Proof of factorization enabling lattice calculation of TMDs

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Support:

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2004.14831
2201.08401
2205.12369

I. Motivation

II. Unified TMD notation

III. Lattice-to-continuum factorization

Non-perturbative contributions to collider physics

EFT: typically, described by matrix elements with lightlike Wilson lines…

- **Phenomenology:** models + fits to account for these effects
- **First principles?** Lightlike paths induce a lattice sign problem…

Thrust

$$\frac{d\sigma}{d\tau} = \frac{d\sigma^{pert}}{d\tau} \otimes \boldsymbol{F_{NP}(k)}$$

$\boldsymbol{B \to X_s + \gamma}$

$$\frac{d\Gamma}{dE_\gamma} = \frac{d\Gamma}{dE_\gamma} \otimes \boldsymbol{F_b}$$

TMD-PDFs

$$\sigma = \sigma^{pert}(b_T)\, \boldsymbol{\sigma^{NP}(b_T)}$$

Goal: circumvent lattice issues for TMDs & gain broader insights

TMD factorization

Experimental data

(e.g. Drell-Yan process)

$$d\sigma = H \int f \otimes f$$

Renormalized continuum QCD

$$f = Z_{UV} \frac{B}{\sqrt{S}}$$

Goal

Lattice-regularized QCD

$$f = C \times \tilde{f}_{lattice}$$

A plethora of TMD definitions…

Modern Collins

$$\tilde{f}_{i/p}(x,\mathbf{b}_T,\mu,\zeta) = \lim_{\epsilon\to 0} Z_{\rm uv}(\mu,\zeta,\epsilon) \lim_{y_B\to-\infty} \frac{\tilde{f}^{0\,(\rm u)}_{i/p}(x,\mathbf{b}_T,\epsilon,y_B,xP^+)}{\sqrt{\tilde{S}^0 \cdots}}$$

Echevarria, Idilbi, Scimemi

$$\tilde{f}_{i/p}(x,\mathbf{b}_T,\mu,\zeta) = \lim_{\substack{\epsilon\to 0\\ \delta^+\to 0}} Z^i_{\rm uv}(\mu,\zeta,\epsilon) \frac{\tilde{f}^{0\,(\rm u)}_{i/p}(x,\mathbf{b}_T,\epsilon,\delta^+/(xP^+))}{\sqrt{\tilde{S}^0_{\rm EIS}(b_T,\epsilon,\delta^+ e^{-y_n})}}$$

Chiu, Jain, Neill, Rothstein

$$\tilde{f}_{i/p}(x,\mathbf{b}_T,\mu,\zeta) = \lim_{\substack{\epsilon\to 0\\ \eta\to 0}} Z^i_{\rm uv}(\mu,\zeta,\epsilon)\, \tilde{f}^{0\,(\rm u)}_{i/p}(x,\mathbf{b}_T,\epsilon,\eta,xP^+)\sqrt{\tilde{S}^0_{\rm CJNR}(b_T,\epsilon,\eta)}$$

Becher & Neubert

$$\lim_{\substack{\epsilon\to 0\\ \alpha\to 0}}\left[\tilde{f}^{0(\rm u),BN}_{i/p}(x_1,\mathbf{b}_T,\epsilon,\alpha,x_a P_A^+)\tilde{f}^{0(\rm u),BN}_{j/p}(x_2,\mathbf{b}_T,\epsilon,\alpha,x_b P_B^-)\right]$$

$$\left(b_T^2 Q^2\right)^{-\gamma^q_\zeta(\mu,b_T)}\left[\tilde{f}^{\rm BN}(\cdots)\,\tilde{f}^{\rm BN}(\cdots{}_T,\mu,\zeta = b_0^2/b_T^2)\right]$$

