Perturbative T-odd proton-helicity asymmetry in SIDIS

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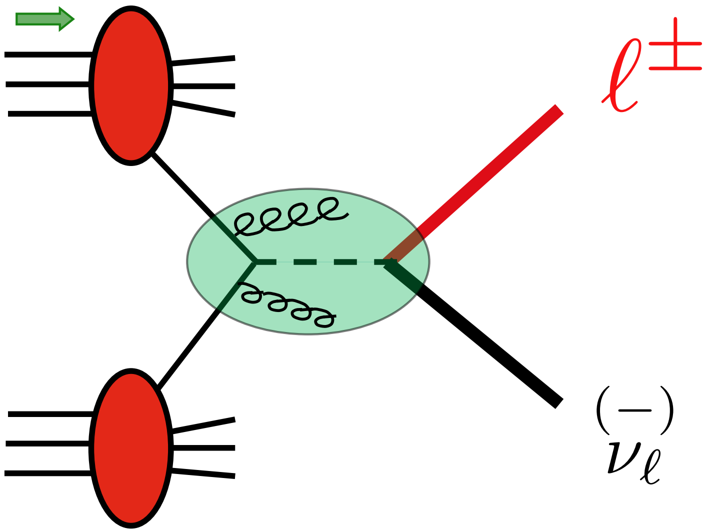
Outline

- Introduction
- Perturbative T-odd observable in long. polarized SIDIS
- Some phenomenology
- Behavior at low q_T and connection to TMD regime

Maurizio Abele, Matthias Aicher, Fulvio Piacenza, Andreas Schäfer, WV

2204.13967 [hep-ph]

W bosons at RHIC:

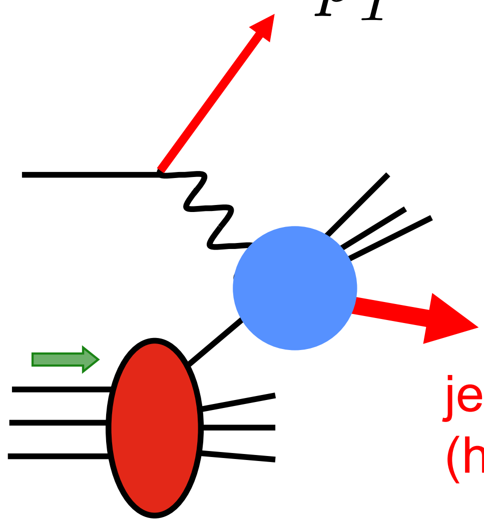


$$\vec{p}\, p \to \ell^{\pm} X$$

$$A_L = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

$A_L \neq 0$ requires parity violation ($\vec{S}_L \cdot \vec{p}_T^{\,\ell}$)

- however:



P even

T-odd

$$\vec{S}_L \cdot \left(\vec{p}_T^{\,\ell} \times \vec{p}_T^{\,\mathrm{jet}}\right) \sim \sin\phi$$

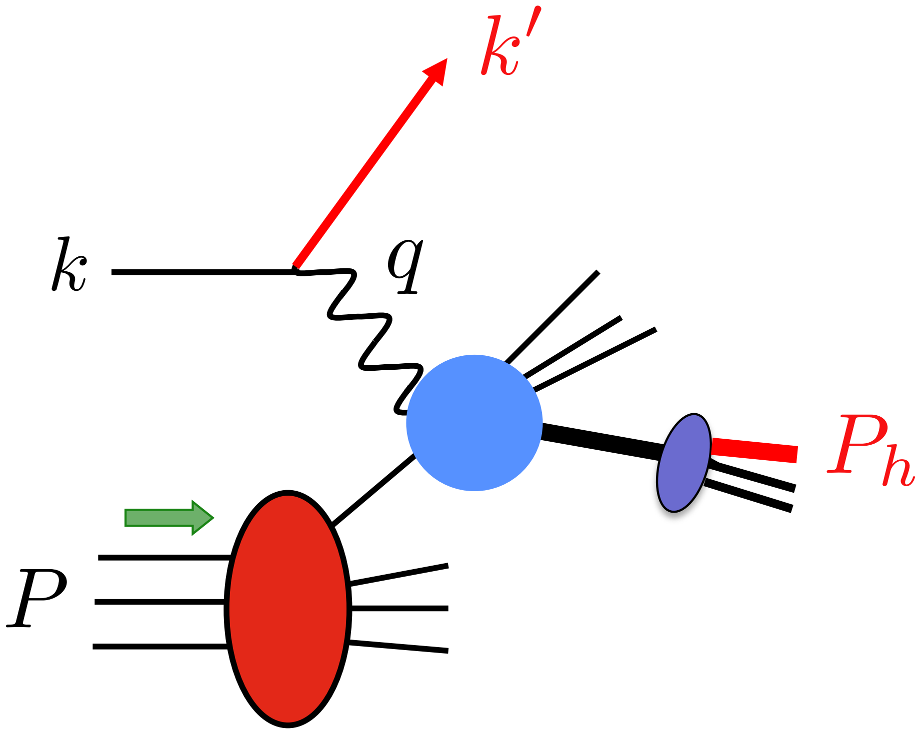
$$\vec{S}_L \cdot \left(\vec{p}_T^{\,\ell} \times \vec{p}_T^{\,\mathrm{jet}}\right) \left(\vec{p}_T^{\,\ell} \cdot \vec{p}_T^{\,\mathrm{jet}}\right) \sim \sin(2\phi)$$

