On the spectral properties of the gauge invariant quark propagator

Caroline S. R. Costa

Transversity, 2022

May 24, 2022

Jefferson Lab
Exploring the Nature of Matter

Outline

- Motivation
- Inclusive jet correlator
- Quark propagator spectral representation
- Conclusions

Introduction

- QCD is characterized by a number of distinct phenomena
  - **Confinement:** Quarks and gluons are not assymptotic states of QCD; are confined inside hadrons
  - **DCSB:** Mass generation

  } **Nonperturbative:** inclusive jet correlators

  - These QCD features are intimately related to *hadronization*
  - How color neutral and massive hadrons emerge out of colored and massless quarks and gluons?

Mass Generation

- Chiral symmetry: approximate symmetry of the light quark sector of QCD

$$m_u \approx 2.16 \text{ MeV}, \quad m_d \approx 4.67 \text{ MeV} : \quad m_u \approx m_d$$

$$\boxed{\mathbf{SU(2)_L \otimes SU(2)_R}}$$

- Mass splitting between parity partners are big ($m_{a_1} - m_\rho \approx 500$ MeV) and cannot be produced by the small current quark masses in the QCD Lagrangian

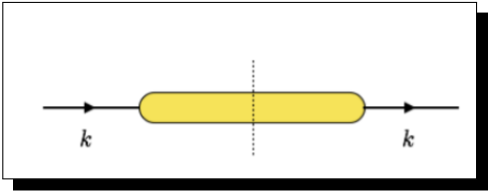
- Chiral symmetry is broken dynamically and gives rise to:
  - the mass splittings observed in hadron spectrum
  - dressed quarks

- The fully inclusive jet correlator can be used to shed light on both of these QCD features

Inclusive jet correlator

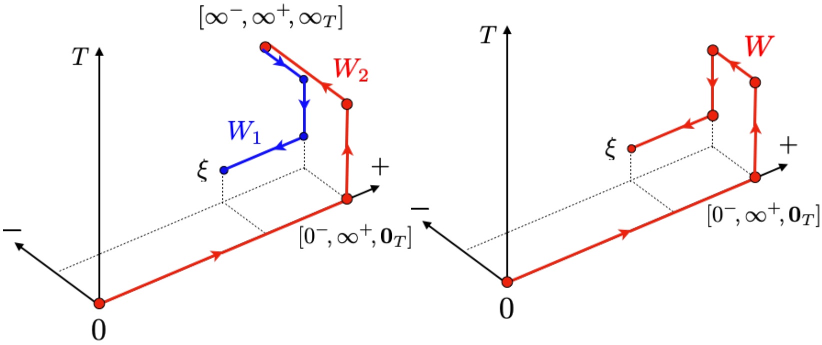
- Fragmentation of a quark into an unobserved jet of particles
- Fully inclusive: no hadrons are observed

$$\Xi_{ij}(k;w) = \mathrm{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \frac{\mathrm{Tr_c}}{\mathrm{N_c}} \langle\Omega|\mathcal{T}\, W_1(\infty,\xi;w)\psi_i(\xi)\bar{\psi}_j(0)W_2(0,\xi;w)|\Omega\rangle$$



- Sum over all the hadronization products crossing the cut

Inclusive jet correlator



- Use of suitable Wilson lines allows to combine the two gauge links into a single staple-like Wilson line
- Formalism presented here applies to a larger class of Wilson lines, not only $w = n^+$

Accardi, Signori (2020)

Inclusive jet correlator

$$\Xi_{ij}(k;w) = \mathrm{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \frac{\mathrm{Tr_c}}{\mathrm{N_c}} \langle\Omega|\psi_i(\xi)\bar{\psi}_j(0) W(0,\xi;w)|\Omega\rangle,$$

- Can be written as the convolution

$$\Xi_{ij}(k;w) = \mathrm{Disc} \int d^4p \, \frac{\mathrm{Tr_c}}{\mathrm{N_c}} \langle\Omega| i\tilde{S}_{ij}(p) \tilde{W}(k-p;w)|\Omega\rangle,$$

$$i\widetilde{S}_{ij}(p) = \int \frac{d^4\xi}{(2\pi)^4} \, e^{i\xi\cdot p} \, \mathcal{T}\psi_i(\xi)\bar{\psi}_j(0)$$

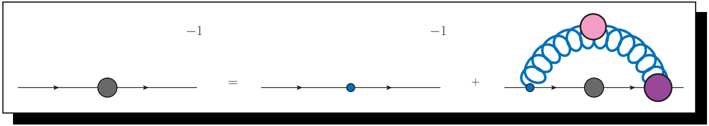
$$\widetilde{W}(k-p;w) = \int \frac{d^4\xi}{(2\pi)^4} \, e^{i\xi\cdot(k-p)} W(0,\xi;w)$$

Quark propagator

- $i\,\tilde{S}(p)$ is nothing but the **full (dressed)** quark propagator

  $\rightsquigarrow$ all possible ways a quark can propagate

- solution of the quark **gap equation**:



- It has the general structure (axial gauges):

  $$\tilde{S}(p) = s_3(p^2, p\cdot v, v^2)\not{p} + \sqrt{p^2}s_1(p^2, p\cdot v, v^2)\mathbb{1} + s_0(p^2, p\cdot v, v^2)\not{v}$$

- $s_0(p^2, p\cdot v, v^2) = 0$ for covariant gauges

- Owing to the rescaling invariance of v and $v^2 = 0$ (for light-like axial gauge), the structure simplifies further to

$$\tilde{S}(p) = s_3(p^2)\not{p} + \sqrt{p^2}s_1(p^2)\mathbb{1} + s_0(p^2)\frac{\not{v}}{v\cdot p}$$

Quark spectral representation

- The convolution representation is convenient because allows to connect the quark propagator spectral functions to the inclusive jet correlator
- The quark propagator in the lcg allows a spectral representation in the form:

$$\tilde{S}(p) = \int_0^\infty d\kappa^2 \frac{\rho_3(\kappa^2)\, \not{p} + \sqrt{p^2}\rho_1(\kappa^2) + \rho_0(\kappa^2)\not{v}/v \cdot p}{p^2 - \kappa^2 + i\epsilon}$$

- The spectral functions encode information about the analytical structure of the propagator
- The study of the analytical structure of QCD propagators has increasingly attracted interest in the past years:
  - Confinement would be associated to dramatic changes in the analytical strucure of QCD propagators: positivity violation, appearance of complex conjugate poles, etc.
- Normalization of the spectral functions are related to the nonperturbative structure of the inclusive jet correlator

TMD jet correlator

- Integrate over subdominant component:

$$J(k^-, \mathbf{k}_\perp) \equiv \frac{1}{2} \int dk^+ \Xi_{ij}(k; w)$$

- Expand in Dirac structures in powers of $1/k^-$:

$$J(k^-, \mathbf{k}_\perp) = \alpha(k^-)\gamma^+ + \frac{\not{k}_\perp}{k^-} + \frac{\mathbf{M_j}}{k^-} + \frac{K_j^2 + \not{k}_\perp^2}{2(k^-)^2}\gamma^-$$

- Generalizes

$$\not{k} + m = \gamma^+ k^- + \not{k}_\perp + m + \frac{m^2 + \not{k}_\perp^2}{2(k^-)^2}\gamma^-$$

- In principle, a structure $\sim \not{v}$ would be present
- However, gauge invariance of $\Xi(k, w, v)$ implies

$$\Xi(k, w, v) = \Xi(k, w, 0)$$

TMD jet correlator

$$\alpha = \int_0^\infty d\kappa^2 \rho_3(\kappa^2) \equiv 1 \quad \text{(normalization condition)}$$

$$M_j \stackrel{\text{lcg}}{=} \int_0^\infty d\kappa^2 \sqrt{\kappa^2}\rho_1(\kappa^2) = \int_0^\infty d\kappa^2 \sqrt{\kappa^2}\rho_1(\kappa^2) + O(1/(k^-)^2)$$

$$\gamma = \int_0^\infty d\kappa^2 \rho_0(\kappa^2) = 0$$

- Jet mass M_j: Gauge invariant quark mass

  Average of all the masses that pass the cut

  Sum over all discontinuities of the quark propagator

  Calculable!

Conclusions

- The jet correlator is directly connected to the quark spectral functions
- This provides a definition of a gauge invariant dressed quark mass: the jet mass, M_j
- The DSE framework can be used to solve the quark propagator for its spectral functions
- With the spectral functions in hand, the jet mass can be directly computed
- The jet mass can be accessed experimentally through sum rules that relates it to the twist-3 collinear FF $\tilde{E}_h(z)$

Thank you!

Backup

Dyson-Schwinger equations

- The Green functions equations of motion of a QFT
- Could in principle be used to solve **QCD**
- An infinite tower of coupled integral equations (DSE)
- Must use symmetry preserving truncation schemes
- It is usually solved in Euclidean space
- The most important DSE is the quark gap equation

$$S_\Lambda^{-1}(p) = \not{p} - m_\Lambda - i \int \frac{d^4q}{(2\pi)^4} g_\Lambda^2 \gamma_\mu D_\Lambda^{\mu\nu}(q) S_\Lambda(p-q) T^a \Gamma^a_{\Lambda\,\nu}(q, p-q, p),$$

- It has the general structure

$$S_\Lambda^{-1}(p) = A(p^2)\not{p} + B(p^2)$$

- Instead of solving for the $A(p^2)$ and $B(p^2)$ functions, we solve for the quark spectral functions

Quark spectral representation

- The quark propagator has a spectral representation in the form:

$$S_\Lambda(p) = \int_0^\infty d\kappa^2 \frac{\rho_{1\Lambda}(\kappa^2)\not{p} + \rho_{2\Lambda}(\kappa^2)}{p^2 - \kappa^2 + i\epsilon} \qquad \text{[covariant gauges]}$$