Ji, Ma, Yuan

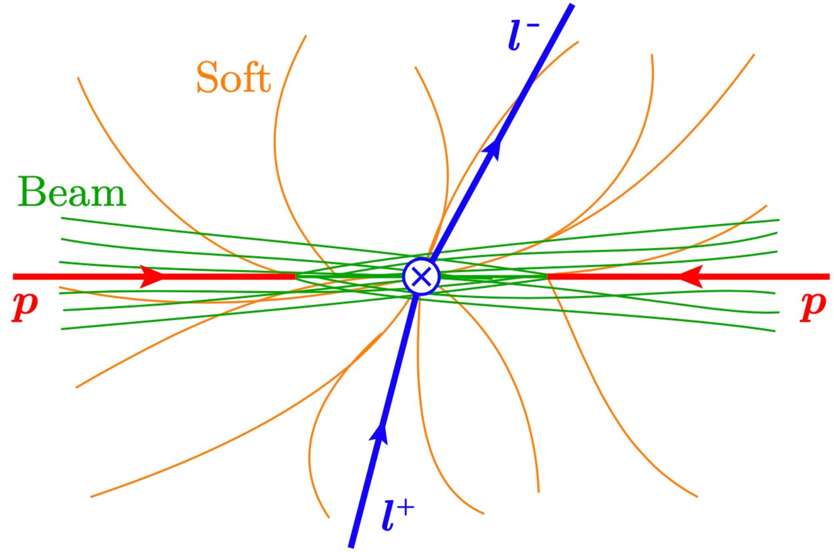
$$\tilde{f}_{i/p}(x_a,\mathbf{b}_T,\mu,x_a\tilde{\zeta}_a;\rho) = \lim_{\epsilon\to 0} Z^i_{\rm uv}(\mu,\rho,\epsilon) \frac{\tilde{f}^{0\,(\rm u)}_{i/p}(x_a,\mathbf{b}_T,\epsilon,v,xP^+)}{\sqrt{\tilde{S}^0_{v\bar{v}}(b_T,\epsilon,\rho)}} + O(v^+,\bar{v}^-)\,.$$

Etc!

Let’s sort this all out!

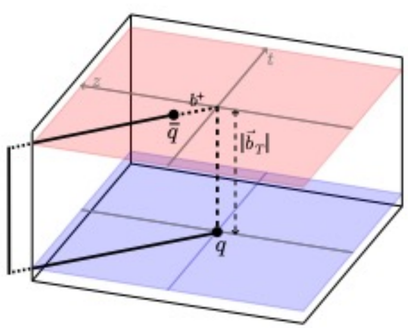
Components of a TMD

$$f = \lim_{\substack{\text{lightcone,}\\ \text{renormalization}}} Z_{UV} \frac{B^{[\Gamma]}_{q_i/H}}{\sqrt{S^R}}$$



Beam function:

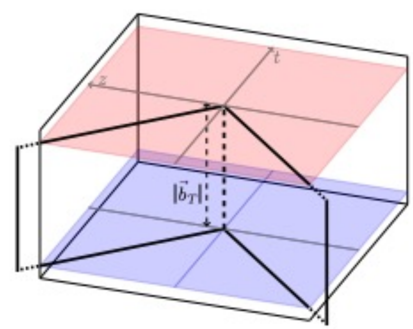
$\langle P|$



$|P\rangle$

Soft factor:

$\langle 0|$



$|0\rangle$

Unified TMD notation

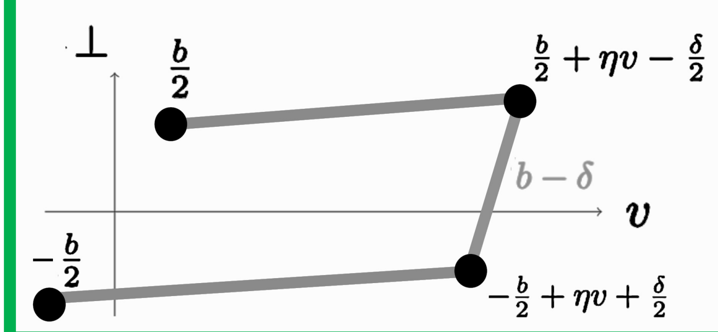
- Quasi-TMD
- Collins TMD
- JMY scheme
- MHENS scheme

$$f = \lim_{\substack{\text{lightcone,}\\ \text{renormalization}}} Z_{UV} \frac{B^{[\Gamma]}_{q_i/H}(b, P, \epsilon, \eta v, \delta)}{\sqrt{S^R(b, \epsilon, \eta v, \bar{\eta}\bar{v})}}$$