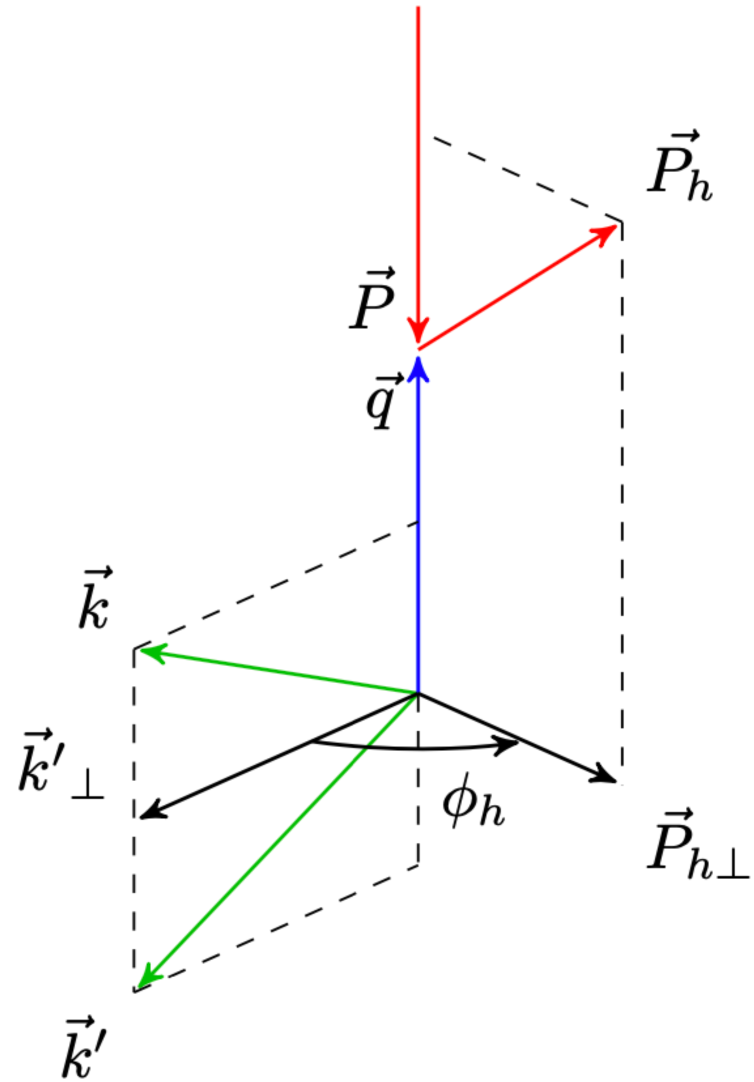
$$A_L = \mathcal{A}\sin\phi + \mathcal{B}\sin(2\phi)$$

Hagiwara, Hikasa, Kai

Perturbative T-odd observable in SIDIS

- SIDIS kinematics:





$$x = \frac{Q^2}{2P \cdot q} \qquad y = \frac{P \cdot q}{P \cdot k}$$

$$z = \frac{P \cdot P_h}{P \cdot q}$$

Gourdin; Kotzinian;
Mulders, Tangerman;
Bacchetta, Diehl, Goeke, Metz,
Mulders, Schlegel

- one-photon exchange:

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dq_T^2} = \frac{\pi\alpha^2}{yQ^4}\,\frac{y^2}{1-\varepsilon}\left\{F_{UU,T} + \varepsilon F_{UU,L}\right.$$

$$+\sqrt{2\,\varepsilon(1+\varepsilon)}\;F_{UU}^{\cos\phi_h}\cos\phi_h + \varepsilon\,F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)$$

$$+ S_\parallel \lambda_e \left[\sqrt{1-\varepsilon^2}\;F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\;F_{LL}^{\cos\phi_h}\cos\phi_h\right]$$

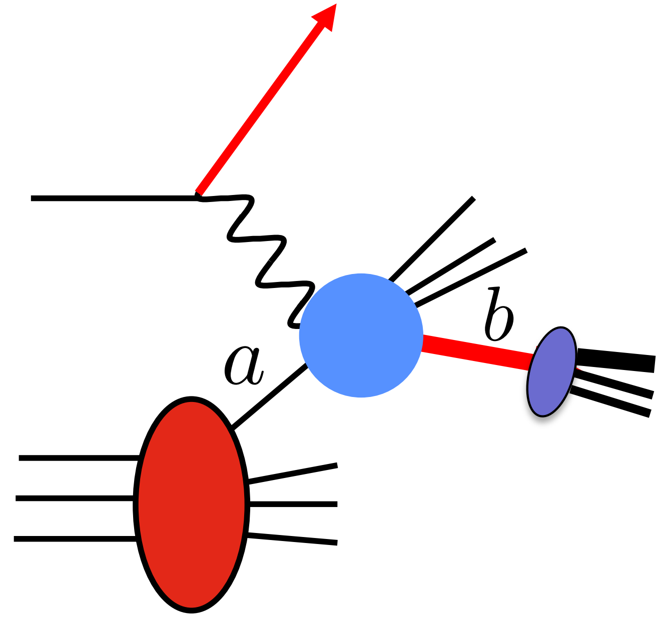
$$+ S_\parallel \left[\sqrt{2\,\varepsilon(1+\varepsilon)}\;F_{UL}^{\sin\phi_h}\sin\phi_h + \varepsilon\,F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)\right]$$