- It is possible to work with only one spectral function by defining

$$\rho_{1\Lambda}(\kappa^2) = \frac{\rho_\Lambda(\kappa) + \rho_\Lambda(-\kappa)}{2\kappa}, \qquad \rho_{2\Lambda}(\kappa^2) = \frac{\rho_\Lambda(\kappa) - \rho_\Lambda(-\kappa)}{2}.$$

$$S_\Lambda(p) = \int_{-\infty}^\infty d\kappa \rho_\Lambda(\kappa) \frac{\not{p} + \kappa}{p^2 - \kappa^2 + i\epsilon}$$

- The spectral function has now support over the entire real axes

- The spectral function satisfy the positivity constraint:

$$\rho_\Lambda(\kappa) \geq 0$$

- Introduce the projectors:

$$P_\pm(p) = \frac{1}{2}\left(1 \pm \frac{\not{p}}{w(p)}\right), \qquad w(p) \equiv \begin{cases} \sqrt{p^2} = \sqrt{(p^0)^2 - \mathbf{p}^2}, & p^2 > 0 \\ i\sqrt{-p^2} = i\sqrt{\mathbf{p}^2 - (p^0)^2}, & p^2 < 0. \end{cases}$$

- They allow to project out the Dirac structure of the quark propagator and write it terms of a scalar funtion,

$$S_\Lambda(p) = P_+(p)\,\widetilde{S}_\Lambda(w(p) + i\epsilon) + P_-(p)\,\widetilde{S}_\Lambda(-w(p) - i\epsilon),$$

$$\widetilde{S}_\Lambda(z) = \int_{-\infty}^{+\infty} d\kappa\,\frac{\rho_\Lambda(\kappa)}{z - \kappa}, \qquad z = \pm(w(p) + i\epsilon).$$

- $\widetilde{S}_\Lambda(z)$ has no zero off the real axes

- The inverse of the full quark propagator

$$S_\Lambda^{-1}(p) = [S_\Lambda^{(0)}(p)]^{-1} - \Sigma_\Lambda(p)$$

also allows a spectral representation as $\widetilde{S}_\Lambda^{-1}(z)$ can only have zero in the real axes,

$$\widetilde{S}_\Lambda^{-1}(z) = z - m_\Lambda - \int_{-\infty}^{+\infty} d\kappa \, \frac{\sigma_\Lambda(\kappa)}{z - \kappa}$$

- And one can write the quark spectral function $\rho(\kappa)$ and the self-energy spectral function in terms of the quark propagator and its inverse

$$\rho(\kappa) = -\frac{1}{2\pi i}\left[\widetilde{S}(\kappa + i\epsilon) - \widetilde{S}(\kappa - i\epsilon)\right],$$

$$\sigma(\kappa) = \frac{1}{2\pi i}\left[\widetilde{S}^{-1}(\kappa + i\epsilon) - \widetilde{S}^{-1}(\kappa - i\epsilon)\right].$$

- Using the identity

$$\widetilde{S}^{-1}(\kappa + i\epsilon) - \widetilde{S}^{-1}(\kappa - i\epsilon) = \widetilde{S}^{-1}(\kappa + i\epsilon)\widetilde{S}^{-1}(\kappa - i\epsilon)\left[\widetilde{S}(\kappa - i\epsilon) - \widetilde{S}(\kappa + i\epsilon)\right],$$

a relationship between the quark propagator spectral function $\rho(\kappa)$ and the self-energy spectral function $\sigma(\kappa)$ can be found

$$\sigma(\kappa) = |\widetilde{S}^{-1}(\kappa + i\epsilon)|^2 \, \rho(\kappa)$$

- The inverse relationship can also be found

$$\rho(\kappa) \quad = R(M_p)\, \delta(\kappa - M_p) + \overline{\rho}(\kappa); \qquad \overline{\rho}(\kappa) = |\widetilde{S}^{-1}(\kappa + i\epsilon)|^{-2} \, \sigma(\kappa)$$

- M_p is a mass pole and $R(M_p)$ the corresponding residue
- The mass pole is found as a zero of $\tilde{S}^{-1}(z)$
- In addition to a real mass pole, there might exist complex-conjugate poles

- The procedure is as follows

1. Start with an ansatz for $\rho(\kappa)$ and find $\sigma(\kappa)$ from

$$\sigma(\kappa) = \frac{1}{2\pi i}\left[\widetilde{S}^{-1}(\kappa + i\epsilon) - \widetilde{S}^{-1}(\kappa - i\epsilon)\right]$$

2. Plug this $\sigma(\kappa)$ into

$$\widetilde{S}^{-1}(z) = Z_\psi\, \widetilde{S}_\Lambda^{-1}(z) = Z_\psi(z - Z_m m) - \int_{-\infty}^{+\infty} d\kappa\, \frac{\sigma(\kappa)}{z-\kappa}$$

3. Find a new $\rho(\kappa)$ from

$$\rho(\kappa) = \frac{i}{2\pi}\left[\widetilde{S}^{-1}(\kappa + i\epsilon)\right]^{-1} - \left[\widetilde{S}^{-1}(\kappa - i\epsilon)\right]^{-1}$$

4. Repeat until achieve convergence