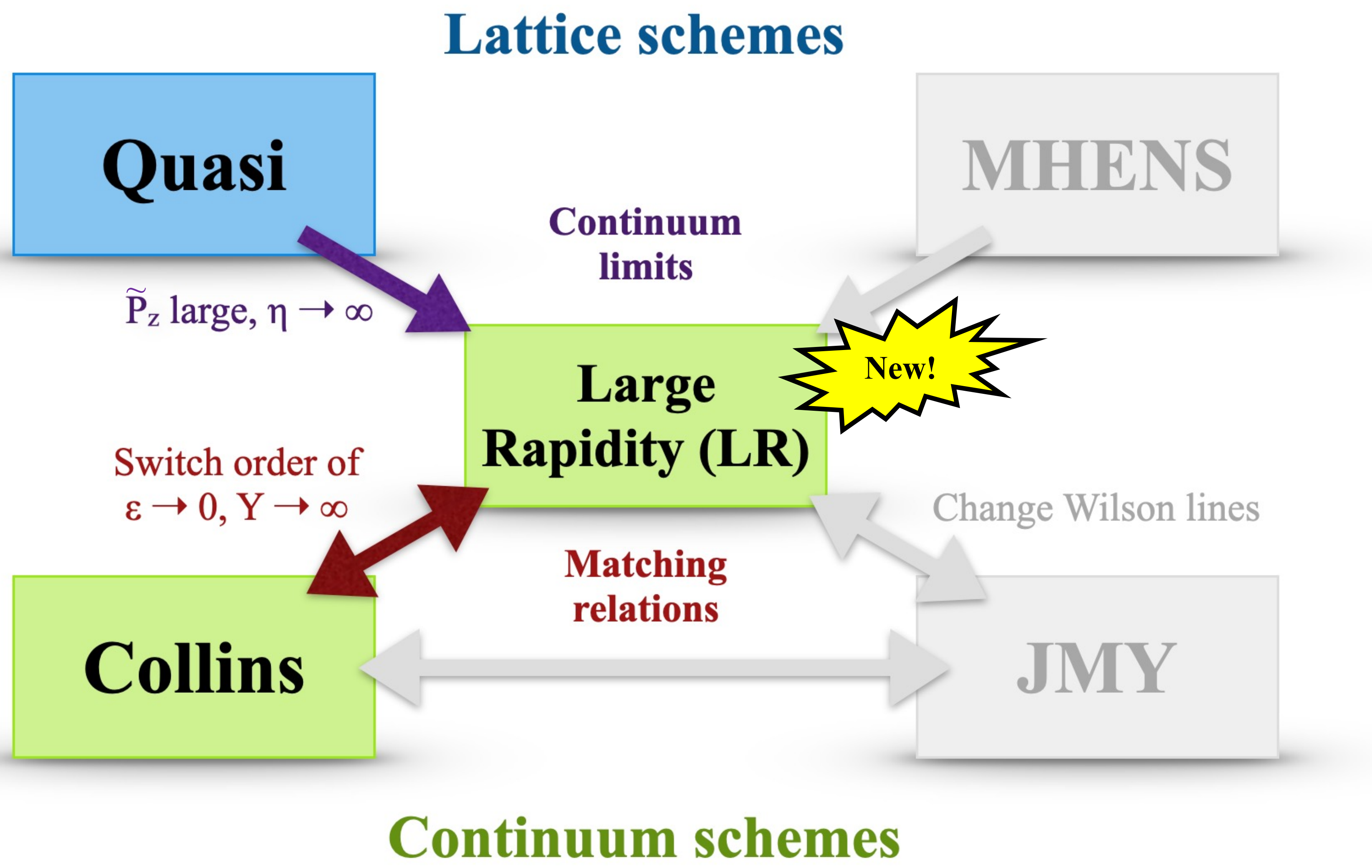
Specify a scheme by choice of arguments & limits

$$\textbf{Beam} = \left\langle P \left| \bar{q}_i \frac{\Gamma}{2} W^F_{\sqsupset}(b, \eta v, \delta) q_i \right| P \right\rangle$$