$$\left. + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\;F_{LU}^{\sin\phi_h}\sin\phi_h\right\}$$

$$q_T^2 \equiv \frac{P_{h\perp}^2}{z^2} \qquad \varepsilon \equiv \frac{1-y}{1-y+y^2/2}$$

$$F_{\mathrm{pol}}^{\mathrm{trig}(\phi)}$$

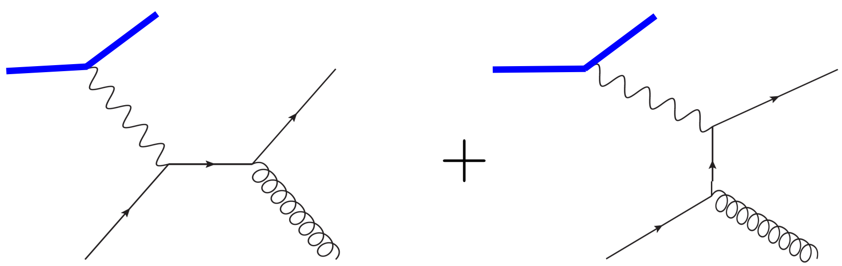
- QCD hard scattering, collinear factorization:



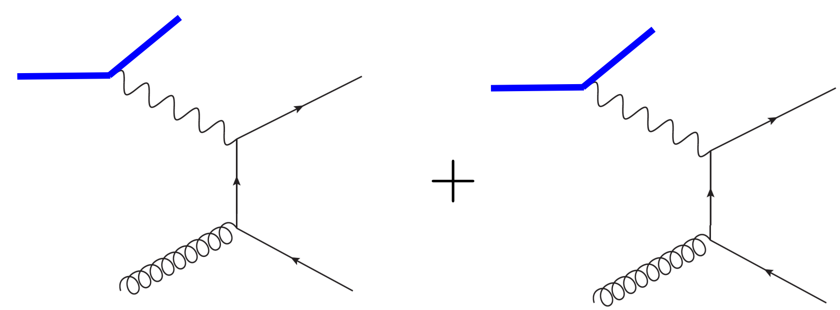
$$\hat{x} = \frac{Q^2}{2p_a \cdot q} \qquad \hat{z} = \frac{p_b \cdot p_a}{p_a \cdot q}$$

$$F_{\rm pol}^{{\rm trig}(\phi_h)}\left(x, z, \frac{q_T^2}{Q^2}\right) = \sum_{a,b} {\rm PDF}_a \otimes C_{\rm pol}^{{\rm trig}(\phi_h), a\to b}\left(\hat{x}, \hat{z}, \frac{q_T^2}{Q^2}, \alpha_s(\mu)\right) \otimes {\rm FF}_b$$

- lowest order, $\mathcal{O}(\alpha_s)$



$\gamma^* q \to qg$



$\gamma^* g \to q\bar{q}$

- all T-even pieces populated by LO pQCD:

Mendez;
Koike, Nagashima, WV

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\pi\alpha^2}{xyQ^2}\frac{y^2}{1-\varepsilon}\Big\{F_{UU,T} + \varepsilon F_{UU,L}$$

$$+\sqrt{2\,\varepsilon(1+\varepsilon)}\,F_{UU}^{\cos\phi_h}\cos\phi_h + \varepsilon F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)$$

$$+ S_\parallel \lambda_e \left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,F_{LL}^{\cos\phi_h}\cos\phi_h\right]$$

$$+ S_\parallel \left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,F_{UL}^{\sin\phi_h}\sin\phi_h + \varepsilon\,F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)\right]$$

$$+ \lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,F_{LU}^{\sin\phi_h}\sin\phi_h\Big\}$$

- how about T-odd?

- compare $|i\rangle \rightarrow |f\rangle$ and $|\tilde{i}\rangle \rightarrow |\tilde{f}\rangle$, where $|\tilde{i}\rangle\,,|\tilde{f}\rangle$ are states with reversed momenta and spins

- T-odd effect: $$0 \neq |\mathcal{M}_{fi}|^2 - |\mathcal{M}_{\tilde{f}\tilde{i}}|^2$$
$$= -2\,\mathrm{Im}\Big(\mathcal{M}^*_{fi}\,\alpha_{fi}\Big) - |\alpha_{fi}|^2$$

absorptive part

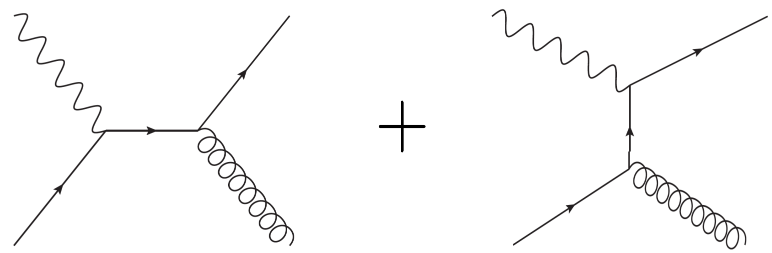
$$i\alpha_{fi} = \mathcal{M}_{fi} - \mathcal{M}^*_{if}$$

