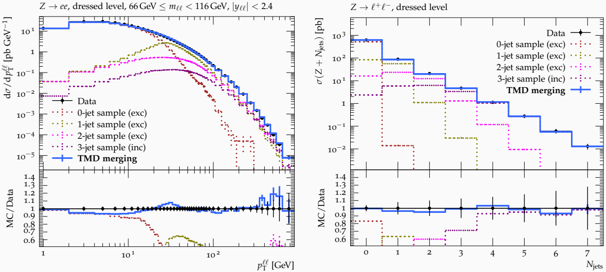
at LHC the contribution from high $k_\perp$ tail to jet emission comparable to perturbative emissions via hard ME!

hys.Lett.B 822 (2021) 136700

New development: TMDs and MLM Multi-Jet Merging

Recall: At high $p_\perp$ large corrections from higher orders

TMD merging procedure developed (at LO) hys.Lett.B 822 (2021) 136700
extension of MLM method NPB 632 (2002) 343–362 to the TMD case

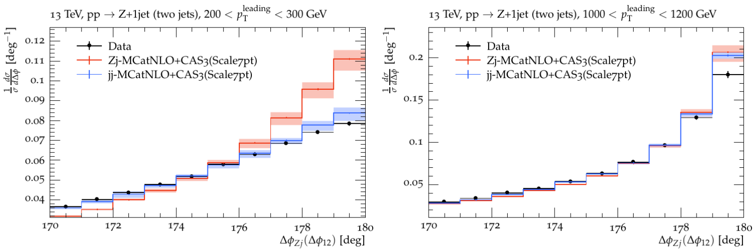


- The merged prediction provides good description of the data in the whole DY $p_\perp$ spectrum
- jet multiplicity in $Z+$ jets production well described, also for multiplicities larger than the maximum nb of jets in MEs

PB TMD in azimuthal correlations

PB method can be also applied to exclusive observables like azimuthal correlations in dijets and Z+jets
arXiv:2204.01528

- probe of colour/spin correlations:
  different flavour composition in initial state, different FSR $\rightarrow$ potential interference between initial and final state different $\rightarrow$ Comparing these two processes one can look for the hints of factorization breaking



dijet data well described by PB TMD + MCatNLO, small deviation in $\Delta\Phi = \pi$ - to be studied further
Still missing: data for Z+jets at high $p_\perp$

Summary & Conclusions

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- PB: TMD evolution equation to obtain TMDs; TMDs can be used in TMD MC generators to obtain predictions
- Some aspects of comparison of PB and standard TMD approach discussed:
  the role of AO in PB to include the A resummation coefficients
- Examples of the PB method applications: DY at different $\sqrt{s}$, m_{DY}, DY+jets, azimuthal correlations in Z+jest and multijets

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Thank you!