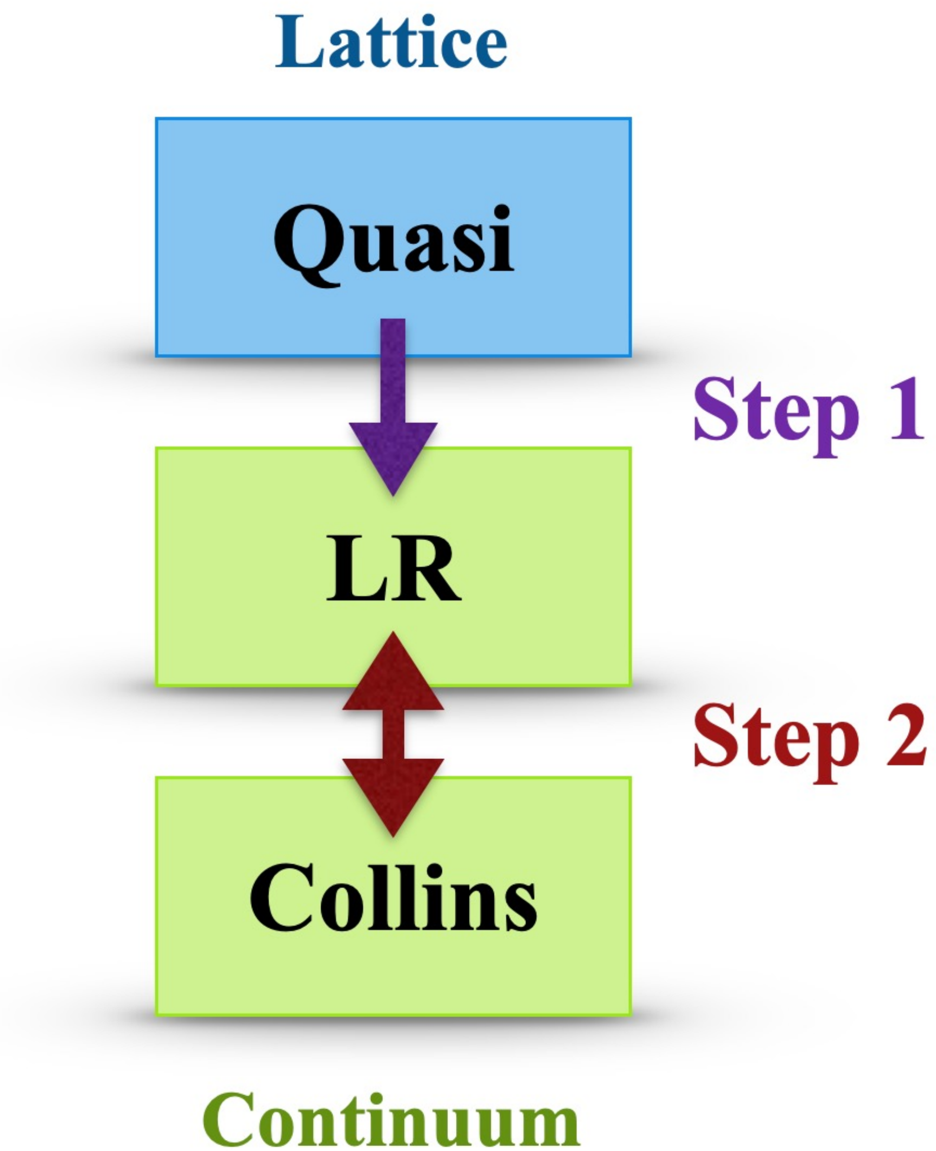
$$\textbf{Soft} = \frac{1}{d_R} \langle 0 | \mathrm{Tr}[S^R_{\bowtie}(b, \eta v, \bar{\eta}\bar{v})] | 0 \rangle$$



Unified notation → straightforward to see relationships



Derivation procedure



(1) Same at large rapidity

$P^z >> \Lambda_{QCD}$

- Map variables after expansion
- Wilson line length $|\eta| \to \infty$

(2) Nontrivial relationship

- Different UV renormalization
- Need matching coefficient

The quasi-soft function is chosen to reproduce the Collins soft function.