- Born contributions are real, but 1-loop amplitudes have imaginary parts

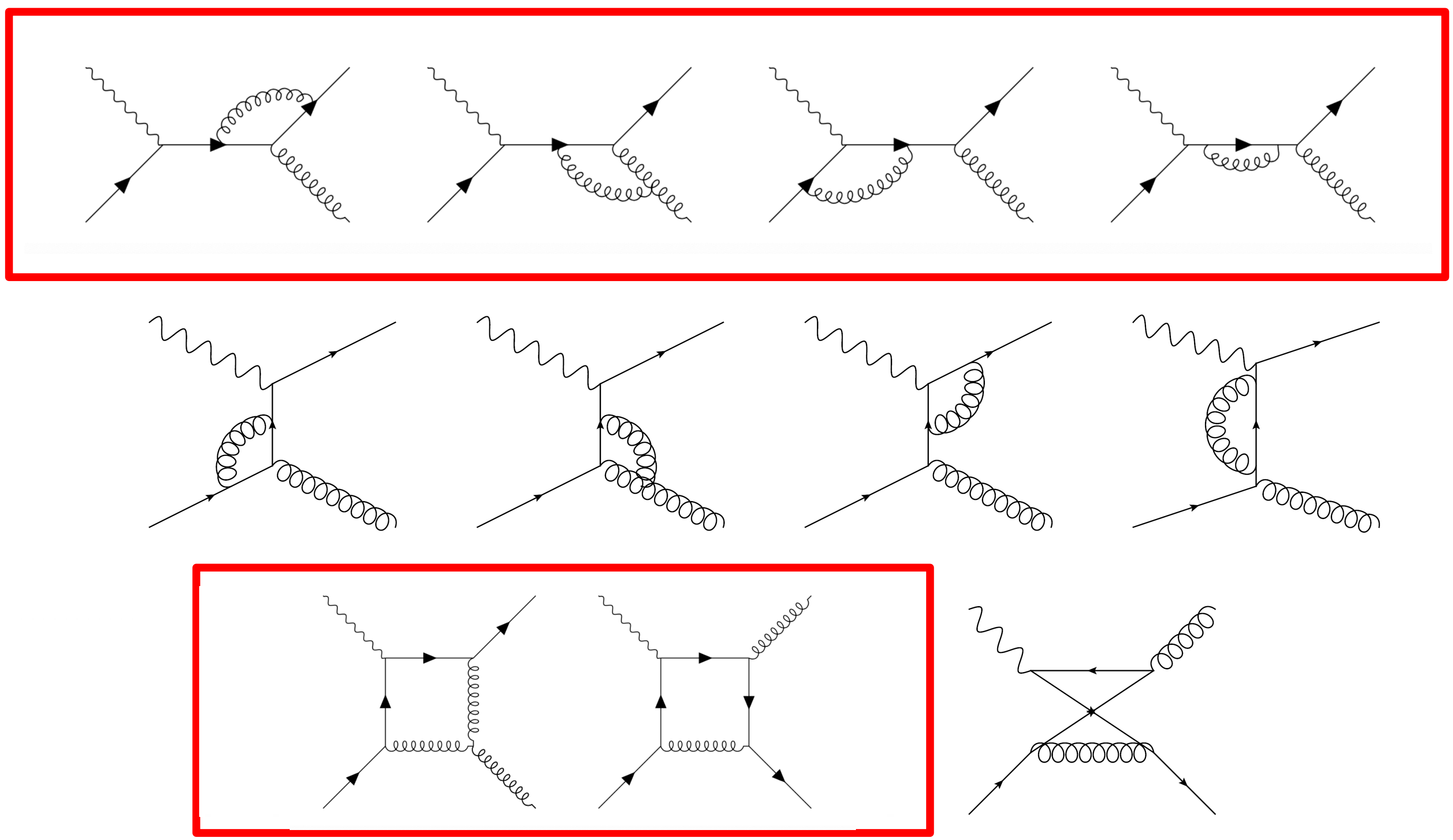
→ consider their interference, $\mathcal{O}(\alpha_s^2)$

De Rujula, Kaplan, De Rafael;
Körner, Kramer, …;
Hagiwara, Hikasa, Kai

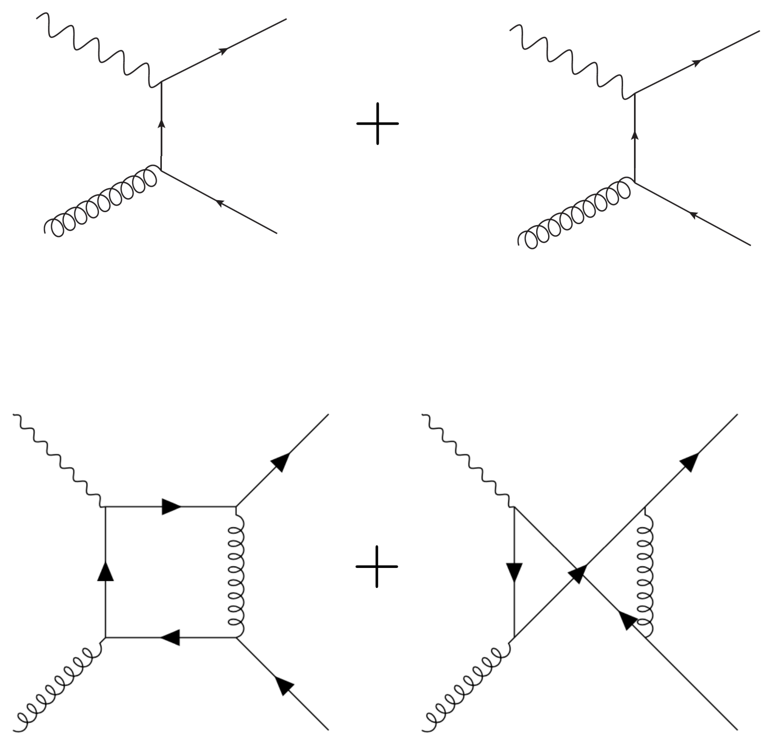
- Born diagrams:



- 1-loop



- plus, gluon channel:



- calculation quite straightforward

- imaginary part must be finite (no place for poles)

- still, individual diagrams have poles

  → dimensional regularization

  → need γ^5 and $\varepsilon^{\mu\nu\rho\sigma}$ ('t Hooft-Veltman-Breitenlohner-Maison)

- structure functions at $\mathcal{O}(\alpha_s^2)$:

$$F_{UL}^{\sin(n\phi_h)}\left(x, z, \frac{q_T^2}{Q^2}\right) = \sum_{a,b}\int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\,\Delta f_a\left(\frac{x}{\hat{x}},\mu^2\right) C_{UL}^{\sin(n\phi_h),a\to b}\left(\hat{x},\hat{z}\right) D_b^h\left(\frac{z}{\hat{z}},\mu^2\right) \times \delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right)$$

where

$$C_{UL}^{\sin\phi_h,q\to q}(\hat{x},\hat{z}) = \left(\frac{\alpha_s}{2\pi}\right)^2 e_q^2\,C_F\left(C_A(1-\hat{x}) + C_F(\hat{x}-1-\hat{z}+3\hat{x}\hat{z}) + \frac{1}{C_A}(1-2\hat{x})\frac{\hat{z}\ln\hat{z}}{1-\hat{z}}\right)\frac{Q}{q_T}$$

$$C_{UL}^{\sin\phi_h,g\to q}(\hat{x},\hat{z}) = \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{e_q^2}{C_A}\,\frac{1-\hat{x}}{2\hat{z}^2}\left(\hat{x}\hat{z}(1-2\hat{z}) - (1-\hat{x})\ln(1-\hat{z}) + (1-\hat{x})\frac{\hat{z}\ln(\hat{z})}{1-\hat{z}}\right)\frac{Q}{q_T}$$

$$C_{UL}^{\sin 2\phi_h,q\to q}(\hat{x},\hat{z}) = \left(\frac{\alpha_s}{2\pi}\right)^2 e_q^2\,C_F(1-\hat{x})\left(\frac{1}{C_A}\,\frac{(1-2\hat{z})\ln\hat{z}}{1-\hat{z}} - (C_A+(1-3\hat{z})C_F)\right)\frac{Q^2}{q_T^2}$$

$$C_{UL}^{\sin 2\phi_h,g\to q}(\hat{x},\hat{z}) = \left(\frac{\alpha_s}{2\pi}\right)^2 e_q^2\,\frac{1}{C_A}\,\frac{(1-\hat{x})^2}{2\hat{z}^3}\left(\hat{z}(2(1-\hat{z})\hat{z}-1) - (1-\hat{z})\ln(1-\hat{z}) - \frac{\hat{z}^2\ln\hat{z}}{1-\hat{z}}\right)\frac{Q^2}{q_T^2}$$

($q \to g$ by crossing)

- so, pQCD at $\mathcal{O}(\alpha_s^2)$ also gives T-odd terms:

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\pi\alpha^2}{xyQ^2}\,\frac{y^2}{1-\varepsilon}\Big\{F_{UU,T} + \varepsilon F_{UU,L}$$

$$+\sqrt{2\,\varepsilon(1+\varepsilon)}\,F_{UU}^{\cos\phi_h}\cos\phi_h + \varepsilon\,F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)$$

$$+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,F_{LL}^{\cos\phi_h}\cos\phi_h\right]$$

$$+S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,F_{UL}^{\sin\phi_h}\sin\phi_h + \varepsilon\,F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)\right]$$

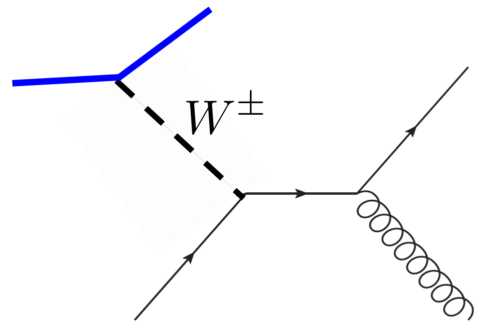
$$+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,F_{LU}^{\sin\phi_h}\sin\phi_h\Big\}$$

Hagiwara, Hikasa, Kai,
Ahmed, Gehrmann

Cross-checks / results in previous literature:

- T-odd (and P-odd) effect in unpol. SIDIS with neutrino beam

Hagiwara, Hikasa, Kai



- crossing of T-odd effects in e^+e^-

Körner, Melic, Merebashvili

- 2021 calculation of SIDIS A_{UT}

Benic, Hatta, Kaushik, Li

involves g_T and its Wandzura-Wilczek part

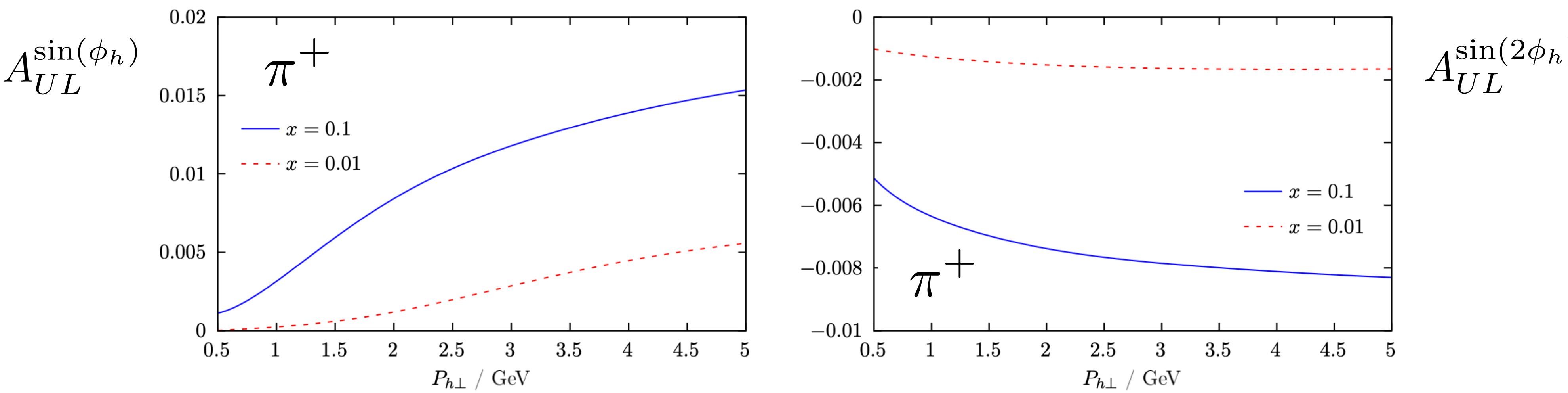
$$g_T^{\mathrm{WW}}(x) = \int_x^1 \frac{dx'}{x'} \Delta q(x')$$

A little bit of phenomenology

$ep \to e\pi^+ X$ at the EIC $\sqrt{s} = 140$ GeV

DSSV pol. PDFs, NNPDF31 unpol., DSS fragmentation

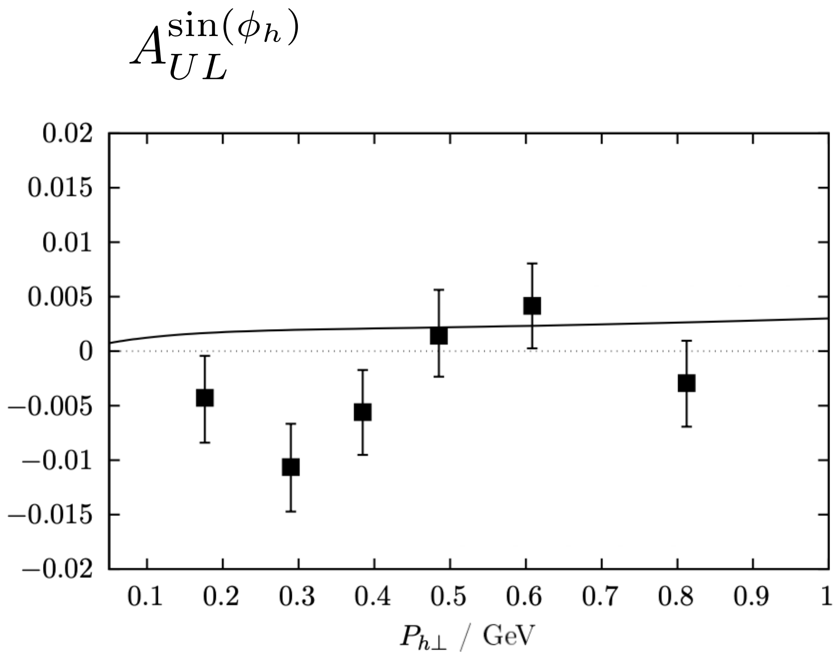
$x = 0.1,\ Q^2 \in [10, 100]\ \mathrm{GeV}^2$ $x = 0.01,\ Q^2 \in [2, 10]\ \mathrm{GeV}^2$