Step 1: Quasi to LR

Compare Lorentz invariants formed from beam function arguments b^μ, P^μ, δ^μ, ηv^μ

Use boosts to show quasi = LR as $|\boldsymbol{\eta}| \to \infty$ **&** $\mathbf{P^z \gg \Lambda_{QCD}}$

	Quasi	**LR**
b^2	$-b_T^2 - (\tilde{b}^z)^2$	$-b_T^2$
$(\eta v)^2$	$-\tilde{\eta}^2$	$-2\eta^2 e^{2y_B}$
$P \cdot b$	$-m_h \tilde{b}^z \sinh y_{\tilde{P}}$	$\frac{m_h}{\sqrt{2}} b^- e^{y_P}$
$\frac{b \cdot (\eta v)}{\sqrt{\lvert(\eta v)^2 b^2\rvert}}$	$\frac{\tilde{b}^z}{\sqrt{(\tilde{b}^z)^2 + b_T^2}} \,\mathrm{sgn}(\eta)$	$-\frac{b^- e^{y_B}}{\sqrt{2}\, b_T} \,\mathrm{sgn}(\eta)$
$\frac{P \cdot (\eta v)}{\sqrt{P^2 \lvert\eta v\rvert^2}}$	$\sinh y_{\tilde{P}} \,\mathrm{sgn}(\eta)$	$\sinh(y_P - y_B)\,\mathrm{sgn}(\eta)$
$\frac{\delta^2}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{b \cdot \delta}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{P \cdot \delta}{P \cdot b}$	1	1
$\frac{\delta \cdot (\eta v)}{b \cdot (\eta v)}$	1	1
P^2	m_h^2	m_h^2

Step 1: Quasi to LR

Examine all 10 Lorentz invariants:

Need $\tilde{\eta} = \sqrt{2}\, e^{y_B} \eta$

Need $y_P - y_B = y_{\tilde{P}}$

As $y_{\tilde{P}} \to -\infty$, $b_T \gg \tilde{b}_z$

Quasi = LR
after large rapidity expansion ✅

	Quasi	LR
b^2	$-b_T^2 - (\tilde{b}^z)^2$	$-b_T^2$
$(\eta v)^2$	$-\tilde{\eta}^2$	$-2\eta^2 e^{2y_B}$
$P \cdot b$	$-m_h \tilde{b}^z \sinh y_{\tilde{P}}$	$\frac{m_h}{\sqrt{2}} b^- e^{y_P}$
$\frac{b \cdot (\eta v)}{\sqrt{\lvert(\eta v)^2 b^2\rvert}}$	$\frac{\tilde{b}^z}{\sqrt{(\tilde{b}^z)^2 + b_T^2}} \operatorname{sgn}(\eta)$	$-\frac{b^- e^{y_B}}{\sqrt{2}\, b_T} \operatorname{sgn}(\eta)$
$\frac{P \cdot (\eta v)}{\sqrt{P^2 \lvert\eta v\rvert^2}}$	$\sinh y_{\tilde{P}} \operatorname{sgn}(\eta)$	$\sinh(y_P - y_B) \operatorname{sgn}(\eta)$
$\frac{\delta^2}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{b \cdot \delta}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{P \cdot \delta}{P \cdot b}$	1	1
$\frac{\delta \cdot (\eta v)}{b \cdot (\eta v)}$	1	1
P^2	m_h^2	m_h^2

Step 2: LR to Collins

	TMD	Beam function	Soft function
Collins	$\lim\limits_{\epsilon\to 0}\lim\limits_{y_B\to -\infty} Z^R_{UV}\frac{\Omega_{i/h}}{\sqrt{S^R}}$	$\Omega^{[\gamma^+]}_{q/h}\left[b, P, \epsilon, -\infty n_B(y_B), b^- n_b\right]$	$S^R\left[b_\perp, \epsilon, -\infty n_A(y_A), -\infty n_B(y_B)\right]$
LR	$\lim\limits_{-y_B\gg 1}\lim\limits_{\epsilon\to 0} Z^R_{UV}\frac{\Omega_{i/h}}{\sqrt{S^R}}$	$\Omega^{[\gamma^+]}_{q/h}\left[b, P, \epsilon, -\infty n_B(y_B), b^- n_b\right]$	$S^R\left[b_\perp, \epsilon, -\infty n_A(y_A), -\infty n_B(y_B)\right]$