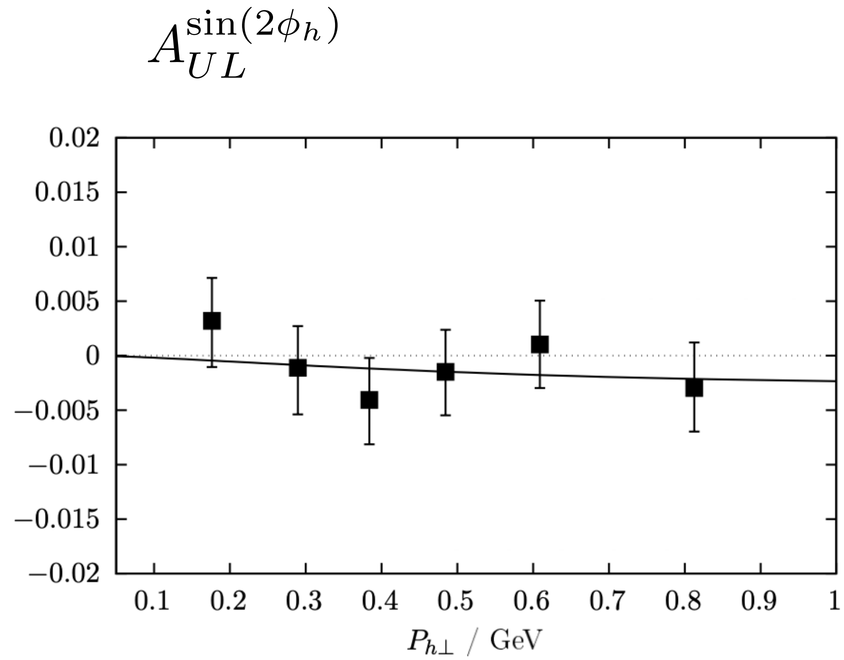


(smaller for π^-)

$\mu d \rightarrow \mu h^+ X$ at COMPASS $\sqrt{s} = 17.4$ GeV

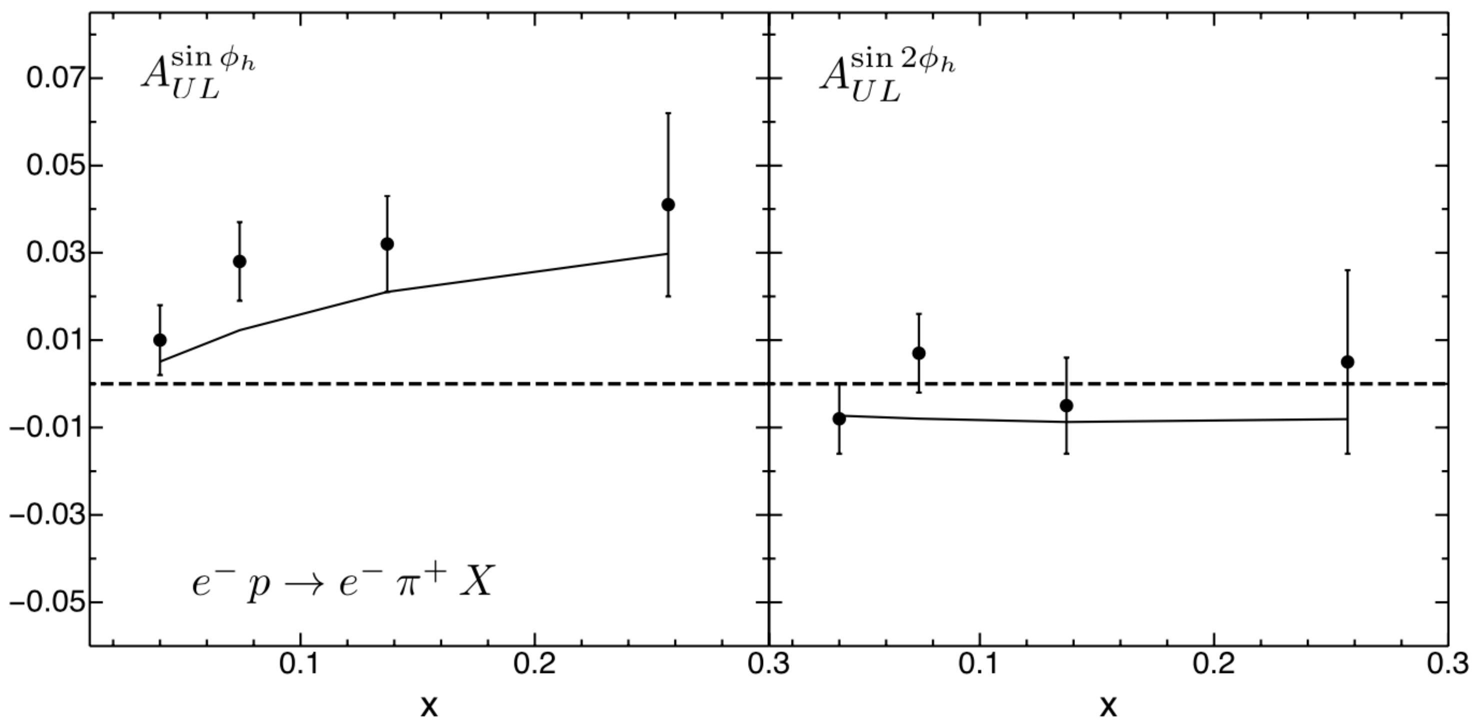
$x \in [0.004, 0.7],\ Q^2 \in [1, 100]$ GeV^2





$ep \to e\pi^+ X$ at HERMES $\sqrt{s} = 7.25$ GeV

$P_{h\perp} \sim 0.5$ GeV



Behavior at low q_T – toward the TMD regime

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\pi\alpha^2}{xyQ^2}\,\frac{y^2}{1-\varepsilon}\Bigg\{F_{UU,T} + \varepsilon F_{UU,L}$$

$$+\sqrt{2\,\varepsilon(1+\varepsilon)}\;F_{UU}^{\cos\phi_h}\cos\phi_h \;+\; \varepsilon\,F_{UU}^{\cos 2\phi_h}\;\cos(2\phi_h)$$

$$+\,S_{\parallel}\lambda_e\left[\sqrt{1-\varepsilon^2}\;F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,F_{LL}^{\cos\phi_h}\;\cos\phi_h\right]$$

$$+\,S_{\parallel}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,F_{UL}^{\sin\phi_h}\;\sin\phi_h \;+\varepsilon\,F_{UL}^{\sin 2\phi_h}\;\sin(2\phi_h)\right]$$

$$+\,\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,F_{LU}^{\sin\phi_h}\;\sin\phi_h\;\Bigg\}$$

- for example,

$$F_{UU,T} \sim \sum_q e_q^2 \int d^2p_T\, d^2k_T\, \delta^{(2)}\left(\vec{p}_T - \vec{k}_T - \vec{q}_T\right) f_q(x, p_T^2)\, D_q^h(z, k_T^2)$$

- on the other hand, can expand pQCD result toward low q_T :

$$\frac{1}{\hat{x}\hat{z}}\,\delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) = \delta(1-\hat{z})\delta(1-\hat{x})\ln\left(\frac{Q^2}{q_T^2}\right) + \frac{\delta(1-\hat{z})}{(1-\hat{x})_+} + \frac{\delta(1-\hat{x})}{(1-\hat{z})_+} + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right),$$

Olness, Meng, Soper

- get

$$F_{UU,T} = \sum_q e_q^2\, \frac{\alpha_s}{2\pi}\, \frac{Q^2}{q_T^2} \left[-C_F \left(2\ln\left(\frac{q_T^2}{Q^2}\right) + 3\right) f_q(x)\, D_q^h(z) \right.$$
$$\left. + f_q(x)\left(P_{qq} \otimes D_q^h\right)(z) + \left(P_{qq} \otimes f_q\right)(x)\, D_q^h(z) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)\right]$$

- in this case, matches TMD evolution

- however, not always known to work

Boer, WV; Ji, Qiu, Yuan, WV;
Bacchetta, Boer, Diehl, Mulders

Bacchetta, Boer, Diehl, Mulders

observable	low-q_T calculation twist	order	power	high-q_T calculation twist	order	power	leading powers match
$F_{UU,T}$	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{UU,L}$	4			2	α_s	$1/Q^2$	
$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s^2	$1/Q^2$	no
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$	
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes

- more precisely, find in q→q channel:

$$F_{UL}^{\sin(\phi_h)} = \sum_q e_q^2 \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{Q}{q_T} \frac{C_A}{2} \left\{ -C_F \left(2\ln\left(\frac{q_T^2}{Q^2}\right) + 3 \right) \Delta f_q(x)\, D_q^h(z) \right.$$

$$+ \Delta f_q(x) \left(\delta P_{qq} \otimes D_q^h\right)(z) + \left(\delta P_{qq} \otimes \Delta f_q\right)(x)\, D_q^h(z)$$

$$\left. - \frac{2C_F}{C_A^2} \Delta f_q(x) \left[\frac{z^2}{1-z} \left(1 + \frac{\ln z}{1-z}\right) \otimes D_q^h \right] \right\} + \mathcal{O}\left(\frac{q_T}{Q}\right)$$

$$F_{UL}^{\sin(2\phi_h)} = -\sum_q e_q^2 \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{3C_A}{4} \left\{ -C_F \left(2\ln\left(\frac{q_T^2}{Q^2}\right) + 3 \right) \Delta f_q(x)\, D_q^h(z) \right.$$

$$+ \Delta f_q(x) \left(\delta P_{qq} \otimes D_q^h\right)(z) + \left(\delta P_{qq} \otimes \Delta f_q\right)(x)\, D_q^h(z)$$

$$\left. - \frac{2C_F}{3C_A^2} \Delta f_q(x) \left[\frac{z^2}{(1-z)^2} \left(1 - 3z - 2(2z-1)\frac{\ln z}{1-z}\right) \otimes D_q^h \right] \right\} + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

- even more, in gluon channel:

$$F_{UL}^{\sin(\phi_h)} = \sum_q e_q^2 \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{Q}{q_T} \frac{1}{2C_A} D_q^h(z)$$

$$\times \left[(1-x) \ln\left(\frac{Q^2}{q_T^2}\right) + (1-x) \ln\left(\frac{1-x}{x}\right) - 1\right] \otimes \Delta f_g$$

- in TMD formalism,

Bacchetta, Boer, Diehl, Mulders

$$F_{UL}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(x h_L H_1^\perp + \frac{M_h}{M}\,g_{1L}\frac{\tilde{G}^\perp}{z}\Big) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(x f_L^\perp D_1 - \frac{M_h}{M}\,h_{1L}^\perp\frac{\tilde{H}}{z}\Big)\Big]$$

Collins fct.

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\Big[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}\,h_{1L}^\perp H_1^\perp\Big]$$

worm gear fct.

- detailed study of perturbative tails of TMDs

Scimemi, Tarasov, Vladimirov; Rodini, Vladimirov

Concluding remarks:

- pQCD predicts T-odd effects in SIDIS at $\mathcal{O}(\alpha_s^2)$
- interesting prospects for phenomenology
- interesting features at low q_T $\leftrightarrow$ something to be learned about TMDs and their evolution
- relevant for matching in TMD studies
- application to other processes?
- higher-order corrections?