Fundamental principle of EFT (here, LaMET):

- Flipping an order of UV limits does not affect IR physics
- However, it may induce a perturbative matching coefficient
- So:

$$\boldsymbol{f_{LR}} = C_i\left(x\tilde{P}^z, \mu\right)\ \boldsymbol{f_{Collins}}$$

Combine steps 1 & 2 $\rightarrow$ factorization

Quasi-TMD (lattice) — Matching — RGE for ζ — **Collins TMD (continuum)**

$$\tilde{f}_{i/H}^{[s]}\left(x, \vec{b}_T, \mu, \tilde{\zeta}, x\tilde{P}^z\right) = C_i\left(x\tilde{P}^z, \mu\right) \exp\left[\frac{1}{2}\gamma_\zeta^i(\mu, b_T) \ln\frac{\tilde{\zeta}}{\zeta}\right] f_{i/H}^{[s]}\left(x, \vec{b}_T, \mu, \zeta\right)$$

$$\tilde{\zeta} = \left(2x\tilde{P}^z\right)^2 e^{2(y_B - y_n)}$$

Power corrections

$$\times\left\{1 + \mathcal{O}\left[\frac{1}{\left(x\tilde{P}^z b_T\right)^2}, \frac{\Lambda_{QCD}^2}{\left(x\tilde{P}^z\right)^2}\right]\right\}$$

Proof works for all choices of spins and for gluons; cross-checked at one-loop.

Matching coefficient

$$\tilde{f}_{i/H}^{[s]}\left(x,\vec{b}_T,\mu,\tilde{\zeta},x\tilde{P}^z\right) = \boldsymbol{C_i\left(x\tilde{P}^z,\mu\right)} \exp\left[\frac{1}{2}\gamma_\zeta^i(\mu,b_T)\ln\frac{\tilde{\zeta}}{\zeta}\right] f_{i/H}^{[s]}\left(x,\vec{b}_T,\mu,\zeta\right)$$

Quasi: convenient for the lattice

- Independent of spin
- No flavor mixing
- No quark/gluon mixing

TMD ratios from beam ratios:

$$\lim_{\tilde{\eta}\to\infty}\frac{\tilde{\boldsymbol{B}}_{q_i/h}^{[\tilde{\Gamma}_1]}}{\tilde{\boldsymbol{B}}_{q_j/h'}^{[\tilde{\Gamma}_2]}} = \lim_{\tilde{\eta}\to\infty}\frac{\boldsymbol{f}_{q_i/h}^{[\tilde{\Gamma}_1]}}{\boldsymbol{f}_{q_j/h'}^{[\tilde{\Gamma}_2]}}$$

One loop: Casimir scaling

$$C_i\left(\mu,x\tilde{P}^z\right) = 1 + \frac{\alpha_s \boldsymbol{C_R}}{4\pi}\left[-\ln^2\frac{(2xP^z)^2}{\mu^2} + \frac{2\ln(2xP^z)^2}{\mu^2} - 4 + \frac{\pi^2}{6}\right] + O(\alpha_s^2)$$

N^{n}LL:

$$C_i\left(x\tilde{P}^z,\mu\right) = C_i[\alpha_s(\mu)]\exp\left[\int_{\alpha_s(\mu)}^{\alpha_s(2x\tilde{P}^z)}\frac{d\alpha}{\beta[\alpha']}2\Gamma_{cusp}^i[\alpha'] + \gamma_C^i[\alpha]\right]$$

Summary

Concrete physics developments:

- New unified TMD notation
- New TMD scheme: Large Rapidity (LR)
- Proof of lattice-to-continuum TMD factorization
- Progress on computing matching coefficients

Broader lessons:

- Need to balance analytic and lattice challenges for computing NP contributions to collider physics
- Useful to consider the full space of possible lattice correlators for an observable